Geodesy: Earth's rotation

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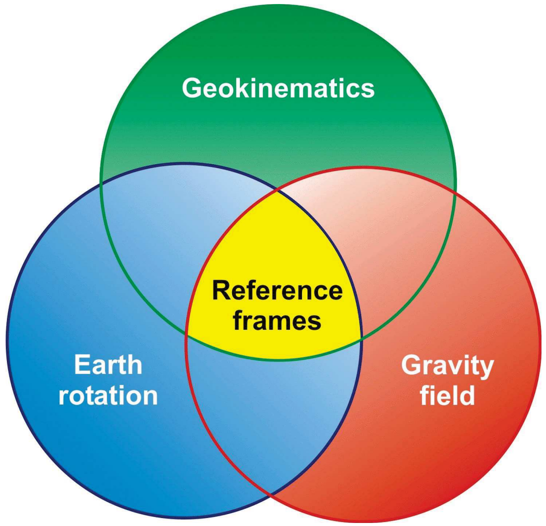
What is geodesy?

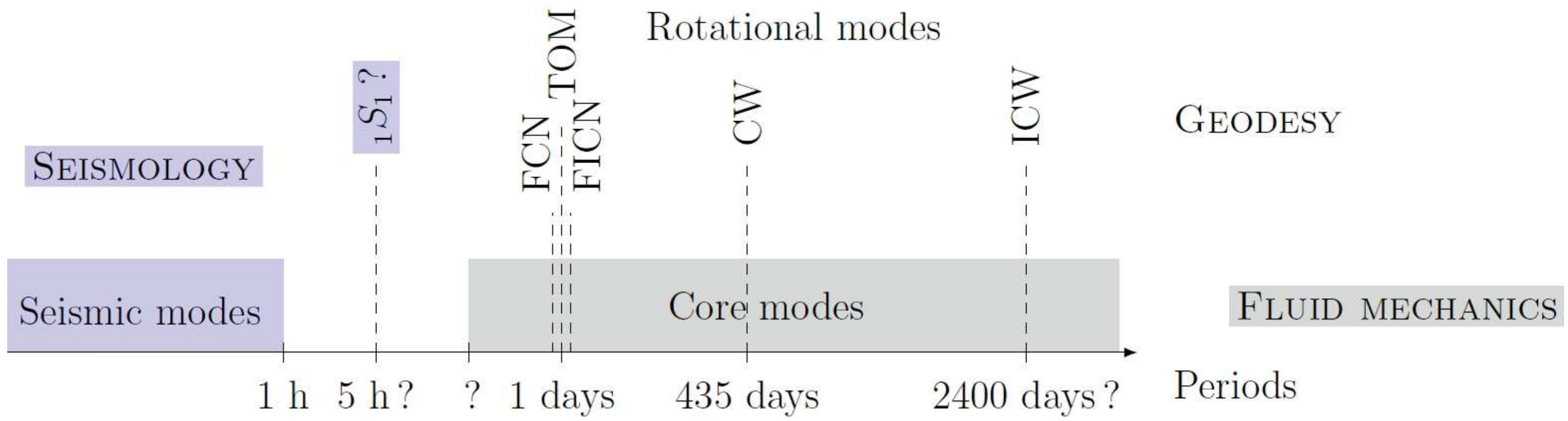
Geodesy is the science of accurately measuring and understanding the Earth's **geometric shape**, **orientation in space**, and **gravity field**.



The three pillars of geodesy. The changes in Earth's shape (geokinematic), gravity field and rotation provide the conceptual and observational basis for the **reference frames** required for Earth observation.

http://ggos.org

Earth's normal modes



- Seismic modes (elastic feedback)

- Core modes:

– Gravity modes (Archimedean)
– Inertial modes (Coriolis)
– Alfven or hydromagnetic modes (Lorentz)

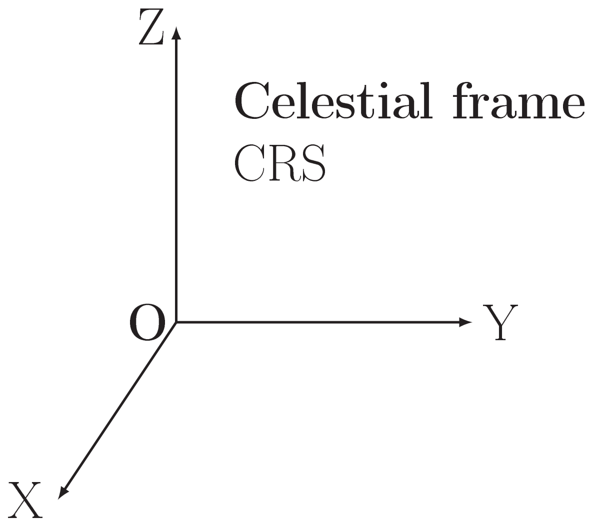
- Rotational modes (torques)

Outline

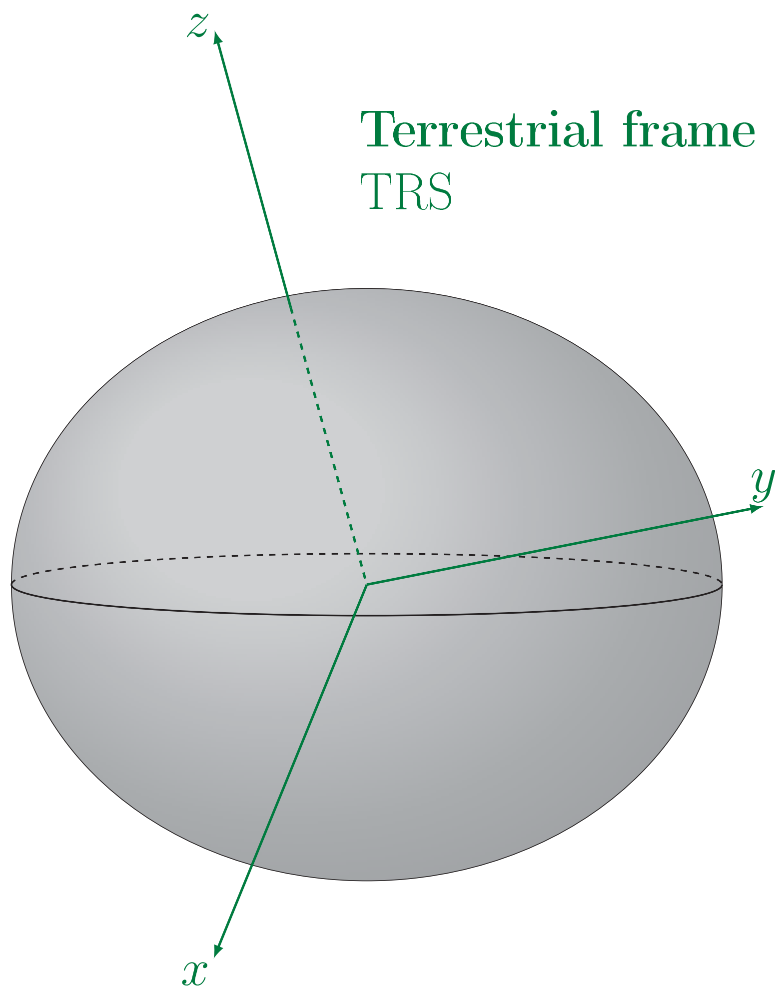
1. Theory
   - Precession-nutation & polar motion
   - Euler-Liouville equations
2. Rotational modes
   - Chandler wobble
   - Free core nutation
   - Free inner core nutation
3. Length of day

Table of Contents

Reference systems



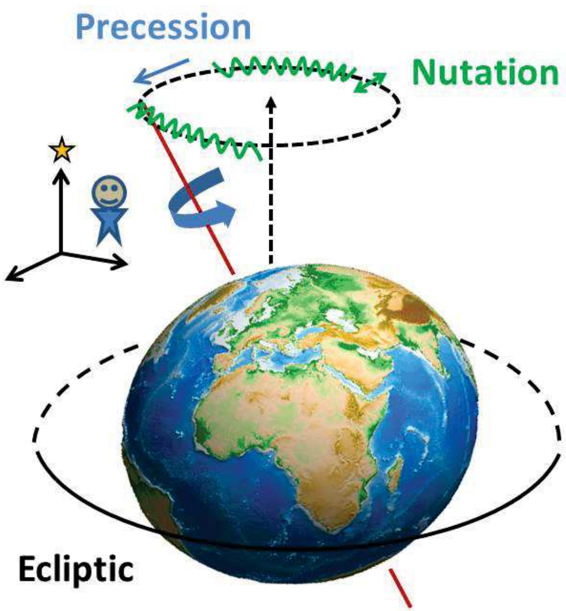
$$[XYZ]_{\mathrm{CRS}} = Q(t)R(t)W(t)[xyz]_{\mathrm{TRS}}$$



Earth's rotation is complex: its rotation axis varies both in space wrt celestial frame and within the Earth wrt geographic reference frame.

Precession-nutation

$\sim$ 1.5 km/yr, nutation $\sim$ 600 m



Celestial Reference system

$\dot{\mathbf{H}} = 0$

Polar motion

$\sim$ 10 m

Terrestrial Reference system

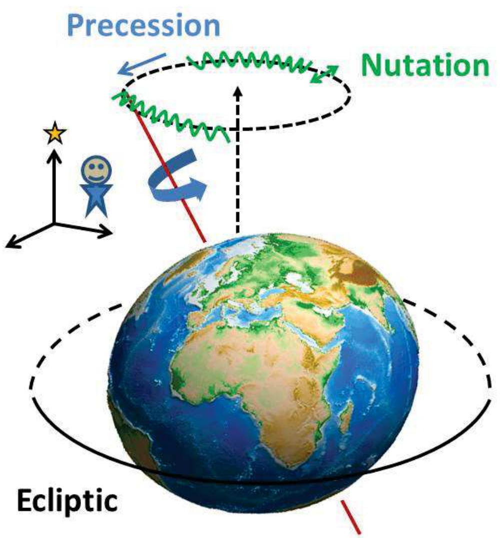
$\dot{\mathbf{H}} + \mathbf{\Omega} \times \mathbf{H} = 0$

Any motion that carry angular momentum or mass redistribution, any external torque (tides, surficial fluid layers...) affect Earth's rotation

Precession - nutation

(long-period)

- precession-nutation: tidal forcing
- rotational modes: Free Core Nutation (**FCN**), Free Inner Core Nutation (**FICN**)



