



## Prediction of system marginal price of electricity using wavelet transform analysis

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### Abstract

This paper proposes a novel wavelet transform based technique for prediction of the system marginal price (SMP) of electricity. Daubechies D1(Haar), D2 and D4 wavelet transforms are adopted, and the numerical results reveal that certain wavelet components can effectively be used to identify the SMP characteristics with relation to the system demand in electric power systems. The wavelet coefficients associated with certain frequency and time localization are adjusted using the conventional multiple regression method and then reconstructed in order to predict the SMP for the next scheduling day through a five scale synthesis technique. The outcome of the study clearly indicates that the proposed wavelet transform approach can be used as an attractive and effective means for SMP forecasting. © 2002 Elsevier Science Ltd. All rights reserved.

**Keywords:** System marginal price; Wavelet transform; Power pool

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### 1. Introduction

The electric power industry is recently in a state of transition worldwide. The notion of the electricity utility being a natural monopoly in its vertically integrated form, which has served nations and economies well for several decades, is being challenged since the independent power

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producer and the broker were introduced in the competitive open access electricity market [1]. Currently, there are two major different architectures for electrical power systems. The first one is a vertically integrated architecture with generation, transmission and distribution being in the hands of a single power utility. The second approach is that of the unbundling of generation, transmission and distribution with competition between different producers/generators feeding into a transmission grid, which is normally handled by a grid authority and then supplies power to bulk purchasers, distribution authorities or large consumers. Thus, the major process in the newly developed electricity market is the power pool. All electricity is sold to and bought through the power pool. The operation of such systems calls for consideration of a number of new issues. The rationale for the movement in a number of countries from vertically integrated utilities towards the unbundled architecture has been to produce competition between different electricity generators feeding into the grid, which it is hoped will encourage initiatives for efficiency improvements. All generators participating in the day ahead bidding of the power pool face a challenge of how to set the prices of their generating units. The system marginal price (SMP) is a major portion of the payment that generators obtain from the power pool for their scheduled generating units. Essentially, it is possible for a high degree of competition to occur in the provision of generator sources. As a result, there is growing interest in the prediction of SMP in the power pool. However, there are few papers published on the topic due to the complexity and newness of the problem. Starting from the POWERCOACH [2], recently, a neural network based technique for the prediction of SMP is proposed for the UK power pool, but it is just beginning in the era [3].

This paper presents a new approach to SMP prediction. In essence, the SMP and demand is strongly coupled. Thus, analysis of the load characteristics is required at first in order to proceed to SMP prediction. The component of the load curve produced by industry has a rather regular profile and exhibits low frequency changes. On the other hand, the consumption of individual consumers may be highly irregular, leading to high frequency components. Judging from these sorts of electric load properties, the wavelet transform, which has received considerable interests in fields such as astronomy, nuclear engineering, sub-band-coding, and earthquake prediction [4–7], is selected as one of the alternatives to establish a novel model of SMP prediction. From the methodological point of view, wavelet transform techniques provide a multi-scale analysis of the signal as a sum of orthogonal signals corresponding to different time scales, allowing a kind of time scale analysis. Recently, several papers have demonstrated the potential applications of wavelet transforms, even in the analysis of electric power systems [8–13]. The approach proposed in this paper decomposes a given signal (SMP curve) into several scales at different levels of resolution. Then, the wavelet coefficients associated with high frequencies are adjusted or predicted using the conventional multiple regression method in order to consider the load conditions through a five scale synthesis technique. The day ahead SMP is then predicted by reconstructing the high frequencies and the low frequency of the wavelet transformed signal. Test results show the satisfactory use of the wavelet transform, and the mean percentage forecasting errors of week days are around 6.03% in the Winter time (January) of 1997. The mean percentage errors of weekend days are 11.11% (Saturday of January), 2.49% (Sunday of March) and 9.72% (Monday of February) of 1997, respectively.

## 2. System marginal price

### 2.1. Day ahead bidding and system marginal price

The definitions and notations on the day ahead bidding settlement process in the pool are as follows:

*Scheduling day*: the period from 05:30 on one day to 05:00 on the next.

*Settlement period*: half-hour.

