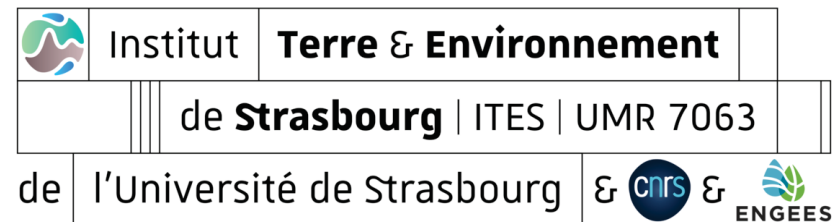


Geodesy: Gravimetry

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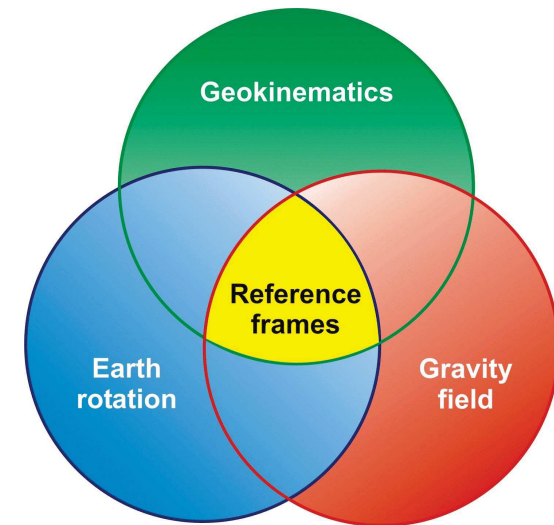
November 1, 2022



Les Houches, November 2022

What is geodesy?

Geodesy is the science of accurately measuring and understanding the Earth's **geometric shape, orientation in space, and gravity field.**



The three pillars of geodesy. The changes in Earth's shape (geokinematic), gravity field and rotation provide the conceptual and observational basis for the **reference frames** required for Earth observation.

<http://ggos.org>

Outline

- 1 Theory
 - Potential theory
 - Earth's figure
 - Ellipsoid of Clairaut
 - Geoid
 - General relativity
- 2 Instruments
 - Absolute Gravimeters
 - Space Gravimetry
 - Relative Gravimeters
- 3 Applications of gravimetry
 - Tides
 - Hydrology
 - Seismology

Table of Contents

1 Theory

- Potential theory
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 - Geoid
- General relativity

2 Instruments

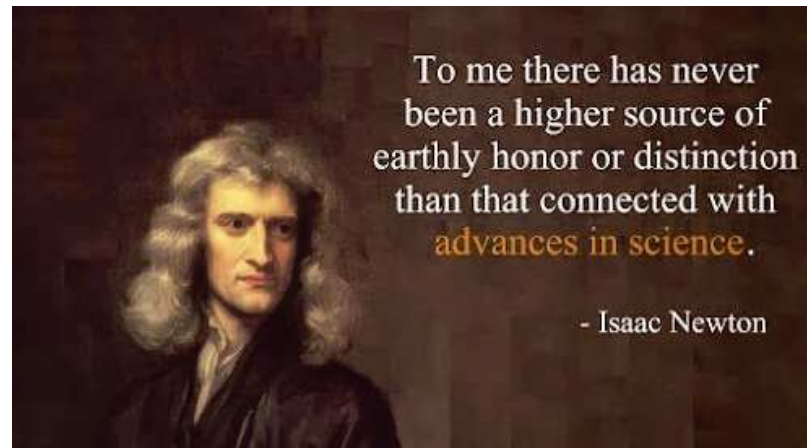
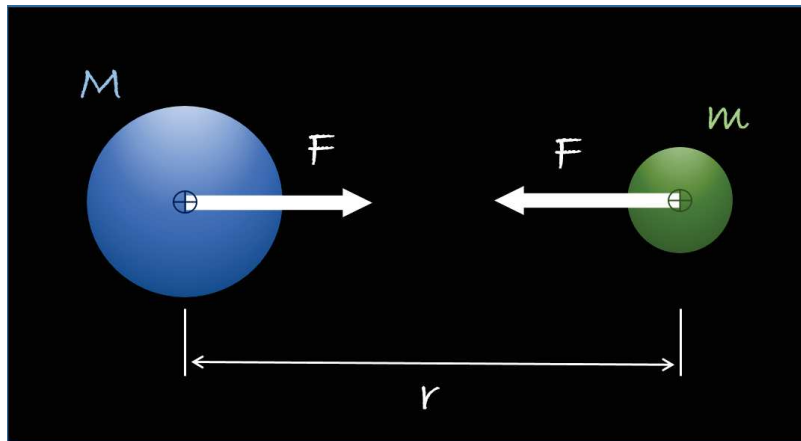
- Absolute Gravimeters
- Space Gravimetry
- Relative Gravimeters

3 Applications of gravimetry

- Tides
- Hydrology
- Seismology

Newtonian law

- Universal gravitational law (Newton, 1687): reciprocal action between two masses via the gravitational force:



$$\vec{F} = -\frac{GmM}{r^2}\vec{e}_r$$

where \vec{e}_r is the unit vector from mass M towards mass m , r the distance between the two masses and G is the universal constant of gravitation ($G = 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ S.I.).

- When $m = 1$, the gravitational field: $\vec{g} = -\frac{GM}{r^2}\vec{e}_r$

Gravitational potential

- Terrestrial gravity (attraction of the Earth): $\vec{g}_0 = -\vec{\nabla}V$
- Potential V (Lagrange, 1777) characterizes the ability of a mass to fall (or rise) (potential energy per unit mass required to move from one equipotential surface to another)
- Spherically symmetric Earth: $g_0 = \frac{GM_0}{R^2}$ (M_0 mass, R radius of the Earth)
- Continuous mass distribution in a volume Γ , $V = G \int_{\Gamma} \frac{\rho}{r} dv$
- Poisson equation:

$$\Delta V = \vec{\nabla} \cdot (\vec{\nabla} V) = \begin{cases} 4\pi G\rho & \text{in the mass} \\ 0 & \text{outside the mass} \end{cases}$$

Gravitational potential

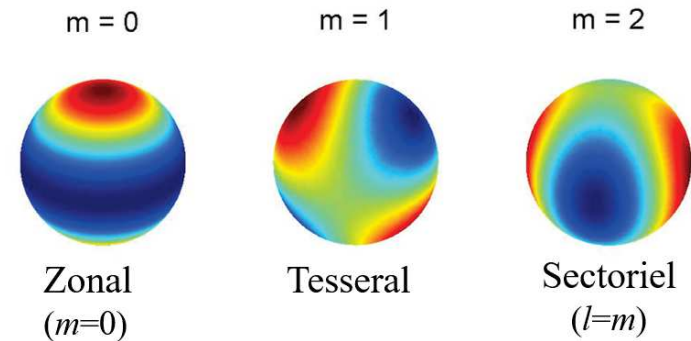
- In a spherical coordinates system (r, θ, λ) , solution to Laplace equation:

$$V(r, \theta, \lambda) = \sum_{l=0}^{\infty} \sum_{m=-l}^l \left(\frac{a}{r}\right)^{l+1} V_{l,m} Y_{l,m}(\theta, \lambda)$$

introducing *Stokes coefficients* $(C_{l,m}, S_{l,m})$ and associate Legendre polynomials,

$$V(r, \theta, \lambda) = \frac{GM}{a} \sum_{l=0}^{\infty} \left(\frac{a}{r}\right)^{l+1} \sum_{m=0}^l (C_{l,m} \cos m\lambda + S_{l,m} \sin m\lambda) P_{lm}(\cos \theta)$$

- $l = 1$: defines center of mass of the Earth
→ *geocenter motion* (seasonal mass transfers of surficial layers)



- $l = 2, m = 0$: $J_2 = -C_{20}$, *dynamic flattening* of the Earth
- Zonal terms: $J_l = -C_{l0}$
- C_{22}, S_{22} : ellipticity of the equator

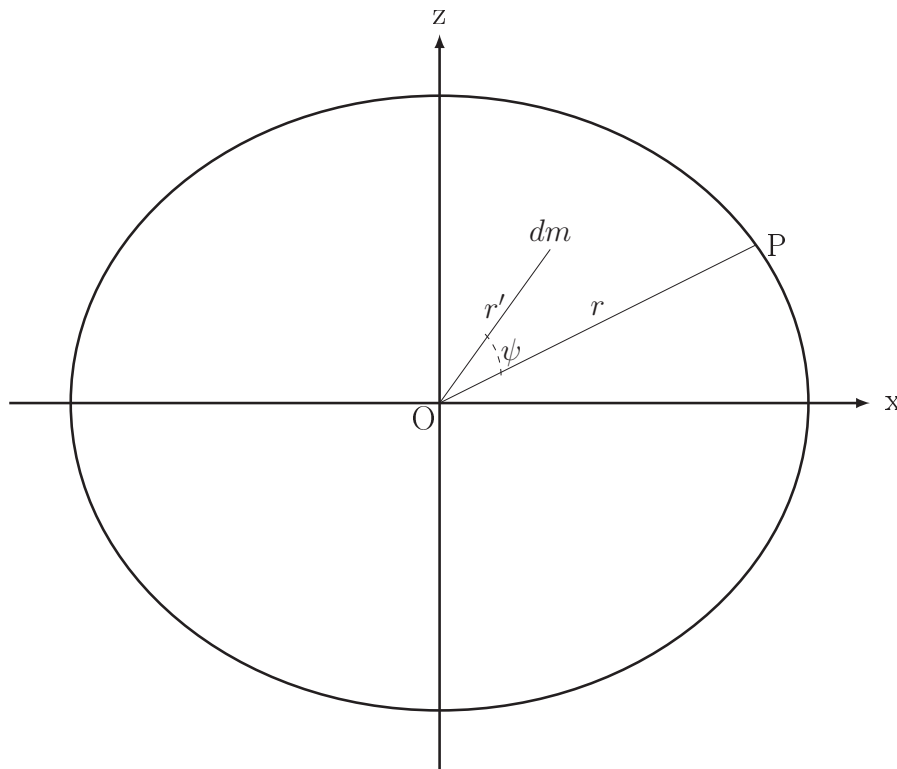
Gravitational potential

Moments of inertia:

$$A = \int_{\Gamma} \rho(x, y, z)(y^2 + z^2)dv$$

$$B = \int_{\Gamma} \rho(x, y, z)(x^2 + z^2)dv$$

$$C = \int_{\Gamma} \rho(x, y, z)(x^2 + y^2)dv$$



$$V^0(\vec{r}) = -G \int_{\Gamma^0} \frac{\rho^0}{|\vec{r} - \vec{r}'|} dv'$$

$$rr' \cos \Psi = xx' + yy' + zz'$$

$$|\vec{r} - \vec{r}'|^{-1} = \frac{1}{|\vec{r}|} \sum_{l=0}^{\infty} \left(\frac{|\vec{r}'|}{|\vec{r}|} \right)^l P_l(\cos \Psi)$$

$$l = 2: \quad V_2^0 = -\frac{GM}{r} - \frac{G}{2r^3} (A + B + C - 3I)$$

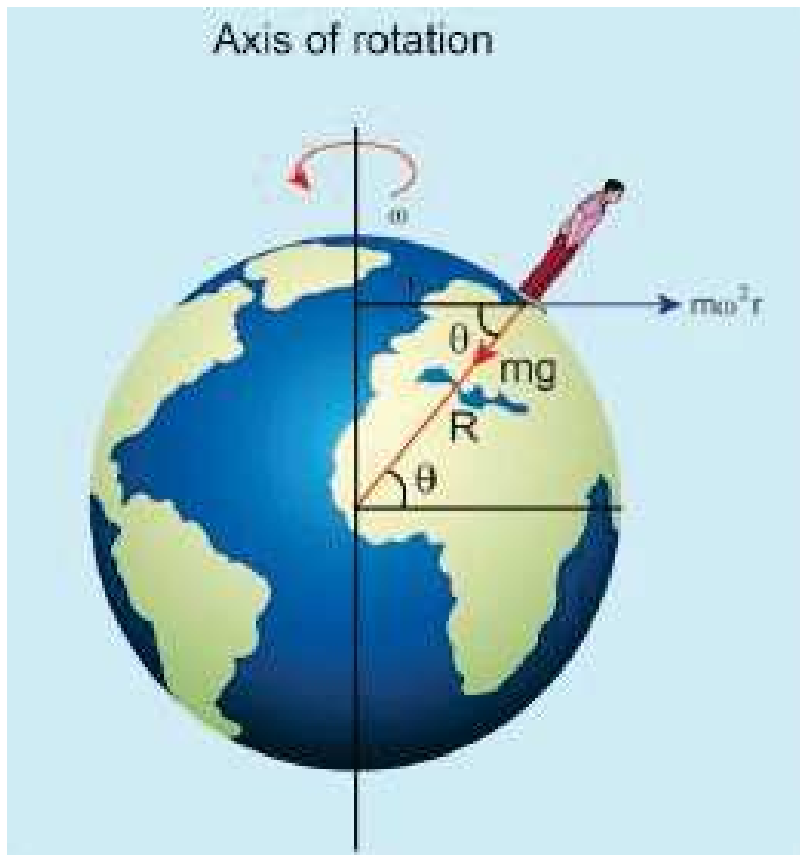
(I moment of inertia around OP axis)

This is **MacCullagh's formula**.