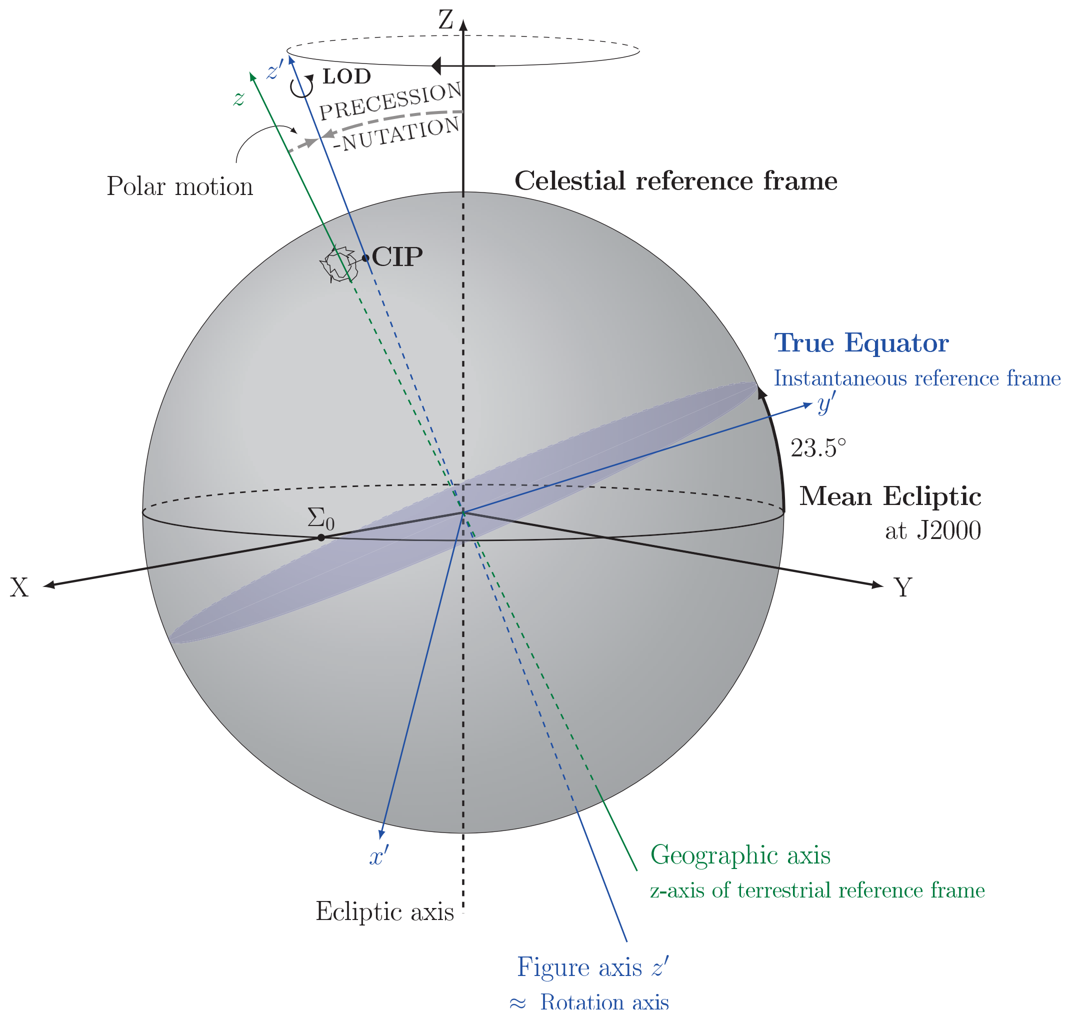
Polar motion

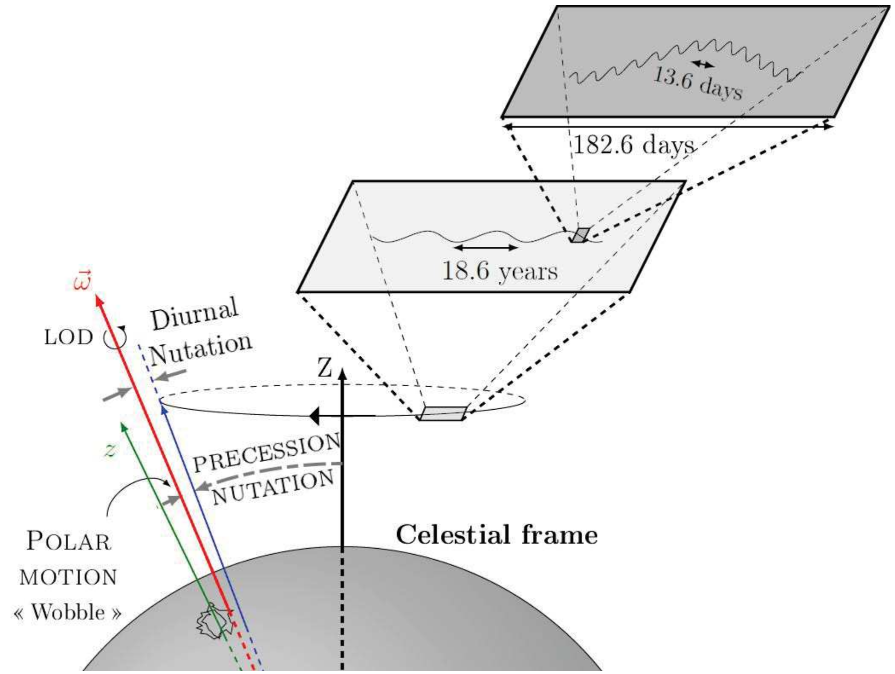
(quasi-diurnal wobble)

- annual polar motion (atmospheric & hydrological forcing)
- secular trend (post-glacial rebound)
- rotational modes: Chandler wobble (**CW**), inner-core wobble (**ICW**)

variations in spin rate = length of day changes (Δ**LOD**)

Precession and Nutations





Precession and Nutations

Fréquences dans le système de référence terrestre ITRS

←RETROGRADE | PROGRADE →

16 h 48 h

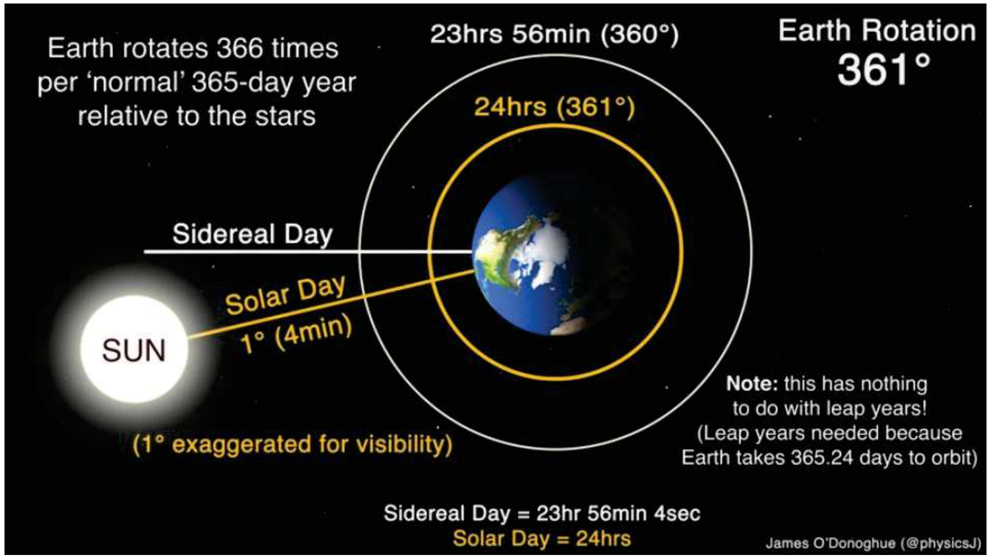
-3.5 -2.5 -1.5 -0.5 +0.5 +1.5 +2.5 (cpsd)

polar motion | | *polar motion*

Fréquences dans le système de référence céleste ICRS

-2.5 -1.5 -0.5 +0.5 +1.5 +2.5 +3.5 (cpsd)

| *nutation* |



Some history

- ∼190-125 BC, **Hipparque**: discovery of *precession* from Timocharis's observations (done between 294 and 283 BC)

- 1532, **Copernic**: this motion is attributed to varying Earth's rotation axis

- 1687, **Newton**: provided a physical explanation (*Principia mathematica*)

- 1748, **Bradley**: discovery of the *18.6 year-nutation*

- 1749, **d'Alembert**: explanation of the 18.6 yr-nutation, 1st analytical theory of precession-nutation for a rigid Earth

- 1749, **Euler**: re-explained it in his work on *Equinox precession and nutation of the Earth's axis*

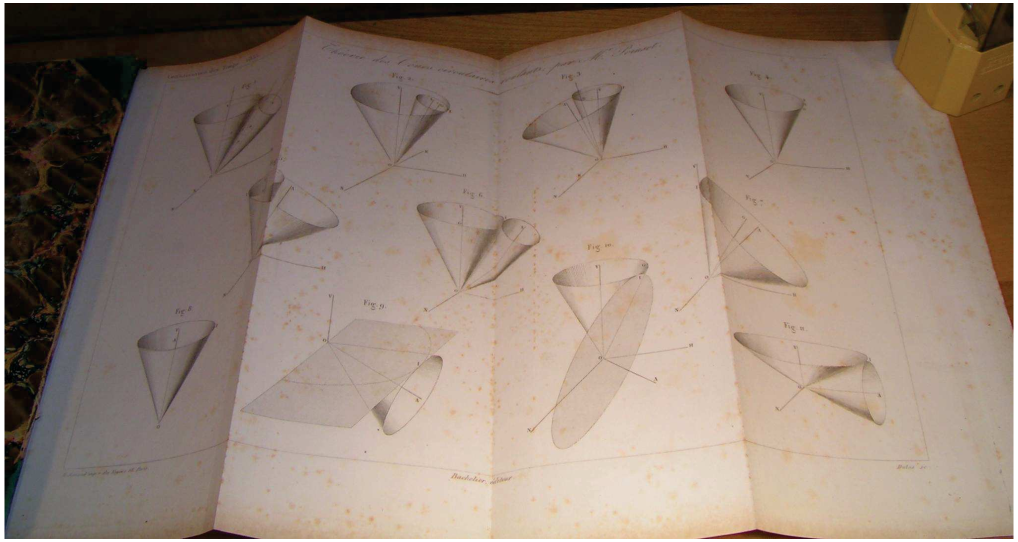
Some history

• 1758, **Euler** (published in 1765): demonstration of the polar motion that exists for any body which main axis of inertia and axis of rotation are different

The rotation axis follows a cone around the axis of inertia → *Eulerian precession*

• 1788, **Lagrange**: from Euler's equations, he presented the equations of a free-rotating body

• 1834, 1851, **Poinsot**: geometrically described the motion of the rotation axis showing that the instantaneous rotation axis follows 2 cones: one cone attached to the body around the principal axis of inertia and one fixed in space (*rolling cones theory*).



Some history

• 1844, **Peters**: predicted that polar motion should appear as **periodic variation of the latitude** of observatories

• 1844, **Bessel**: observed a variation of latitude in his observations realized between 1842 and 1844 but he attributed them to internal variations of the Earth

• 1876, **Thomson**: suggested that polar motion should be more complicated because of **mass redistributions inside the Earth and at the surface**

• 1888, **Küstner**: observations of variations of the latitude (1884 - 1885), **1st observational proof** of the polar motion and its **annual component** (Küstner 1888, Brosche 2000)

This discovery is often attributed to Küstner (1888) but indeed it results from a long process that lasted the whole XIXth century to which numerous (German) astronoms participated.

Some history

Existence of polar motion was however not yet unanimously accepted.

• 1890, **Tisserand**: (2nd volume of Celestial Mechanics) developped first quasi-diurnal terms of the polar motion induced by luni-solar torque and of a few cm at the surface

• published by von Oppolzer (1882): terms corresponding to precession, 13.6 day and semi-annual nutation

• 1891, **Chandler**: discovered the polar motion can be decomposed into a free circular motion, now on called *Chandler wobble* and an elliptical forced annual motion

→ numerous fundamental publications about Earth's elasticity (Newcomb 1892, Hough 1895, Love 1909, Larmor 1909) and about the presence of an Earth's fluid core (Sloudsky 1895, Hough 1895, Poincaré 1910, Jeffreys 1926).

Dynamical equations of the Earth's rotation

Conservation of angular momentum

- Inertia tensor of a mass body of volume V and density ρ is

$$I_{ij} = \int_V \rho \left(r^2 \delta_{ij} - x_i x_j\right) dv, \tag{1}$$

where δ is the Kronecker symbol and r is the distance to the center of mass.

- if A and C main inertia moments of the ellipsoidal Earth, c_{ij} small perturbations, then

$$I_{ij} = \begin{pmatrix} A + c_{11} & c_{12} & c_{13} \\ c_{12} & A + c_{22} & c_{23} \\ c_{13} & c_{23} & C + c_{33} \end{pmatrix}, \tag{2}$$

where we have assumed that the c_{ij} are symmetric. The perturbation of the inertia tensor is trace-free so that $c_{11} + c_{22} + c_{33} = 0$.

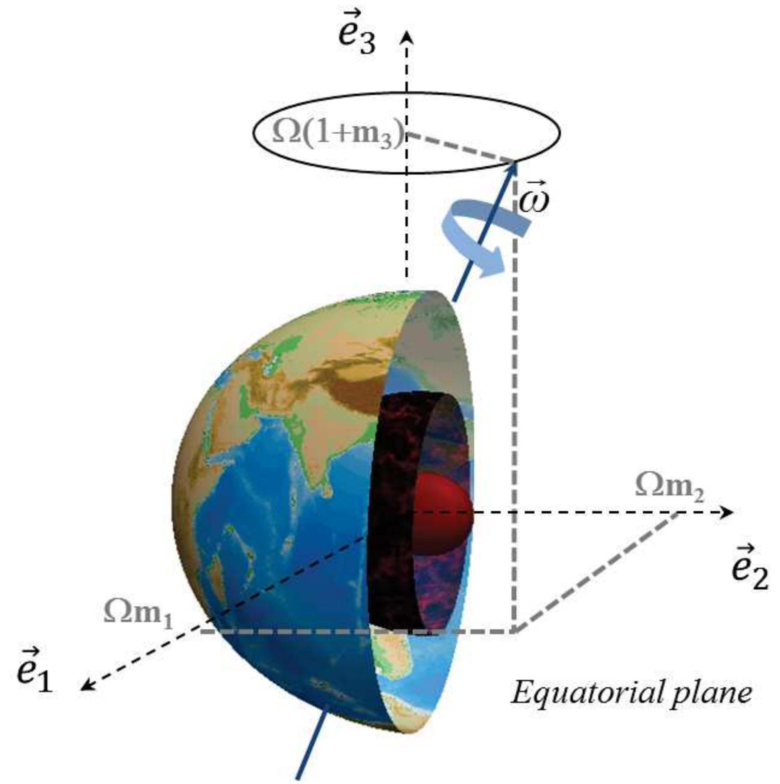
Dynamical equations of the Earth's rotation

The action of any external torque or any mass redistribution within or at the surface of the Earth will perturb the Earth's rotation.

- $\vec{\omega} = \vec{\Omega}(m_1, m_2, 1+m_3)$ instantaneous rotation vector in the steadily rotating terrestrial frame, $(m_1, m_2, 1)$ are the direction cosines of the rotation axis

- Equatorial directions (m_1 direction of Greenwich, m_2 direction of 90°E) define variations in Earth's rotation axis position: $\tilde{m} = m_1 + im_2$ is the Earth's **polar motion**

- m_3 = spin rate changes (**length of day** changes: ΔLOD)



Steady rotating at Ω about the Earth's figure axis $\vec{e}_3$, fixed in inertial space.

Dynamical equations of the Earth's rotation

- $H_j = I_{ij}\omega_i$ (definition of angular momentum)
- $\dot{\mathbf{H}} + \boldsymbol{\Omega} \times \mathbf{H} = 0$ (conservation of angular momentum)

After linearization and neglecting 2nd-order quantities in m_i and c_{ij}, the varying Earth's rotation is governed by the linearized form of the so-called *Euler-Liouville equations* (Munk and McDonald 1960, Lambeck 1980).

$$\boxed{\left(1 - \frac{k_2}{k_s}\right)\tilde{m} + \frac{i}{\sigma_e}\dot{\tilde{m}} = 0}$$

$\boxed{\dot{m}_3 = 0} \rightarrow \Delta LOD = -\overline{LOD}\; m_3$ ($\overline{LOD} = 86400$ s)

$\sigma_e = \alpha\Omega$ (**Eulerian nutation** frequency), $\alpha = \frac{C-A}{A}$ (dynamic ellipticity)

- $\boxed{\sigma_{CW} = \left(1 - \frac{k_2}{k_s}\right)\alpha\Omega}$ (**Chandler wobble** frequency)

The degree-2 k_2 and secular $k_s = \frac{3G(C-A)}{\Omega^2 a^5} \sim 0.938$ (for a fluid Earth in hydrostatic equilibrium) **Love numbers** characterize elastic deformations that take place under polar motion.

