北京航空航天大學
BEIHANG UNIVERSITY

Image processing in frequency domain

Experiment Report

School	School of Automatic Science
Major	Pattern Recognition and Intelligent System
Student	Tianxiang Lan
Student ID	15231087
Teacher	Hong Zheng

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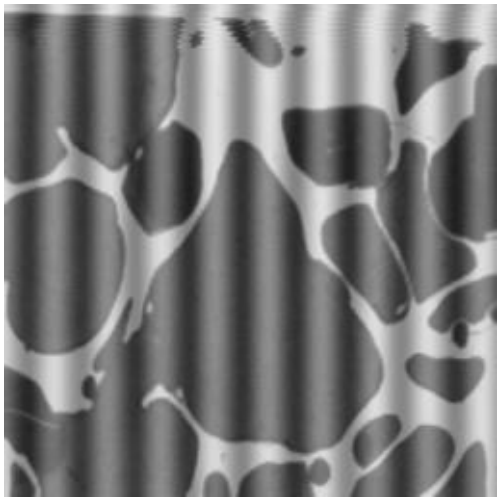
1 Introduction

Sometimes it's difficult to process image in spatial domain,such as remove the sine wave of the following picture.So we need to transform from spatial domain to frequency domain to do some process.



2 Objective

The goals of the experiment are as follows:

(1) To understand the advantages of processing image in frequency domain.

(2) To understand the 2D Fourier transform towards image.

(3) To understand the high-pass filtering and low-pass filtering algorithm.

(4) To understand the homomorphism filtering algorithm.

3 Theory

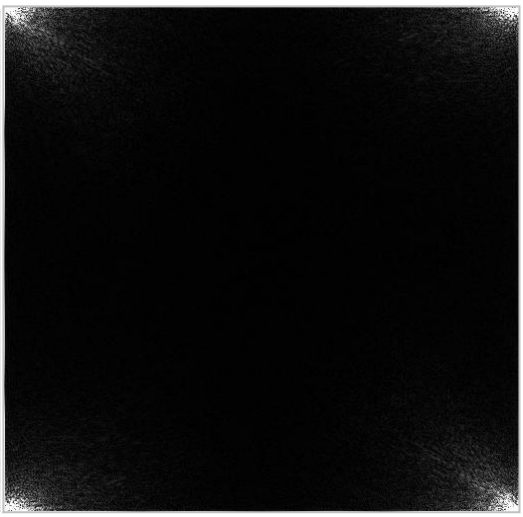
(1) Fourier

Any signal can be represented as a series of sine signal superposition. In the field of image, the gray level change of the image is taken as sine variable.The frequency of image is the index of the intensity of the grayscale changes in the image, and it is the gradient of the gray level in the plane space.The frequency at the noise point and the image edge is high frequency.Fourier transformation and its inverse transformation formula are as follows.

$$F(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)\exp[-j2\pi(ux+vy)]dxdy$$

$$f(x,y)=\int_{-\infty}^{\infty}\int F(u,v)\exp[j2\pi(ux+vy)]dudv$$

For an image, the relatively slow gray area of the image corresponds to a lower frequency spectrum, while the edge region with larger gray scale changes corresponds to a higher frequency spectrum. In addition, most of the images are slow - changing regions with only a small portion of the edge. Therefore, the image of the transform domain is mainly concentrated in the low frequency part (higher corresponding value), and only a small portion of the energy is concentrated at the high frequency part (the corresponding value is lower).The picture on the left is the Lena,and the one on the right is its frequency spectrum.

Then we need to move the origin to the center of the frequency spectrum.

