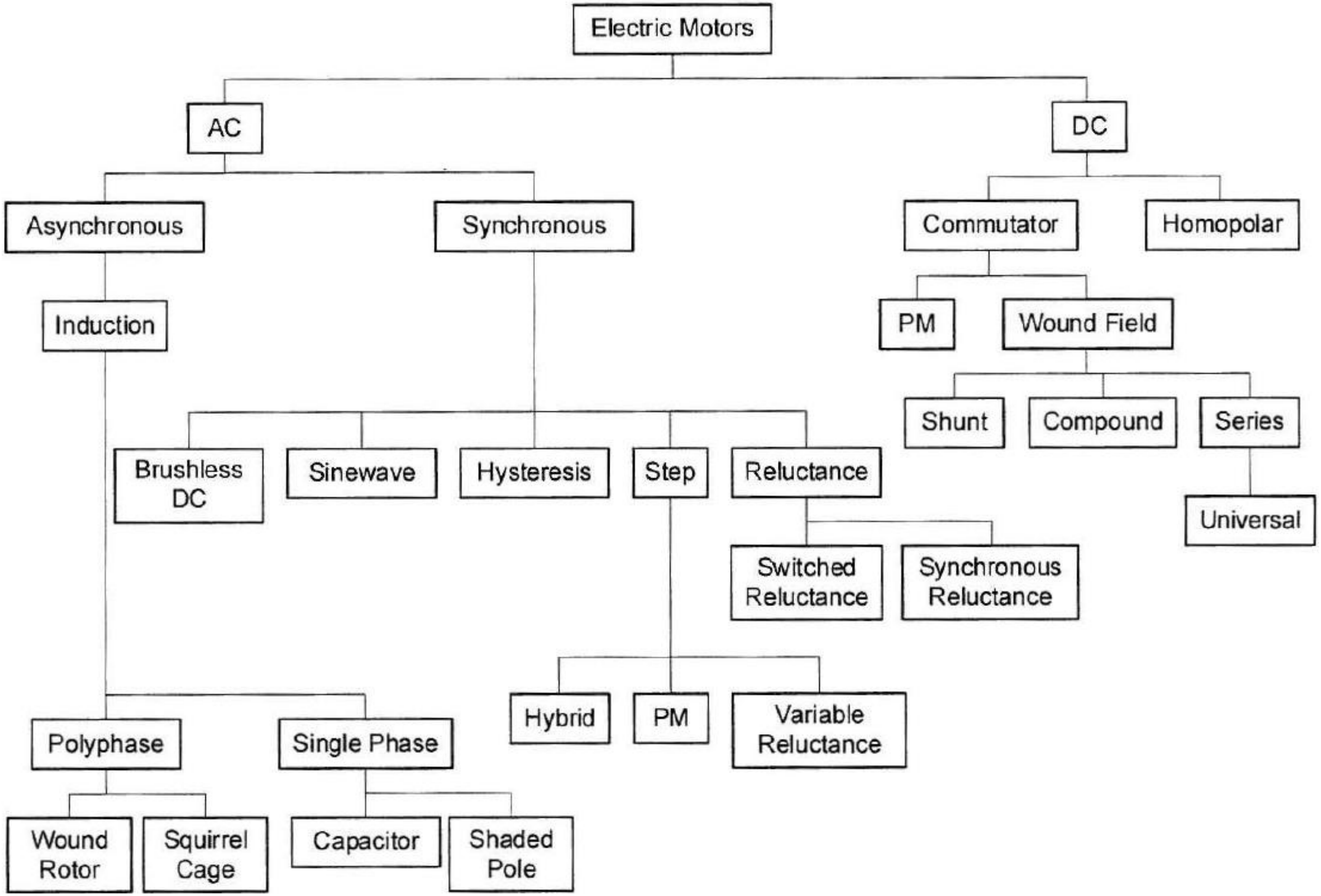
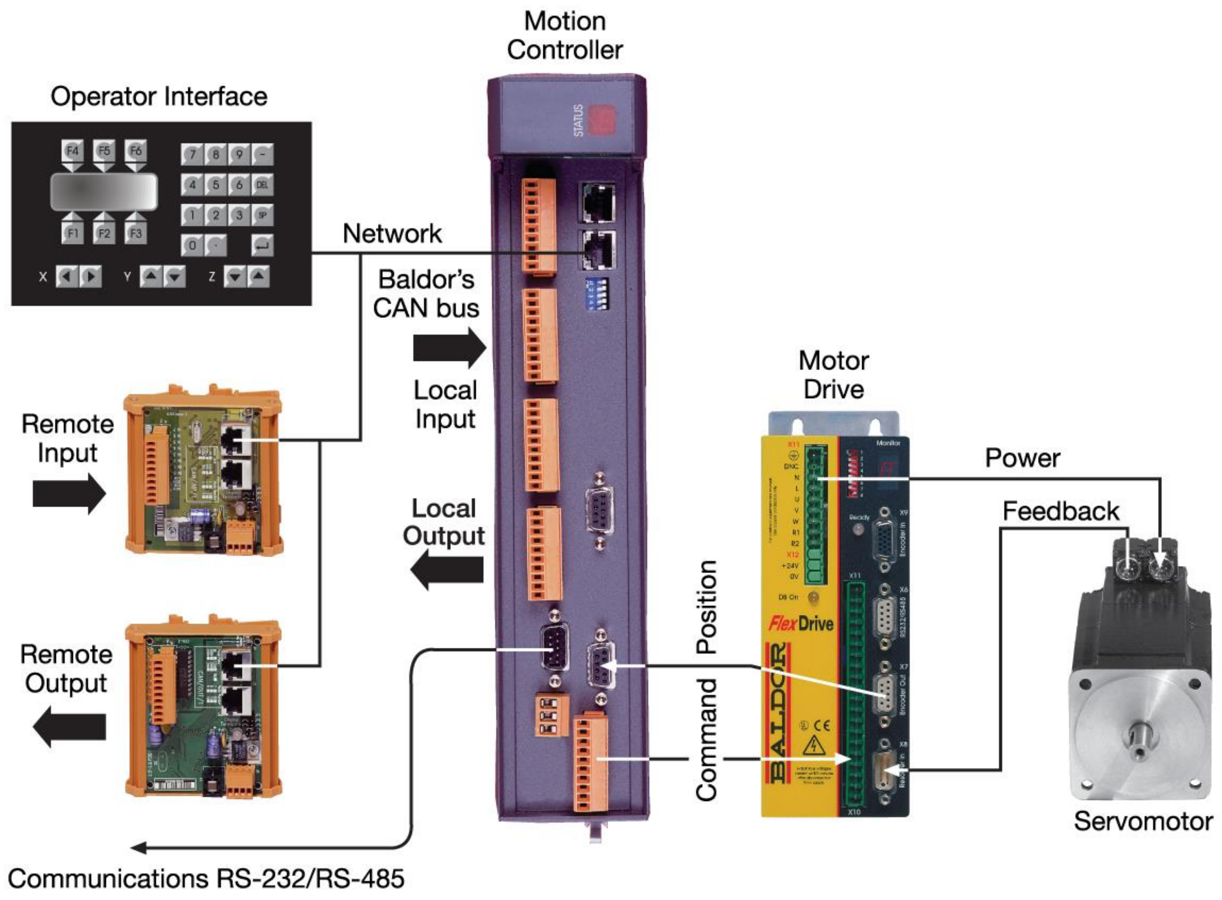
SERVO MOTORS AND MOTION CONTROL SYSTEMS

(Servo Motors, Drives, and Control Methods: Part I-Lecture 5)

Dr. Caglar UYULAN

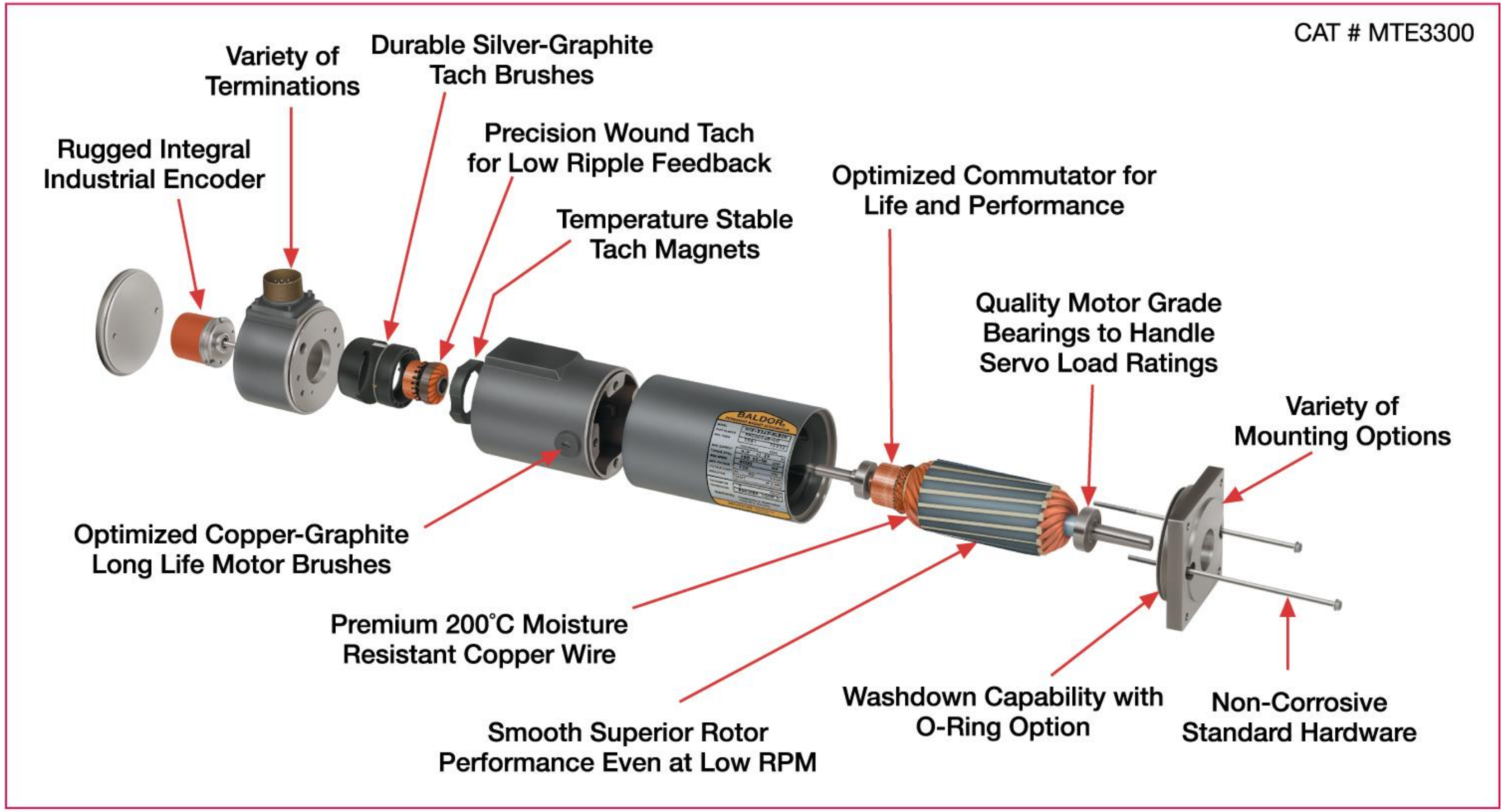

Electric Motors
AC
DC
Asynchronous
Synchronous
Commutator
Homopolar
Induction
PM
Wound Field
Shunt
Compound
Series
Universal
Brushless DC
Sinewave
Hysteresis
Step
Reluctance
Switched Reluctance
Synchronous Reluctance
Hybrid
PM
Variable Reluctance
Polyphase
Single Phase
Wound Rotor
Squirrel Cage
Capacitor
Shaded Pole

Interface to Servo



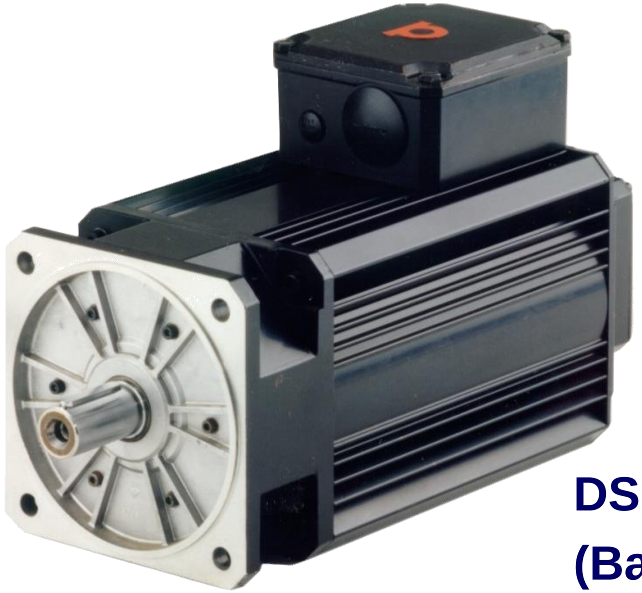
Baldor

DC PM Servo Motor Construction



State of the Art: AC Servos

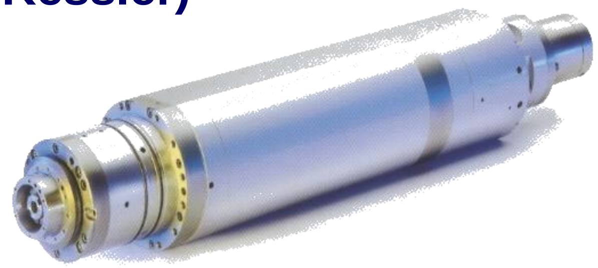
High Speed Spindle
(Siemens)



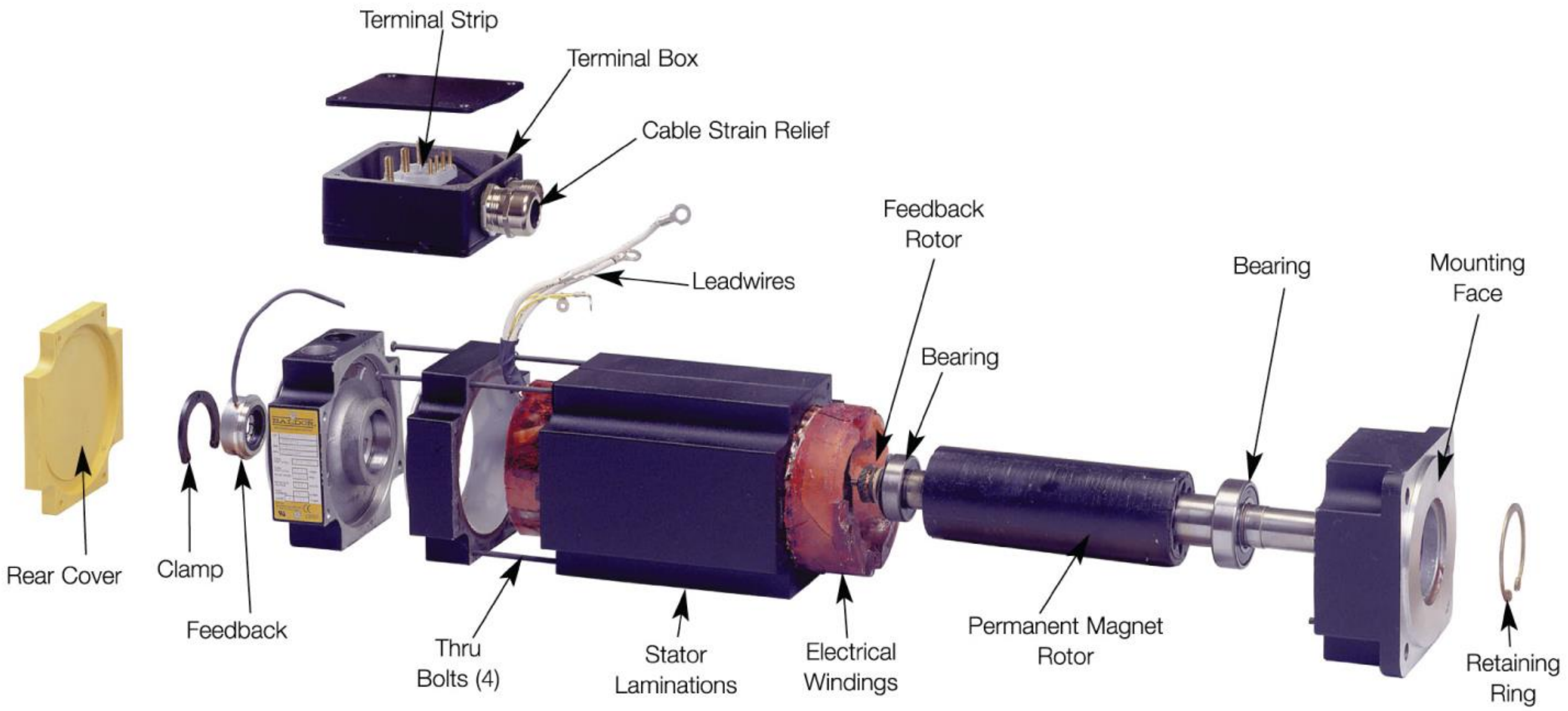
DSD Servo
(Baumüller)

High Torque Motor
(Baumüller)

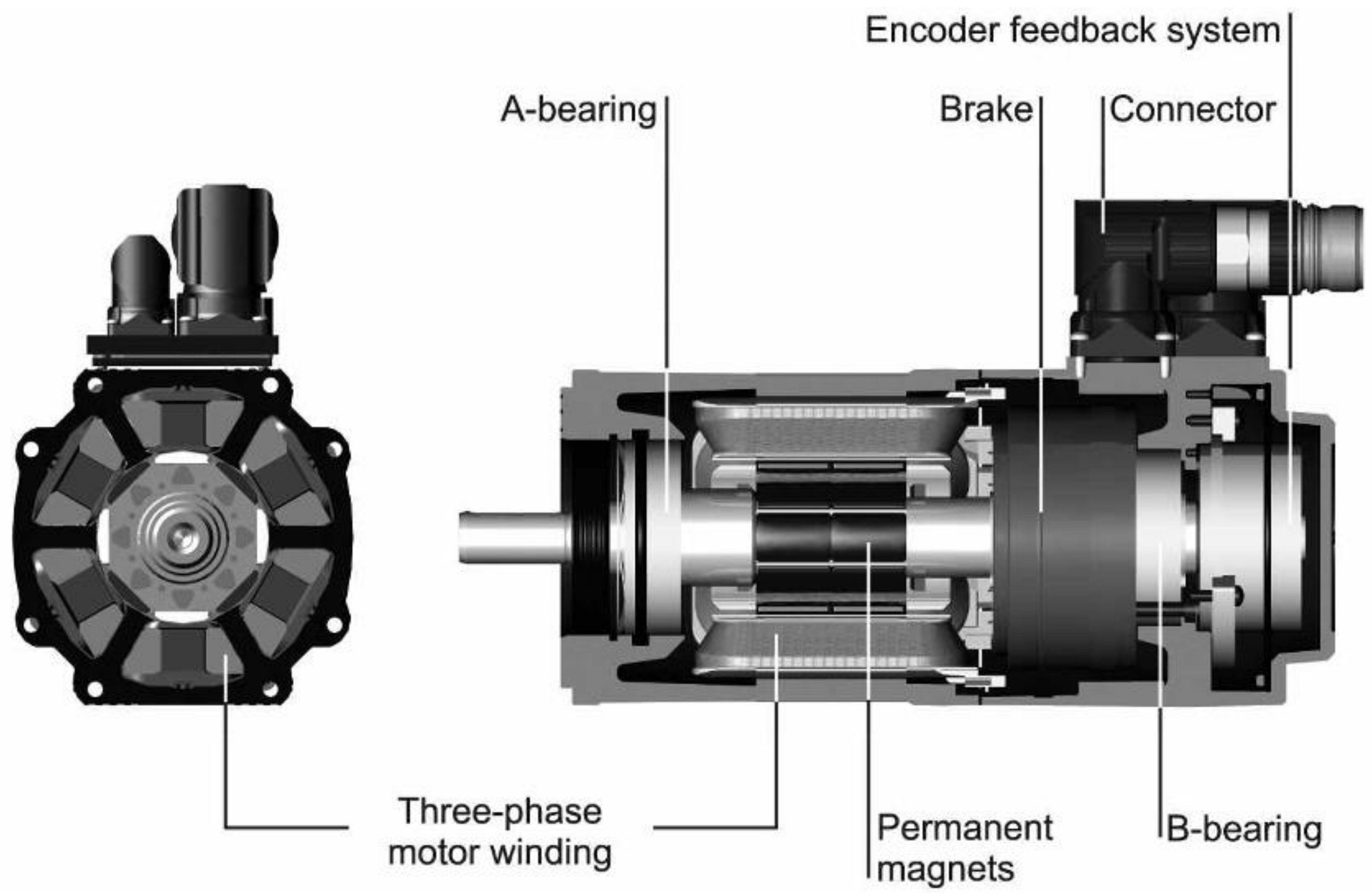
Spindle Motor
(Franz Kessler)



Brushless Servo Motor
Exploded View



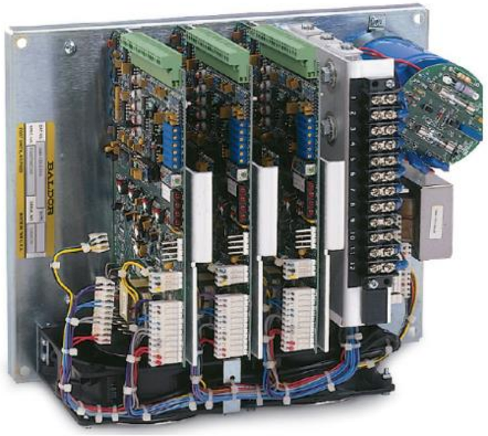
Baldor



Encoder feedback system
A-bearing
Brake
Connector
Three-phase
motor winding
Permanent
magnets
B-bearing

Controls are Available in Various Packages

UMH Servo Control



Controls (amplifiers) originally were designed for a particular application, to handle a specific load. About the late 70's a designer figured that a product could be made available to handle a wide variety of applications if potentiometers were provided for adjustments. Today, digital adjustments via keypad, and with PC setup, are also available.

TSD Servo Controls

H-Series Controls

H2 Controls

Flex+Drive[II]

Baldor

Baldor Brushless Servo Motors

- Standard, custom and Stainless Steel motors
- Standard and low inertia models
- Potted stator – superior protection for high voltage and current spikes
- Premium 200°C moisture resistant, multi-coated wire
- Superior bearing grease provides 4 times greater life
- Worldwide acceptable mounting – IEC, NEMA & customs – UL/CSA/CE/BISSC
- Variety of feedback – resolver, encoder, Halls, SSI, EnDat, etc.

Baldor

Stainless Steel Brushless Servos

- Stainless steel housing
- Non-corrosive
- FDA, BISSC, UL, CSA, CE
- Watertight 1500 psi & IP67
- Potted stator
- Exxon Polyrex®EX Polyurea grease
- Laser etched nameplate

Precision Gearheads for BSM Servos

- Mount directly onto BSM
- Planetary - high efficiency
- Low backlash
- Inertia matching
- Torque multiplication
- GBSM Models
  - Standard 8-15 arc min
  - Higher torque / Lower backlash
  - Right angle

AC Brushless Servo Control

- Rotary or linear brushless servos
- Analog or pulse/direction input
- Intuitive WindowsTM front end provides full auto-tuning
- ActiveX libraries supplied free

MicroFlex

- Cont 3, 6, 9 amp (pk = 2X)
- Anti-resonance filters

FlexDrive^{II}

- Cont 2 - 7.5 amp 1ϕ; 2 - 27.5 3ϕ; (pk=2X)
- 8/3 I/O; 1-14 bit analog input
- Master encoder input for gearing
- DeviceNet, ProfiBus, CANOpen

Motion Controllers

- Coordinated motion control
- Programmable in MintMT Multi-Tasking, or 'C'
- CANOpen for distributed control

NextMovePCI

- 1 – 8 axis servo/stepper
- USB & RS232 communications

NextMoveBX^{II}

- 2 - 4 axis servo
- RS232/485 communications

NextMoveESB

- 3 axis servo / 4 axis stepper
- USB & RS232 communications

Accessories to Complete the Package

- Cable assemblies

- HMI operator panels

- Auxiliary breakout boards

Programmable Positioners Come in Wide Selection of Configurations...

MintDrive[II]

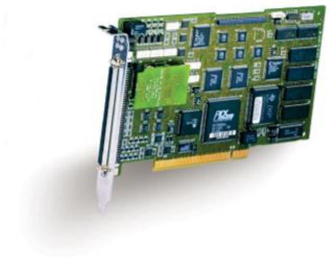
Programmable motion controls are available in many different configurations. some include the motor control (amplifier) with an internal power supply, and typically control one motor. Other are multi-axis. These are commercially available in enclosures or printed circuit boards, which fit inside a PC.

NextMoveBX[II]

Motion Controller
2, 3, 4 Axes

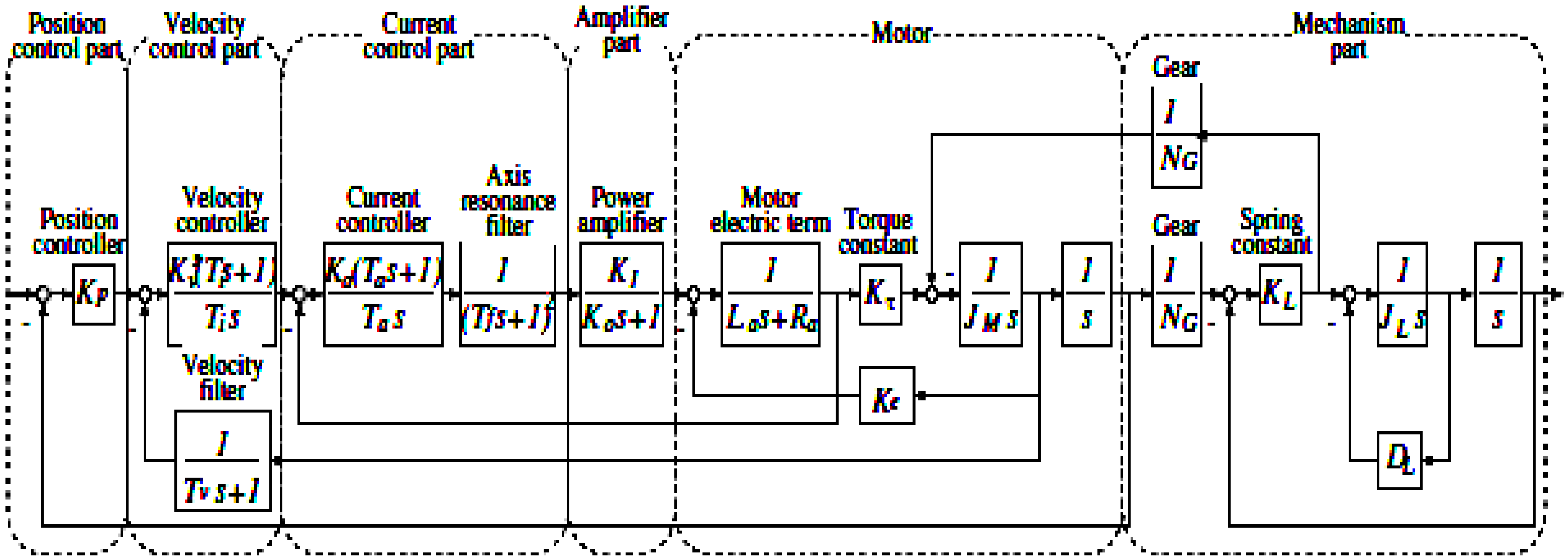
NextMovePCI

Motion Controller
1-12 Axes PCI Bus



Baldor

Construction of position control system of one-axis mechatronic servo system



General Concepts in Motor Control

Servomotors are specialized motors used for precise control of angular or linear position, velocity, and acceleration. These motors are essential in applications where precise positioning is crucial. In general, servomotors are controlled through position, velocity, and torque (current) controllers.

Position, Speed, and Torque Control

Position Control: The most fundamental aspect of a servomotor. It involves moving the motor to a desired angle or position. Position controllers typically use feedback mechanisms, like encoders, to determine the motor's current position and adjust the control input accordingly.

Speed Control: This involves controlling the rate at which the motor shaft rotates. Speed control is vital in applications where the motor needs to maintain a constant speed, despite varying loads.

Torque Control: Torque, or rotational force, control is essential in scenarios where the motor must apply a specific amount of force. It is typically achieved by controlling the motor's current.

Types of Servomotors

Servomotors are broadly categorized into two types based on their construction and operation principles:

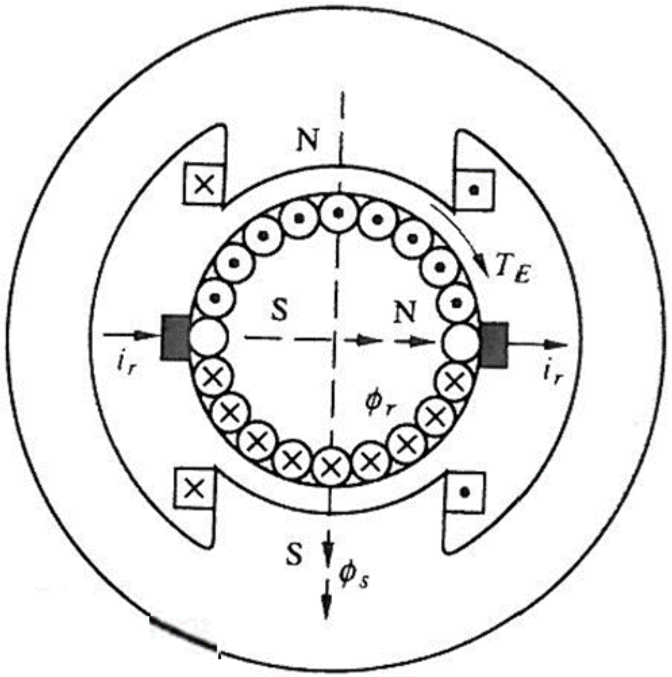
Brushed DC Motors: These motors use brushes and a commutator to deliver current to the motor windings. The main advantage of brushed DC motors is their simplicity of control. However, they suffer from wear and tear due to brush friction and are typically less efficient.

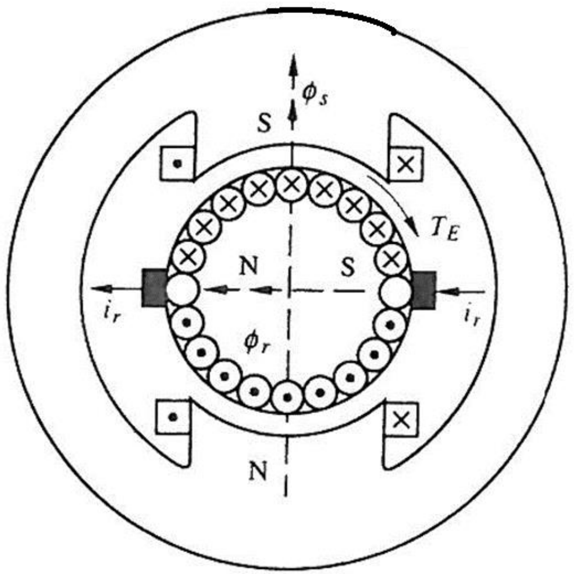
Brushless Motors: These include Brushless DC Motors (BLDCM) and Permanent Magnet Synchronous Motors (PMSM).

BLDCM: These motors do not have brushes, reducing wear and tear and increasing efficiency. They use a permanent magnet rotor and require an electronic controller for phase shifting.

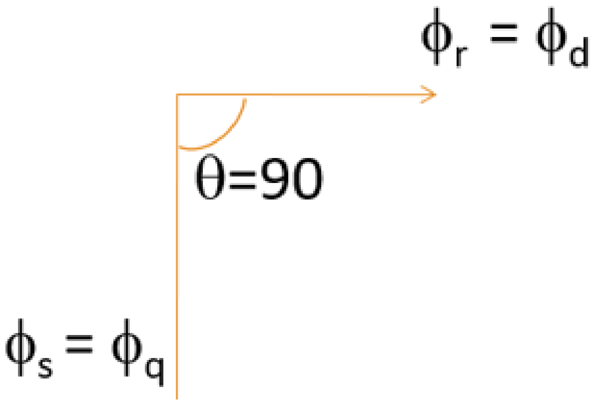
PMSM: Similar to BLDC motors but with a different rotor structure. They are known for high efficiency, better heat dissipation, and quieter operation.