Dynamical equations of the Earth's rotation

With source of perturbations...

$$\boxed{\left(1-\frac{k_2}{k_s}\right)\tilde{m}+\frac{i}{\sigma_e}\dot{\tilde{m}}=\tilde{\Psi}}$$

- $\tilde{\Psi}=\Psi_1+i\Psi_2$ equatorial projection of the *excitation function*

$$\boxed{\dot{m}_3=\dot{\Psi}_3}\rightarrow \Delta LOD=-\overline{LOD}\,m_3$$

- Ψ_3 axial component of the *excitation function*

The *excitation function* Ψ_i contains all possible geophysical effects on the rotation of the Earth, due to motion that carry angular momentum or to redistribution of mass, and to any external torque (tides, surficial fluid layers, etc...).

- Unit for m_i: **mas** (milliarcsecond) (1 mas ∼3 cm on Earth)

Dynamical equations of the Earth's rotation

Linearized Euler-Liouville equations with a fluid core

Conservation of angular momentum
$A = A_f + A_m$ moments of inertia

- for the global Earth:

$$-\Omega\tilde{m} + \frac{A_f}{A}\sigma_{CW}\tilde{m}_f = \frac{i\Gamma}{\Omega^2 A}$$

- for the core:

$$-\Omega\tilde{m} + (\beta - \alpha_f)\Omega\tilde{m}_f = \frac{i\Gamma_{CMB}}{\Omega A_f} = \left(K^{Re}_{CMB} + iK^{Im}_{CMB}\right)\tilde{m}_f$$

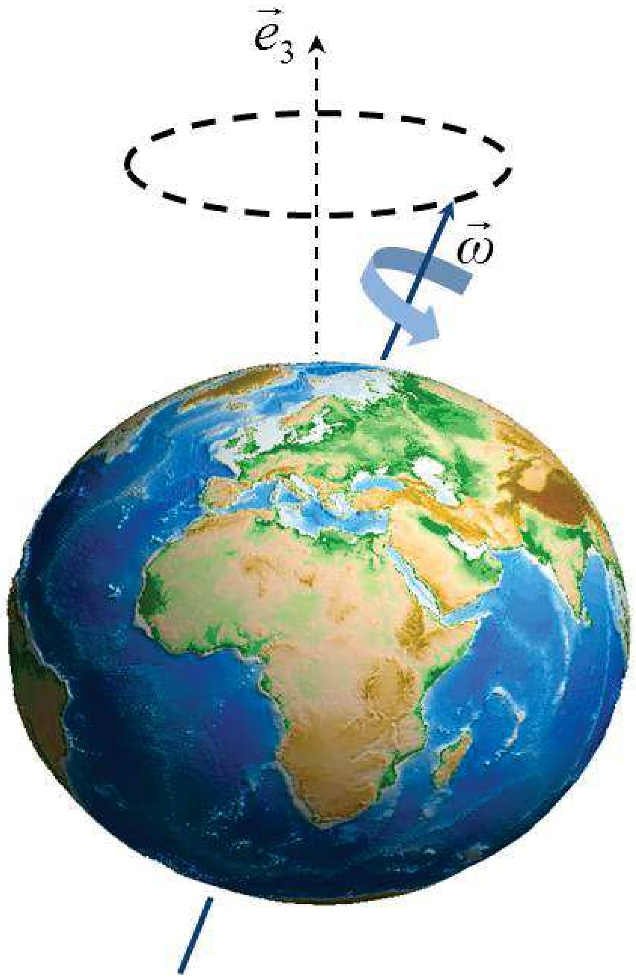
β compliance accounting for the deformation of the Earth at the CMB.

- and for the inner core...

- $\Gamma = 0 \rightarrow$ **normal modes**

Table of Contents

Chandler wobble



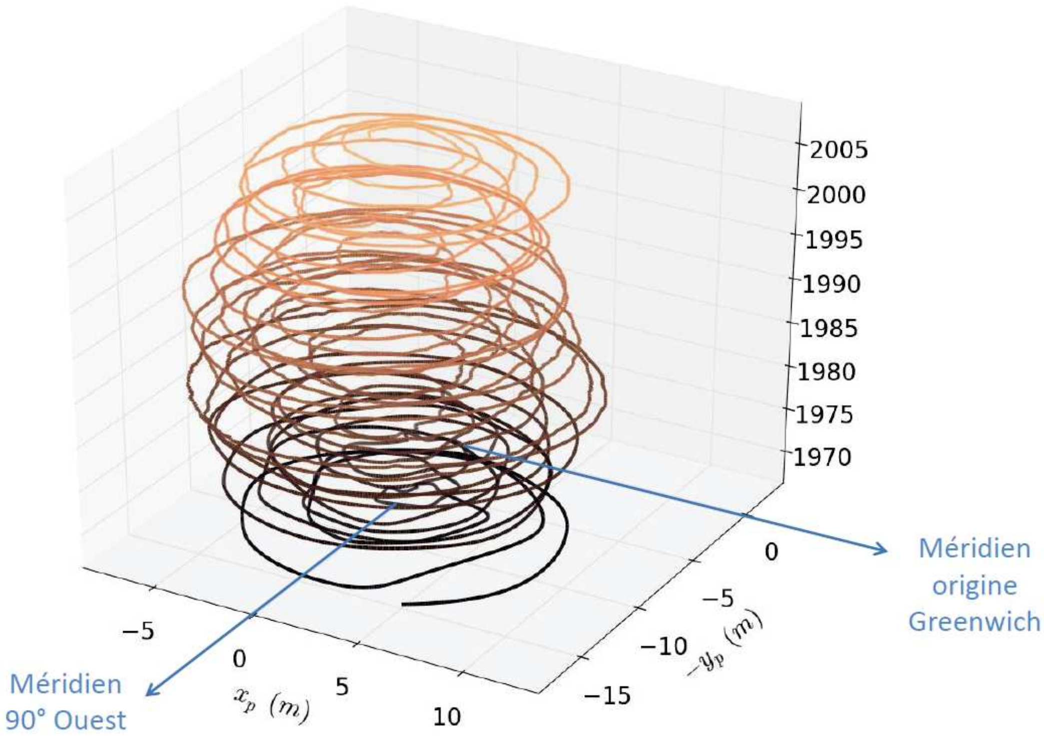
- Earth's flattening

- Eulerian precession (Euler 1975): the Earth spins so that related to its surface, the instantaneous rotation axis moves around the main inertia axis
$\sigma_e = \alpha\Omega$

- Chandler wobble (Chandler 1891)
$\sigma_{CW} = \left(1 - \frac{k_2}{k_s}\right)\alpha\Omega$

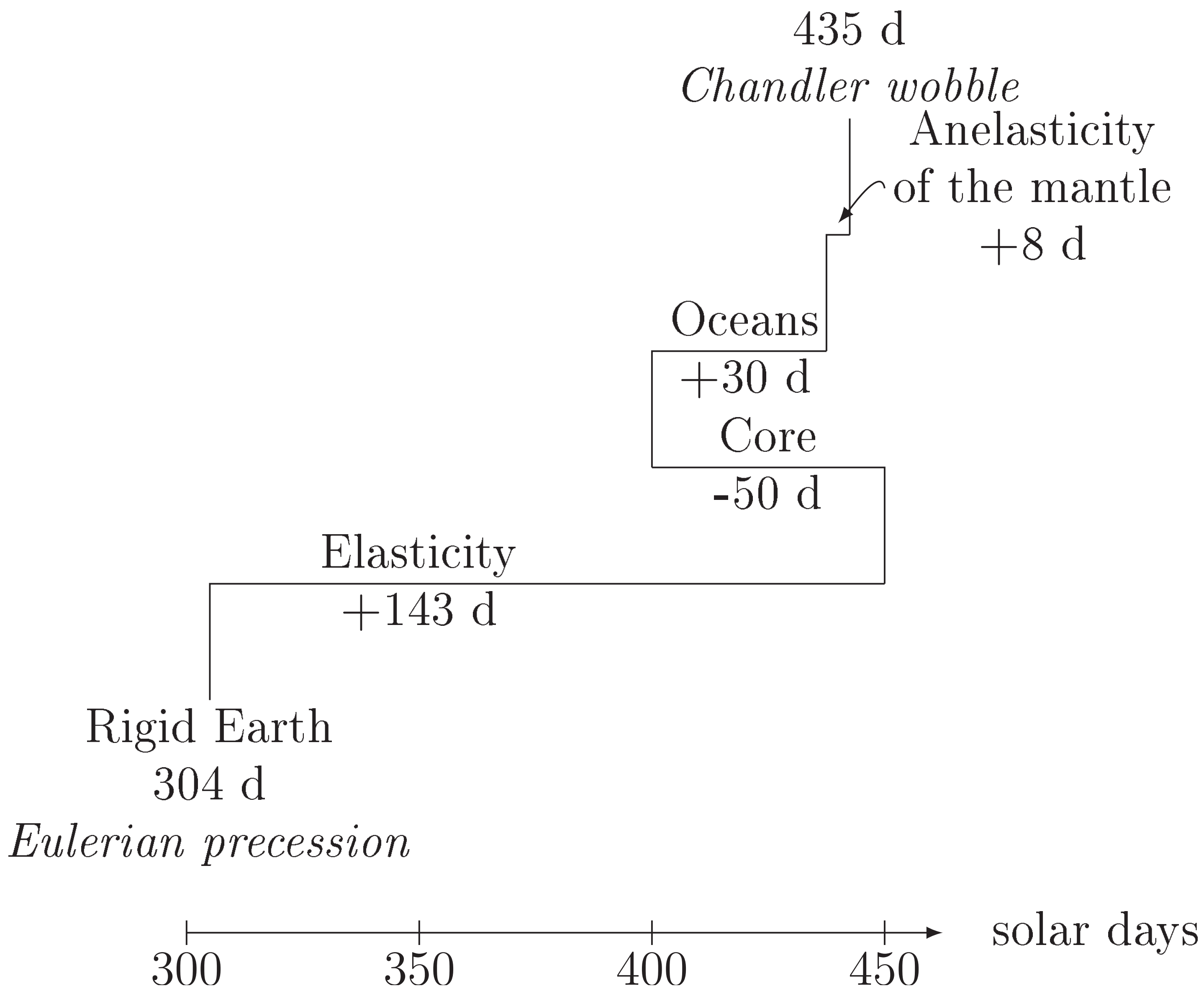
Chandler wobble

Chandler wobble, a component of the polar motion



Polhody

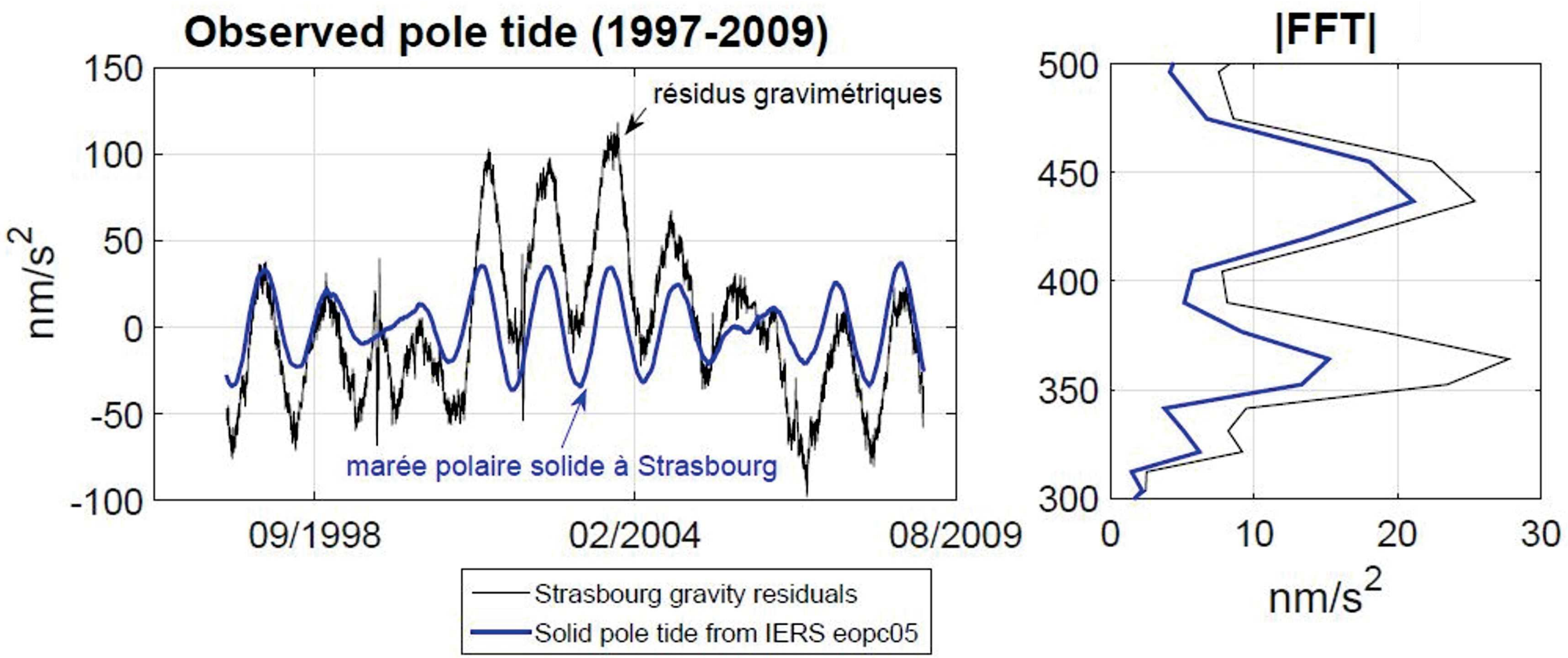
Chandler wobble



Chandler wobble

- Gravimetric observations of **Chandler wobble** (centrifugal effect)

$$\Delta g(r,\theta,\lambda) = \tilde{\delta}\Omega^2 r \left[\sin 2\theta(m_1 \cos\lambda + m_2 \sin\lambda) - 2m_3 \sin^2\theta\right],$$
$$m_3 \ll (m_1, m_2)$$