If Earth is a rotating ellipsoid with $A = B$,

$$V(r, \theta, \lambda) = -\frac{GM}{r} \left(1 - \sum_{l=2}^{\infty} \left(\frac{a}{r} \right)^l J_l P_l(\cos \theta) \right)$$

Gravity vs. *pesanteur*



Total gravity:

$$\vec{g} = \underbrace{\vec{g}_0}_{\text{gravitational}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{e}_r)}_{\text{centrifugal}}$$

Total gravity potential:

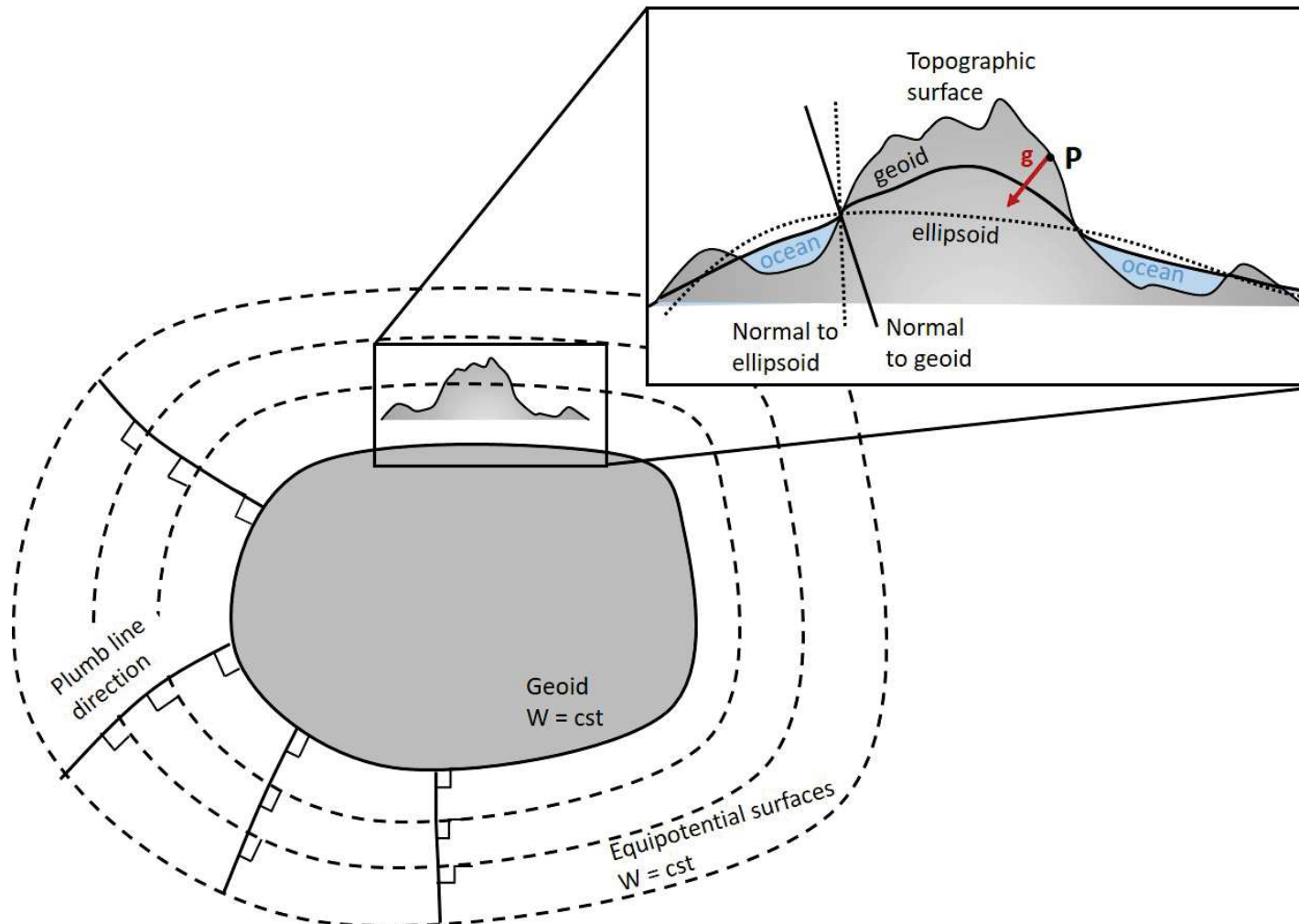
$$W = V + \Phi = G \int_{\Gamma} \frac{\rho}{r} dv + \frac{1}{2} \omega^2 r^2$$

$$\vec{g} = -\vec{\nabla} W$$

norm: acceleration in *Gal*

(1 Gal = 1 cm/s²)

Geoid vs. Ellipsoid



Homogeneous non-rotating spherical Earth: $g = \text{cst}$ at the surface
Real Earth: rotation, flattening \rightarrow *ellipsoid of reference*

Hydrostatic Earth's figure

[...] I indeavour'd to shew that the form of the Earth was probably somewhat flatter towards the Poles than towards the Equinoctial.

Robert Hooke, *A Discourse of Earthquakes*, 1705

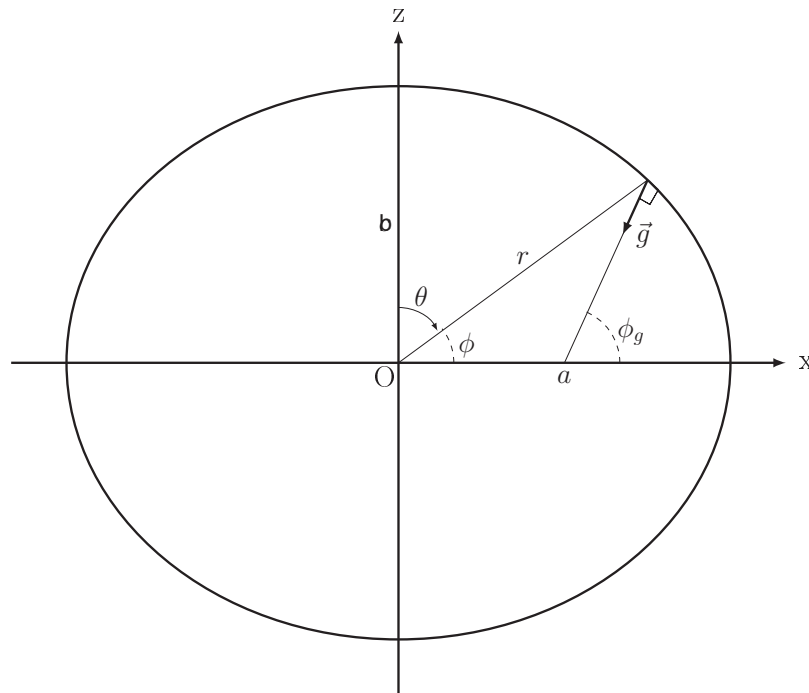
- 1666: Cassini observed that Jupiter was flattened
- 1671: Picard suggested that the Earth was not perfectly spherical
- 1675: Hooke assumed the Earth was a flattened ellipsoid
- 1686-1687: Hooke clearly claimed that the seas had an ellipsoidal figure because of the Earth's rotation. He proposed to prove it by *pendulum* and geodetic measurements at different latitudes.
- 1687: Newton supported this idea in his *Principia mathematica* and estimated a $1/230$ flattening for a homogeneous Earth, based on his general attraction theory
- 1690: Huygens proposed a new flattened model with a value $1/578$
- 1743: Clairaut reconciled equilibrium principa of Newton and Huygens

Ellipsoid of Clairaut

Fundamental hypothesis: Earth in hydrostatic equilibrium, its surface is an equipotential, its rotation is uniform about the polar axis, ellipsoidal $A = B$.

Principal moments of inertia:

$$A = \int_V \rho(x, y, z)(y^2 + z^2)dV, \quad C = \int_V \rho(x, y, z)(x^2 + y^2)dV$$



geographic latitude ϕ_g , geocentric latitude ϕ

Flattening: $\epsilon = 1 - \frac{b}{a}$

q : centrifugal force/gravity at equator

$J_2 = \frac{C-A}{Ma^2}$, *dynamic shape factor*

Theorem of Clairaut

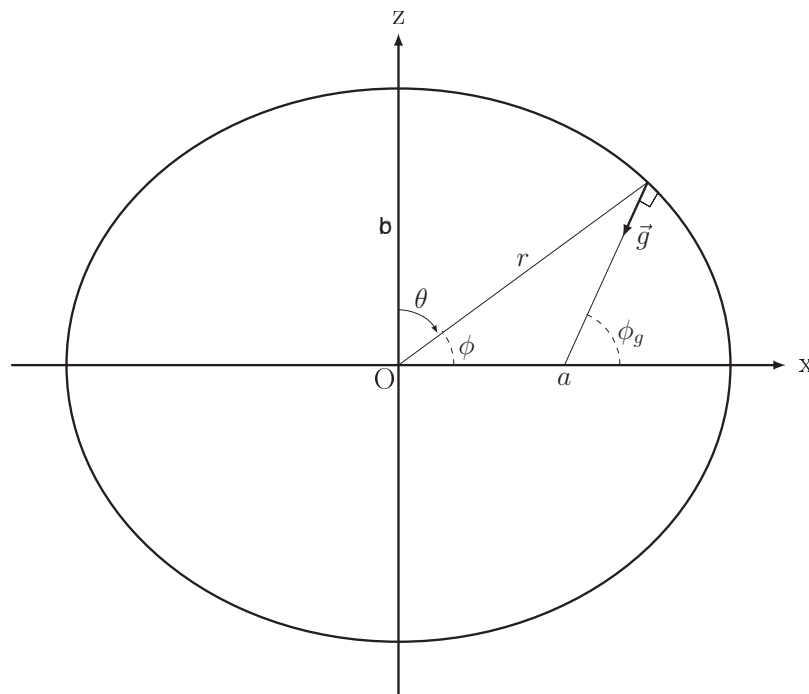
1st order in ϵ and J_2

$$\boxed{\epsilon = \frac{3}{2}J_2 + \frac{q}{2}} \quad (\text{1st eq. of Clairaut})$$

This equation connects a geometric ϵ , dynamic J_2 and kinetic q quantity.

Ellipsoid of Clairaut

- Newton: homogeneous Earth, $J_2 \approx \frac{2}{5}\epsilon$ and $\epsilon = \epsilon_H = \frac{5}{4}q \approx \frac{1}{230}$
- Huygens: Earth's model for which all masses are concentrated in the center ($J_2 = 0$), $\epsilon = \frac{q}{2} = \frac{1}{578}$



geographic latitude ϕ_g , geocentric latitude ϕ

To 1st order,

$$r = a \left[1 - \left(\frac{3}{2} J_2 + \frac{q}{2} \right) \cos^2 \theta \right] \text{ or}$$

$$r = a (1 - \epsilon \cos^2 \theta)$$

This is the *ellipsoid of Clairaut*,
1st-order approximation of the geoid.

Gravity on the ellipsoid of Clairaut

- Gravity = gradient of potential along the normal of the ellipsoid

$$g = -|\nabla \tilde{\phi}_2^0| = -\left[\left(\frac{\partial \phi_2^0}{\partial r}\right)^2 + \left(\frac{1}{r} \frac{\partial \phi_2^0}{\partial \theta}\right)^2\right]^{1/2}$$

- Deviation to the vertical wrt radial direction $\sim \epsilon$

$$g \approx -\frac{\partial \tilde{\phi}_2^0}{\partial r} = \frac{GM}{r^2} \left[1 - 3 \left(\frac{a}{r}\right)^2 J_2 \frac{3\cos^2\theta - 1}{2} - q \left(\frac{r}{a}\right)^3 \sin^2\theta\right]$$

- $\left(\frac{a}{r}\right)^2 \sim 1$, $\left(\frac{r}{a}\right)^3 \sim 1$ ($r = a$), to 1st order in J_2 and q ,

$$g \approx \frac{GM}{a^2} \left(1 + \frac{3}{2} J_2 - q + \cos^2\theta (2q - \frac{3}{2} J_2)\right)$$

- At the equator ($\theta = 90^\circ$), $g_E = \frac{GM}{a^2} \left(1 + \frac{3}{2} J_2 - q\right)$

- At the pole ($\theta = 0^\circ$), $g_P = \frac{GM}{c^2} \left(1 - 3 \frac{a^2}{c^2} J_2\right) \approx \frac{GM}{a^2} (1 - 3J_2 + 2\epsilon)$

- To 1st-order in ϵ , $\frac{1}{b} \approx \frac{1+\epsilon}{a}$, $\frac{1}{b^2} \approx \frac{1+2\epsilon}{a^2}$, $g = g_E \left[1 + (2q - \frac{3}{2} J_2) \cos^2\theta\right]$ or

$$g = g_E \left[1 + \left(\frac{5}{2} q - \epsilon\right) \cos^2\theta\right] \text{ or } g = g_E \left[1 + \left(\frac{g_P}{g_E} - 1\right) \cos^2\theta\right]$$

Gravity on the ellipsoid of Clairaut

$$g = g_E \left[1 + \left(\frac{g_P}{g_E} - 1 \right) \cos^2 \theta \right]$$

→ Expression of the gravity in the theory of Clairaut

- $\epsilon_g = \left(\frac{g_P}{g_E} - 1 \right)$ is the **gravimetric flattening**.

