

ÉCOLE DE
PHYSIQUE
DES HOUCHES

October 2022

Earth's rheology: insights from climate (un)loading

Kristel Chanard



Perito Moreno, Argentina, 2017

What is rheology?

What is rheology?

Rheology is the study of the flow of matter
Or the ability of stressed materials to deform or flow



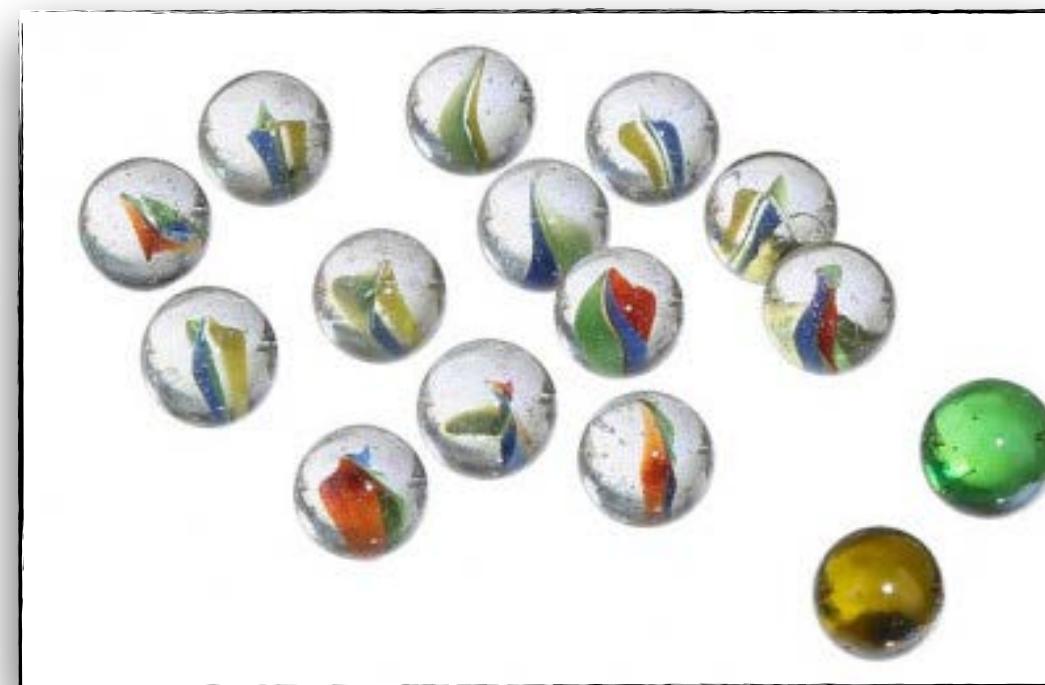
Brittle behaviour



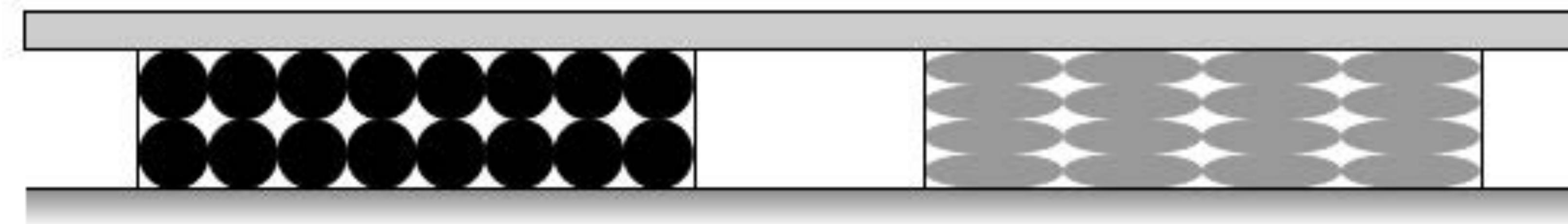
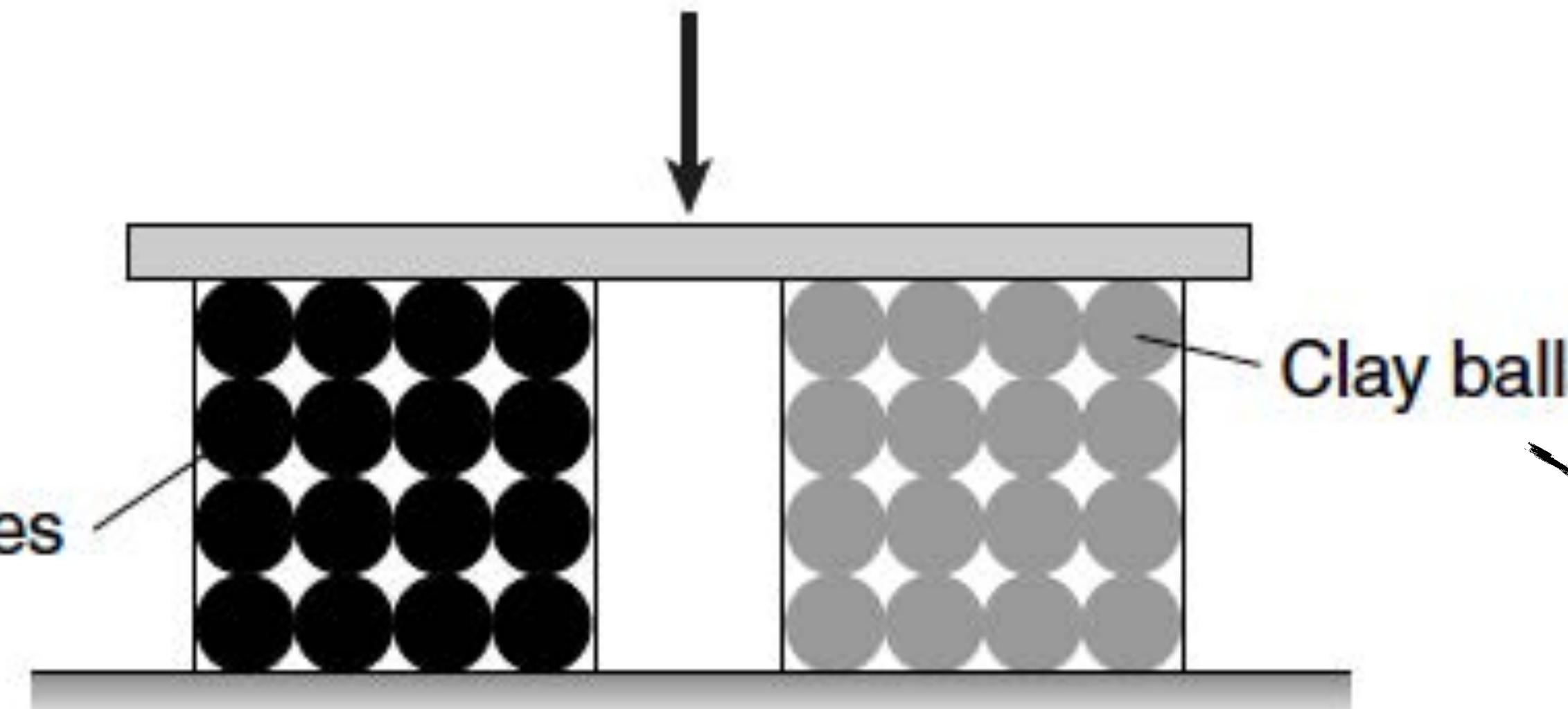
Ductile behaviour

What is rheology?

behavior vs mechanisms of deformation

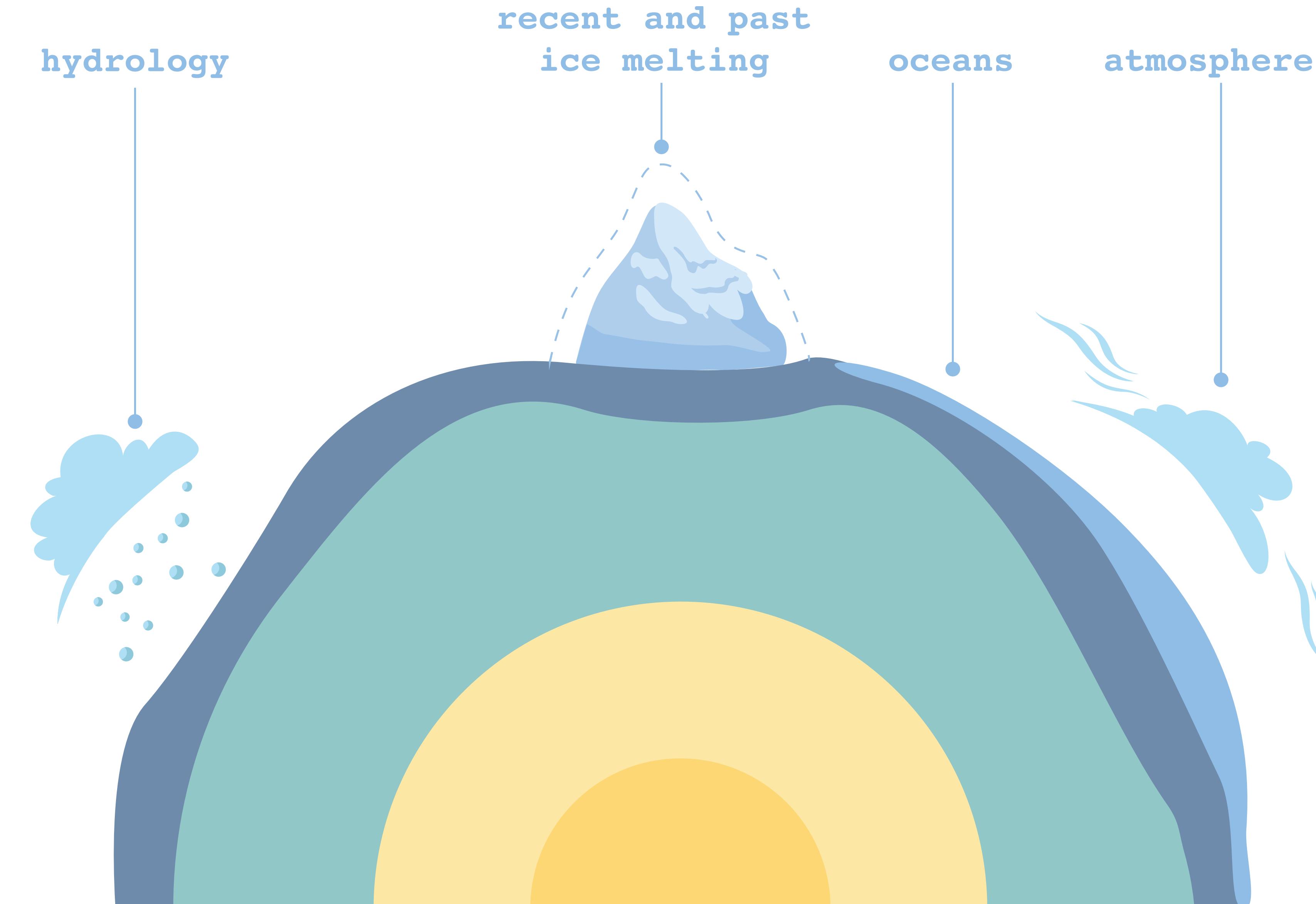


Marbles



What is climate (un)loading?

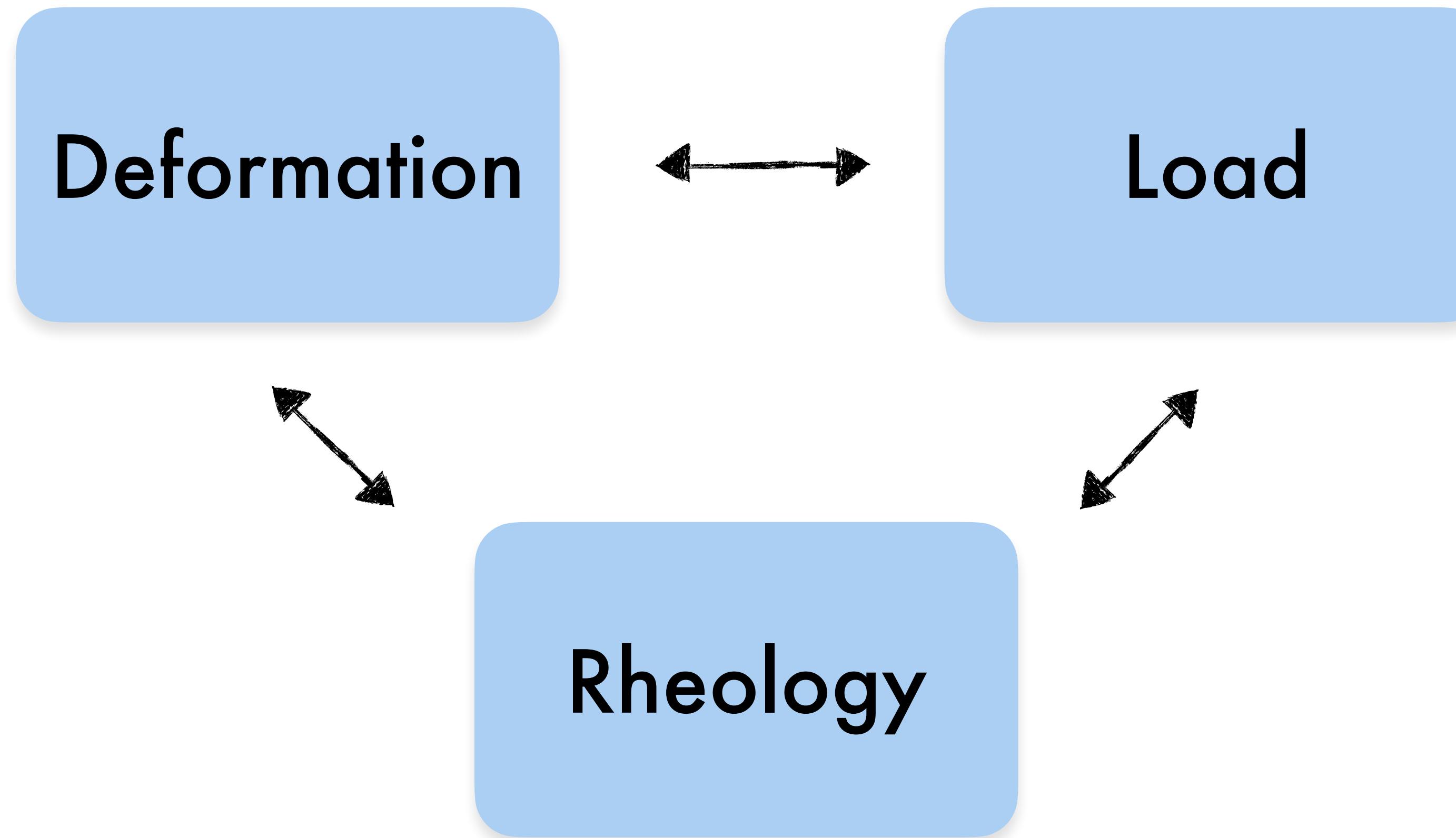
What is climate (un)loading?