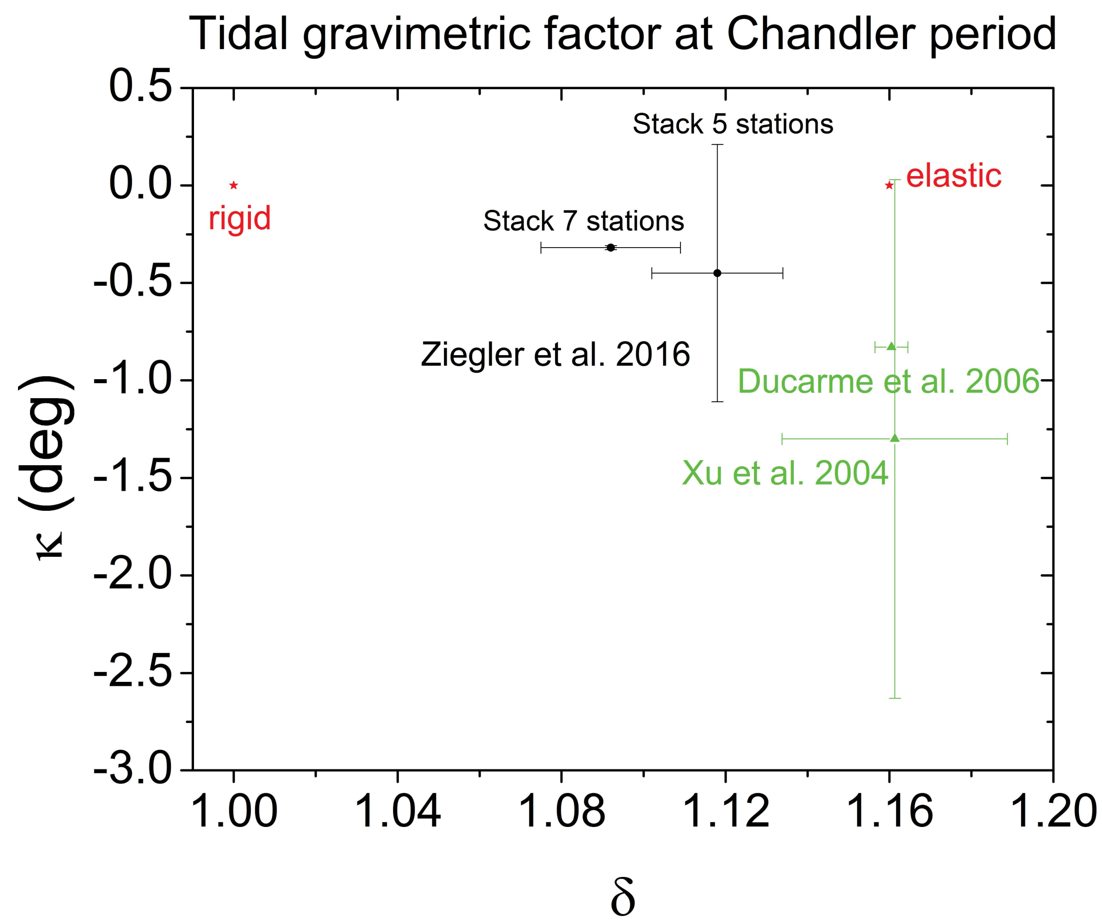
T $\sim$ 432 solar days, Q $\sim$ 130

$\rightarrow$ mantle rheology

[Ziegler et al. 2016]

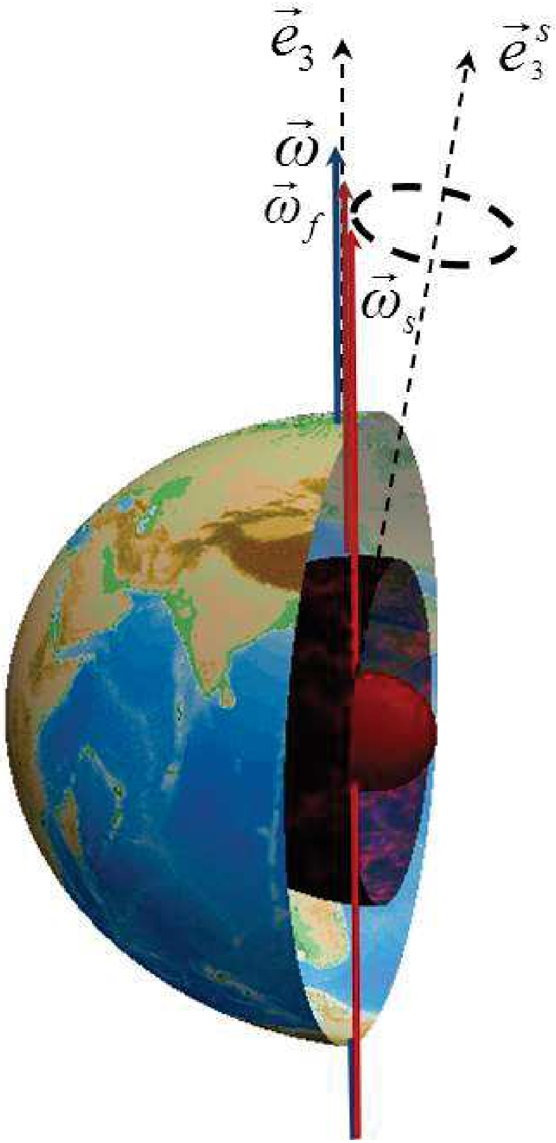
Chandler wobble

$\tilde{\delta}$ gravimetric factors = observed tidal amplitude/theoretical amplitude for a rigid Earth (complex transfer function of the Earth)



Inner core wobble

Inner core wobble, a component of the polar motion



Never observed... till maybe recently

Neither in polar motion data, nor in gravimetric observations

- $T_{\mathrm{ICW}} \sim$ [6 - 8] years for PREM model
- Prograde motion

$\rightarrow$ Density jump at the inner-core boundary

$\rightarrow$ Elasticity of the inner-core

Inner core wobble

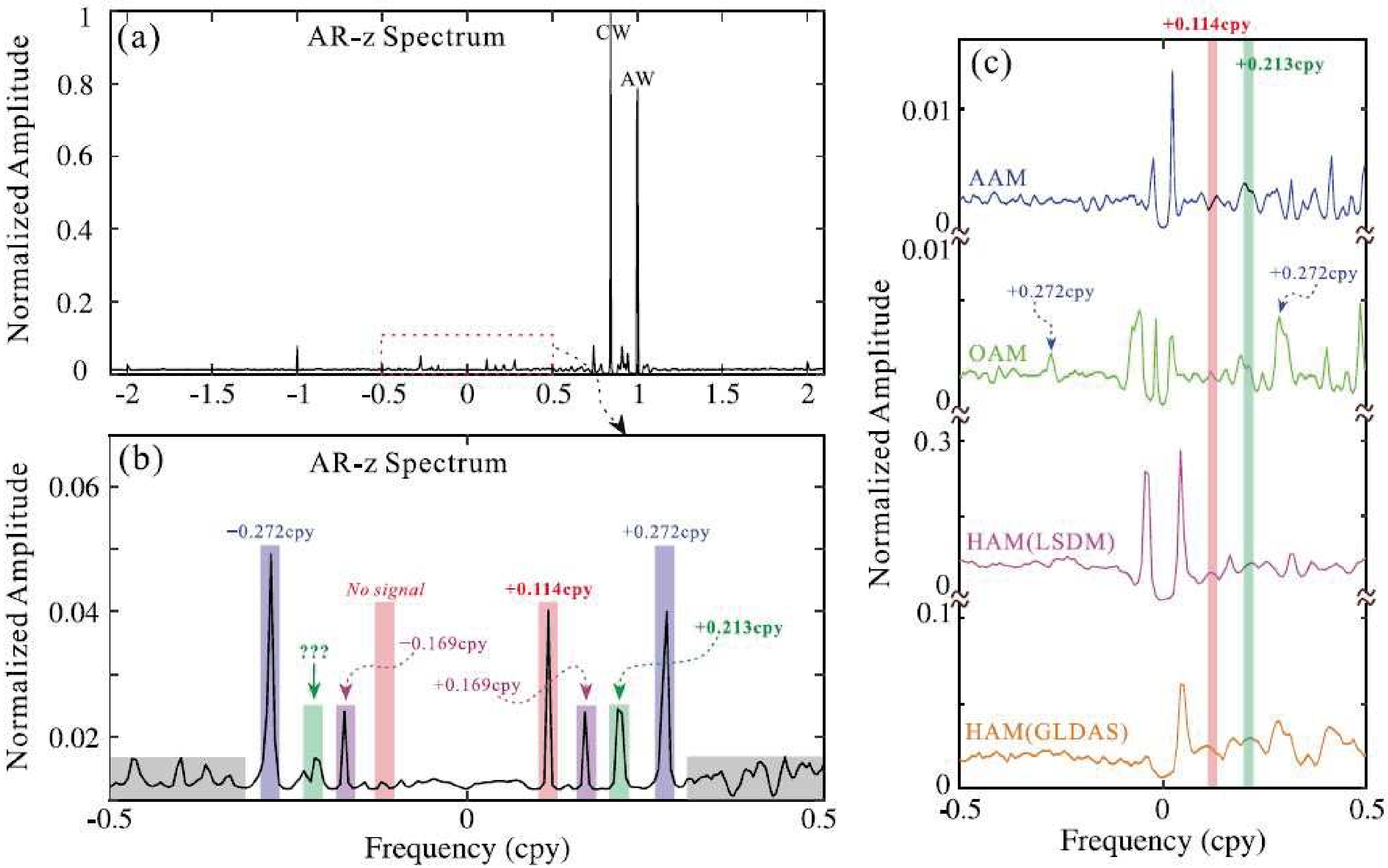


Figure 2. (a) The AR-z spectrum of the residual 1960–2017 PM time series; (b) an enlargement of the −0.5 to +0.5 cpy frequency band. (c) The AR-z spectra of the AAM/OAM/HAM-excited PM time series.

Prograde signal at 0.114 cpy (∼8.7 years)

$\rightarrow \Delta\rho_{ICB} = 507$ kg/m^3 (PREM: 598 kg/m^3)

Ding et al. (2019)

Free core nutation

T = -430 sid. days
Q $\sim$ 20,000
Retrograde in space

Presence of an elliptical fluid core
$\rightarrow$ periodic variations of Earth's rotation axis in space (nutation)

- Nearly Diurnal Free Wobble (in a rotating frame)
- raw egg $\neq$ cooked egg

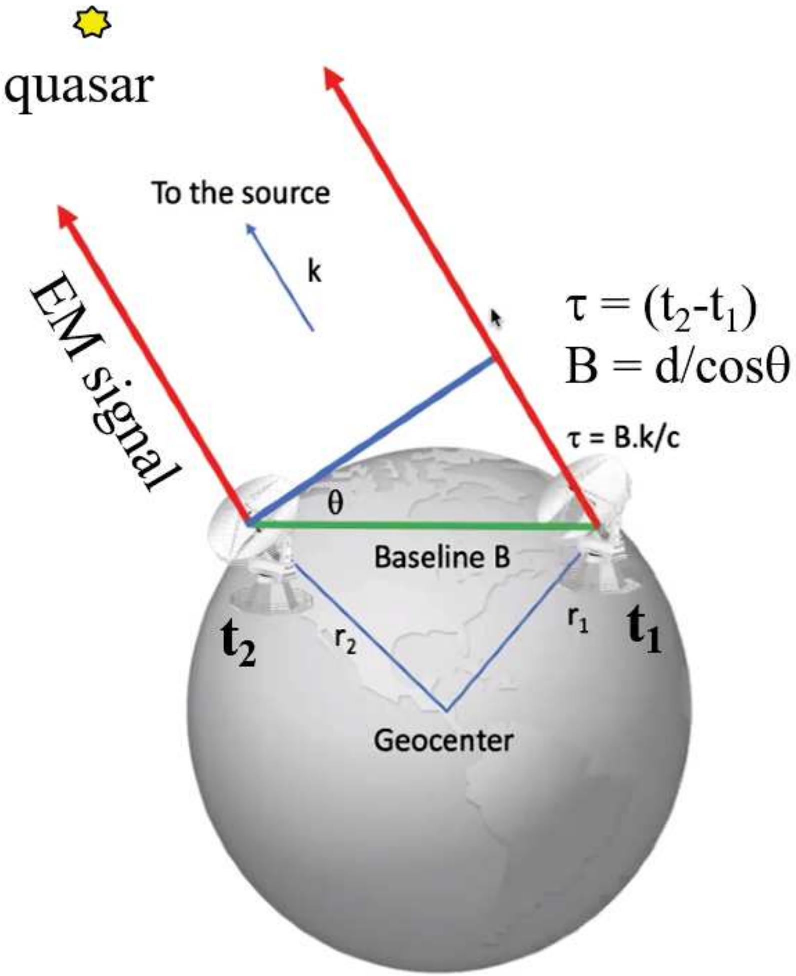
$$\boxed{\omega_{FCN} = -\Omega\left[1 + \frac{A}{A_m}\left(\beta - \alpha_f - (K^{Re}_{CMB} + iK^{Im}_{CMB})\right)\right]}$$

- Ellipticity of CMB?
- Viscosity at CMB?
- Magnetic field at CMB?
- Magnetic field at ICB?
- Topography at CMB?
- Other coupling effects at CMB?...