*Settlement day*: the period from 00:00 on one day to 00:00 on the next, i.e. calendar day.

*Genset*: generating unit.

*System marginal price*: the highest price of qualified units scheduled in the unconstrained schedule in each settlement period.

*Availability declaration period*: the period from 21:00 on bidding day to 12:00 on the day after next.

*Table B period*: when the aggregate offered availability of scheduled units exceeds the demand to be met at that time by more than a pre-determined level.

*Table A period*: all other settlement periods of a scheduling day.

Day ahead bidding starts by 10:10 am on each day. The grid operator produces forecasted demand in MW for each settlement period in the availability declaration period. Each generator provides to the grid operator the offer data in respect of each centrally dispatched genset for the following scheduling day; this offer data covers:

- offered availability;
- price: start-up cost, no-load cost, incremental cost;
- unit dynamic running characteristics: run-up rates, run-down rates, minimum and synchronizing generation;
- declared inflexibility.

The generator ordering and loading (GOAL) program schedules units from a merit order derived from the generator's offered prices and considers the factors: offered availability, declared inflexibility and dynamic running characteristics. In order to work out the SMP, the unit price is calculated for each genset scheduled in the GOAL. The calculation is different depending on whether the settlement period is determined to be a Table A or B period. It gives

$$P_{UAj}(t) = C_{Sj}(t) + C_{Nj}(t) + C_{Ij}(t) \quad (1)$$

$$P_{UBj}(t) = C_{Ij}(t) \quad (2)$$

where  $t$  is the settlement period, there are forty eight settlement periods within a scheduling day,  $P_{UAj}(t)$  the unit price of scheduled genset  $j$  at  $t$  in Table A period,  $P_{UBj}(t)$  the unit price of

scheduled genset  $j$  at  $t$  in Table B period,  $C_{Sj}(t)$  the start-up cost of scheduled genset  $j$  at  $t$ ,  $C_{Nj}(t)$  the no-load cost of scheduled genset  $j$  at  $t$ ,  $C_{Ij}(t)$  the incremental cost of scheduled genset  $j$  at  $t$ .

The unit-price is an average price allocated to the whole scheduled period in proportion to the output in that period. For each settlement period in a scheduled day, the unit-prices of all flexible units are placed in a stack in increasing price order. The price of the most expensive scheduled unit (marginal unit) in each settlement period is taken as the SMP. Thus,

$$\text{SMP}(t) = \max \{P_{U1}(t), P_{U2}(t), \dots, P_{Un}(t)\} \quad (3)$$

$$P_{Uj}(t) = P_{UAj}(t) \quad t \text{ within Table A period} \quad (4)$$

$$P_{UBj}(t) \quad \text{otherwise} \quad (5)$$

where  $n$  is the total number of scheduled gensets at settlement period  $t$ ,  $\text{SMP}(t)$  the system marginal price at settlement period  $t$ ,  $P_{Uj}(t)$  the unit-price at  $t$ .

## 2.2. The characteristics of the SMP

To understand the nature of power pool transactions, it is important first to understand day ahead bidding. The definitions and notations on the day ahead bidding settlement process in the power pool and SMP are described in the previous section in detail.

Fig. 1 shows a one week SMP curve. There are forty eight SMP values within one scheduling day. The shape of the SMP curve follows daily and weekly cycles, like the load behavior for week days (Monday through Friday), with big random variations due to changes of total bidding generating units availability, industrial activities, community life style and so on. The trends of electricity demand influences on the trends of SMP curves are represented in Fig. 2, where the demand is converted from MW to GW for convenience of depiction.

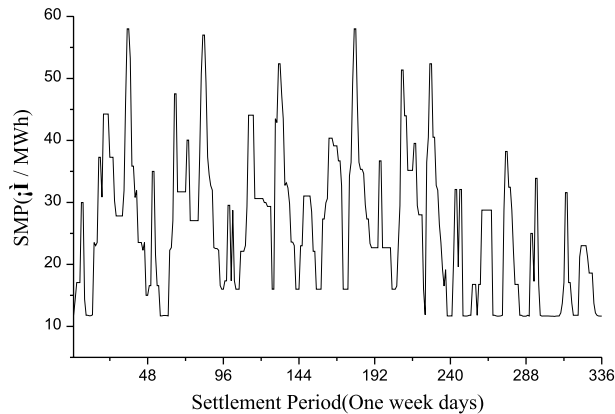


Fig. 1. One week SMP curve.

