The University of Queensland
School of Mechanical & Mining Engineering

MECH2700 Computational Engineering & Data Analysis (S2, 2022)

Assignment 1: Computational Mechanics

Introduction

This assignment asks you to simulate the descent trajectory of a recreational skydiver. You will write a Python program to simulate the skydiver's jump, based on some given conditions. That program can then be used to investigate how the trajectory is affected by various parameters.

Theory

The continuous motion of an object in the x, y-plane can be described by specifying the object's position (x, y) and velocity (v_x, v_y) as functions of time, t. If we know the forces (F_x, F_y) acting on the object then we can compute the state of motion for all times by integrating the equations of motion, shown here in Cartesian coordinates.

$$m\, a_x = \Sigma F_x \quad ; \quad m\, a_y = \Sigma F_y \tag{1}$$

The object is assumed to have a fixed mass, m, concentrated at its centroid and the components of acceleration are defined as

$$a_x = \frac{d^2x}{dt^2} \quad ; \quad a_y = \frac{d^2y}{dt^2} \tag{2}$$

In your Applied Mechanics course in first year, you usually had a simple functional form for the force components and integrated the equations of motion analytically to obtain a closed-form solution. For most situations that you encounter in your engineering practice this analytic approach will not be feasible [1] and you will have to resort to some form of numerical scheme to integrate the equations of motion approximately. This is the basis of nearly all simulation packages. In this assignment, you will develop a Python program to compute a discrete approximation to the trajectory of a skydiver.

Task 0 Read Section 14.6 *Numerical Solutions* of an old version of Bedford & Fowler's Engineering Mechanics text. You may have this book from your Applied Mechanics course or find it in the library. In case the numbering is different in your copy of the text, Chapter 14 is labelled "Force, Mass and Acceleration".

Task 1 **(3 marks)**

Using the tabulated data from the U.S. standard atmosphere shown in Table 1, write a python function to interpolate the air density (kg.m^{-3}) as a function of the altitude (m) in the range of 0 to 5000 m. Plot the resulting interploting function over that range of altiudes along with the tabulated data. This function will be used estimate the density at any altitude in later calculations.

Task 2 **(3 marks)**

Show that the state of motion of the skydiver at $t = t_0 + \Delta t$ (*i.e.* a short time after some

[1] Possibly because you do not have the forcing functions in analytic form or because they are too complicated to integrate.

Table 1: U.S. standard atmosphere air properties - SI units

Altitude (m)	**Temperature (Celsius)**	**Density (kg.m^{-3})**
-1000	21.5	1.347
0	15.0	1.225
1000	8.5	1.112
2000	2.0	1.007
3000	-4.5	0.9093
4000	-11.0	0.8194
5000	-17.5	0.7364
6000	-24.0	0.6601
7000	-30.5	0.5900
8000	-36.9	0.5258
9000	-43.4	0.4671
10000	-49.9	0.4135
15000	-56.5	0.1948
20000	-56.5	0.08891
25000	-51.6	0.04008
30000	-46.6	0.01841
40000	-22.8	0.003996
50000	-2.5	0.001027
60000	-26.1	0.0003097
70000	-53.6	0.00008283
80000	-74.5	0.00001846

arbitrary time t_0) can be approximated by

$$x(t_0+\Delta t) = x_0 + v_{x0}\Delta t + \frac{1}{2}a_{x0}\Delta t^2 \tag{3}$$

$$y(t_0+\Delta t) = y_0 + v_{y0}\Delta t + \frac{1}{2}a_{y0}\Delta t^2 \tag{4}$$

$$v_x(t_0+\Delta t) = v_{x0} + a_{x0}\Delta t \tag{5}$$

$$v_y(t_0+\Delta t) = v_{y0} + a_{y0}\Delta t \tag{6}$$

where the subscript 0 indicates a value at $t = t_0$. State any assumptions used.

The simulation

We will begin the simulation of the jump from when the skydiver leaves the aircraft. We will only consider motion in the $x - y$ plane, and so we ignore any initial velocity in the direction perpendicular to the aircraft's motion. At the start of the jump, the skydiver's horizontal velocity (x-velocity) is equal to the speed of the aircraft, while the skydiver's vertical velocity (y-velocity) is zero. When the skydiver begins the jump ($t = 0$), the aircraft is flying at a speed of 40 m/s and at an altitude of 4000 m.

