Jungla – soluţie

(Em. Cerchez) (problema a fost pregătită pentru BOI 2003, deci citiţi descrierea soluţiei în limba engleză !)

It is obvious that we can associate to this problem an undirected graph (the vertices of this graph are the tribes, and the edges are the existing direct paths between the tribes' locations). The task is to determine the shortest even length elementary cycle in this graph (SELEC).

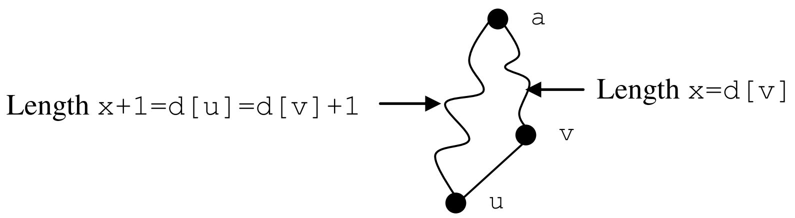
The algorithm starts a BFS from every vertex. As soon as an even length elementary cycle (ELEC) is detected, it is compared with the current SELEC.

The BFS is improved to detect ELEC. For this purpose, we record for every vertex the following informations:

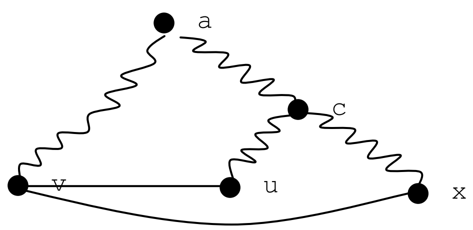
- `d[v]`=the distance of `v` from the start vertex of BFS (denoted from now on by `a`), that means the level of `v` in the BFS tree; if `v` has not yet been visited, `d[v]=+∝`;
- `p[v]`=the parent of `v` in the BFS tree; `p[v]=0` if `v=a` or `v` has not yet been visited. Obviously, if `p(v}≠0, d[v]=1+d[p[v]]`.
- `o[v]`=the match of `v`; if `o[v]≠0` then `o[v]` is a vertex in the same level of `v` such that `[v,o[v]]` is an edge in the graph. A vertex is said to be matched if `o[v]≠0`. If `v` is matched, then `o[v]` is also matched and `o[o[v]]=v`.
- `r[v]`=the highest proper ancestor of `v` in the BFS tree that is matched. If `v` has no matched proper ancestor, then `r[v]=0`.

We now describe how we process a vertex `v` that has been popped out of the BFS queue. The action to take for the edge `[v,u]` depends on the values of `d[u]`, `o[v]` and `o[u]` in the following way:

1. if `d[u]=d[v]-1`, do nothing (this edge has been procesed before, in the opposite direction);
2. if `d[u]=+ ∝`; let `d[u]=d[v]+1`, `p[u]=v` and enqueue `u` to the BFS queue.
3. if `d[u]=d[v]+1`, halt the BFS, as an ELEC was found. Let `c` be the lowest common ancestor in the BFS tree of `u` and `v` (denoted by `LCA(v,u)`). Then the paths from `c` to `v` and from `c` to `u` and the edge `[v,u]` form an ELEC.



4. if `d[u]=d[v]` and `o[v]=u`, do nothing (this edge has been processed before in its opposite direction);
5. if `d[u]=d[v]` and `o[v]≠u`, and `o[v]` and `o[u]` are not both zero, halt the BFS an ELEC was found. Assume for example that `o[v]=x≠0`. Let `c` be the `LCA(x,u)`. The path from `c` to `x` and the path from `c` to `u` and the edges `[x,v]` and `[v,u]` form an ELEC.



6. if `d[u]=d[v]` and `o[u]=o[v]=0`, test wether `r[u]=r[v]`. If they are equal, let `o[v]=u`, `o[u]=v`. If they are not equal, halt the BFS an ELEC was found. Assume for example that `r[v]=x≠0` and let `y=o[x]`. Let `c=LCA(y,u)`. The path between `c` and `y`, followed by the edge `[y,x]`, followed by the path from `x` to `v`, followed by the edge `[v,u]`, followed by the path from `u` to `v` represent an ELEC.

After we finish scanning all the neighbours of `v`, we rescan them to set `r[u]` for every `u` that became a child of `v`.

We put `r[u]=r[v]` unless `r[v]=0` and `o[v]≠0` in which case we put `r[u]=v`.

Observation

The improved BFS scans no more than `3|V|/2` edges and therefore runs in `O(V)` time. Further more, if C is a SELEC of length `2k` and `a` is the root of it, then the improved BFS that starts from `a` finds an ELEC of length `2k`.