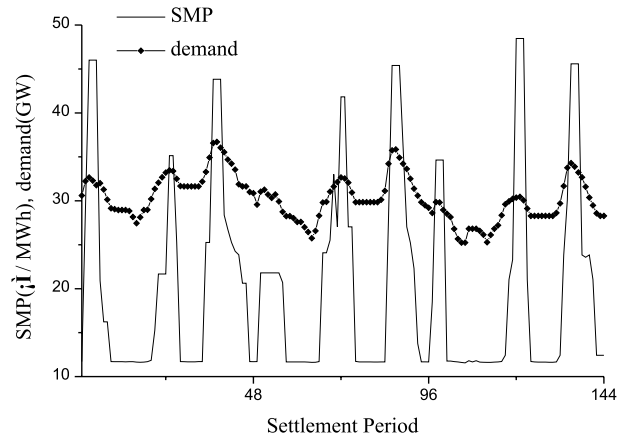


Fig. 2. Trends of SMP and demand.

### 3. Wavelet transform analysis

Wavelet theory provides a unified framework for a number of techniques, which have been developed independently for various signal processing applications. In particular, the wavelet transform is of interest for the analysis of non-stationary signals, because it provides an alternative to the classical short-time Fourier transform (STFT) or Gabor transform. The basic differences in contrast to the STFT, which uses a single analysis window, is that the wavelet transform uses short windows at high frequencies and long windows at low frequencies and is also related to time frequency analysis. Thus, the windowing of wavelet transforms is adjusted automatically for low or high frequencies and each frequency component gets treated in the same manner without any reinterpretation of the results. This difference is that the wavelet transform provides an alternative way of breaking a signal down into its constituent parts. The basic functions in the wavelet transform employ time compression or dilation rather than a variation in frequency of the modulated signal. Fig. 3(a) shows an example of the time frequency plane tiling for the STFT. The shaded squares in the figure correspond to waveforms, which are localized in

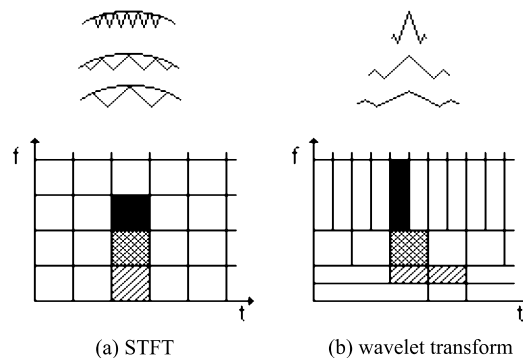


Fig. 3. Basis function and corresponding tiling of the frequency plane.

the same time interval and in three adjacent frequency levels. On the other hand, wavelets offer, as is shown in Fig. 3(b), a different compromise, the frequency localization is logarithmic, that is, proportional to the frequency level. As a consequence, time localization gets finer in the highest frequencies.

The wavelet analysis procedure is to adopt a wavelet prototype function (mother wavelet). Temporal analysis is performed with a contracted, high frequency version of prototype wavelet, while frequency analysis is performed with a dilated, low frequency version of the prototype wavelet. Because the original signal or function can be represented in terms of a wavelet expansion using coefficients in a linear combination of the wavelet functions, data operations can be performed using just the corresponding wavelet coefficients. There are several types of wavelet transforms. Depending on the application, one may be preferred to the others. For a continuous input signal, the time and scale parameters can be continuous, leading to the continuous wavelet transform. On the other hand, the discrete wavelet transform can also be defined for discrete time signals.