The (unfortunate) circle of knowledge

*How to measure deformation
associated with surface loads?*

*How to measure past and
present-day climate loads?*

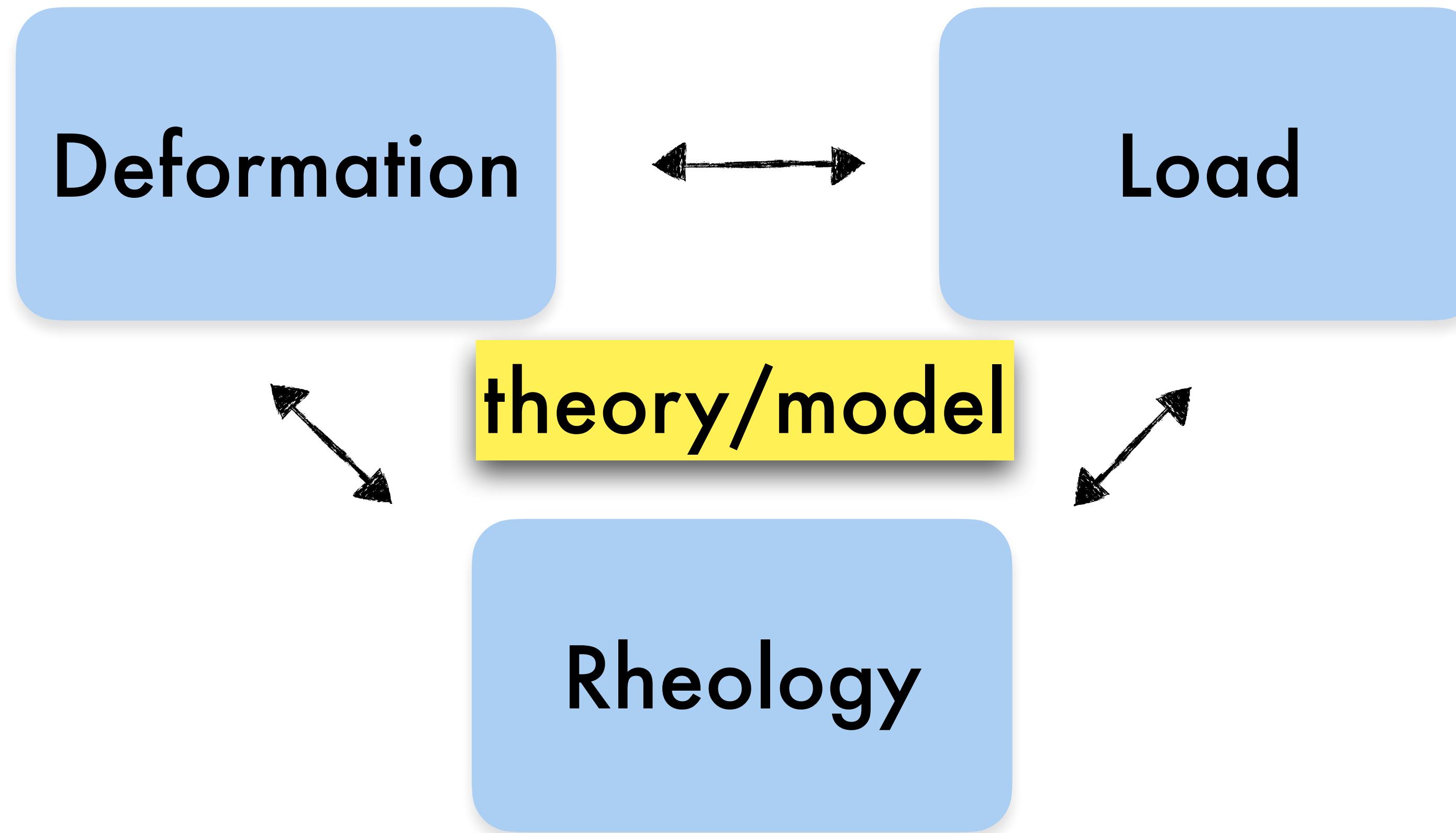


*How do the Earth deform at
the mineral and global scale?*

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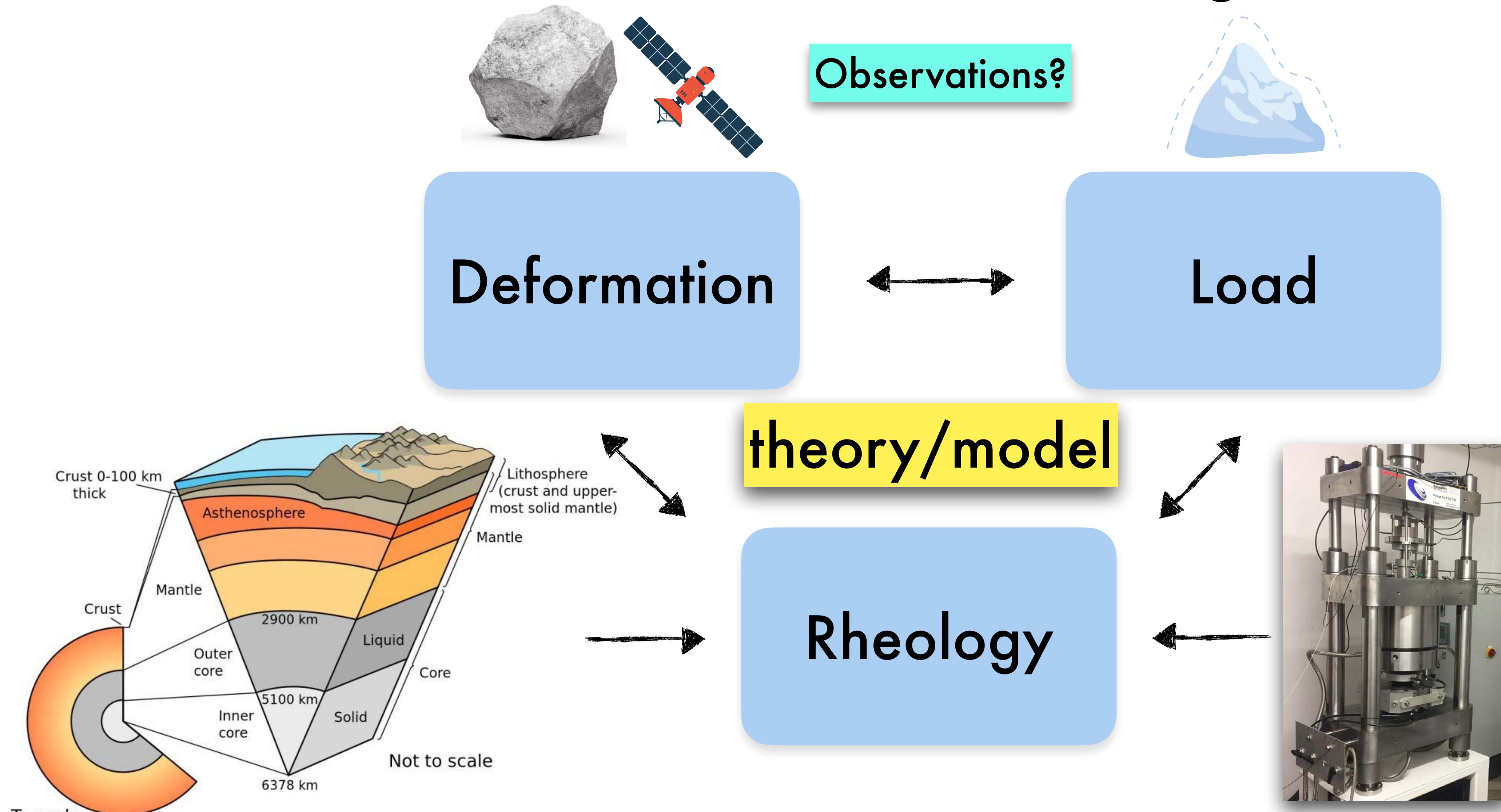
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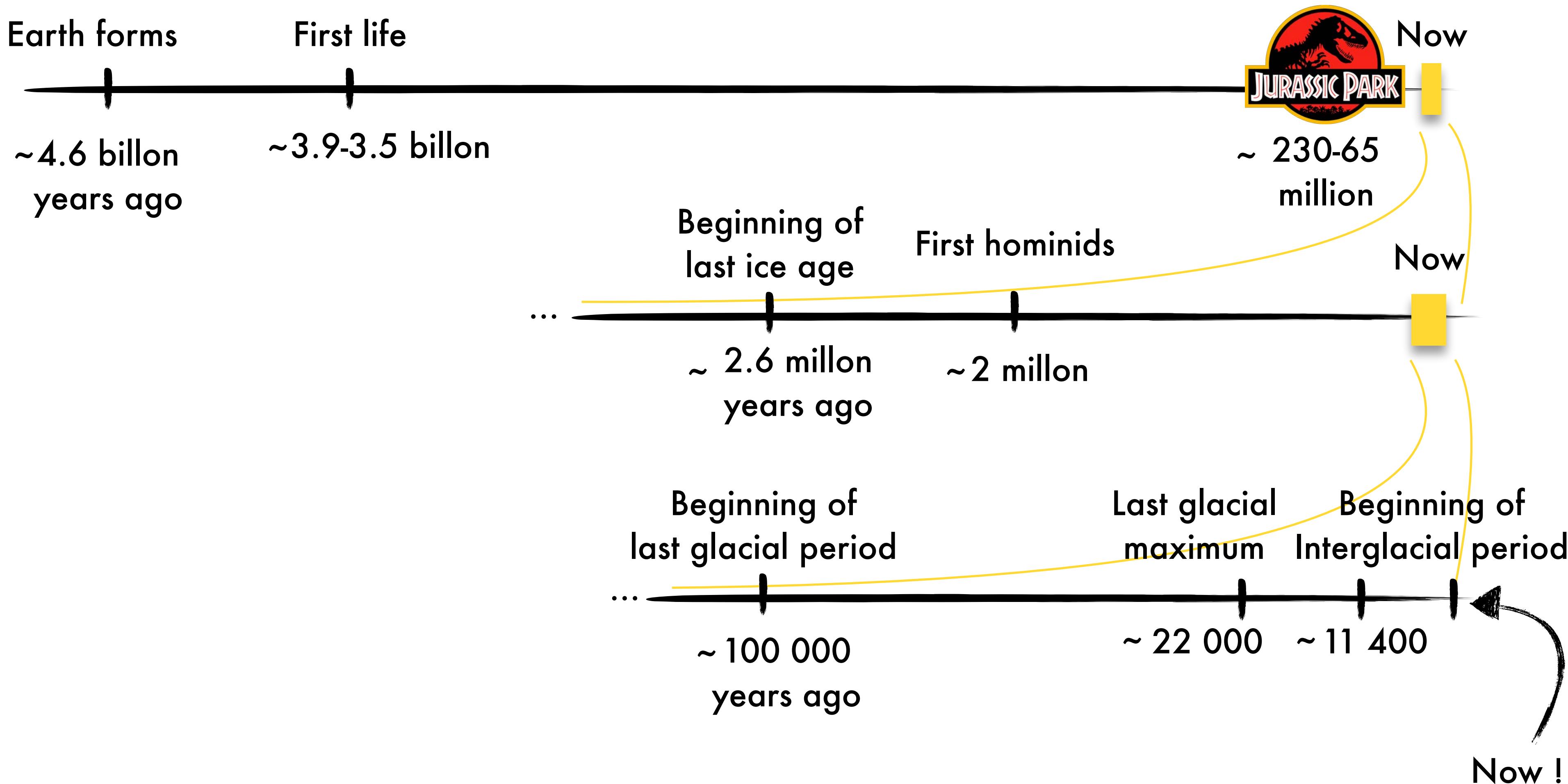
Rheology of the Earth's mantle

Link to laboratory derived rheologies?

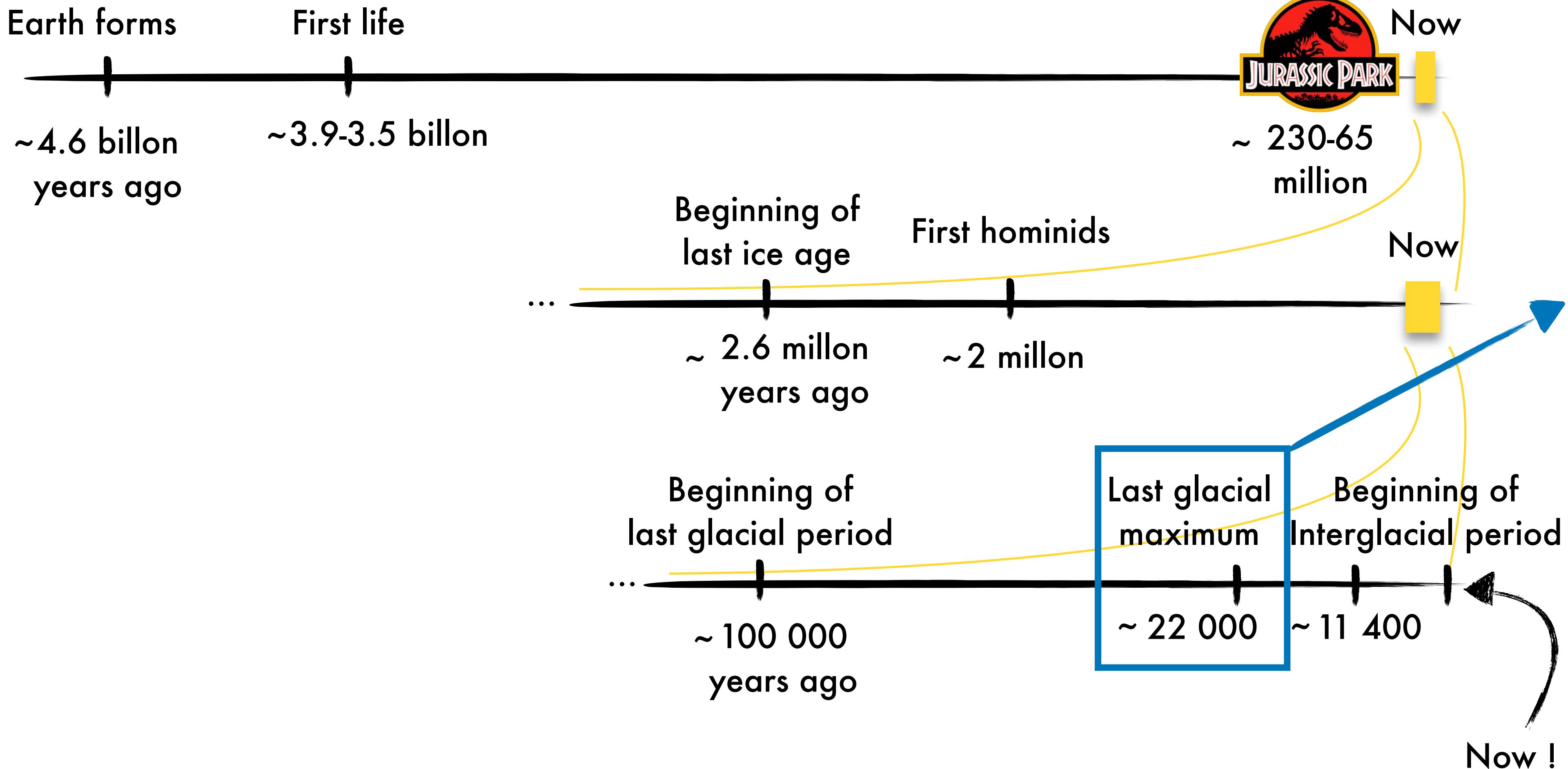
The (almost) perfect case of Glacial Isostatic Adjustment (GIA)

1. What is GIA and why study it?
2. Theory: general concepts, modelling and rheology
3. Observations : constraining ice history and monitoring deformation

The Glacial... what?



The Glacial... what?



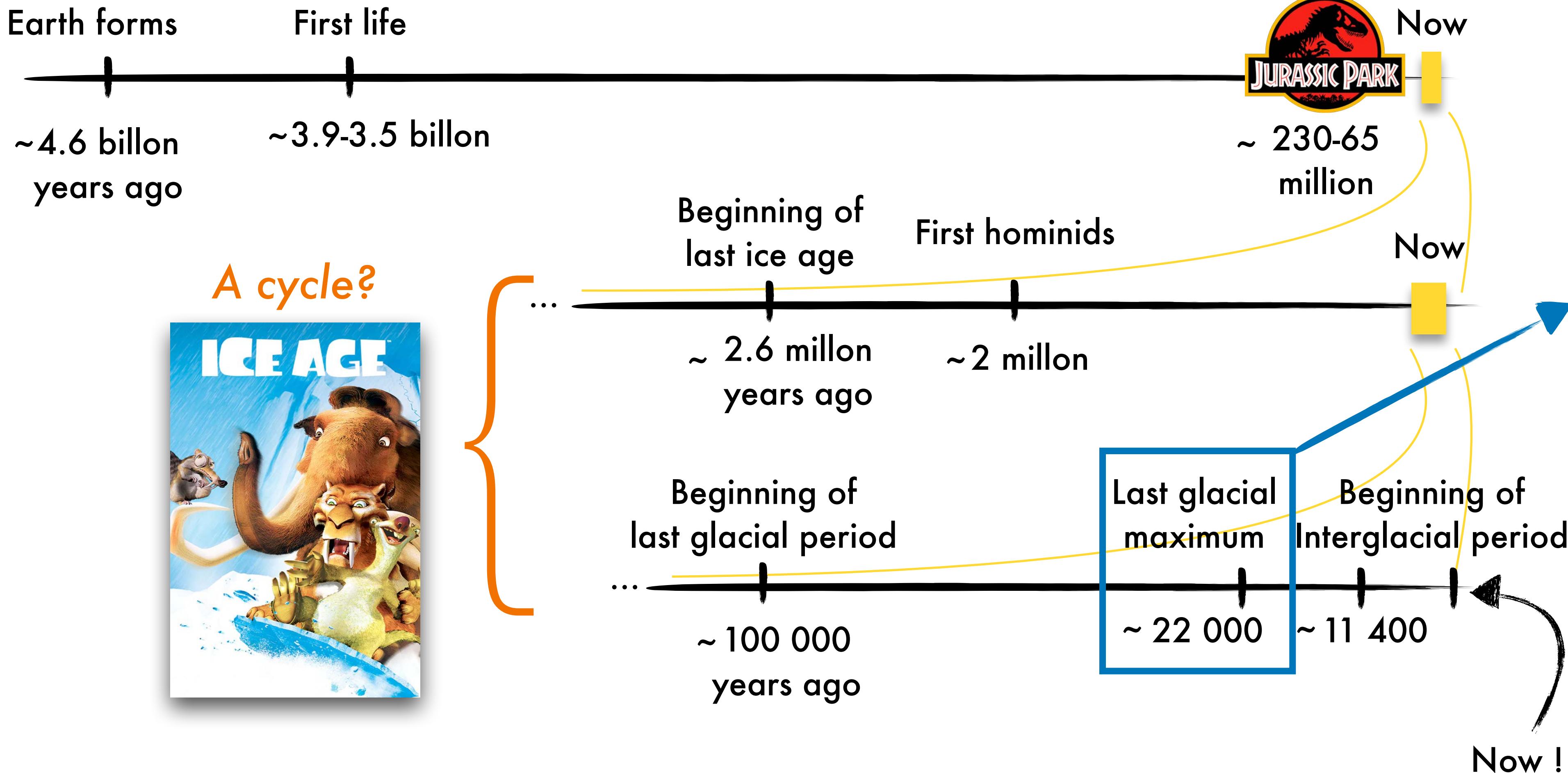
The last glacial maximum
LGM



@NOAA

- ▶ Thick ice sheets
- ▶ Sea level 120m lower than today

The Glacial... what?



The last glacial maximum
Ice Age

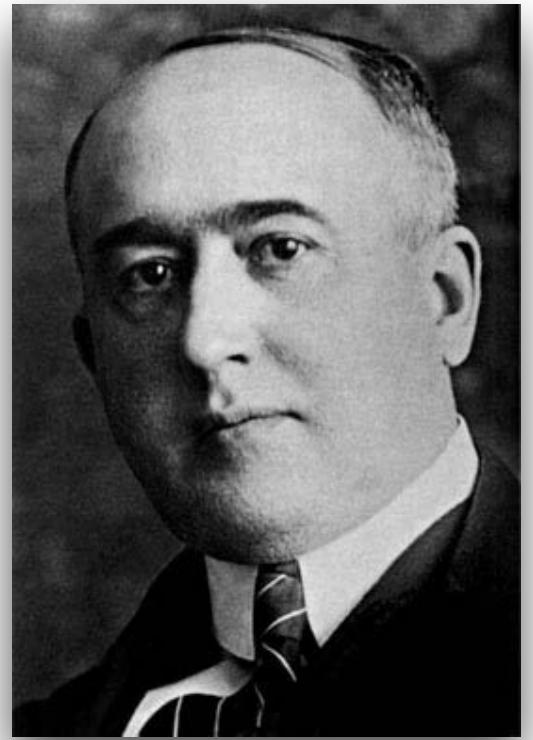


@NOAA

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Why cycles of glacial periods?

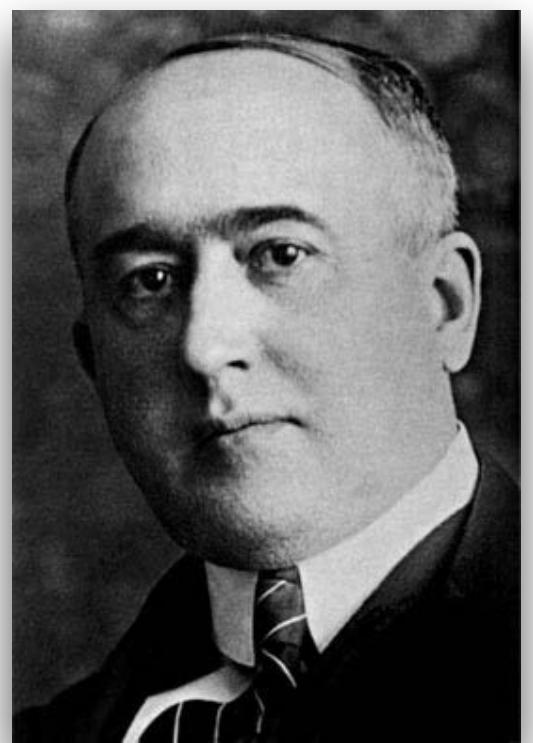
Why cycles of glacial periods?