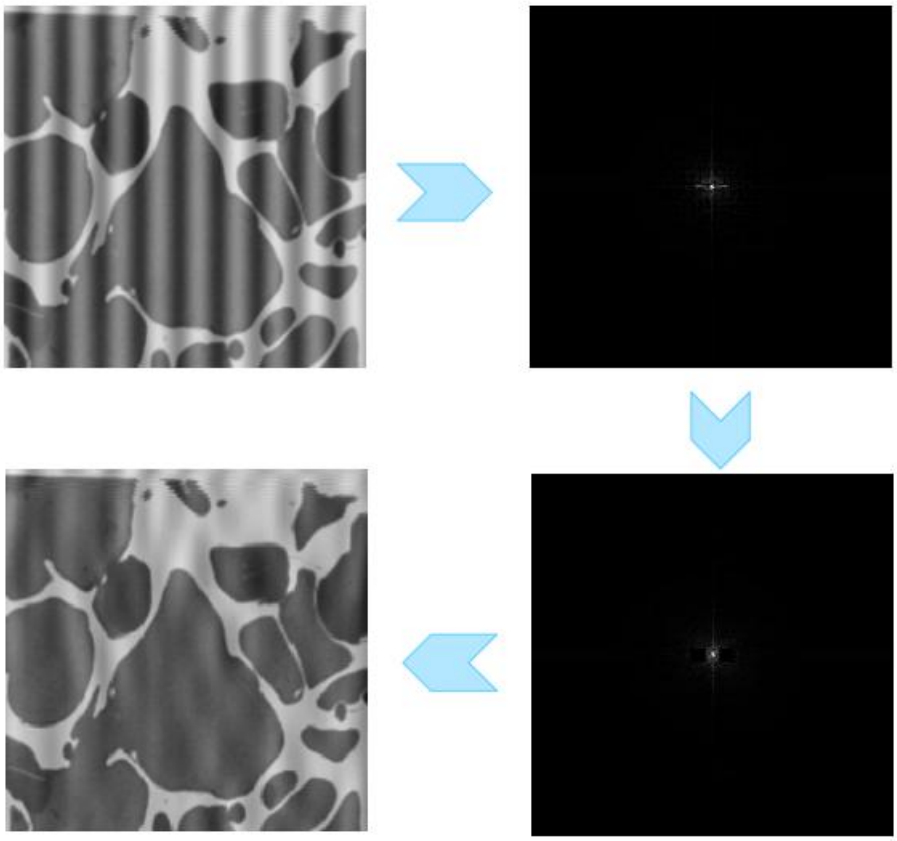


（a）lina 图　　（b）无平移的傅立叶谱　　（c） 原点移到中心的傅立叶谱

(2) High-pass and low-pass filter

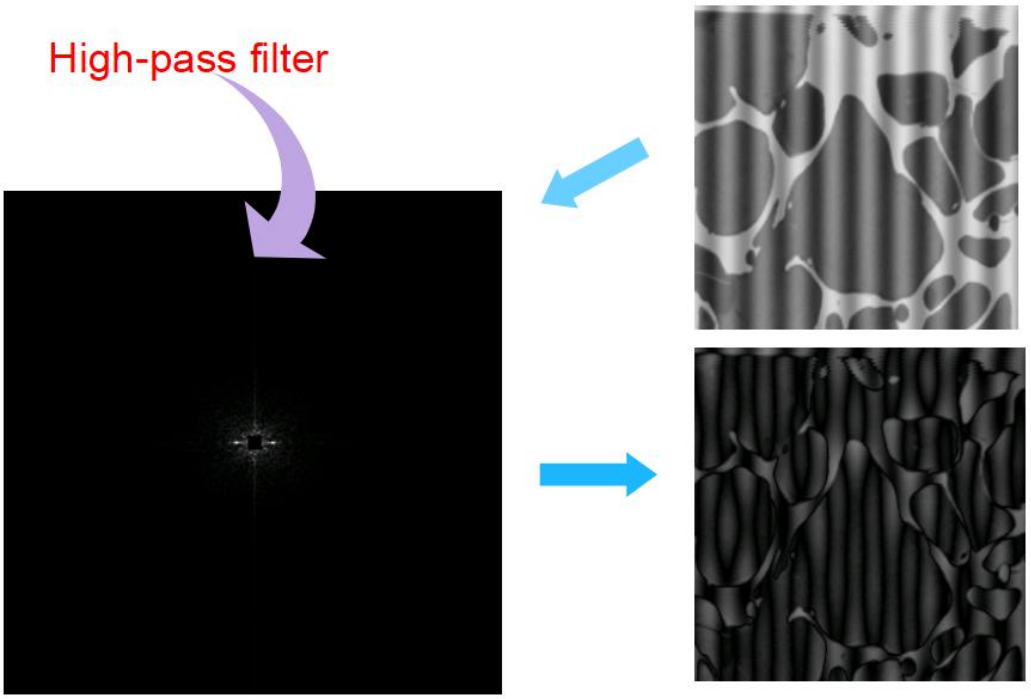
(a) Low-pass filter

We need to remove the sine wave of the picture.So we need to remove the correlative frequency,which is the high frequency.



(b) High-pass filter

We remove the low frequency component this time,getting back the picture as follow.



(3) Homomorphism filter

Eliminate the uneven illumination in the image, increase the details of the image in the dark area without losing the details of the bright area. It compresses the brightness range of the image and enhances the contrast of the image in the frequency domain.

Image f(x,y) could be denoted as $f(x,y) = I(x,y) \cdot R(x,y)$,in which I(x,y)(low frequency) is the irradiation component and R(x,y)(high frequency) is the reflection component.

The procedure of the algorithm is as follows.

Step：

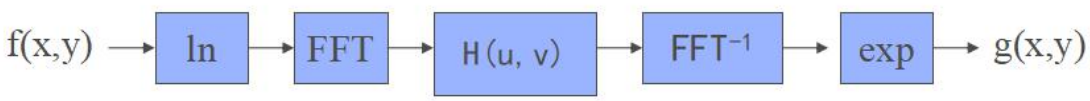
（1）z(x,y) = ln [f(x,y)] = ln[I(x,y)] + ln[R(x,y)]