In the case of wavelet transforms, the original domain is the time domain. The transformation process from time domain to time scale domain is a wavelet transform, technically known as signal decomposition because a given signal is decomposed into several other signals with different levels of resolution. From these decomposed signals, it is possible to recover the original time domain signal without losing any information. This reverse process is called the inverse wavelet transform or signal reconstruction. To illustrate the transform, let  $s(t)$  be the time domain signal to be decomposed or analyzed. The dyadic wavelet transform of  $s(t)$  is defined as follows:

$$\text{DWT}_\varphi s(m, n) = 2^{-m/2} \int_{-\infty}^{\infty} s(t) \varphi^* \left( \frac{t - n2^m}{2^m} \right) dt \quad (6)$$

where the asterisk denotes a complex conjugate,  $m$  and  $n$  are scale and time shift parameters, respectively, and  $\varphi(t)$  is a given basis function (mother wavelet).

The dyadic wavelet transform is implemented using a multi-resolution pyramidal decomposition technique. In principle, a recorded digitized time signal  $S(n)$  is decomposed into its detailed  $cD_1(n)$  and smoothed  $cA_1(n)$  signals using filters HiF\_D and LoF\_D, respectively, as in Fig. 4. Filter HiF\_D has a bandpass filter response. Thus, the filtered signal  $cD_1(n)$  is a detailed version of  $S(n)$  and contains higher frequency components than the smoothed signal  $cA_1(n)$ , because filter LoF\_D has a lowpass frequency filter response. The decomposition of  $S(n)$  into  $cA_1(n)$  and  $cD_1(n)$

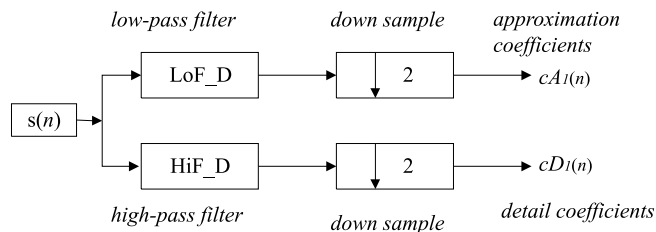


Fig. 4. The first scale signal decomposition of  $S(n)$  in  $cA_1(n)$  and  $cD_1(n)$ .

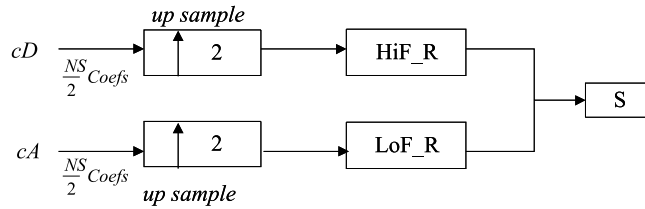


Fig. 5. Reconstruction of the approximations and the details.

is a first-scale decomposition. It is possible to reconstruct the original signal from the coefficients of the approximations and details, as in Fig. 5, for the number of samples (NS) case for example.

It is also possible to reconstruct the approximations and details themselves from their coefficient vectors. The coefficient vector  $cA_1$  passes through the same process that is used to reconstruct the original signal. However, instead of combining it with the level one detail  $cD_1$ , a vector of zeros is fed in place of the details, as in Fig. 6.

The process yields a reconstructed approximation  $A_1$ , which has the same length as the original signal  $S(n)$  and which is a real approximation of it. Similarly, the first level detail  $D_1$  can be reconstructed using the analogous process, as in Fig. 7.

The reconstructed details and approximations are then true constituents of the original signal as

$$A_1 + D_1 = S \quad (7)$$

Note that the coefficient vectors  $cA_1$  and  $cD_1$ , because they were produced by down sampling, contain aliasing distortion and are only half the length of the original signal, cannot directly be combined to reproduce the signal. It is necessary to reconstruct the approximations and details before combining them. Extending this technique to the components of a multi-level analysis, it is

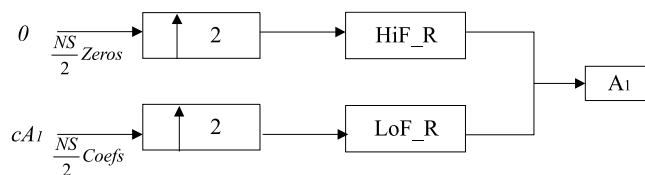


Fig. 6. Reconstruction of the approximations.