- $\epsilon_g = 2\epsilon - \frac{9}{2}J_2 + q$ (2nd equation of Clairaut)

- $\epsilon_g + \epsilon = \frac{5}{2}q$ (Clairaut's formula)

→ link between geometric flattening and gravimetric flattening

→ flattening is determined by gravity measurements (Ω_0 , a known)

→ to evaluate ϵ , we **do not need to know the Earth's internal structure**.

Gravity on the ellipsoid of reference

- With 2nd-order terms,

$$g = g_E \left[1 + \left(\frac{5}{2}q - \epsilon - \frac{17}{4}q\epsilon \right) \cos^2 \theta_g + \left(\frac{\epsilon^2}{8} - \frac{5}{8}q\epsilon \right) \cos^2 2\theta_g \right]$$

θ_g geographic co-latitude

- **International gravity formula** in the GRS80 reference ellipsoid

$$g = 9.780327 \left(1 + 0.0053024 \cos^2 \theta_g - 0.0000059 \cos^2 2\theta_g \right) \text{ m.s}^{-2}$$

$$g_E = 9.78 \text{ m.s}^{-2}, g_P = 9.83 \text{ m.s}^{-2}$$

- GRS80: 1980 Geodetic Reference System = reference ellipsoid (a , GM , J_2 , ω)
- WGS84: 1984 World Geodetic System (used in cartography, geodesy, satellite navigation including GPS) originally used the GRS80 reference ellipsoid, but undergone some minor refinements (Doppler, satellite laser ranging (SLR) and very-long-baseline interferometry (VLBI) observations).

Ellipsoid	Semi-major axis a	Semi-minor axis b	Inverse flattening $1/f$
GRS80	6378137.0 m	~ 6356752.314140 m	298.257222100882711...
WGS84	6378137.0 m	~ 6356752.314245 m	298.257223563

Dynamic effects of the Earth's flattening

- Homogeneous sphere: orbit of a satellite is a Keplerian ellipse
- Oblateness of the Earth: slow variation of the orbital plane with time, variation of the direction of the Earth's axis of rotation in space (*precession*)
- Law of mechanics, angular momentum and forces that tend to make orbital and equatorial planes coincide

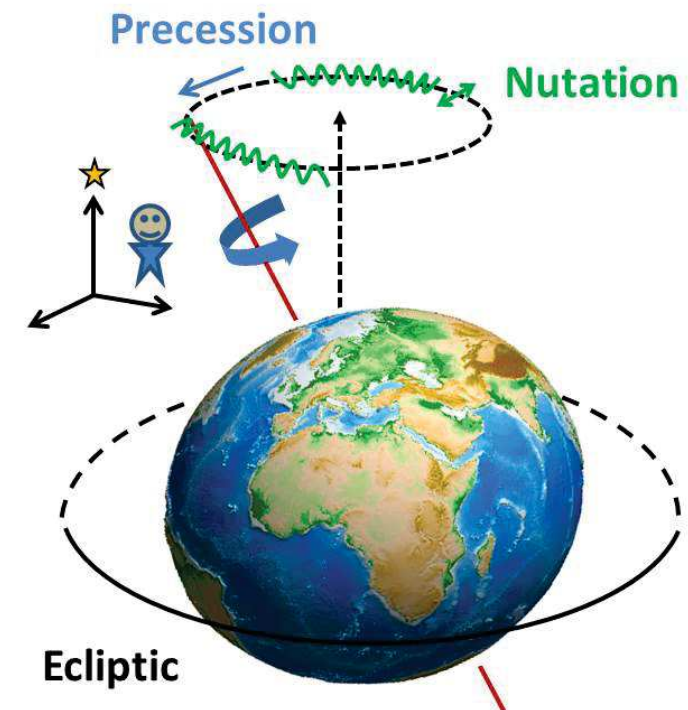
→ precession is proportional to $\alpha = \frac{C - \bar{A}}{C}$,
mechanical ellipticity

→ regression of orbital plane to $J_2 = \frac{C - \bar{A}}{Ma^2}$,
spherical harmonic coefficient

\bar{A} mean equatorial moment of inertia

C moment of inertia about rotation axis

[Heiskanen & Moritz (1967). Physical Geodesy]



Dynamic effects of the Earth's flattening

- For a heterogeneous ellipsoid, the *mechanical ellipticity* α is

$$\alpha = \frac{C-A}{C} = \frac{\int_0^1 \rho \, d(\epsilon r'^5)}{\int_0^1 \rho \, d(r'^5)} \rightarrow \text{depends on density}$$

- Equilibrium figure of the Earth depends on its density
- Although iso-density surfaces are equipotential surfaces, their **ellipticity decreases with depth**, when density increases.
- Ellipticity of equipotential surfaces is then affected by the composition of the layer surrounding the equipotential surface, making the computation of the ellipticity more difficult
- Link between ϵ and α complicated and indirect (\neq between J_2 and ϵ)

Equilibrium shape of the Earth

- In order to determine ϵ from α , we must assume the Earth in *hydrostatic equilibrium*
- Hydrostatic equilibrium: surfaces of equal density are equipotential

Theory of spheroidal figures governed by:

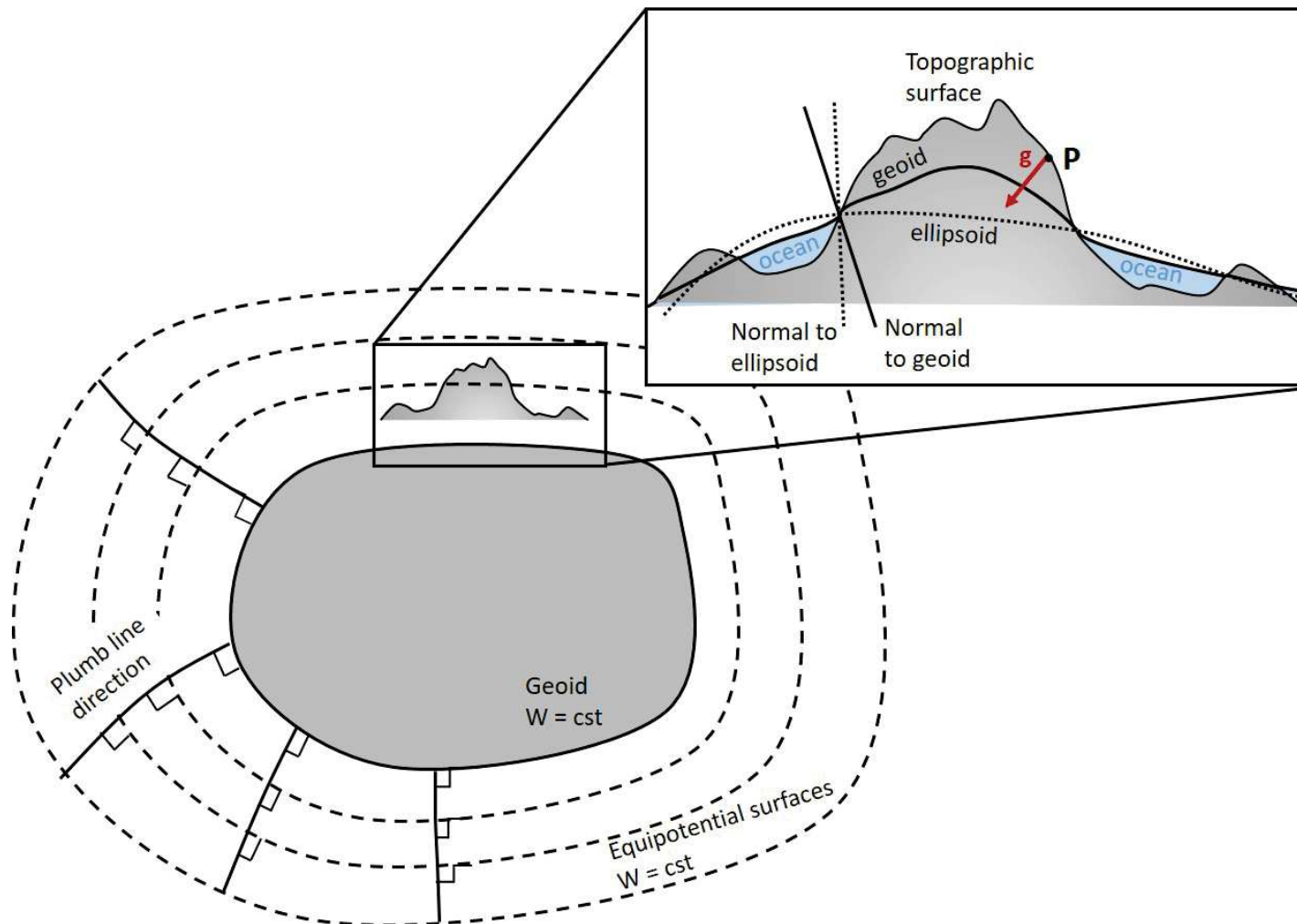
- $$\frac{d^2\epsilon}{dr^2} + \frac{6\rho}{r\bar{\rho}} \frac{d\epsilon}{dr} + \frac{6\epsilon}{r^2} \left(\frac{\rho}{\bar{\rho}} - 1 \right) = 0$$
 Differential equation of Clairaut

with $\bar{\rho} = \frac{3}{r^3} \int_0^r \rho(r') r'^2 dr'$, mean density of sphere of radius r

→ when integrated (concentric spheres), gives the shape of equipotential surfaces inside a body in equilibrium as a function of the density distribution

[Heiskanen and Moritz (1967). Physical Geodesy]

Geoid vs. Ellipsoid



True Earth: density distribution is more complex (non-homogeneous envelopes, convective motions...) than in a concentric model

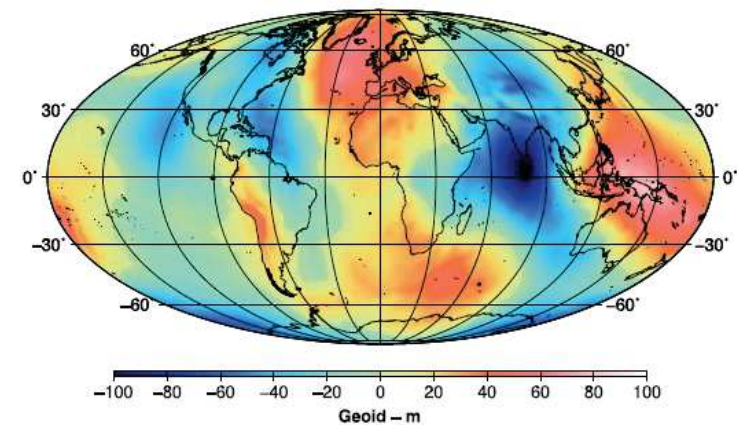
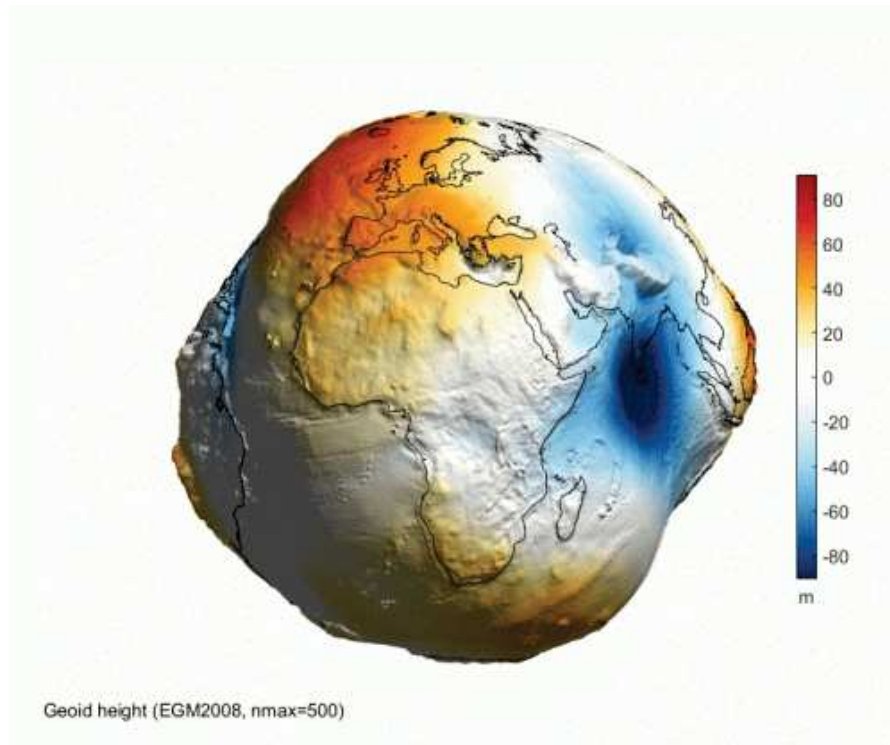
Separation between the geoid and the ellipsoid: *geoid undulation*.