Free Core Nutation

- 1895 theoretical discovery (Hough, 1895; Sloudsky 1896)
- 1959 observational evidence by Lecolazet from gravimetric records (1957-1958) at the Observatory of Strasbourg (France)
- 1974 determination of the FCN frequency by Lecolazet & Steinmetz (1974) from 3-year continuous gravimetric records (1964-1967) at the seismological Observatory of Strasbourg (North-American AG 138 gravimeter)
- Since then, numerous determinations of its period and damping from gravimetric and/or VLBI measurements.

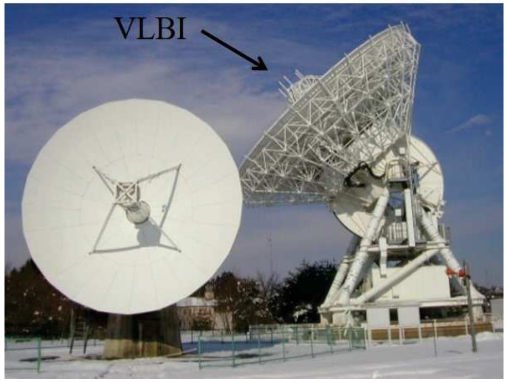
Very Long Baseline Interferometry



- geometric space technique

- position and orientation of the Earth in a celestial reference frame
$\rightarrow$ precession and nutation
$\rightarrow$ spin rate changes (ΔLOD vs. 86400 s)

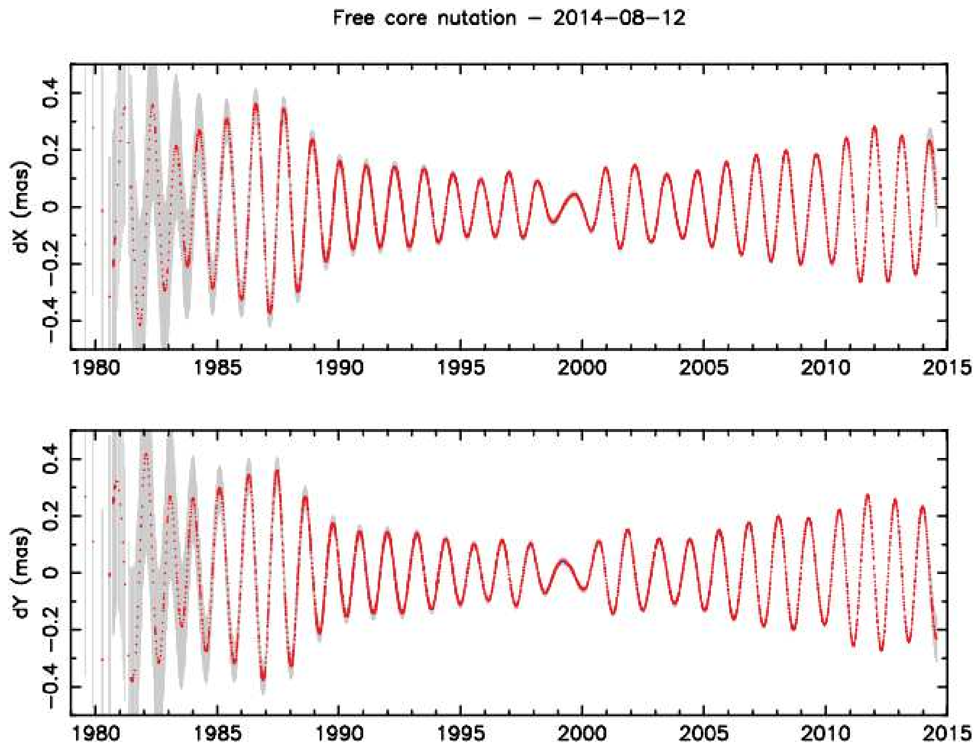
- precision $\sim$3 mm
- $\sim$32 antenna worldwide



Free Core Nutation

Direct observation from VLBI data

Amplitude $\sim$200 μas
1 mas $\sim$3 cm
Δg $\sim$1.2 nGal $(10^{-12}g)$

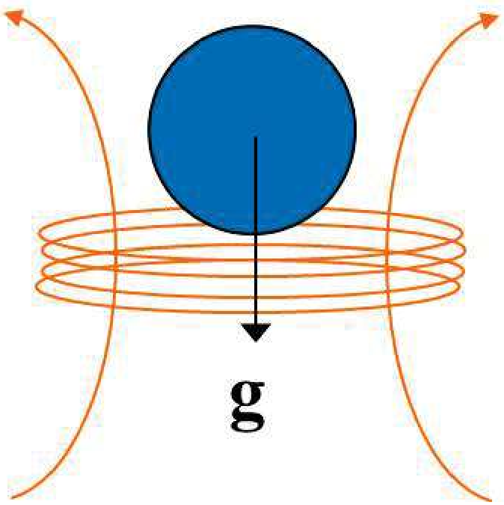


Source: http://ivsopar.obspm.fr/nutation/index.php

Excitation mechanism: atmosphere alone not enough

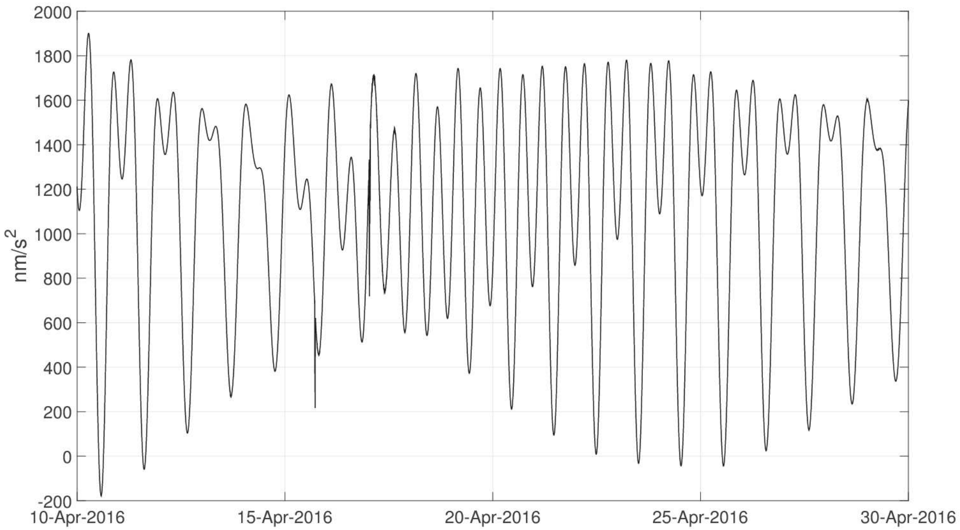
Superconducting gravimeter

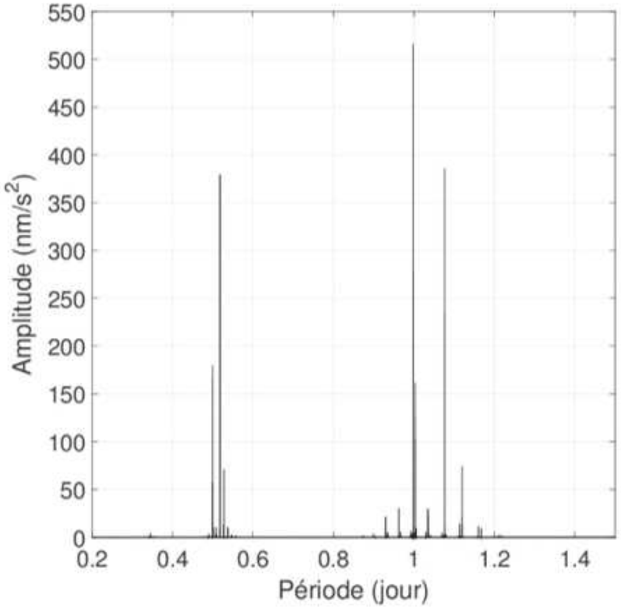
Rigid rotation + deformation
→ effect on Earth's gravity field (centrifugal effect)



magnetic levitation of a Niobium sphere in a cryogenic Helium bath
- sensitivity ∼1 nGal ($\sim 10^{-12} g$)
- weak instrumental drift (a few μGal/yr)

Free Core Nutation resonance

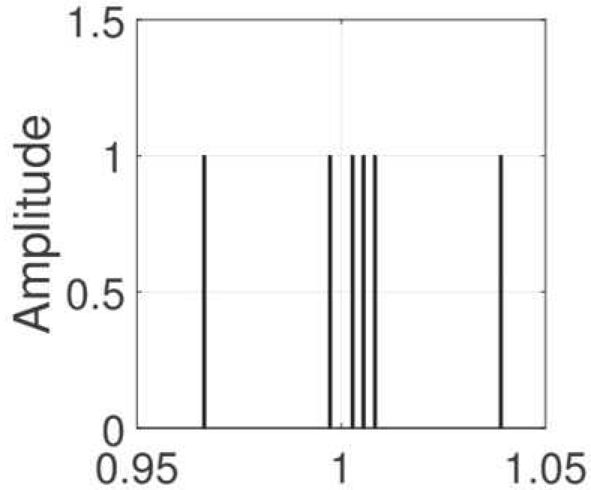




Tides: δg of several 100 of nm/s² $(10^{-10} g)$

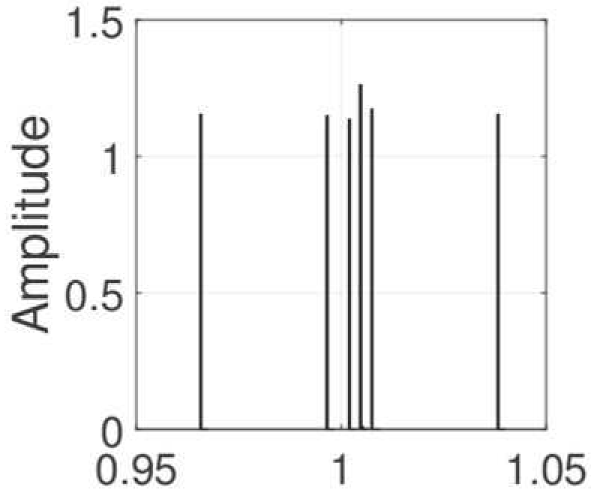
Free Core Nutation resonance

Rigid Earth



Earth with a fluid core

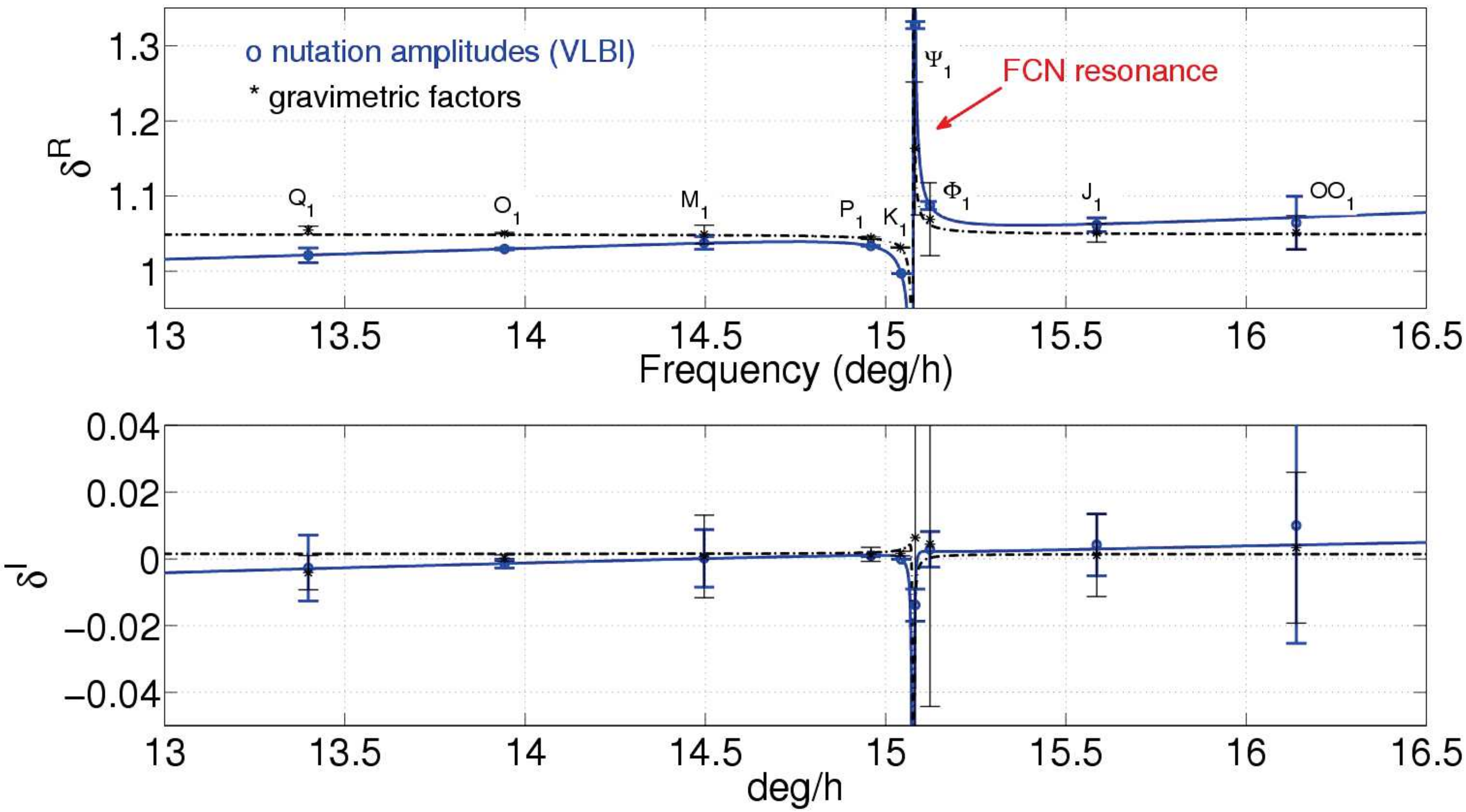
Tidal forcing $\rightarrow$ Amplification by **resonance** of some diurnal waves