"the long-term, collective effects of changes in Earth's position relative to the Sun are a strong driver of Earth's long-term climate, and are responsible for triggering the beginning and end of glaciation periods"

Milutin Milanković
(1879-1958)

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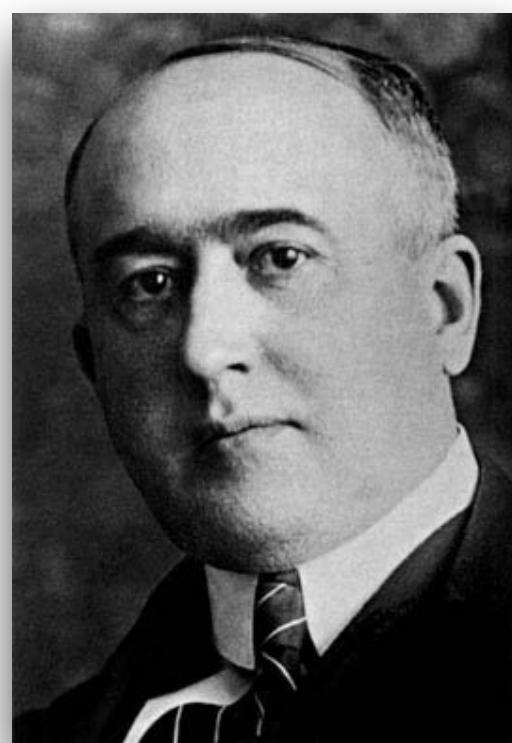
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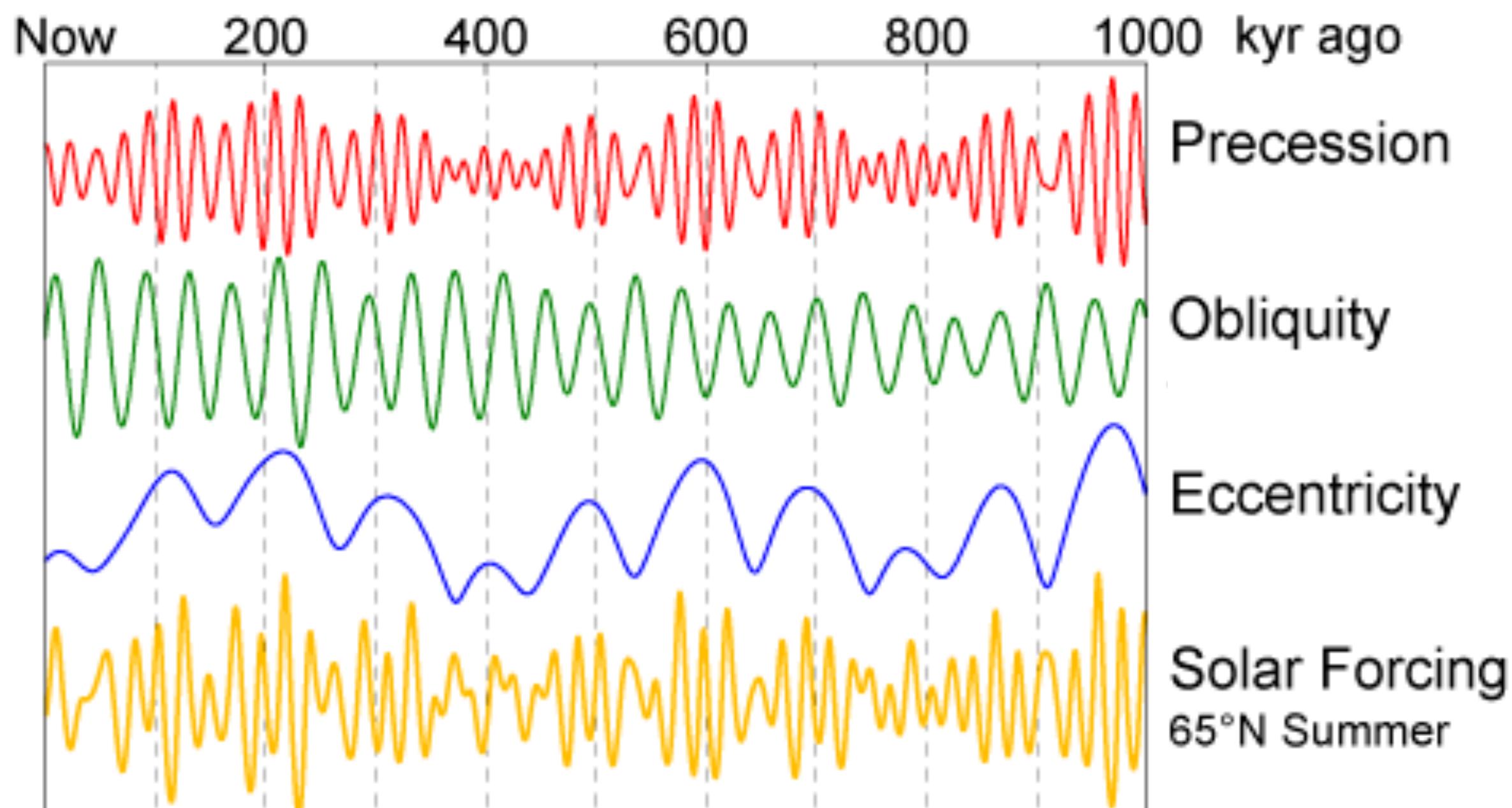


GEOODESY !!!

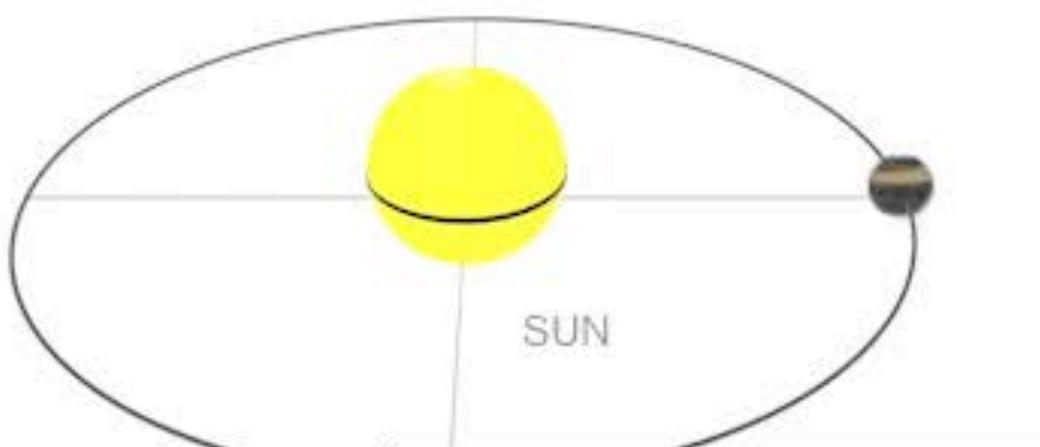
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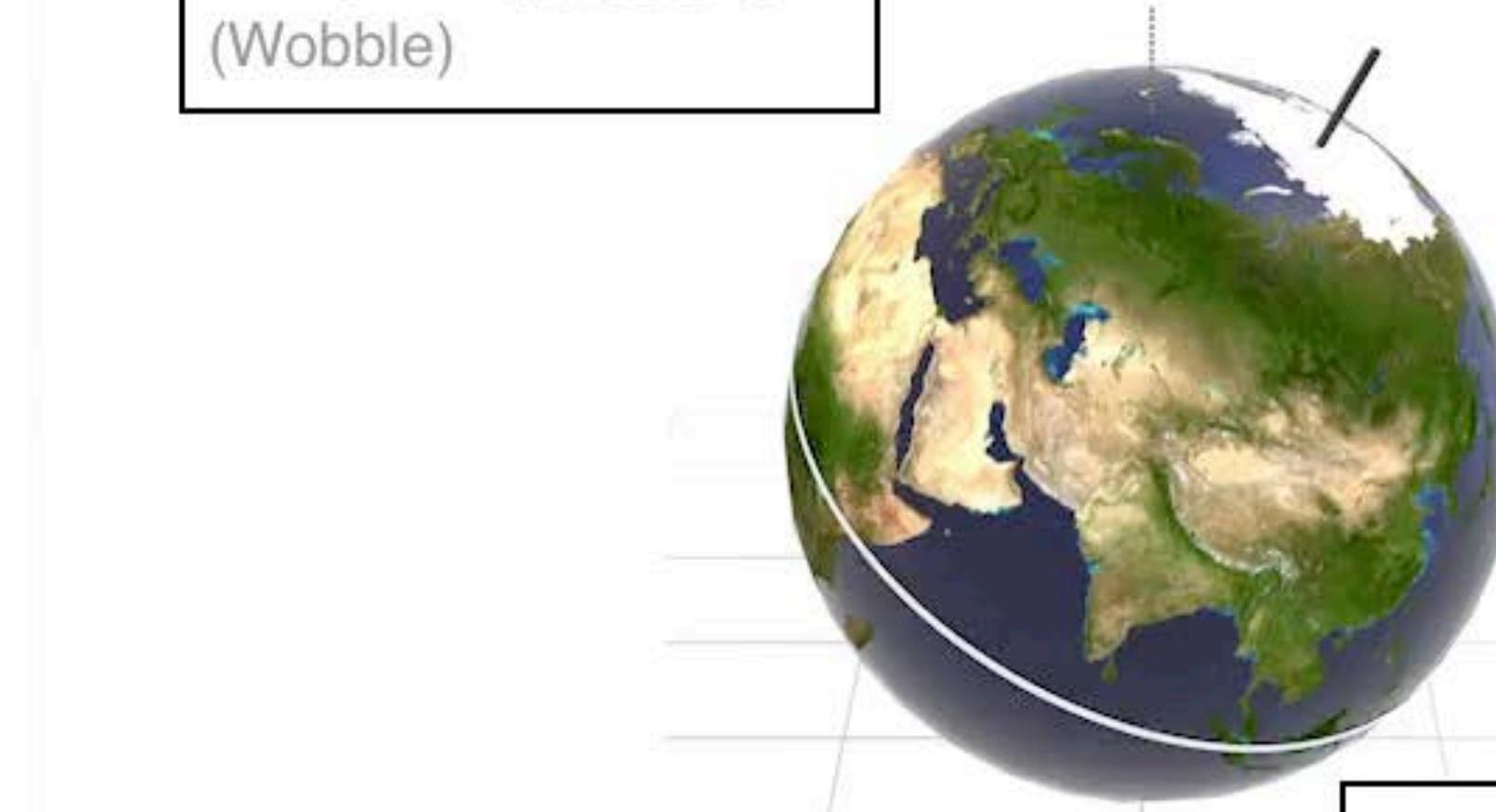


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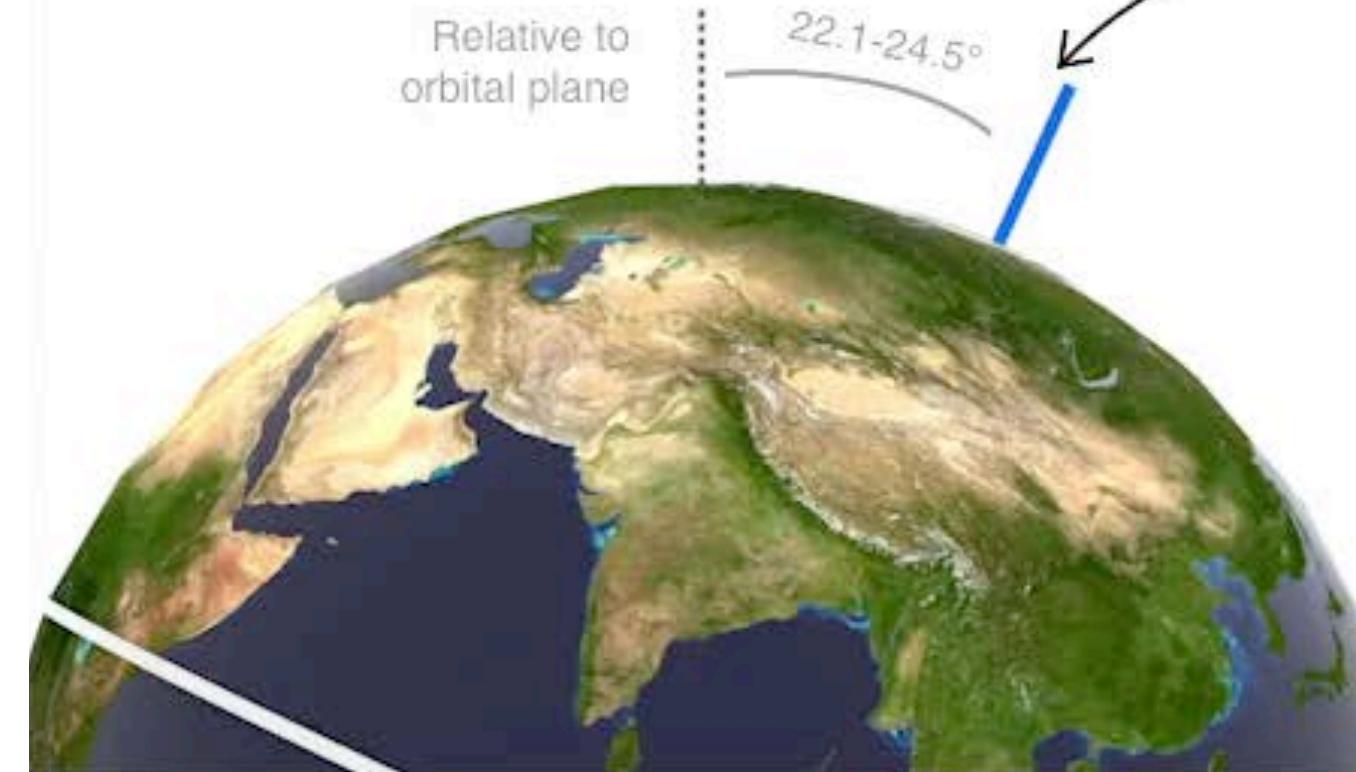


100,000-year cycles
Changes in Eccentricity
(Orbit Shape)

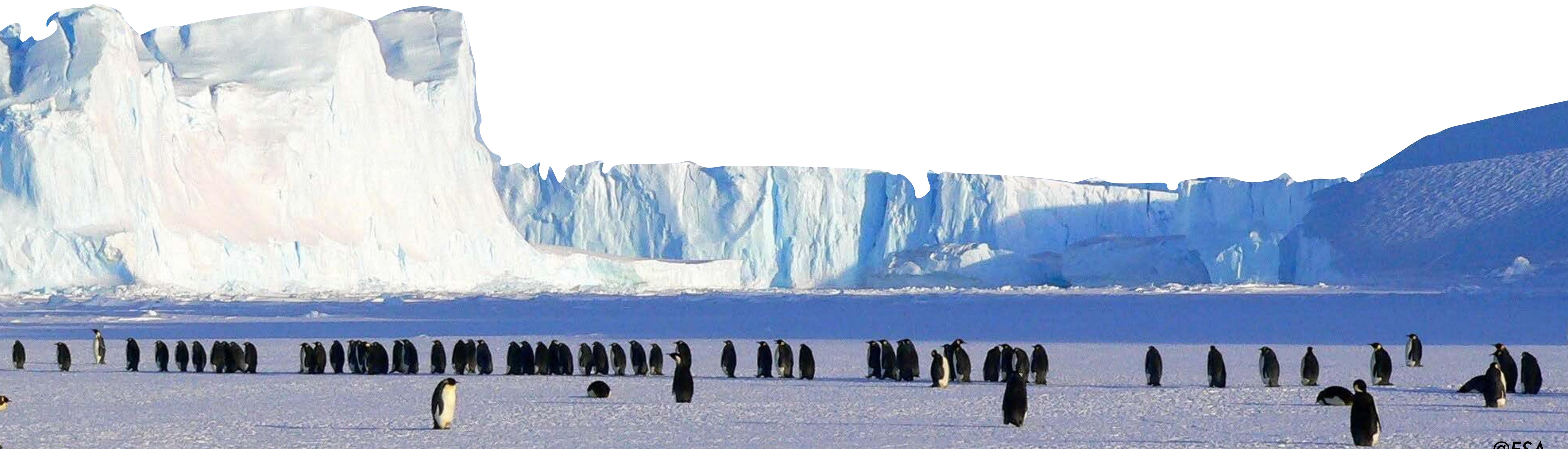
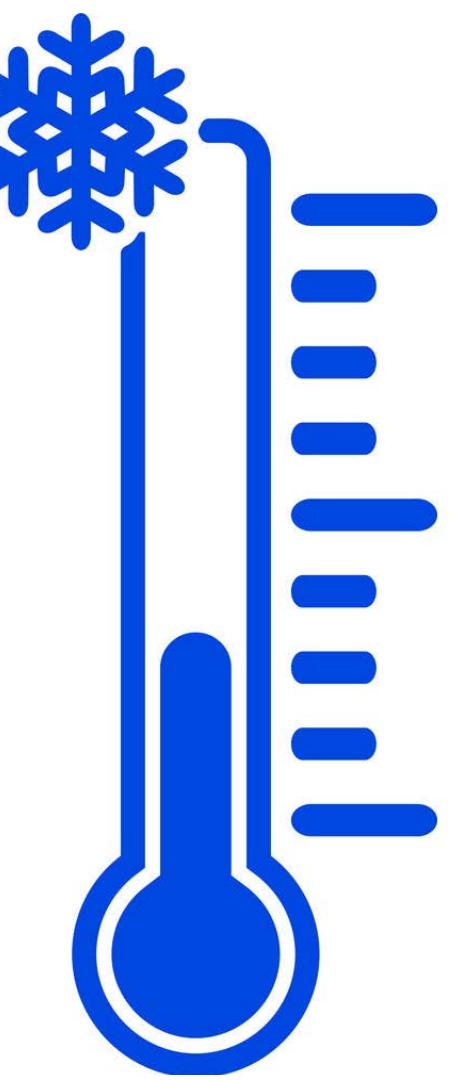
26,000-year cycles
Axial Precession
(Wobble)