Earth's flattening

- Bullard (1948): $\frac{1}{\alpha} = 305.59$, using 2nd-order solution to Clairaut's equation (density distribution): $\frac{1}{\epsilon} = 297.34$
- Satellite geodesy: $\epsilon^{-1} \in [298.2 - 298.3]$ from J_2 **determination**
- Question of a **significant deviation of the Earth's interior from hydrostatic equilibrium** is old. Poincaré (~ 1885) recognized that the results of precession under the assumption of hydrostatic equilibrium were incompatible with the value of $1/\epsilon = 293$ at that time.
- The difference between the polar and equatorial radii appears to be 113 ± 1 m (instead of 98 m) larger than the hydrostatic value [Chambat et al. 2010]

Earth's geoid

Geoid heights with respect to the reference ellipsoid surface established by World Geodetic System 1984 (WGS84)



Mean geoid (between 2003 and 2008),
after GRACE satellites (GRGS
Release-3 solutions)

→ Obtained by combination of various gravity measurements (static ground, dynamic airborne and marine and satellite gravity and surface topography)



Isaac Newton (1687): gravity is a central force, instantaneous, proportional to inverse of square of distance

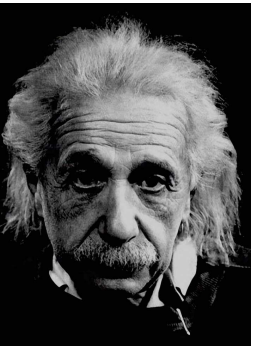
- Conservation of energy of a system
- Orbit of a body around another one is elliptical
- Time and space are independent



Pierre Simon de Laplace (1805): gravity may have a finite speed. . .

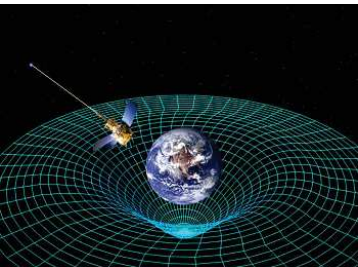
- Energy is no more conserved (dissipation) ? *but how?*

→ *good feeling but not good theory*



Albert Einstein (1905/1916): the force of gravity does not exist. . .

- Time and space are not independent
- A mass deforms the geometry of space-time
- Trajectories of bodies are straight but in a curved space
- Gravitation propagates at the speed of light
- Gravitation is the manifestation of the curvature of space-time



Energy dissipated through deformation of space time in the form of a **gravitational wave**.

Gravitational waves

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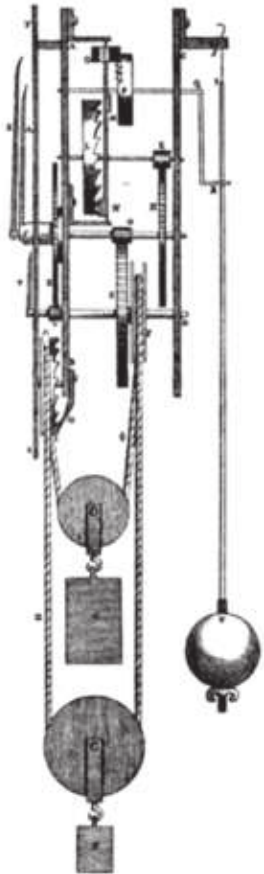
How to measure gravity?

- Absolute measurements of its intensity
- Relative measurements of its variations in time and/or space

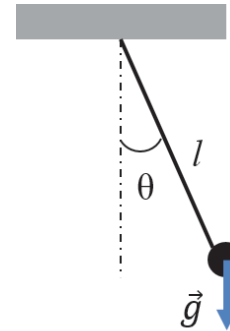
→ Two types of gravimeters:

- 1 absolute gravimeters: intensity of g
- 2 relative gravimeters: variations of g

From pendulum...



$$T = 2\pi\sqrt{l/g} \text{ (small } \theta \text{)}$$



- ~1602, **Galileo Galilei**: 1st scientific investigations of the pendulum
- 1620, British scientist **Francis Bacon**: 1st to propose using a pendulum to measure gravity, suggesting taking one up a mountain to see if gravity varies with altitude
- 1656, Dutch scientist **Christiaan Huygens** built the 1st pendulum clock
- 1666, English scientist **Robert Hooke** suggested that the pendulum could be used to measure the force of gravity

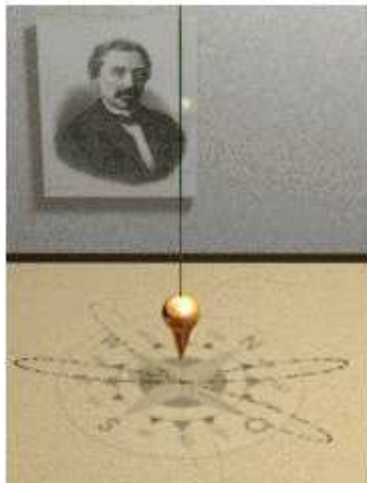
Regular motion of pendulums used for timekeeping (world's most accurate timekeeping technology until the 1930s (quartz clocks)).

From pendulum...

- 1671, **Jean Richer**: **1st observation that gravity varied at different points on Earth**. A pendulum clock was $2\frac{1}{2}$ minutes per day slower at Cayenne (French Guiana) than at Paris → force of gravity lower at Cayenne.
- 1687, **Isaac Newton** (*Principia Mathematica*): this was because the Earth was slightly oblate (centrifugal force due to its rotation), causing gravity to increase with latitude. He also found that equal length pendulums with bobs made of different materials had the same period, proving that the gravitational force on different substances was exactly proportional to their mass (inertia)
→ **Equivalence principle**
- 1737, **Pierre Bouguer** made a sophisticated series of pendulum observations in the Andes mountains, Peru → first rough estimate of the density of the Earth
- 1743, **Alexis Claude Clairaut**: 1st hydrostatic model of the Earth, allowing the ellipticity of the Earth to be calculated from gravity measurements

Portable pendulums began to be taken on voyages to distant lands, as precision gravimeters to measure the acceleration of gravity at different points on Earth, eventually resulting in **accurate models of the shape of the Earth**.

From pendulum...



Foucault pendulum

- 1851, Jean Bernard Léon Foucault showed that the plane of oscillation of a pendulum, like a gyroscope, tends to stay constant regardless of the motion of the pivot, and that this could be used to demonstrate the rotation of the Earth.

→ first demonstration of the Earth's rotation that didn't depend on celestial observations

... to ballistic gravimeters

- Time-duration and distance of free fall of an object in vacuum

$$\rightarrow d \sim \frac{1}{2}gt^2$$



$$a = g = \frac{F}{m} \text{ (no drag)}$$



Ballistic Gravimeters

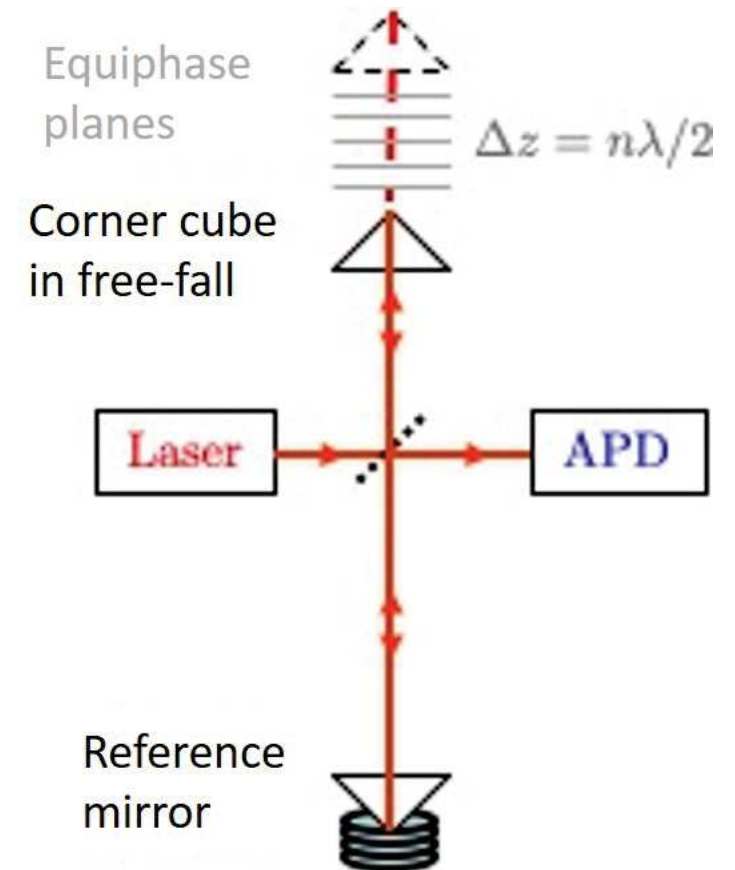


FG5#206 at the
gravimetric observatory
of Strasbourg (France)



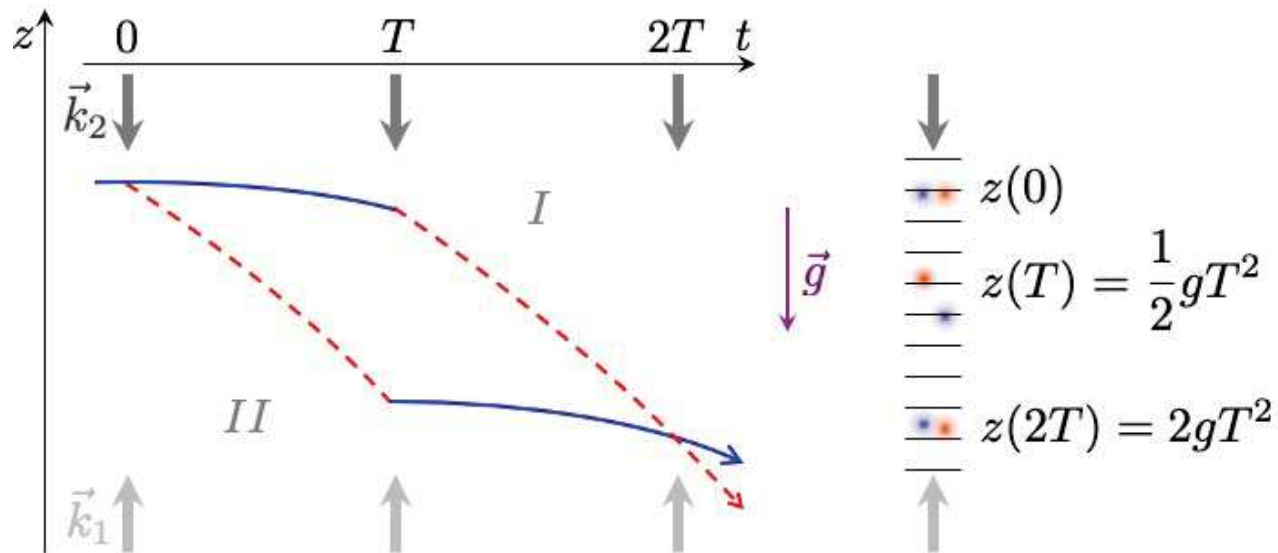
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MICROg
LACOSTE
A DIVISION OF LRS



Atomic Gravimeters

- Matter-wave interferometer



Space Gravimetry

- Free-fall of satellites

$$\vec{a} = \vec{g} + \vec{g}^* + \underbrace{\vec{a}_{SP} + \vec{a}_D}_{\text{measured}} + \underbrace{\vec{a}_{th}}_{\text{modeled}}$$

- \vec{a}_{SP} acceleration vector due to solar pressures (from the Sun and reflected by the Earth - *albedo*)

- \vec{a}_D drag due to the atmosphere

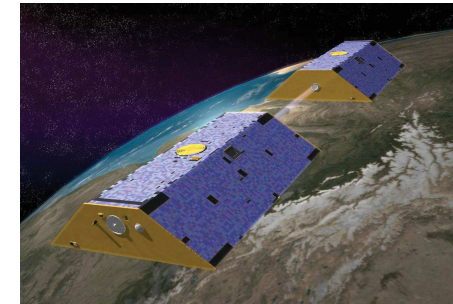
→ \vec{a}_{SP} and \vec{a}_D may be measured by micro-accelerometers as in CHAMP, GRACE, GRACE-FO and GOCE satellites