Frequency (cycle per day)

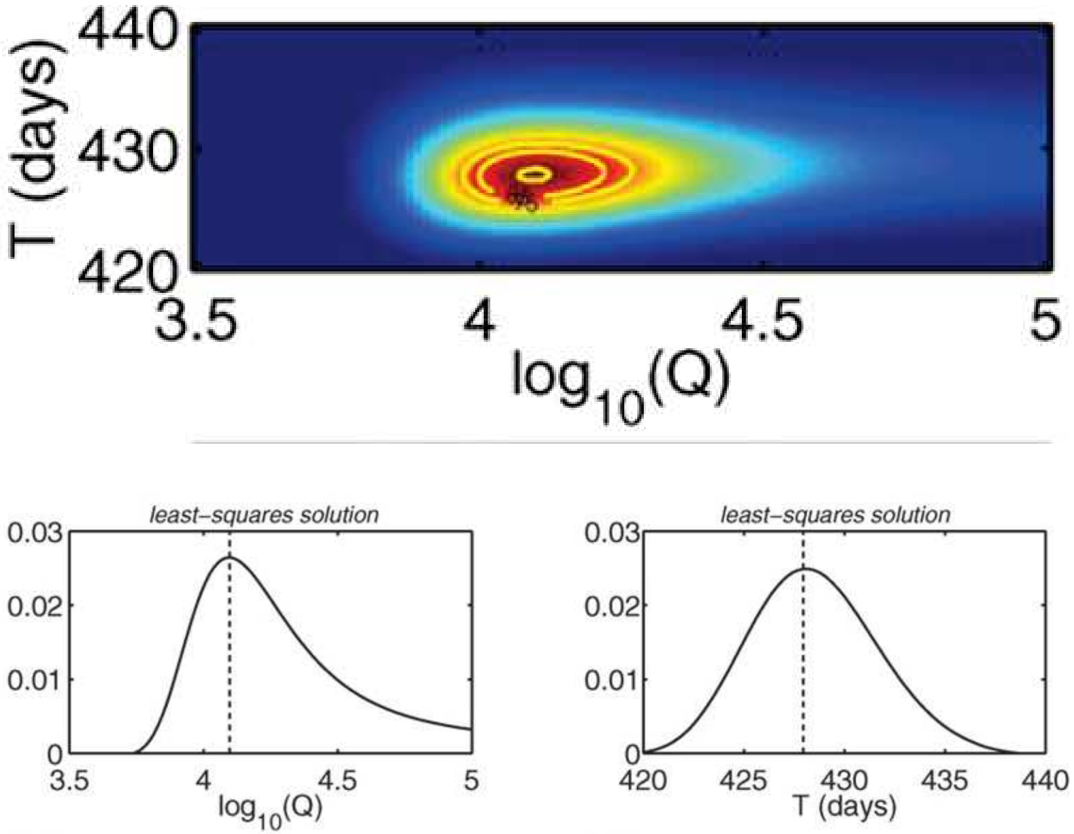
Free Core Nutation resonance

δ-factors = observed/theoretical tidal amplitudes for a rigid Earth



Free Core Nutation resonance

$$T(\sigma) = \delta_0 + \frac{N_2}{\sigma' - \sigma_{FCN}}$$



Bayesian inversion $\rightarrow T \sim -429.6 \pm 0.1$ days, $Q \sim 19,700 \pm 300$

Free Core Nutation

- FCN period

→ core dynamic flattening
~1/400 (a~3482 km, b~3491 km)
(Earth: ~1/300, a~6378 km, b~6357 km)

- FCN damping

→ **dissipation** at CMB due to coupling mechanisms (topography, viscosity, electromagnetic)
→ **electric conductivity** of lower mantle
→ **magnetic field** value at CMB
(~ 0.7 mT, Mathews et al. 2002)
→ **viscosity** at CMB
($\sim 4\ 10^{-2}$m^2/s, Deleplace & Cardin 2006)

Free inner core nutation

T $\sim$ [400 - 1200] sid. days?
Q $\sim$ [400 - 500]?
Prograde in space

Never "clearly" detected

Neither the mode nor its resonance (by tidal forcing)

IC angular momentum $\sim$ 1/1400 global Earth

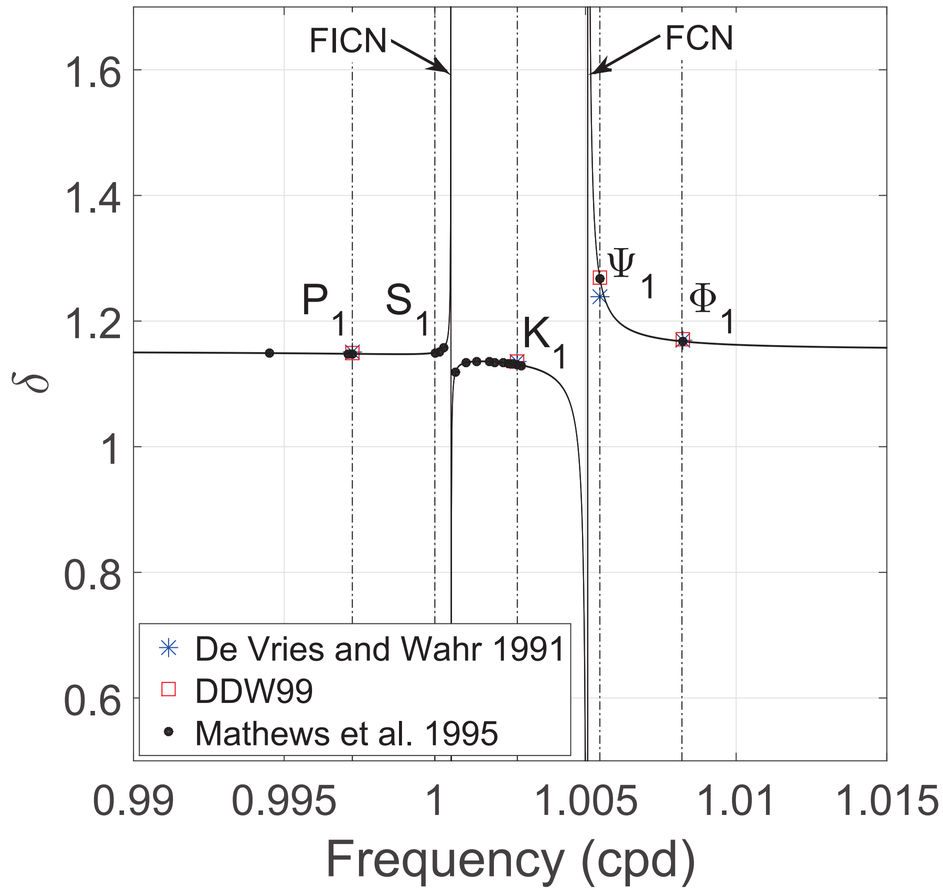
$\sim$ a few tens of µas ($\sim$**0.3 mm**)

$\rightarrow \Delta g \sim$**0.1 nGal** $(10^{-13} g)$

- Ellipticity of ICB?
- Density jump at ICB?
- Magnetic field at ICB?
- Viscosity at ICB?
- Other coupling effects at ICB?...

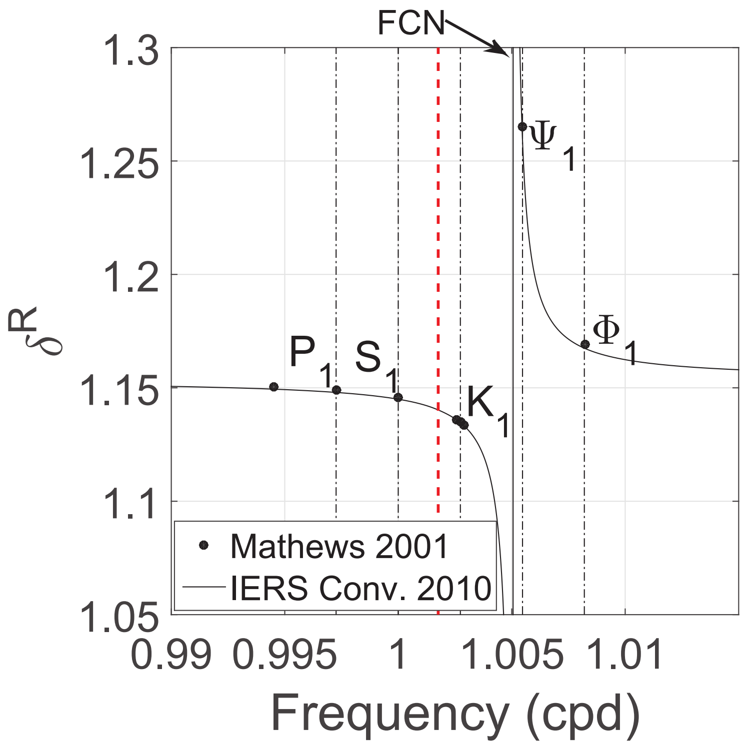
Free inner core nutation

No dissipation, $T_{FICN} = 455$ days



Free inner core nutation

Dissipation, $T_{FICN} = 1030$ days



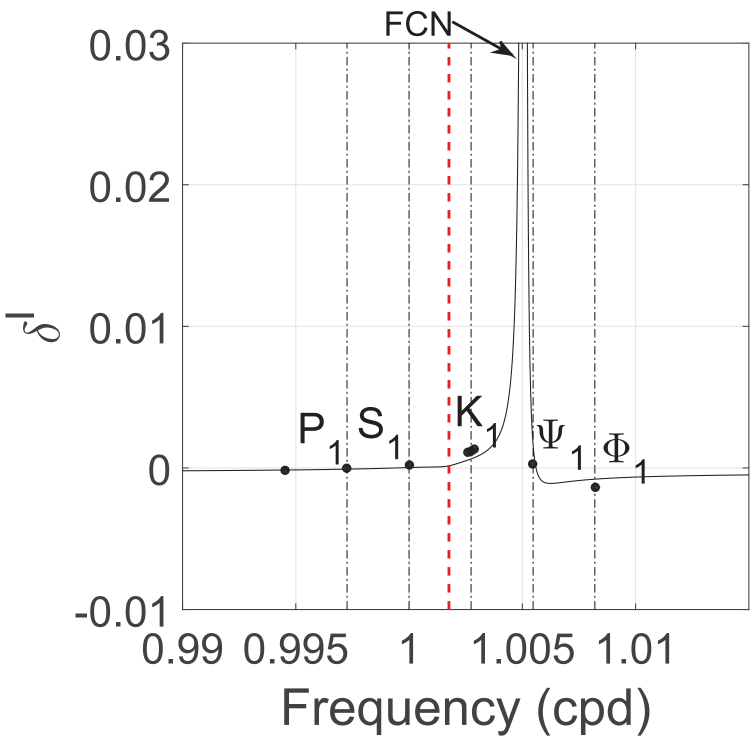
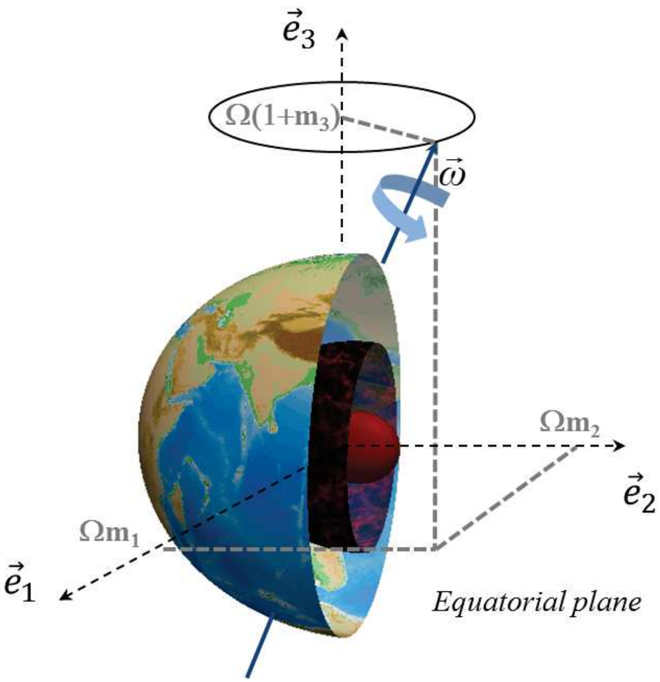
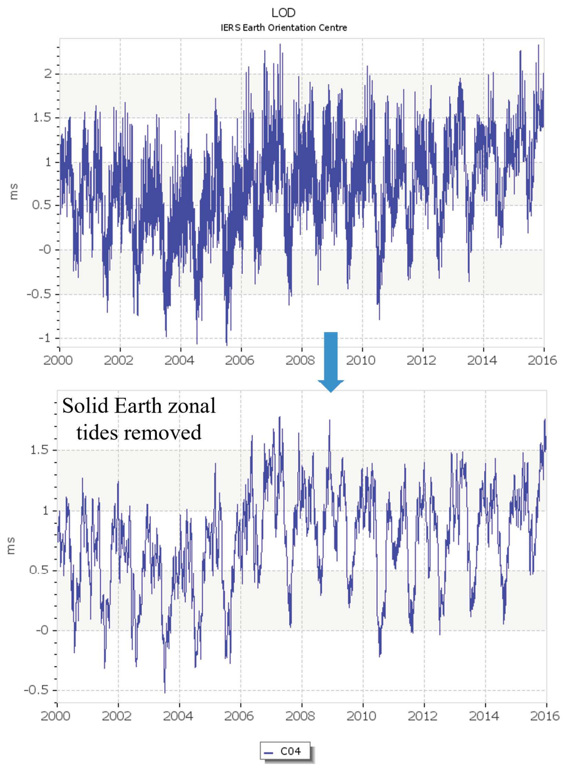