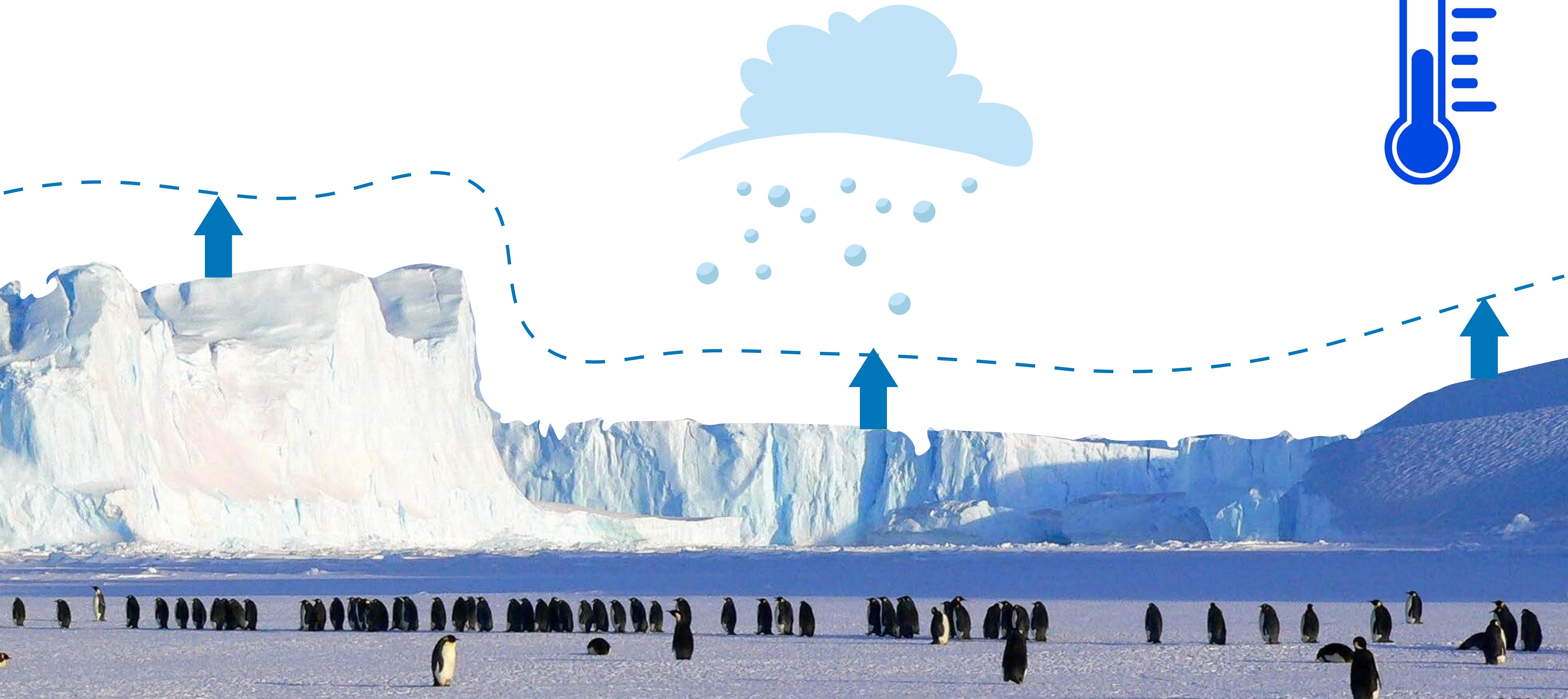
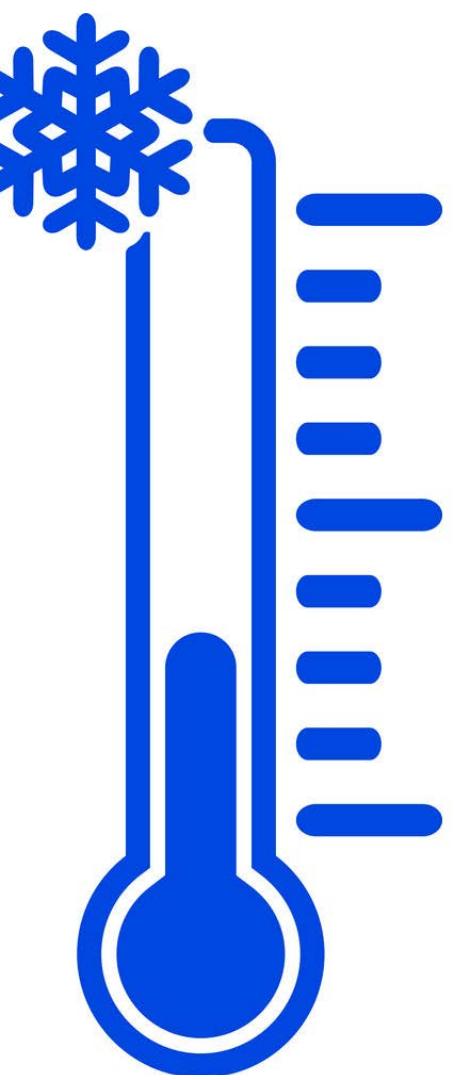
41,000-year cycles
Changes in Obliquity
(Tilt)



Building ice sheets season by season



Building ice sheets season by season

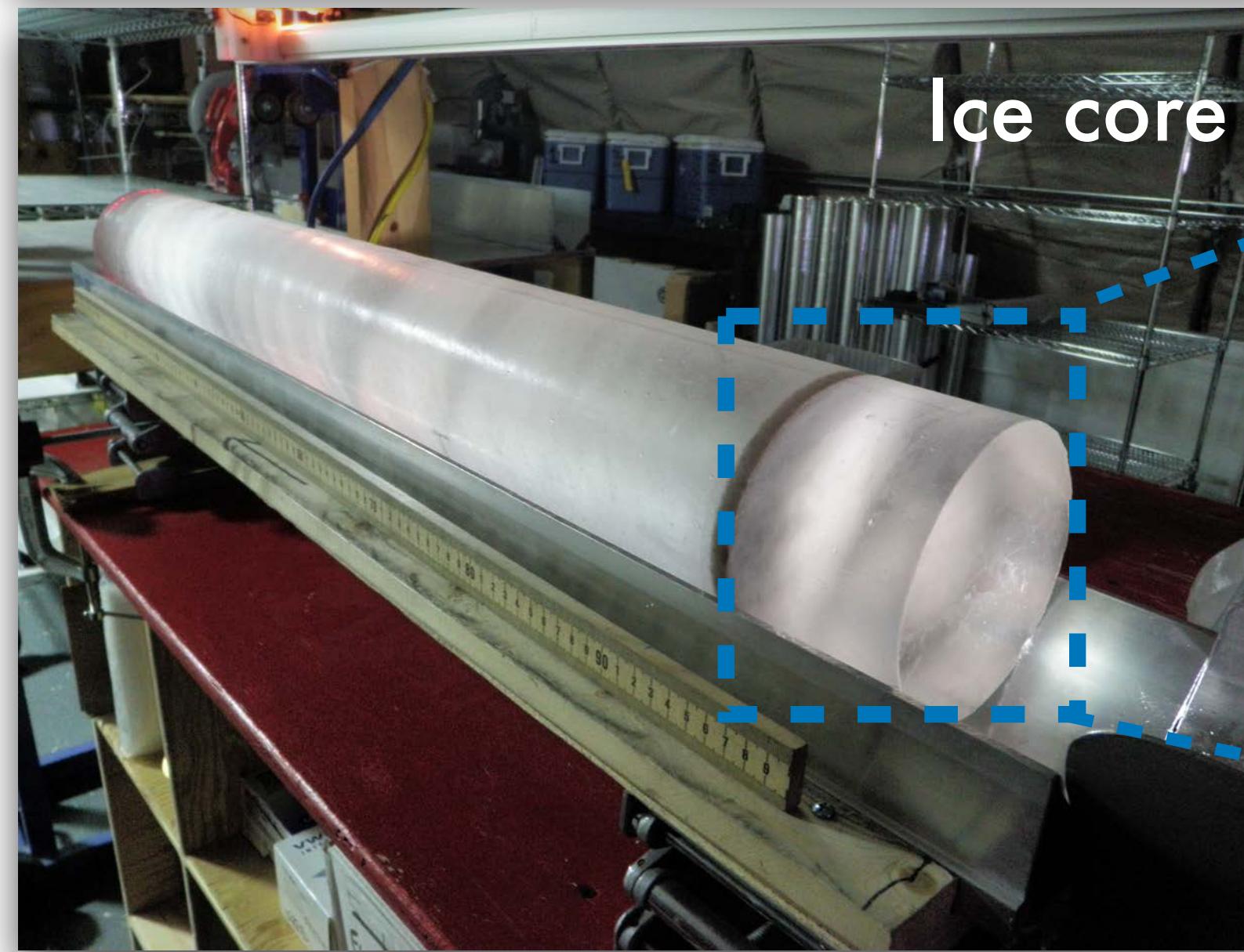


Temperature record is written into the ice

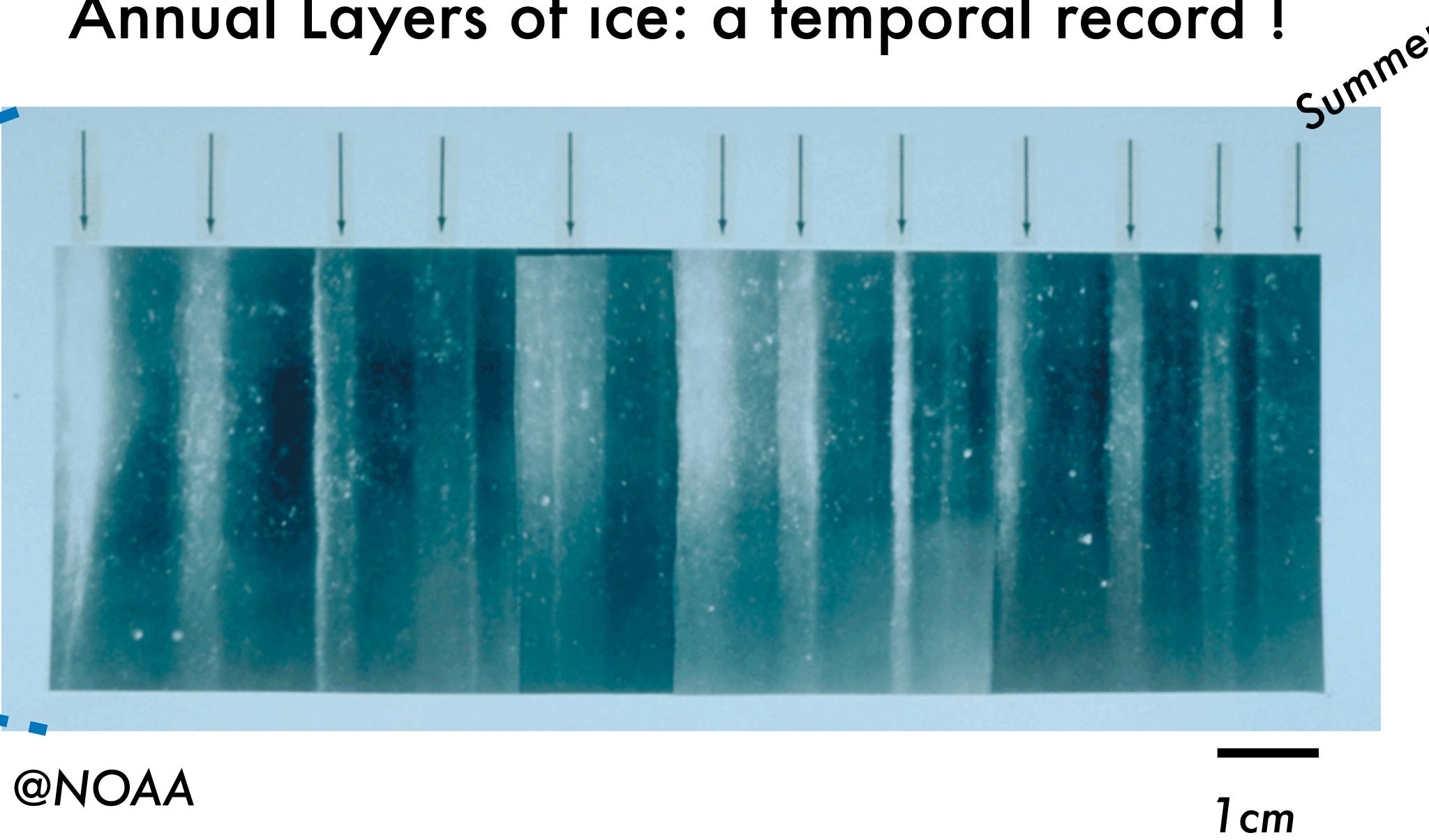
@ Clark, Univ. Washington



@Roop, NSF



Annual Layers of ice: a temporal record !





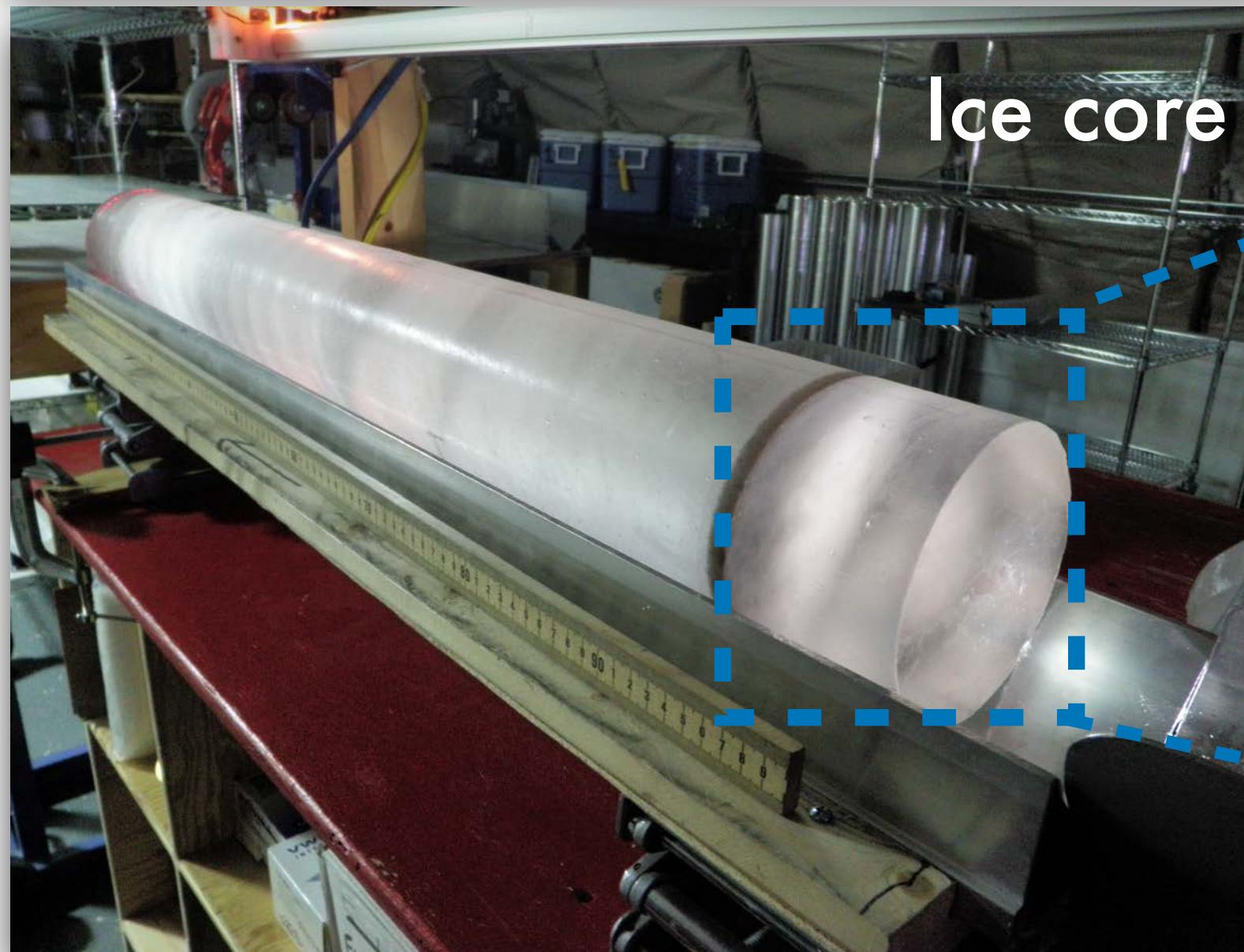
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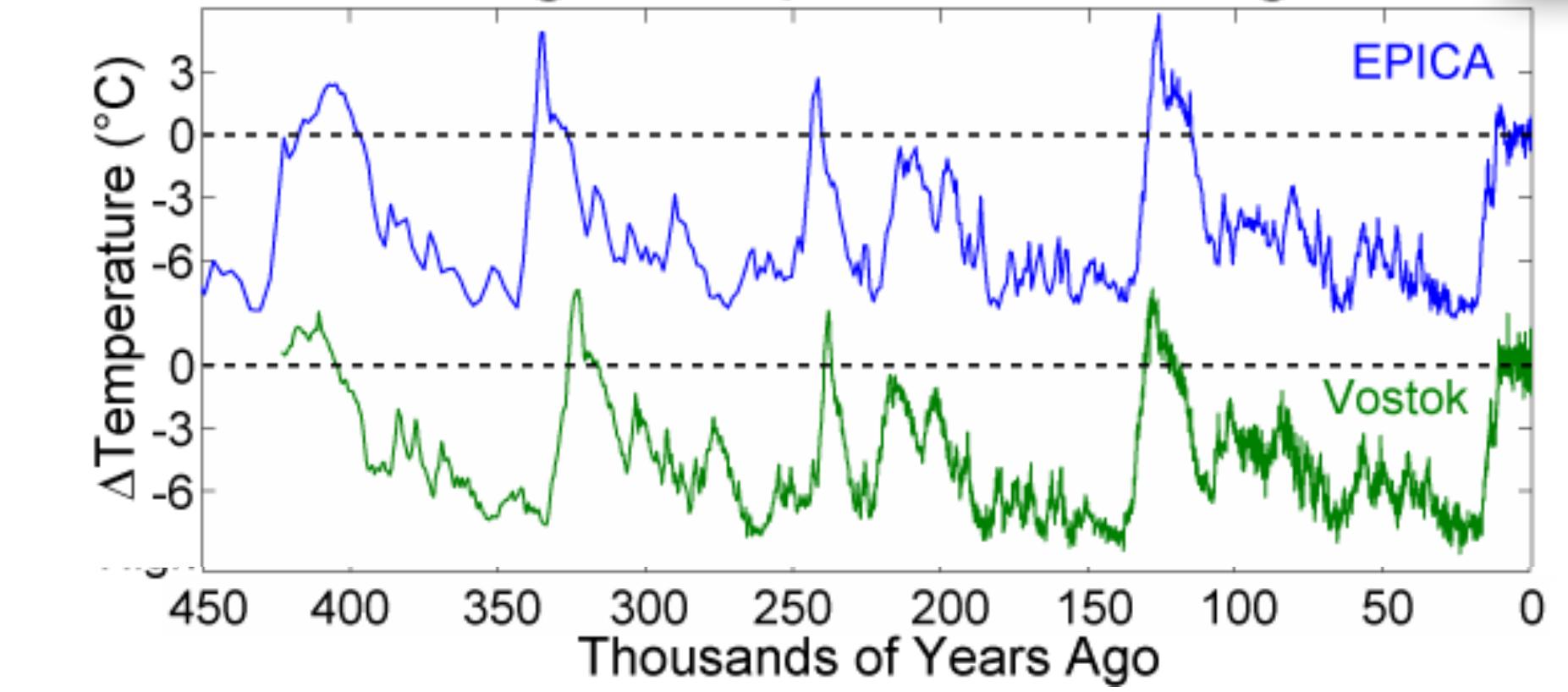
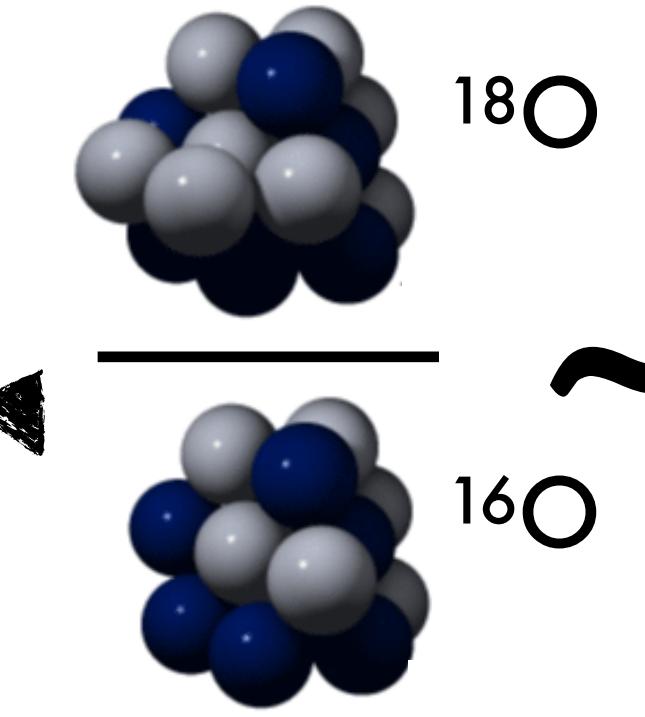
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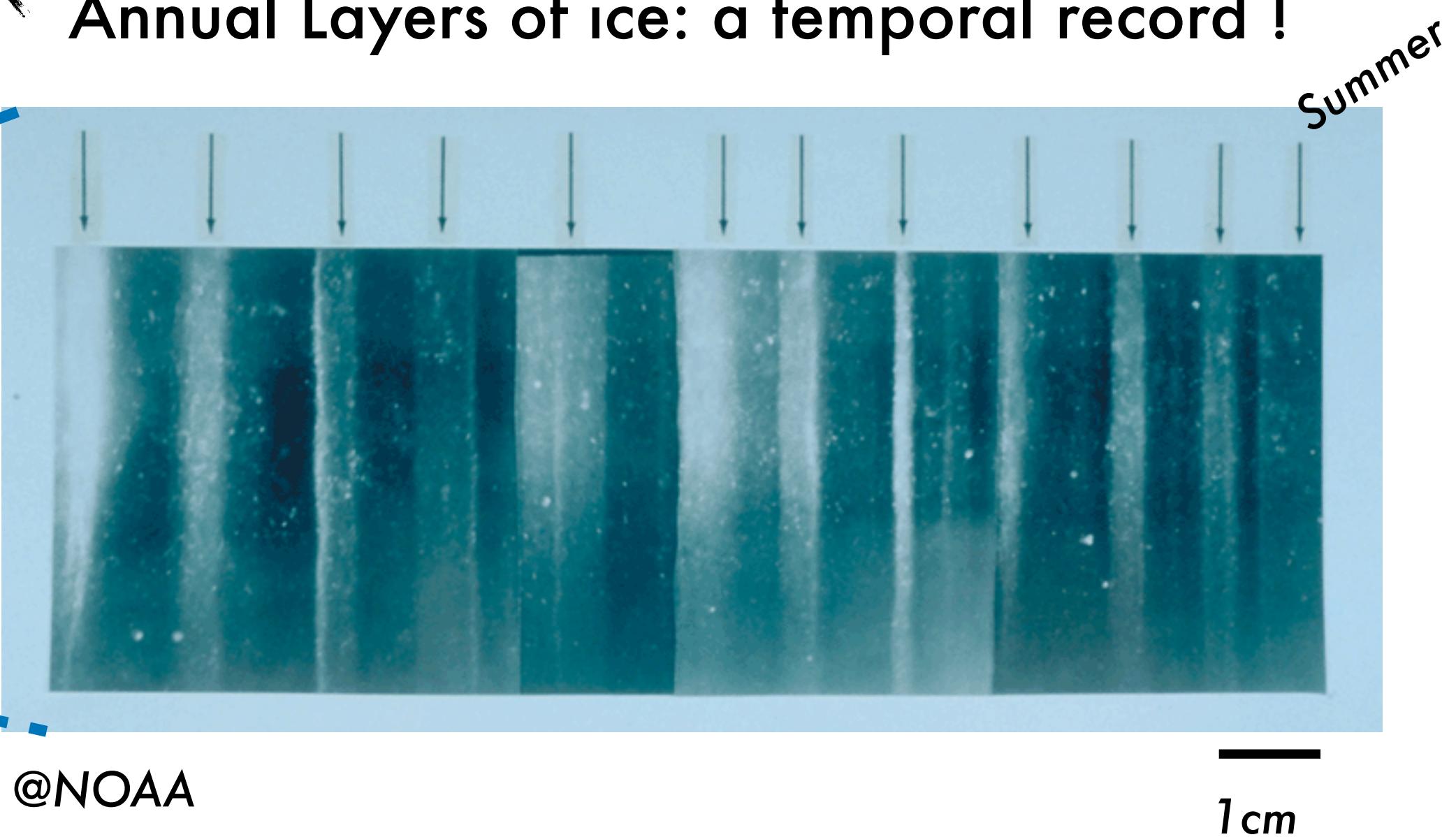
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HEAVY!