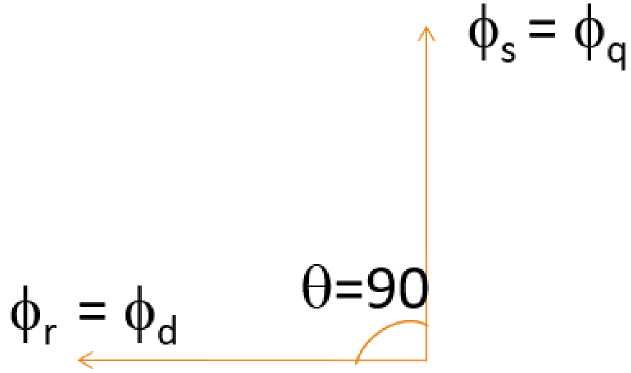
Brushed DC Motor

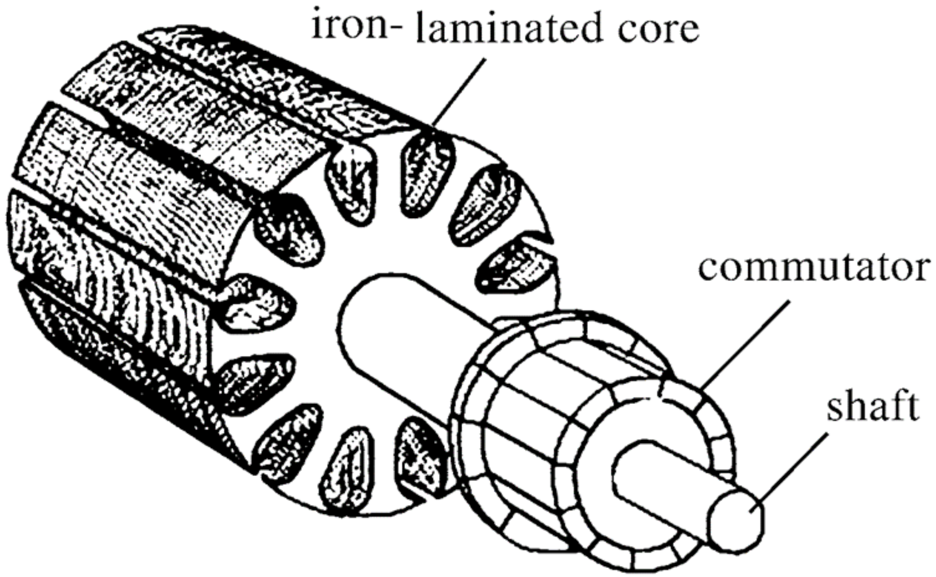


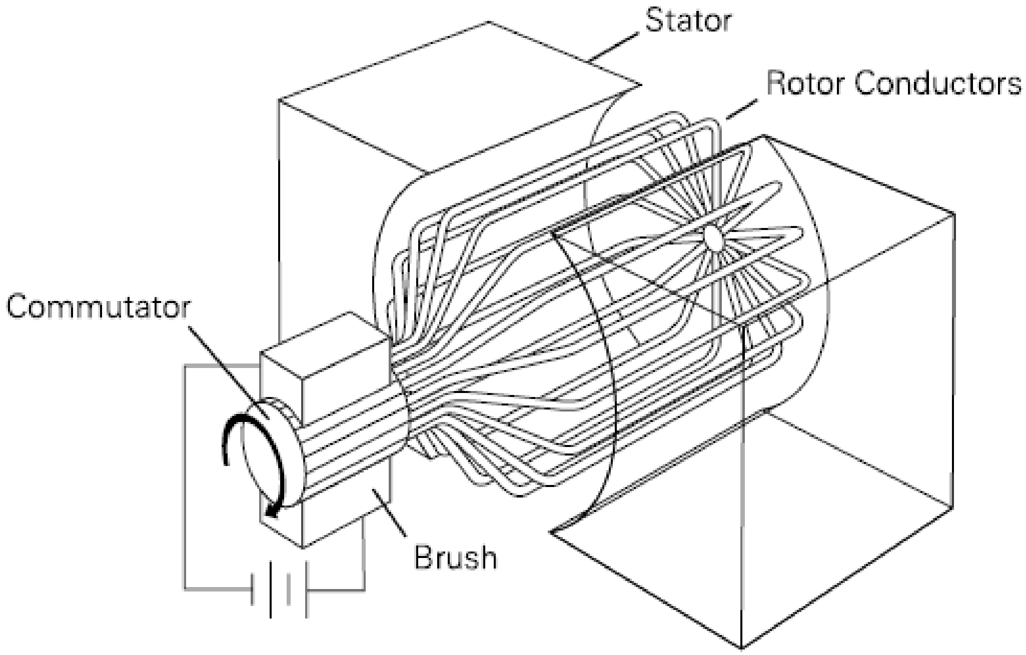


$$T_m = K\,\phi_s\,\phi_r \sin\theta$$
$$= K\,\phi_q\,\phi_d$$

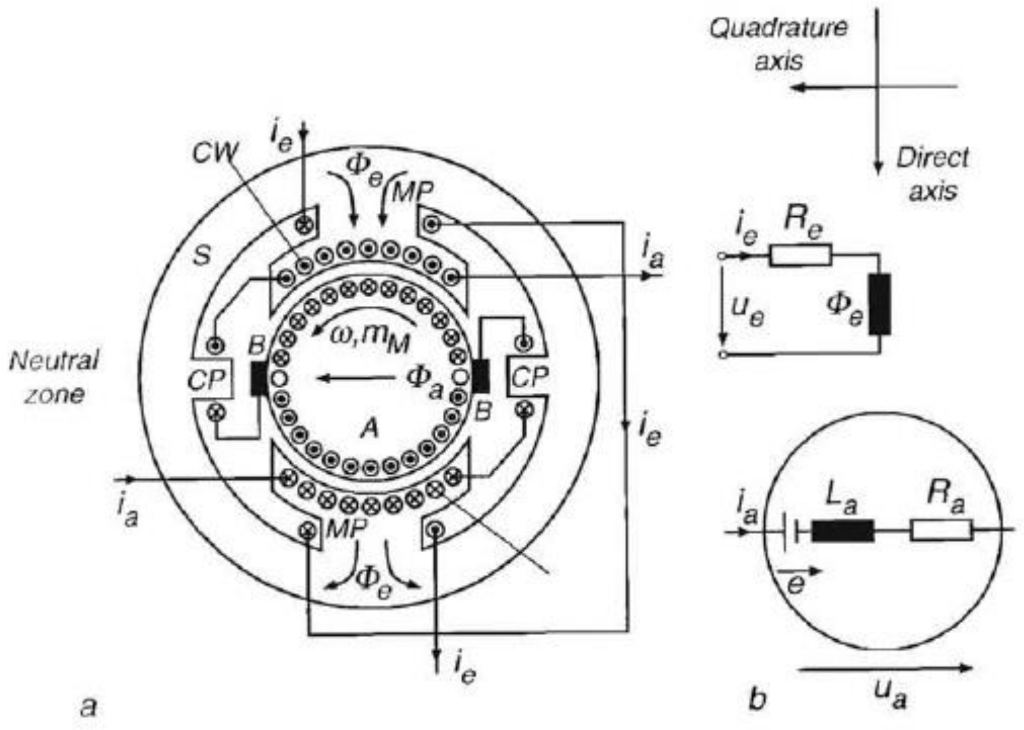


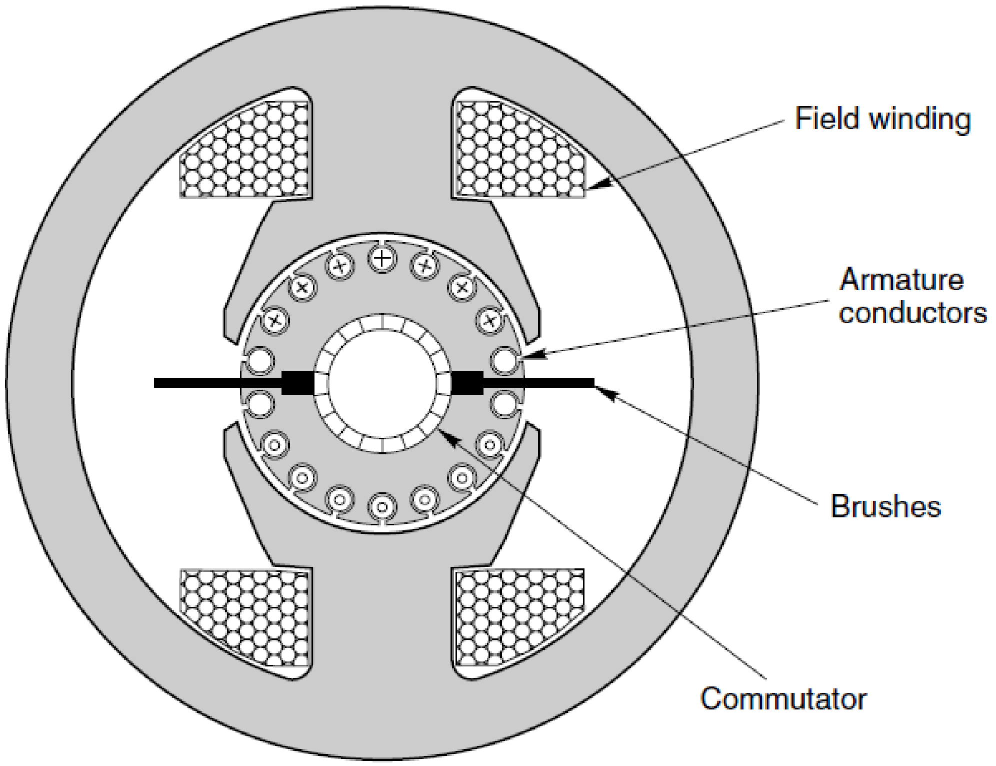






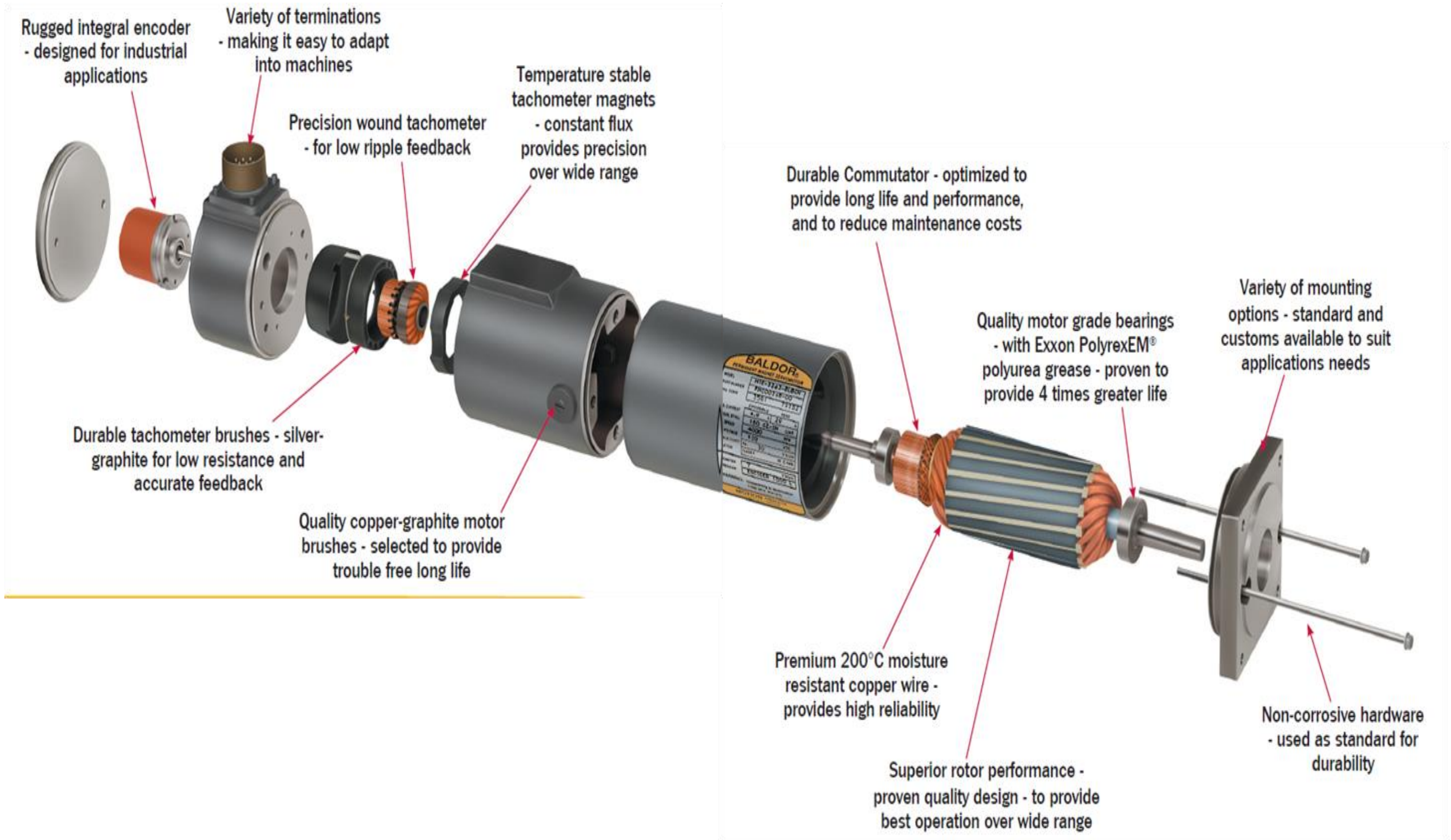
DC Motor: rotor body and windings

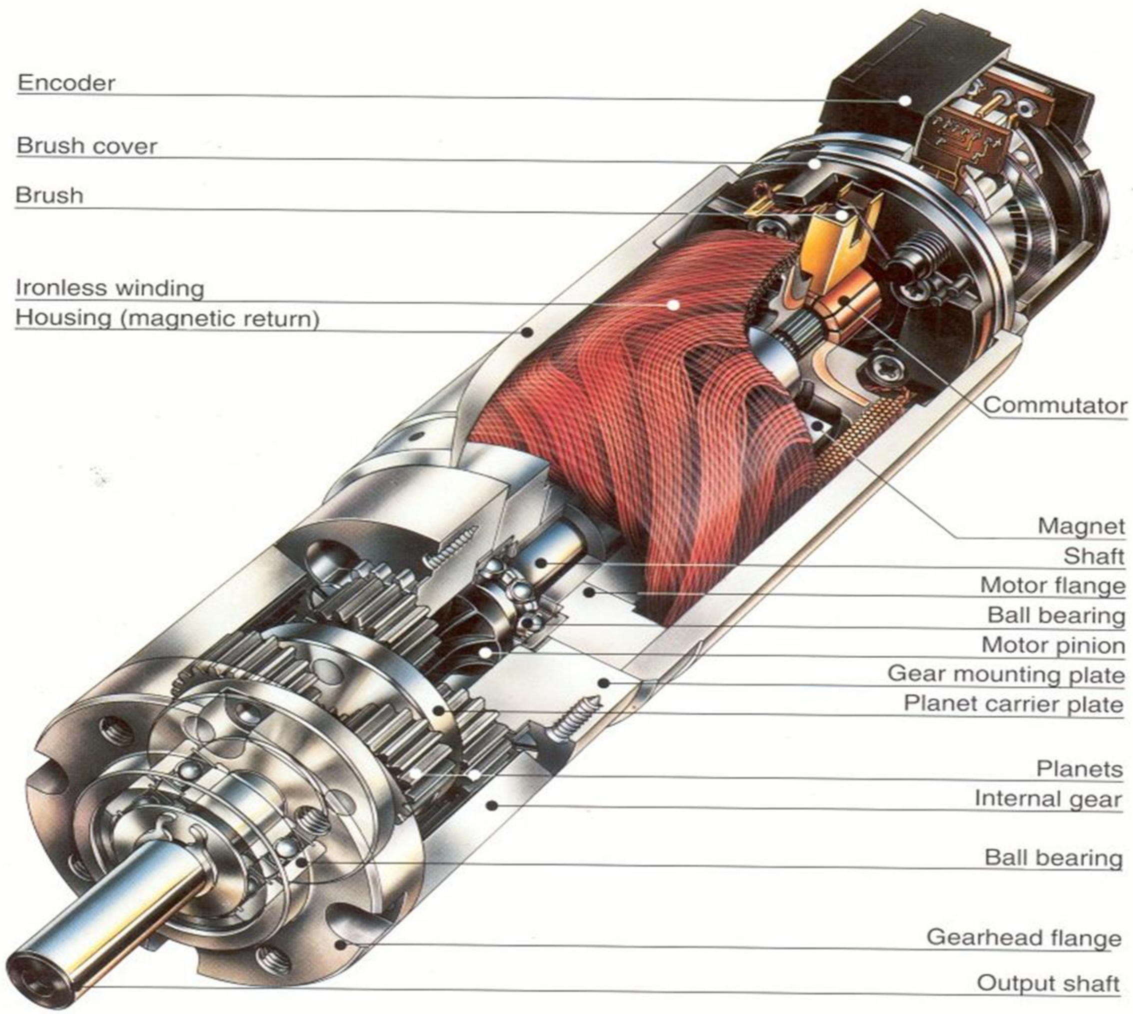




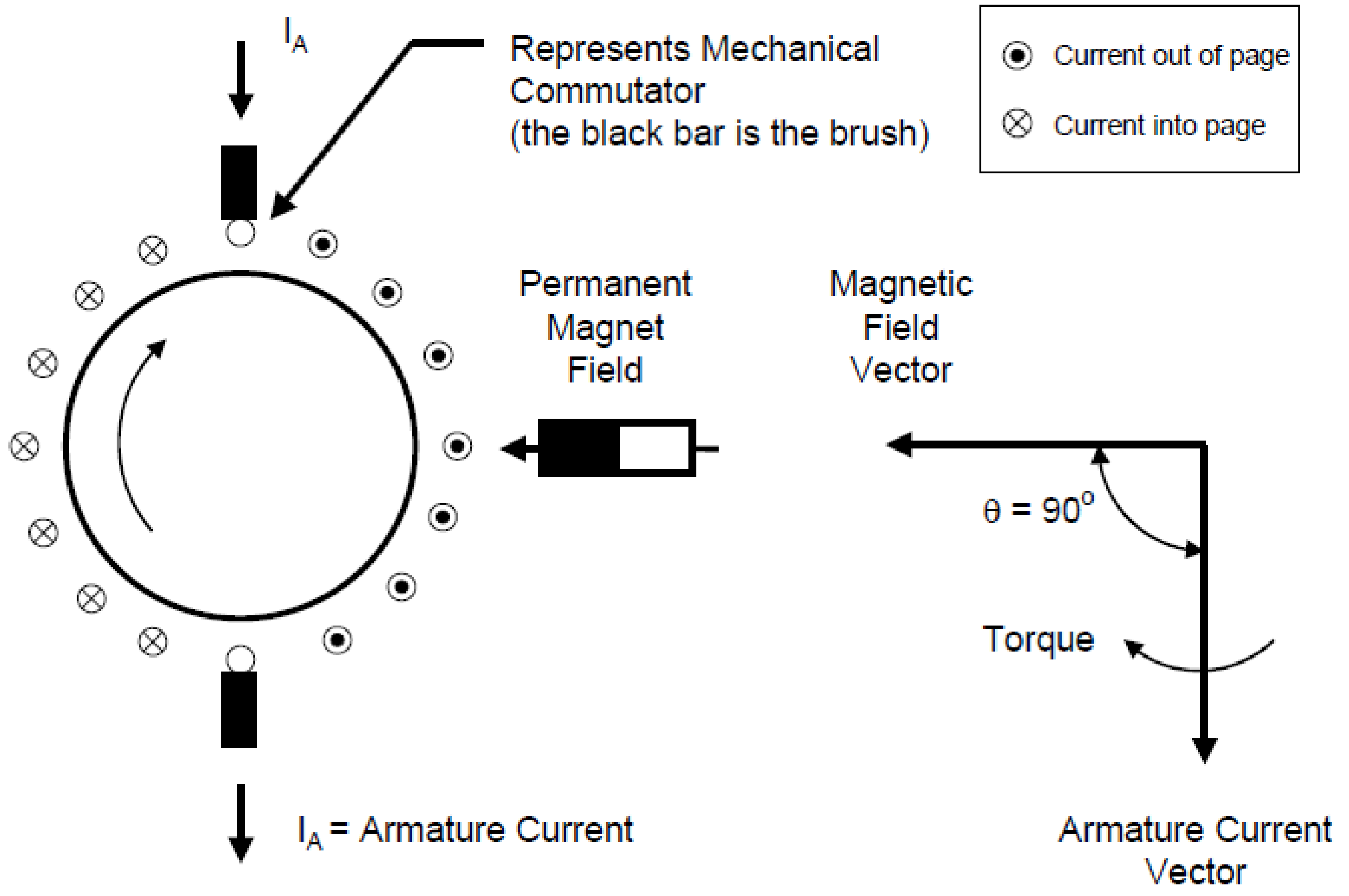
Brushed DC motor equivalent representation

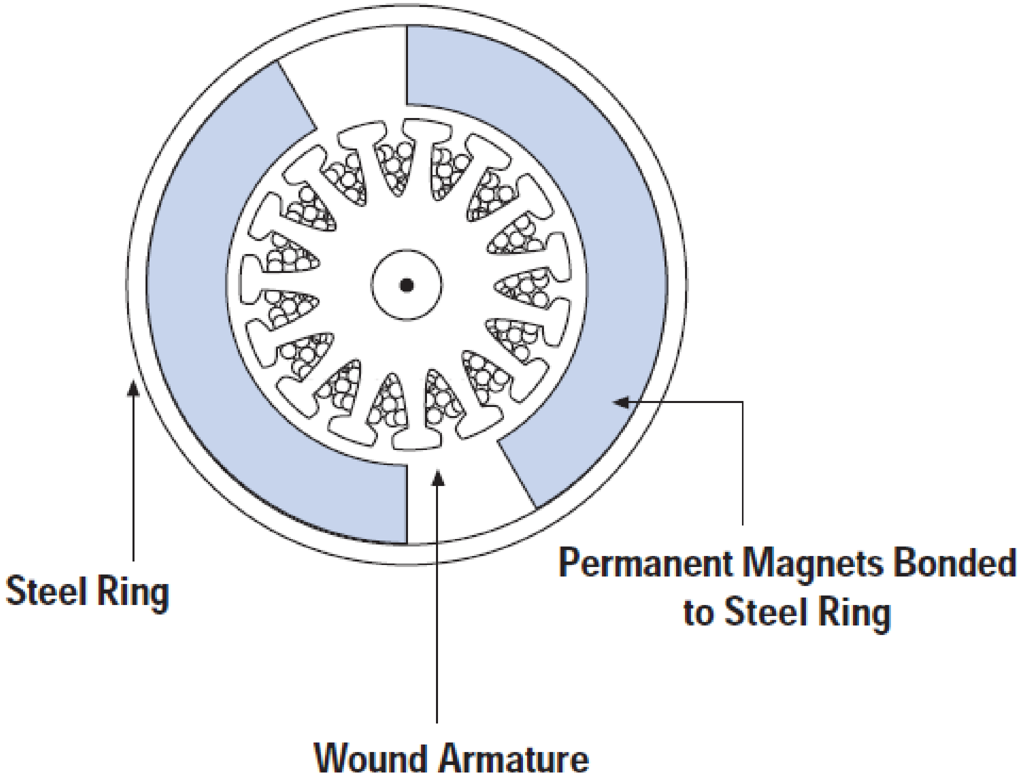
Elements of a brushed DC Motor used in practice



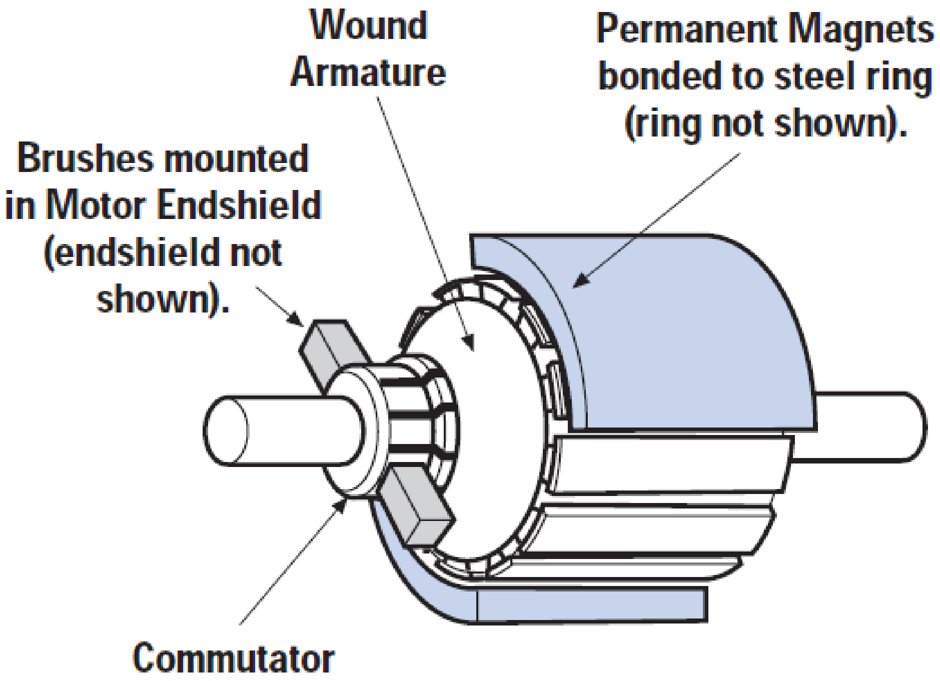

Encoder
Brush cover
Brush
Ironless winding
Housing (magnetic return)
Commutator
Magnet
Shaft
Motor flange
Ball bearing
Motor pinion
Gear mounting plate
Planet carrier plate
Planets
Internal gear
Ball bearing
Gearhead flange
Output shaft

Permanent magnet DC Motor





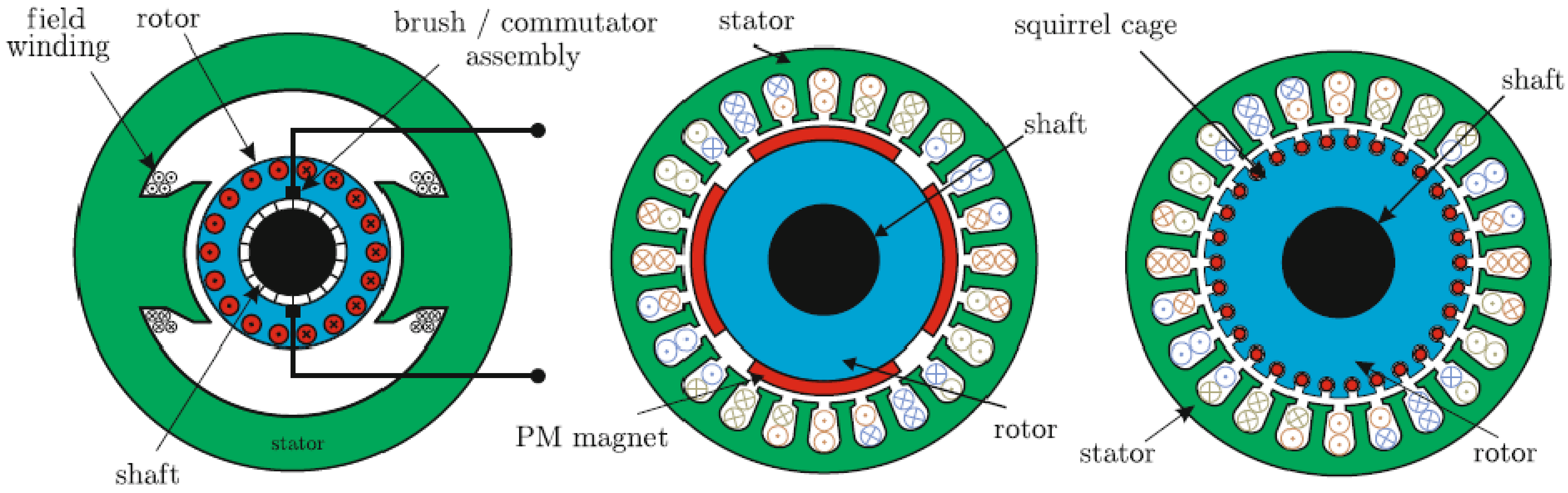
PMDC MOTOR

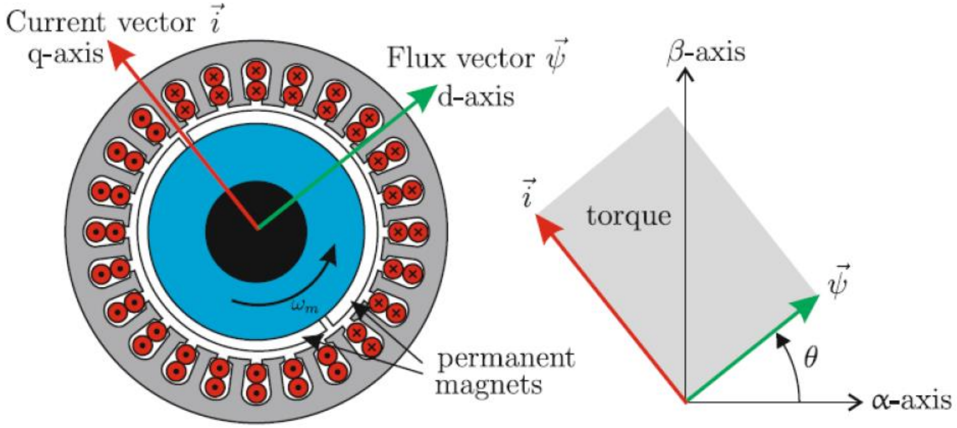


PMDC MOTOR

DC Brushed Servo Motors

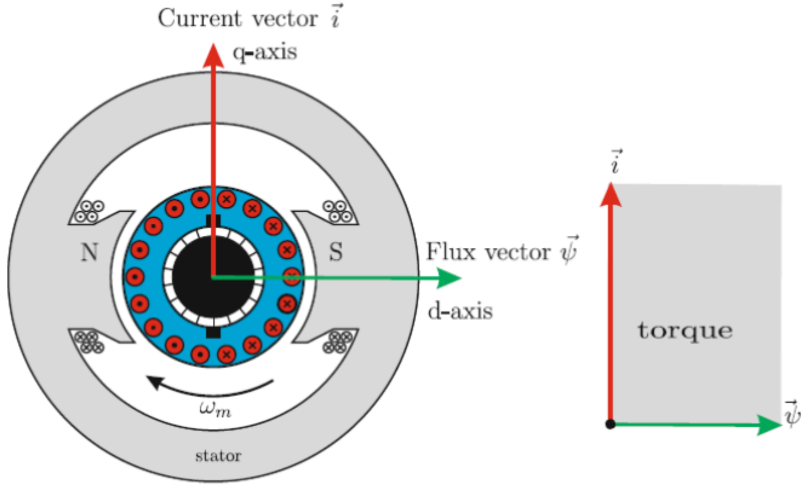
- **Sizes from 5 cm to 10 cm**
- **Continuous Torques from 0.3 to 6 Nm**
- **Not used often for new applications**
- **Stock and custom designs**


field
winding
rotor
brush / commutator
assembly
stator
shaft
stator
shaft
PM magnet
rotor
squirrel cage
shaft
stator
rotor



Torque production mechanism in a PM synchronous machine

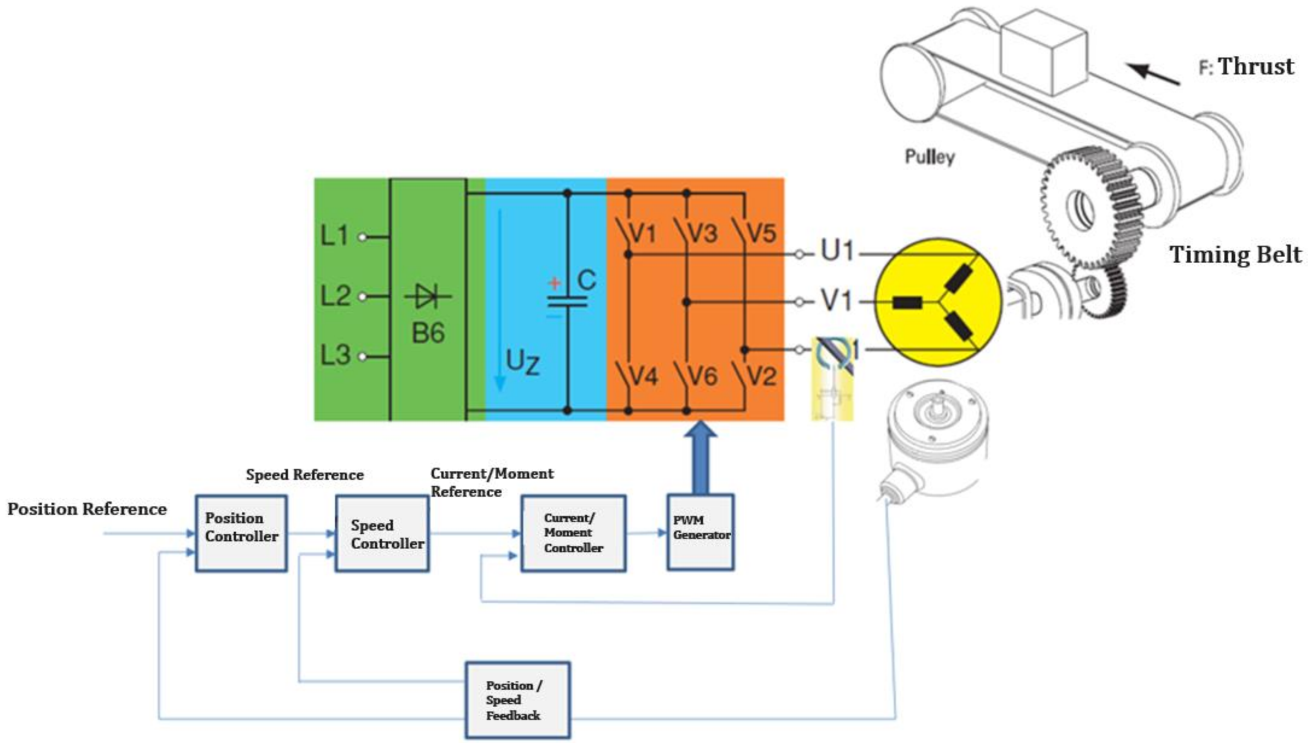
$$T_e = \frac{3}{2}\,\psi\, i$$



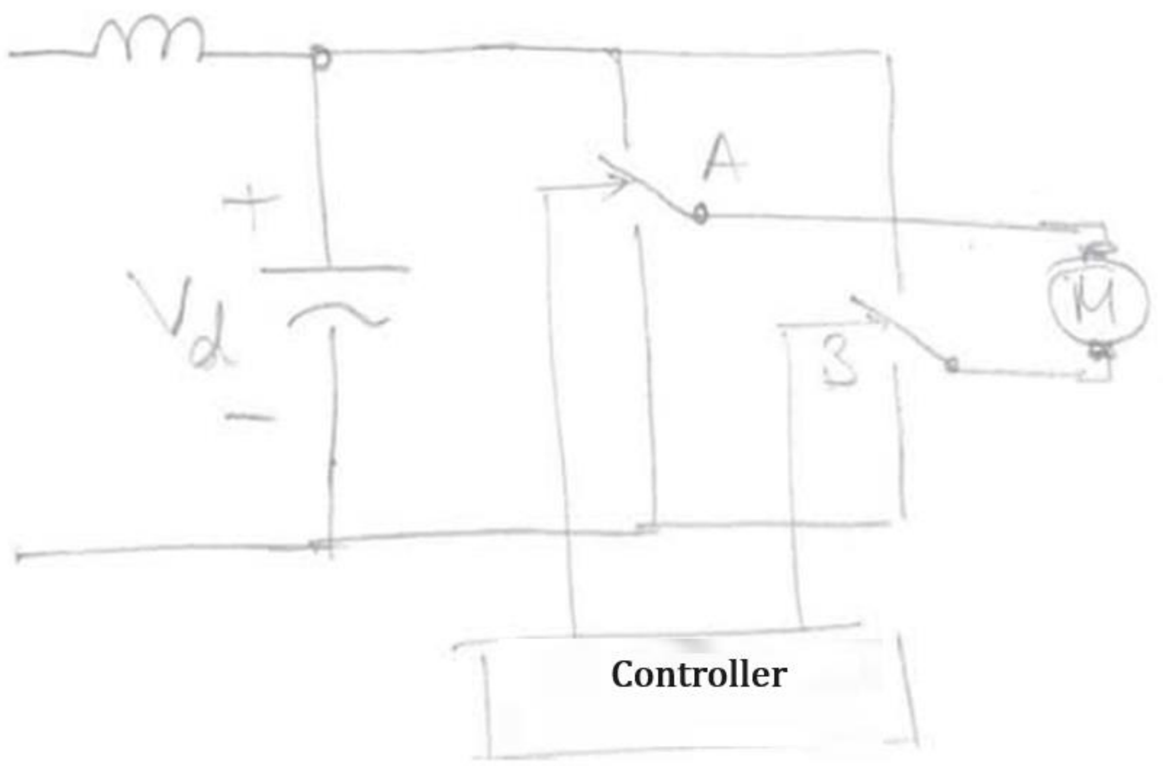
Torque production mechanism in a DC machine

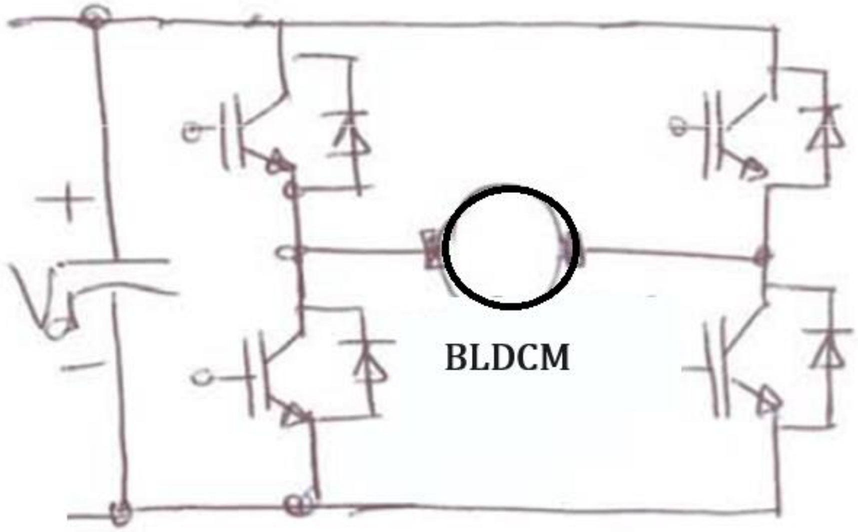
$$T_e = \frac{3}{2}\left(\vec{\psi} \times \vec{i}\right)$$

Servo System Control Scheme



Switched mode inverters for DC and AA motors





DC Motor Control System

1. **Mathematical Models for Different Mechanisms**:
    1. Each mechanical component in a servo system, like gears, shafts, and linkages, has its unique dynamics.
    2. Mathematical models of these components typically involve equations representing their inertia, friction, stiffness, and other mechanical properties.
2. **Servo Motor Mathematical Model**:
    1. Focusing first on DC motors, the mathematical model captures the relationship between input voltage, generated torque, motor speed, and electrical characteristics.
    2. The model includes equations for electromotive force, resistance, inductance, and torque constants.
3. **Servo Motor Driver Modeling**:
    1. Considering a controlled rectifier for a DC motor, which can operate in two quadrants, the model addresses the conversion of AC to DC power and how it controls the motor's speed and torque.
    2. This involves understanding the power electronics, switching mechanisms, and control strategies used in the driver.

Integration into a Closed-Loop Servo System

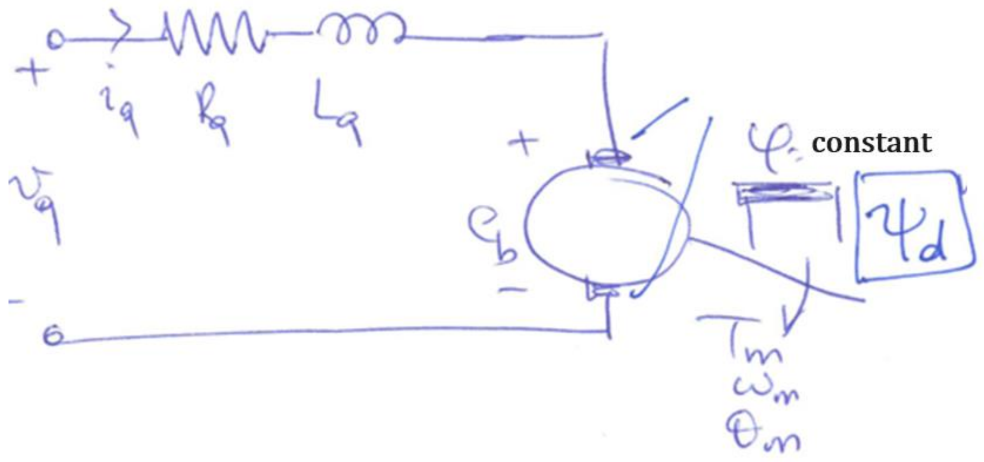
With the individual components modeled, we now look at integrating these into a closed-loop servo system. In a closed-loop system, feedback is used to adjust the control action, enhancing accuracy and stability.

The Cascaded (Sequential) Control System

The most commonly used control architecture in high-performance servo systems is the cascaded control system. This system layers different control loops, each with its own function:

1. **Inner Loop - Current (Torque) Control**:
    1. This loop responds to changes in load and commands from the speed control loop.
    2. It ensures that the motor produces the required torque to maintain the desired speed.
2. **Middle Loop - Speed Control**:
    1. This loop controls the motor's speed, responding to commands from the position control loop.
    2. It adjusts the current setpoint to meet the speed requirements.
3. **Outer Loop - Position Control**:
    1. The top layer in the cascade, this loop is responsible for the motor's position.
    2. It sends speed commands to the speed control loop to bring the motor to the desired position.

Modeling of Permanent Magnet Brushed DC Motor



$$V_q = R_q i_q + L_q \frac{di_q}{dt} + k_b \omega_m$$

$$k_t i_q = J \frac{d\omega_m}{dt} + B\omega_m + T_L$$

$$\frac{d\theta_m}{dt} = \omega_m$$

$$\frac{di_q}{dt} = -\frac{R_q}{L_q} i_q - \frac{k_b}{L_q}\omega_m + \frac{1}{L_q} V_q$$

$$\frac{d\omega_m}{dt} = -\frac{B}{J}\omega_m - \frac{1}{J} T_L$$

$$\frac{d\theta_m}{dt} = \omega_m$$

$$T_L = 0 \Rightarrow$$

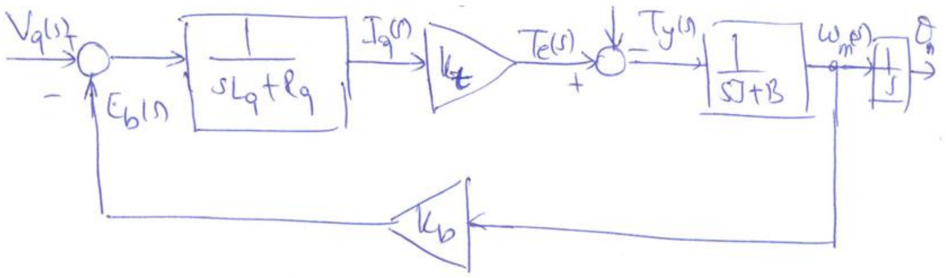
Input

$$V_q(s) = \left(R_q + sL_q\right) I_q(s) + k_b \omega_m(s)$$

Output

$$k_t I_q(s) = \left(Js + B\right)\omega_m(s)$$

$$s\theta_m(s) = \omega_m(s)$$



$$\frac{\theta_m(s)}{V_q(s)} = \frac{k_t}{\left[s^2 JL_q + s\left(JR_q + BL_q\right) + BR_q + k_t k_b\right]s}$$

$$v_q = R_q i_q + k_b \omega$$

$$k_t i_q = \underbrace{B \omega_m + T_{load}}_{T_y} = T_m$$

$$i_q = \frac{T_y}{k_t} = \frac{T_m}{k_t}$$

$$v_q = \frac{R_q}{k_t} T_m + k_b \omega$$

$$\omega = \frac{v_q}{k_b} - \frac{R_q}{k_t k_b} T_m$$

At Equilibrium (Steady regime)

$$\frac{di_q}{dt} = -\frac{R_q}{L_q} i_q - \frac{k_b}{L_q}\omega_m + \frac{1}{L_q} V_q \Rightarrow = 0$$

$$\frac{d\omega_m}{dt} = -\frac{B}{J}\omega_m - \frac{1}{J} T_L \Rightarrow = 0$$

$$\frac{d\theta_m}{dt} = \omega_m$$

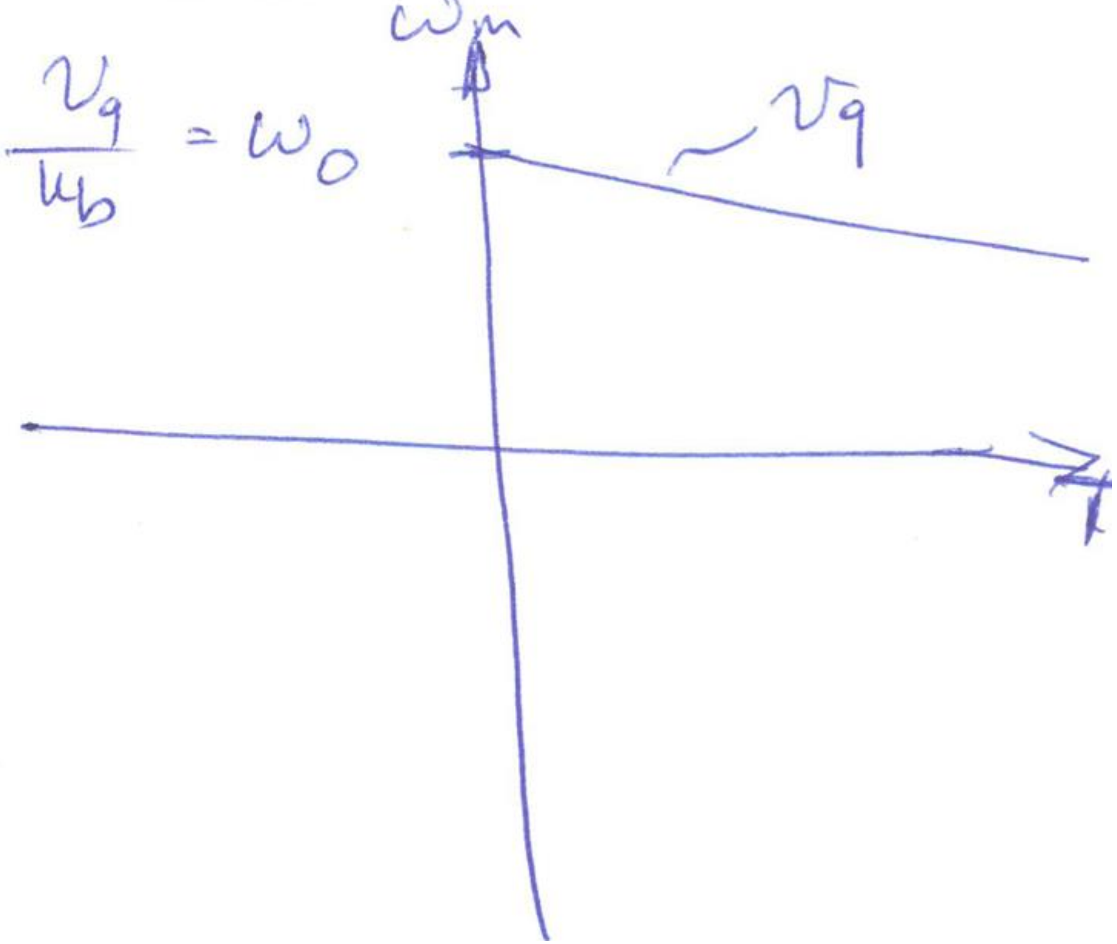
Idle

$$\omega = \omega_0$$

$$i_q = 0 \longrightarrow T_m \equiv 0$$

$$\omega_0 = \frac{v_q}{k_b}$$

$$\frac{v_q}{k_b} = \omega_0$$



$$\omega = \omega_0 - a . T_m$$

$$= \frac{v_q}{k_b} - a T_m$$

Problem 1:

$$V_{qn} = 40V$$

$$n_n = 2000rpm$$

$$R_q = 2.86\Omega$$

$$L_q = 10mH$$

$$k_t = 0.15Nm / A$$

$$k_b = 0.15Ns / rad$$

$$J_m = 5*10^{\wedge -4} kgm^2$$

$$B_m = 2*10^{\wedge -4} Nms / rad$$

a*) *Find the nominal current

$$V_{qn} = R_q I_{qn} + k_b \omega_n$$

$$I_{qn} = \frac{V_{qn} - k_b \omega_n}{R_q} \simeq 3A$$

b) ***Find the nominal speed in idle mode***

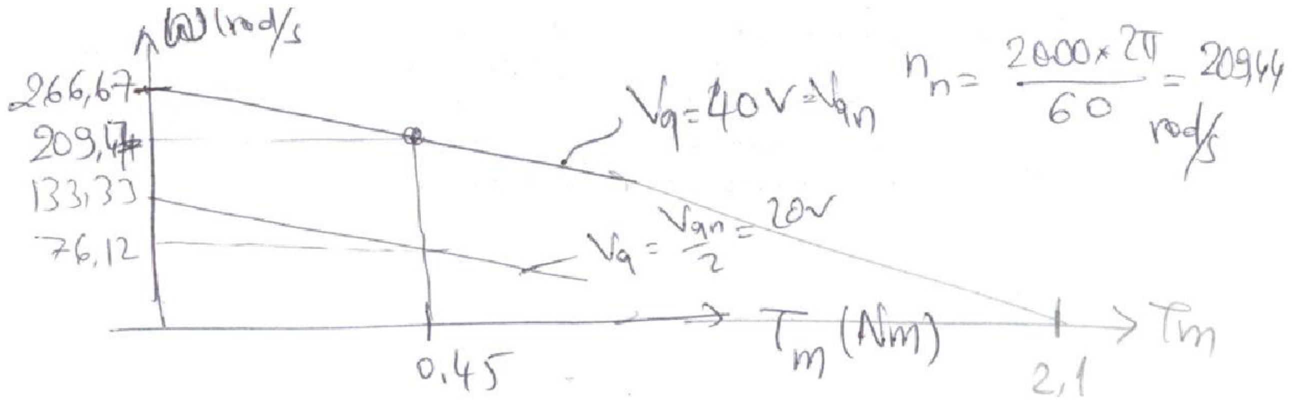
$$T_m = k_t\, i_q = T_y = 0$$

$$i_q = 0$$

$$V_{qn} = R_q \cdot 0 + K_b\, \omega_{on}$$

$$\frac{V_{qn}}{K_b} = \omega_{on} = 266{,}67 \text{ rad/s} \rightarrow n_{on} = \frac{60 * 266{,}67}{2\pi} = 2546 \text{ rpm}$$

Speed − Torque Characteristic



c) *Find the nominal torque*

$$T_{mn} = k_t \cdot i_{qn} = 0.15 \times 3 = \underline{\underline{0.45\,Nm}}$$

d) *Speed − Torque Relation*

$$V_{qn} = R_q i_q + k_b \omega$$

$$T_m = k_t i_q = T_y$$

$$i_q = \frac{T_m}{k_t} \longrightarrow V_{qn} = R_q \frac{T_m}{k_t} + k_b \omega$$

$$\boxed{\omega = \frac{V_{qn}}{k_b} - \frac{R_q}{k_t k_b} T_m} \Rightarrow \omega = \omega_{0n} - \frac{2{,}86}{0{,}225} T_m$$

($\frac{V_{qn}}{k_b} = \omega_{0n}$)

$$\omega = 266{,}67 - 127{,}11\, T_m$$

$T_{mn} = 0{,}45$ iuin $\quad \omega_n = 266{,}67 - 127{,}11 \cdot 0{,}45 \cong 209.6$

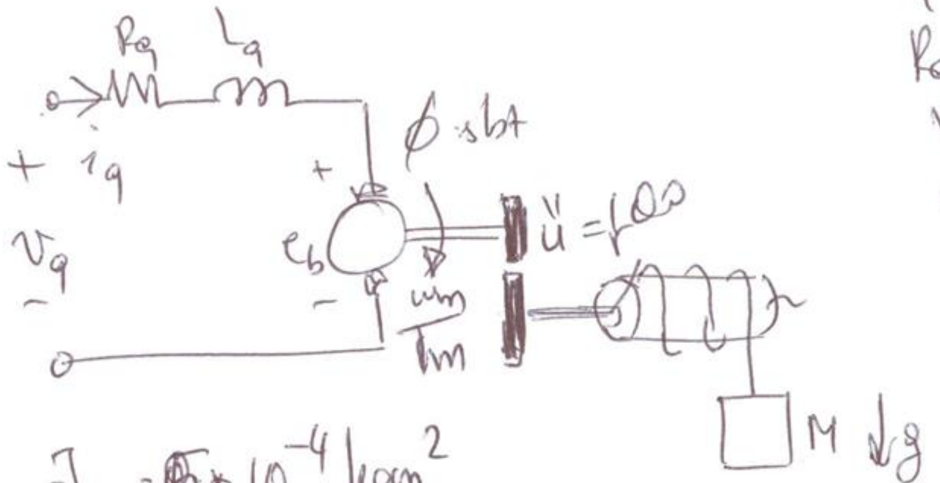
$V_q = \frac{V_{qn}}{2}$ yapılırsa aynı tork için hız değerini hesaplayınız. $V_q = 20V$

$$\omega = \frac{20}{0.15} - 127.11 \cdot 0.45 =$$

$$= 133,33 - 57,2 = 76,12 \text{ rad/s}$$

e) Nominal Efficiency

$$\eta_n = \frac{P_n}{U_{qn} \cdot i_{qn}} \hat{=} 0.785$$



$K_t = K_b = 0.15$

$R_a = 2.86\,\Omega$

$V_{an} = 40V$, $I_{an} = 3A$

$n_n = 2000$ rpm

$T_{mn} = 0.45$ Nm

$n = 100$ seçelim

$r = 0{,}3$ m , $M = 15$ kg

$J_m = 5 * 10^{-4}$ kgm^2

$B_m = 2 * 10^{-4}$ Nm s/rad

Yük tarafına aktarılan nominal motor torku $0.45 \times 100 = 45$ Nm

hızı : $\dfrac{209{,}44}{100} \cong 2.09$ rad/s

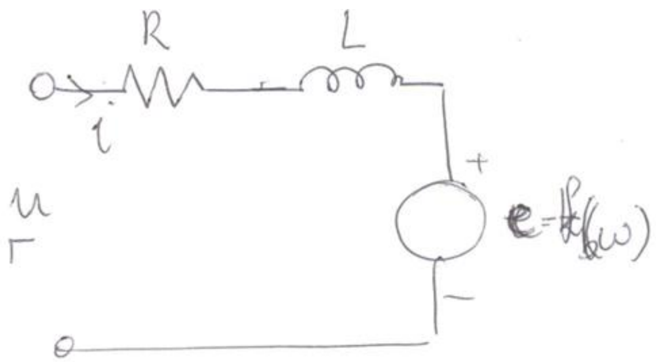
$\eta_n = 1$ ideal dişli

$$a = 0.1\ m/s^2$$

$$T_m = J_m \frac{d\omega_m}{dt} + B_m \omega_m + \frac{Mr^2}{\ddot{n}^2} \frac{d\omega_m}{dt} + \frac{Mgr}{\ddot{n}}$$

$$= \left(\cancel{J_m}^{0} + \frac{Mr^2}{\ddot{n}^2} \right) \frac{d\omega_m}{dt} + \cancel{B_m \omega_m}^{0} + \frac{Mgr}{\ddot{n}}$$

$$= \left(5 \times 10^{-4} + \frac{10 \times 0.3^2}{100^2} \right) \frac{d\omega_m}{dt} + \frac{10 \times 9,81 \times 0.3}{100}$$



$$T_m = f(i)$$

$$u - e = Ri + L\frac{di}{dt}$$

$$\frac{(U(s) - E(s))/R}{s\tau_{el} + 1} = I(s)$$

$$U(s) = \frac{U}{s}$$

$$E(s) = \frac{E}{s}$$

$$\tau_{el} << \tau_{meh}$$

$$i(t) = \frac{U - E}{R}\left(1 - e^{-t/\tau_{el}}\right)$$

$$T_m = f(i)$$

$$T_m - T_L = J\frac{d\omega}{dt} + B\omega$$

$0 \leq t < 1$ $\omega_m = 200t$ $i_q = 3 + 0.27t$

$t = 1$ $\omega_m = 200\ rad/s$ $i_q = 3.27\ A$

$1 \leq t \leq 3$ $\omega_m = 200$ $i_q = 3,2\ A$

$3 < t < 4$ $\omega_m = 200(4-t)$ $i_q = 2.97 + 0.27(4-t)$

$t = 3$ $\omega_m = 200$ $i_q = 3.2$

$t = 4$ $\omega_m = 0$ $i_q = 2.97$

$$T_m - T_L = J\frac{d\omega}{dt} + B\omega$$

$$\frac{\left(T_m(s) - T_L(s)\right)/B}{s\,\tau_{meh} + 1}$$

$T_L(s) = \frac{T_L}{s}$

$T_m(s) \cong \frac{T_m}{s}$

$$\omega(t) = \frac{(T_m - T_L)/B}{}\left(1 - e^{-t/\tau_{meh}}\right)$$

$\tau_{meh} \gg \tau_{el}$

$0 \le t < 1$

$$v_q = 2.86 \cdot [3 + 0.27t] + 10^{-2} \cdot 0.27 \cdot 2.86 + 0.15 * 200t$$

$t = 0$ $\quad v_q = 8.58\ V$

$t = 1$ $\quad v_q = 39.36\ V$

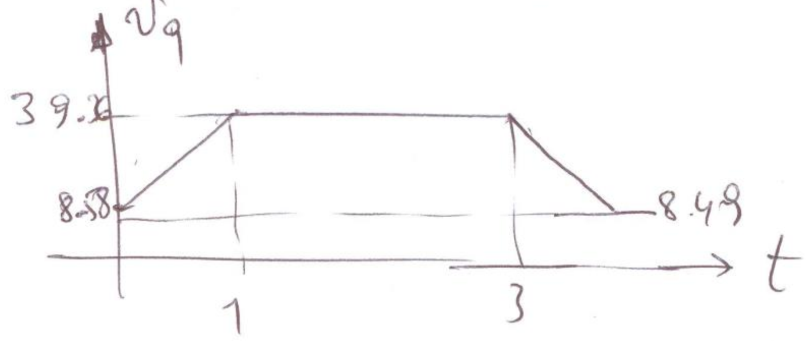
$1 < t \le 3$ $\quad v_q = \underline{\underline{39.36\ V}}$

$3 < t \le 4$ $\quad v_q = 2.86\,[2.97 + 0.27(4-t)] + 10^{-2} \cdot 0.27 \cdot 2.86$

$$+ 0.15 * 200\,(4-t)$$

$t = 4s$

$v_q = 8.49\ V$



Since the excitation is constant, the speed of the motor is changed by the voltage applied to the motor.