Table of Contents

Length of day (LOD)

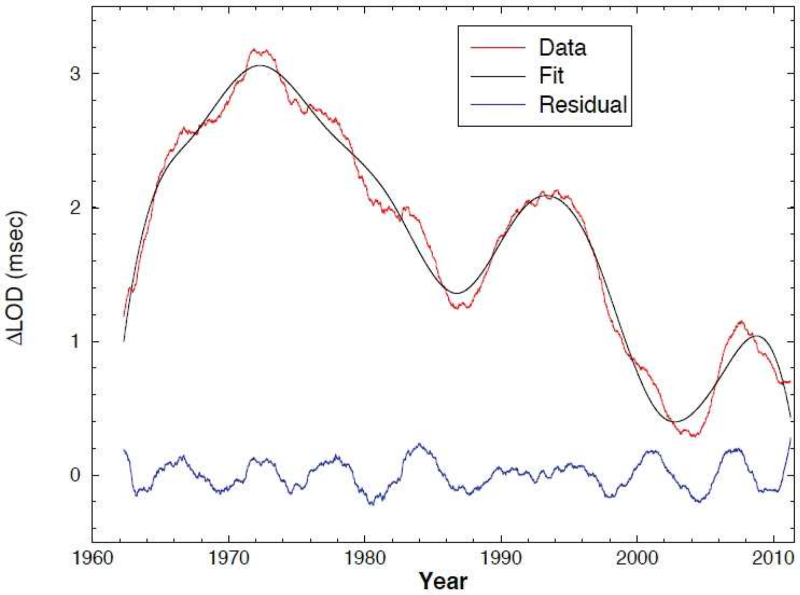




Observed effect	Amplitude	Origin
Secular trend	< 2 ms/century	Tidal friction
Decadal fluctuations	~ 2 ms	Core-mantle coupling
Seasonal variations	~ 0.5 ms	Tides, atmosphere oceans, hydrology
Monthly, fortnightly variations	~ 0.5 ms	Luni-solar tides

A 5.9-yr oscillation in length of day (LOD)

- A decadal fluctuation in LOD ($\approx$ 2 ms) well-explained by core-mantle coupling (e.g. Jault et al. 1988)
- A 5.9-yr oscillation in LOD ($\approx$ 0.12 ms): mechanism?



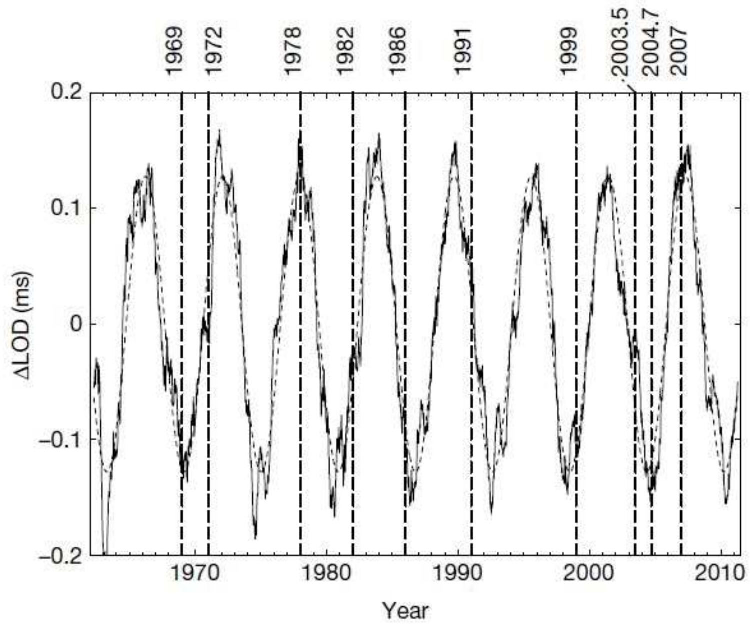


Figure 2 | Decadally detrended LOD data (with 6-month running average), plotted with 5.9-year oscillation fit (dashed line). Vertical lines show best determinations of geomagnetic jerk timings.

Abarca del Rio et al. (2000), Holme & de Viron (2013)

Core-Mantle Angular Momentum exchanges

- Geostrophic flow velocity:

$$U_G = -\sum_{n=0}^{\infty} t_{2n+1}^0 P_{2n+1}$$

- Core angular momentum (C_c core moment of inertia):

$$H_c \simeq C_c \left(t_1^0 + 1.776 t_3^0 + 0.0796 t_5^0 + 0.002 t_7^0 + 4.10^{-5} t_9^0 + ...\right),$$

- Conservation of total angular momentum of Earth:

$$\rightarrow \boxed{\Delta LOD = -H_c \frac{2\pi}{\Omega^2 C_m} \simeq 1.232 \left(\delta t_1^0 + 1.776 \delta t_3^0\right)}$$ (LOD in ms, flows in km/yr)

[Jault & Finlay (2015)]

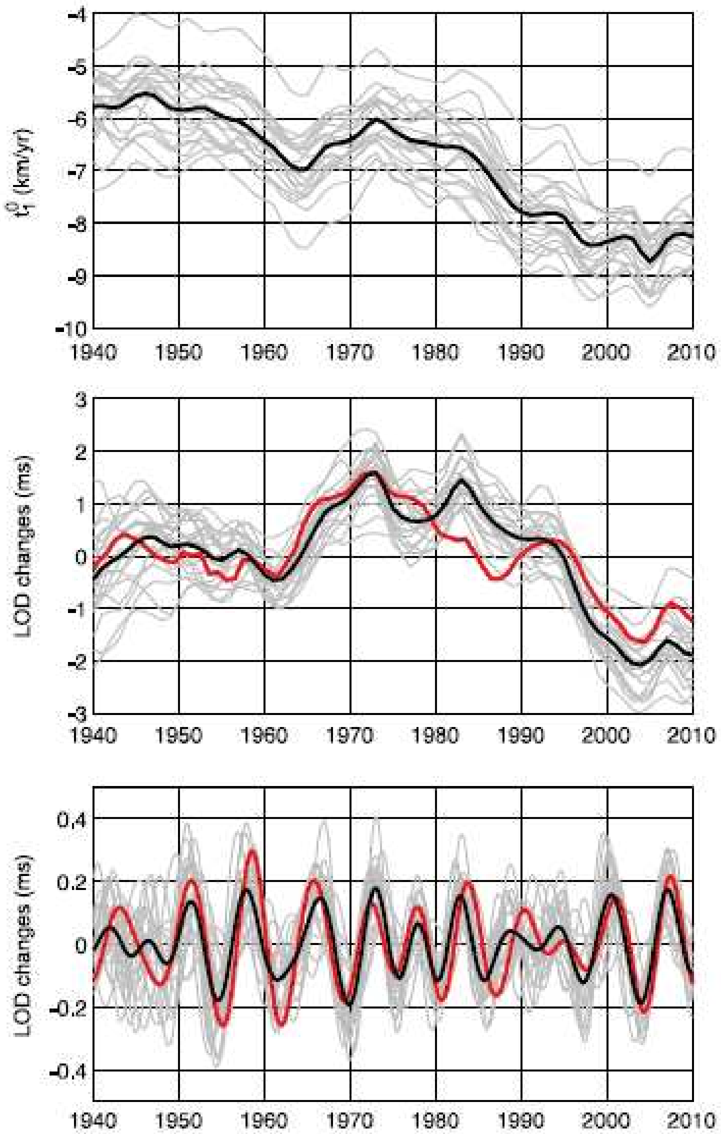
- Steady or slow flows ~ 15 km/yr, decadal & interannual flows: few km/yr

Inter-annual LOD changes and core flows

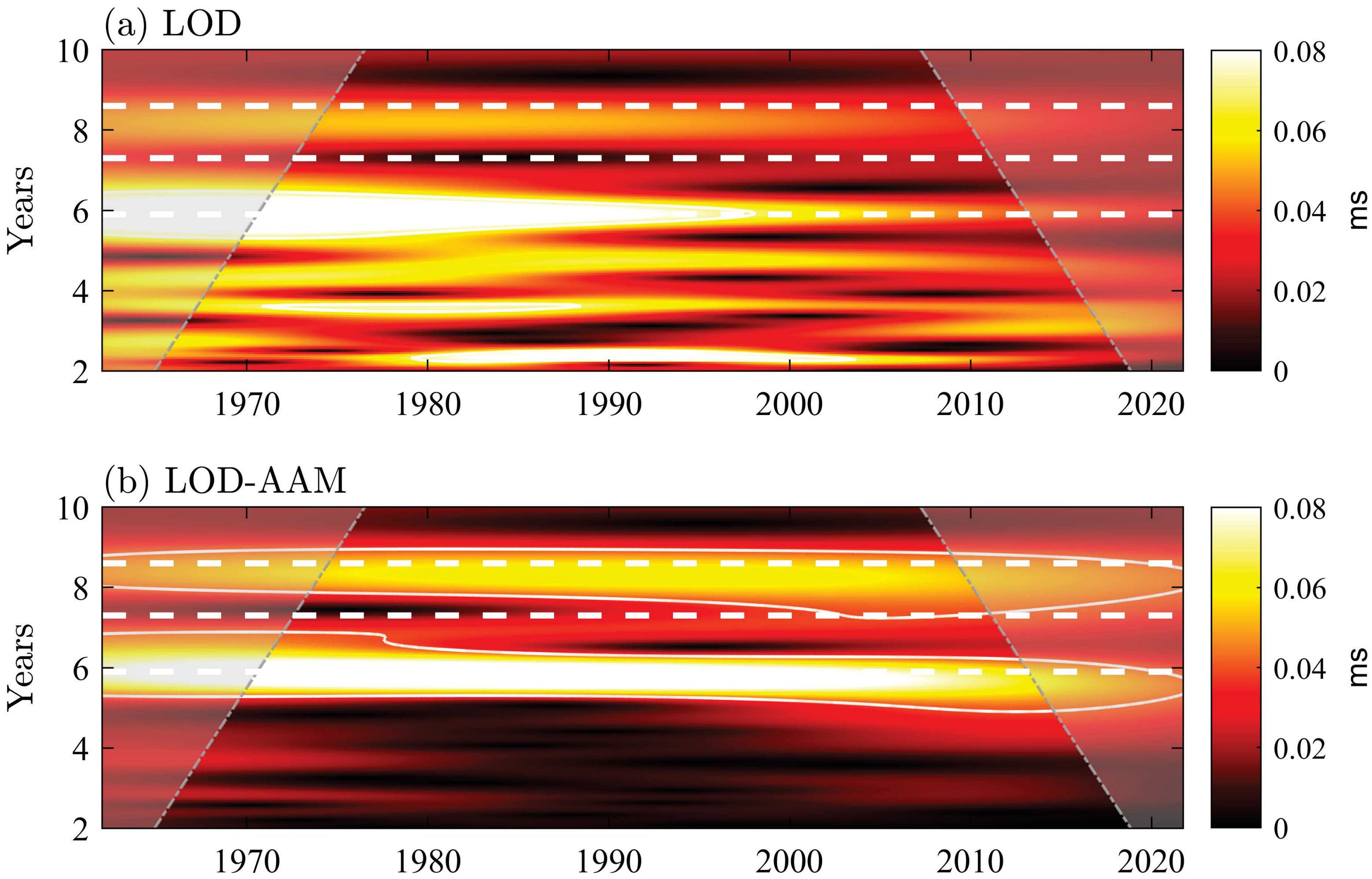
(top) Flow coefficient t_0^1 (km/yr)
(middle) predicted (black) and observed LOD changes (red) (ms)
(bottom) LOD band-pass filtered between 4 and 9.5 years.