- \vec{a}_{th} residual acceleration due to poorly known effects (e.g. thruster acceleration). It may be modeled via time-series or as window functions at specific epochs (e.g. times of attitude maneuvers)

- \vec{g}^* gravitational acceleration due to the Sun, Moon and other planets

→ numerical integration of the equations of motions

Space Gravimetry



- CHAMP (CHAllenging Minisatellite Payload, 2000-2010): geoid with a 10-cm resolution, near-polar orbit ~ 400 km

- GRACE (Gravity Recovery And Climate Experiment, 2002-2017): 1-cm resolution, near-polar orbit ~ 450 km

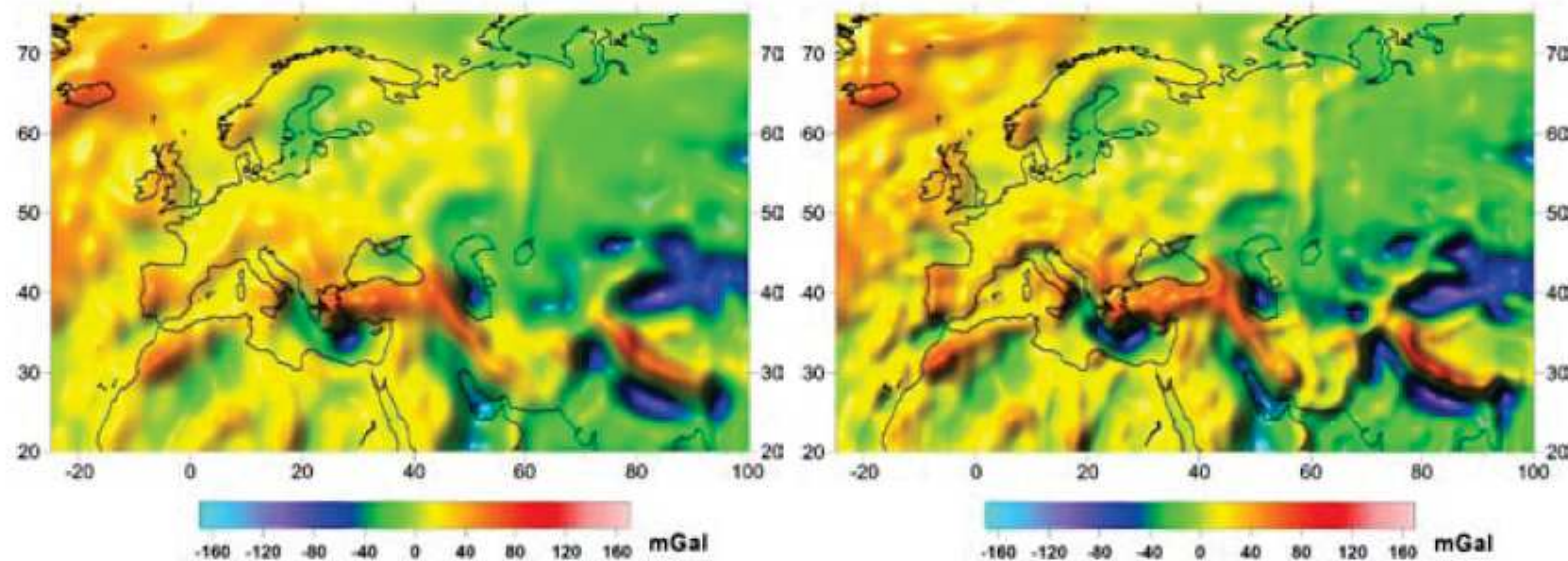


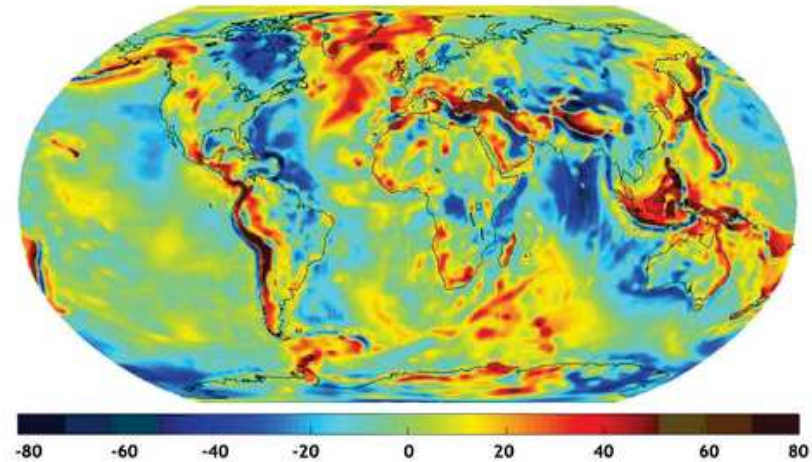
Figure 2.5: Gravity anomalies over Europe derived from 33 months of CHAMP data (left) and 110 days of GRACE data (right).

Space Gravimetry

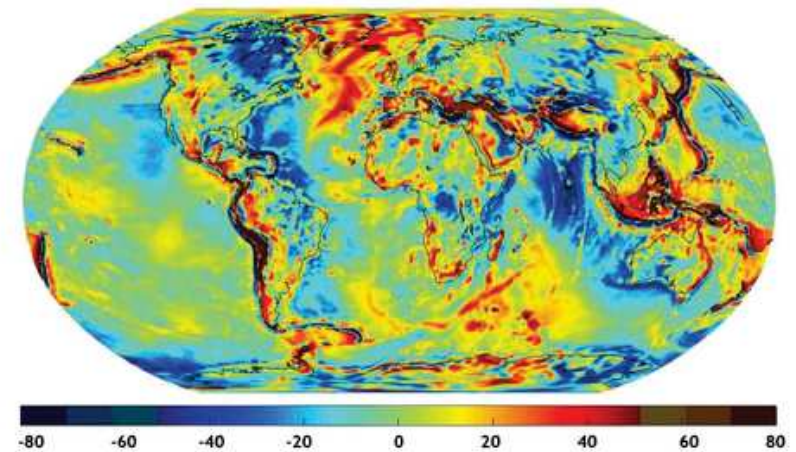
- Gravity field and steady-state Ocean Circulation Explorer (GOCE): ESA satellite (2009-2013), geoid measured with a 2-cm resolution, orbiting at 260 km



Gravity anomalies from ten years (2003-2013) of GRACE data (GGM05S)

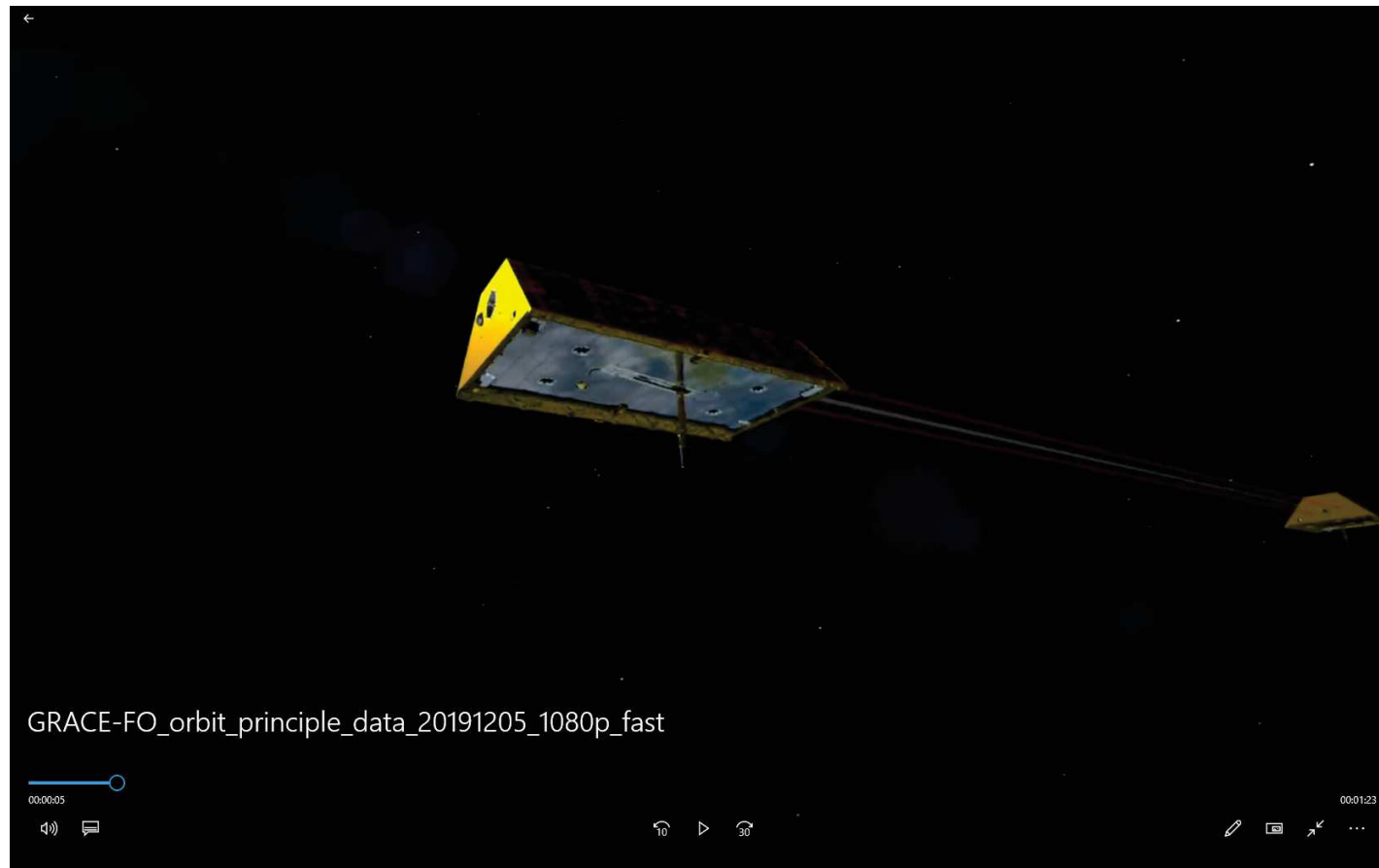


Gravity anomalies from ten years (2003-2013) of GRACE data and four years of GOCE data (GGM05G)



Gravity satellites

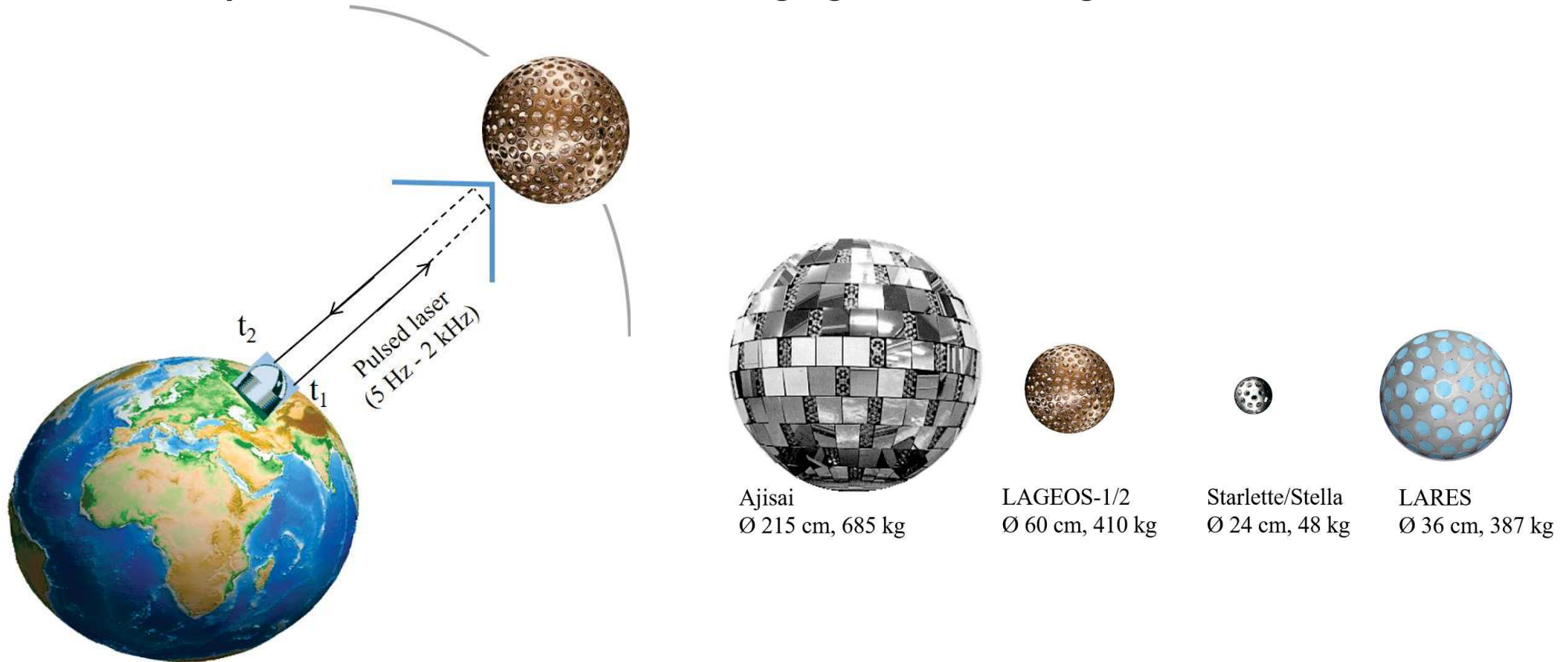
GRACE-FO (GRACE Follow-On, 2018-):