（2）Z(u,v) = I(u,v) + R(u,v)

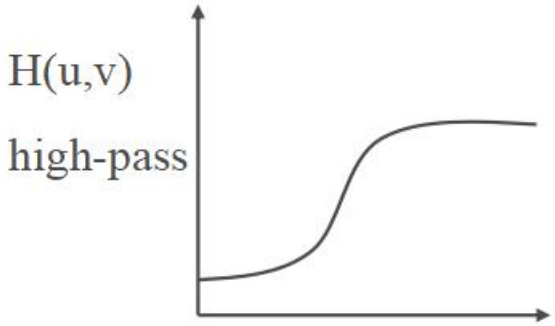
（3）S(u,v) = H(u,v) • Z(u,v)

（4）s(x,y) = F-1[S(u,v)] = i(x,y) + r(x,y)

（5）g(x,y) = exp [s(x,y)]= exp [i(x,y) + r(x,y)]= i0(x,y) • r0 (x,y)



The homomorphism filter function H(u,v) need to select.The irradiation components of illuminance components are weakened in the range of gray scale or frequency domain. Enhance the contrast of reflection components or increase the reflection spectrum components in frequency domain, enhance the details of dark areas, and retain the details of bright area images.



$$H(u,v) = (\gamma_H - \gamma_L)[1 - e^{-c(D^2(u,v)/D_0^2)}] + \gamma_L$$

4 Content

(1) By observing the Fourier frequency spectrum maps of pictures ”rect.bmp”,”rect-45 度 ” and ”rect-move”,you should understand the features,which are rotation,translation and so on, of image Fourier transforming.

(2)You should finish the ideal high-pass filtering and low-pass filtering towards “grid.bmp”

(3)Import any sine-wave towards “lena.bmp”.