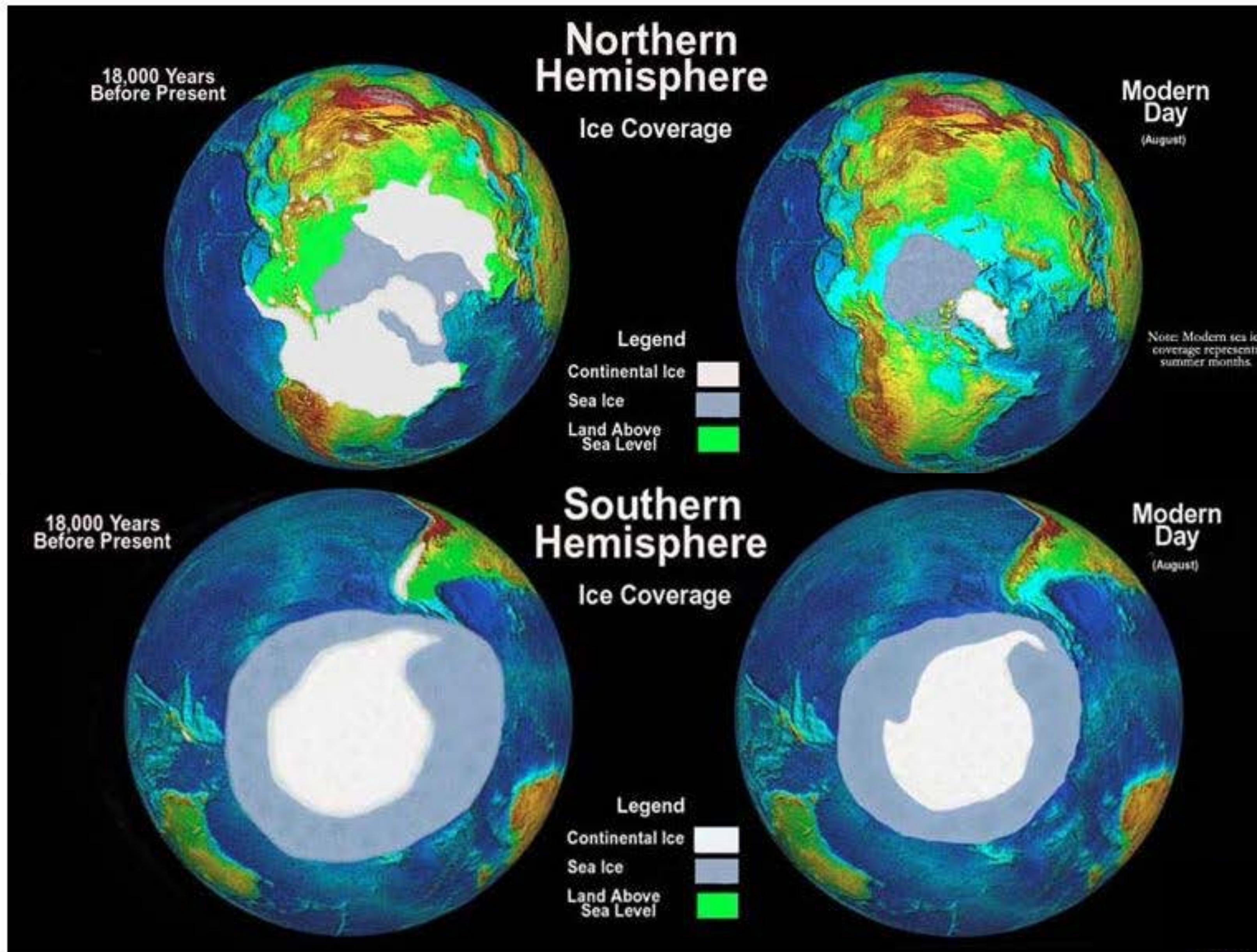


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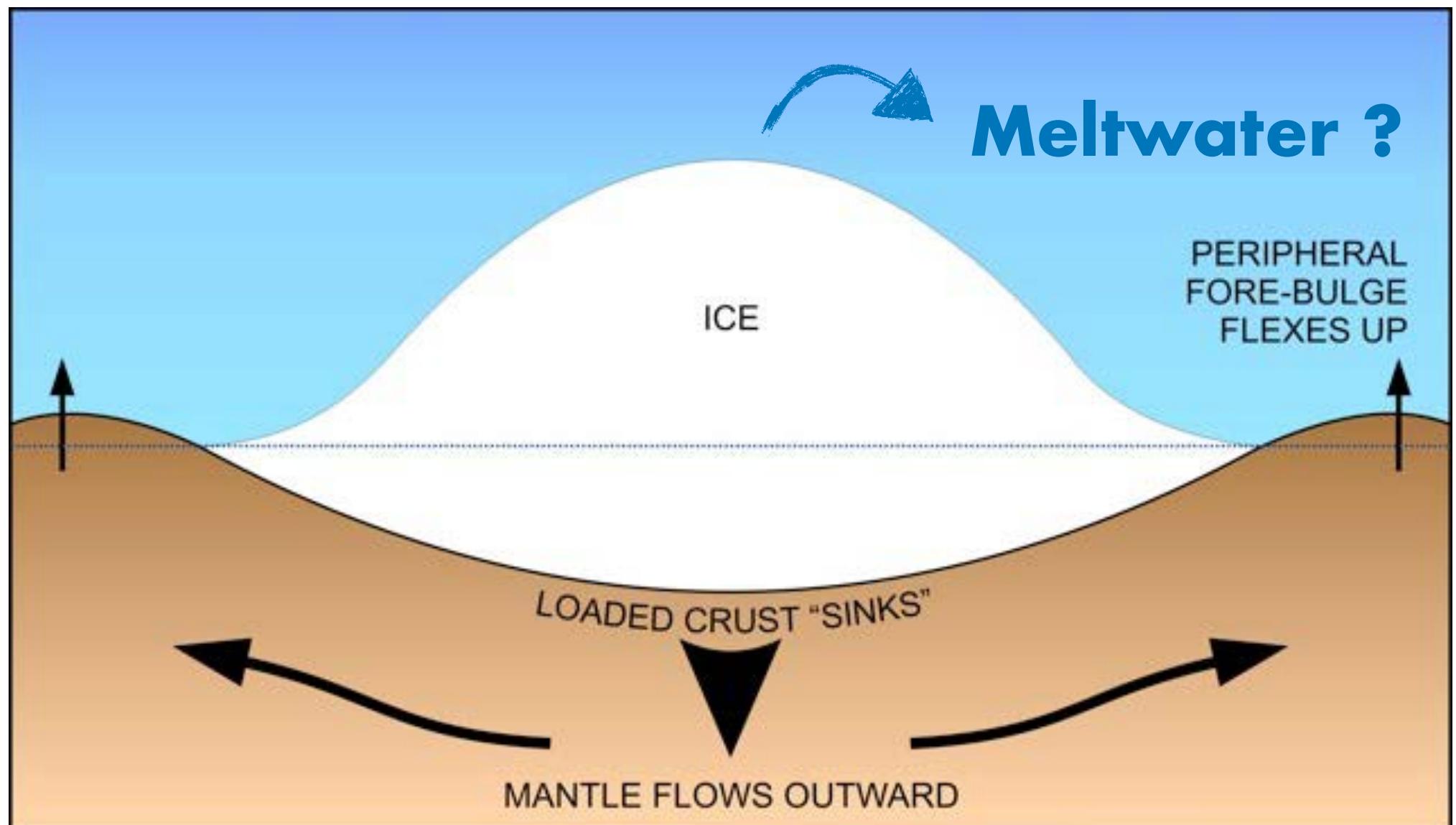
@NOAA

Ice Sheets “recent” evolution



Glacial Isostatic Adjustment (finally!)

A classic geodynamical problem of determining the Solid Earth response to changes in ice and ocean surface loading



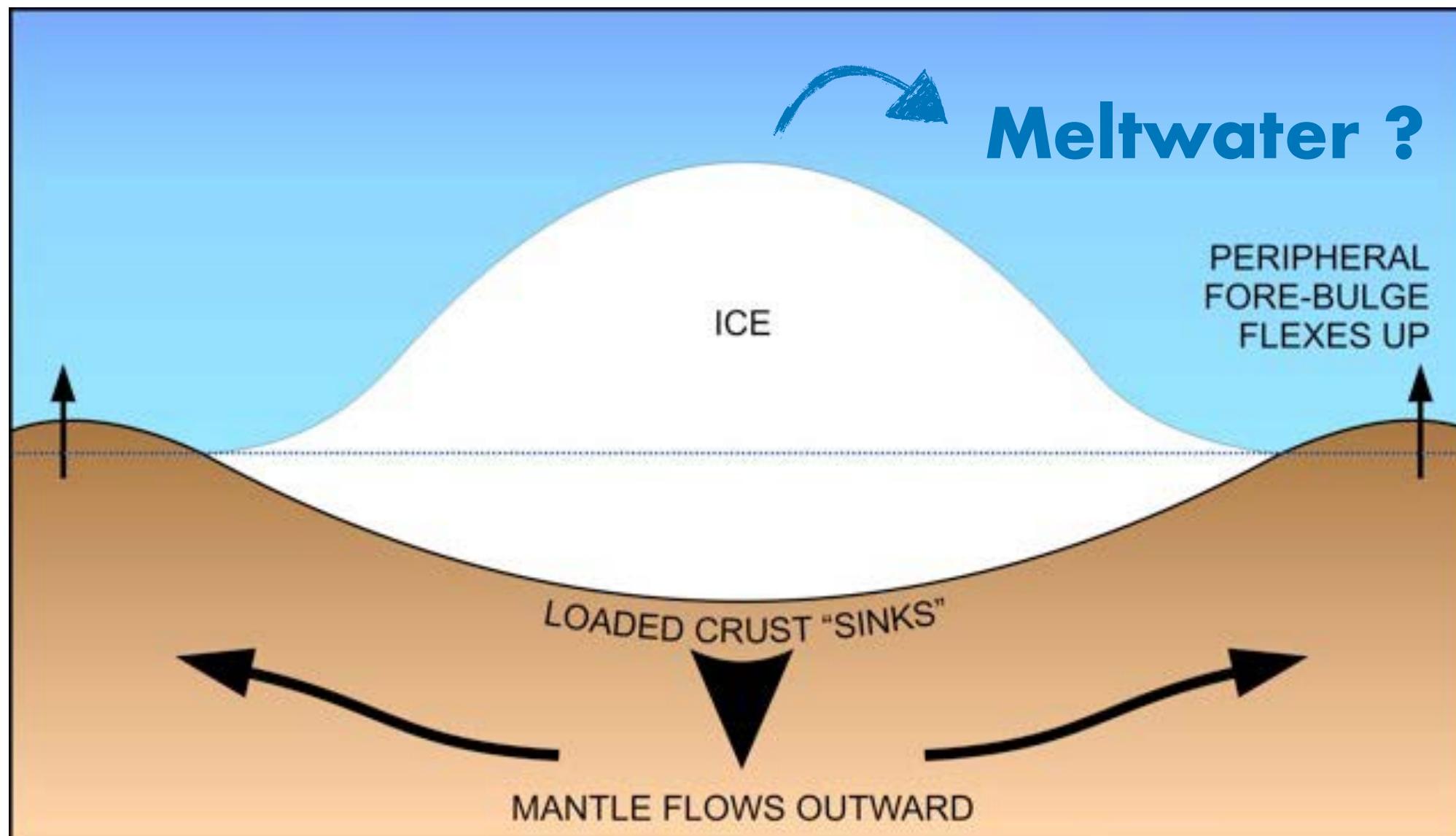
@UNAVCO

Gravity change, 3D crustal motion

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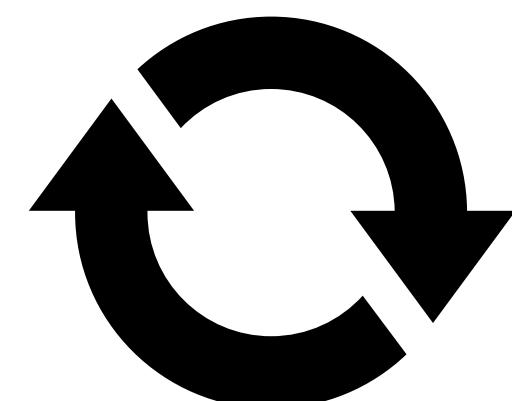
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... and solving for a gravitationnally-consistent redistribution of ice and meltwater across the ocean