Movements of the two GRACE-FO satellites as they orbit Earth. Credit: NASA/JPL

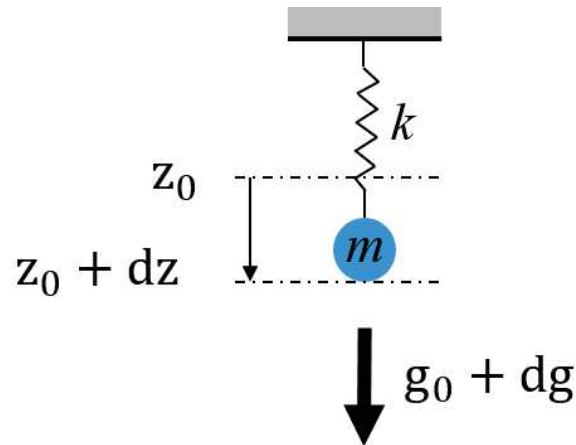
Satellite Laser Ranging (SLR)

- Gravity field from Satellite Laser Ranging: cannonball geodetic satellites



- better than GRACE/GRACE-FO for low-degree ($l = 2, 3$) Stokes coefficients

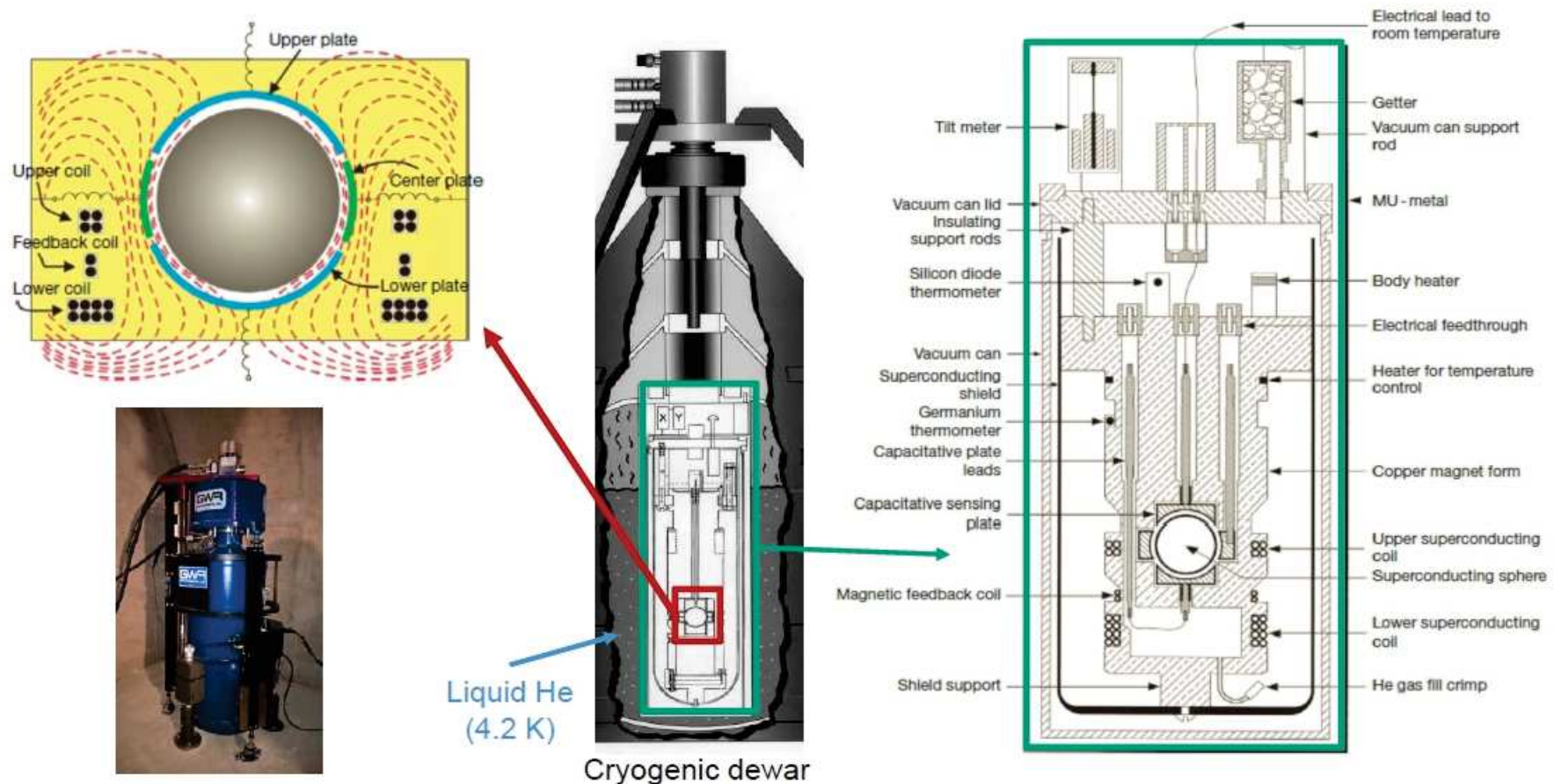
Mass-spring Gravimeters



- Tension $\vec{T} = -k dz \vec{u}$, where k stiffness, \vec{u} unit vector from fix to mobile
- Gravity force $m\vec{g}$
- Fundamental principle of dynamics on mass m : $m\ddot{x}(t) = -kx(t)$
 $\rightarrow m dg = k dz$
- Small, cheap, convenient for field measurements, fast gravity measurement (automatic leveling, tides correction)
- Precision of a few μGal ($1\mu\text{Gal} = 10 \text{ nm/s}^2$), cm-resolution of geoid
- BUT sensitive to temperature variations, spring wears out over time, large drift (a few $\text{nm/s}^2/\text{day}$)

Superconducting Gravimeters

Components of the SG sensor (GWR Instruments, Inc.)



The levitation force is produced by the interaction between the magnetic field from the coils and the currents induced on the surface of the superconducting sphere.

Superconducting Gravimeters



<https://www.gwrinstruments.com/>

- Transportable but not portable, more expensive, not easy-to-use

BUT:

- Precision at the nGal ($\sim 10^{-12}g$)
- Small drift: a few $\mu\text{Gal}/\text{year}$, even less than 1 $\mu\text{Gal}/\text{year}$ for iGrav

→ a wide-range of scientific applications

Satellite vs. Ground gravimetry

	Satellite (GRACE)	Ground SG/Absolute
	Newtonian attraction (global) + mass redistribution	Newtonian attraction (global + local <i>e.g. soil moisture</i>) + mass redistribution + vertical displacement in gravity field
Resolution	10 μGal /0.5 μGal	0.1 μGal / 1-3 μGal
Spatial resolution	400 km/2,000 km	Point
SH coefficients	nmax = 60/10	N/A
Temporal resolution	10 days - 1 month	1 s / variable
Long-term stability	No drift	$\sim 3\mu\text{Gal}/\text{year}$ / no drift

$$g = \frac{GM}{r^2}, \quad \frac{dg}{dr} = -\frac{2GM}{r^3} = -\frac{2g}{r}$$

$$\text{At } 45^\circ \text{ latitude, } \frac{2g}{r} = 0.3086 \text{ mGal/m} = 3.086 \mu\text{Gal/cm} = 30.86 \text{ nm/s}^2/\text{cm}$$