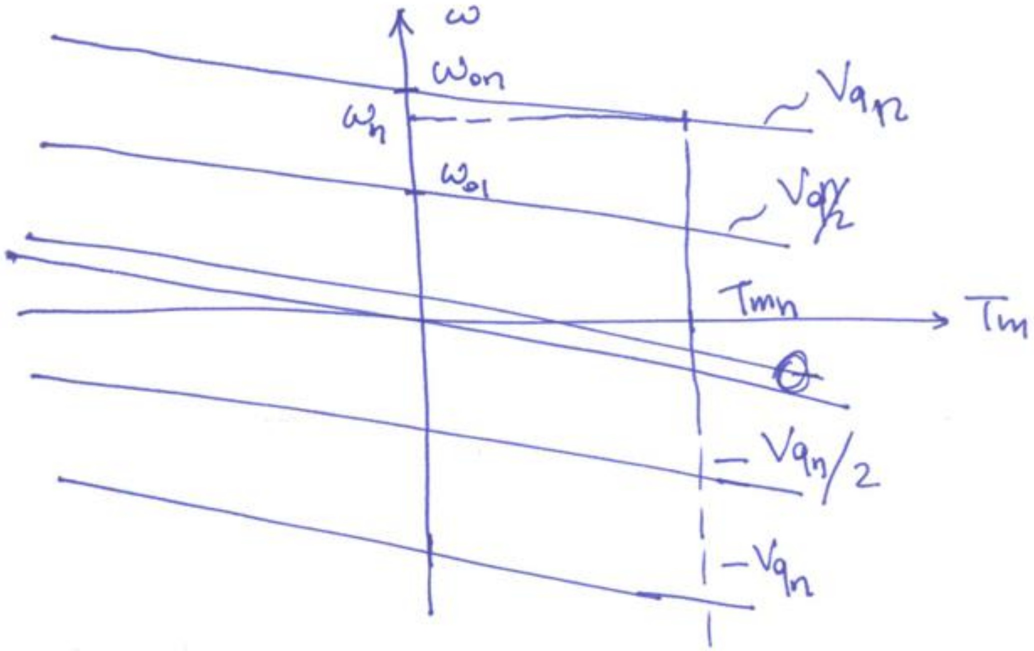
$$\omega = \frac{V_q - \frac{R_q}{K_t} T_m}{K_b} = \frac{V_q}{K_b} - \frac{R_q}{K_t K_b} T_m$$

ω_{on} : initial nominal speed

$$\omega_{on} = \frac{V_{qn}}{K_b}$$

$$\omega_n = \omega_{on} - \frac{R_q}{K_t K_b} T_{mn}$$

$$\tau_{elek} = \frac{L}{R} \sim 0 \qquad \tau_{mek} = \frac{J}{B} > 0$$



The speed of the DC motor is changed by the direct voltage applied to the machine.

$$v_q = R_q\, i_q + L_q \frac{di_q}{dt} + K_b\, \omega_m$$

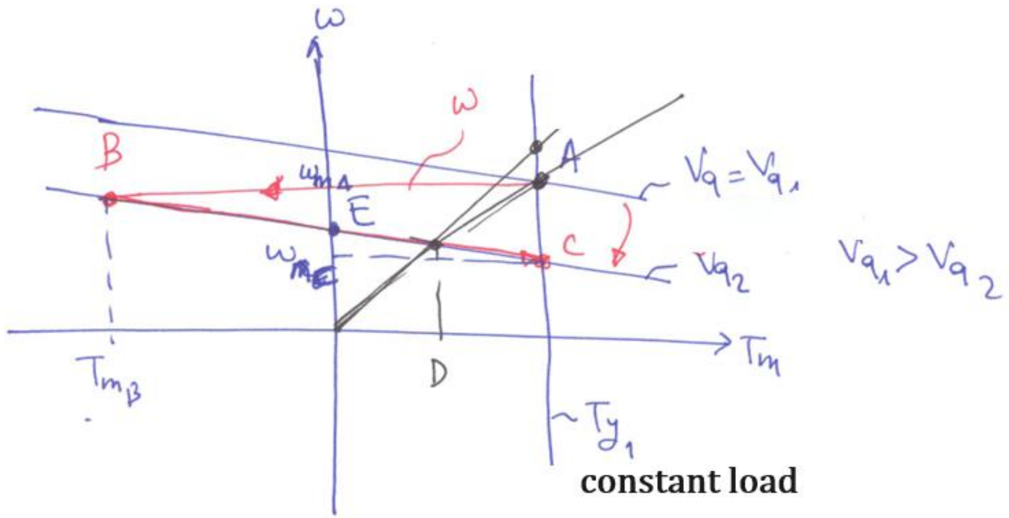
$$K_t\, i_q = J \frac{d\omega_m}{dt} + \underbrace{B\, \omega_m + T_{fr} + T_{gr} + T_y}_{T_{load}}$$

In steady state: (in case the electrical time constants are small compared to the mechanical time constants and there is no fluctuation in the current)

$$v_q = R_q\, i_q + K_b\, \omega_m$$

$$K_t\, i_q = T_{yük} = T_m \rightarrow i_q = \frac{T_m}{K_t}$$

$$\omega_m = \frac{1}{K_b} v_q - \frac{R_q}{K_t\, K_b} T_m$$



If we switch to Vq1→Vq2 while working at point A, Ty=Ty1=constant in this case; The aim is to reduce the speed of the motor to a speed value smaller than w1. When the voltage changes to Vq1→Vq2 while operating at point A, the speed of the motor cannot change quickly, but the torque goes to point B in a stepwise manner. The generator slowly moves to the operating region from point B to point C on the curve Vq2.

A

$$V_q = V_{q_A} = R_q i_{q_A} + k_b \omega_{m_A}$$

$$\omega_{m_A} = \frac{V_{q_A}}{k_b} - \frac{R_q}{k_t k_t} T_{m_A}$$

B

$$V_q = V_{q_C} \qquad V_{q_C} < V_{q_1}$$

$$i_q = \frac{V_q - k_b \omega_{m_A}}{R_q}$$

$$V_{q_C} < k_b \omega_{m_A}$$

$i_q < 0$ $\quad T_m = k_t \cdot i_q < 0$

$T_m < 0 \quad \omega_m > 0$

$$\frac{-T_m - T_y}{J} = \frac{d\omega_m}{dt}$$

$$\int_{\omega_{mA}}^{\omega_m} d\omega_m = -\frac{1}{J}\int_0^t (T_m + T_y)\,dt$$

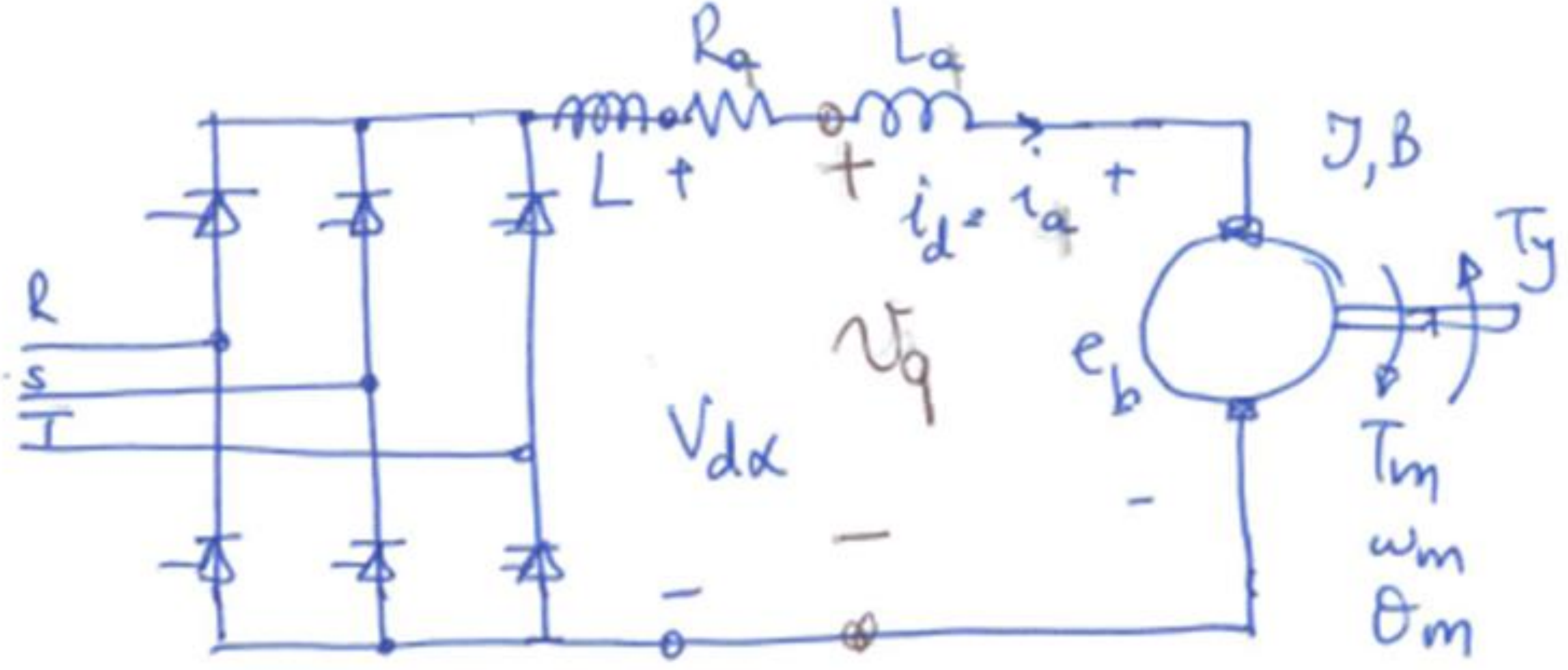
$$\omega_m = \omega_{mA} - \frac{1}{J}\int_0^t (T_m + T_y)\,dt$$

$V_{q_2} = k_b \omega_{mE}$ $\quad T_{mE} = 0$

$V_{q_2} > k_m \omega_m \quad 0 < \omega_m < \omega_{mE}$

$i_q > 0 \rightarrow T_m > 0$ $\quad \omega_m > 0$

$\omega_m = \omega_{mC}$ de. $\quad T_{mC} = T_{yC} = T_{y_1}$



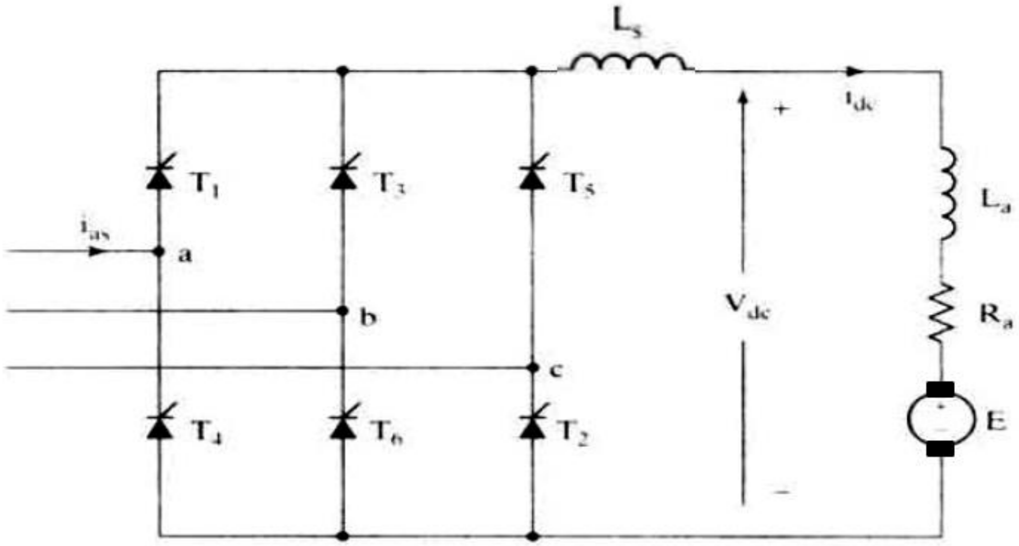
Ra
La
L
+
+
+
J, B
R
S
T
Ty
Vdα
eb
Tm
ωm
θm
-
-
-
-

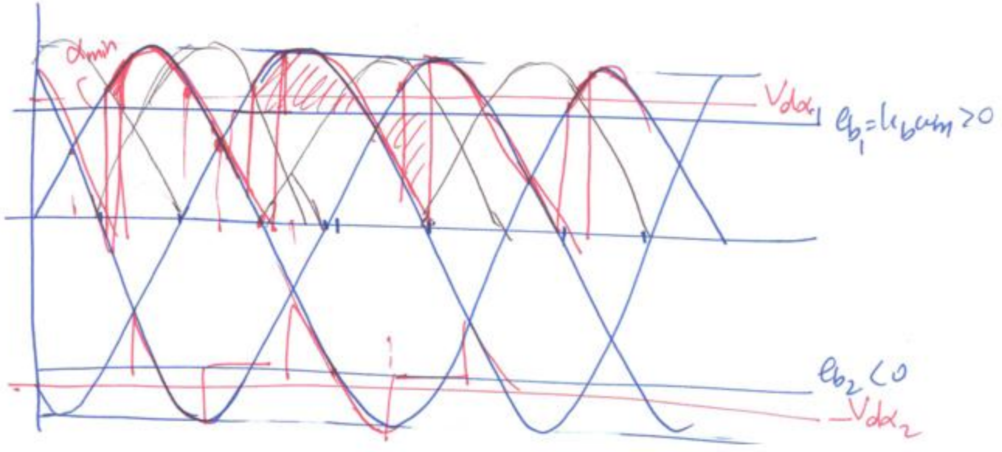
Controlled rectifier in driving DC Motor

With a 3-phase full-wave controlled rectifier, the average value of the voltage applied to the motor will be changed. Here Ls is used to reduce the ripple in the current. The average value of the voltage applied to the motor as a function of the trigger angle applied to the thyristors:

Here p is the number of peaks in a period, V_{LL} is the effective value of the inter-phase voltage, α is the trigger angle.

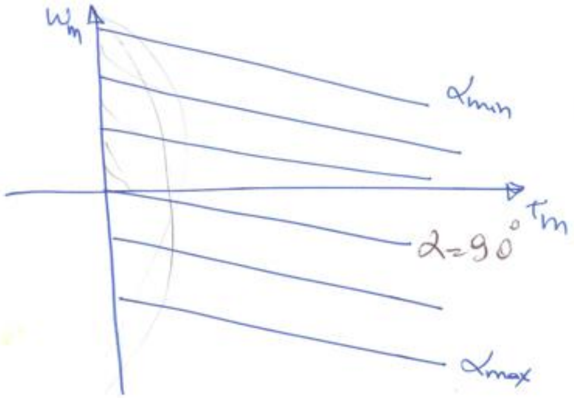
$$V_{dc} = V_{d\alpha} = \frac{p}{\pi}\sqrt{2}V_{LL}\sin\frac{\pi}{p}\cos\alpha = 1.35V_{LL}\cos\alpha = V_{d0}\cos\alpha$$





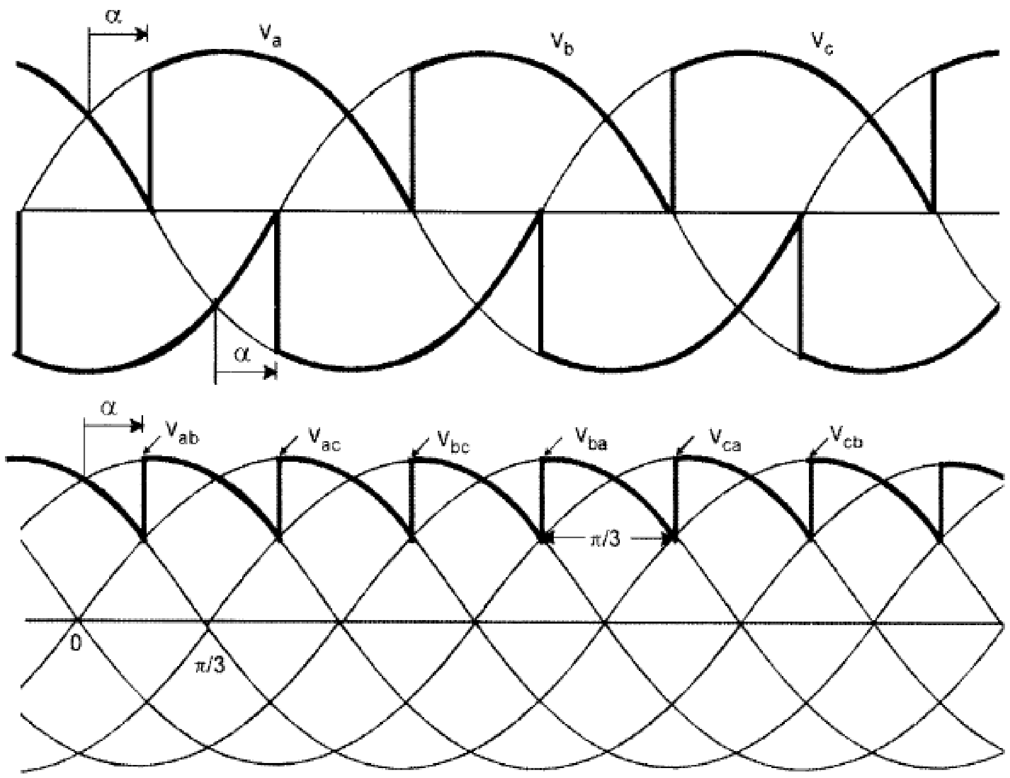
$$\omega_m = \frac{V_{d\alpha}}{K_b} - \frac{R_a I_d}{K_b}$$

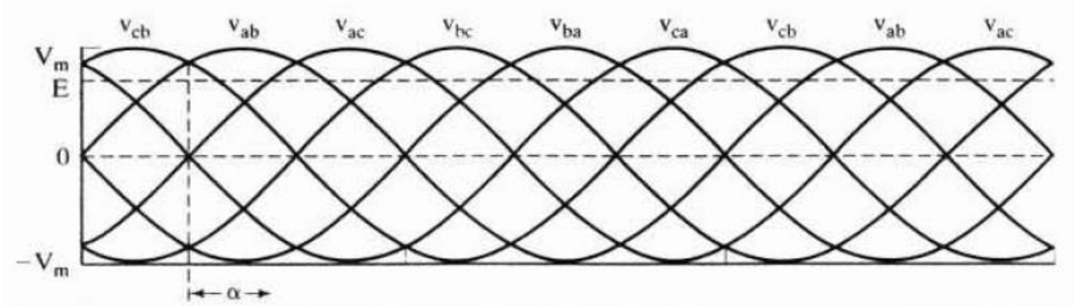
$$= \frac{V_{d\alpha}}{K_b} - \frac{R_a}{K_t K_b} T_m$$



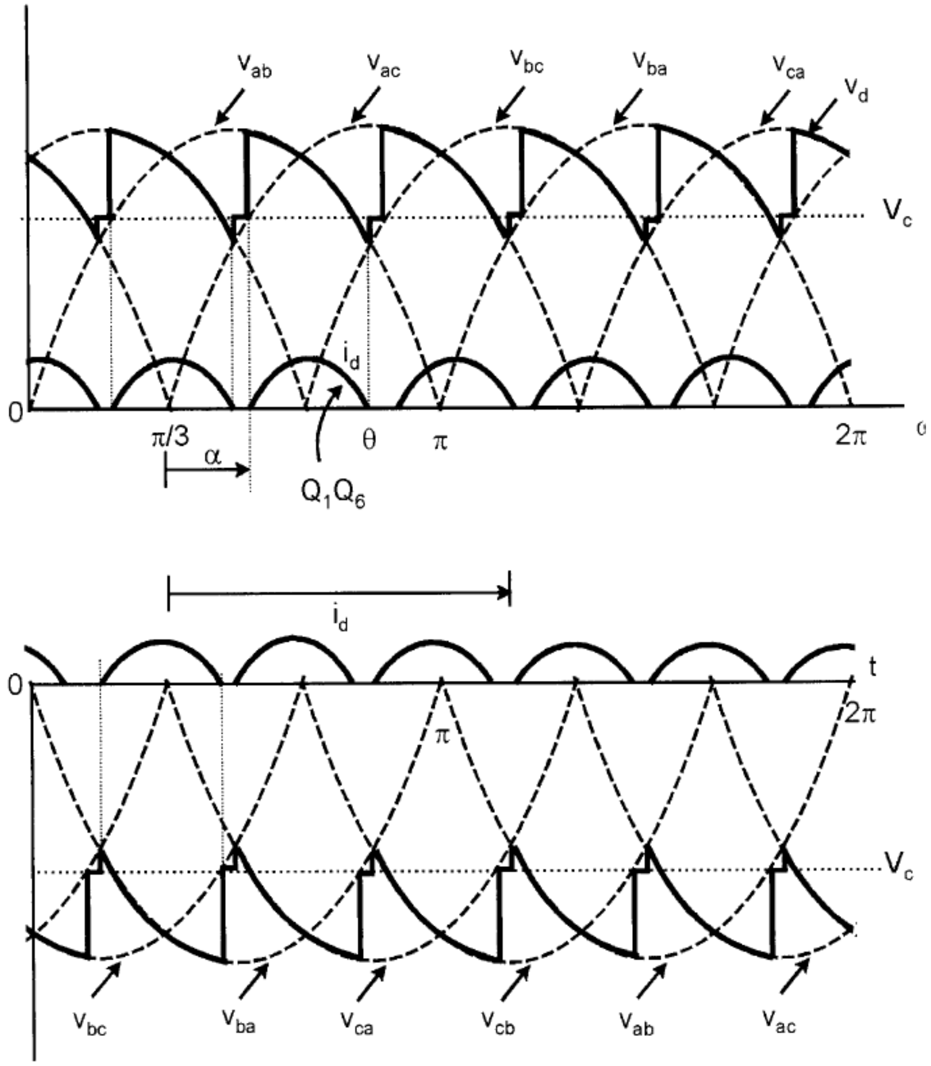
$$\omega_m = \underbrace{\frac{V_{do}}{K_b}}_{\text{sabit } k_1} \cos\alpha - \underbrace{\frac{R_a}{K_t K_b}}_{k_2} T_m$$

$$= k_1 \cos\alpha - k_2 T_m$$

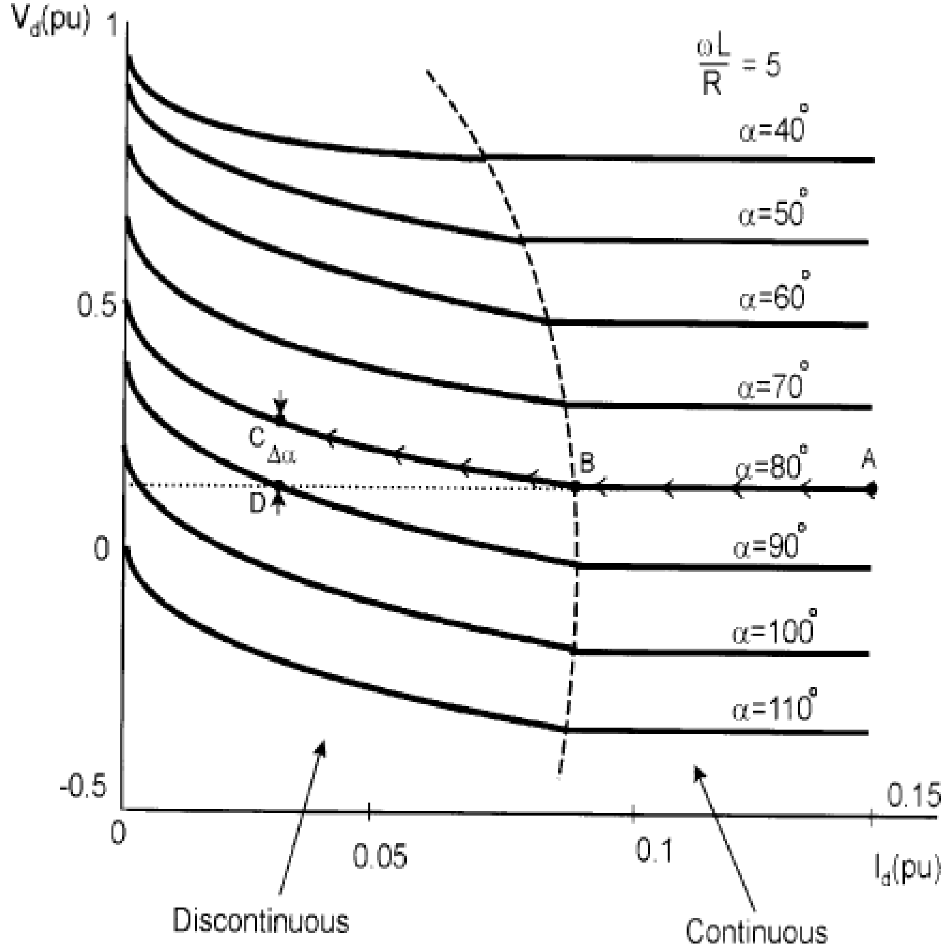

α
v_a
v_b
v_c
α
α
v_{ab}
v_{ac}
v_{bc}
v_{ba}
v_{ca}
v_{cb}
π/3
0
π/3

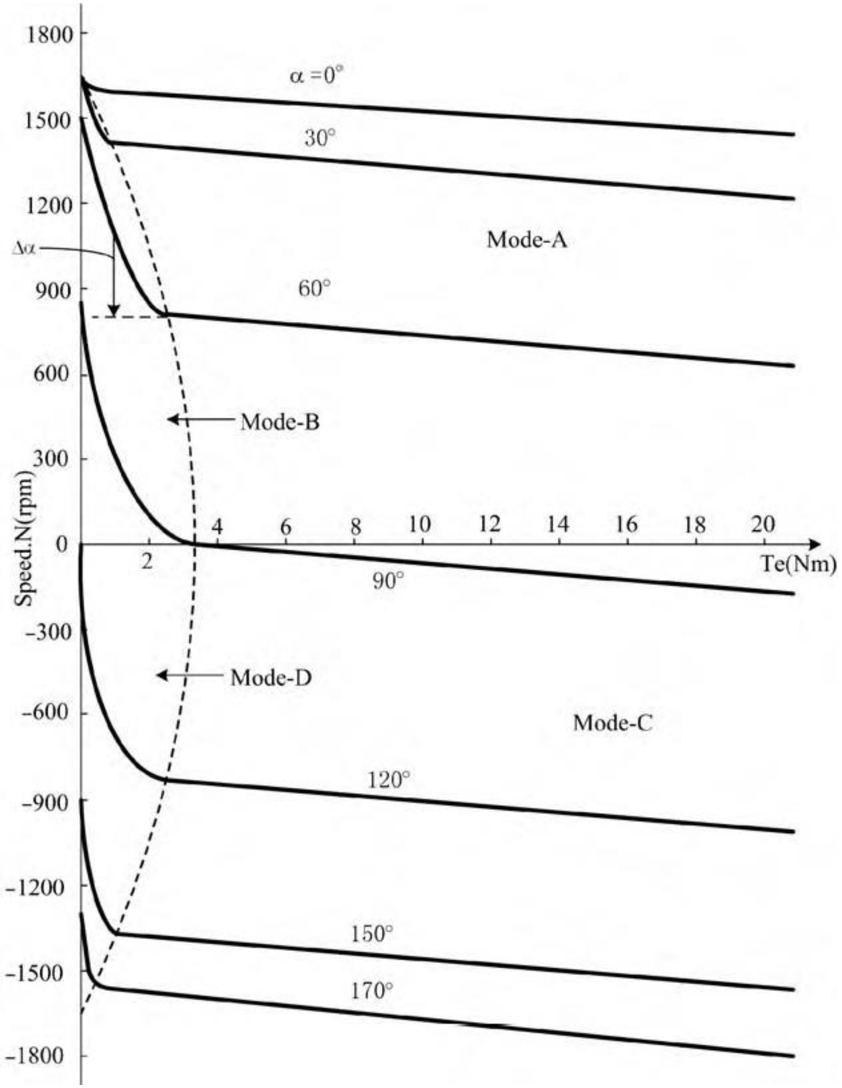

v_{cb}
v_{ab}
v_{ac}
v_{bc}
v_{ba}
v_{ca}
v_{cb}
v_{ab}
v_{ac}
V_m
E
0
$-V_m$
α

Motor 1 and generator operation in zone 4, or operation in rectifying and inverting mode when viewed from the full-wave bridge side

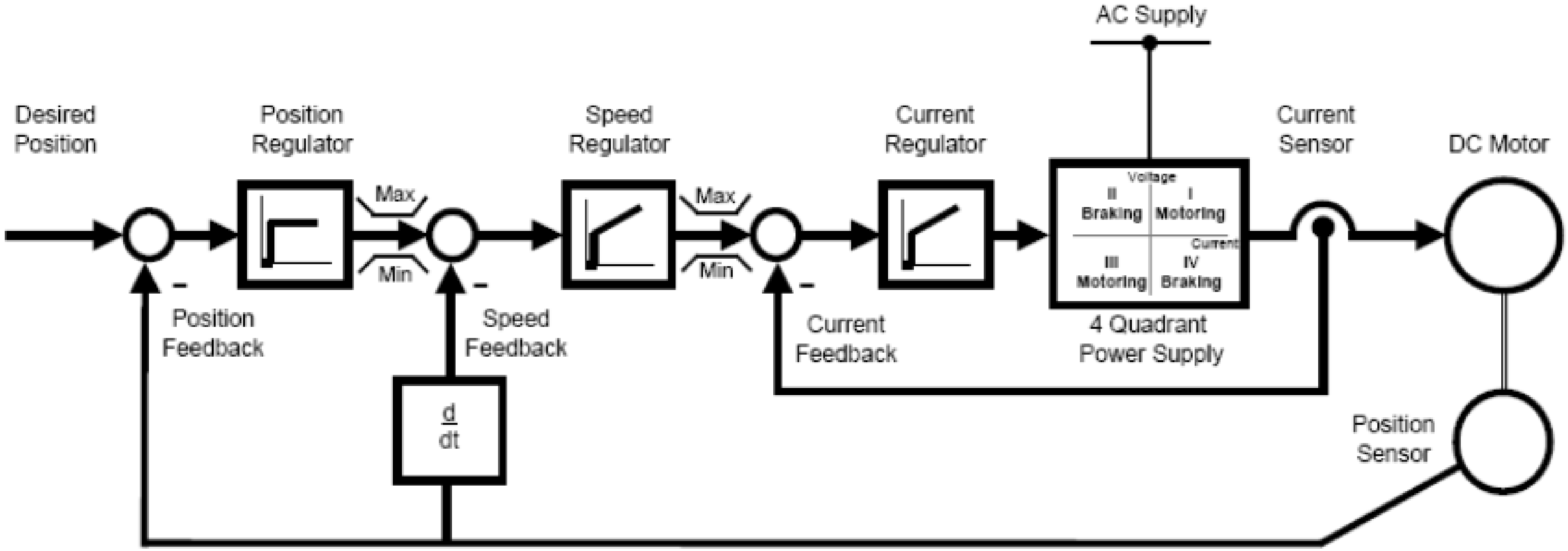


Rectifier average voltage-average current characteristics showing detailed discontinuous and continuous regions in the current





Cascade Control Structure



DC servo systems are vital in applications requiring precise control of motion, such as robotics, CNC machines, and automated manufacturing systems. A key feature of these systems is their ability to accurately control position, speed, and torque.

Cascaded Control Structure

The cascaded control structure is universally acknowledged as the most effective approach for high-performance servo systems. This structure comprises three layers of regulation:

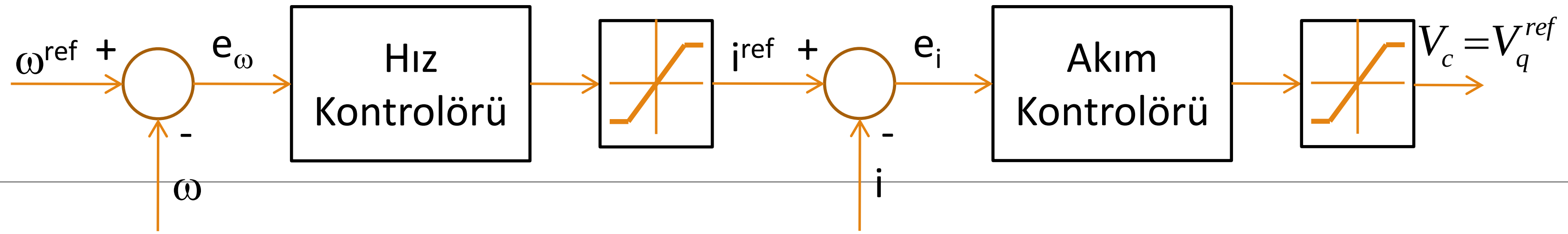
1. **Innermost Layer - Current (or Torque) Regulator**
    1. This layer directly controls the motor's torque, which is essential for immediate responses to command changes.
    2. Torque is proportional to the current in DC motors, hence the term current regulator is often used.
2. **Middle Layer - Speed Regulator**
    1. This wraps around the current regulator.
    2. Controls the motor's speed and acts as an intermediary between the torque and position control layers.
3. **Outermost Layer - Position Regulator**
    1. This layer ensures the motor reaches and maintains the desired position.
    2. The position is the integral of speed, making this the highest layer in the control hierarchy.

The Principle of Operation

- **Four Quadrant Power Supply**: A critical component in DC servo systems. It allows the motor to operate under all combinations of torque (current) and speed (voltage), enabling both forward and reverse motion, as well as motoring and braking in both directions.
- **Bandwidth Considerations**:
  - Bandwidth in control systems refers to the range of frequencies over which the system can accurately track command signals.
  - In the cascaded structure, bandwidth hierarchy is crucial:
    - The current regulator must have the highest bandwidth for rapid response.
    - The speed regulator follows with a lower bandwidth.
    - The position regulator has the lowest bandwidth, focusing on overall system stability and accuracy.

System Tuning and Benefits

- **Tuning Process**:
  - Tuning starts from the innermost current regulator, progressing outward.
  - This ensures that each layer responds correctly to the commands from the outer layers.
- **Variable Limiting**:
  - Each variable (current, speed, position) can be individually limited, providing fine control over the system's behavior.
- **Benefits of Cascade Control**:
  - Improved precision in controlling torque, speed, and position.
  - Enhanced system stability.
  - Ability to handle disturbances and changes in load conditions effectively.



The current controller and speed controller are arranged in a cascade structure, as seen.
The output of the current controller is the average value of the voltage that will be applied to the DC motor.
This value indicates the change in the average voltage that needs to be applied to the motor, both in transient and steady-state conditions.
The voltage applied to the motor is expected to be the same as Vdc, with a scale difference.

$$V_c = V_q^{ref}$$

$$V_{dc} = V_{d0} \cos\alpha$$

$$V_q^{ref} = V_c = V_{c\max} \cos\alpha$$

$$\cos\alpha = \frac{V_{dc}}{V_{d0}} = \frac{V_c}{V_{c\max}}$$

$$\alpha = \cos^{-1}\left(\frac{V_c}{V_{c\max}}\right)$$

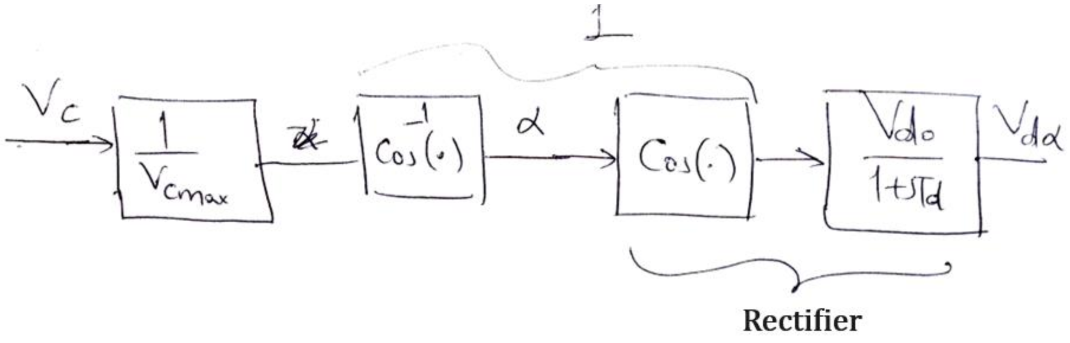
$$v_c = V_{cmax} \cdot \cos\alpha$$

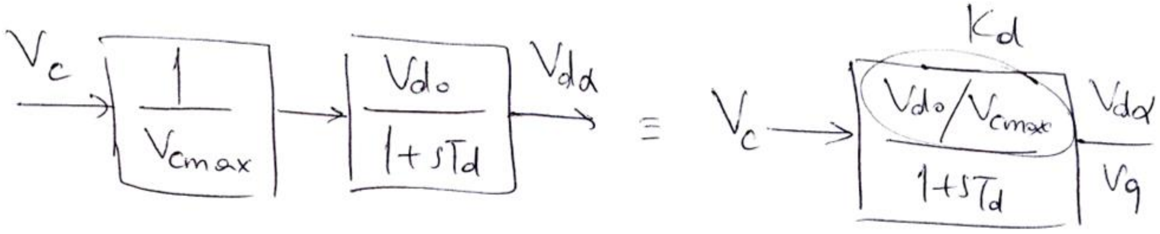
$$v_q = V_{do} \cos\alpha$$

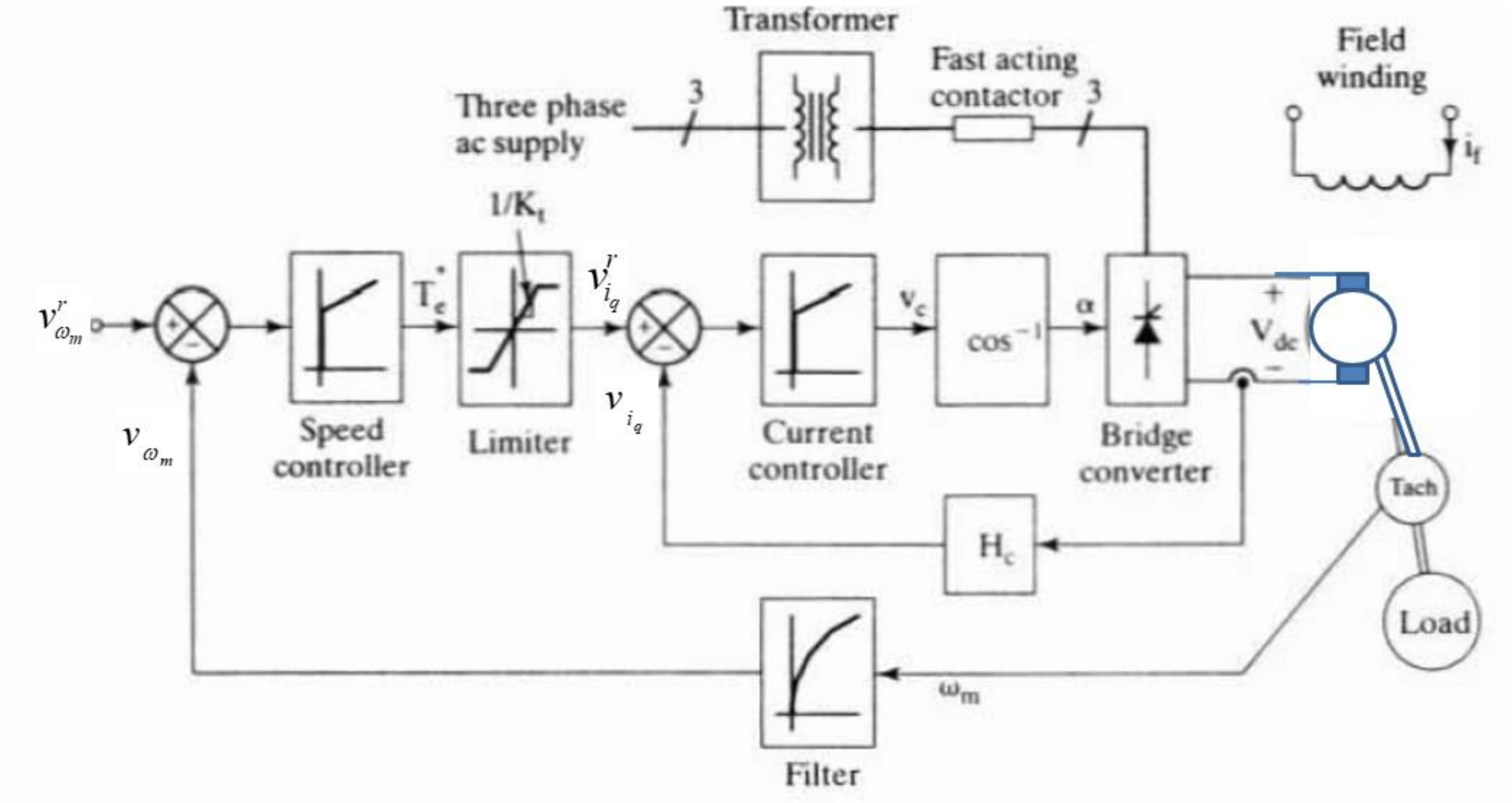
$$\frac{v_q}{v_c} = \frac{V_{do}}{V_{cmax}} = K_d$$

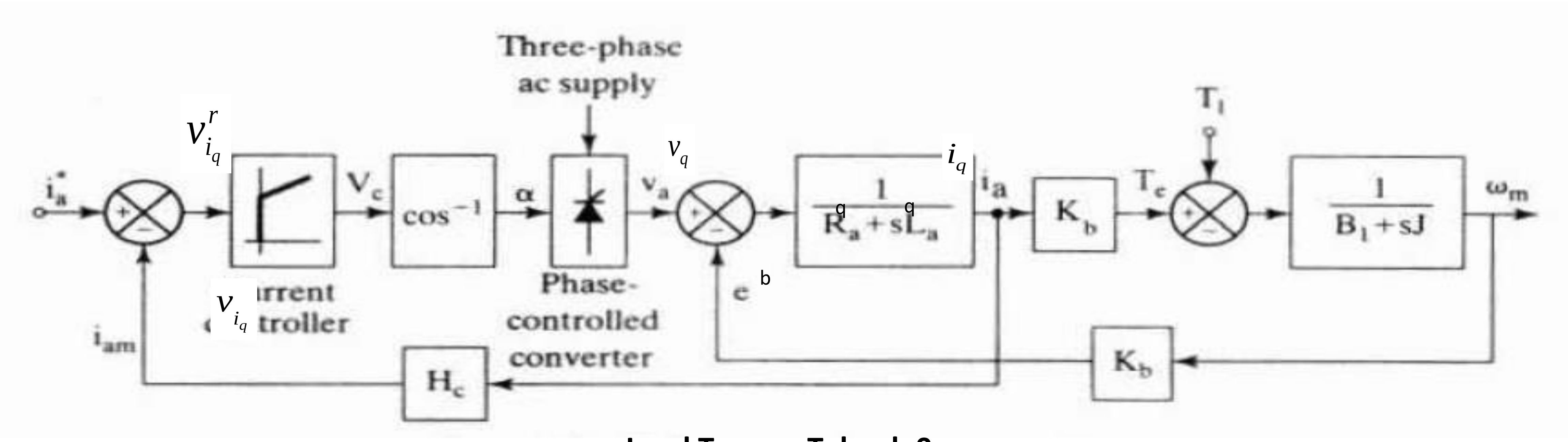
$$v_q = v_c \cdot K_d$$

$$V_{do} = K_d V_{cmax}$$

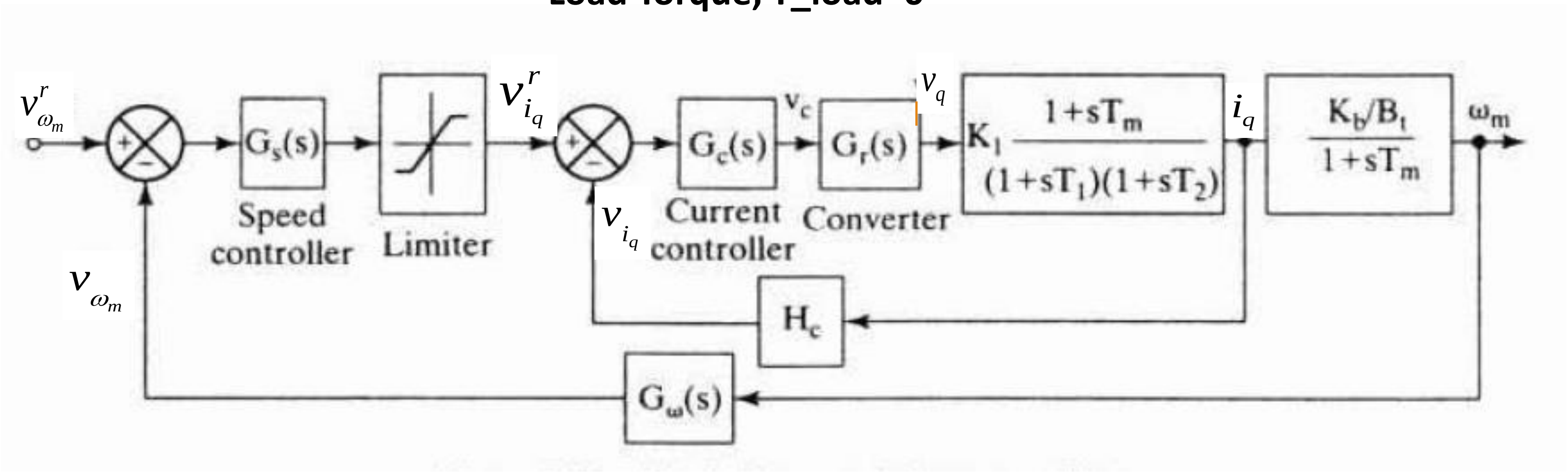





Transformer
Three phase
ac supply
3
Fast acting
contactor
3
Field
winding
i_f
$1/K_t$
$v^r_{\omega_m}$
T_e^*
$v^r_{i_q}$
v_c
$\cos^{-1}$
α
+
V_{dc}
−
v_{i_q}
v_{ω_m}
Speed
controller
Limiter
Current
controller
Bridge
converter
Tach
H_c
Load
ω_m
Filter



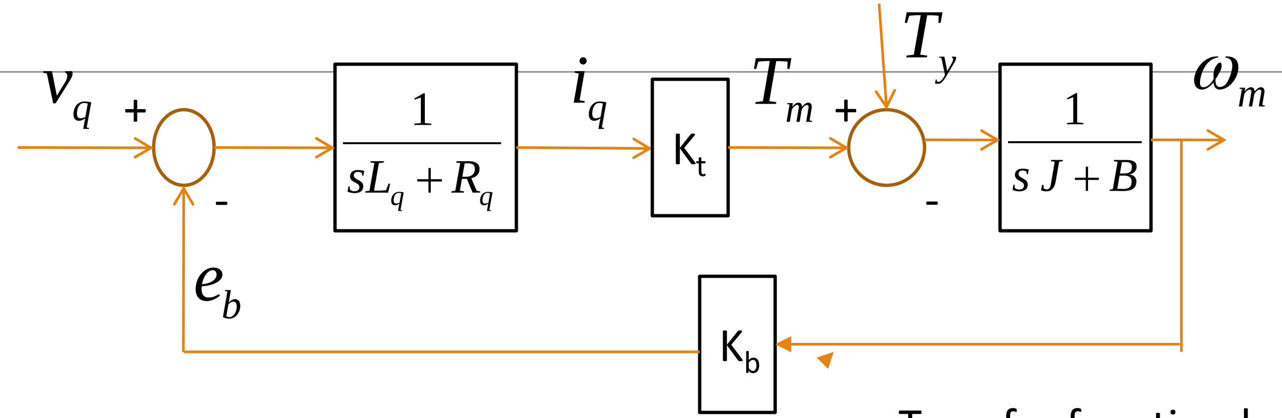
Load Torque, T_load=0



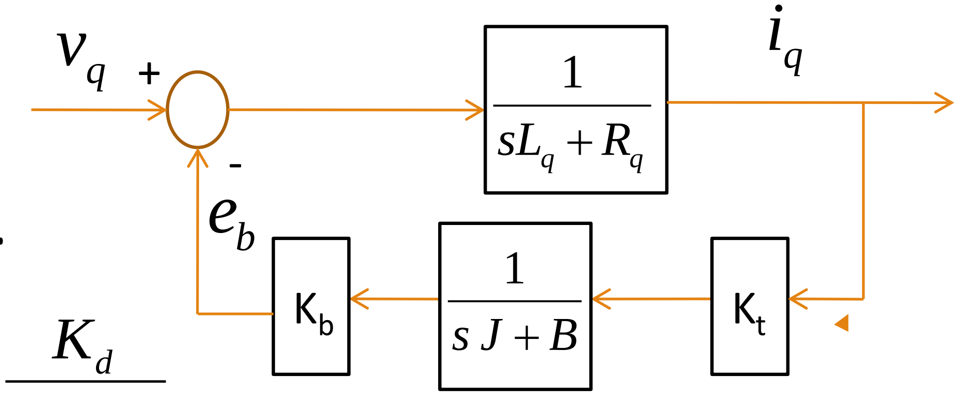
The current loop is the inner loop in cascade control design.