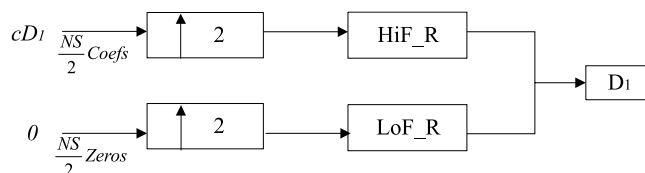


Fig. 7. Reconstruction of the details.

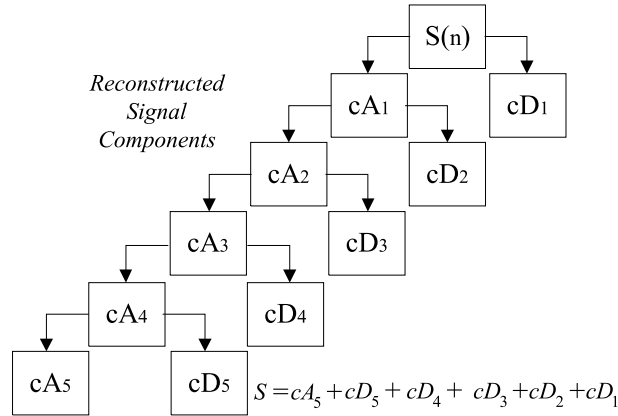


Fig. 8. Reconstruction of the five scaled approximations and details.

possible to find that similar relationships hold for all the reconstructed signal constituents. That is, there are several ways to reassemble the original signal.

In this paper, the five scaled reconstruction technique, as in Fig. 8, is applied to predict the SMP, and the second order regression polynomial is used to describe the relationship between demand and SMP.

$$S_F = a_0 + a_1 L^1 + a_2 L^2 \quad (8)$$

where  $S_F$  represents the forecasted load,  $a_0$ ,  $a_1$  and  $a_2$  are the coefficients and  $L^1$ ,  $L^2$  are the forecasted demand. For convenience, the coefficients  $a_0$ ,  $a_1$  and  $a_2$  are included in  $A$ . Matrix algebra, for the SMP forecasting, can be used as follows [14]:

$$S_F = LA + e \quad (9)$$

where  $S_F$  is an  $n \times 1$ ,  $L$  is an  $n \times k$ ,  $A$  is a  $k \times 1$  and  $e$  is a  $k \times 1$  matrix, respectively. In order to obtain the values of  $A$ , the sum of the squared deviations must be minimized

$$\sum e_i^2 = e'e = (S_F - LA)'(S_F - LA) \quad (10)$$

where  $e'$ ,  $(S_F - LA)'$  is the transpose of  $e$ . Thus, the coefficients are finally calculated as

$$A = (L'L)^{-1}L'S_F \quad (11)$$

where  $(L'L)^{-1}$  is the inverse of  $(L'L)$ .

The process of SMP forecasting using the wavelet transform is shown in Fig. 9. Input data consist of settlement period (forty eight intervals) for the Winter (January) of 1997, and the forecasted demands, considered for the same period, are also given. The wavelet transform is performed to seek the approximation and details. The regression coefficients are calculated using  $d_1$ ,  $d_2$ ,  $d_3$ ,  $d_4$ ,  $d_5$  and forecasted demand, and the demand applied wavelet coefficients is predicted. The SMP forecasting is then finally implemented using the previous day's low frequency and the forecasted high frequency components.



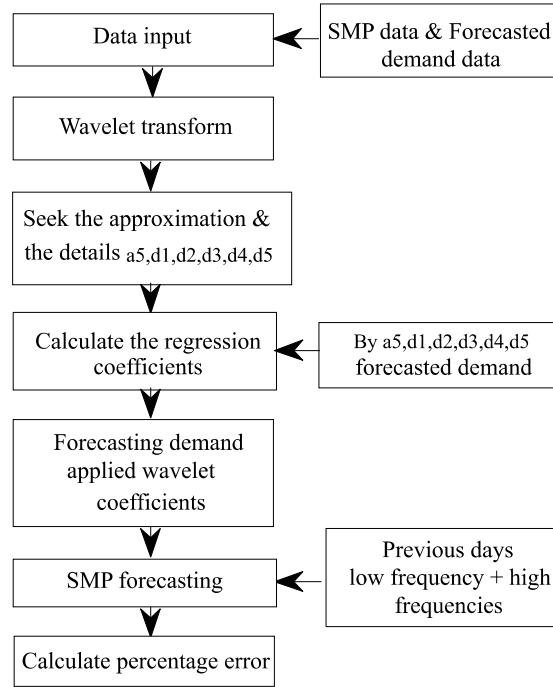


Fig. 9. Flow of the proposed SMP predicting method.