$\rightarrow$ inter-annual LOD changes well-explained by core flow models inverted from (independent) geomagnetic data

(Gillet et al. 2015)

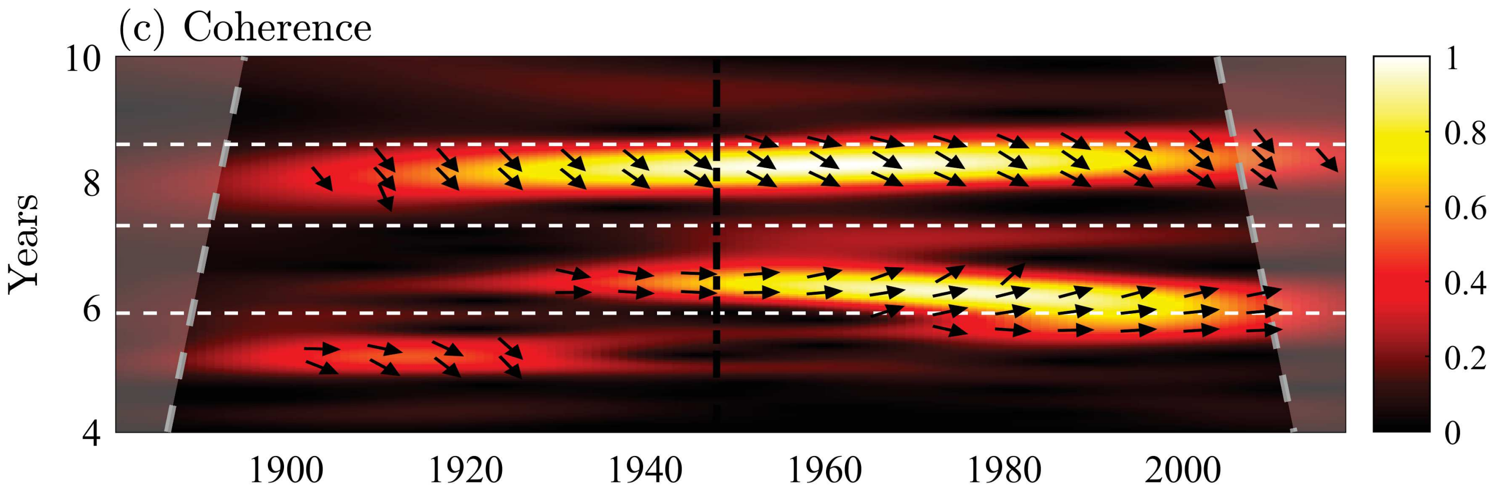


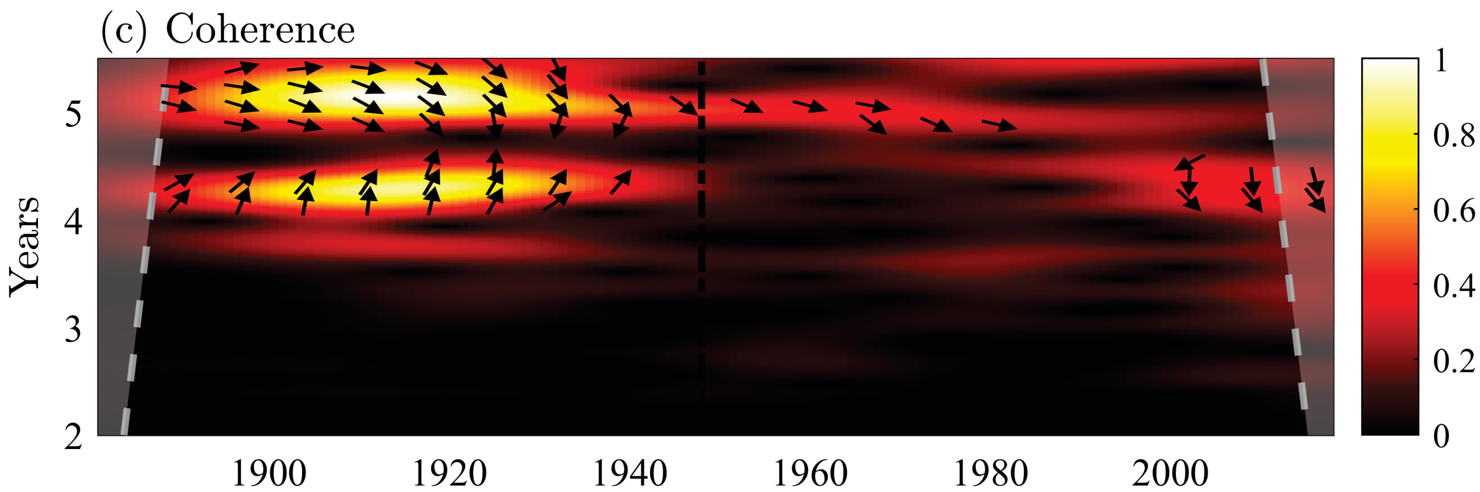
Inter-annual LOD changes and Atmospheric Angular Momentum



[Rosat & Gillet, in prep.]

Inter-annual LOD changes and core flows



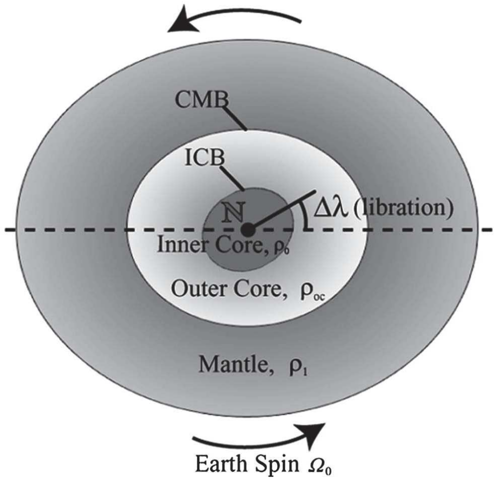


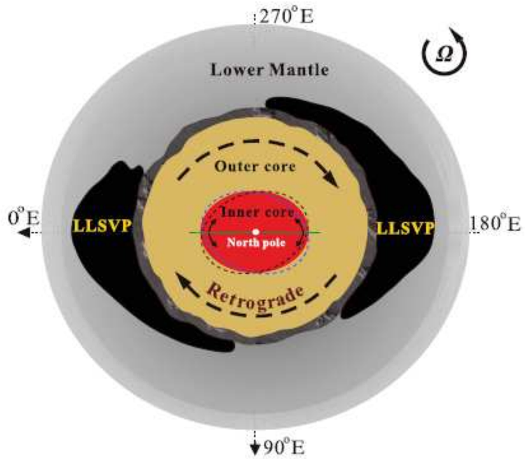
[Rosat & Gillet, in prep.] $\rightarrow$ 5.9, 8.5-yr signals

A 5.9-yr oscillation in length-of-day (LOD)

Free mode of the Mantle-Inner Core Gravitational coupling (MICG)

Buffett (1996, 1997), Mound & Buffett (2006)





Chao (2017) Ding & Chao (2018)

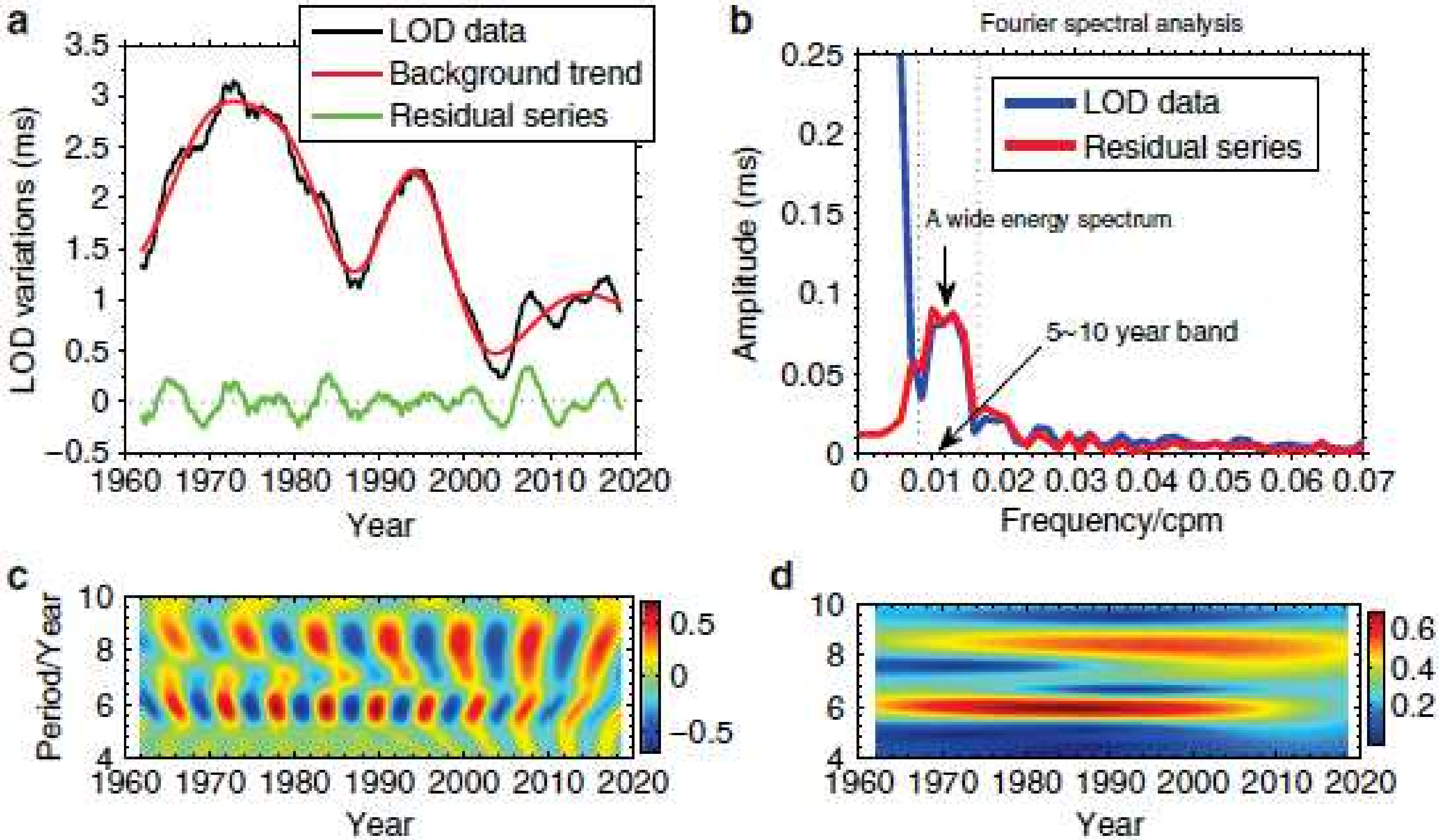
Large Low Shear Velocity Provinces

Torsional waves

Gillet et al. (2010, 2017)

Quasi-geostrophic torsional waves travelling from ICB to CMB in 4 years

A 8.6-yr oscillation in length-of-day (LOD)



amplitude ~ 0.08 ms   Mechanism?

Duan & Huang (2020)

A 8.6-yr oscillation in length-of-day (LOD)

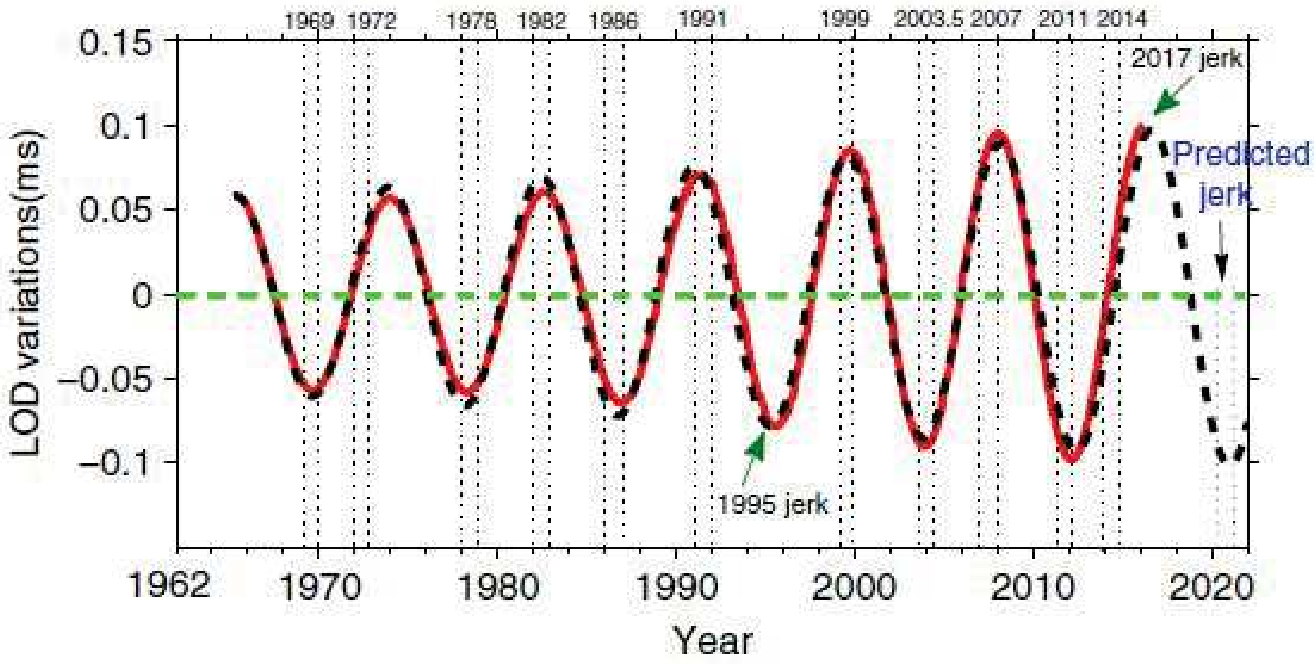


Fig. 4 Correspondence between the 8.6-year signal and geomagnetic jerks. In this figure, the red curve expresses the recovered 8.6-year signal in LOD, while the black dashed curve shows the fitting result (i.e., an exponentially increasing model with the expression of $y(t) = A_0 \exp[\alpha(t - t_0)]\cos(2\pi f(t - t_0))$, where the initial amplitude $A_0 \approx 0.06$ ms; the currently observed exponential rate $\alpha \approx +0.00131$/month; $f \approx 0.00969$ cpm; the initial time t_0 is set to be at June 1982) of the red curve, which may be used to predict the time when the next new jerk (i.e., the predicted jerk in blue fonts) will probably happen.

Questions?

Earth orientation data available from the International Earth Rotation and Reference Systems Service

https://www.iers.org

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