Controller design will begin with this cycle, which has much smaller time constants than the speed cycle.

For this purpose, first the transfer function between current and voltage will be obtained:



Transfer function between iq and vq, taking Ty = 0



$$\frac{I_q(s)}{V_q(s)} = \frac{(sJ + B)}{(sJ + B)(sL_q + R_q) + K_t K_b}$$

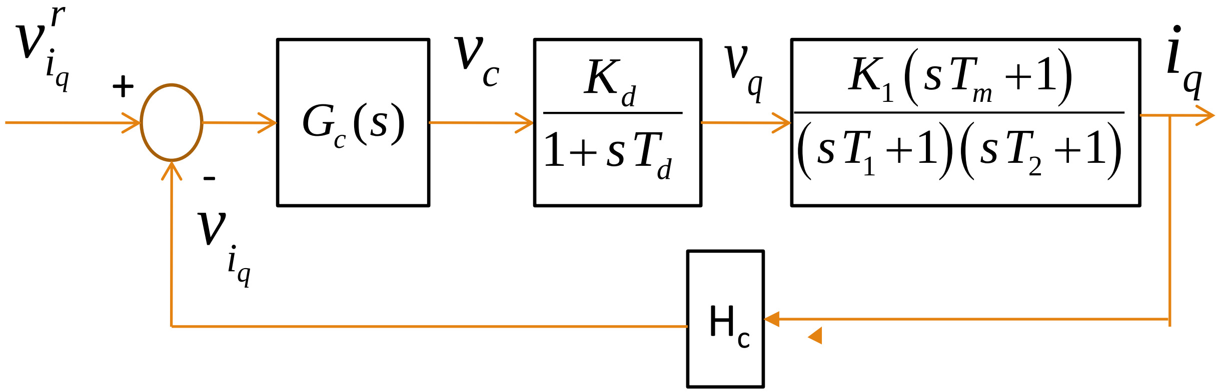
Rectifier's T.F.

$$G_{DC}(s) = \frac{V_q(s)}{V_c(s)} = \frac{K_d}{1 + sT_d}$$

$$\frac{I_q(s)}{V_q(s)} = \frac{(sJ+B)}{(sJ+B)(sL_q+R_q)+K_t K_b} = \frac{K_1(sT_m+1)}{(sT_1+1)(sT_2+1)}$$

K_t=K_b

$$G_{DC}(s) = \frac{V_q(s)}{V_c(s)} = \frac{K_d}{1+sT_d}$$



$$T_m = \frac{J}{B}$$

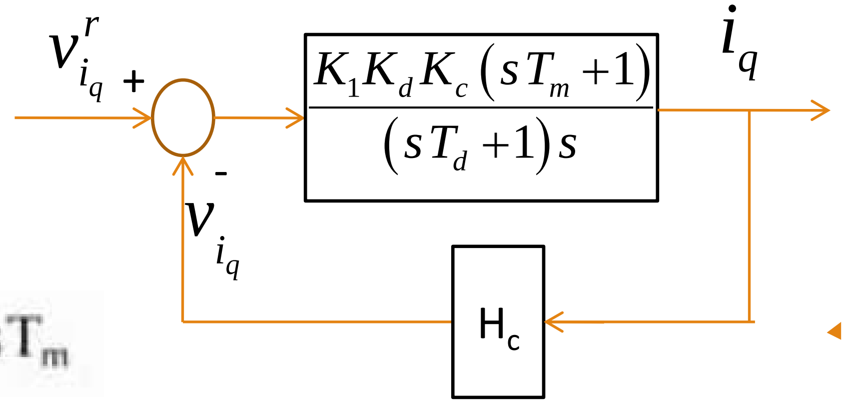
PID Controller Structure: $G_c(s) = \frac{K_c(sT_{c1}+1)(sT_{c2}+1)}{s}$

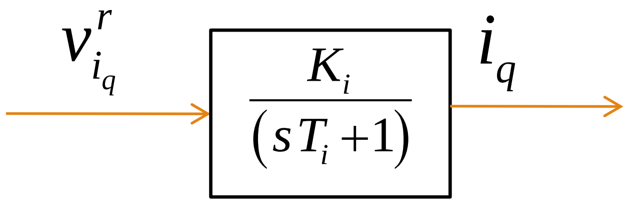
$$-\frac{1}{T_1}, -\frac{1}{T_2} = -\frac{1}{2}\left[\frac{B}{J}+\frac{R_q}{L_q}\right] \pm \sqrt{\frac{1}{4}\left(\frac{B}{J}+\frac{R_q}{L_q}\right)^2 - \left(\frac{K_b^2+R_qB}{JL_q}\right)}$$

$$K_1 = \frac{B}{K_b^2 + R_qB}$$

Pole-Zero Cancellation

$$T_{c1} = T_1;\ T_{c2} = T_2$$

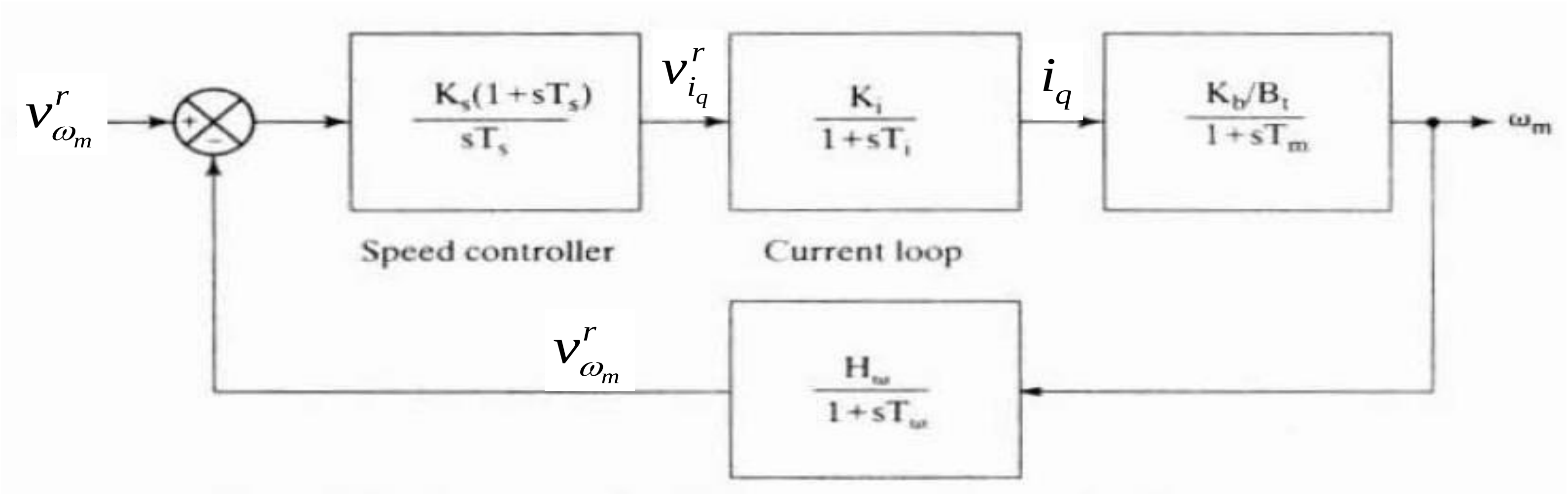




$$(1 + sT_m) \cong sT_m$$

$$K_i = \frac{K_1\,K_d\,K_c\,T_m}{1+K_1\,K_d\,K_c\,T_m\,H_c}; \qquad T_i = \frac{T_d}{1+K_1\,K_d\,K_c\,T_m\,H_c}$$

Speed Controller Loop:



System Transfer Function

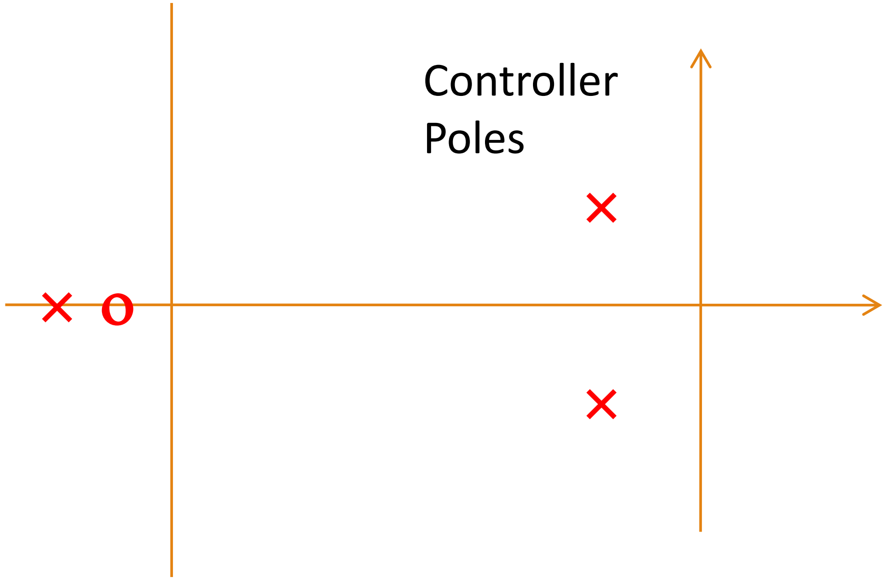
$$G(s)=\frac{\omega_m(s)}{V^r_{\omega_m}}=\frac{\frac{K_s\left(1+sT_s\right)}{sT_s}\frac{K_i}{\left(1+sT_i\right)}\frac{K_b/B}{\left(1+sT_m\right)}}{1+\frac{K_s\left(1+sT_s\right)}{sT_s}\frac{K_\omega}{\left(1+sT_\omega\right)}\frac{K_i}{\left(1+sT_i\right)}\frac{K_b/B}{\left(1+sT_m\right)}}$$

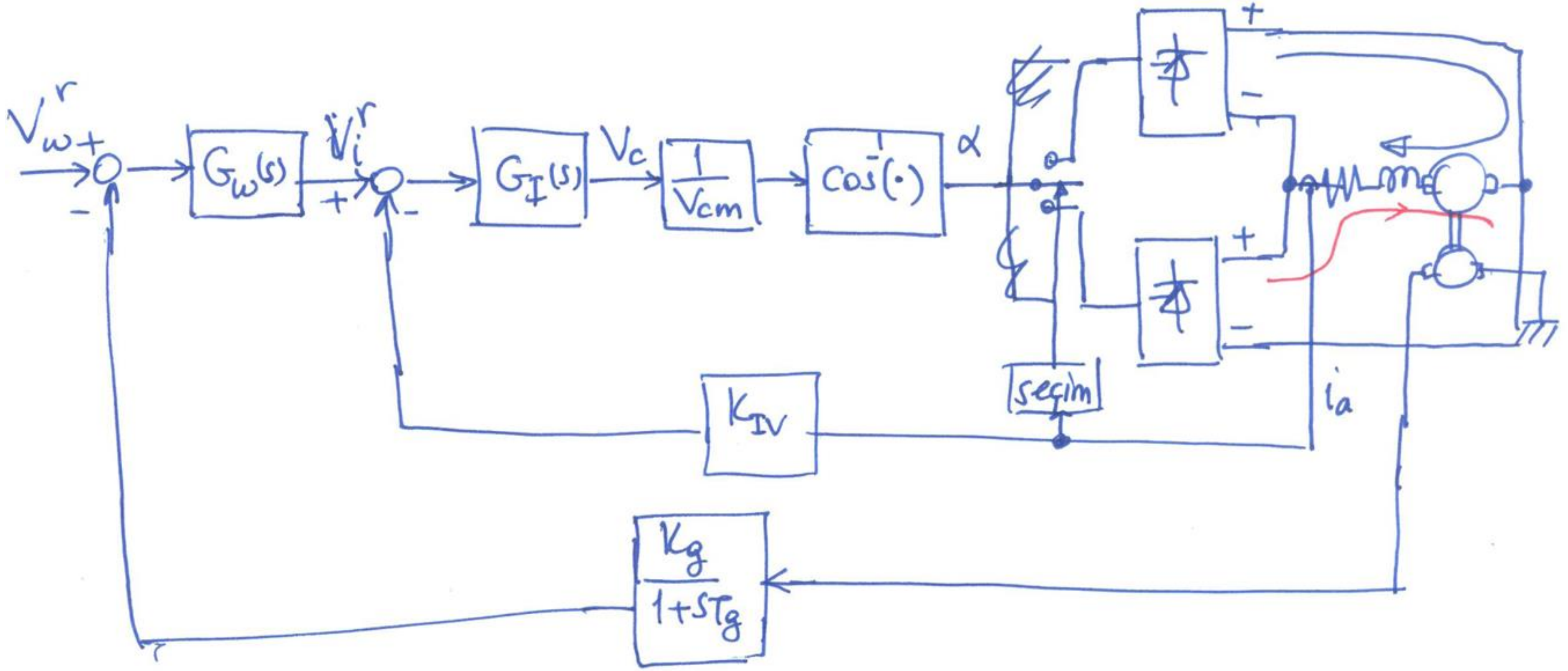
$$K_\alpha=K_s\,K_i\,{K_b}/{B}$$

$T_s=T_m$

$$G(s)=\frac{\omega_m(s)}{V^r_{\omega_m}}=\frac{\frac{K_\alpha}{T_i}\left(\frac{1}{T_\omega}+s\right)}{s^3+s^2\left(\frac{1}{T_\omega}+\frac{1}{T_i}\right)+s\,\frac{1}{T_\omega T_i}+\frac{K_\alpha K_\omega}{T_\omega T_i}}$$

Outside of the 2 control poles, the 3rd pole is 5-6 times larger than the real parts of the 2 control poles.
The additional zero should also be at least 5-6 times further away.





V_ω^r
$G_\omega(s)$
V_i^r
$G_I(s)$
V_c
$\frac{1}{V_{cm}}$
$\cos^{-1}(\cdot)$
α
seçim
K_{IV}
i_a
$\frac{K_g}{1+sT_g}$

A separately-excited dc motor with the following parameters: $R_a = 0.5\ \Omega$, $L_a = 0.003$H, and $K_b = 0.8$ V/rad/sec, is driving a load of $J = 0.0167$ kg-m^2, $B_l = 0.01$ N·m/rad/sec with a load torque of 100 N·m. Its armature is connected to a dc supply voltage of 220 V and is given the rated field current. Find the speed of the motor.

Solution The electromagnetic torque balance is given by

$$T_e = T_l + B_l\omega_m + J\frac{d\omega_m}{dt}$$

In steady state, $\frac{d\omega_m}{dt} = 0$

$$T_e = T_l + B_l\omega_m = 100 + 0.01\omega_m$$

$$T_e = K_b i_a = 100 + 0.01\omega_m$$

$$i_a = \frac{(100 + 0.01\omega_m)}{K_b} = (125 + 0.0125\omega_m)$$

$$e = V - R_a i_a = 220 - 0.5 \times (125 + 0.0125\omega_m) = 157.5 - 0.00625\omega_m = K_b\omega_m$$

Rearranging in terms of ω_m,

$$\omega_m(0.8 + 0.00625) = 157.5$$

Hence $\omega_m = \frac{157.5}{0.80625} = 195.35$ rad/sec

Example

Design a speed-controlled dc motor drive maintaining the field flux constant. The motor parameters and ratings are as follows:

220 V, 8.3 A, 1470 rpm, $R_a = 4\ \Omega$, $J = 0.0607\ kg - m_2$, $L_a = 0.072$ H, $B_1 = 0.0869$ N·m / rad/sec, $K_b = 1.26$ V/rad/sec.

The converter is supplied from 230V, 3-phase ac at 60 Hz. The converter is linear, and its maximum control input voltage is ±10 V. The tachogenerator has the transfer function $G_\omega(s) = \frac{0.065}{(1 + 0.002s)}$. The speed reference voltage has a maximum of 10V. The maximum current permitted in the motor is 20 A.

f=60 Hz T=1/60=16.67 ms

There are 6 peaks in a period 16.67/6= 2.778 ms = T is the time it takes for the applied control signal to change 0<Td<2.778 ms.

From here Td=1.38 ms is selected.

Solution (i) Converter transfer function:

$$K_r = \frac{1.35\ V}{V_{cm}} = \frac{1.35 \times 230}{10} = 31.05\ V/V$$

$$V_{dc}(max) = 310.5\ V$$

The rated dc voltage required is 220 V, which corresponds to a control voltage of 7.09 V. The transfer function of the converter is

$$G_r(s) = \frac{31.05}{(1 + 0.00138s)}\ V/V$$

(ii) Current transducer gain: The maximum safe control voltage is 7.09 V, and this has to correspond to the maximum current error:

$$i_{max} = 20\ A$$

$$H_c = \frac{7.09}{I_{max}} = \frac{7.09}{20} = 0.355\ V/A$$

(iii) Motor transfer function:

$$K_1 = \frac{B_t}{K_b^2 + R_a B_t} = \frac{0.0869}{1.26^2 + 4 \times 0.0869} = 0.0449$$

$$-\frac{1}{T_1}, -\frac{1}{T_2} = -\frac{1}{2}\left[\frac{B_t}{J} + \frac{R_a}{L_a}\right] \pm \sqrt{\frac{1}{4}\left(\frac{B_t}{J} + \frac{R_a}{L_a}\right)^2 - \left(\frac{K_b^2 + R_a B_t}{J L_a}\right)}$$

$$T_1 = 0.1077 \text{ sec}$$

$$T_2 = 0.0208 \text{ sec}$$

$$T_m = \frac{J}{B_t} = 0.7 \text{ sec}$$

The subsystem transfer functions are

$$\frac{I_a(s)}{V_a(s)} = K_1 \frac{(1 + sT_m)}{(1 + sT_1)(1 + sT_2)} = \frac{0.0449(1 + 0.7s)}{(1 + 0.0208s)(1 + 0.1077s)}$$

$$\frac{\omega_m(s)}{I_a(s)} = \frac{K_b/B_t}{(1 + sT_m)} = \frac{14.5}{(1 + 0.7s)}$$

(iv) Design of current controller:

$$T_c = T_2 = 0.0208 \text{ sec} \qquad T_{c1} = T_1 = 0.1077 \text{ sec}$$

$$K_i = \frac{K_1 K_d K_c T_m}{1 + K_1 K_d K_c T_m H_c} = \frac{0.0449 \times 31.05 \times 2 \times 0.7}{1 + 0.0449 \times 31.05 \times 2 \times 0.7 \times 0.355} = 1.153;$$

$$T_i = \frac{T_d}{1 + K_1 K_d K_c T_m H_c} = 0.0000815 \text{ sec}$$

$$K_\alpha = K_s K_i \frac{K_b}{B} = K_s \times 1.153 \times 14.5 = 16.72 K_s$$

$$G(s) = \frac{\omega_m(s)}{V^r_{\omega_m}} = \frac{\frac{K_\alpha}{T_i}\left(\frac{1}{T_\omega} + s\right)}{s^3 + s^2\left(\frac{1}{T_\omega} + \frac{1}{T_i}\right) + s\frac{1}{T_\omega T_i} + \frac{K_\alpha}{T_\omega T_i}}$$

$T_s = T_m$

$$G_\omega(s) = \frac{0.065}{(s\,0.002 + 1)}$$

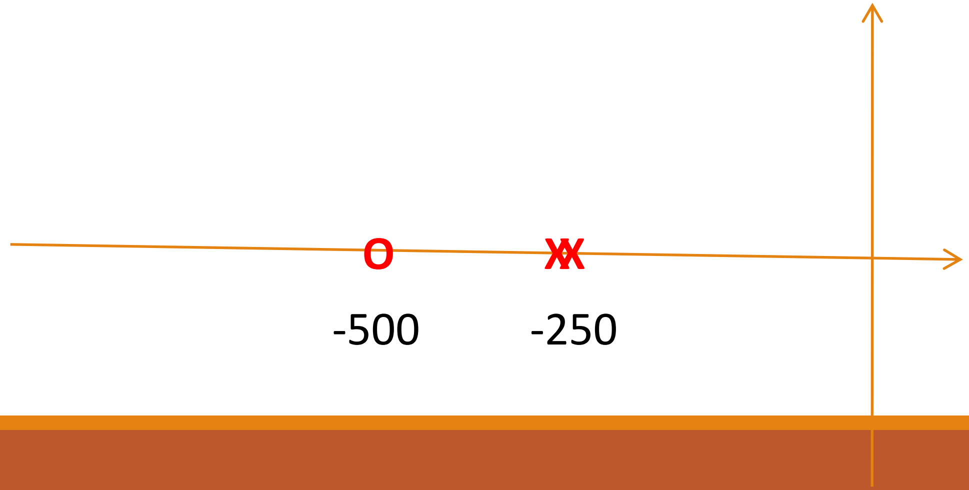
$$\frac{1}{T_\omega} << \frac{1}{T_i}$$

$$G(s) = \frac{16.72 K_s \left(\frac{1}{T_\omega} + s\right)}{T_i s^3 + s^2 + s\frac{1}{T_\omega} + \frac{1.087 K_s}{T_\omega}}$$

0

$$T_\omega = 0.002\, san$$

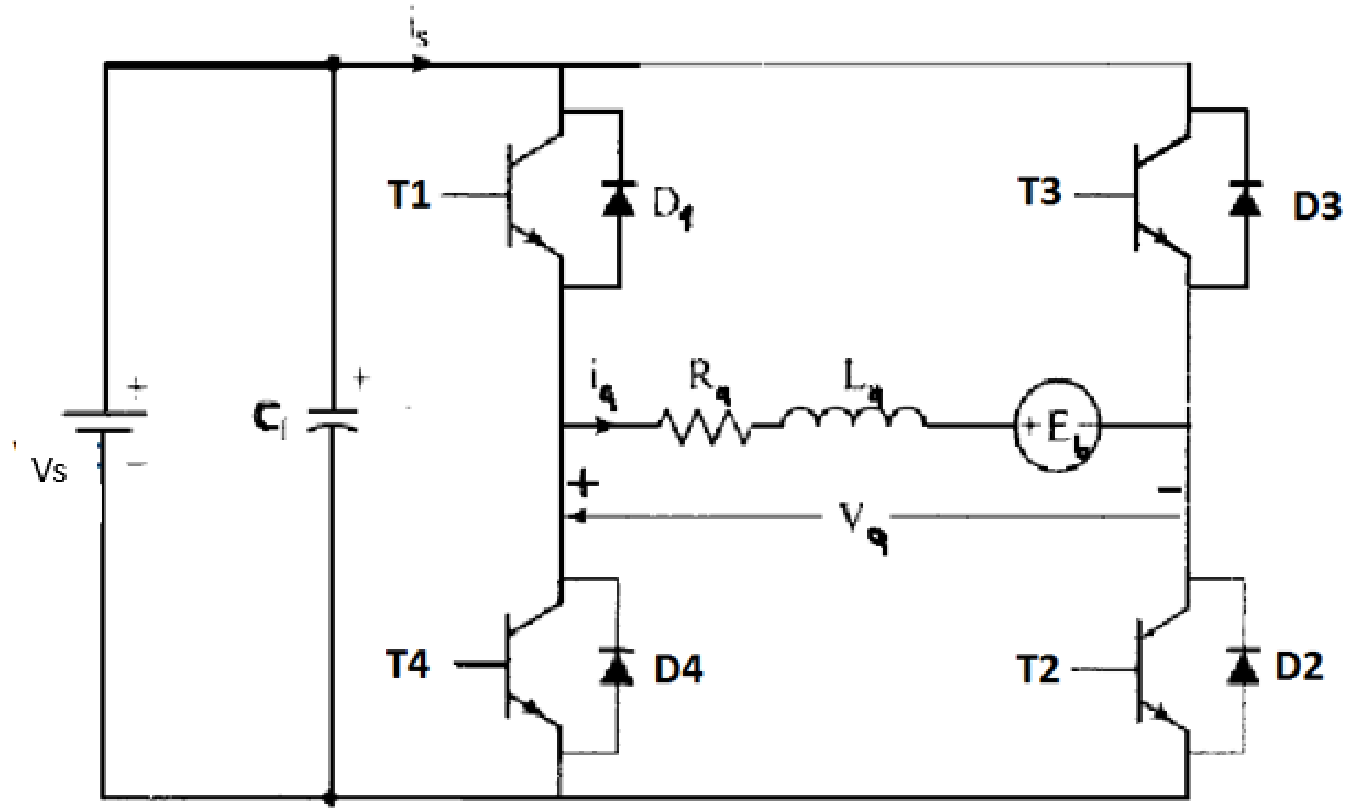
$$G(s) = \frac{16.72 K_s (s + 500)}{s^2 + 500 s + 543.4 K_s}$$



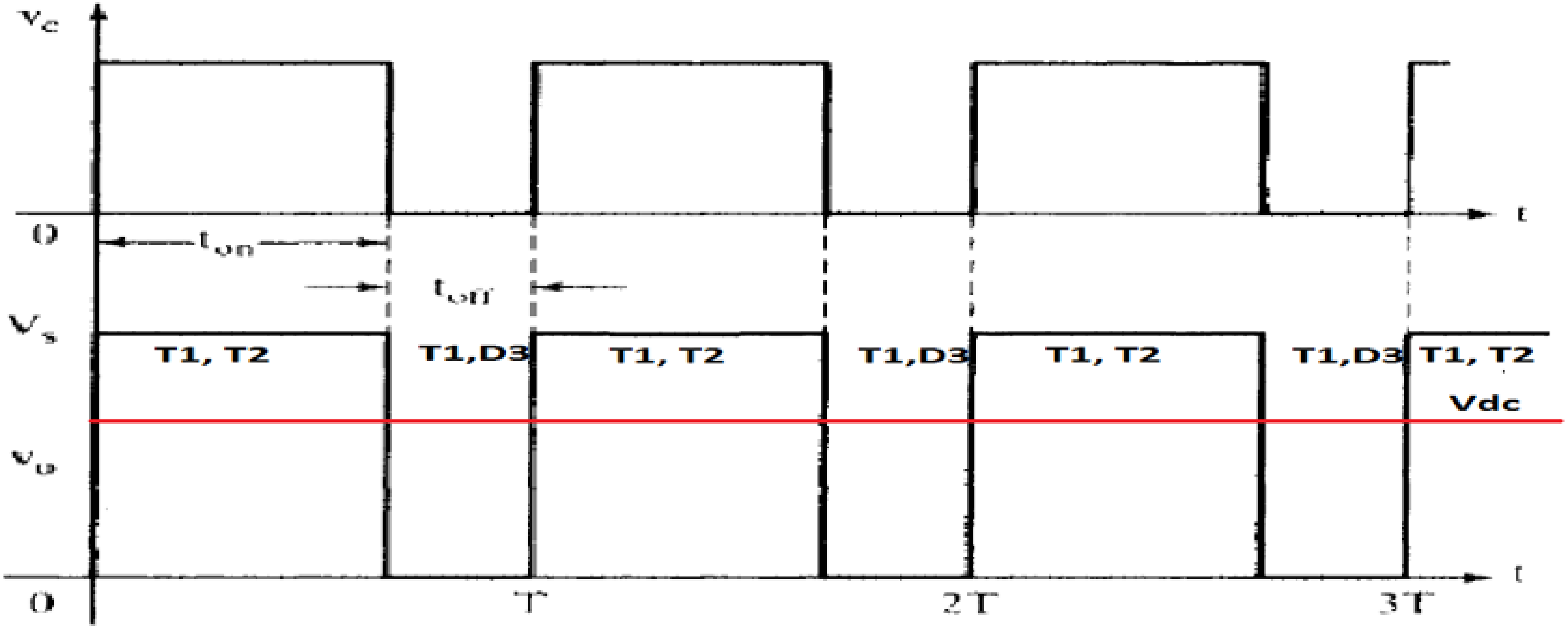
If Ks=115.016 is selected, the system will have a multiple root at -250. Since zero at -500 is closer than 5 times, the critical damped response expected from the system cannot be obtained.

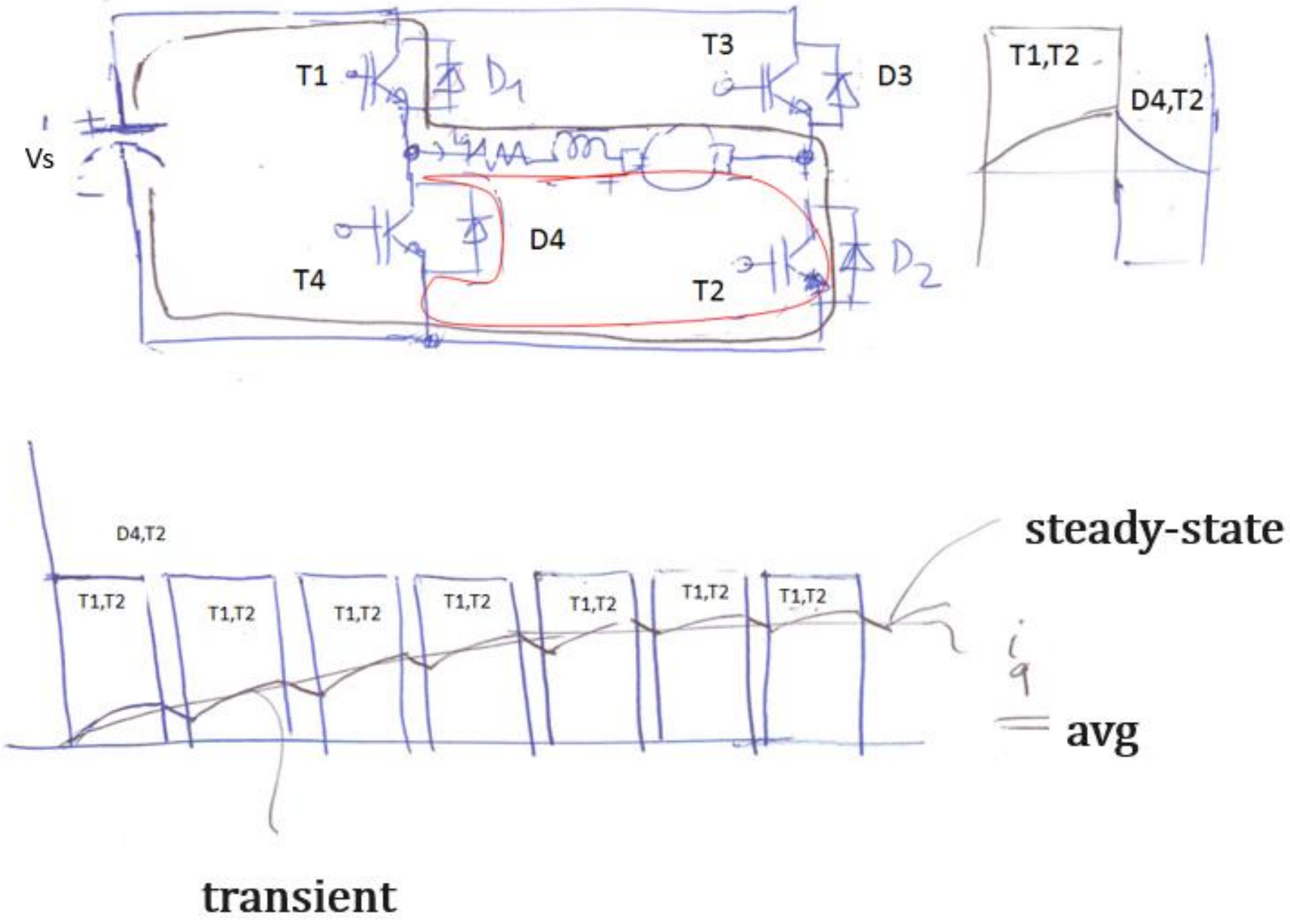
Permanent Magnet DC Motor with DC-DC Drive

DC-DC chopper is preferred in DC motor control systems in order not to give too much harmonics to the network and to reduce motor torque oscillations.

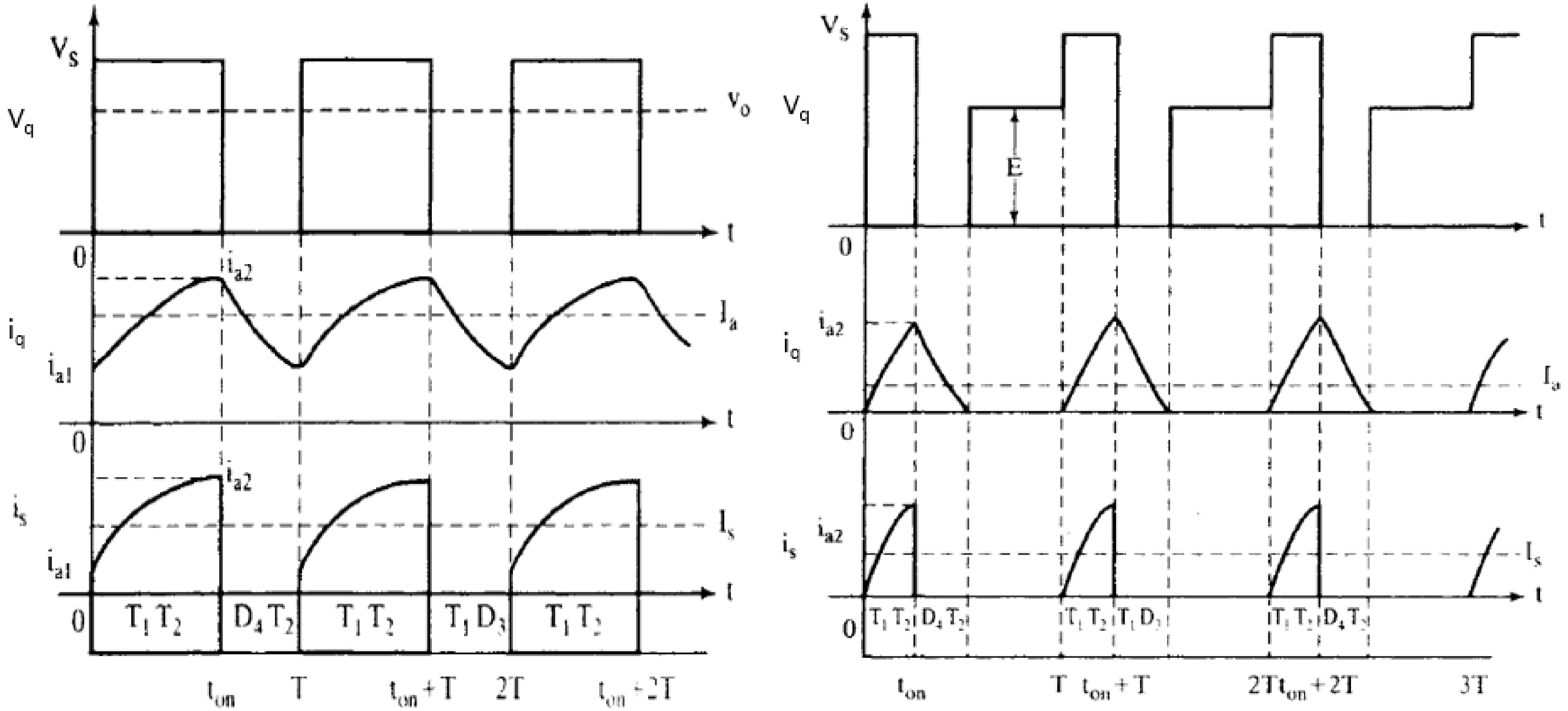


In order for current to flow in one direction from the motor, T1 and T2 or T3 and T4 must be in conduction at the same time.