The accuracy of the forecasted results is estimated by the following percentage error calculation:

$$\text{Percentage error} = \frac{|\text{actual SMP} - \text{forecasted SMP}|}{\text{actual SMP}} \times 100(\%) \quad (12)$$

#### 4. Numerical results

Practical data from the UK power pool have been used to test the proposed technique. Fig. 10 illustrates the five level decomposition coefficients, one low and five high frequency components, of a daily SMP curve analyzed using a D1(Haar) wavelet transform. Fig. 11 depicts the synthesis (reconstruction) of the predicted five level coefficients by which the forecasted SMP is obtained eventually.

An interesting observation is shown from the results that the mean percentage error of week days (Tuesday through Friday) is the lowest at db2(lev5). On the other hand, the mean percentage error of weekend days (Saturday through Monday) is the lowest at db1(lev5). That is why the db2 on the week days and db1(Haar) on the weekend days are used as dominant wavelet transform coefficients in these cases. Table 1 shows the mean percentage errors of week days and weekend days at lev5 (Winter and Spring, 1997), where db1, db2 and db4 denote Daubechies D1(Haar), D2 and D4 wavelet transforms, respectively. It is clear that the mean percentage errors of db2(lev5) for weekdays and db1(lev5) for weekend days are the lowest.

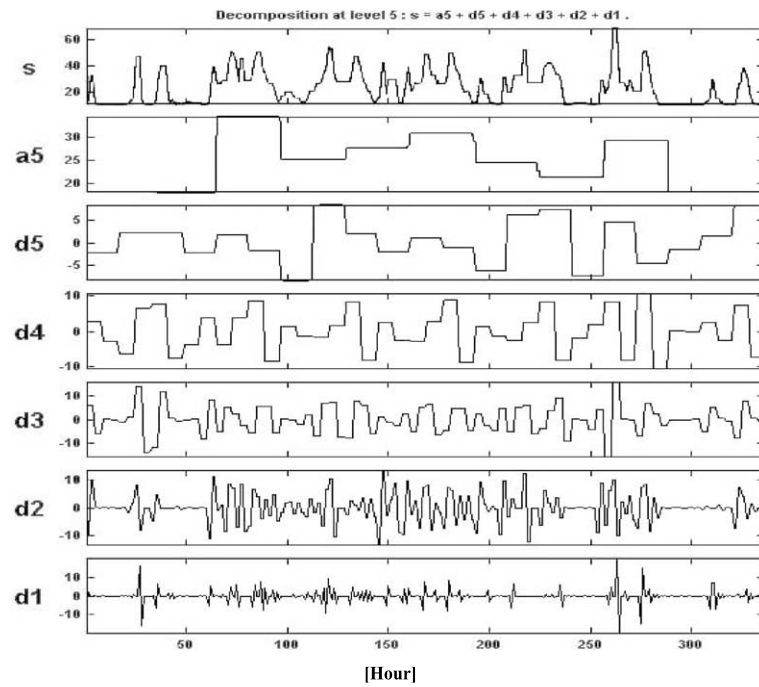


Fig. 10. Five level decomposition of a daily SMP curve.