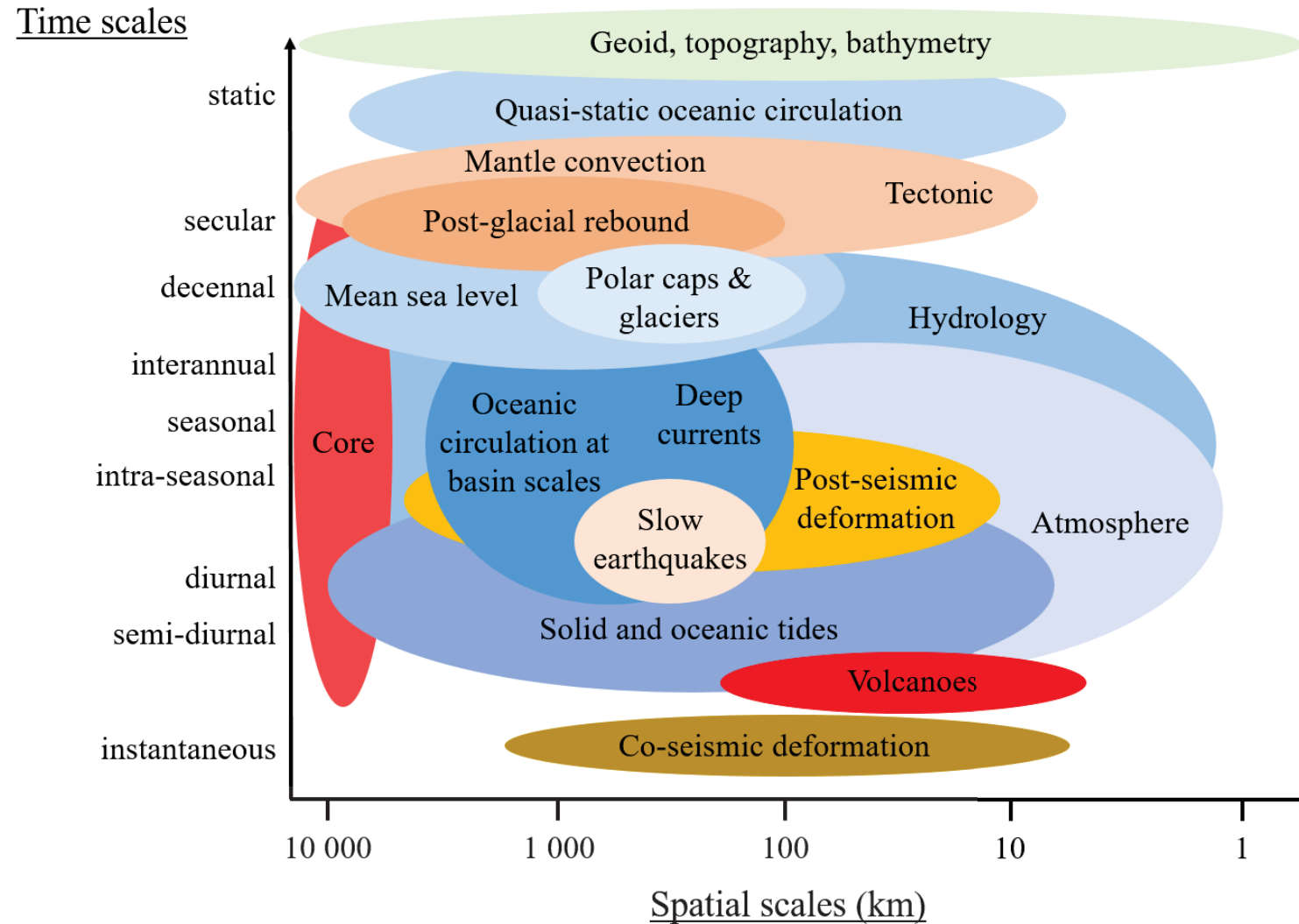
Ground vs. Satellite gravimetry

Recorded gravity signals are different

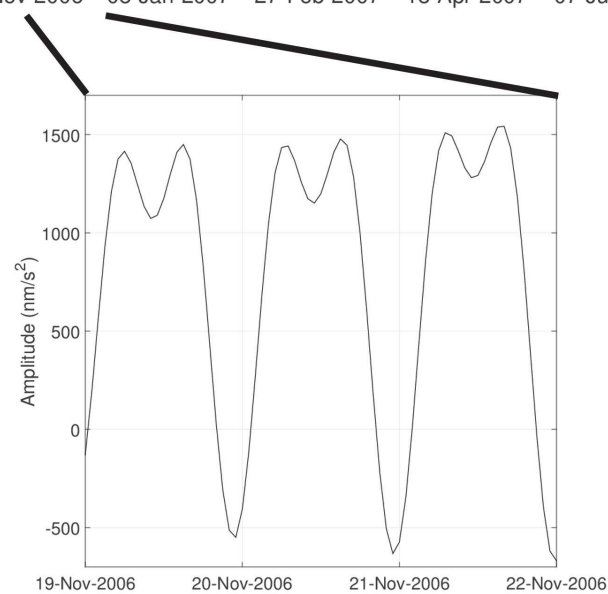
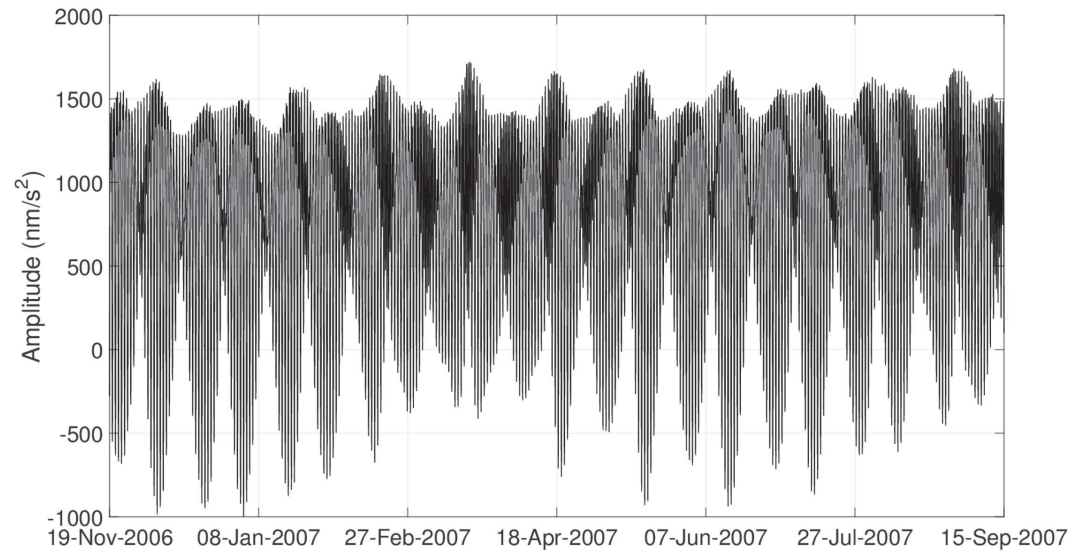
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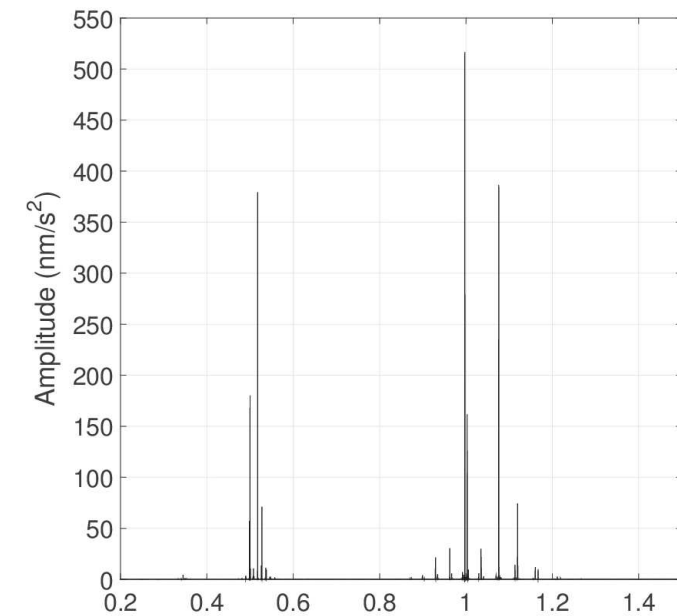
Applications of gravimetry



Tides



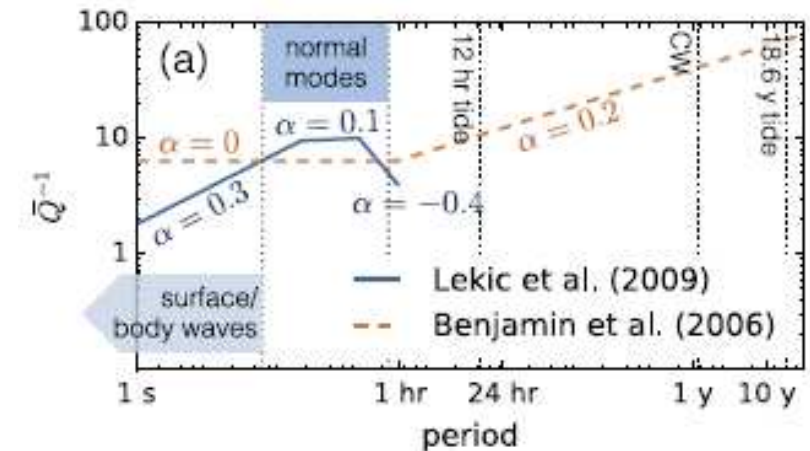
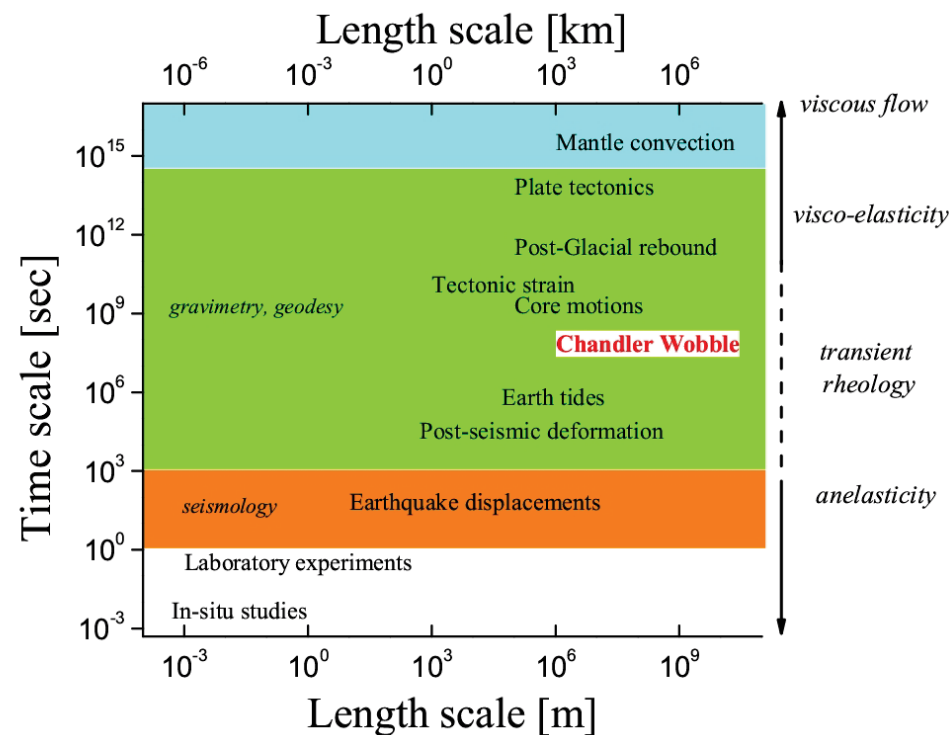
~ 30 cm



Frequency in cycle per day
(cpd)

Tides

- Mantle rheology

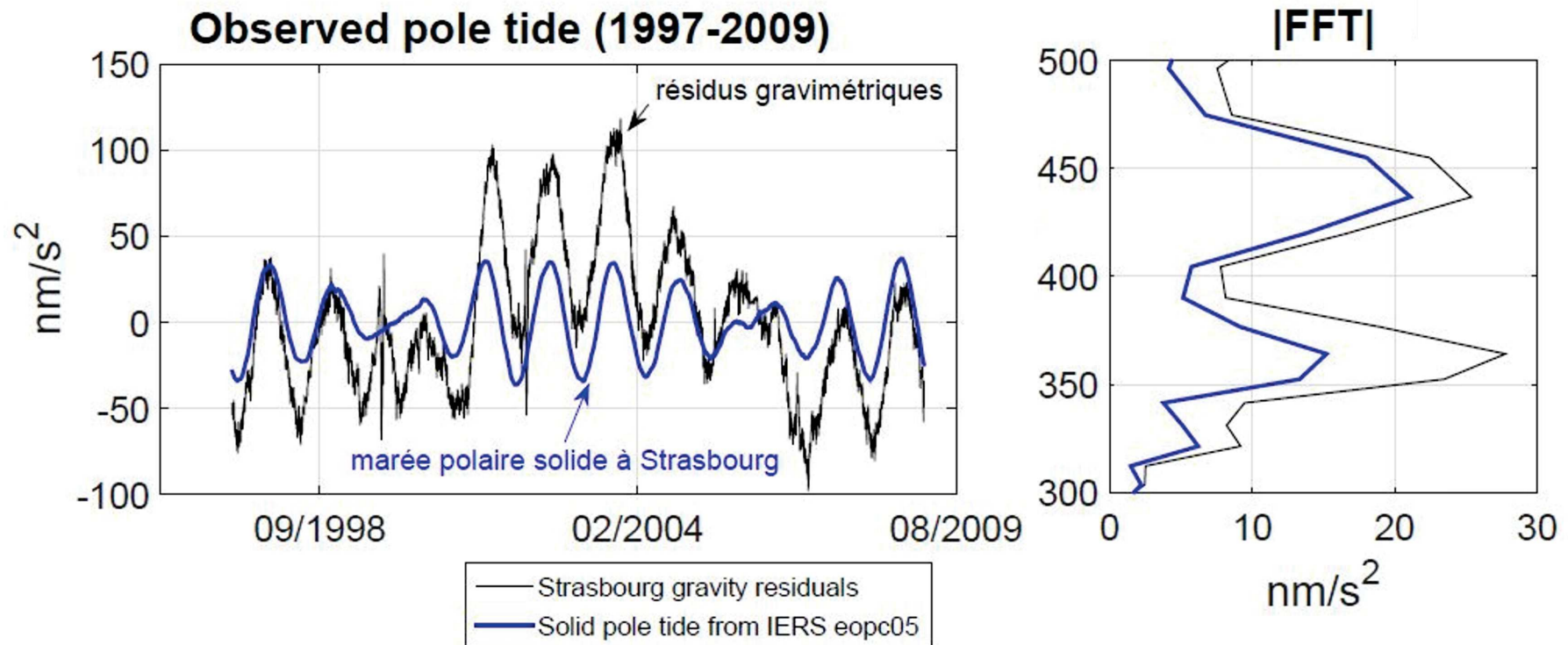


[Lau & Faul, 2019]

Frequency dependence of normalized attenuation Q^{-1} , displaying the contrasting absorption bands

Tides

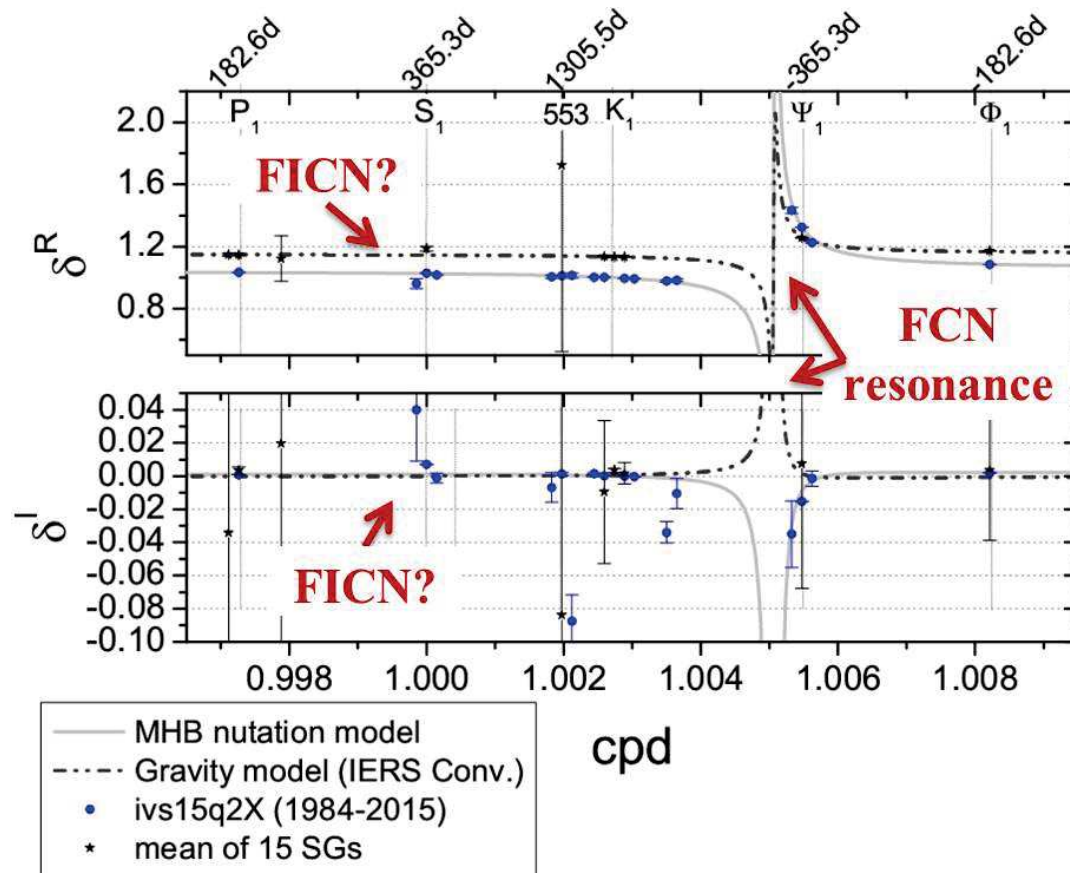
- Mantle rheology: **Chandler wobble**



$T \sim 432$ solar days, $Q \sim 130$

Tides

- Free Core Nutation: resonance in diurnal tides



Nearly Diurnal Free Wobble

$T \sim -430$ sidereal days
(in space)