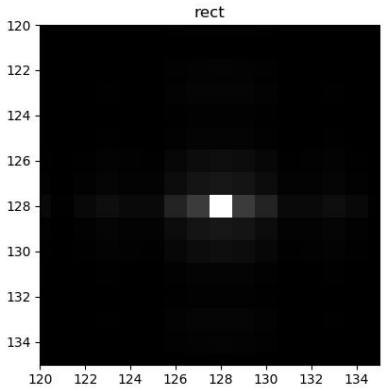
(4)Finish homomorphism filtering algorithm towards “cave.jpg”.Then observe the effect of illumination correction.(using the template of high-pass filtering of Gauss type)

5 Results and Analyzation

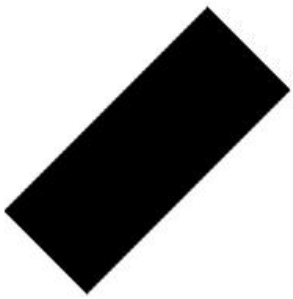
(1)By observing the Fourier frequency spectrum maps of pictures ”rect.bmp”,”rect-45 度 ” and ”rect-move”,you should understand

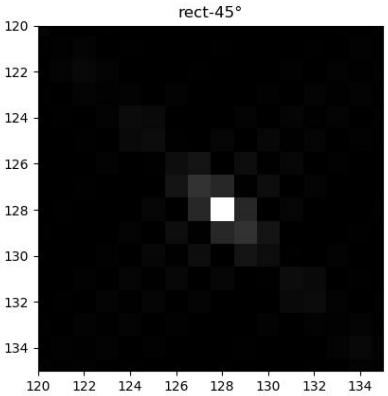
the features,which are rotation,translation and so on, of image Fourier transforming.

”rect.bmp” and its frequency spectrum:

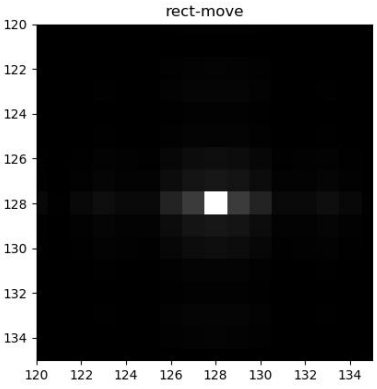


”rect-45 度.bmp” and its frequency spectrum:





”rect-move.bmp” and its frequency spectrum:

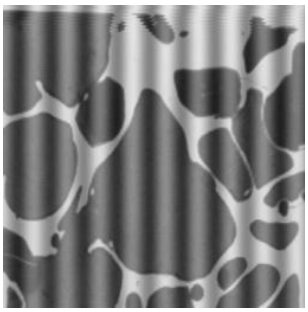


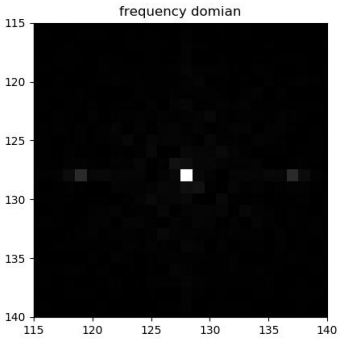
Analyzation:

Translation dosen't change frequency spectrum of Fourier transformation ,but rotation does.The frequency spectrum map would rotate 45° after rotating.

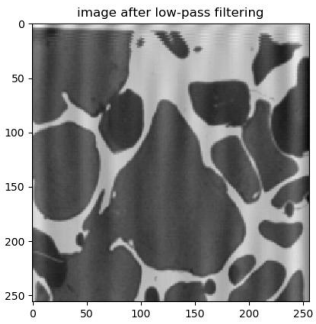
(2)Finish the ideal high-pass filtering and low-pass filtering towards "grid.bmp"

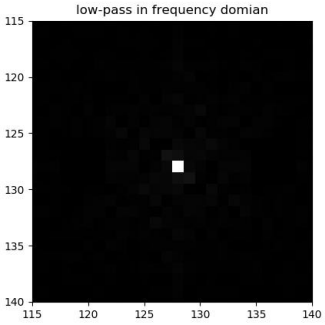
Origin image and its frequency spectrum:



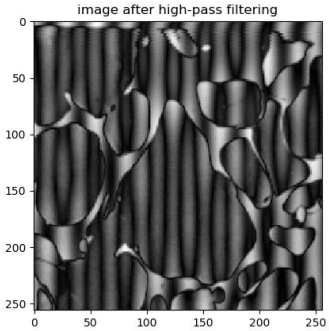


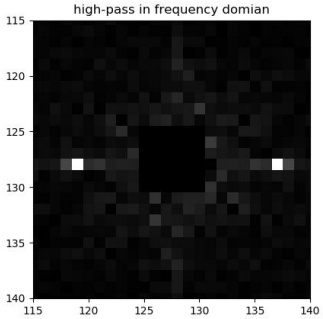
Low-pass image and its frequency spectrum:





High-pass image and its frequency spectrum:



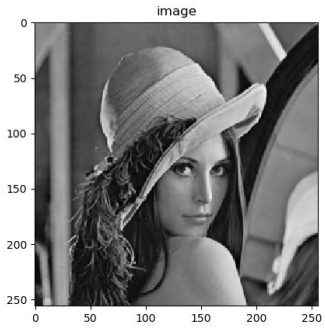


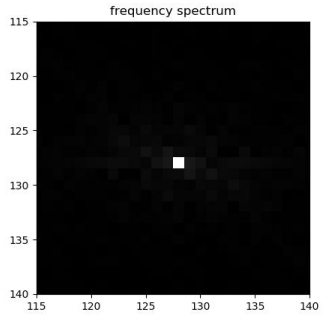
Analyzation:

There are 3 clear bright points in frequency spectrum of original image.The brightest point is in the center.The other two bright points beside the center are the sine wave of the image.We use the low-pass filter to remove the two points,then the image lost the sine wave component.If we use high-pass filter to remove the two points brightest point,we get back picture like ugly one.

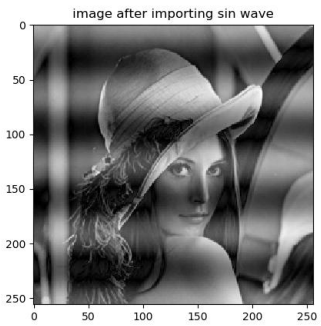
(3)Import any sine wave towards “lena.bmp”.

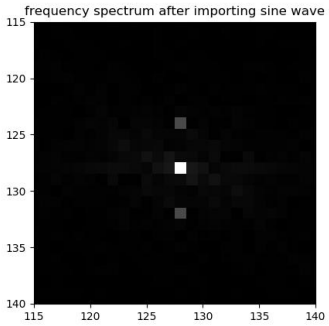
“lena” and its frequency spectrum:





“lena” and its frequency spectrum after import sine wave:



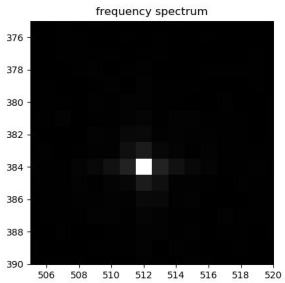


Analyzation:

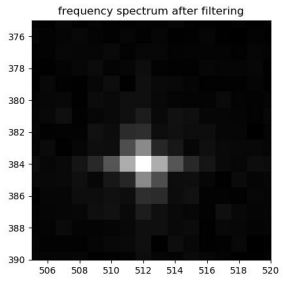
The principle is the same as (2),just import high frequency component will OK.

(4)Finish homomorphism filtering algorithm towards “cave.jpg”.

”cave.jpg” and its frequency spectrum:



”cave.jpg” and its frequency spectrum after homomorphism filter:



Analyzation:

You can see,after homomorphism filtering,the picture become clearer in dark region.Because the low frequency component are weakened,and the high frequency component are enhanced.

6 Summary

Sometimes processing image in frequency domain would be a more powerful method than processing image in spatial domain,especially for

filtering.In this way,we needn’t care local region,but global area to finish processing image in frequency domain.

All the code and instruction files are on my github.You can run them if you want:
https://github.com/Deep-Lan/Image-processing-in-frequency-domain.git