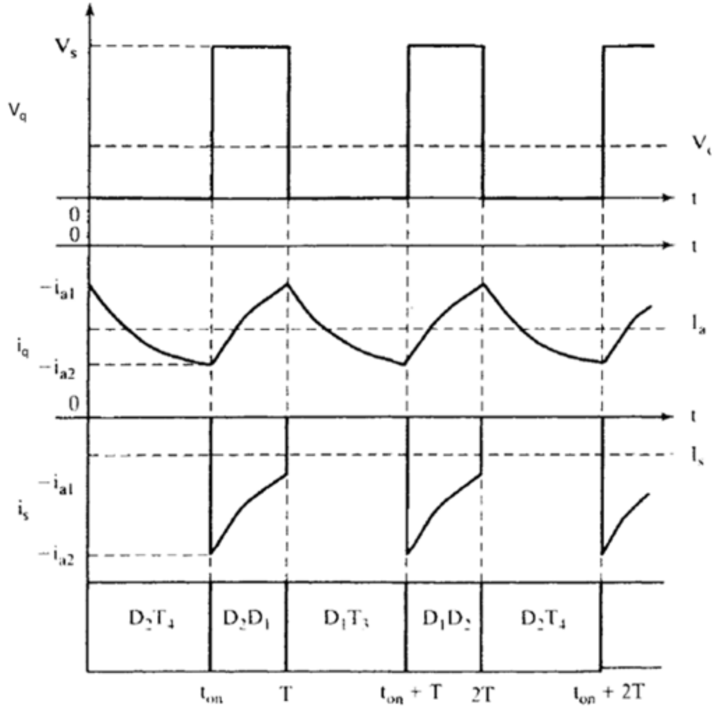

Vs
T1
D1
T3
D3
T4
D4
T2
D2
T1,T2
D4,T2
D4,T2
T1,T2
T1,T2
T1,T2
T1,T2
T1,T2
T1,T2
T1,T2
steady-state
avg
transient

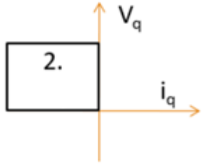
First Quadrant



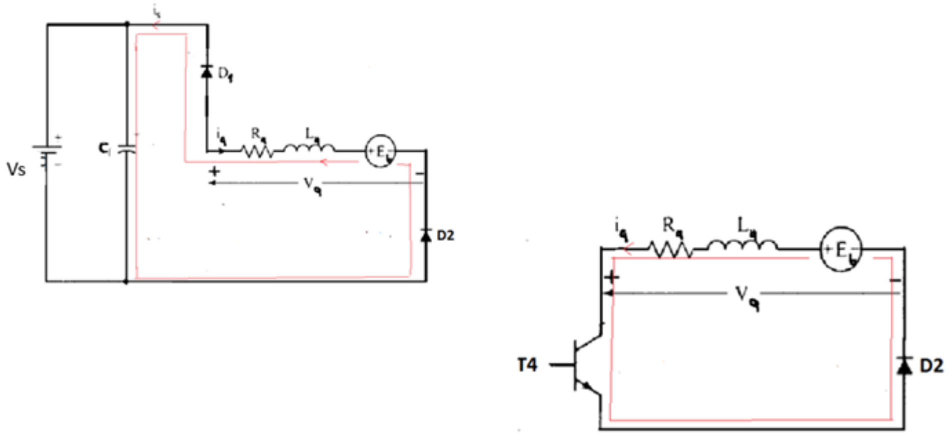
(i) Continuous Conduction

(ii) Discontinuous Conduction

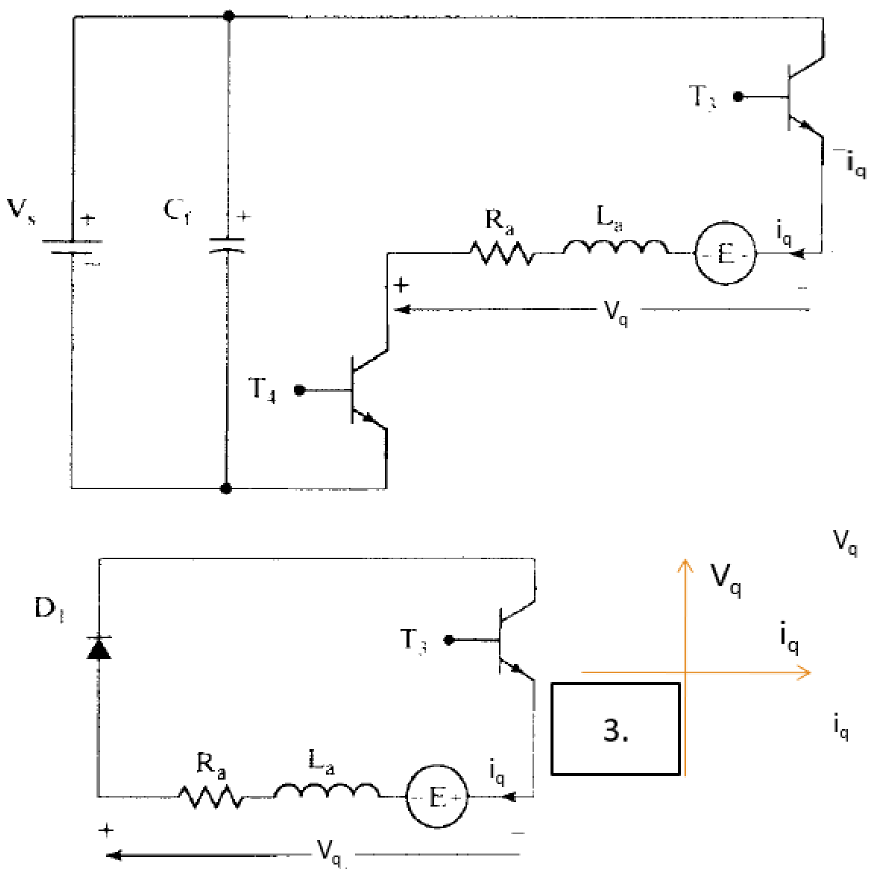


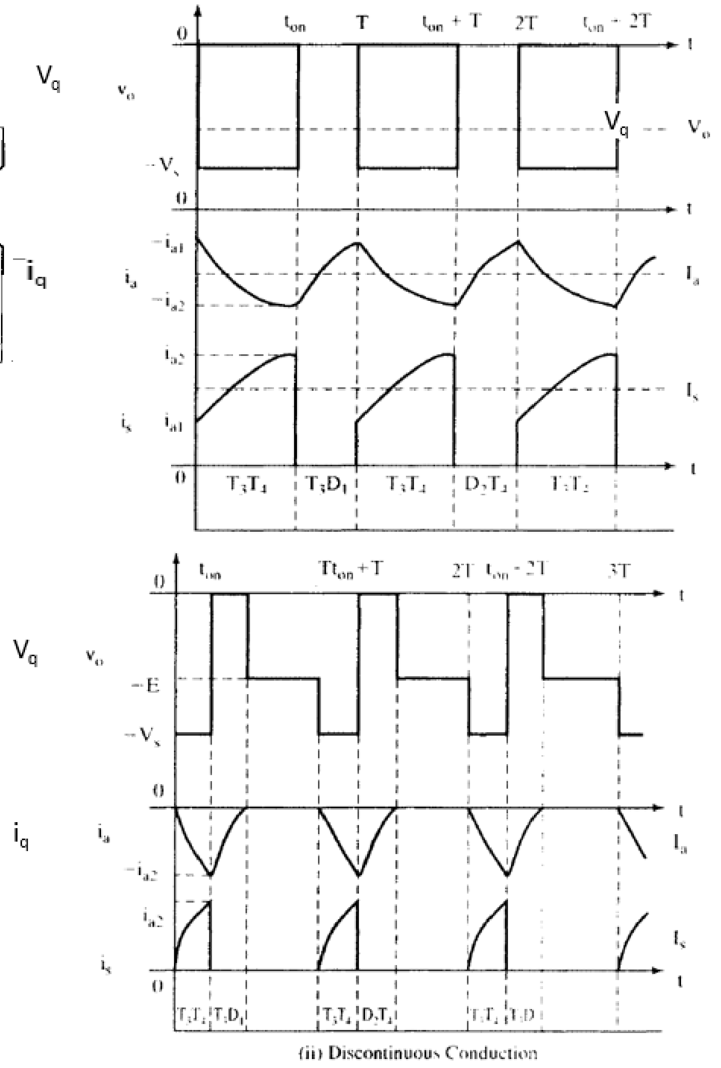


Second Quadrant

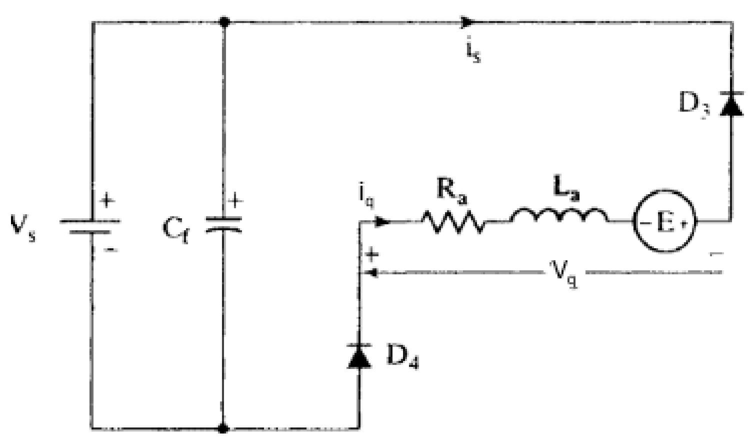


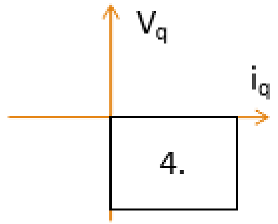
Third Quadrant

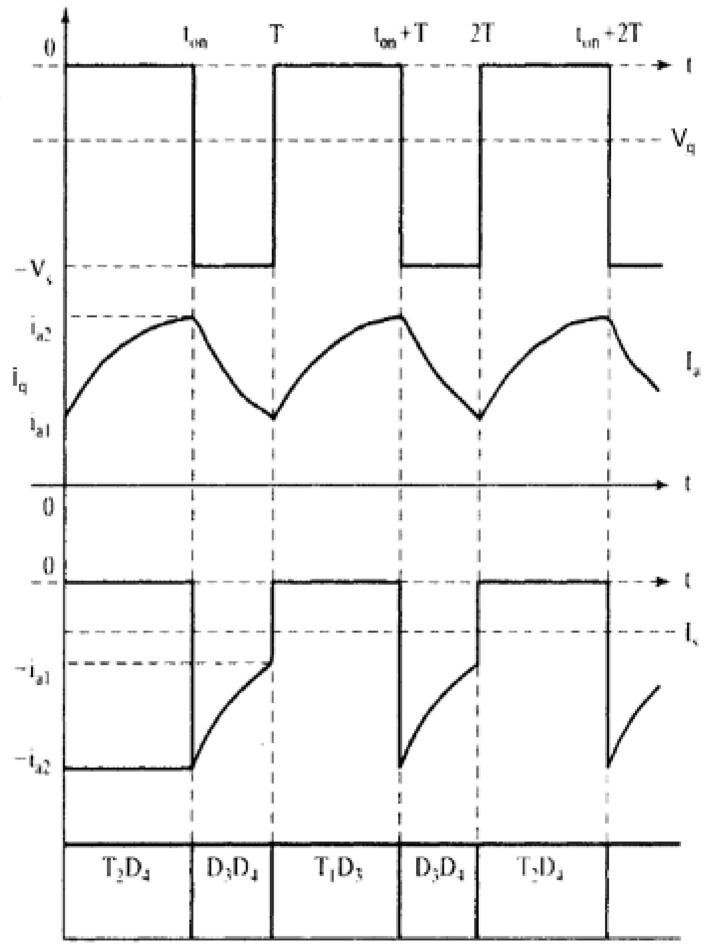


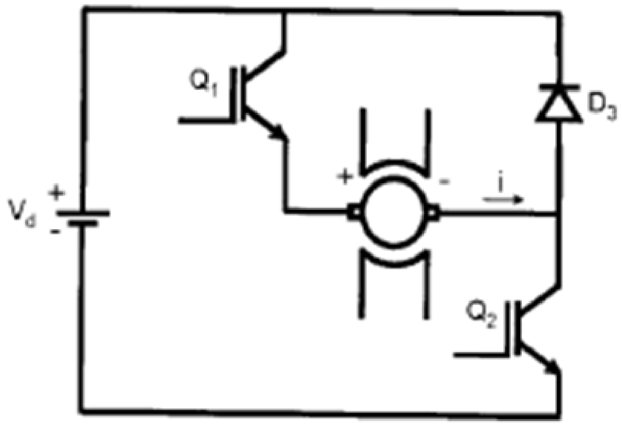


Fourth Quadrant

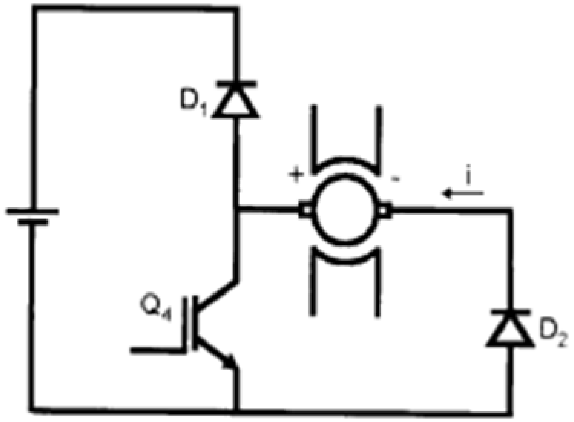




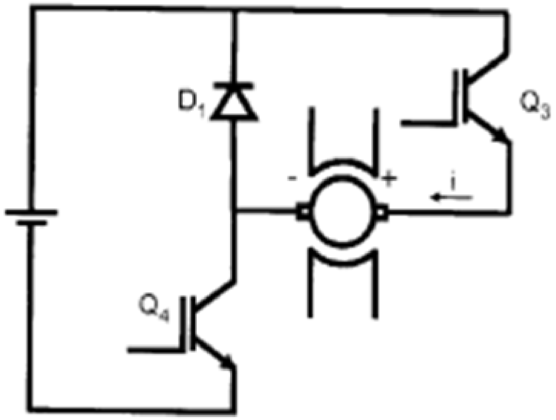




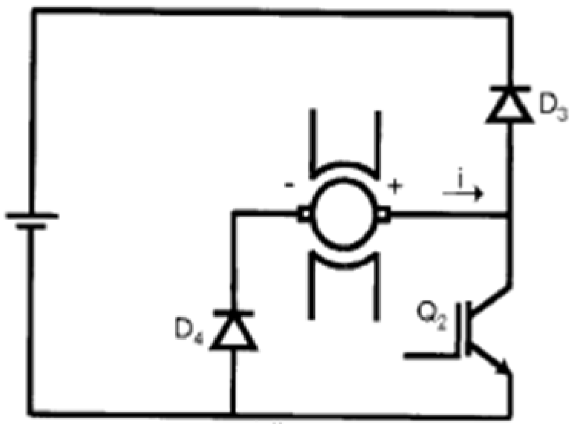
I



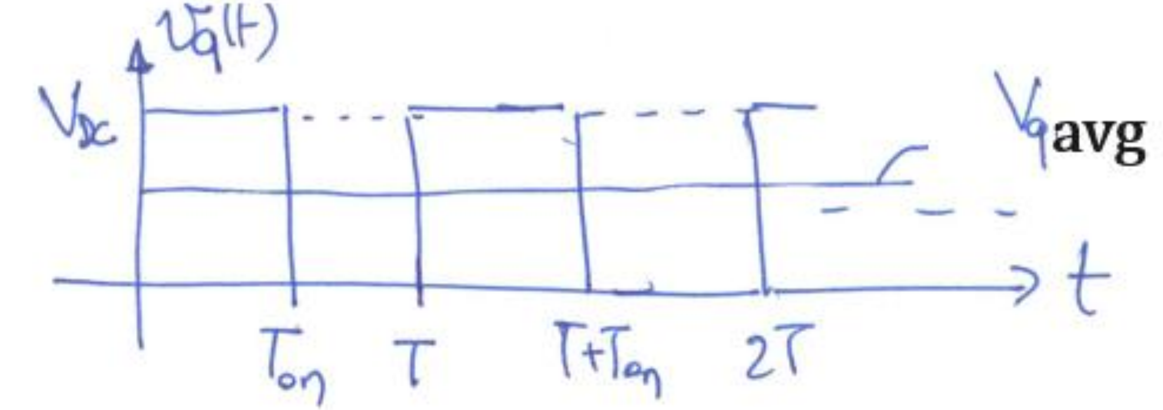
II



III



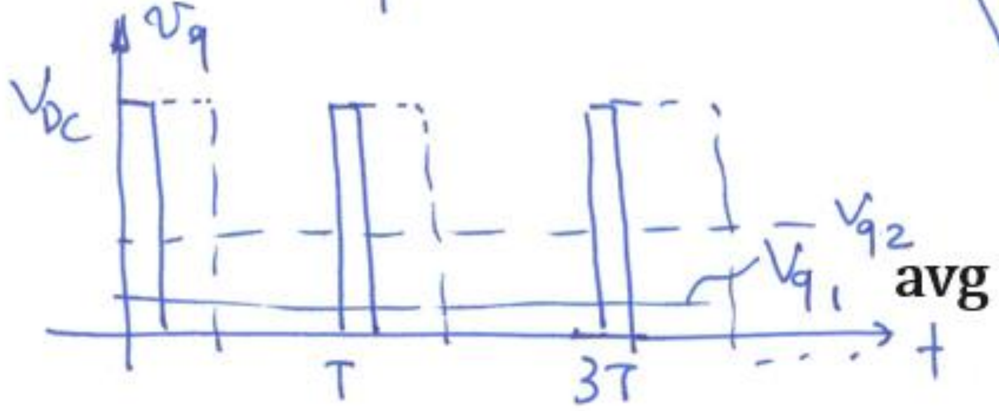
IV



$$V_{q\,avg} = \frac{T_{on}}{T} \cdot V_{DC}$$

$V_{q\,avg} = 0 \qquad T_{on} = 0$

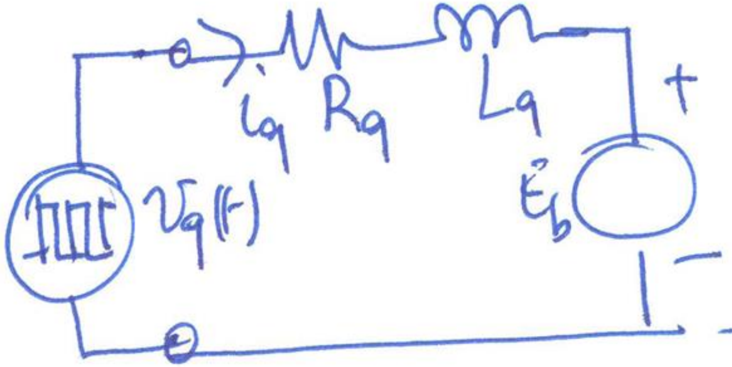
$V_{q\,avg} = V_{DC} \qquad T_{on} = T$

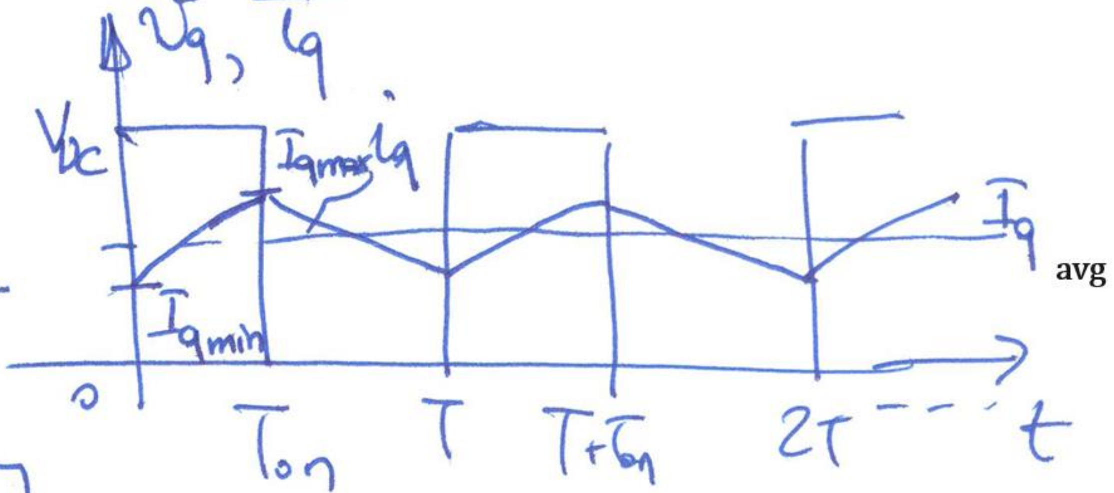


$$\frac{T_{on}}{T} = d \quad \text{duty cycle}$$

$$0 \le d \le 1$$

$$V_{q_{avg}} = d \cdot V_{DC}$$





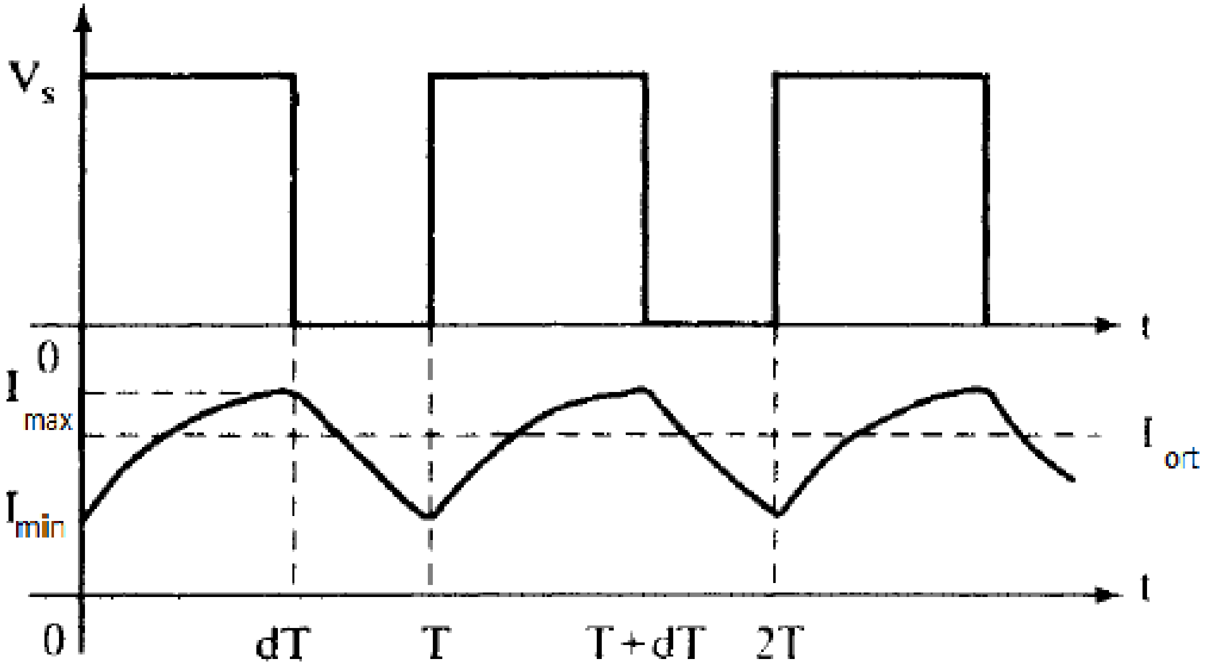
$$I_{q_{avg}} = \frac{V_{q_{avg}} - E_b}{R_q}$$

$$\text{duty cycle} \quad d = \frac{T_{on}}{T} \rightarrow (dT = T_{on})$$

Average Armature Current

$$I_a = \frac{V_a - E_b}{R_a} = \frac{d\,V_{dc} - E_b}{R_a}$$

$$T_m = k_t I_a = \frac{k_t (d\,V_{dc} - E_b)}{R_a}$$



$$\tau_q = \frac{L_q}{R_q}$$

T_{on}

$$V_{DC} = R_q\, i_q + L_q \frac{di_q}{dt} + e_b$$

Mechanical time constant is bigger than T

T_{on} and T_{off} steadily changes.

T_{off}

$$0 = R_q\, i_q + L_q \frac{di_q}{dt} + e_b$$

$$0 = R_a i_a + L_a \frac{di_a}{dt} + k_b \omega$$

$$T_{on} < t \leq T$$

$$\frac{V_{DA}}{s} = (sL_a + R_a) I_a(s) - L_a I_a(0) + \frac{E_b}{s}$$

$$L_a I_a(0) + \frac{V_{DA} - E_b}{s} = (sL_a + R_a) I_a(s)$$

$$I_a(s) = \frac{V_{DA} - E_b}{s(sL_a + R_a)} + \frac{L_a I_a(0)}{sL_a + R_a}$$

$$I_a(s) = \frac{\frac{V_{DA} - E_b}{L_a}}{s\left(s + \frac{R_a}{L_a}\right)} + \frac{I_a(0)}{s + \frac{R_a}{R_a}}$$

k

$$= \frac{k_1}{s(s+a)} + \frac{k_2}{s+a}$$

$$= \frac{k_1/a}{s} - \frac{k_1/a}{s+a} + \frac{k_2}{s+a}$$

$$= \frac{\frac{V_{DA} - E_b}{L_a}}{\frac{R_a}{L_a}}\left(1 - e^{-t/\tau_e}\right) + I_{a_{min}} e^{-t/\tau_e}$$

$$i_q(t) = \frac{V_{dc} - E_b}{R_q}\left(1 - e^{-t/\tau_q}\right) + I_{min} e^{-t/\tau_q} \qquad 0 \leq t < dT$$

$$i_q(t) = -\frac{E_b}{R_q}\left(1 - e^{-(t-dT)/\tau_q}\right) + I_{max} e^{-(t-dT)/\tau_q}$$

$$dT \leq t < T$$

$$i_q(t) = i_q(t+T)$$

$$I_q(t) = \frac{V_{DA} - E_b}{R_q}\left(1 - e^{-t/\tau_e}\right) + I_{q_{min}} e^{-t/\tau_e}$$

$$t = 0 \qquad i_q(0) = I_{q_{min}}$$

$$t = T_{on} \quad I_{q_{max}} = i_q(T_{on}) = \frac{V_{DA} - E_b}{R_q}\left(1 - e^{-T_{on}/\tau_e}\right) + I_{q_{min}} e^{-T_{on}/\tau_e}$$

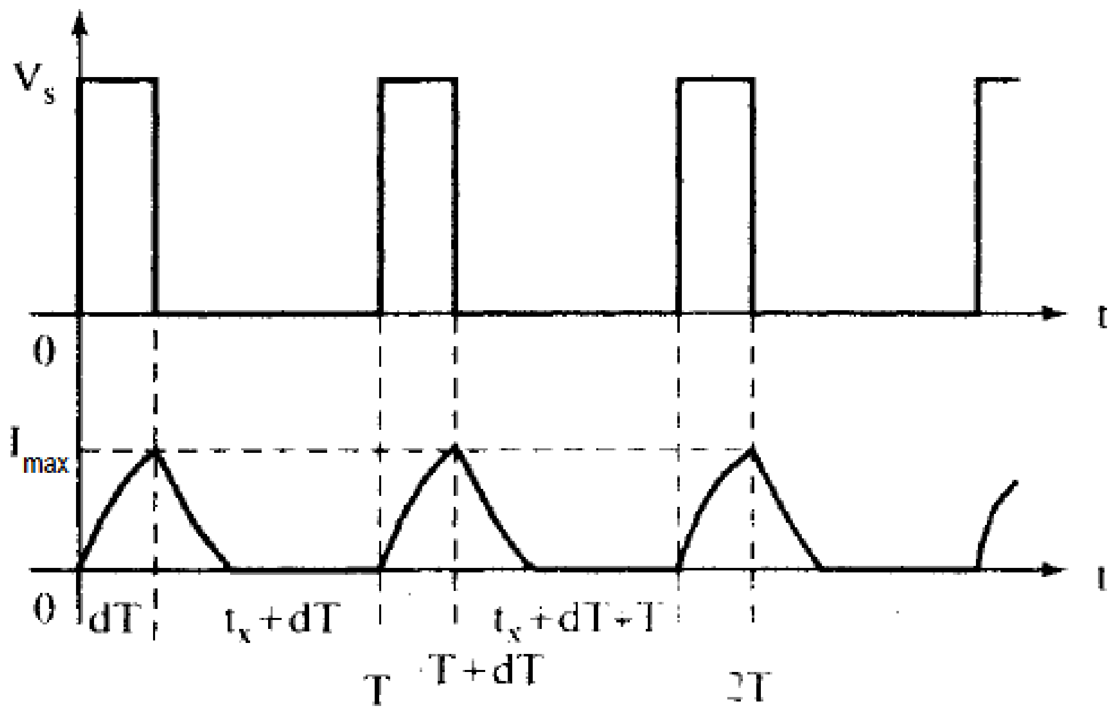
$$I_{min} = \frac{V_{dc}\left(e^{dT/\tau_a} - 1\right)}{R_a\left(e^{T/\tau_a} - 1\right)} - \frac{E_b}{R_a}$$

$$I_{max} = \frac{V_{dc}\left(1 - e^{-dT/\tau_a}\right)}{R_a\left(1 - e^{-T/\tau_a}\right)} - \frac{E_b}{R_a}$$

$$i_a(0) = 0$$

$$V_{dc} = E + R_a i_a + L_a \frac{di_a}{dt} \qquad 0 < t < dT$$

Discontinuity in Current



$$V_s = E + R_a i_a + L_a \frac{di_a}{dt}, \qquad 0 < t < dT$$

$$0 = E + R_a i_a + L_a \frac{di_a}{dt}, \qquad dT < t < (t_x + dT)$$

$$i_a(t_x + dT) = 0$$

$$i_a(0) = 0$$

$$I_{a1} = \frac{V_s - E}{R_a}\left(1 - e^{-dT/T_a}\right)$$

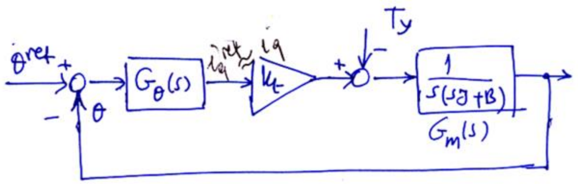
$$i_a(t_x + dT) = -\frac{E}{R_a}\left(1 - e^{-\frac{t_x}{T_a}}\right) + I_{a1} e^{-\frac{t_x}{T_a}}$$

$$t_x = T_a \log_e\left[1 + \frac{I_{a1} R_a}{E}\right]$$

$$i_a(t) = \frac{V_s - E}{R_a}\left(1 - e^{-t/T_a}\right), \qquad 0 < t < dT$$

$$i_a(t) = I_{a1} e^{-\frac{(t - dT)}{T_a}} - \frac{E}{R_a}\left(1 - e^{-\frac{(t - dT)}{T_a}}\right), \qquad dT < t < t_x + dT$$

$$i_a(t) = 0, \qquad (t_x + dT) < t < T$$



$$\theta(s) = \theta^{ref}(s)\,\frac{G_\theta(s)\,K_t\,G_m(s)}{1+G_\theta(s)\,K_t\,G_m(s)} - \frac{T_y(s)\;G_m(s)}{1+G_\theta(s)\,K_t\,G_m(s)}$$

$$\theta^{ref}(s) = \frac{\theta^r}{s} \qquad T_y(s) = \frac{T_y}{s}$$

$$\theta_\infty = \lim_{s\to 0} \frac{G_\theta(s)\,K_t\,G_m(s)\cdot\theta^r}{1+G_\theta(s)\,K_t\,G_m(s)} - \frac{G_m(s)\cdot T_y}{1+G_\theta(s)\,G_m(s)\,K_t}$$

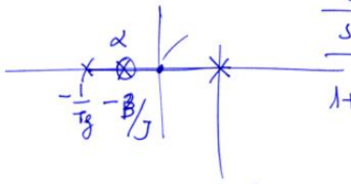
$$\lim_{s\to 0} G_m(s) \longrightarrow \infty$$

$$\theta_\infty = \theta^r - \lim_{s\to 0} \frac{T_y}{G_\theta(s)\,K_t}$$

$$G_\theta(s) = \frac{Q_\theta(s)}{s\,P_\theta(s)} \qquad \lim_{s\to 0} G_\theta(s) \to \infty$$

$$\theta_\infty \to \theta^r$$

$$G_\omega(s) = \frac{K_\omega(s+\alpha)}{s} \qquad T_y = 0$$



$$\frac{\dfrac{K_t K_\omega (s+\alpha)}{s(sJ+B)}}{1+\dfrac{K_t K_\omega K_g (s+\alpha)}{s(sJ+B)(sT_g+1)}} = T(s)$$

$$T(s) = \frac{K_t K_\omega (sT_g+1)}{s(sT_g+1)+K_t K_\omega K_g} = \frac{K_1(s+\beta)}{(s+\delta)(s+\gamma)}$$

$$\beta = \frac{1}{T_g}$$

Problem: In a DC machine with permanent magnet brushes given as Rq=1 Ohm, Lq=10 mH, Kt=Kb=0.8, J=0.01 kgm2, B=0.04 Nm/s, position control will be made via the DC-DC chopper.

A controller with hysteresis structure was used as the current controller.

In the system that moves a fixed load of Ty = 10 Nm

Design a controller that will make the steady-state error zero and provide a critically damped response.

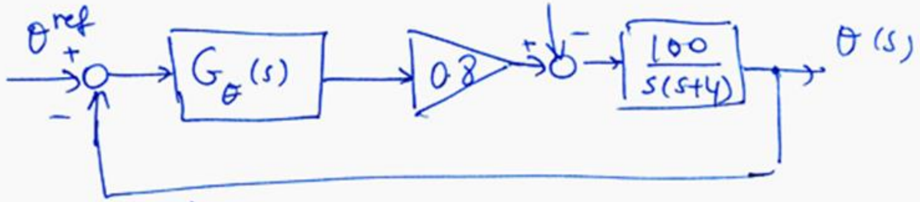
SOLUTION: Since it is a hysteresis current controller, the reference current can be considered as the approximate motor current. In this case, the system block diagram can be simplified as follows.

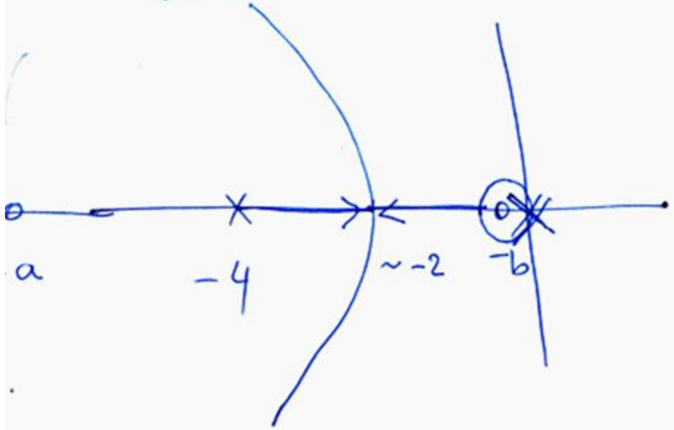
$k_t = 0.8$

$J = 0.01\ kgm^2$

$B = 0.04\ Nm/s$

$$G_m(s) = \frac{100}{s(s+4)}$$





$$G_\theta(s) = \frac{(s+a)(s+b)}{s} k$$

$b = 0,001$

$a = -10$

If it is assumed that zero at b = 0.001 and one of the poles at s = 0 are approximated,
K = 0.005 is found when s = 2-fold poles.

$$1+\frac{80K\left(s+10\right)}{s\left(s+4\right)}=0$$

$$\frac{G_\theta(s)\,K_t\,G_m(s)}{1+G_\theta(s)\,K_t\,G_m(s)} =$$

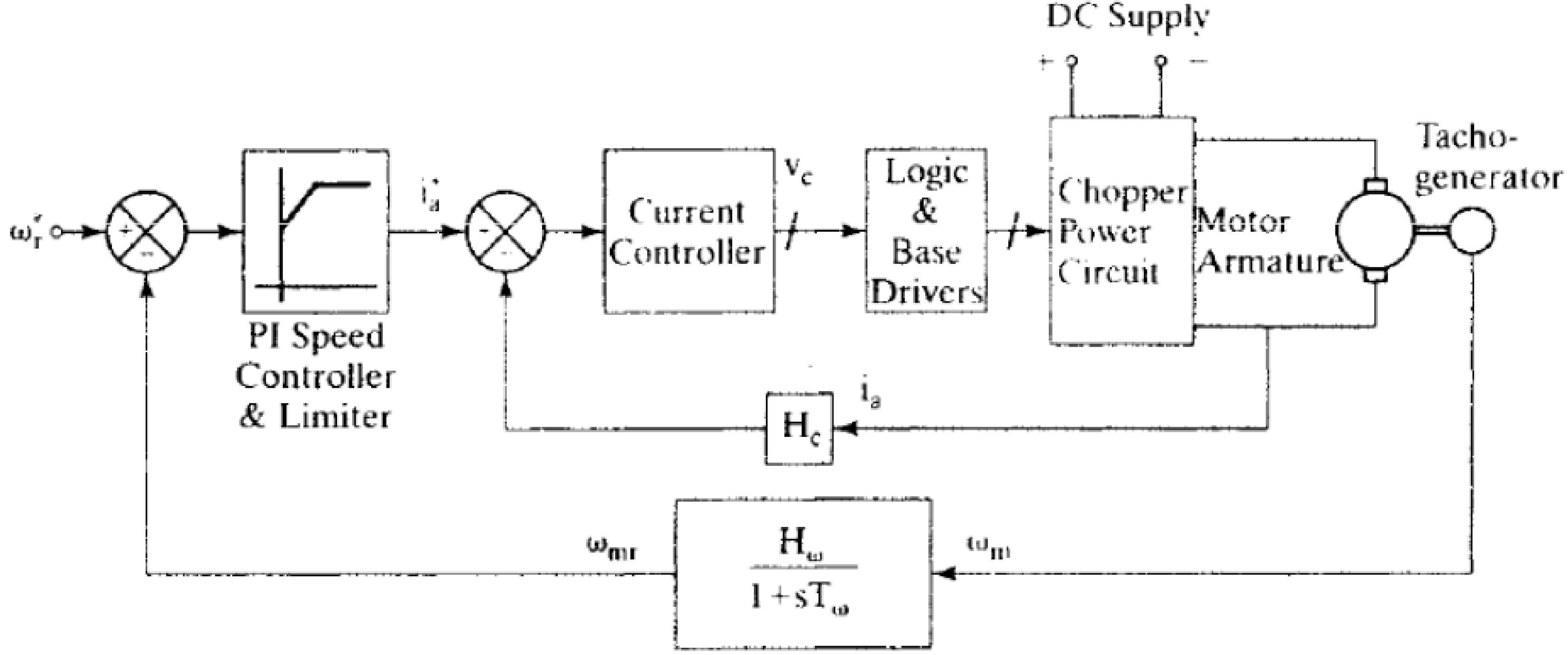
$$s^2(s+4)+80K(s+10)(s+0{,}01)$$

$$s^3+s^2(80K+4)+s\,800K+8K=0$$

$$(s^4+4s+4)(s+p)=s^3+(p+4)s^2+(4p+4)s+4p$$

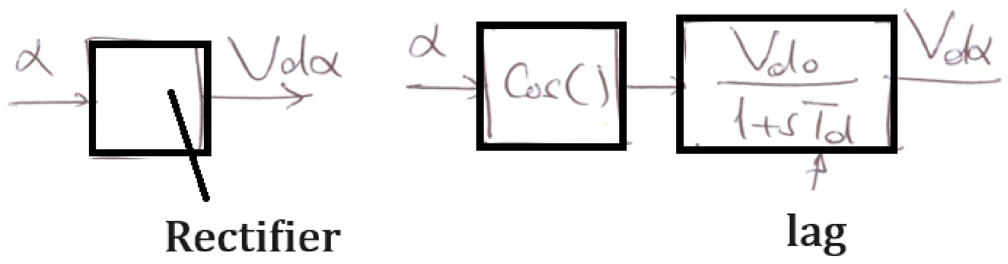
$$p+4=80k+4 \qquad 4p+4=800k \qquad 480k=4$$

$$320k+4=800k \rightarrow \qquad k=\frac{4}{48}$$


ω_r^*
PI Speed Controller & Limiter
i_a^*
Current Controller
v_c
Logic & Base Drivers
DC Supply
Chopper Power Circuit
Motor Armature
Tacho-generator
i_a
H_c
ω_{mr}
$\frac{H_\omega}{1+sT_\omega}$
ω_m

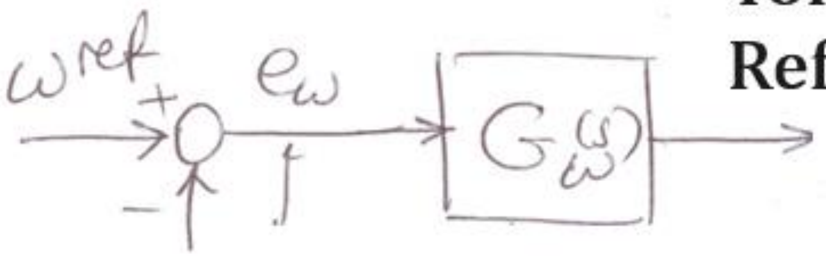
$$V_{d\alpha} = V_{d0} \cdot \cos\alpha$$

$$T_d \simeq 1{,}67\,\text{ms}$$

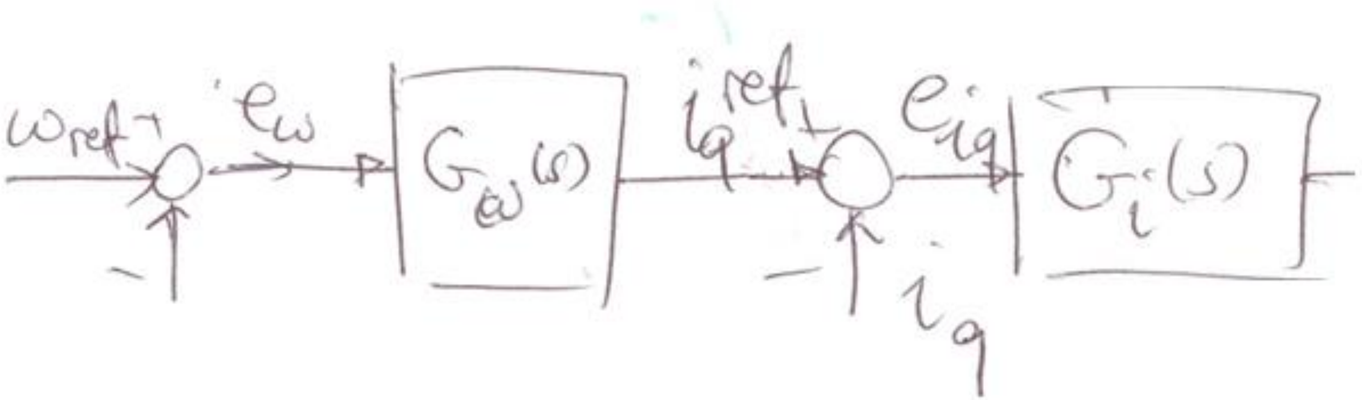


Torque is proportional with the current

Torque Reference



$T_m = k_t i_q$



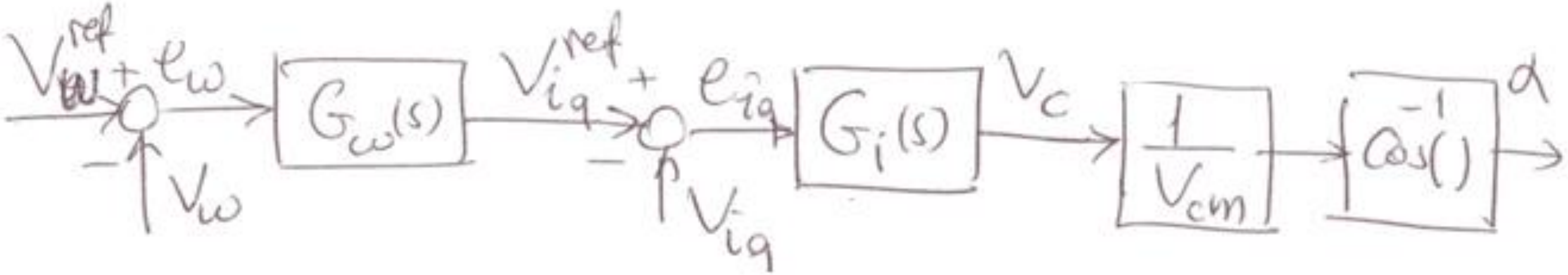
$$v_c = V_{cmax} \cdot \cos\alpha$$

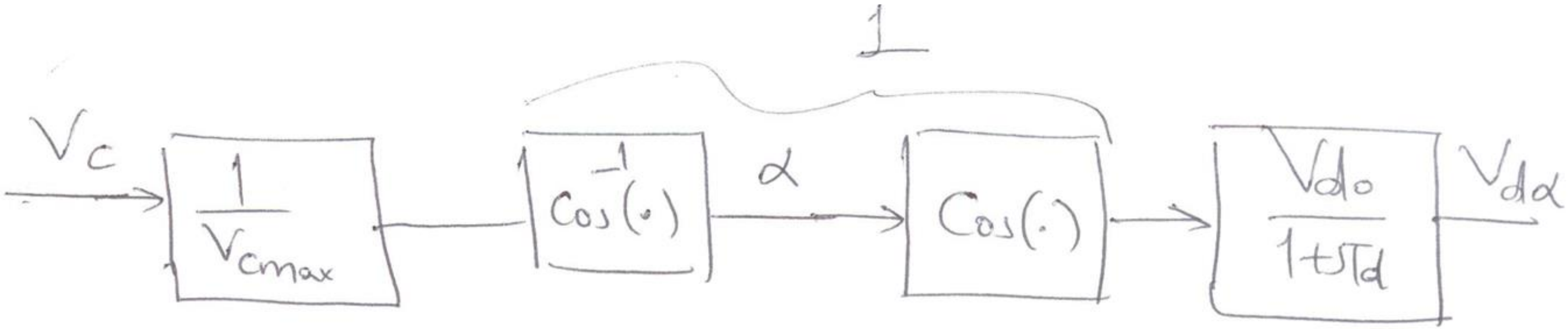
$$v_q = V_{do} \cos\alpha$$

$$\frac{v_q}{v_c} = \frac{V_{do}}{V_{cmax}} = K_d$$

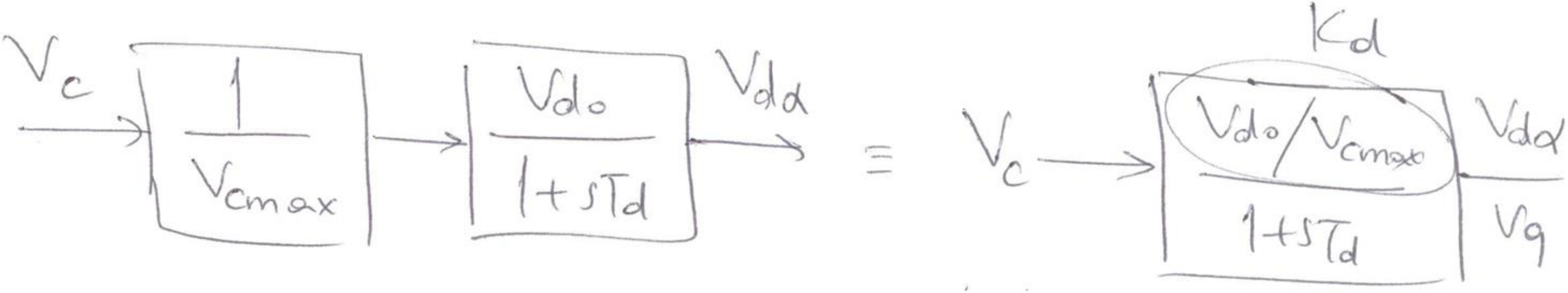
$$v_q = v_c \cdot K_d$$

$$V_{do} = K_d V_{cmax}$$

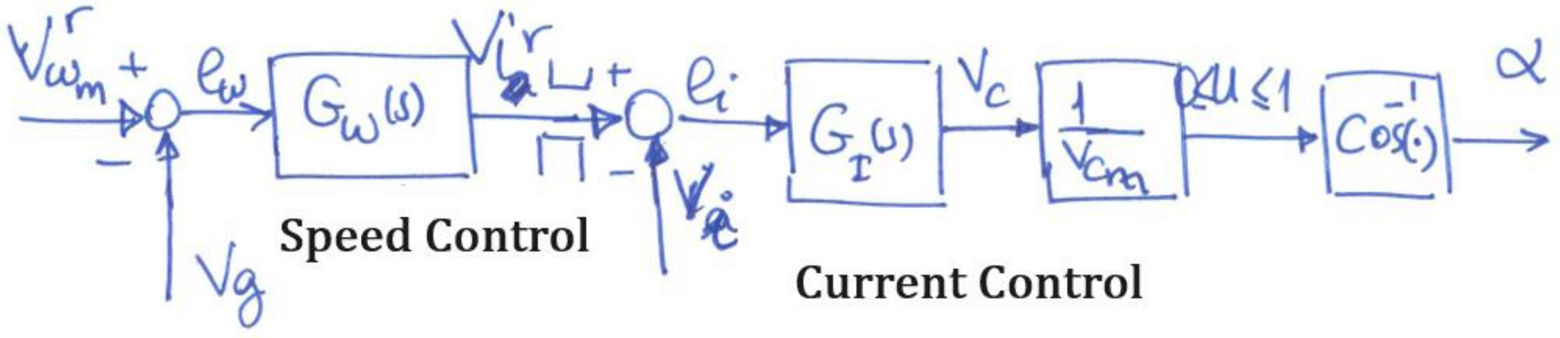




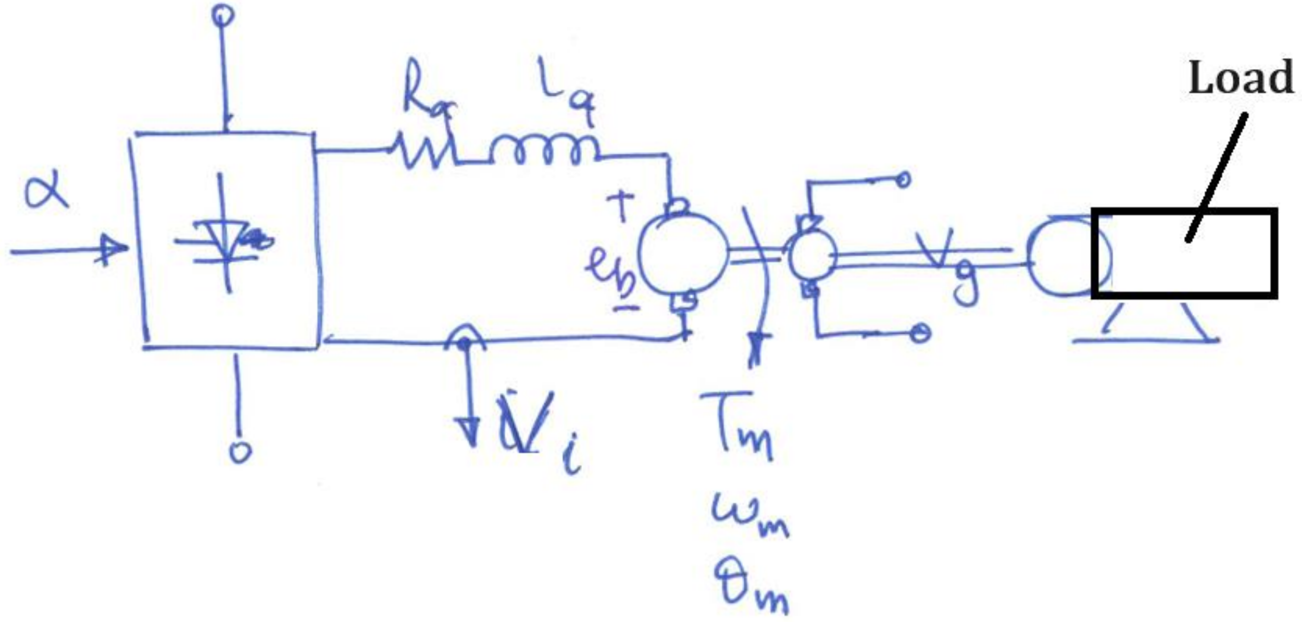
1
V_c
$\frac{1}{V_{cmax}}$
$\cos^{-1}(\cdot)$
α
$\cos(\cdot)$
$\frac{V_{do}}{1+sT_d}$
$V_{d\alpha}$