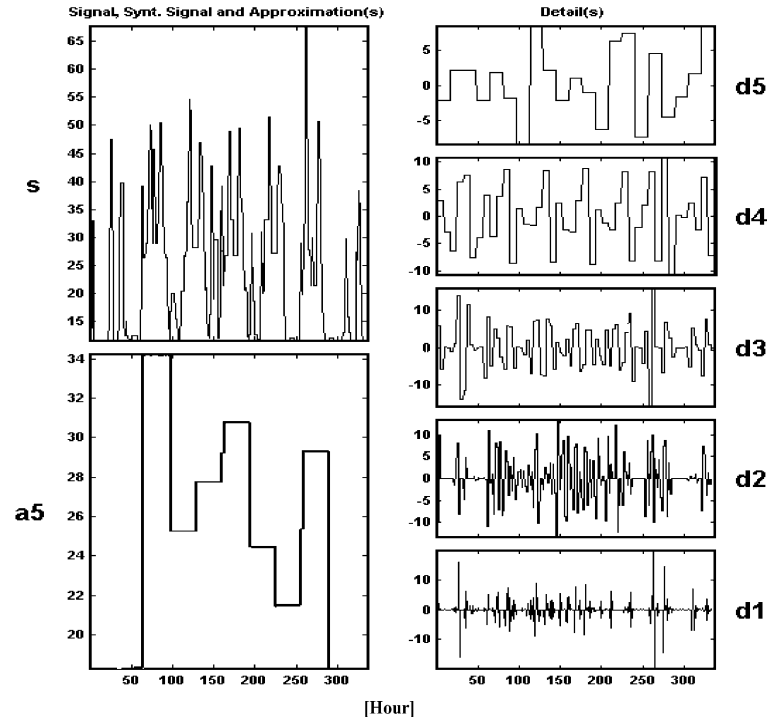


Fig. 11. Synthesis of the predicted five level coefficients.

Table 1

The mean percentage errors of week days and weekend days at level five

	Day					
	Week days			Weekend days		
	db1	db2	db4	Saturday	Sunday	Monday
Mean percentage error (%)	6.24	6.03	6.31	11.11	2.49	9.72

Fig. 12 depicts comparison results of the actual and the forecasted SMP for one day, which is predicted by the db2(lev5) wavelet transform in the Winter (17 January 1997). The mean absolute percentage error is 6.21(%).

Fig. 13 gives the forecasted results for one week days (21 through 24) in the Winter (January) of 1997.

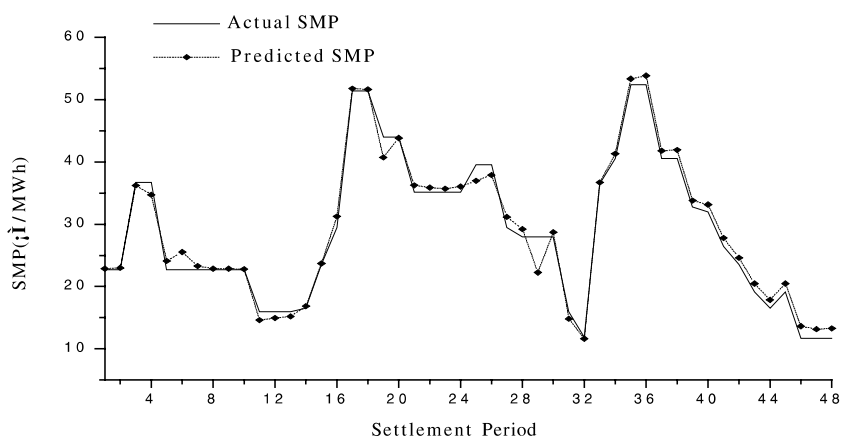


Fig. 12. Comparison of the actual and the forecasted SMP for one day in the Winter (17 January 1997).

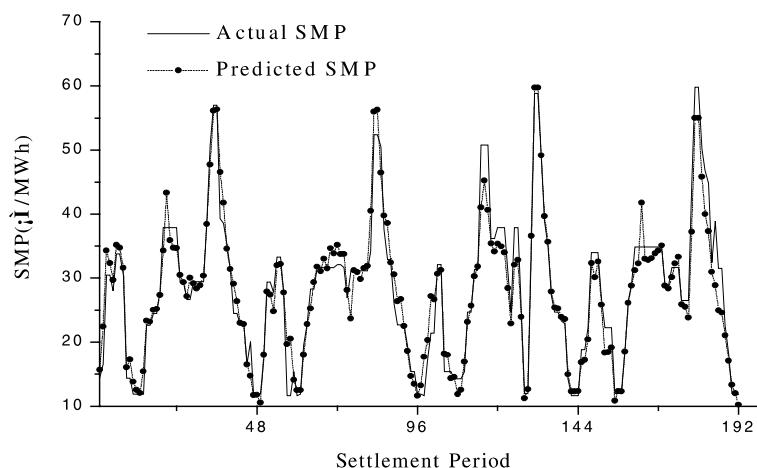


Fig. 13. Comparison of the actual and the forecasted SMP for week days in the Winter (January 1997).