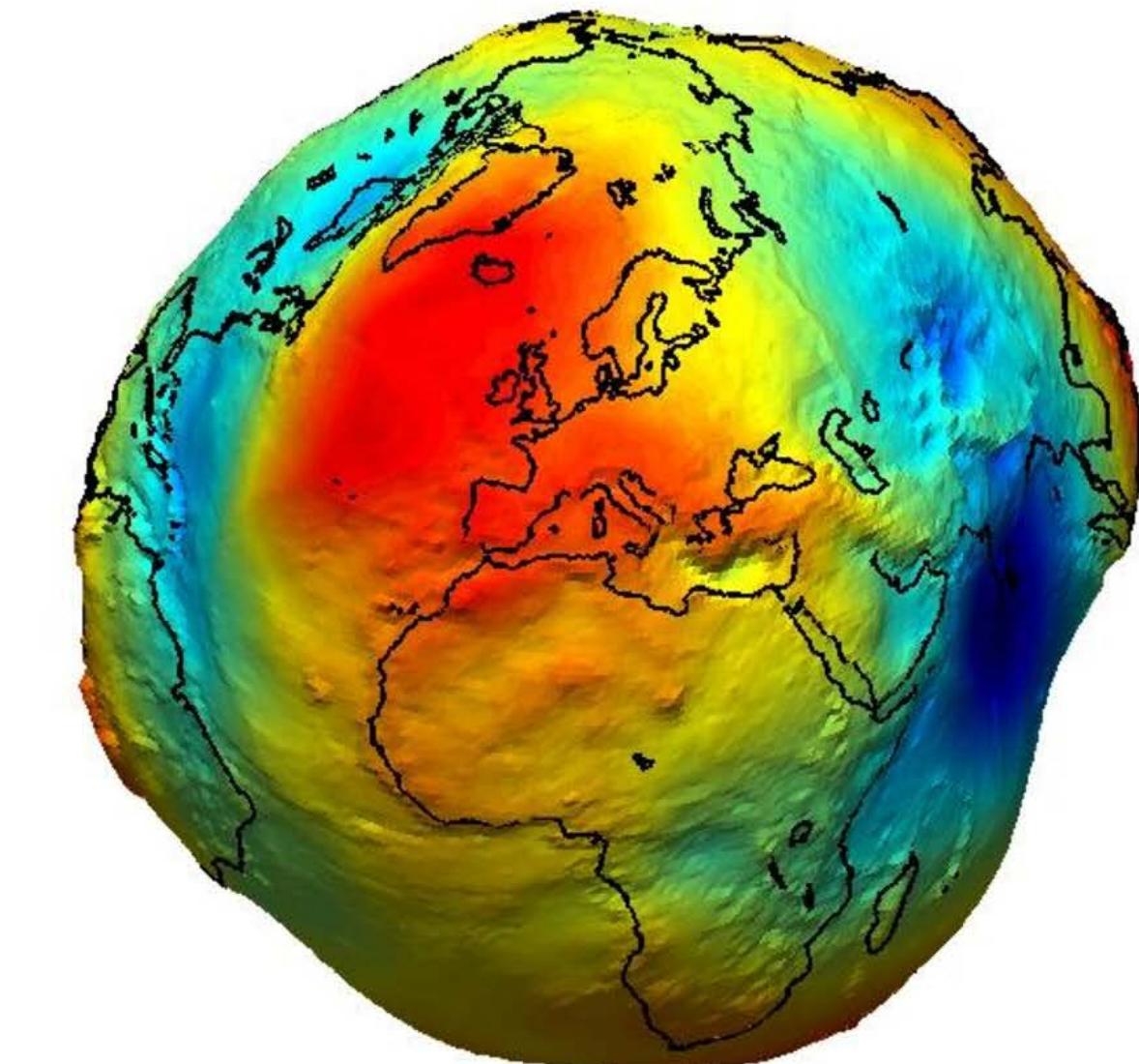


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Iterative equation



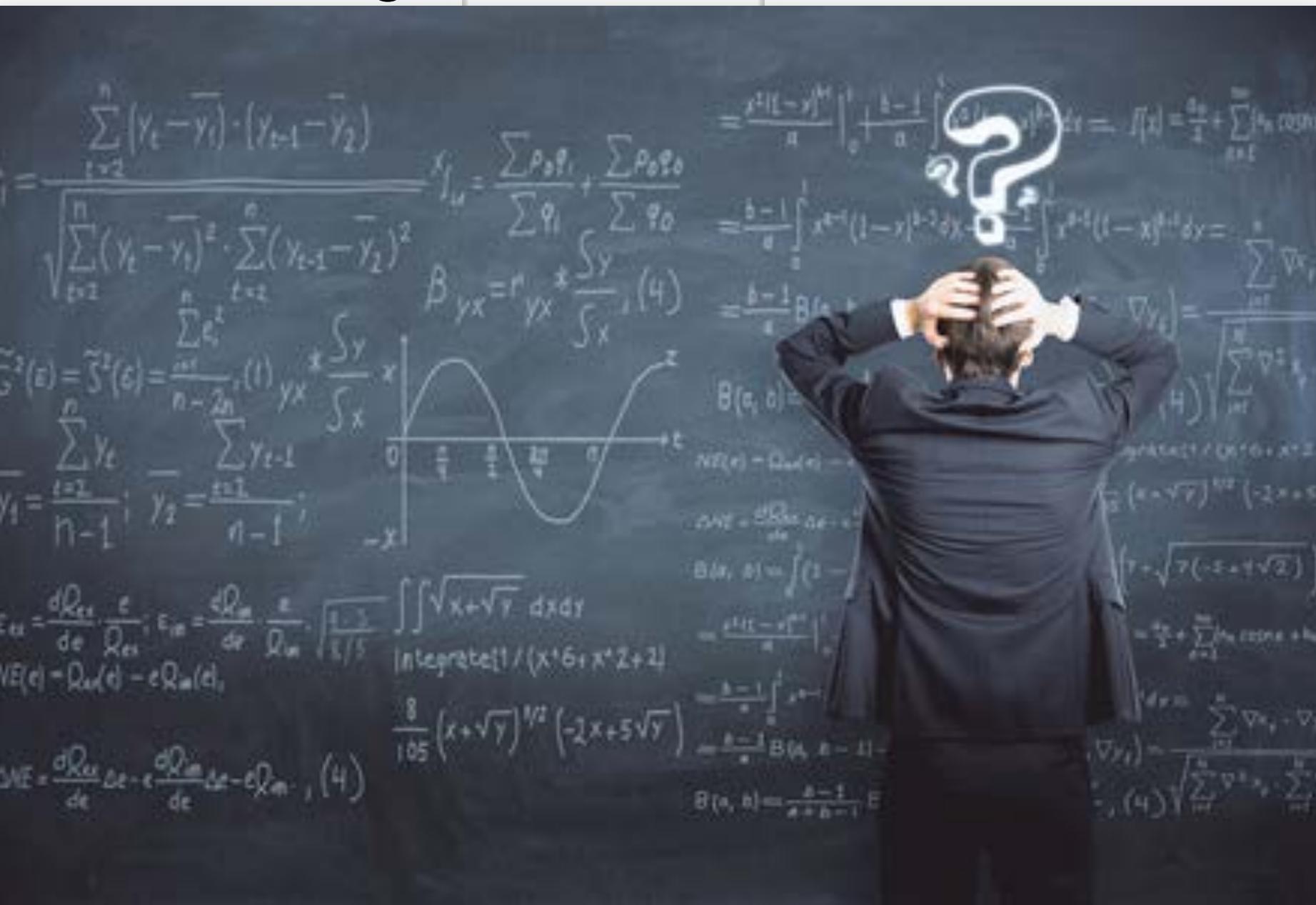
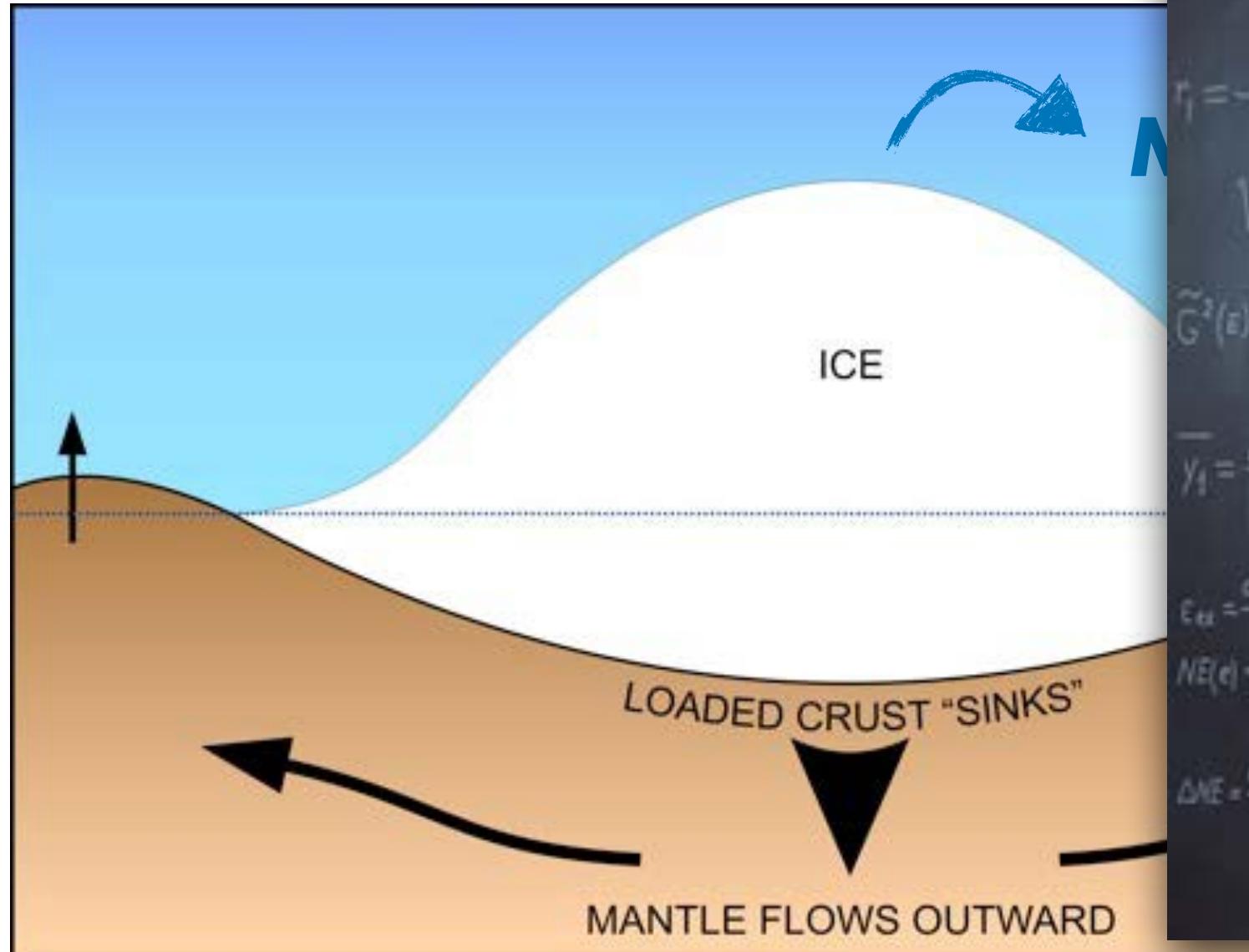
@ESA

Sea level change, solve for the shape of geoid, Earth rotation perturbation, changing ocean is also a load

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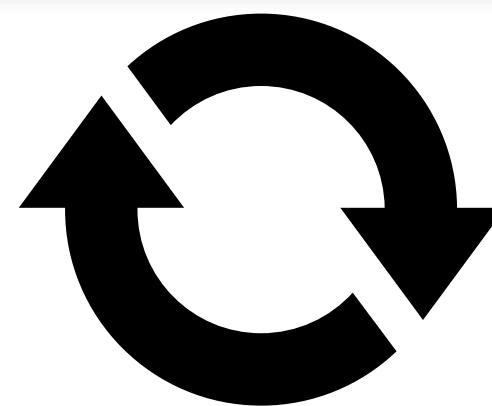
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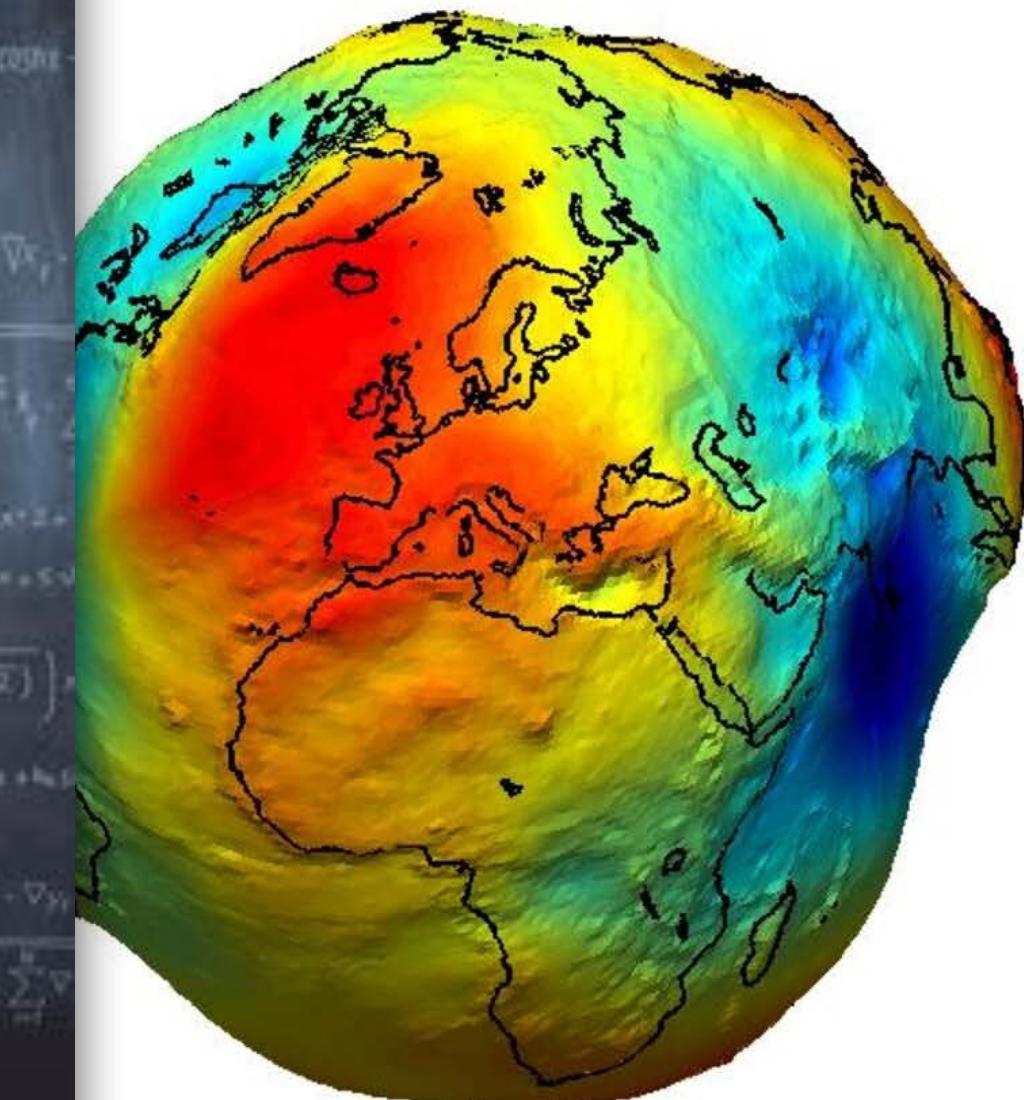


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GIA, should I care ?

1491

The Swedish government decides to relocate the town of Östhammar because of harbour problems

"During recent years the land has grown outside the town at the sea, so that where some years ago a cargo boat of five or six Swedish läster [about 15 tons] could come from the sea into the town of Östhammar not even a fishing-boat can go nowadays. And the land is still growing and rising every year."

*What is going on?
Why is it going on?
How is it going on?*

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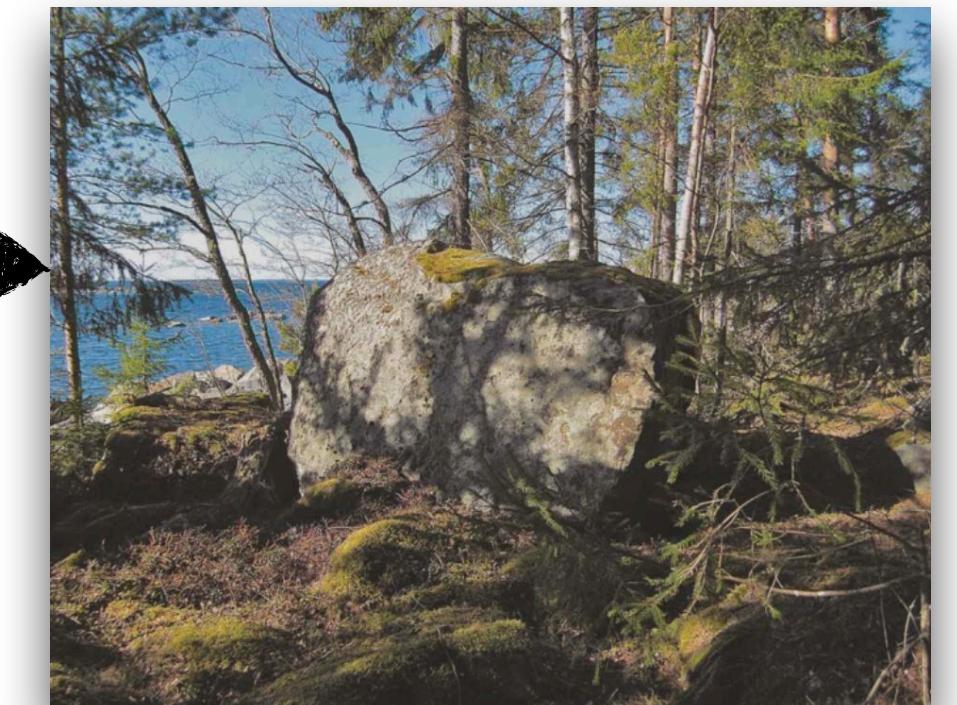
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Anders Celsius
(1701-1744)

1743 – Land uplift rate: ~1 cm/yr



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Land uplift: up to several hundreds meters since LGM
up to a few cm/yr



Kiluhiqtuq (Bathurst Inlet), Canada @Beauregard



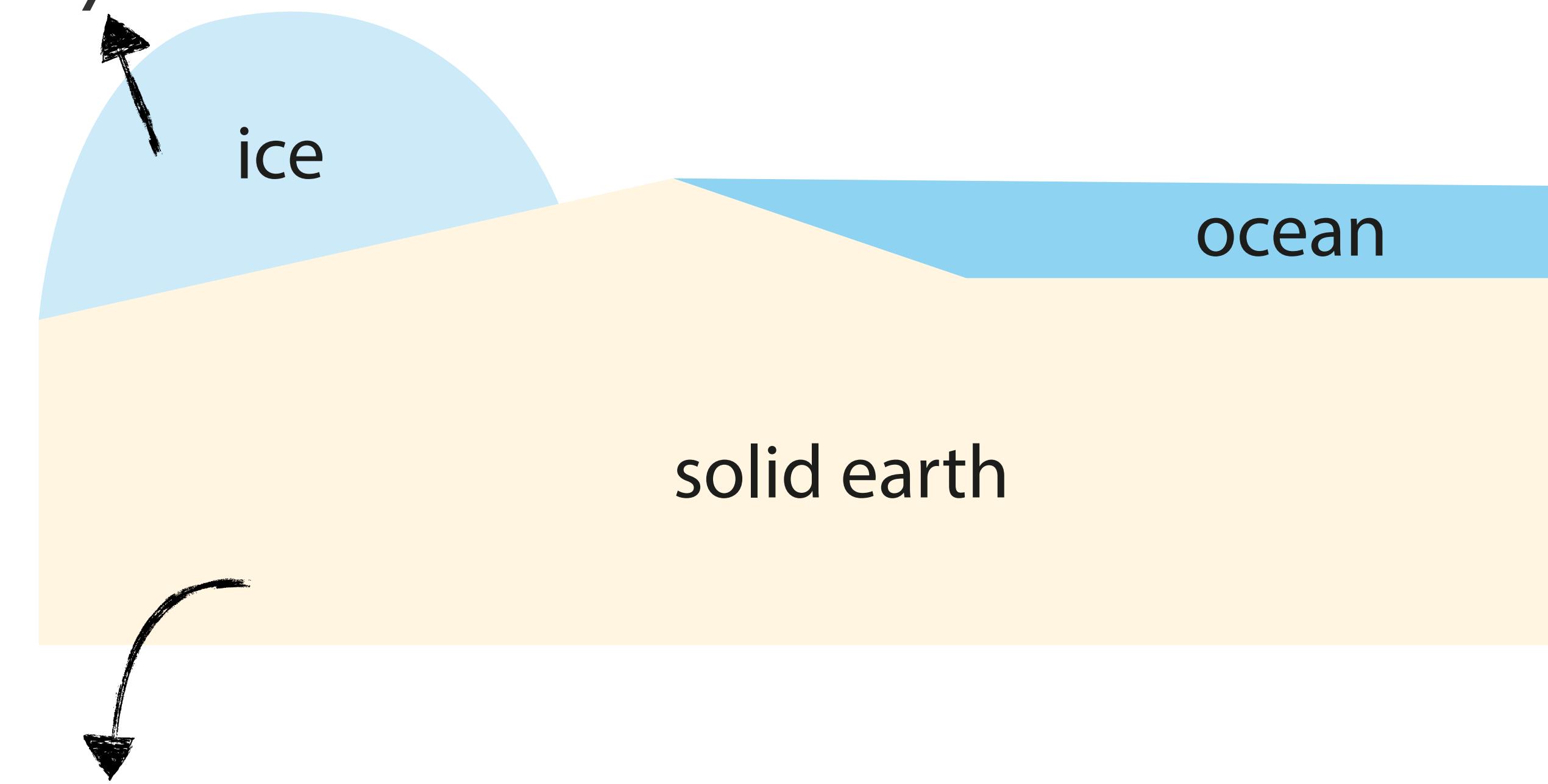
Lantmäteriet, Sweden @Rehn

Why study GIA today?