V_c
$\frac{1}{V_{cmax}}$
$\frac{V_{do}}{1+sT_d}$
$V_{d\alpha}$
$\equiv$
V_c
K_d
$\frac{V_{do}/V_{cmax}}{1+sT_d}$
$V_{d\alpha}$
V_q



Speed Control
Current Control



Load

$$\alpha = \cos^{-1}\left(\frac{V_c}{V_{cm}}\right) \longrightarrow V_c = V_{cm} \cdot \cos\alpha$$

$$V_{d\alpha} = \underbrace{\frac{3}{\pi} V_m}_{V_{do}} \cos\alpha = \overset{V_{do}}{\cancel{1.35\,V}} \cos\alpha$$

$$= \underset{V_{do}}{\cancel{1.35\,V}} \cdot \frac{V_c}{V_{cm}}$$

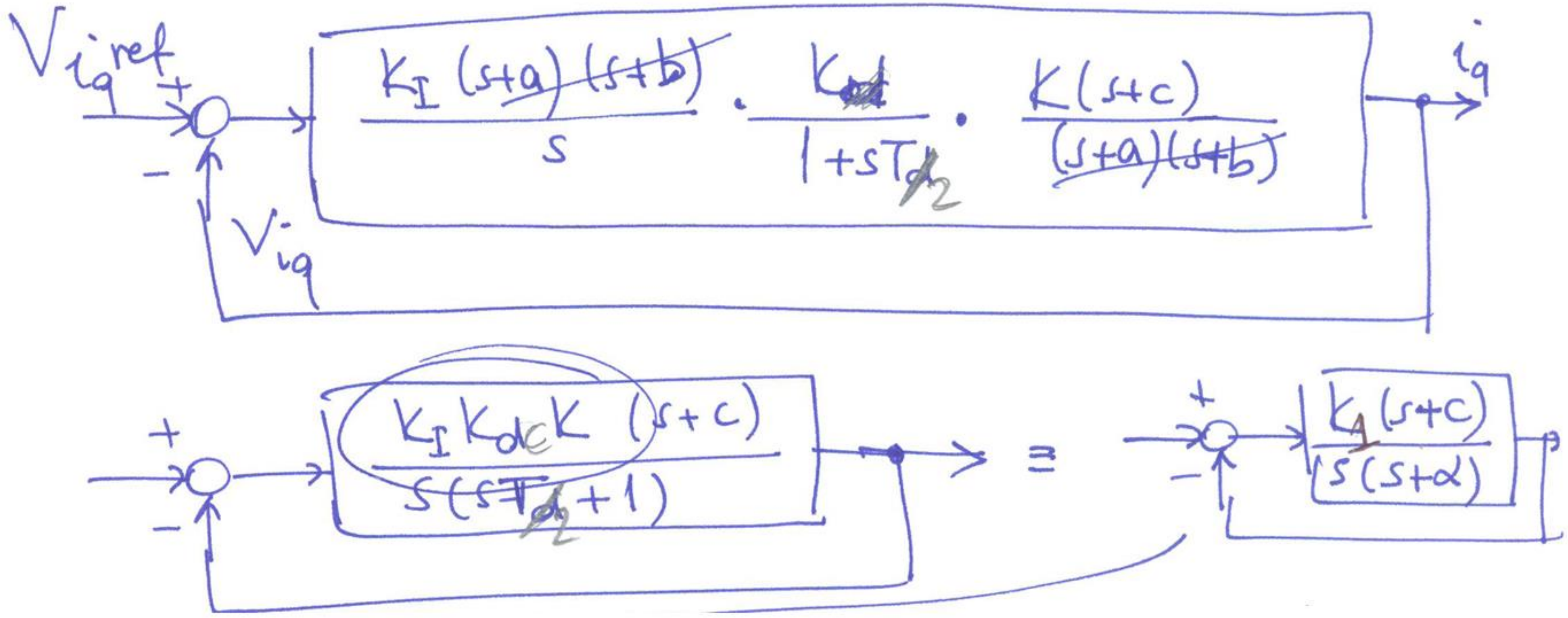
$$K_d = \frac{\overset{V_{do}}{\cancel{1.35 \cdot V}}}{V_{cm}}$$ olarak tanımlanırsa

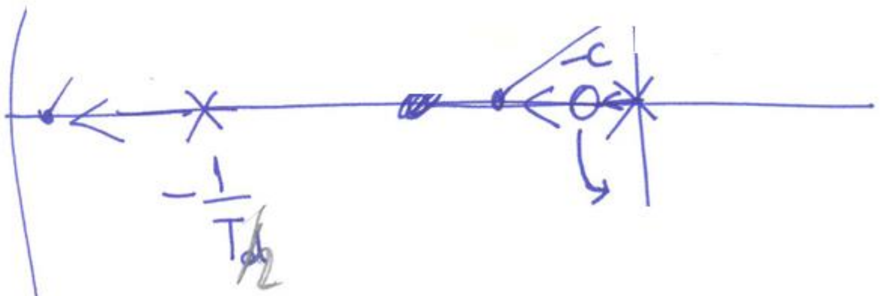
$$V_{d\alpha} = K_d \cdot V_c = G_d(s) \cdot V_c$$

$$G_d(s) = K_d\, e^{-sT_d} \cong \frac{K_d}{1 + sT_d}$$

PID Controller

$$G_I(s) = \frac{K_I (s+a)(s+b)}{s}$$





$$c = \frac{B}{J} = \frac{1}{\tau_{mek}}$$

$$G_I(s) = \frac{K_I\,(s+a)\,(s+b)}{s}$$

$$\frac{\dfrac{K_1(s+c)}{s(s+\alpha)}}{1+\dfrac{K_1(s+c)}{s(s+\alpha)}} = \frac{K_1(s+c)}{s^2+s(\alpha+K_1)+K_1 c} \approx \frac{K_1\cancel{(s+c)}}{\cancel{(s+c)}(s+\beta)}$$

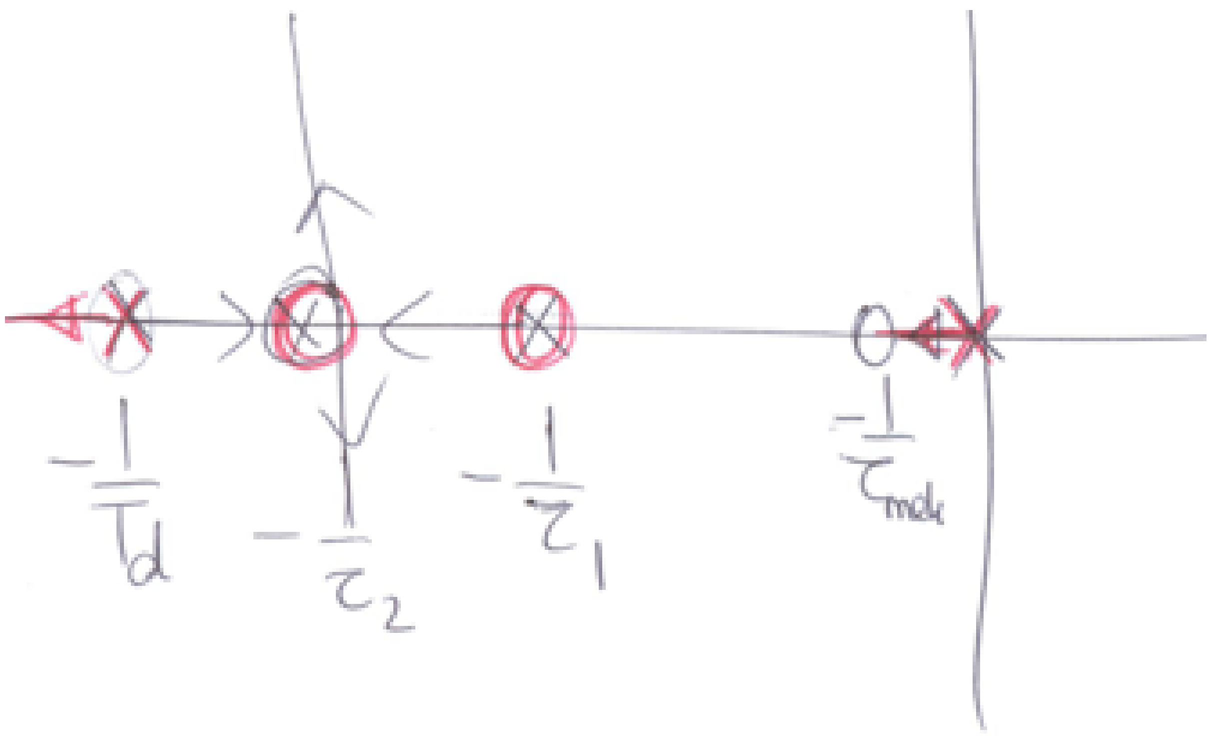
$$\frac{I_q(s)}{V_q(s)} = \frac{J\left(s + \overset{1/\tau_{mek}}{B/J}\right)}{s^2 J L_q + s\left(R_q J + B L_q\right) + \left(R_q B + K_t K_b\right)}$$

$$\frac{I_q(s)}{V_q(s)} = \frac{K\,(s\tau_m + 1)}{(s\tau_1 + 1)\,(s\tau_2 + 1)}$$

$$G_I(s) = \frac{K_A\,(s\tau_A + 1)}{s} = \frac{K_I\,(s + \alpha)}{s}$$

$$s^2 + s\left(\frac{R_q}{L_q} + \frac{B}{J}\right) + \frac{R_q B + K_t K_b}{L_q J}$$

$$\frac{1}{\tau_{el}} + \frac{1}{\tau_{mek}} \qquad \tau_{mek} \gg \tau_{el}$$



$$\frac{1}{\tau_{el}} \gg \frac{1}{\tau_{mek}}$$

$$s^2 + s\frac{1}{\tau_{el}} + \frac{1}{\tau_{el}\tau_{mek}} + \frac{k_t k_b}{L_q J}$$

$$-\frac{1}{2\tau_{el}} \mp \sqrt{\frac{1}{4\tau_{el}^2} - \frac{1}{\tau_{el}\tau_{mek}} - \frac{k_t k_b}{L_q J}}$$

$$\tau_1 > \tau_2 > T_d$$

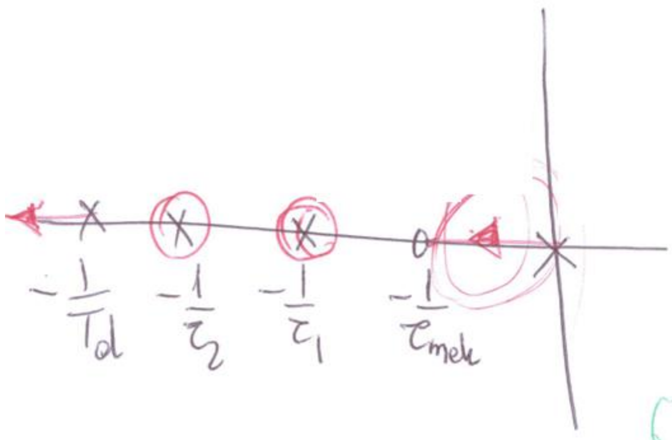
$$G_I(s) = \frac{K_A (1 + s\tau_2)(1 + s\tau_d)}{s}$$

$$GH(s) = \frac{K_A (1 + s\tau_2)(1 + s\tau_d)}{s} \cdot \frac{K (s\tau_m + 1)}{(s\tau_1 + 1)(s\tau_2 + 1)} \frac{K_{dc}}{(1 + s\tau_d)}$$

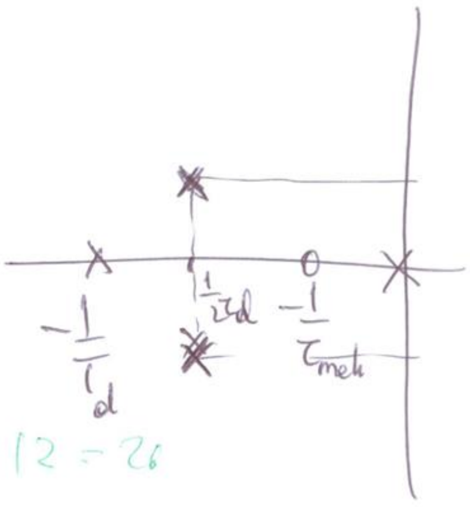
$$s\tau_m + 1 \simeq s\tau_m$$

$$GH(s) = \frac{K_A K K_d \tau_m}{(1 + s\tau_d)}$$

$$\frac{I_q(s)}{I_q^{ref}(s)} = \frac{GH}{1 + GH} = \frac{K_{AI}}{1 + s\tau_i}$$



vagy



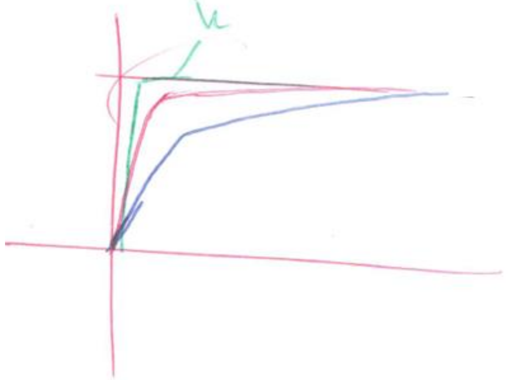
$$6+8+12=26$$

$$(s+2)(s+3)(s+4)$$

$$(s^2+5s+6)(s+4)$$

$$s^3+9s^2+26s+24$$

$$s^3+(a_1+a_2+a_3)s^2+(a_1a_2+a_1a_3+a_2a_3)s+a_1a_2a_3$$



$$s(s+4)$$

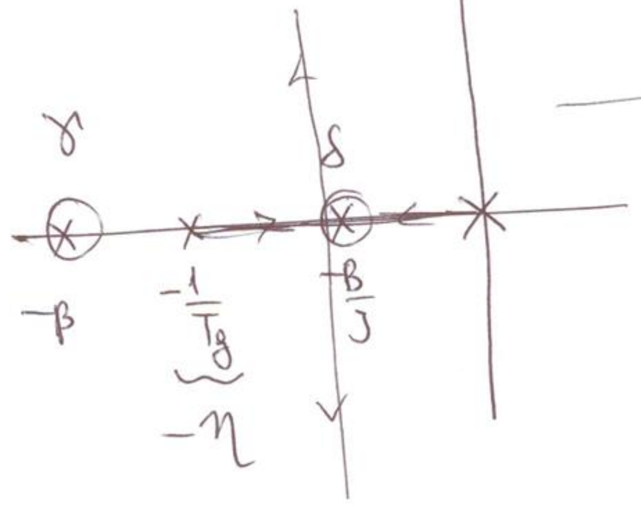
$$s^2+4s$$

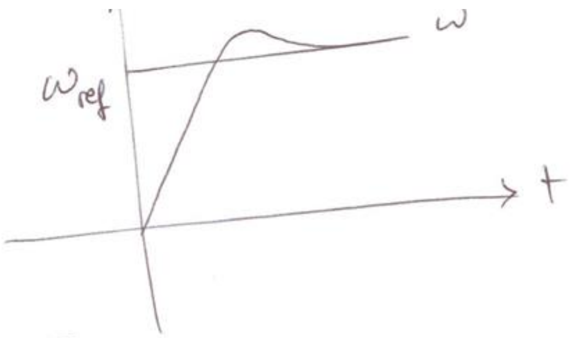
$$G_\omega(s) = \frac{(s+\delta)\,K_\omega\,(s+\gamma)}{s}$$

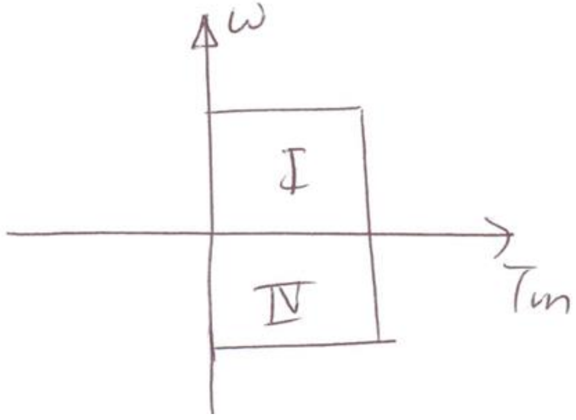
K_2, K_3, η

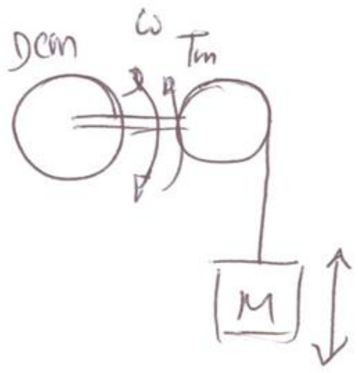
$$T(s) = \frac{\dfrac{\frac{K_\omega K_1 K_t}{J}(s+\delta)(s+\gamma)}{(s+\beta)\,s\,\left(s+\frac{B}{J}\right)}}{1+\dfrac{\frac{K_\omega K_1 K_t}{J}(s+\delta)(s+\delta)}{(s+\beta)\,s\,\left(s+\frac{B}{J}\right)}\cdot\dfrac{K_g/T_g}{\left(s+\frac{1}{T_g}\right)}} = \frac{\frac{K_2}{s}}{1+\frac{K_3}{s(s+\eta)}}$$

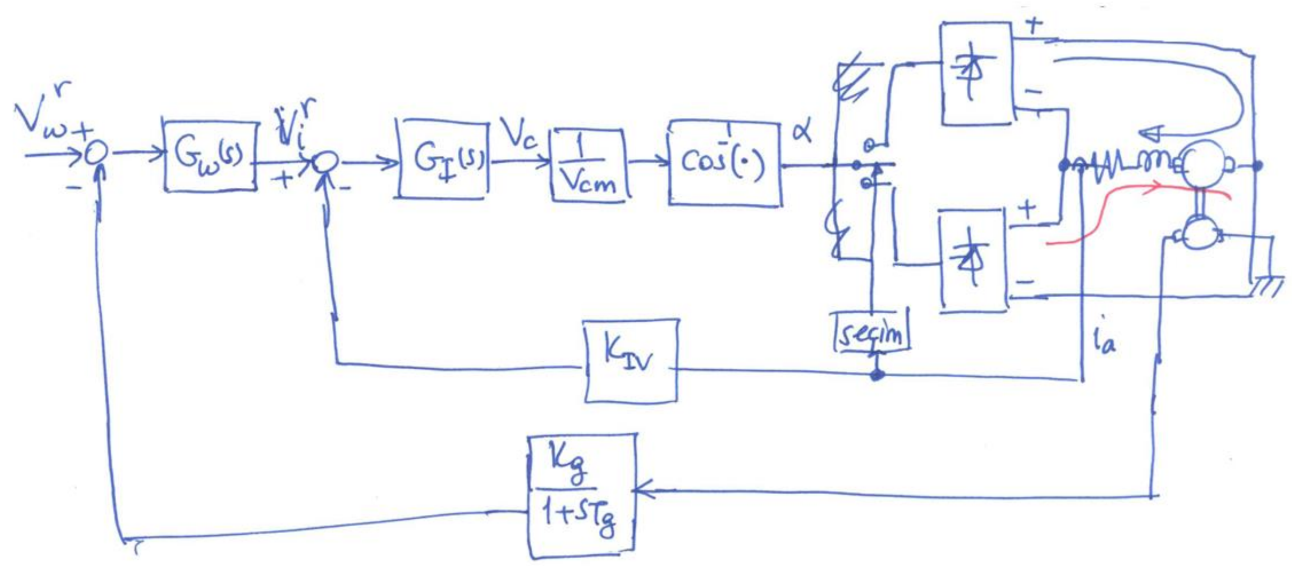
$$T(s) = \frac{K_2(s+\eta)}{s^2+\eta s+K_3}$$


γ
δ
−β
−1/Tg
−B/J
−η

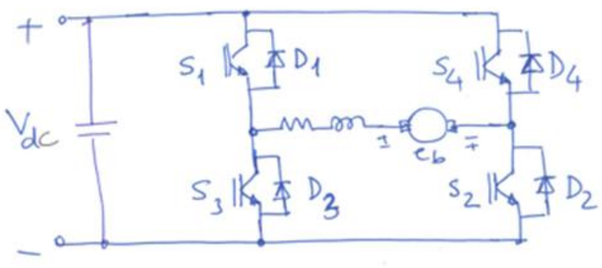

ω
ωref
t


ω
I
IV
Tm

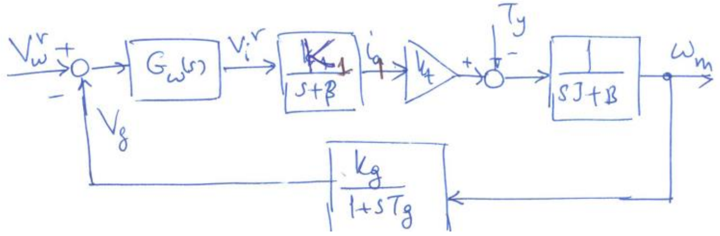

Dcm
ω
Tm
M



Vω r
Gω(s)
Vi r
GI(s)
Vc
1/Vcm
cos(·)
α
seçim
KIV
ia
Kg/(1+sTg)

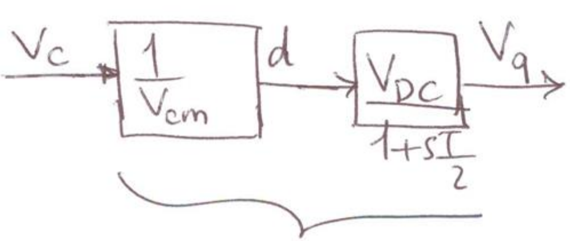


Vdc
S1 D1
S4 D4
S3 D3
S2 D2
eb



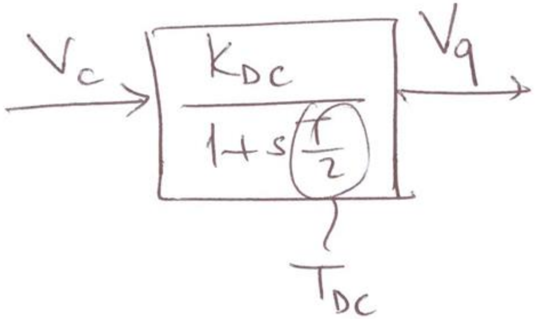
Vω r
Gω(s)
Vi r
K1/(s+β)
iq
Kt
Ty
1/(sJ+B)
ωm
Vg
Kg/(1+sTg)

duty cycle $d = \frac{T_{on}}{T} \rightarrow (dT = T_{on})$

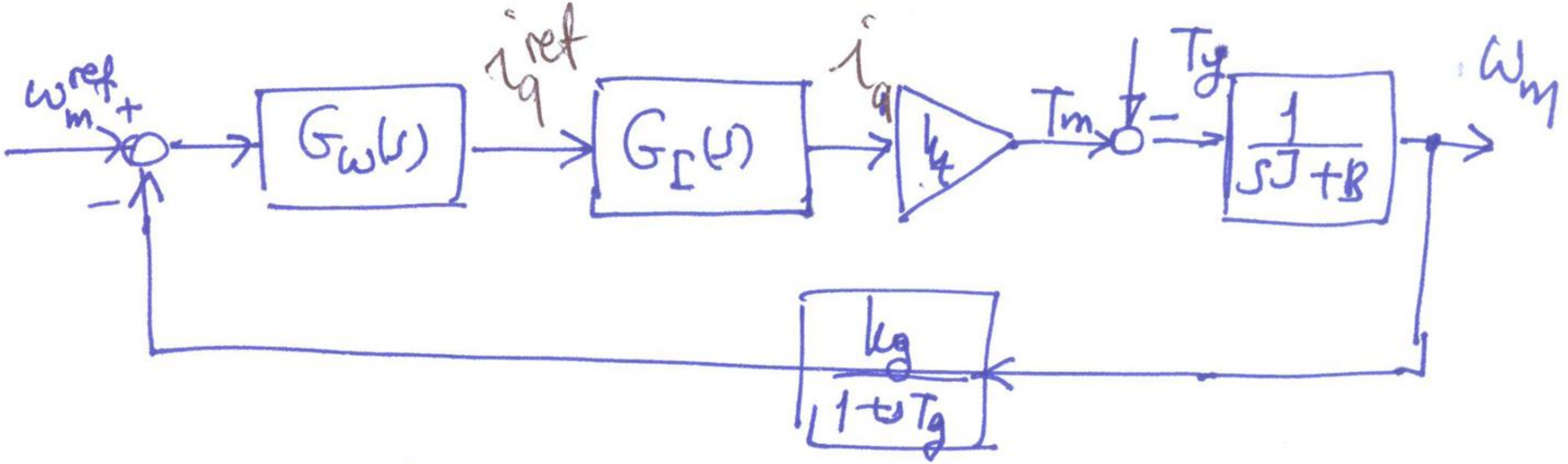


$$\frac{V_{DC}}{V_{cm}} = k_{DC}$$

$$I_q = \frac{V_q - E_b}{R_q} = \frac{d V_{dc} - E_b}{R_q}$$



$$T_m = k_t I_q = \frac{k_t (d V_{dc} - E_b)}{R_q}$$



$$G_\omega(s) = \frac{k_\omega\,(s+\delta)(s+\gamma)}{s}$$

PID

Brushed Permanent Magnet DC Servo Motors

1. Construction and Principle:

•**Permanent Magnets:** These motors use permanent magnets to create the magnetic field. This design simplifies the motor and eliminates the need for external field excitation.

•**Brushes and Commutator:** The brushes provide current to the commutator, which in turn energizes different armature coils as the motor rotates, ensuring continuous rotation.

2. Operation:

•When voltage is applied, the current flowing through the armature interacts with the magnetic field from the permanent magnets, creating a force that rotates the armature.

Drive and Control Methods

1. DC-DC Choppers:

•**For Small Power:** Power MOSFETs are typically used. MOSFETs are efficient and suitable for lower power applications due to their fast switching speed and lower losses.

•**For Higher Power:** IGBTs (Insulated Gate Bipolar Transistors) are employed. IGBTs combine the high-current handling capability of a bipolar transistor with the ease of control of a MOSFET.

2. Voltage Control:

•**Adjustable Output Voltage:** The output voltage of the DC-DC choppers, usually fed from a constant voltage source, needs to be adjusted within the range of -Vda < Vq < Vda. This adjustment is crucial for controlling the motor speed.

•**Operating Regions:** The motor operates mainly in the I and III regions, as per your provided information. This implies that the motor works effectively across a range of voltages and speeds.

3. Speed Control:

•**PWM (Pulse Width Modulation):** A common method to control the speed of these motors is through PWM, where the average voltage and thus the speed of the motor is controlled by adjusting the duty cycle of the voltage applied to the motor.

4. Feedback Mechanisms:

•To achieve precise control, feedback mechanisms like encoders or tachometers are often used. These provide real-time feedback on the motor's position or speed, enabling more accurate control.

Application and Advantages

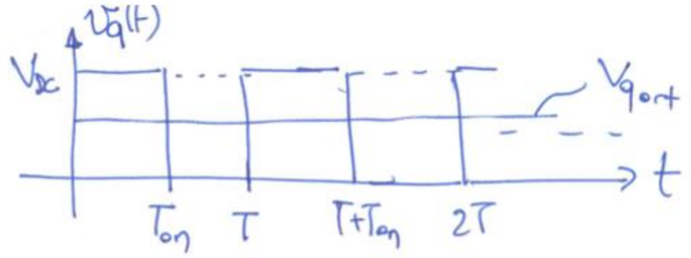
•**Precision and Simplicity:** These motors are popular in applications requiring precise control of speed and position, such as in robotics and automation.

•**Reliability:** The simplicity of their design makes them reliable and easy to maintain, albeit with the need to periodically replace brushes.

Challenges

•**Brush Maintenance:** The brushes in these motors wear out over time and need regular replacement.

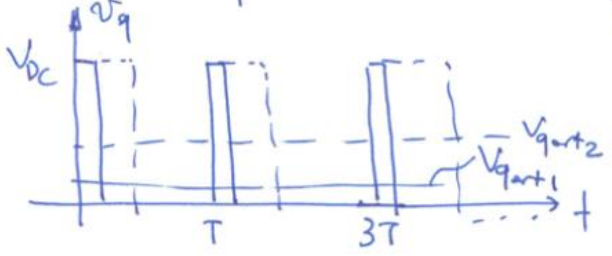
•**Electrical Noise:** Brushed motors generate more electrical noise due to the commutation process.

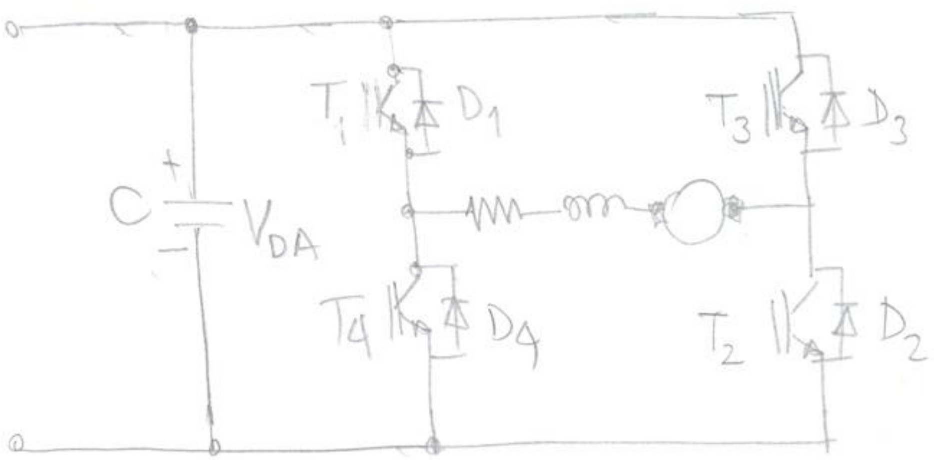


$$V_{qort} = \frac{T_{on}}{T} \cdot V_{DC}$$

$V_{qort} = 0 \qquad T_{on} = 0$

$V_{qort} = V_{DC} \qquad T_{on} = T$

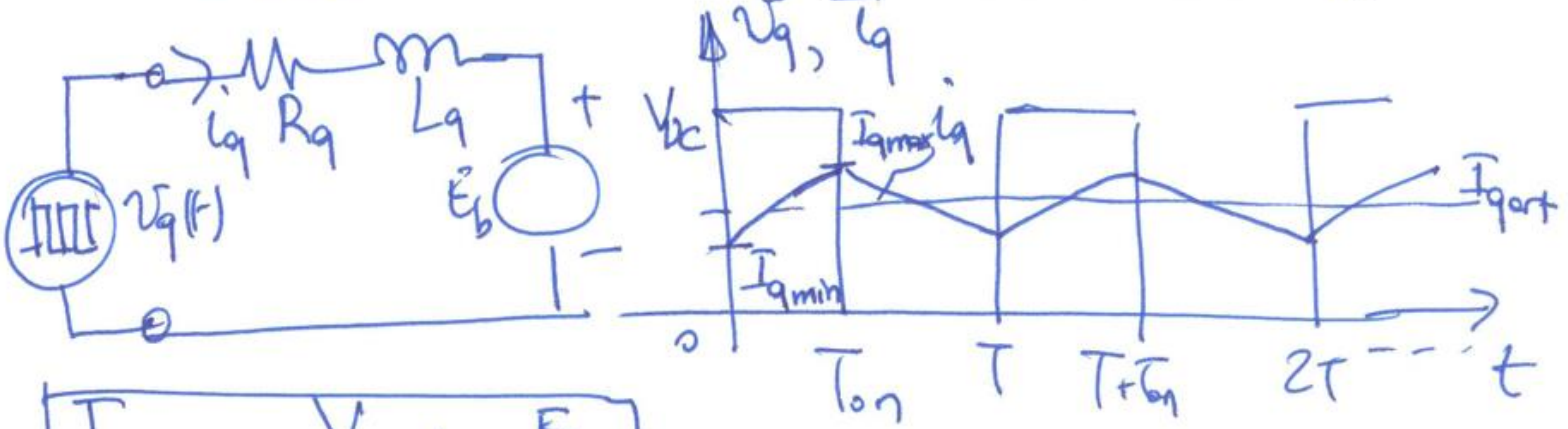




$$\frac{T_{on}}{T} = d \quad \text{duty cycle}$$

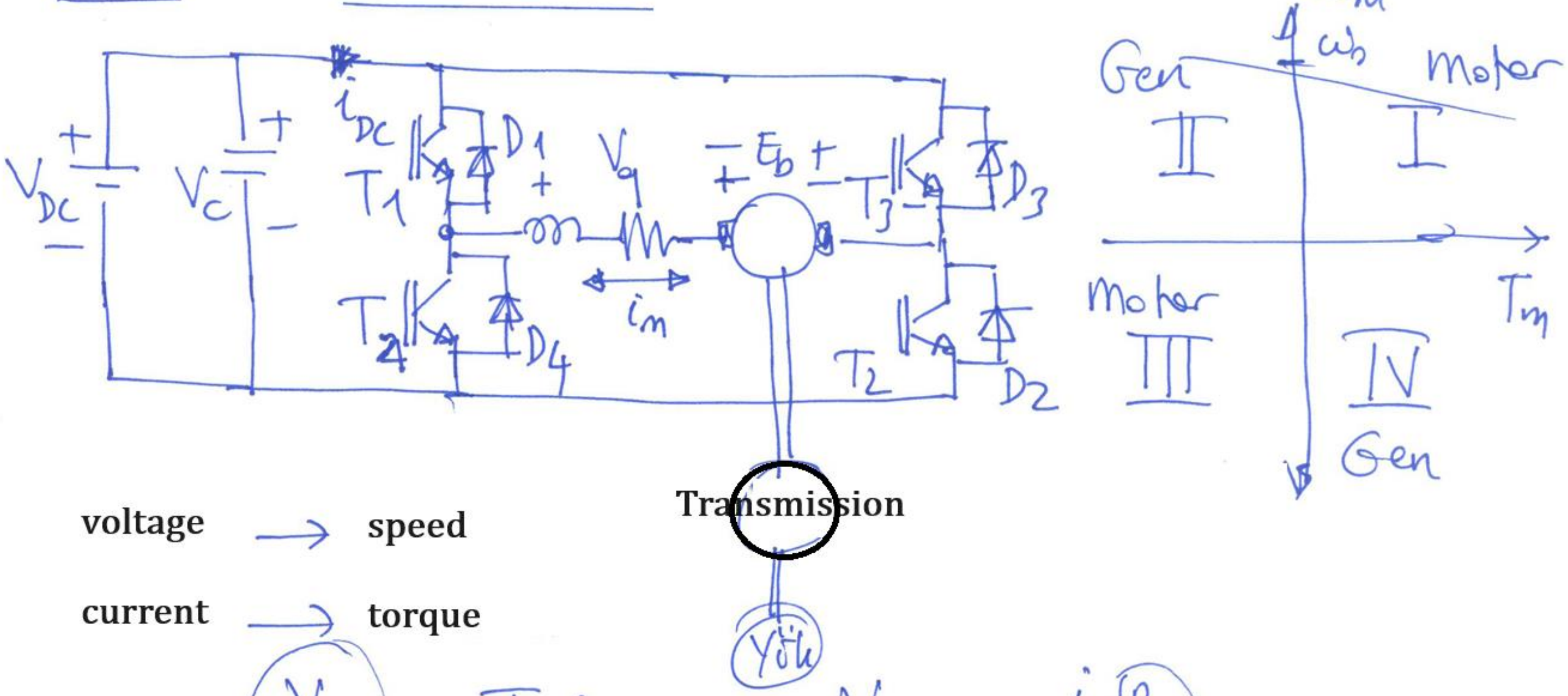
$$0 \leq d \leq 1$$

$$V_{qort} = d \cdot V_{DC}$$



$$I_{qort} = \frac{V_{qort} - E_b}{R_q}$$

4 DC- DC



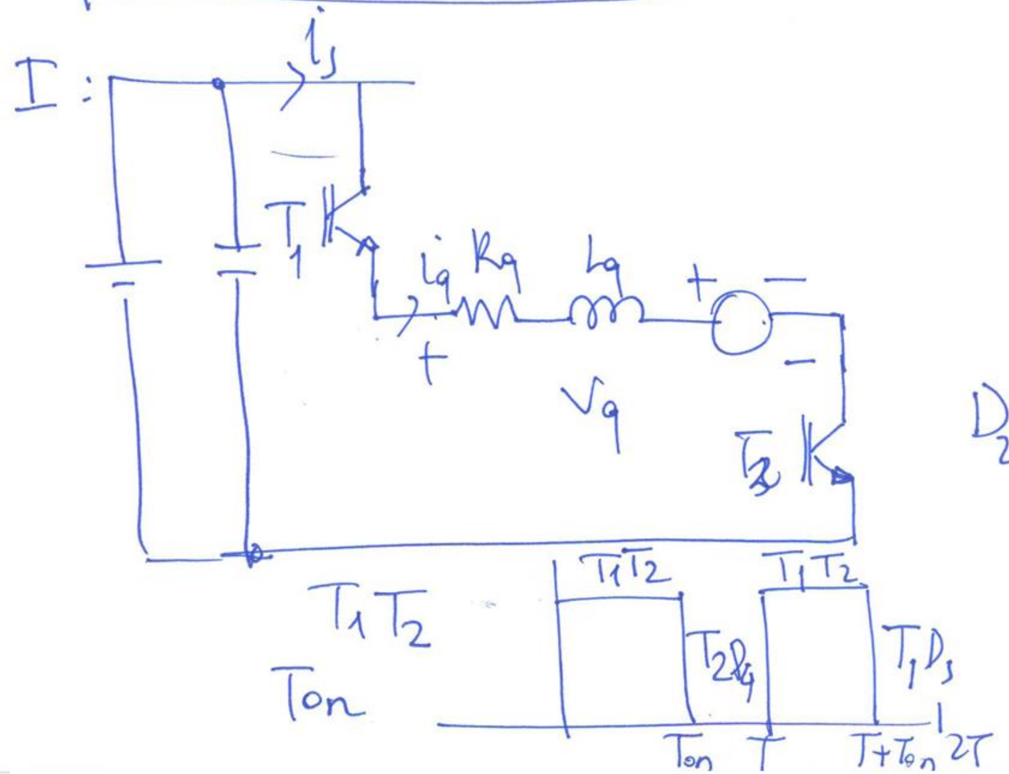
voltage → speed

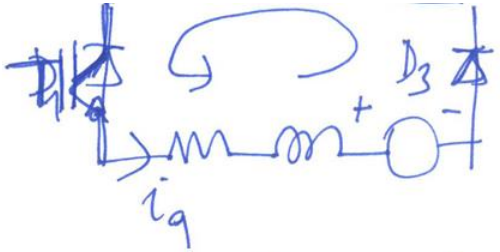
current → torque

$$\omega = \underbrace{\frac{V_q}{K_b}}_{\omega_0} - \frac{T_m R_q}{k_t k_b} = \frac{V_q}{K_b} - \frac{i_q R_q}{K_b}$$

$$\omega = \omega_0 - b\,T_m$$

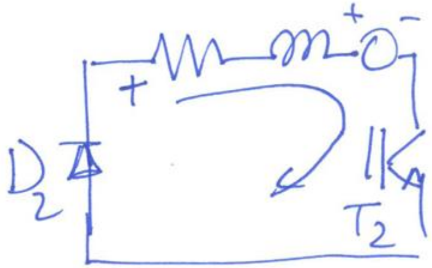
$$\boxed{\omega = a V_q - b i_q}$$

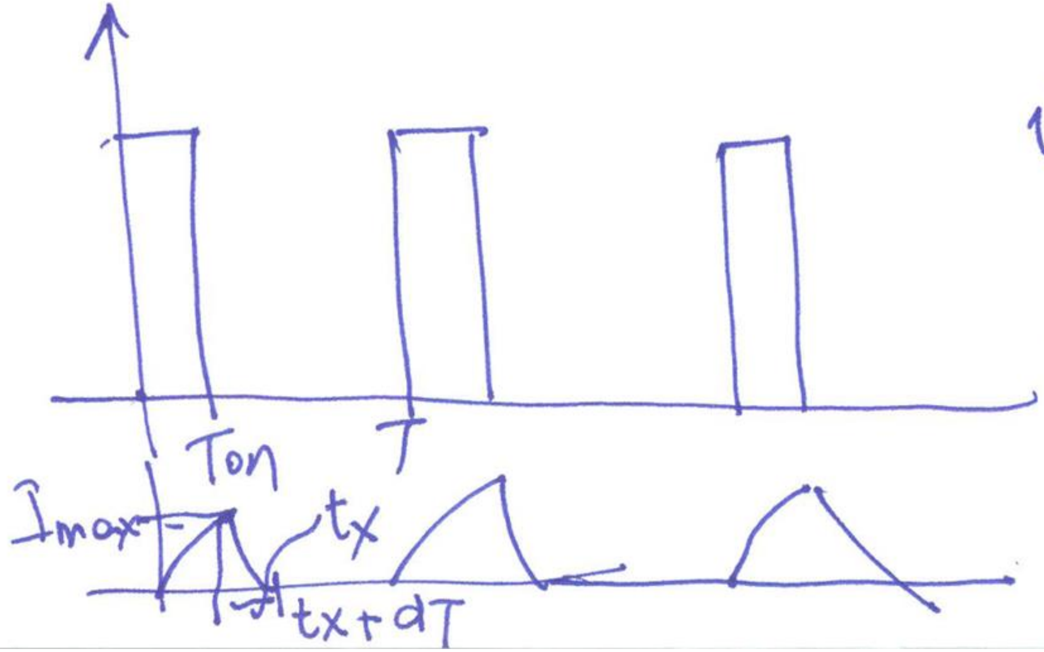




$V_q > 0$ ($\omega_0 > 0$)

$i_q > 0$ ($T_m > 0$)

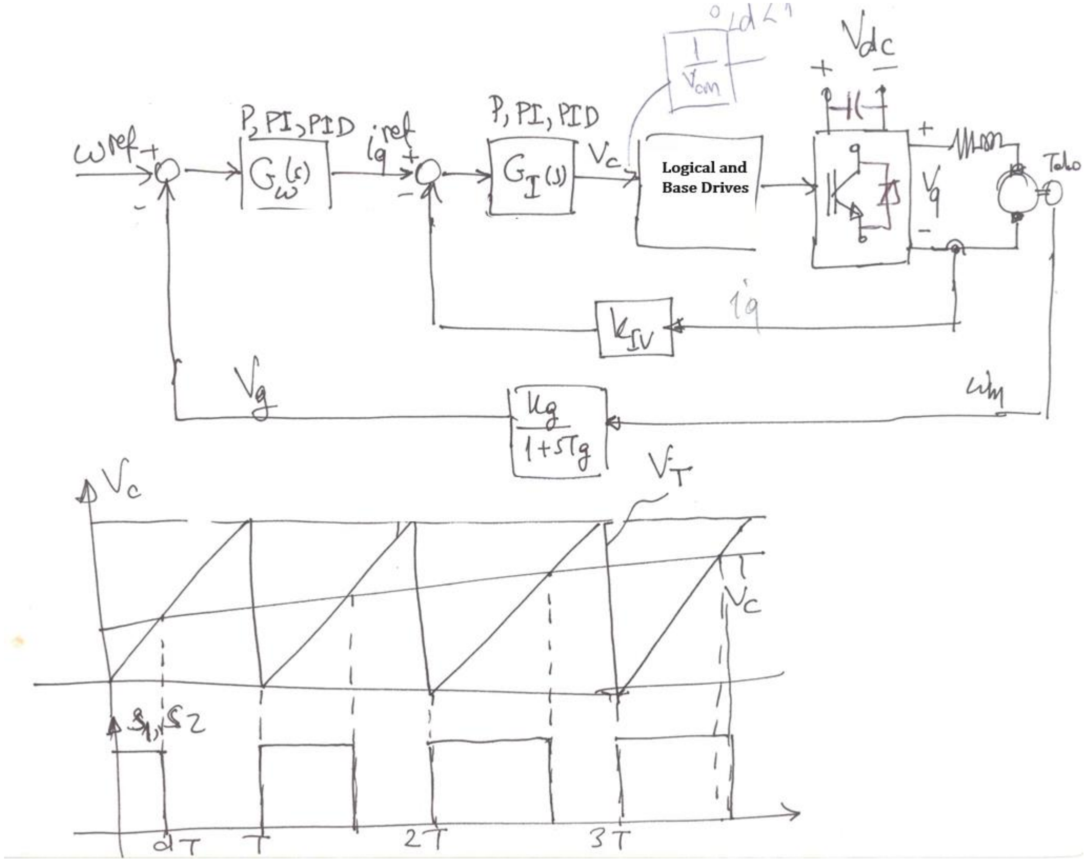




$$i_a(t) = \frac{V_{dc} - E_b}{R_a}\left(1 - e^{-t/\tau_a}\right) \quad 0<t<dT$$

$$i_a(t) = I_{max}\, e^{-(t-dT)/\tau_a} - \frac{E_b}{R_a}\left(1 - e^{-(t-dT)/\tau_a}\right) \quad dT \le t < t_x + dT$$

$$i_a(t) = 0 \quad t_x + dT \le t < T$$



$$G_{DA} = \frac{K_{DA}}{sT_{DA}+1} = \frac{K_{DA}}{s\frac{T}{2}+1}$$

PWM Generator
+ Power

Example:

A brushed DC motor lifts a load via a DC-DC converter.