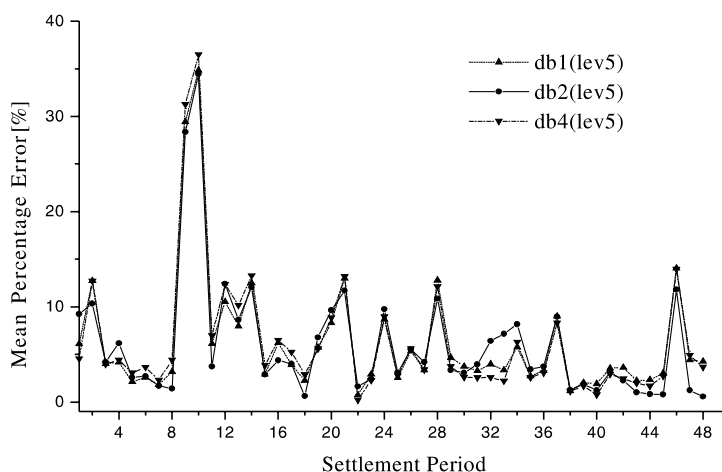


Fig. 14. The mean percentage error of week days in the Winter (January, 1997).

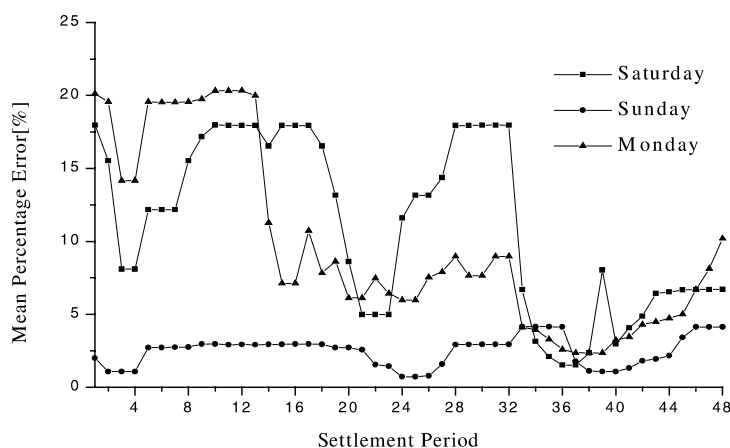


Fig. 15. The mean percentage error of weekend days in 1997 (January, February and March).

Fig. 14 gives the mean percentage error of week days in the Winter (January, 1997). The week days mean percentage errors are obtained as 6.24% at db1(lev5), 6.03% at db2(lev5) and 6.31% at db4(lev5), respectively. For the settlement period of 8–12, the percentage error represented much higher than others because of a wide variation of power demand.

Fig. 15 shows the mean percentage error of weekend days obtained by db1(lev5) on January, February and March in 1997, respectively. The weekend days mean percentage errors of 11.11% (Saturday), 2.49% (Sunday) and 9.72% (Monday) are obtained, respectively. It is clear that the Saturday's and Monday's errors are larger than that of Sunday. It is also observed that the difference of peak and valley value of the SMP on Sunday is less than Saturday's and Monday's, relatively, which is possibly influenced by the smooth load variation of Sunday.

## 5. Conclusions

A novel prediction technique for the SMP using the wavelet transform is proposed in this paper. The influence of demand on SMP forecasting is effectively considered via the conventional multiple regression method, and the complete daily SMP prediction model has been developed using wavelet transform analysis, signal decomposition and synthesis techniques. The outcome of the study clearly indicates that the wavelet transform approach can be used as an attractive and effective means of SMP prediction. The percentage prediction error of week days of approximately 6.03% is experienced for the winter time of 1997. In addition, the weekend days mean percentage error of 11.11% (Saturday), 2.49% (Sunday) and 9.72% (Monday) are obtained, respectively.

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