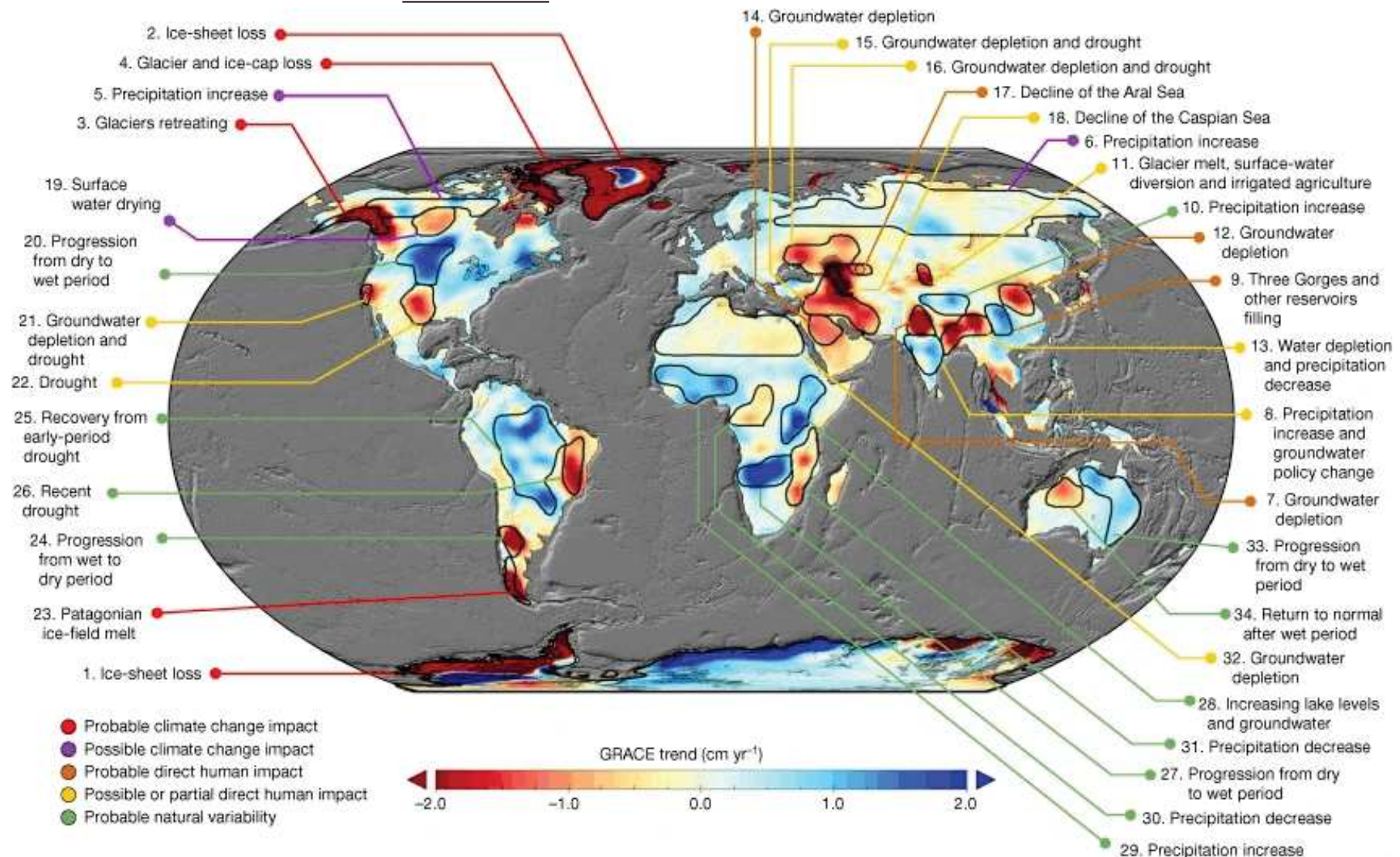
$Q \sim 20,000$

Coupling at core boundaries

e.g. viscomagnetic, topographic

Hydrology

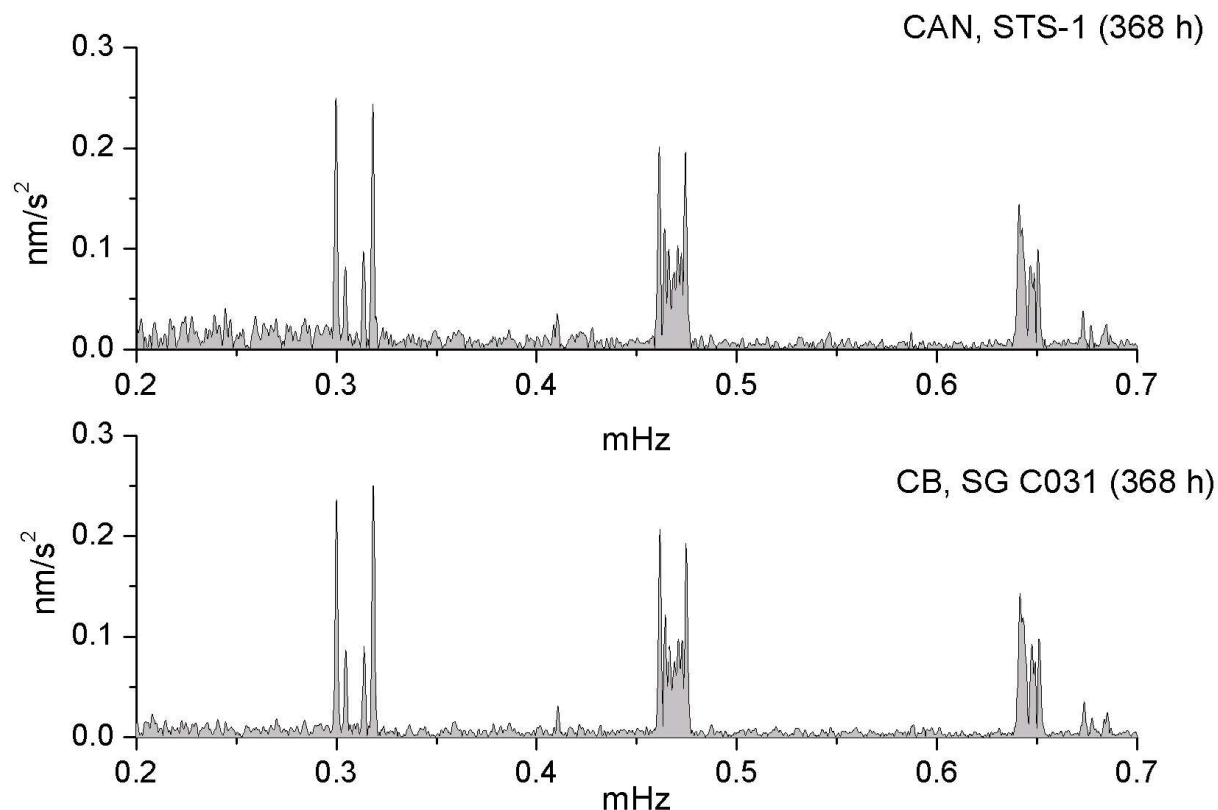
Example: GRACE and GRACE-FO



Trends in TWS (cm/yr) from April 2002 to March 2016 [Rodell et al. (2018)]

Seismic modes

- Linear response to atmospheric pressure changes (\neq seismometers)
→ higher precision for study of seismic modes below 1 mHz



Example: 2004 M_w 9.1 Sumatra-Andaman earthquake at Canberra (Australia)

- Constraints on density profile

Seismic cycle

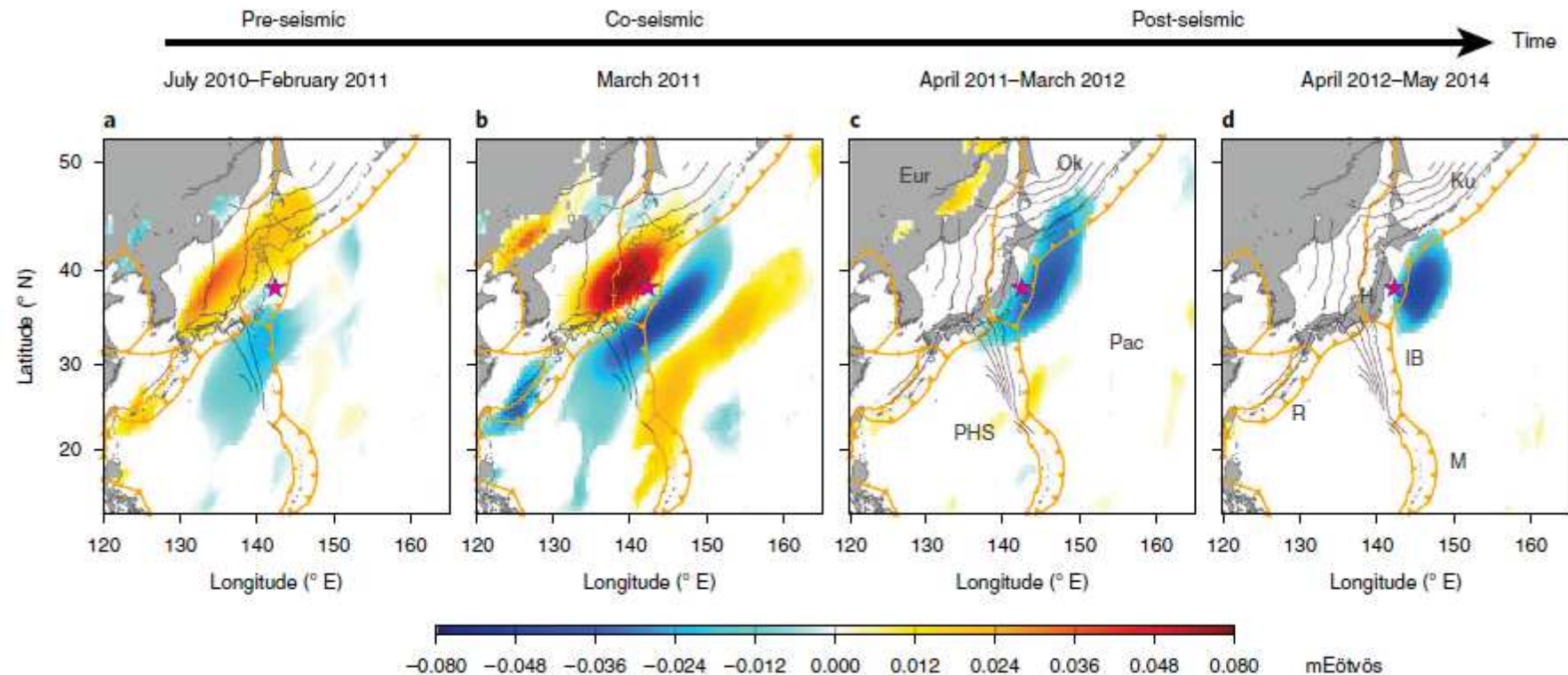


Fig. 1 | Pre-, co- and post-seismic variations of the Tohoku-Oki earthquake gravity signal. a–d, Time sequence of the 1,400-km-scale $\varphi\varphi$ gravity gradients in the local south-east-up spherical frame of unit vectors (e_θ, e_φ, e_r), rotated in the direction Az_i . They correspond to $\varphi\varphi$ gravity gradients averages for 20–55° clockwise rotations (defining the direction range Az_i) of the frame about the radial axis (Supplementary Section 2.2). Star: 11 March 2011 earthquake epicentre; orange lines: plate boundaries⁴¹; thin violet lines: Pacific slab isodepth contours⁴², every 200 km (**a,b**) or 100 km (**c,d**). Tectonic plates: Pacific (Pac), Philippine Sea (PHS), Okhotsk (Ok), Eurasian (Eur); island arcs: Kuril (Ku), Izu-Bonin (IB), Marianna (M), Ryukyu (R), Honshu (H).

[Panet et al. 2021]

Tomorrow: rotation of the Earth

Questions?

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