Given parameters:

$$V_{q,n} = 100V$$

$$n_n = 1000 rpm$$

$$R_q = 1\Omega$$

$$L_q = 10 mH$$

$$k_b = k_t = 0.9$$

$$V_{DC} = 120V$$

$$T_L = 5 Nm (constant)$$

$$B = 0$$

$$J = 1 kgm^2$$

a) Find the duty cycle for n=0 and n=n_nominal.
b) At n_nominal speed, evaluate Imin and Imax
T=1ms (switching frequency)

n=0 Imin

$$V_q = d V_{DC} = R_q i_q + k_b \omega \quad (\omega = 0)$$

$$d \cdot 120 = 1 \cdot I_q$$

$$T_L = T_m = k_t i_q = 5$$

$$i_q = \frac{5}{0.9} = 5.55 A$$

$$d = \frac{5.55}{120} = 0.046 = d_{min}$$

$n = 1000$ rpm $\quad \omega = 104.72$ rad/sec

$T_L = T_m \longrightarrow i_q = 5.55A$

$$dV_{DC} = 0.9 \cdot 104.72 + 5.55$$

$$d = \frac{0.9 * 104.72 + 5.55}{120} = 0.83 ~~17~~ = d_{max}$$

$$\frac{~~E_b~~}{~~R_q~~} = \frac{V_{DC}}{R_q} \frac{(e^{dT/\tau_q} - 1)}{(e^{T/\tau_q} - 1)}$$

$$\frac{E_b}{V_{DC}} = \frac{e^{dT/\tau_q} - 1}{e^{T/\tau_q} - 1}$$

$$E_b (e^{T/\tau_q} - 1) = V_{DC} e^{dT/\tau_q} - V_{DC}$$

$$\frac{E_b (e^{T/\tau_q} - 1) + V_{DC}}{V_{DC}} = e^{dT/\tau_q}$$

$$\frac{E_b}{V_{DC}}\left(e^{T/\tau_q}-1\right)+1 = e^{dT/\tau_q}$$

$$\ln\left(\frac{E_b}{V_{DC}}\left(e^{T/\tau_q}-1\right)+1\right) = \frac{dT}{\tau_q}$$

$$d_c = \frac{\tau_q}{T}\ln\left[\left(\frac{E_b}{V_{DC}}\left(e^{T/\tau_q}-1\right)+1\right)\right]$$

$$I_{q_{av}} = 5.547 A$$

$f = 1 kHz$

$T = 10^{-3}$ san

$d = 0,8317$

$$I_{q_{min}} = \frac{120\left(e^{0.8317 * 10^{-3}/10 \cdot 10^{-3}} - 1\right)}{\left(e^{0.1}-1\right)} - \frac{0.9 * 104.72}{1}$$

$$\tau_q = \frac{L_q}{R_q} = 10 ms$$

$$I_{q_{min}} = 4.707 A$$

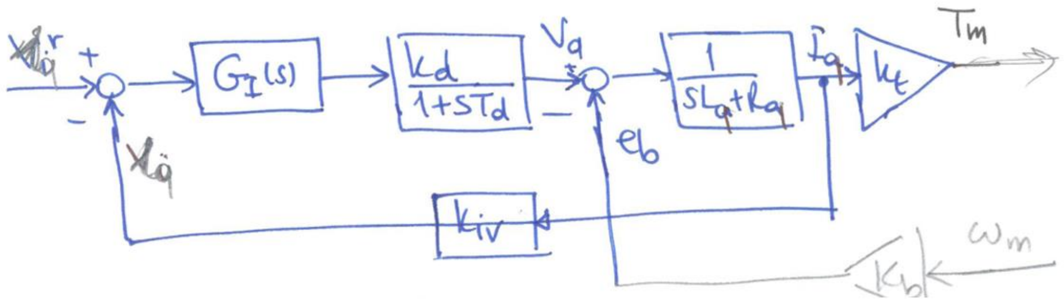
$$I_{q_{max}} = \frac{120\left(1-e^{-0.817/10}\right)}{\left(1-e^{-0.1}\right)} - 0.9 * 104.72$$

$$= 6.387 A$$

$$V_a(s) - k_b\,\omega_m(s) = (R_a + sL_a)\,I_a(s)$$

$$T_y(s) = 0 \qquad (sJ + B)\,\omega_m(s) = k_t I_a(s)$$

$$\omega_m(s) = \frac{k_t}{sJ + B} I_a(s)$$

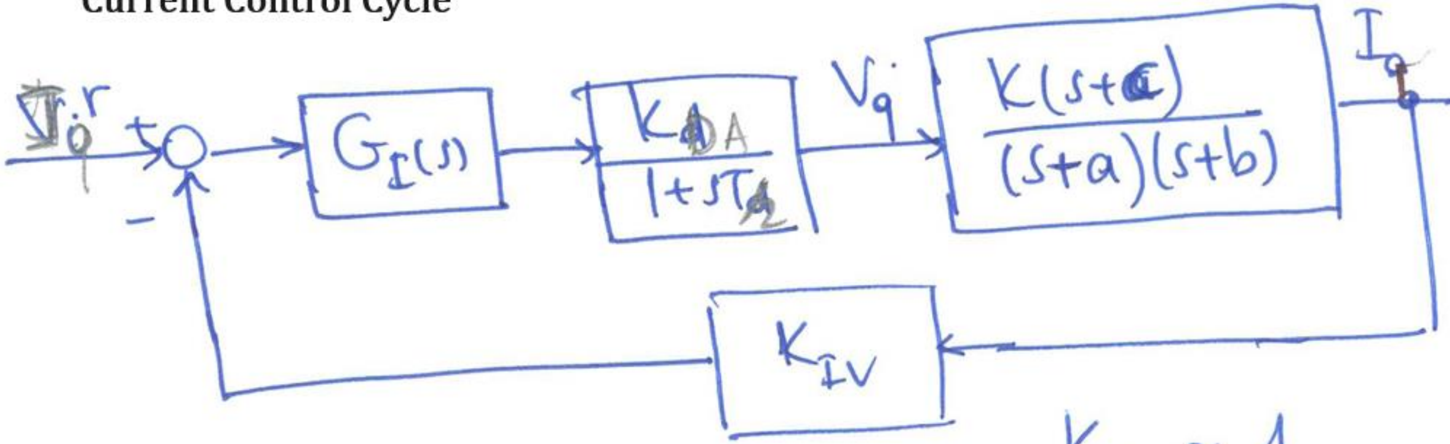


$$V_q(s) = \frac{(sL_q+R_q)(sJ+B)+K_tK_b}{sJ+B}\, I_q(s)$$

$$\frac{I_q(s)}{V_q(s)} = \frac{sJ+B}{s^2JL_q+s(JR_q+BL_q)+(R_qB+K_tK_b)} = \frac{K(s+c)}{(s+a)(s+b)}$$

$K=\frac{1}{L_q}$ $\quad c=\frac{B}{J}$ $\left(\frac{1}{\tau_{mek}}\right)$

Current Control Cycle



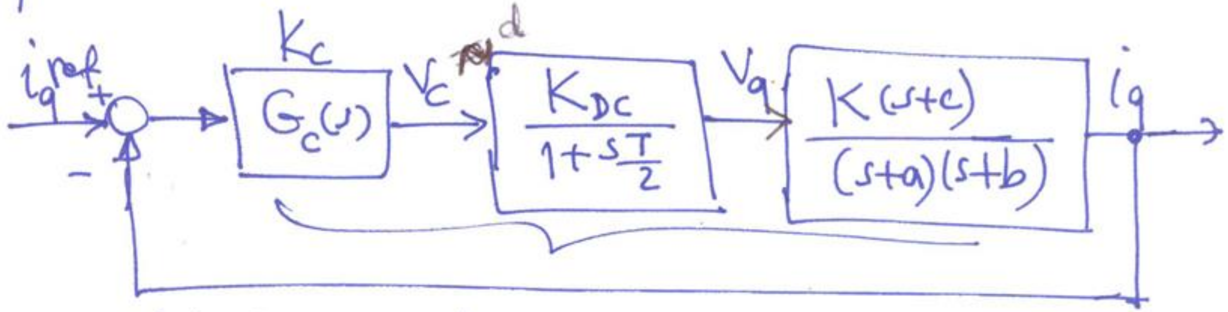
$$-\frac{1}{2}\left(\frac{R_q}{L_q}+\frac{B}{J}\right) \mp \sqrt{\left(\frac{R_a}{L_a}+\frac{B}{J}\right)^2 - \left(\frac{R_aB+K_tK_b}{L_aJ}\right)}$$

$\frac{1}{\tau_{el}}$ $\quad \frac{1}{\tau_{mek}}$

$K_{IV} \simeq 1$

DC-DC **Converter** $\dfrac{K_{DC}}{1+s\frac{T}{2}}$

$$\frac{i_q}{i_q^{ref}} = \frac{K(s+c)}{(s+a)(s+b)} \rightarrow$$



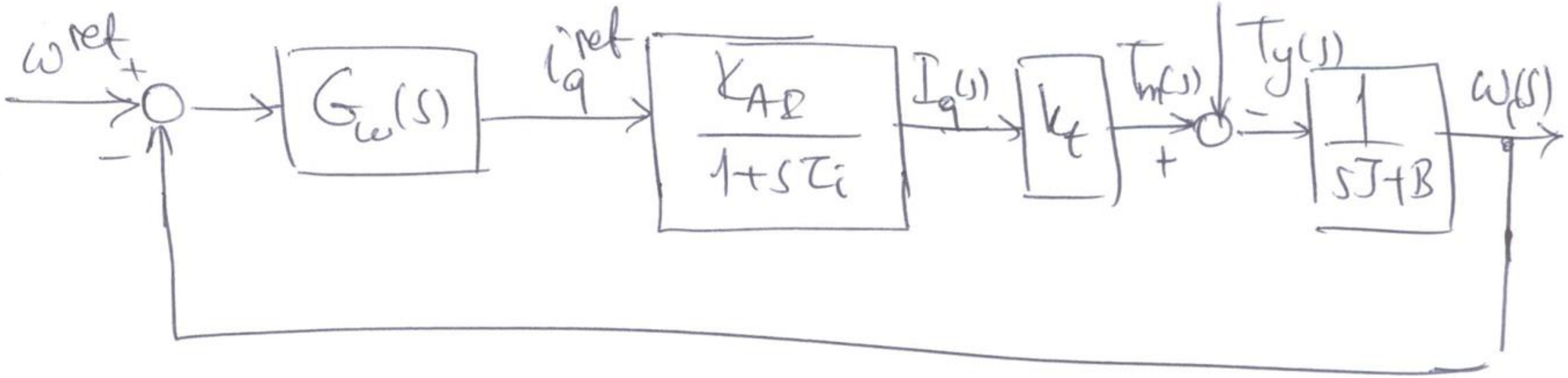
$$G_c(s) = \frac{K_c(s+a)(s+b)}{s}$$

PID

$$\frac{i_q}{i_q^{ref}} = \frac{\dfrac{K_c K_{DC} K (s+c)}{\left(1+s\frac{T}{2}\right)s}}{1 + \dfrac{K_c K_{DC} K (s+c)}{\left(1+s\frac{T}{2}\right)s}} = \frac{K_c K_{DC} K (s+c)}{s\left(1+s\frac{T}{2}\right) + K_c K_{DC} K (s+c)}$$

$$= \frac{K_c K_{DC} K (s+c)}{(s+\alpha)(s+\beta)} = G_I(s) \simeq \frac{K_1}{(s+\beta)}$$

Speed Control Loop



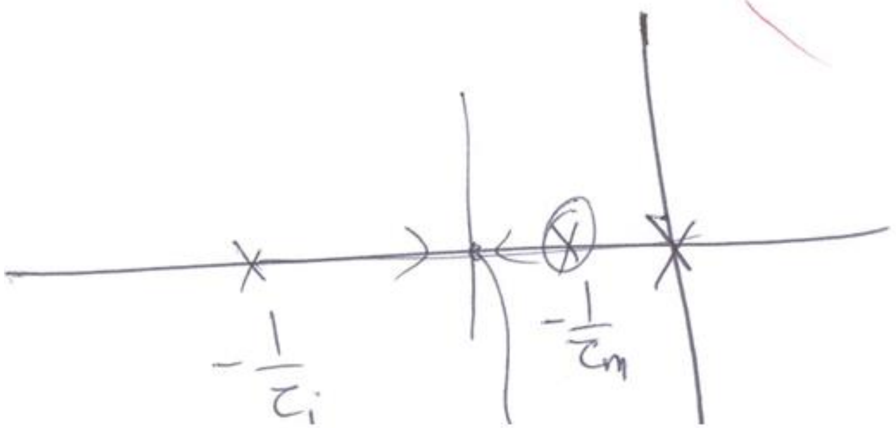
$T_y(s) = 0$

$$\frac{G_\omega(s)\, K_{AI} \cdot K_t}{(sJ+B)\,(s\tau_i+1)}$$

$$G_\omega(s) = \frac{K_\omega (1+\tau_\omega s)}{s}$$

$$\frac{K_\omega K_{AI} K_t (1+s\tau_\omega)}{s(sJ+B)(s\tau_i+1)} = \frac{K_\omega K_{AI} \frac{K_t}{B}(1+s\tau_\omega)}{s(s\tau_{mec}+1)(s\tau_i+1)}$$

$$\tau_\omega = \tau_m$$



Pulse Width Modulation (PWM) Basics

- **Definition**: PWM is a technique used in electronic circuits to control the amount of power delivered to a load. It achieves this by rapidly switching the power source on and off.
- **Application in Motor Control**: In electric motors, PWM is used to regulate the speed and torque by varying the average voltage sent to the motor.

DC-DC Converters

- **Role**: These converters are essential for transforming a source of direct current (DC) from one voltage level to another.
- **Importance in Motor Applications**: They are crucial in electric vehicles and various motor control applications, providing the required voltage levels for efficient operation.

Hysteresis Current Control

Now, let's focus on the core topic: Hysteresis Current Control.

Concept of Hysteresis in Current Control

- **Hysteresis Loop**: A hysteresis loop in the context of current control represents the allowable deviation of current from its desired value. When the actual current deviates from the setpoint by a certain threshold, the control system reacts.
- **Rapid Response**: This form of control is known for its fast response, making it ideal for applications where immediate current adjustment is necessary.

Integration with PWM and DC-DC Converters

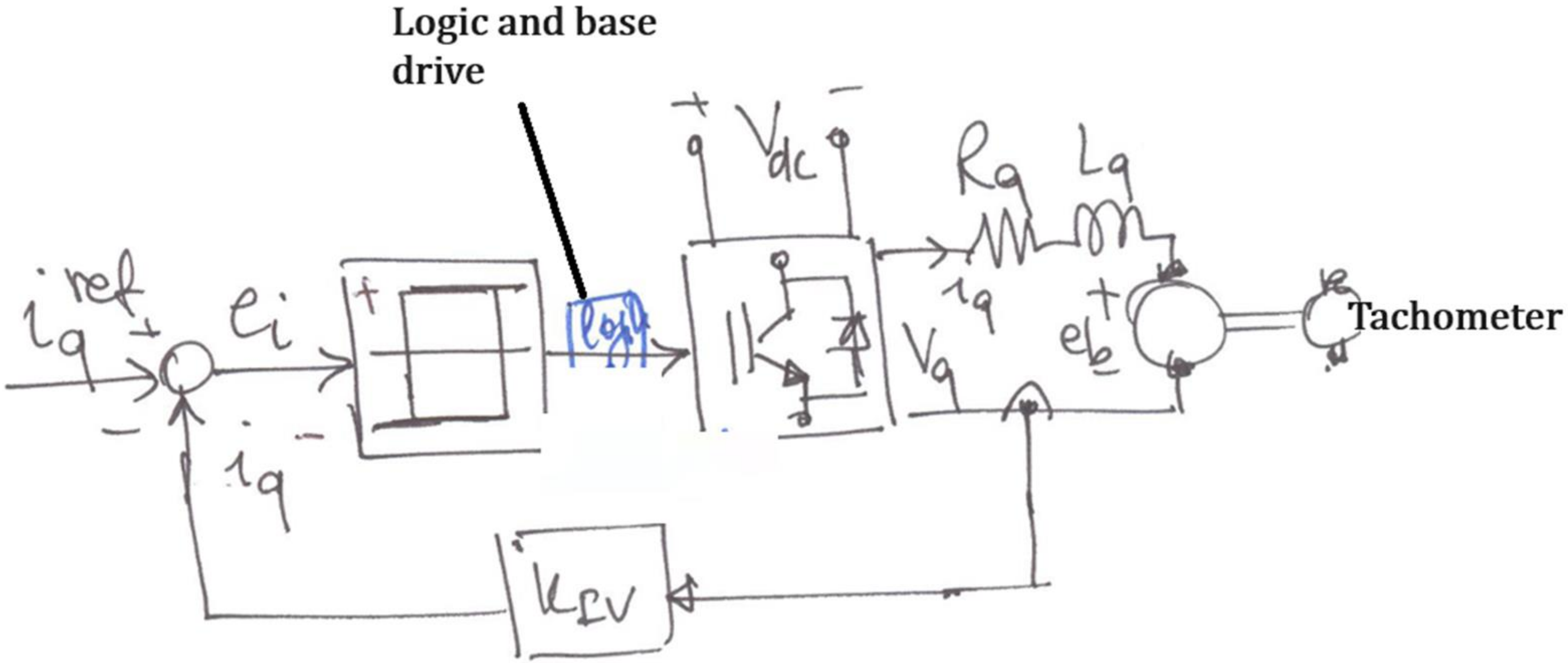
- **Duty Cycle Control**: In a PWM system, the duty cycle—the proportion of 'on' time to the regular interval or 'period' of time—directly affects the average power delivered to the load.
- **Average Current Control**: In a DC-DC converter, managing the average current is crucial. This is where hysteresis control plays a significant role. It ensures that the current stays within the desired range, adjusting the duty cycle of the PWM as needed.

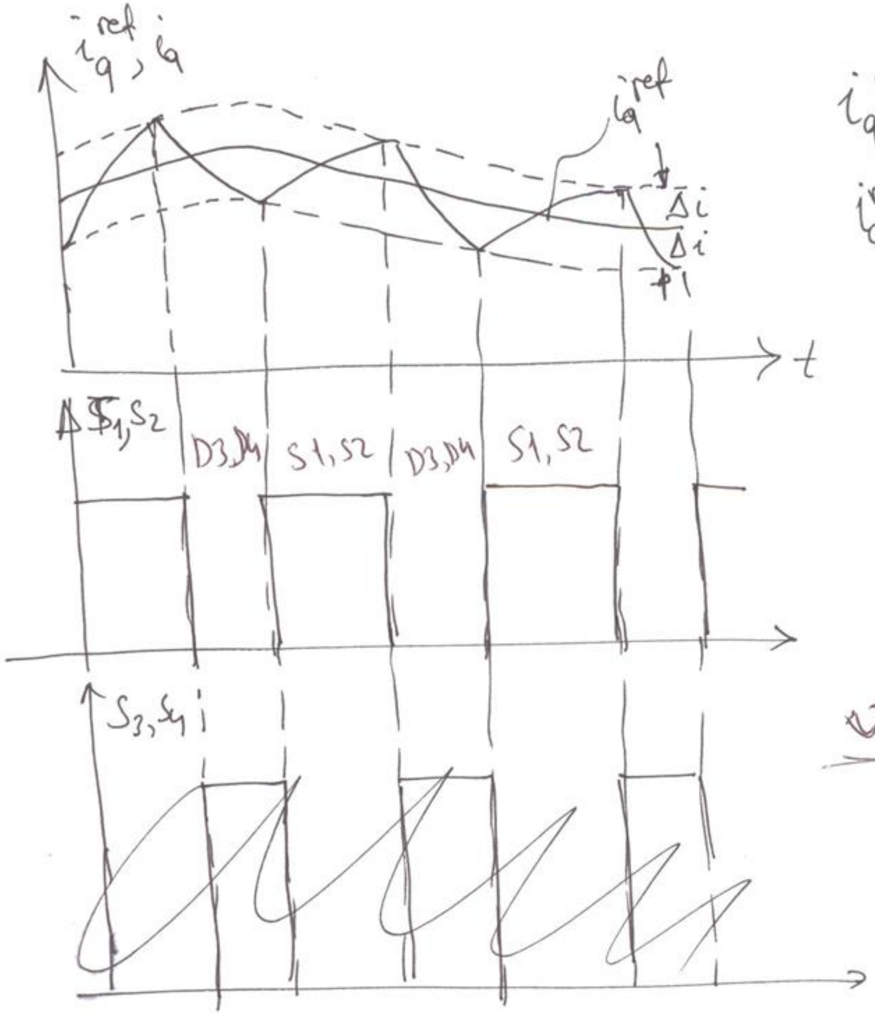
Applications and Advantages

•**Precision in Motor Control**: Hysteresis current control is particularly beneficial in scenarios where precise motor control is required. Electric vehicles, robotic arms, and precision machining tools are prime examples.

•**Efficiency**: By minimizing the deviation from the desired current, efficiency in energy usage is maximized, which is crucial in battery-operated systems like electric vehicles.

•**Reduced Noise and EMI**: The controlled approach to managing current can also lead to reduced electromagnetic interference (EMI) and noise, which is vital in sensitive electronic applications.





$i_q^{ref} - \Delta i \geq i_q \quad v_q = V_{dc}$

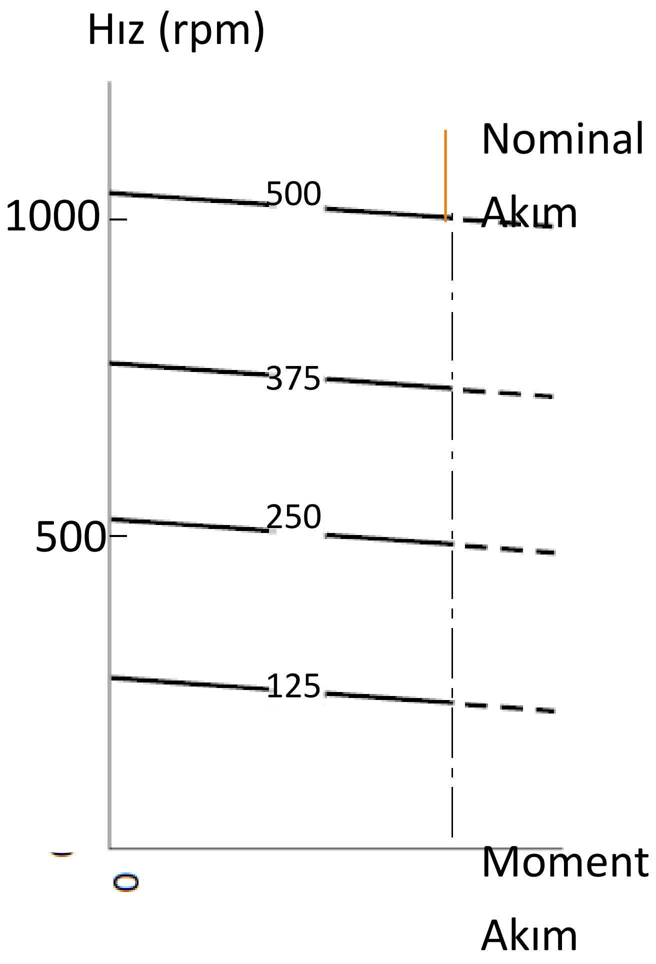
$i_q^{ref} + \Delta i \leq i_q \quad v_q = 0$

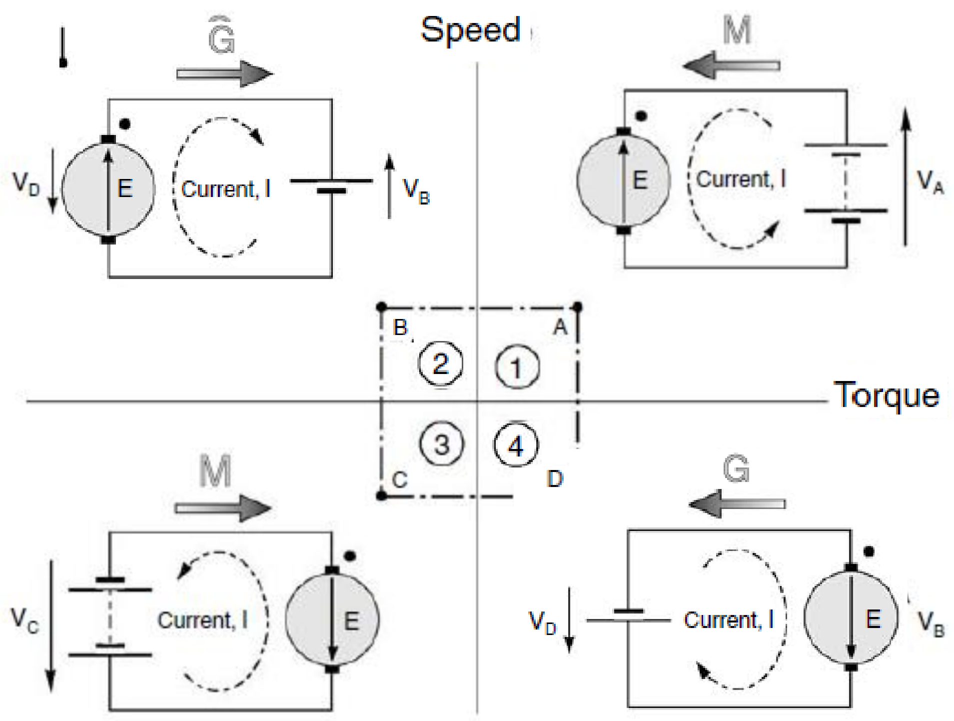
Δi

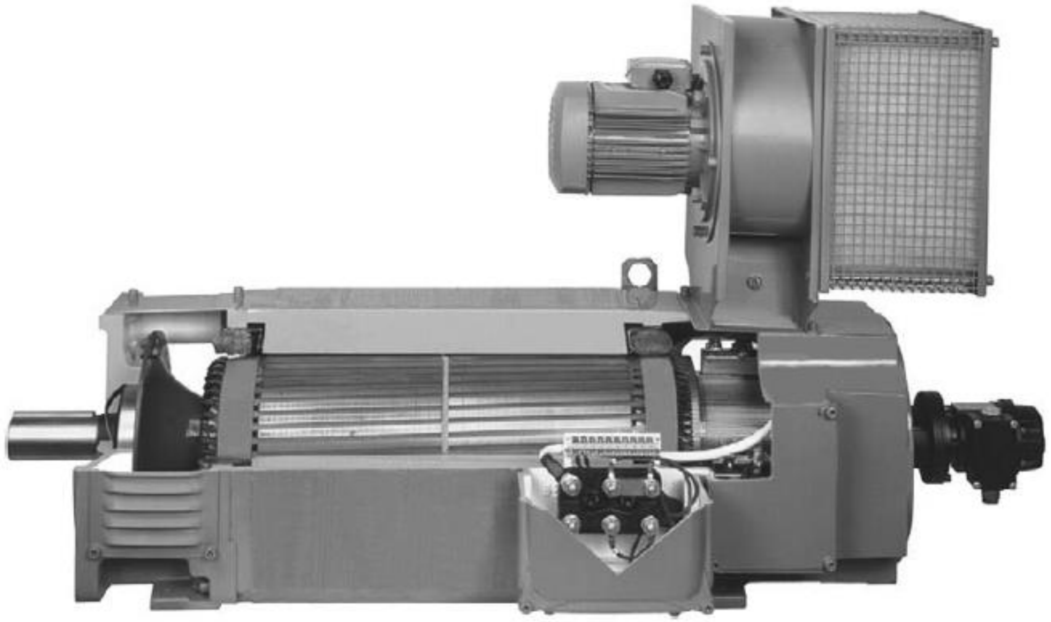
$v_q = K e_i$

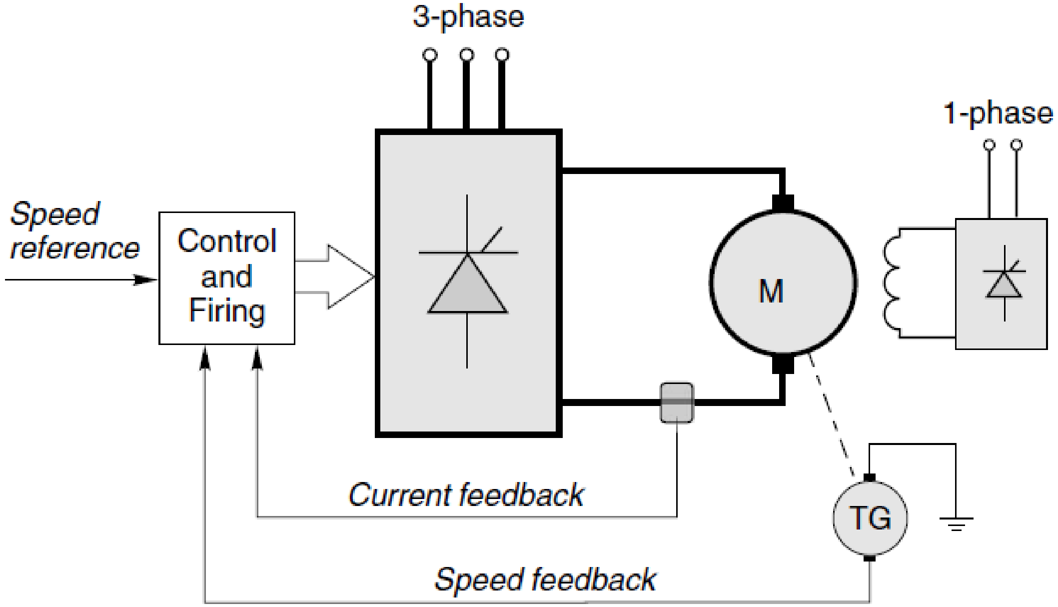
$|e_i| \leq \Delta i$

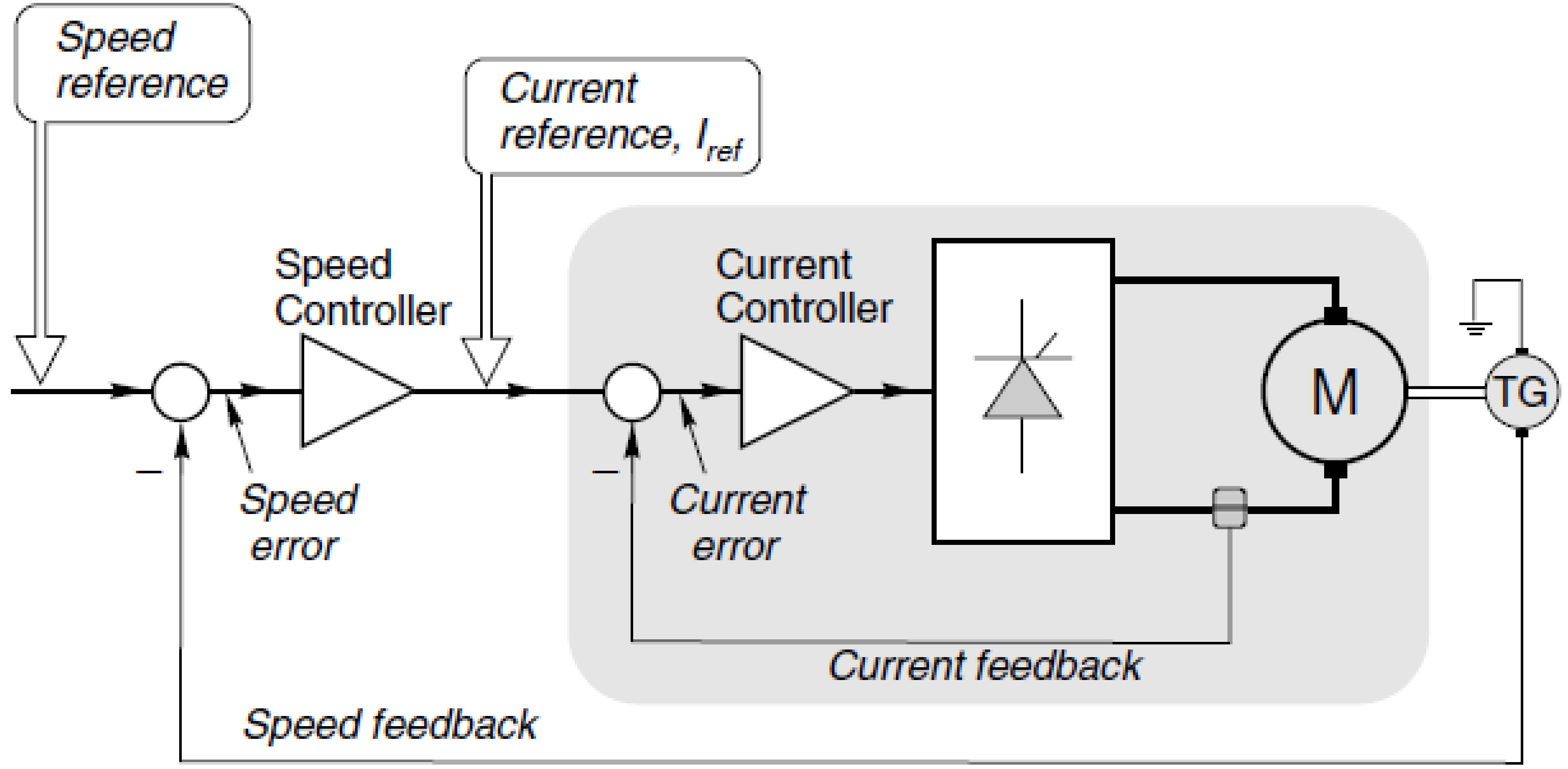
$K >>$

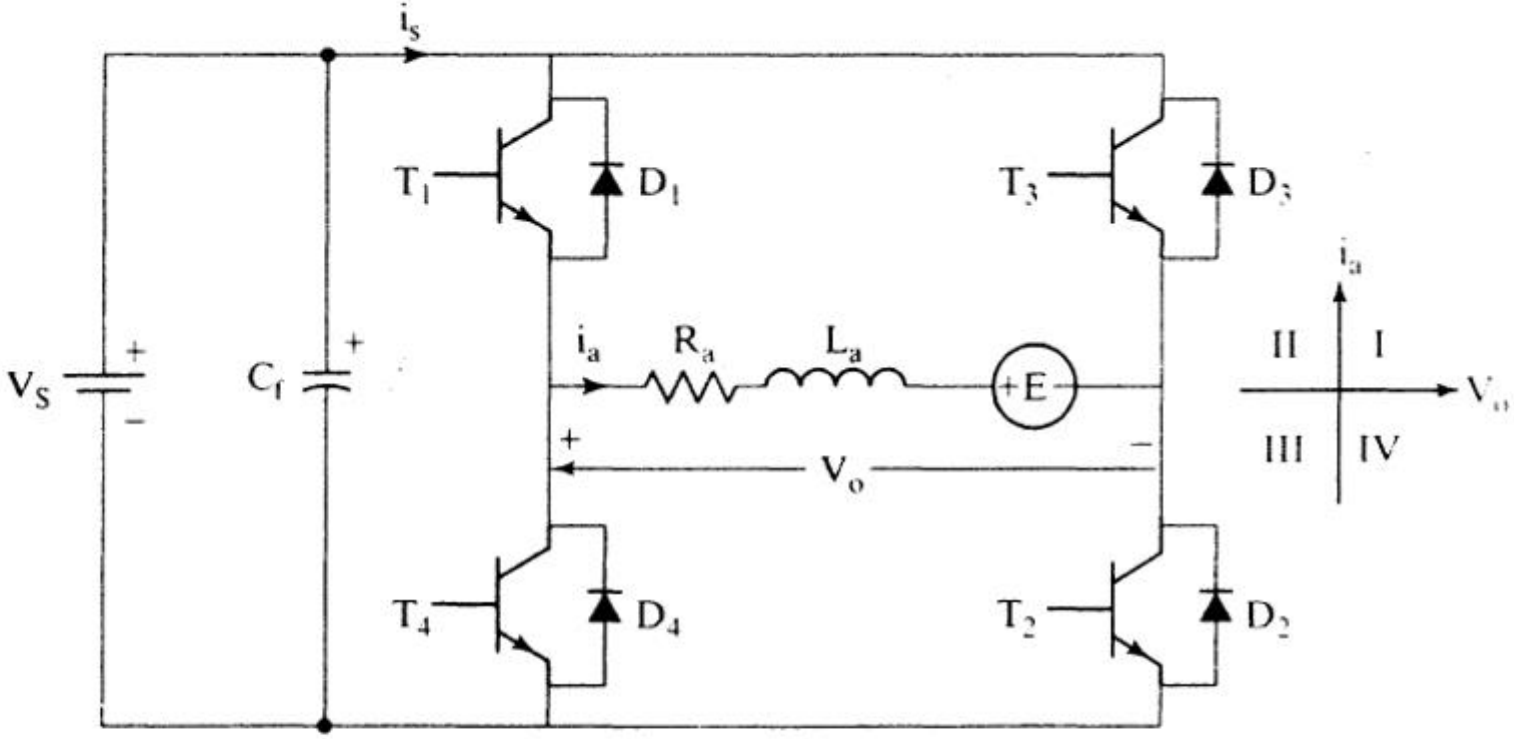

Hız (rpm)
Nominal
Akım
1000
500
500
375
250
125
0
Moment
Akım


G
Speed
M
E
Current, I
V_D
V_B
V_A
B
A
2
1
Torque
3
4
C
D
M
G
V_C

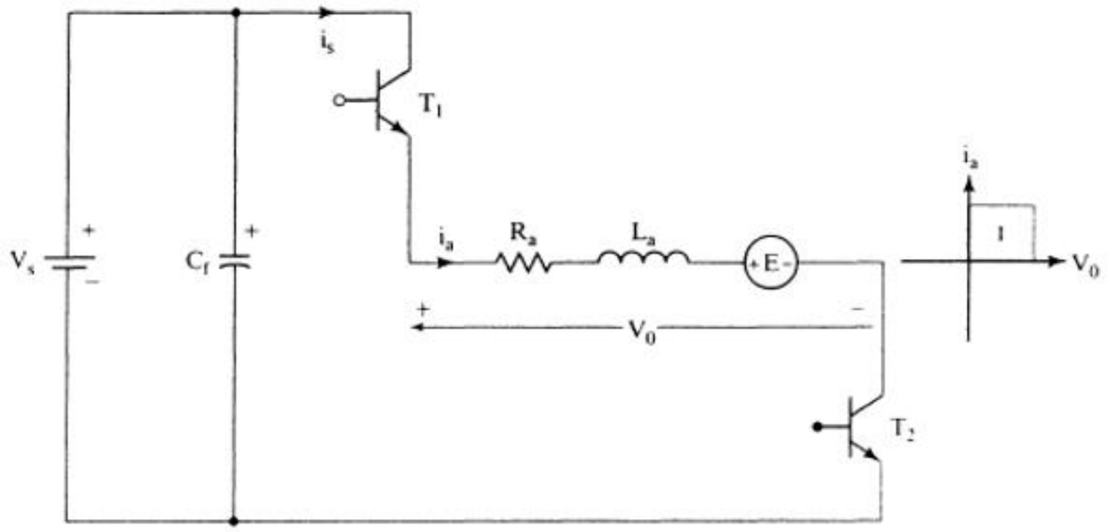



3-phase
1-phase
Speed
reference
Control
and
Firing
M
TG
Current feedback
Speed feedback

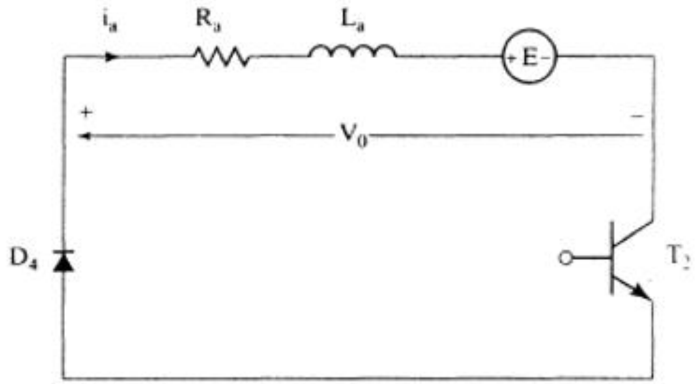

Speed reference
Current reference, I_{ref}
Speed Controller
Current Controller
M
TG
–
Speed error
–
Current error
Current feedback
Speed feedback