We will analyse the descent trajectory in two phases: (1) freefall; and (2) with parachute deployed. During the jump, we will consider two forces acting on the skydiver: self-weight due to the Earth's gravity ($g = 9.81\,\mathrm{m/s}$); and aerodynamic drag. We will neglect any lift force generated by the skydiver in our analysis. During freefall, the magnitude of the drag force is given by

$$F_{\mathrm{drag}} = \tfrac{1}{2}\rho v^2 A_{\mathrm{s}} C_{D_s} \tag{7}$$

where ρ is the density of the air, v is the speed of the skydiver, A_{s} is the effective area of the skydiver in the direction of travel, and C_{D_s} is a dimensionless drag coefficient for the skydiver. The value of C_{D_s} may be taken as 0.92, and the area of the skydiver is $A_{\mathrm{s}} = 0.5\,\mathrm{m}^2$. The skydiver follows safe practices and deploys the parachute at an altitude of 1200 m. As soon as the parachute is deployed, the total drag increases dramatically and the descent is slowed. During the descent with the parachute deployed, the magnitude of the drag force includes the drag on the skydiver and on the parachute:

$$F_{\mathrm{drag}} = \tfrac{1}{2}\rho v^2 \left(A_{\mathrm{s}}C_{D_s} + A_{\mathrm{p}}C_{D_p}\right) \tag{8}$$

where A_{p} is the effective area of the parachute in the direction of travel, and C_{D_p} is a dimensionless drag coefficient for the parachute. The area of the parachute (A_{p}) is $45.0\,\mathrm{m}^2$ and the drag coefficient (C_{D_p}) is 1.33. The drag forces are defined such that their direction is opposite to the direction of the skydiver's velocity at any instant in time.

The forces on the skydiver will vary throughout the trajectory. We require some additional information to compute those forces at any instant in time. The total mass of the skydiver is 90 kg. This includes the skydiver's mass plus the parachute, chute lines and harness. The density of the air (ρ) in the atmosphere varies from the point of leaving the aircraft to the ground. The interpolating function developed in task 1 can be used to estimate the density. If you were not able to complete task 1, you may model that variation of density using a simple exponential model:

$$\rho = \rho_0 e^{-y/7249.0} \tag{9}$$

where ρ_0 is sea-level density of $1.225\,\mathrm{kg/m^3}$ and y is the height above the ground in metres.

Task 3 **(2 marks)**

Draw a free-body diagrams showing the forces acting on the skydiver during the jump when: (a) in freefall, and (b) with the parachute deployed.

Task 4 **(3 marks)**

Write expressions for the acceleration components a_x and a_y. Each of these may be built up as a set of expressions. Write expressions for the two phases of the jump:

(a) During freefall.

(b) With the parachute deployed.

State any assumptions used in the model for acceleration.

Task 5 **(5 marks)**

Write a Python program that computes the trajectory of the skydiver after leaving the aircraft until landing on the ground. Your program should start with the initial state as discussed above. It should then integrate the equations of motion with small increments of time and save the skydiver's position, velocity and acceleration at discrete points in time. Terminate the integration process when the skydiver has landed (safely, hopefully).

Task 6 **(2 marks)**

For $C_{D_s} = C_{D_p} = 0.0$, integrate the equations of motion analytically to determine an exact solution for the case of no aerodynamic drag. Demonstrate that your program agrees with your analytic solution.

Task 7 **(3 marks)**

Determine a time-step that gives a sufficiently accurate estimate of the trajectory for the case of non-zero aerodynamic drag. Use the time of jump and distance down range in the drop zone as the test criteria. Explain why you consider this to be good enough. What was the landing speed of the skydiver and how long did the jump take?

Task 8 **(5 marks)**

Produce some plots of the skydiver's jump.

1. A plot of the trajcetory (x vs y).
2. A plot of the velocity with time. Use time on the x-axis and velocity on the y-axis. Show two curves on this plot: x-velocity and y-velocity.
3. A plot of the acceleration with time. As above but with acceleration on the y-axis and two curves showing x-acceleration and y-acceleration.

Task 9 **(4 marks)**

As a skydiving operator, you advertise that the jump will last 6 minutes from aircraft exit to landing. On one trip, the maximum mass amongst a group of customers is 80 kg. You instruct the jumpers to deploy their parachutes at a height of 1400 m. To what altitude does the aircraft need to climb at the beginning of the jump so that the customers get a 6-minute jump time?

Use your program to determine an answer. Explain the method you used to arrive at your answer.

Assessment Criteria

The assignment is worth **30 marks total**. There are **27 marks** associated with Tasks 1–9. The marks per task are listed beside each task. The marks are awarded in each task based on the extent to which your response is correct.

There are **3 marks** associated with the quality of your submitted code. The tutors will mark code quality looking at the following aspects:

- Is the code logical, readable and clearly matches the numerical method?
- Is the code adequately documented with comments?
- Is the code general enough for reuse on similar applications?

Submission Requirements

Submit your responses to Tasks [1–3, 5–8] in a single PDF. Put your responses in order and clearly marked as Task 1, Task 2, etc. so that the tutors may easily find your work. Equations and text responses may be handwritten (legibly) or typeset. Plots should be produced using Python.

Tasks 1 and 4 require submission of Python code. Submit this as separate files.