What is the history of ice-sheets?

How did they respond to climate change?

How fast is ice melting today?



What is the rheology of the Earth?

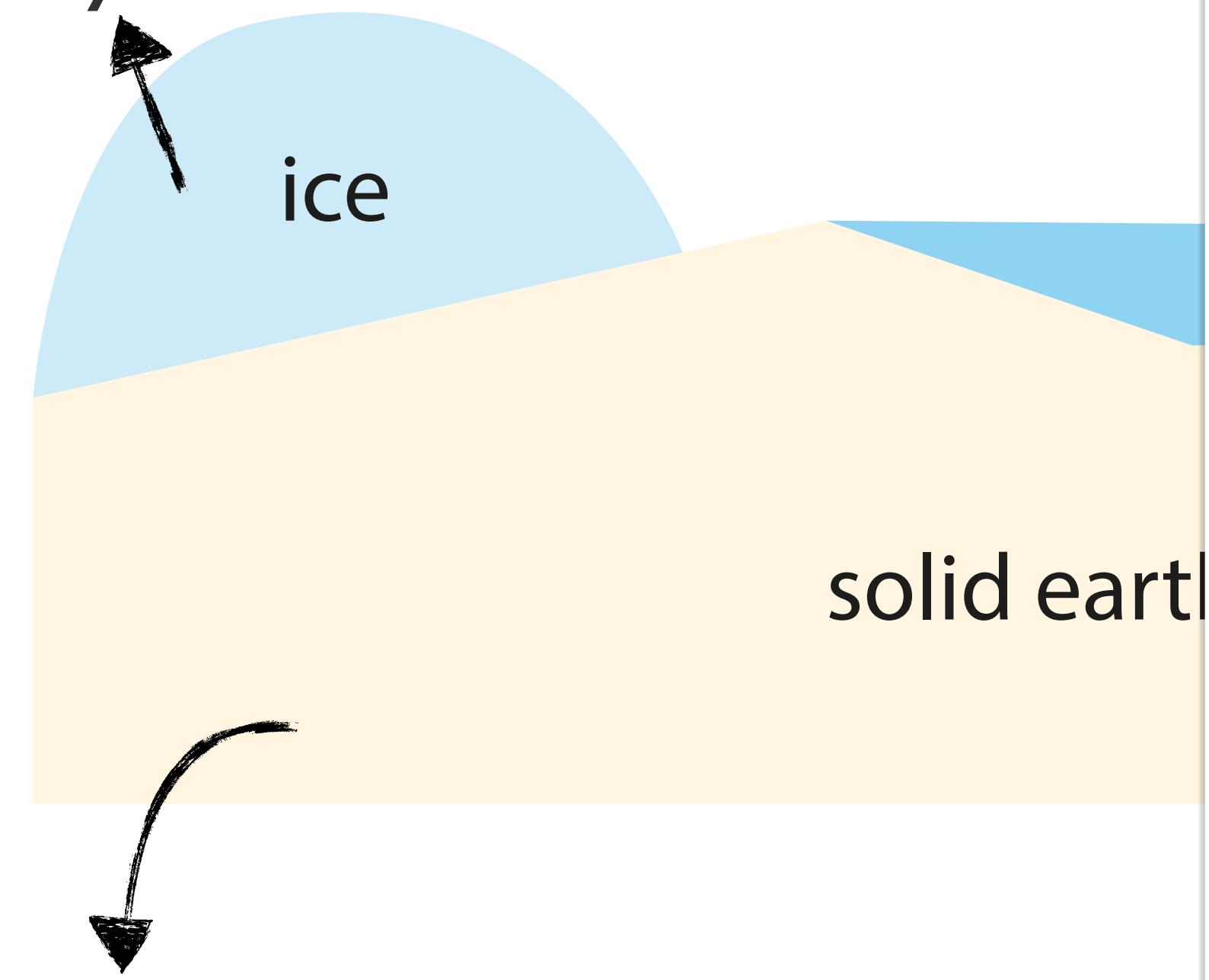
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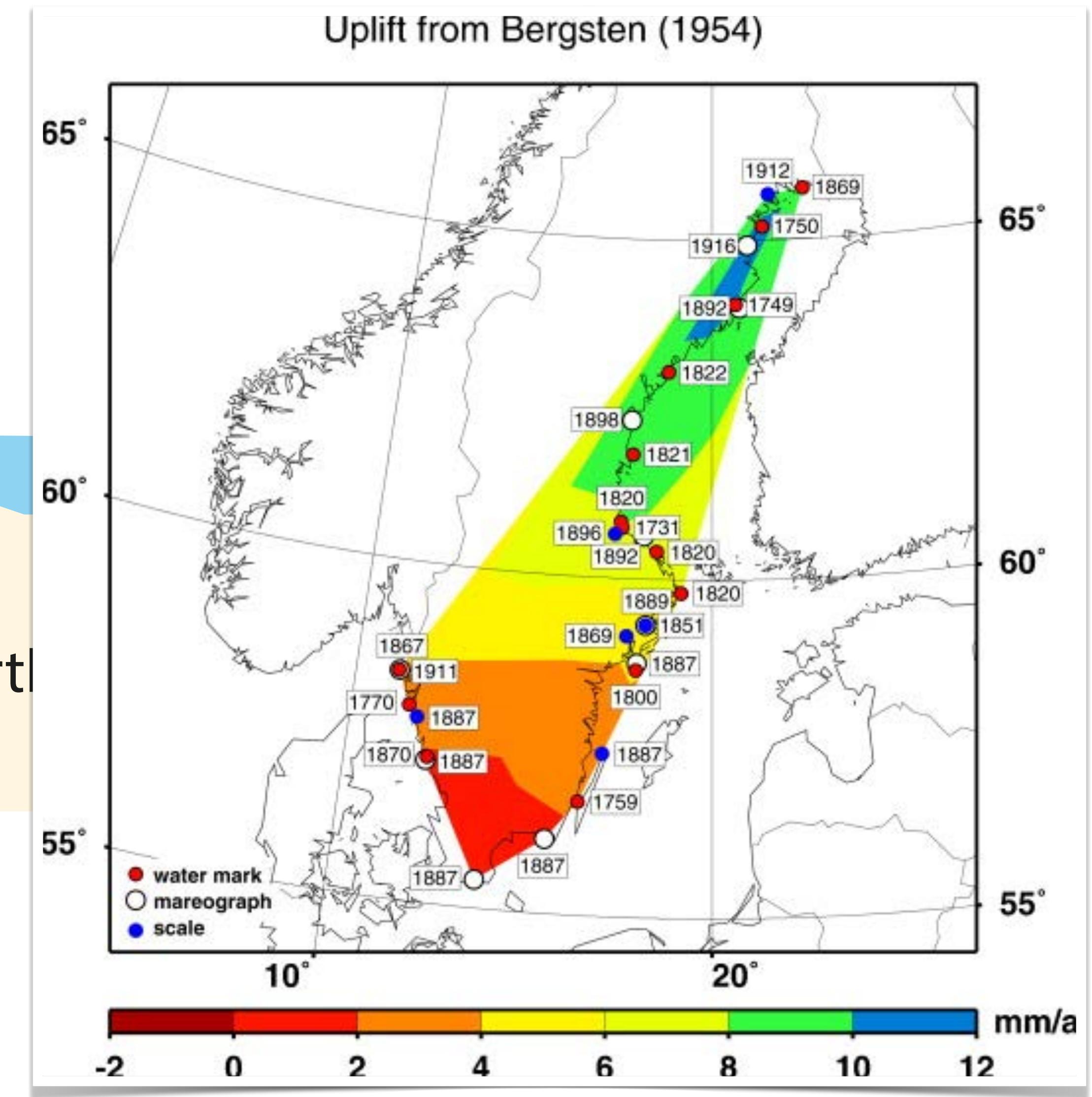
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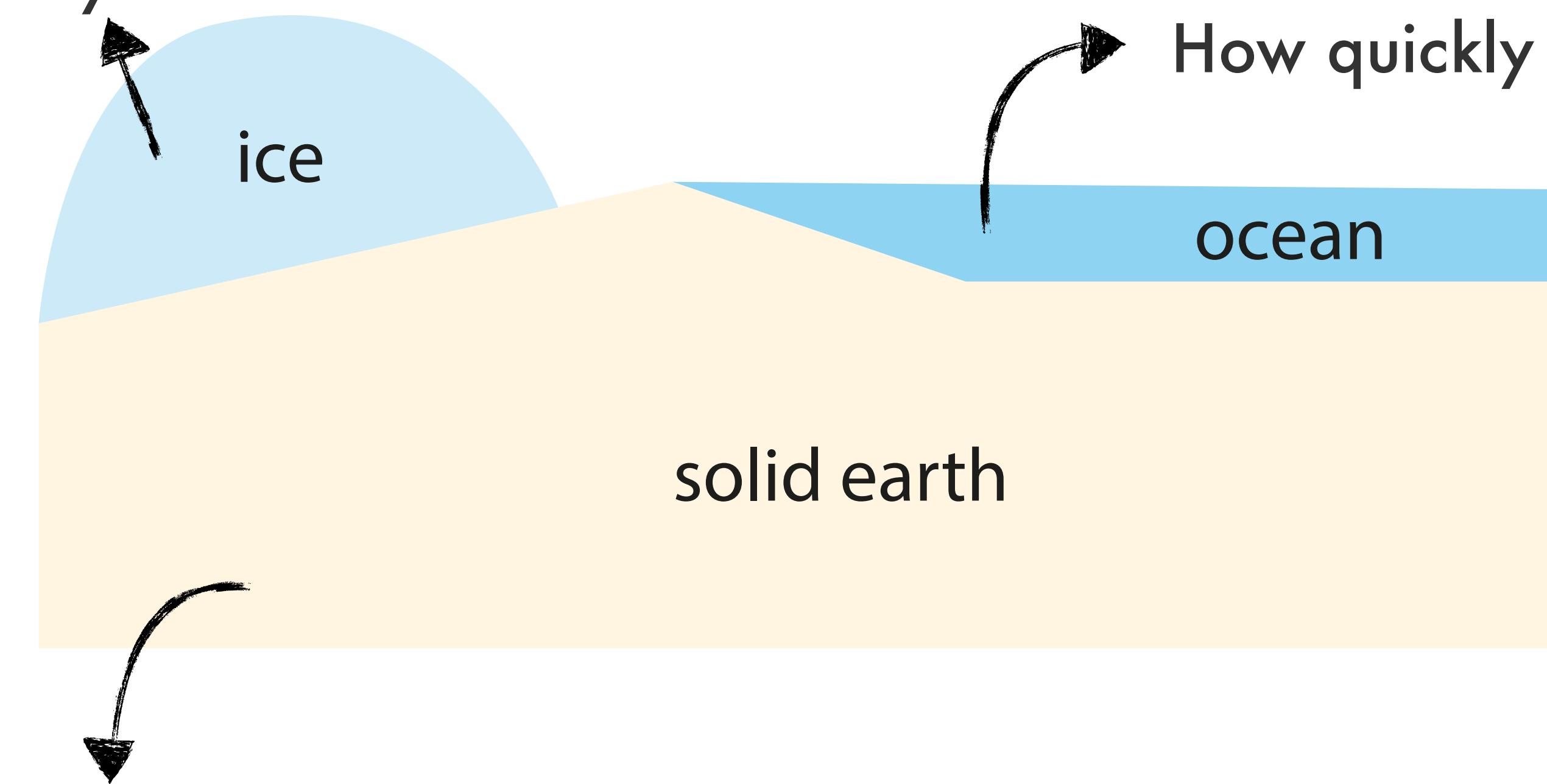
Steffen & Wu (2011)

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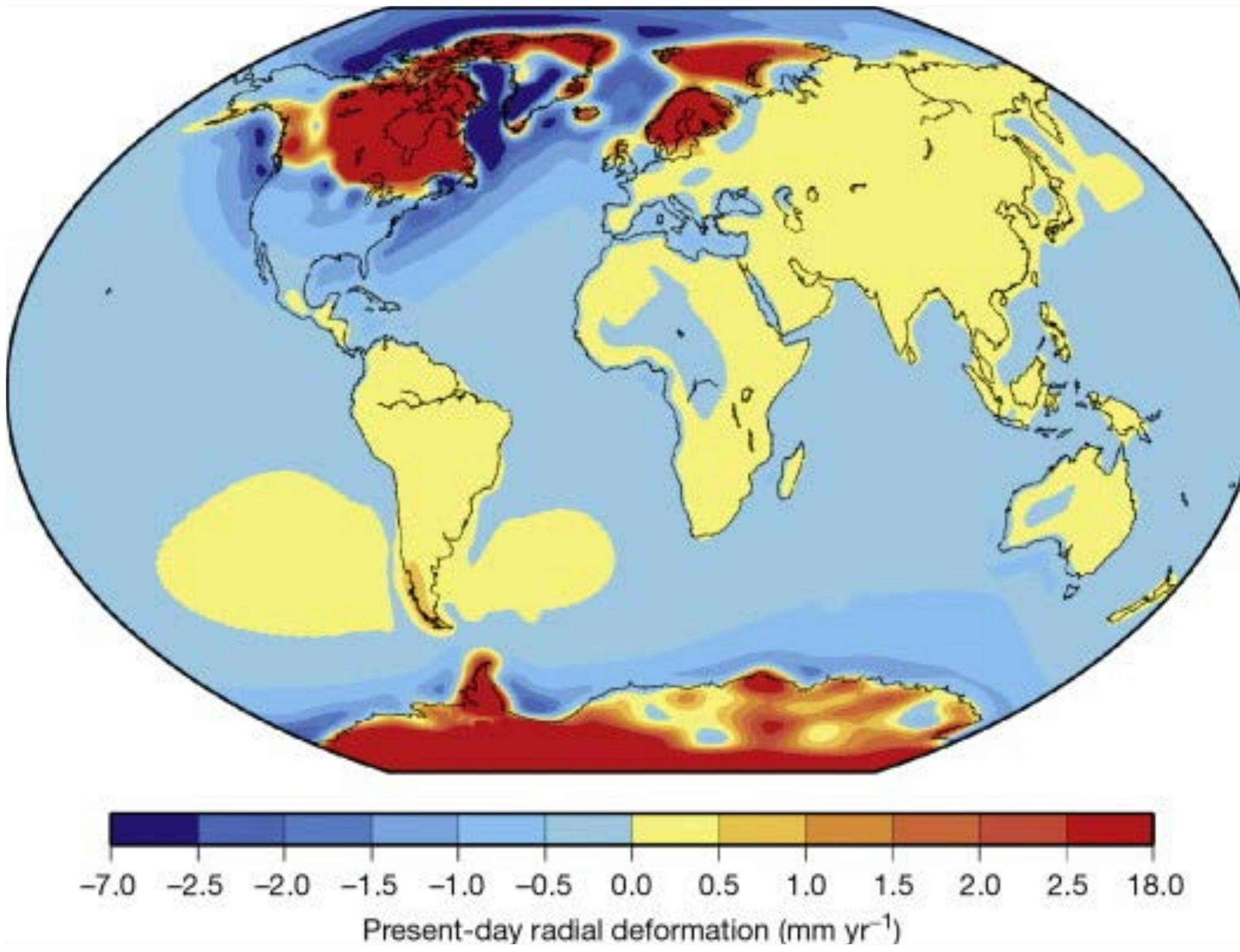


What drives sea-level change today?
How quickly can sea-level change?

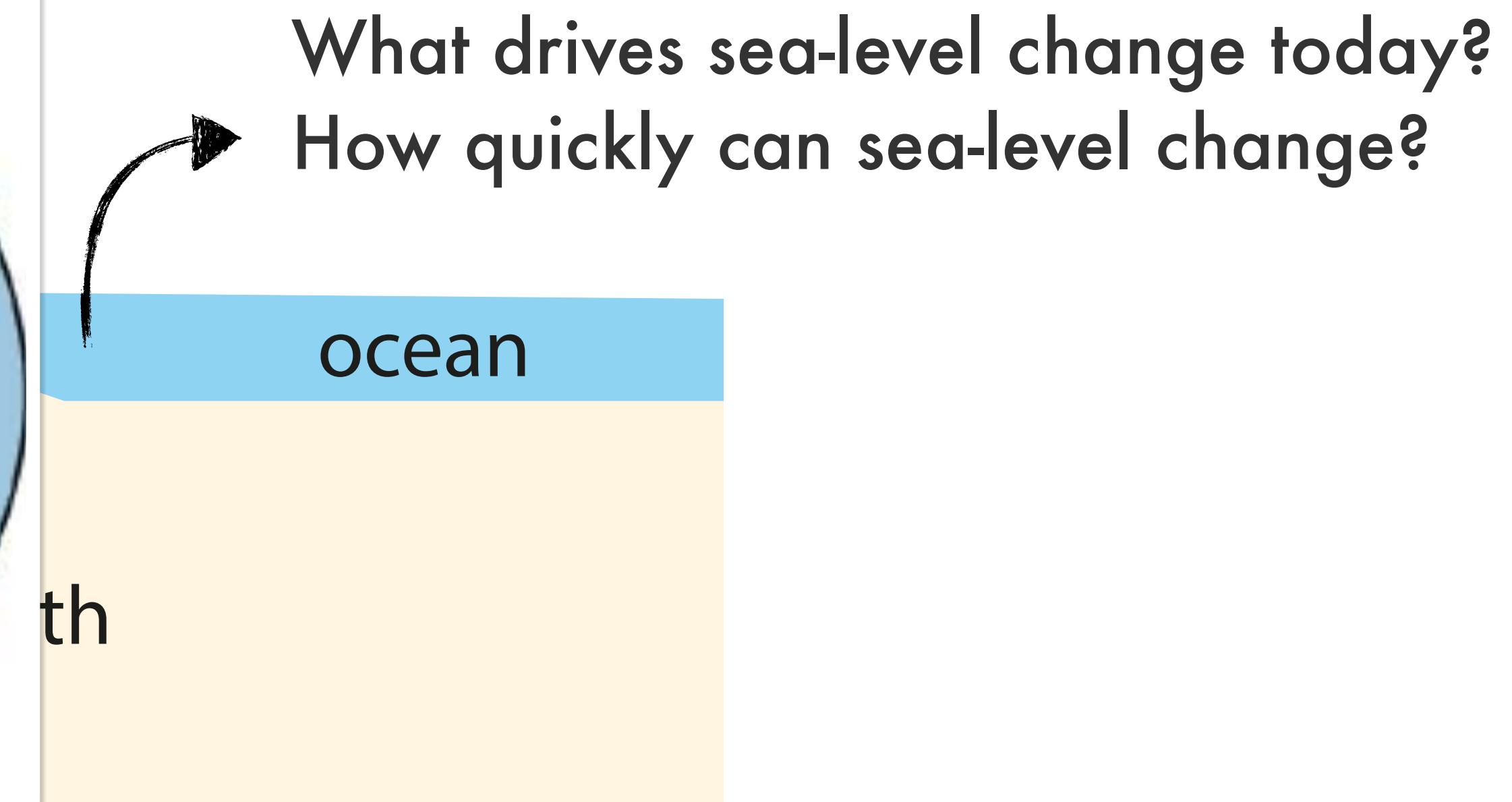
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Milne & Shennan (2011)