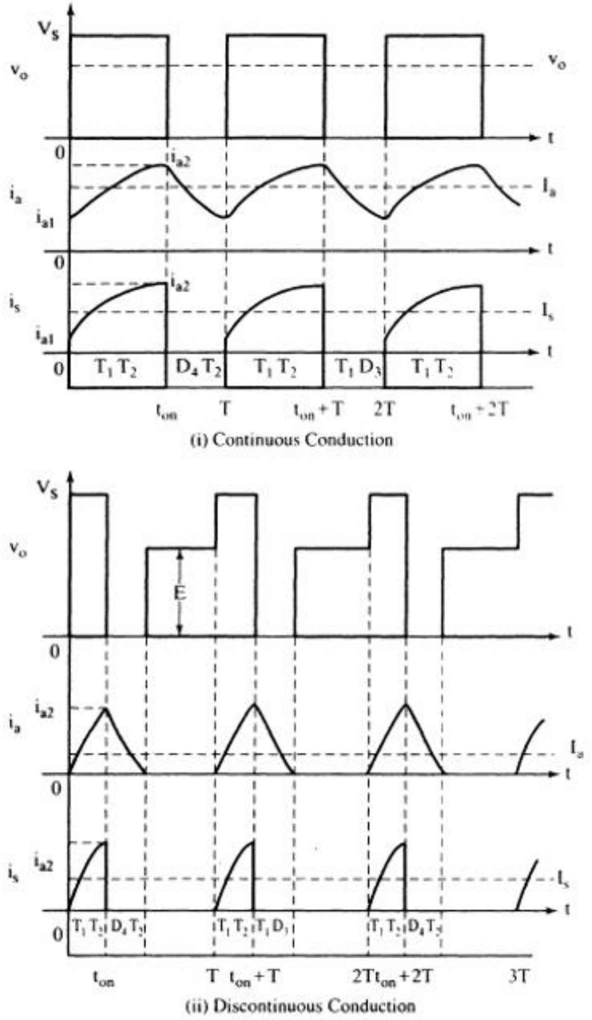
A four-quadrant chopper circuit



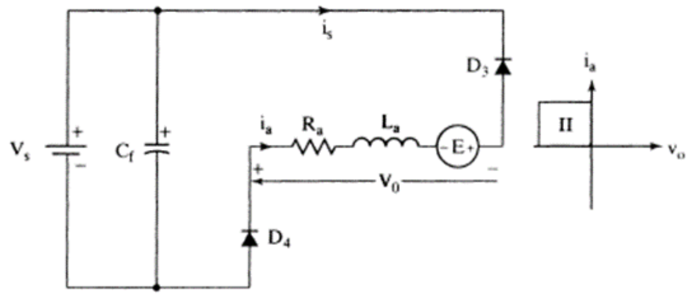
First-quadrant operation with positive voltage and current in the load



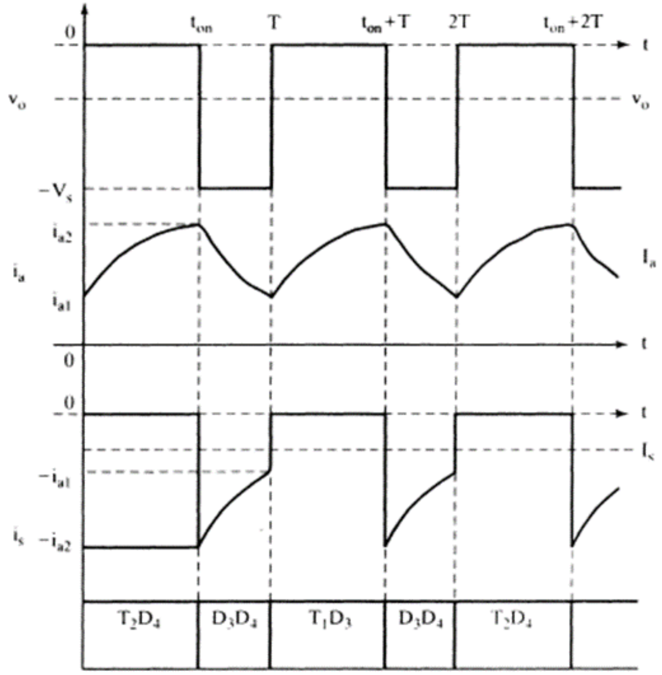
First-quadrant operation with zero voltage across the load



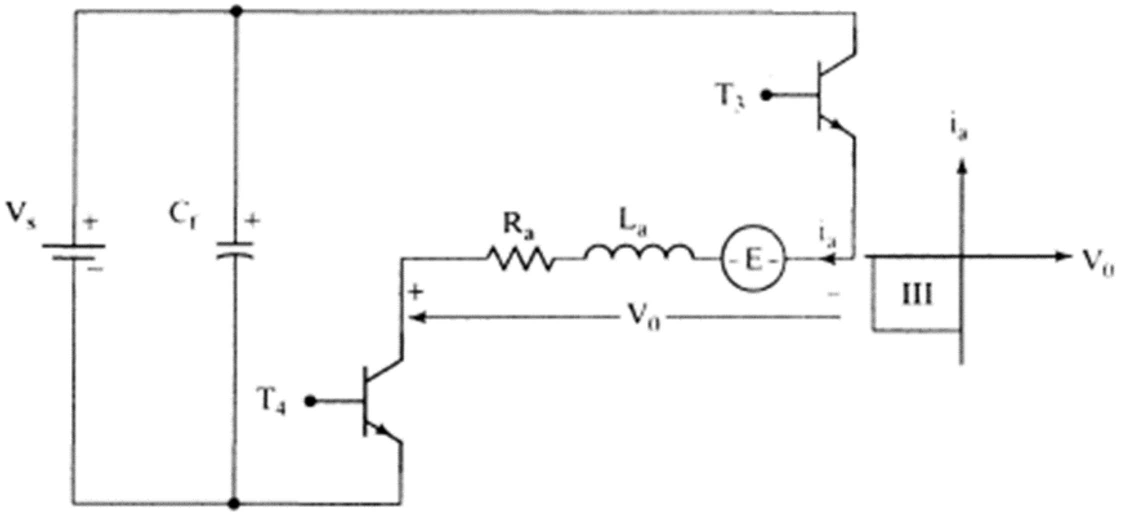
Voltage and current waveforms in first-quadrant operation



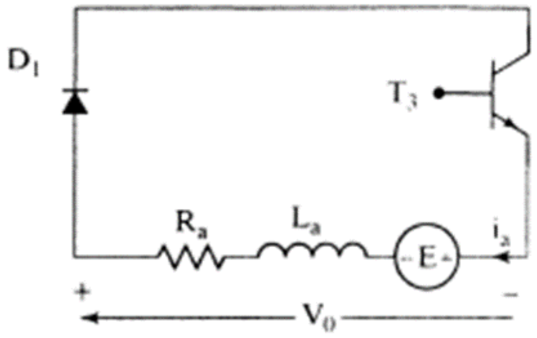
Second-quadrant operation, with negative load voltage and positive curren



Second-quadrant operation of the chopper

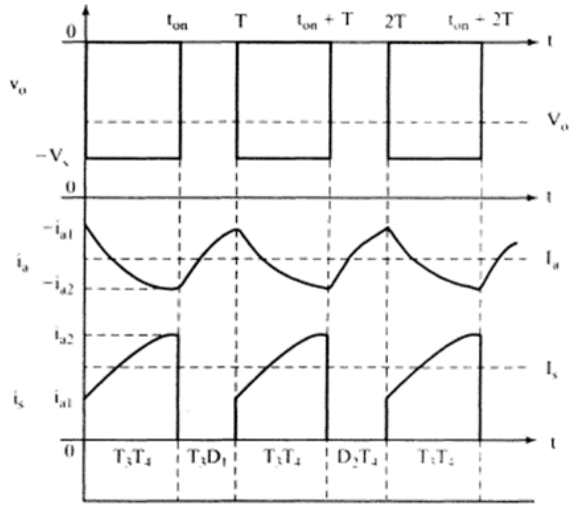


(i) Increasing load current

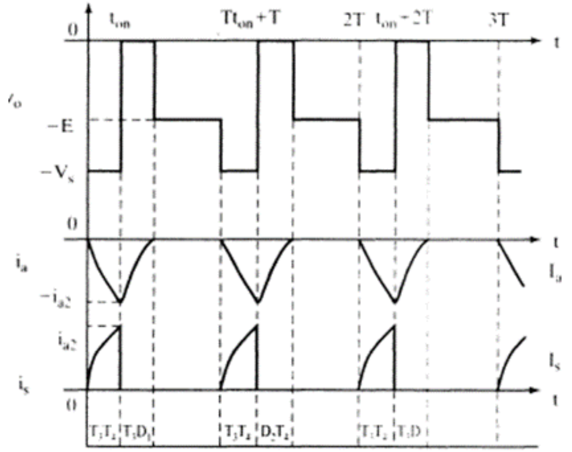


(ii) Decreasing load current

Modes of operation in the third quadrant

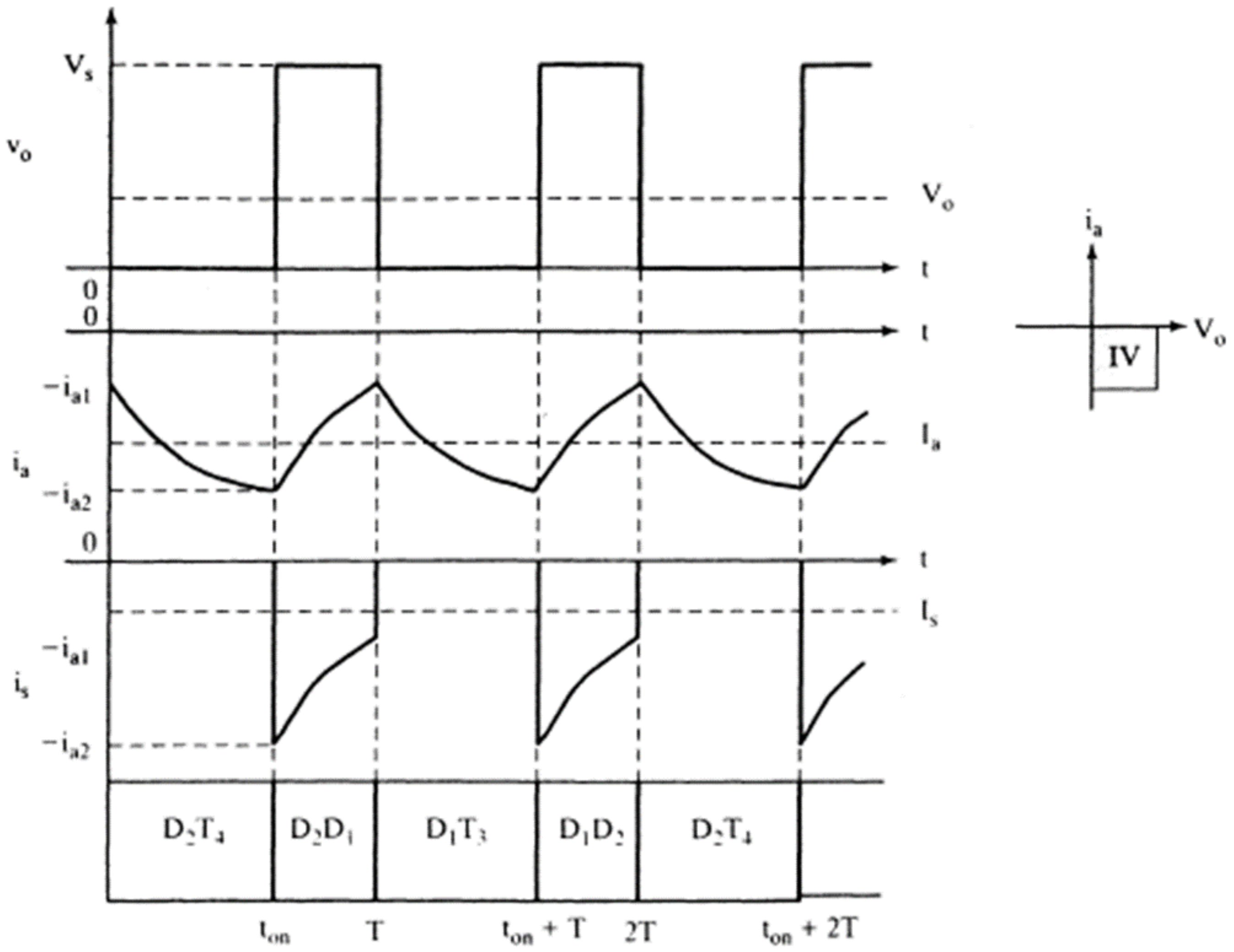


(i) Continuous Conduction

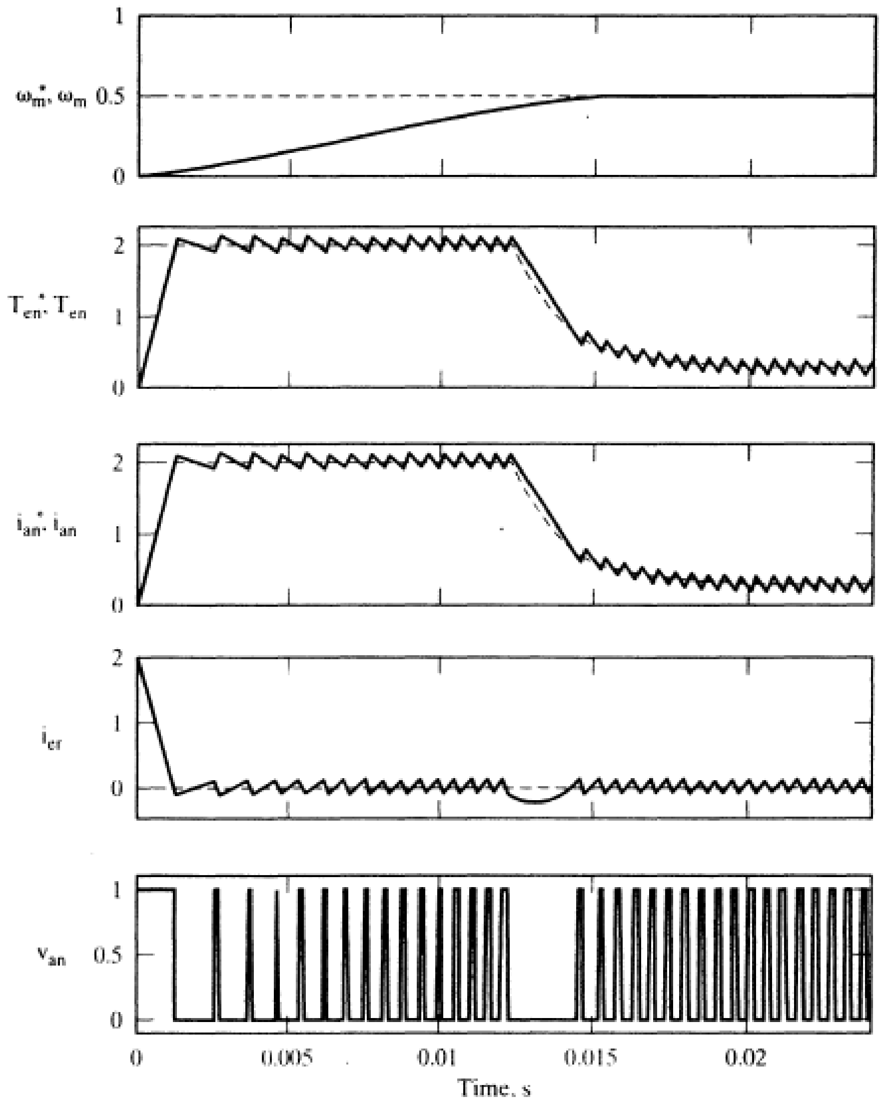


(ii) Discontinuous Conduction

Third-quadrant operation

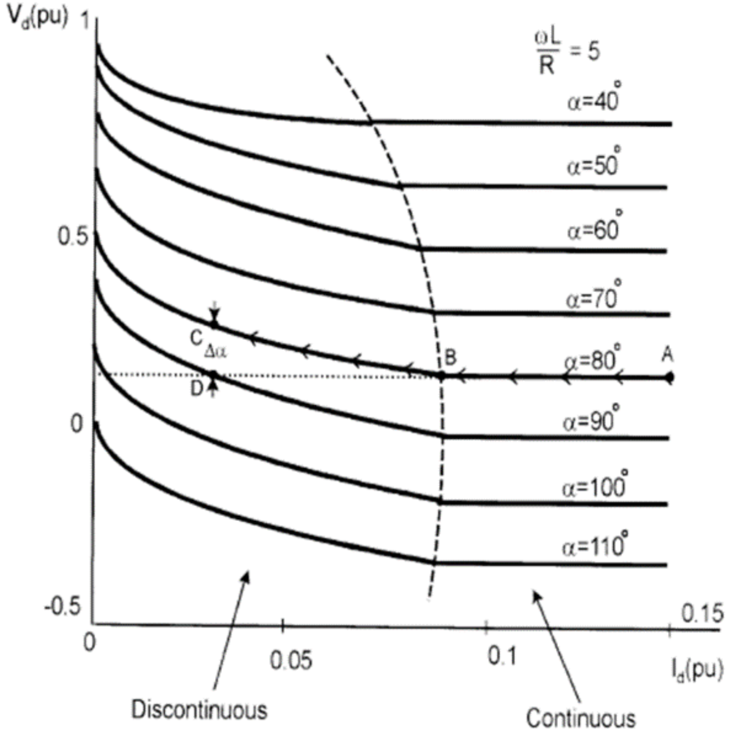


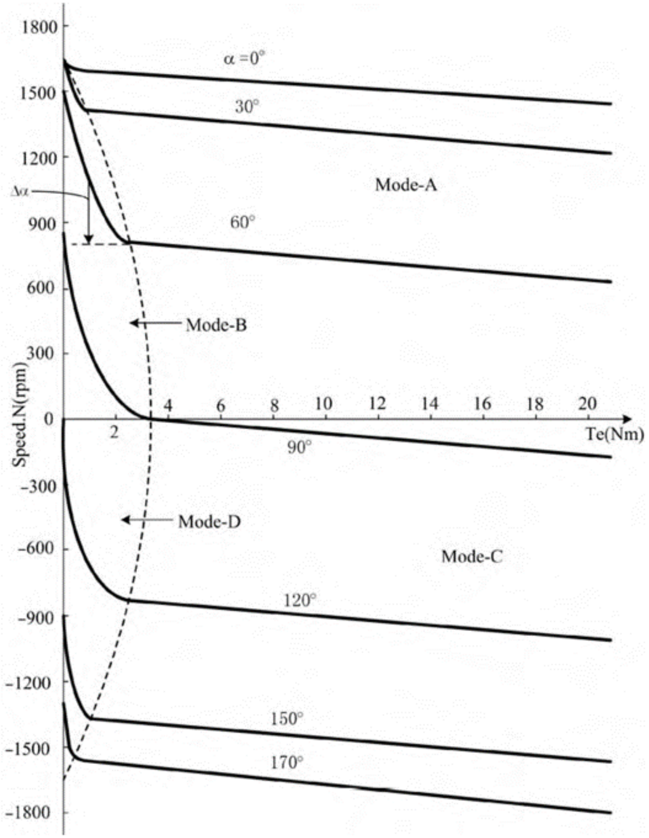
Fourth-quadrant operation of the chopper

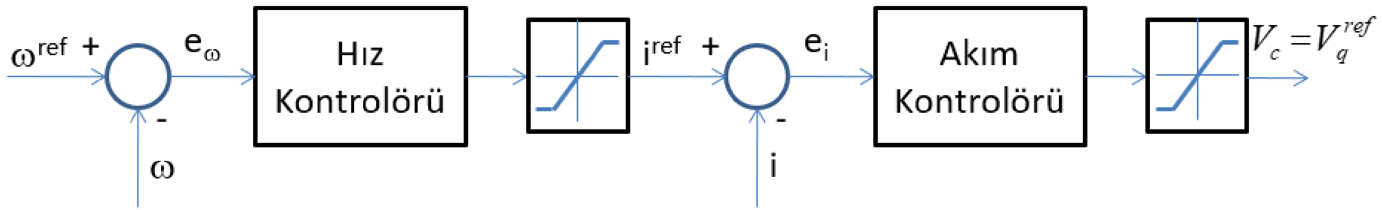


ω_m^*, ω_m
1
0.5
0
T_{en}^*, T_{en}
2
1
0
i_{an}^*, i_{an}
2
1
0
i_{er}
2
1
0
v_{an}
1
0.5
0
0
0.005
0.01
0.015
0.02
Time, s

Rectifier average voltage-average current characteristics, where discontinuous and continuous regions in the current are shown in detail.





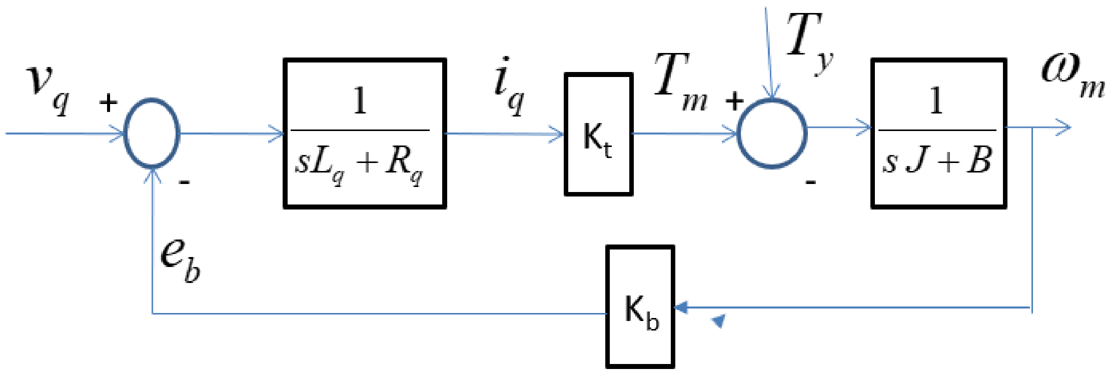


- As can be seen, the current controller and speed controller have a cascade structure.
- The output of the current controller is the average value of the voltage to be applied to the DC motor.
- This value shows the change in the average voltage that should be applied to the motor in both transient and continuous regimes.
- They are expected to be the same as the voltage Vdc applied to the motor, with a scale difference. $V_c = V_q^{ref}$

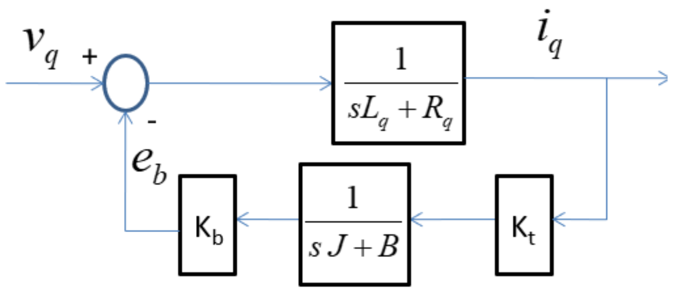
$V_{dc} = V_{d0} \cos\alpha \qquad V_q^{ref} = V_c = V_{c\max} \cos\alpha$

$$\cos\alpha = \frac{V_{dc}}{V_{d0}} = \frac{V_c}{V_{c\max}} \qquad \alpha = \cos^{-1}\left(\frac{V_c}{V_{c\max}}\right)$$

The current loop is the inner loop in cascade control design. Controller design will begin with this cycle, which has much smaller time constants than the speed cycle. For this purpose, first the transfer function between current and voltage will be obtained:



Transfer function between iq and vq, taking Ty = 0:



$$\frac{I_q(s)}{V_q(s)} = \frac{(sJ+B)}{(sJ+B)(sL_q+R_q)+K_t K_b}$$

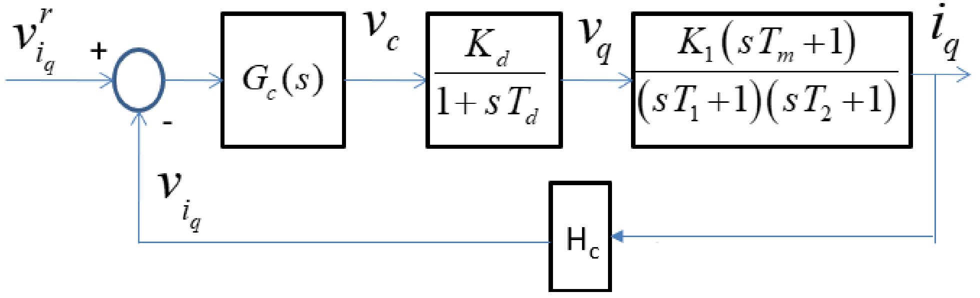
Transfer function of rectifier: $G_{DC}(s) = \frac{V_q(s)}{V_c(s)} = \frac{K_d}{1+sT_d}$

$$\frac{I_q(s)}{V_q(s)} = \frac{(sJ+B)}{(sJ+B)(sL_q+R_q)+K_t K_b} = \frac{K_1(sT_m+1)}{(sT_1+1)(sT_2+1)}$$

$$-\frac{1}{T_1}, -\frac{1}{T_2} = -\frac{1}{2}\left(\frac{B}{J}+\frac{R_q}{L_q}\right) \pm \sqrt{\frac{1}{4}\left(\frac{B}{J}+\frac{R_q}{L_q}\right)^2 - \left(\frac{K_t K_b + R_q B}{J L_q}\right)}$$

$$K_1 = \frac{B}{K_t K_b + R_q B} \qquad T_m = \frac{J}{B}$$

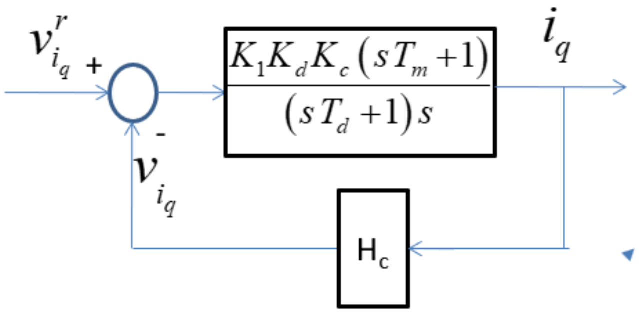
The model of the rectifier: $G_{DC}(s) = \dfrac{V_q(s)}{V_c(s)} = \dfrac{K_d}{1+sT_d}$

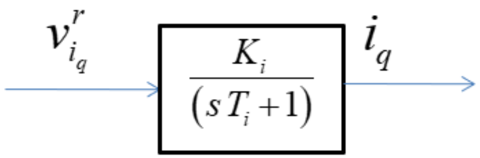


If PID controller is selected as the controller:

$$G_c(s) = \frac{K_c\,(sT_{c1}+1)(sT_{c2}+1)}{s}$$

- If zero-pole cancellation is carried out: $T_{c1} = T_1\,;\ T_{c2} = T_2$

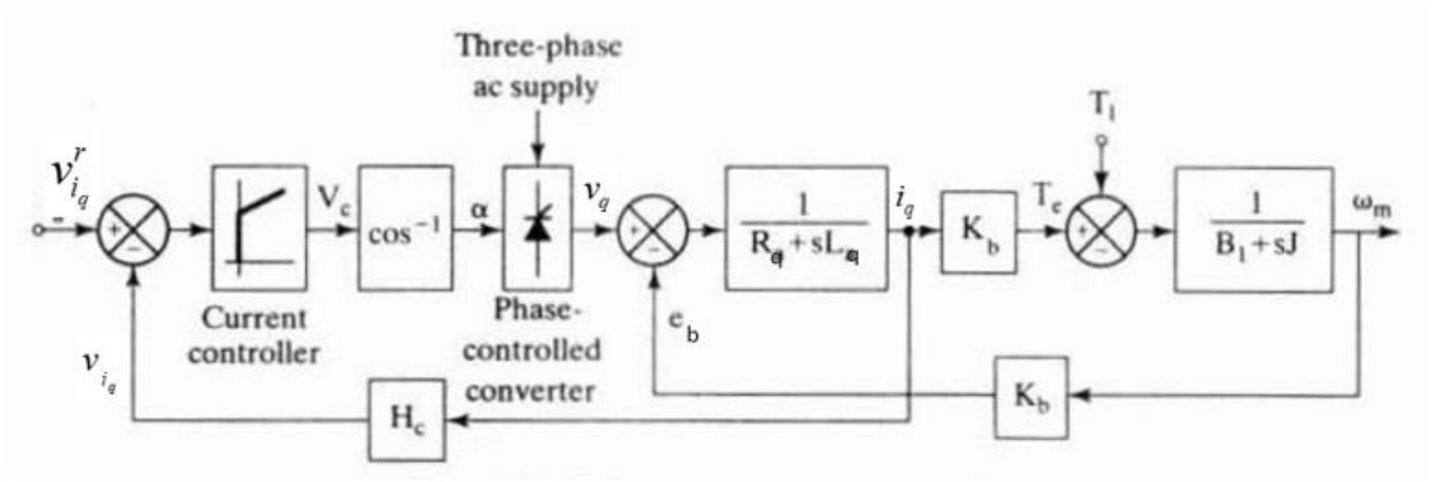




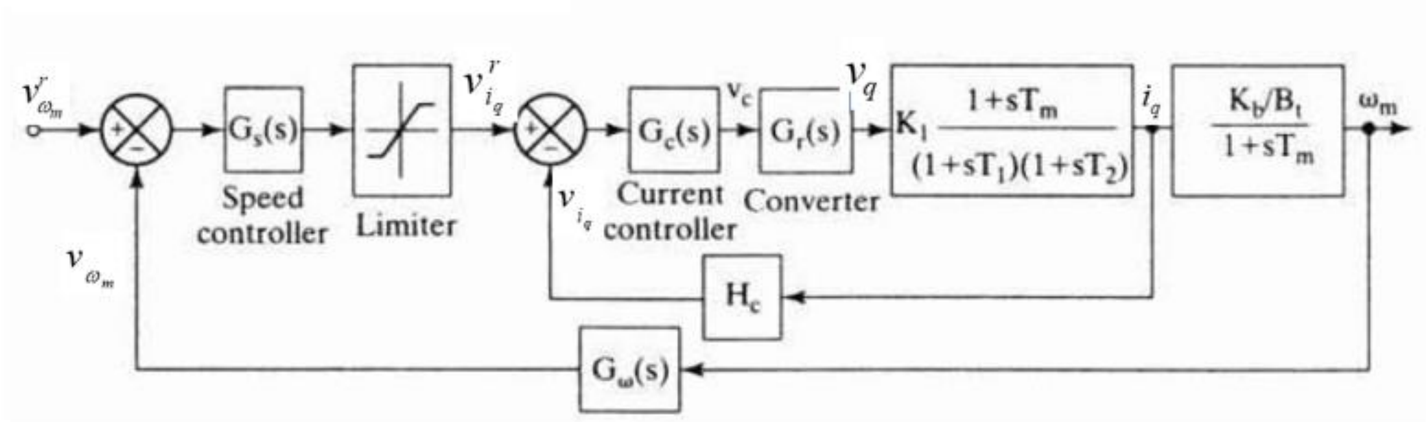
$$(1 + s\,T_m) \cong s\,T_m$$

$$K_i = \frac{K_1\,K_d\,K_c\,T_m}{1+K_1\,K_d\,K_c\,T_m\,H_c}\,; \qquad T_i = \frac{T_d}{1+K_1\,K_d\,K_c\,T_m\,H_c}$$

Hc=1

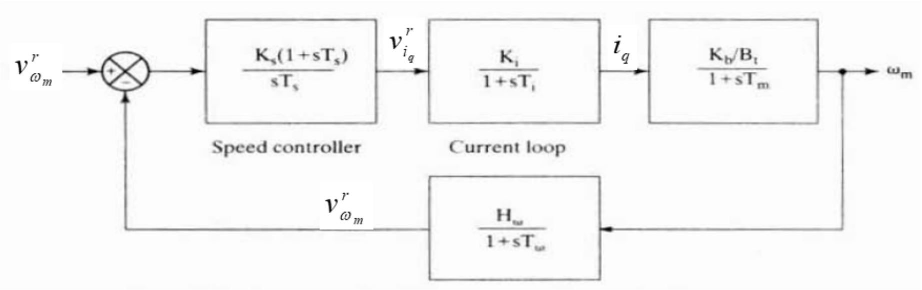


Three-phase ac supply
$v_{i_q}^r$
V_c
$\cos^{-1}$
α
v_q
$\frac{1}{R_q + sL_q}$
i_q
K_b
T_e
T_l
$\frac{1}{B_1 + sJ}$
ω_m
Current controller
Phase-controlled converter
e_b
v_{i_q}
H_c
K_b



$v_{\omega_m}^r$
$G_s(s)$
$v_{i_q}^r$
$G_c(s)$
v_c
$G_r(s)$
v_q
$K_1 \frac{1+sT_m}{(1+sT_1)(1+sT_2)}$
i_q
$\frac{K_b/B_t}{1+sT_m}$
ω_m
Speed controller
Limiter
v_{i_q}
Current controller
Converter
v_{ω_m}
H_c
$G_\omega(s)$

Speed Control Loop



Transfer Function of the System:

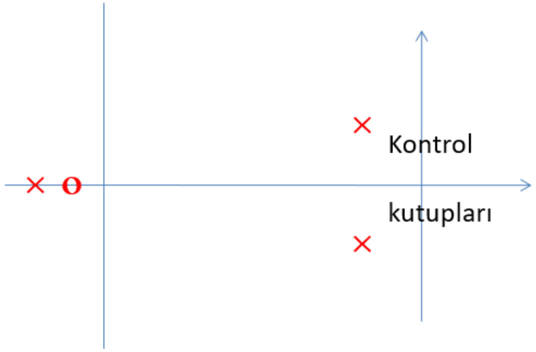
$$G(s)=\frac{\omega_m(s)}{V^r_{\omega_m}}=\frac{\frac{K_s(1+sT_s)}{sT_s}\frac{K_i}{(1+sT_i)}\frac{K_b/B}{(1+sT_m)}}{1+\frac{K_s(1+sT_s)}{sT_s}\frac{K_\omega}{(1+sT_\omega)}\frac{K_i}{(1+sT_i)}\frac{K_b/B}{(1+sT_m)}}$$

$$K_\alpha = K_s K_i \, K_b/B$$

$T_s = T_m$

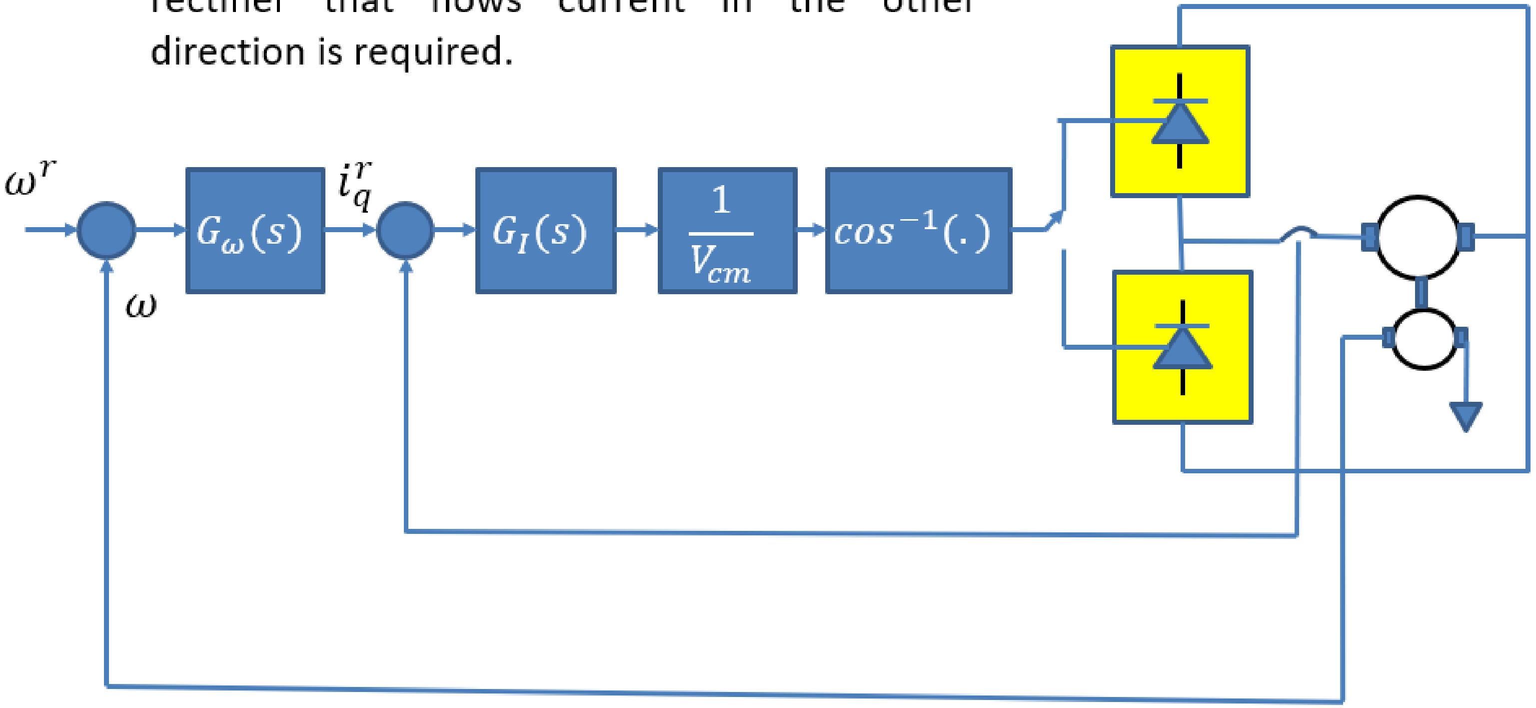
$$G(s)=\frac{\omega_m(s)}{V^r_{\omega_m}}=\frac{\frac{K_\alpha}{T_i}\left(\frac{1}{T_\omega}+s\right)}{s^3+s^2\left(\frac{1}{T_\omega}+\frac{1}{T_i}\right)+s\frac{1}{T_\omega T_i}+\frac{K_\alpha K_\omega}{T_\omega T_i}}$$

Apart from the 2 control poles, the 3rd pole should be 5-6 times further away from the real parts of the 2 control poles, and the additional zero should be at least 5-6 times further away.



For operation in 4 zones, the direction of the motor torque and therefore the motor current must change.

In a controlled rectifier that can only flow current in one direction, a second controlled rectifier that flows current in the other direction is required.



A separately-excited dc motor with the following parameters: $R_a = 0.5\ \Omega$, $L_a = 0.003$H, and $K_b = 0.8$ V/rad/sec, is driving a load of $J = 0.0167$ kg-m^2, $B_l = 0.01$ N·m/rad/sec with a load torque of 100 N·m. Its armature is connected to a dc supply voltage of 220 V and is given the rated field current. Find the speed of the motor.

Solution The electromagnetic torque balance is given by

$$T_e = T_l + B_l\omega_m + J\frac{d\omega_m}{dt}$$

In steady state, $\frac{d\omega_m}{dt} = 0$

$$T_e = T_l + B_l\omega_m = 100 + 0.01\omega_m$$

$$T_e = K_b i_a = 100 + 0.01\omega_m$$

$$i_a = \frac{(100 + 0.01\omega_m)}{K_b} = (125 + 0.0125\omega_m)$$

$$e = V - R_a i_a = 220 - 0.5 \times (125 + 0.0125\omega_m) = 157.5 - 0.00625\omega_m = K_b\omega_m$$

Rearranging in terms of ω_m,

$$\omega_m(0.8 + 0.00625) = 157.5$$

Hence $\omega_m = \frac{157.5}{0.80625} = 195.35$ rad/sec

A 3-phase full-wave controlled rectifier is fed from a 230 V, 60 Hz network.
The control input of the rectifier is +/- 10V.
The maximum value of the speed reference voltage is 10 V.
The maximum motor current is 20 A.
Speed feedback is received with the help of a tacho generator.
The transfer function of the tacho generator and filter is given below.

$$G_\omega(s) = \frac{0.065}{(1 + 0.002s)}.$$

f=60 Hz T=1/60=16.67 msThere are 6 peaks in a period16.67/6= 2.778 ms = TThe time taken for the applied control signal to change is 0<Td<2.778 ms. Here Td=1.38 ms is selected.

Solution (i) Converter transfer function:

$$K_r = \frac{1.35\,V}{V_{cm}} = \frac{1.35 \times 230}{10} = 31.05\ V/V$$

$$V_{dc}(max) = 310.5\ V$$

The rated dc voltage required is 220 V, which corresponds to a control voltage of 7.09 V. The transfer function of the converter is

$$G_r(s) = \frac{31.05}{(1 + 0.00138s)}\ V/V$$

(ii) Current transducer gain: The maximum safe control voltage is 7.09 V, and this has to correspond to the maximum current error:

$$i_{max} = 20\ A$$

$$H_c = \frac{7.09}{I_{max}} = \frac{7.09}{20} = 0.355\ V/A$$

(iii) Motor transfer function:

$$K_1 = \frac{B_t}{K_b^2 + R_a B_t} = \frac{0.0869}{1.26^2 + 4 \times 0.0869} = 0.0449$$

$$-\frac{1}{T_1}, -\frac{1}{T_2} = -\frac{1}{2}\left[\frac{B_t}{J} + \frac{R_a}{L_a}\right] \pm \sqrt{\frac{1}{4}\left(\frac{B_t}{J} + \frac{R_a}{L_a}\right)^2 - \left(\frac{K_b^2 + R_a B_t}{JL_a}\right)}$$

$$T_1 = 0.1077 \text{ sec}$$

$$T_2 = 0.0208 \text{ sec}$$

$$T_m = \frac{J}{B_t} = 0.7 \text{ sec}$$

The subsystem transfer functions are

$$\frac{I_a(s)}{V_a(s)} = K_1 \frac{(1 + sT_m)}{(1 + sT_1)(1 + sT_2)} = \frac{0.0449(1 + 0.7s)}{(1 + 0.0208s)(1 + 0.1077s)}$$

$$\frac{\omega_m(s)}{I_a(s)} = \frac{K_b/B_t}{(1 + sT_m)} = \frac{14.5}{(1 + 0.7s)}$$

(iv) Design of current controller:

$$T_c = T_2 = 0.0208 \text{ sec}$$

$$K = \frac{T_1}{2T_r} = \frac{0.1077}{2 \times 0.001388} = 38.8$$

$$K_c = \frac{KT_c}{K_1 H_c K_r T_m} = \frac{38.8 \times 0.0208}{0.0449 \times 0.355 \times 31.05 \times 0.7} = 2.33$$

$$K_\alpha = K_s K_i \, {K_b}/{B} = K_s \times 1.153 \times 14.5 = 16.72 K_s$$

$$G(s) = \frac{\omega_m(s)}{V^r_{\omega_m}} = \frac{\frac{K_\alpha}{T_i}\left(\frac{1}{T_\omega} + s\right)}{s^3 + s^2\left(\frac{1}{T_\omega} + \frac{1}{T_i}\right) + s\frac{1}{T_\omega T_i} + \frac{K_\alpha}{T_\omega T_i}}$$

$T_s = T_m$

$$G_\omega(s) = \frac{0.065}{(s\,0.002 + 1)}$$

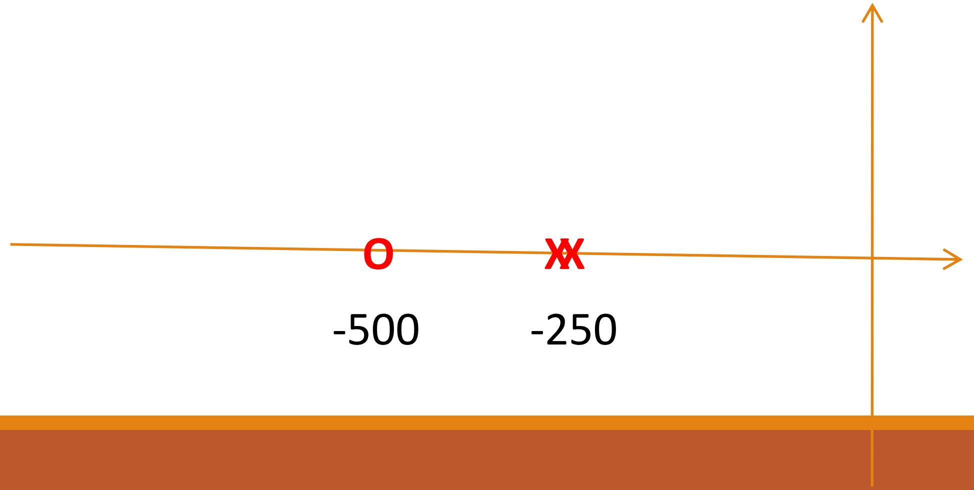
$$\frac{1}{T_\omega} << \frac{1}{T_i}$$

$$G(s) = \frac{16.72 K_s\left(\frac{1}{T_\omega} + s\right)}{T_i s^3 + s^2 + s\frac{1}{T_\omega} + \frac{1.087 K_s}{T_\omega}}$$

0

$$T_\omega = 0.002 \sec$$

$$G(s) = \frac{16.72 K_s (s + 500)}{s^2 + 500 s + 543.4 K_s}$$



If Ks=115.016 is selected, the system will have a multiple root at -250. Since zero at -500 is closer than 5 times, the critical damped response expected from the system cannot be obtained.