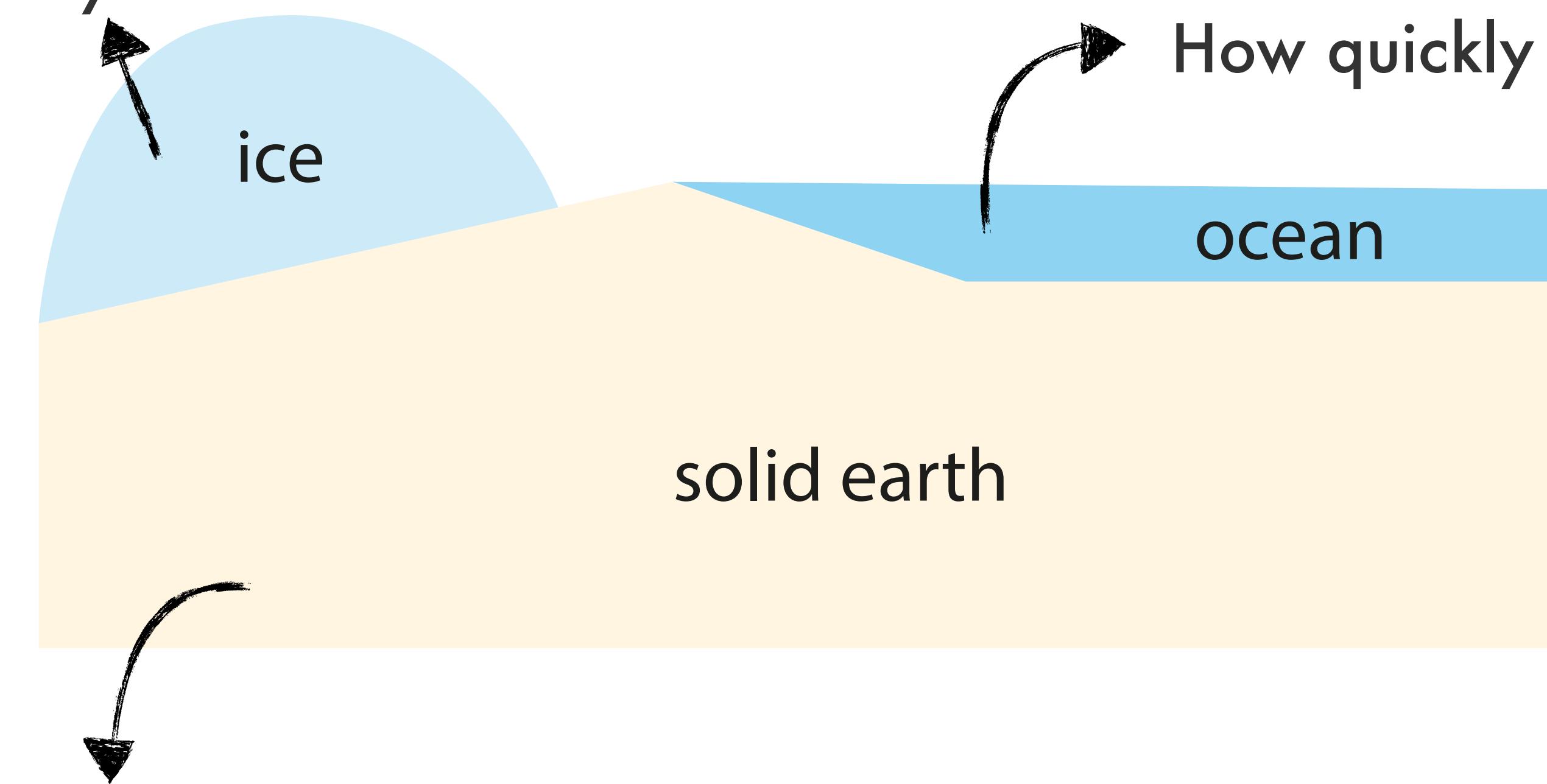


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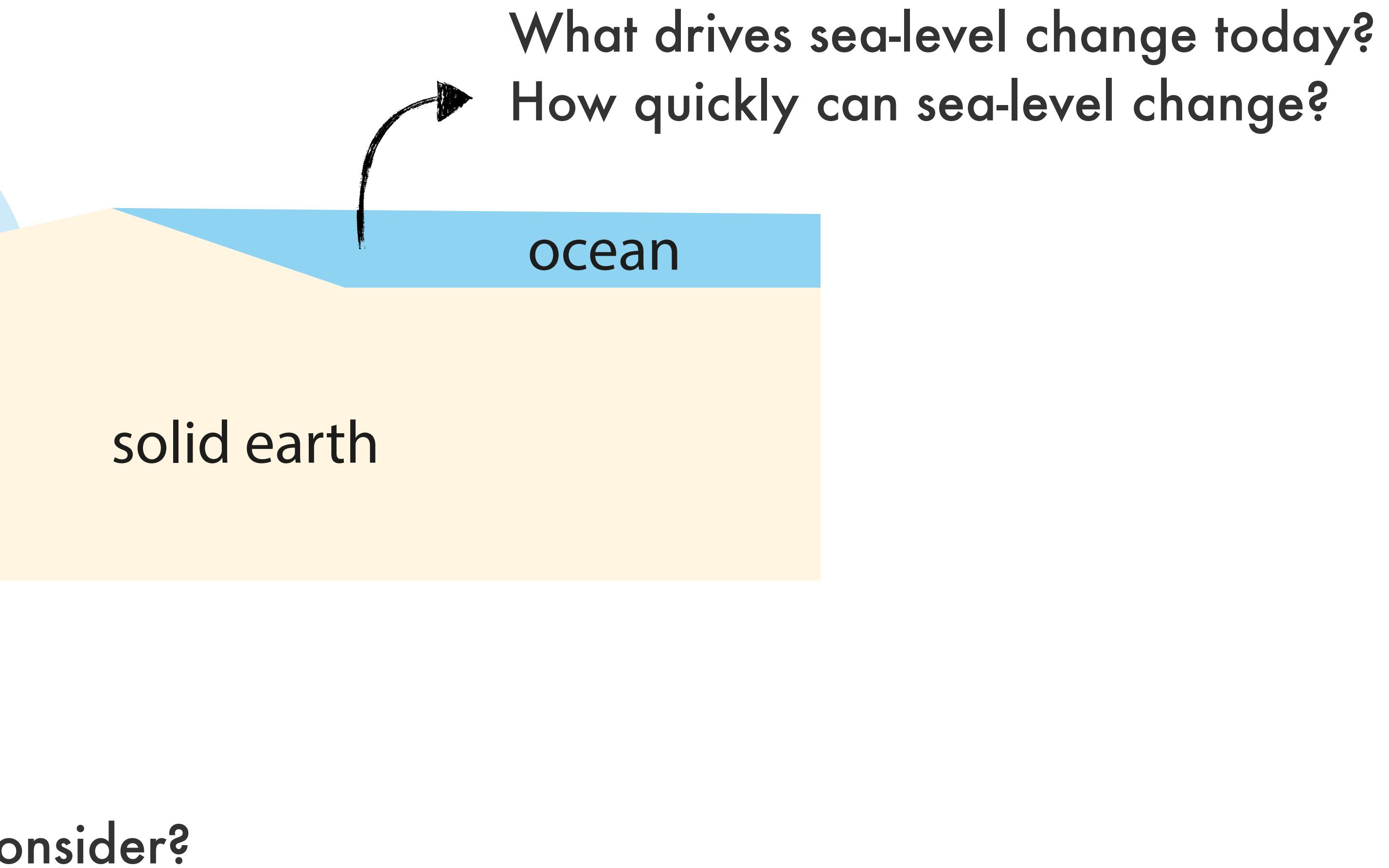
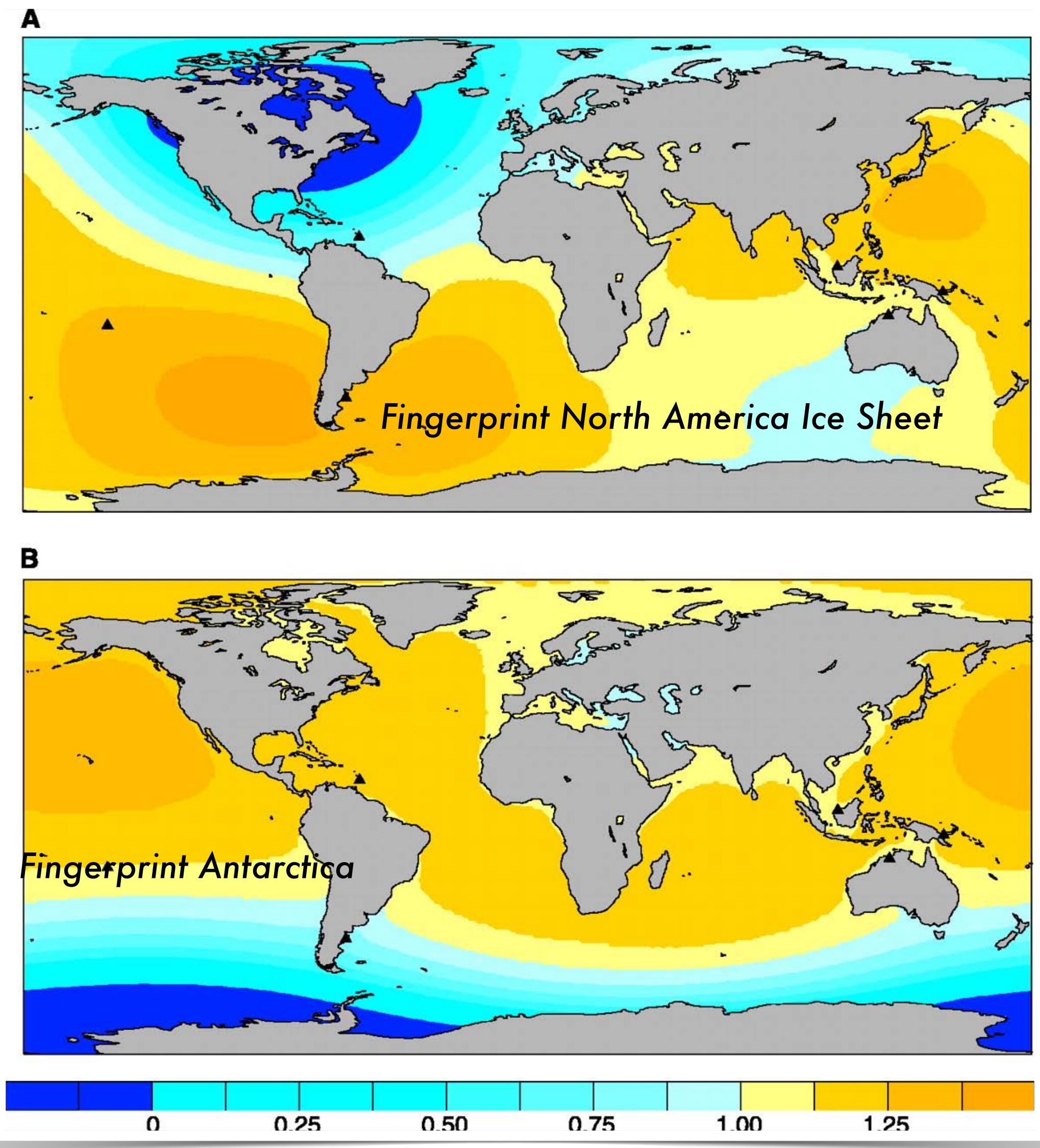


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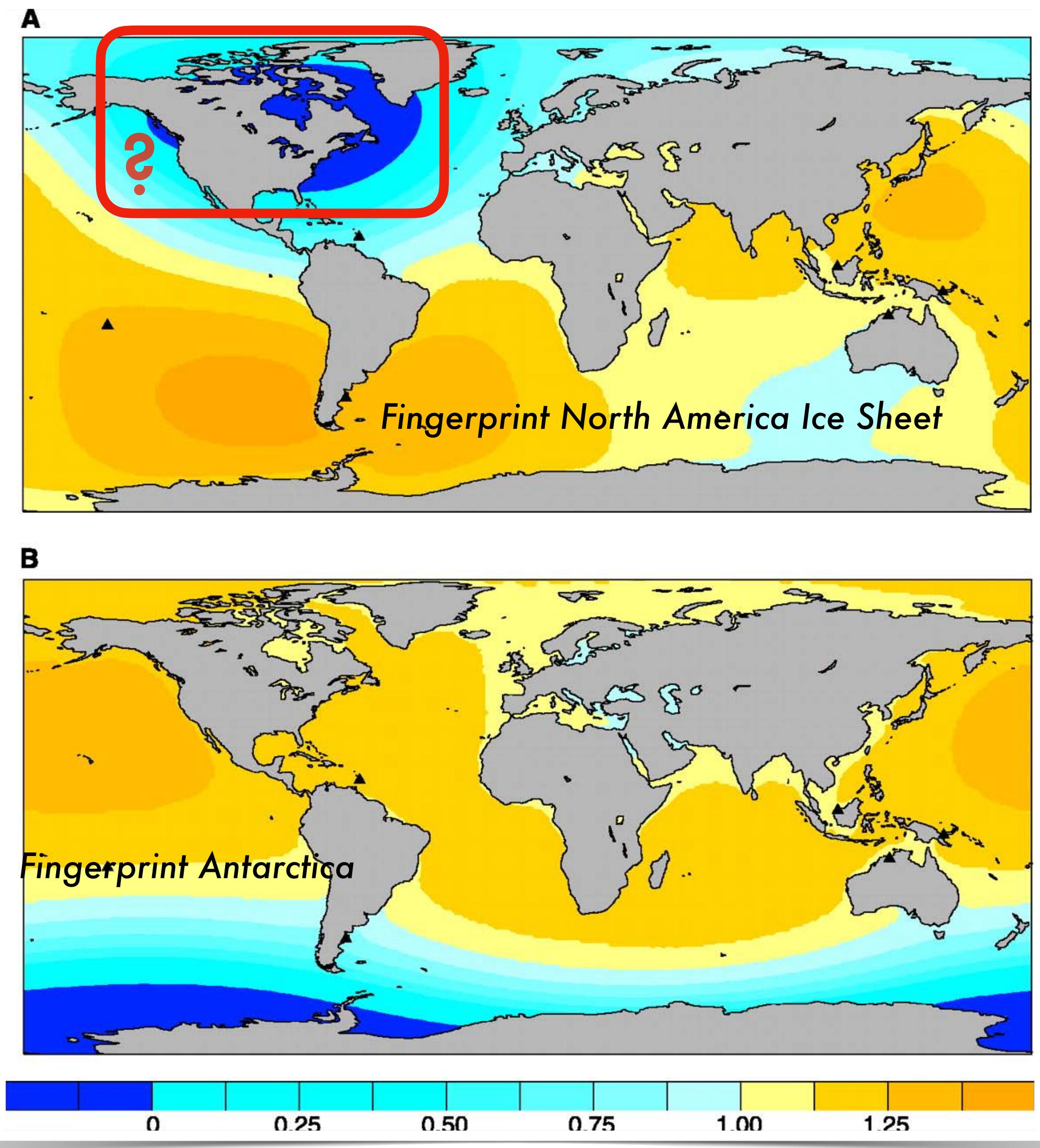
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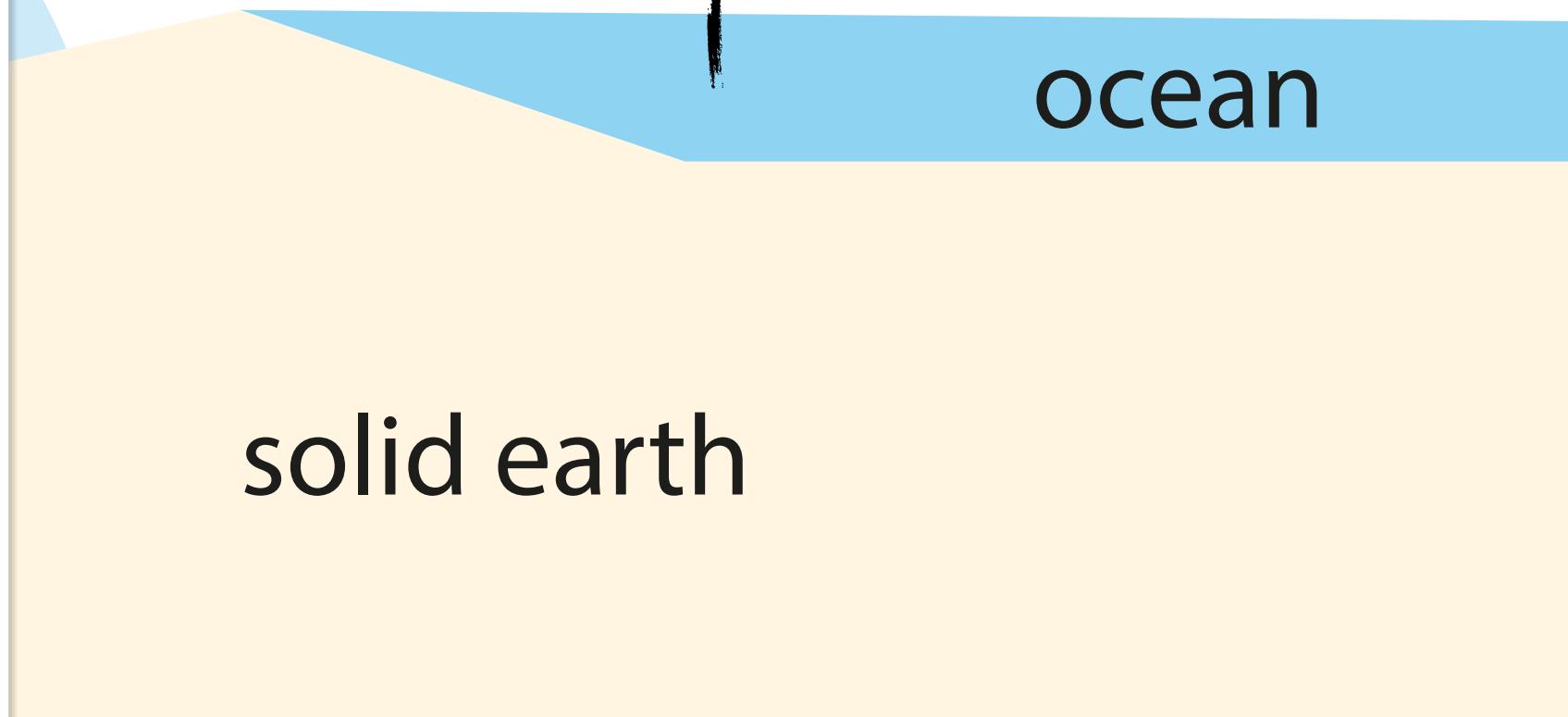
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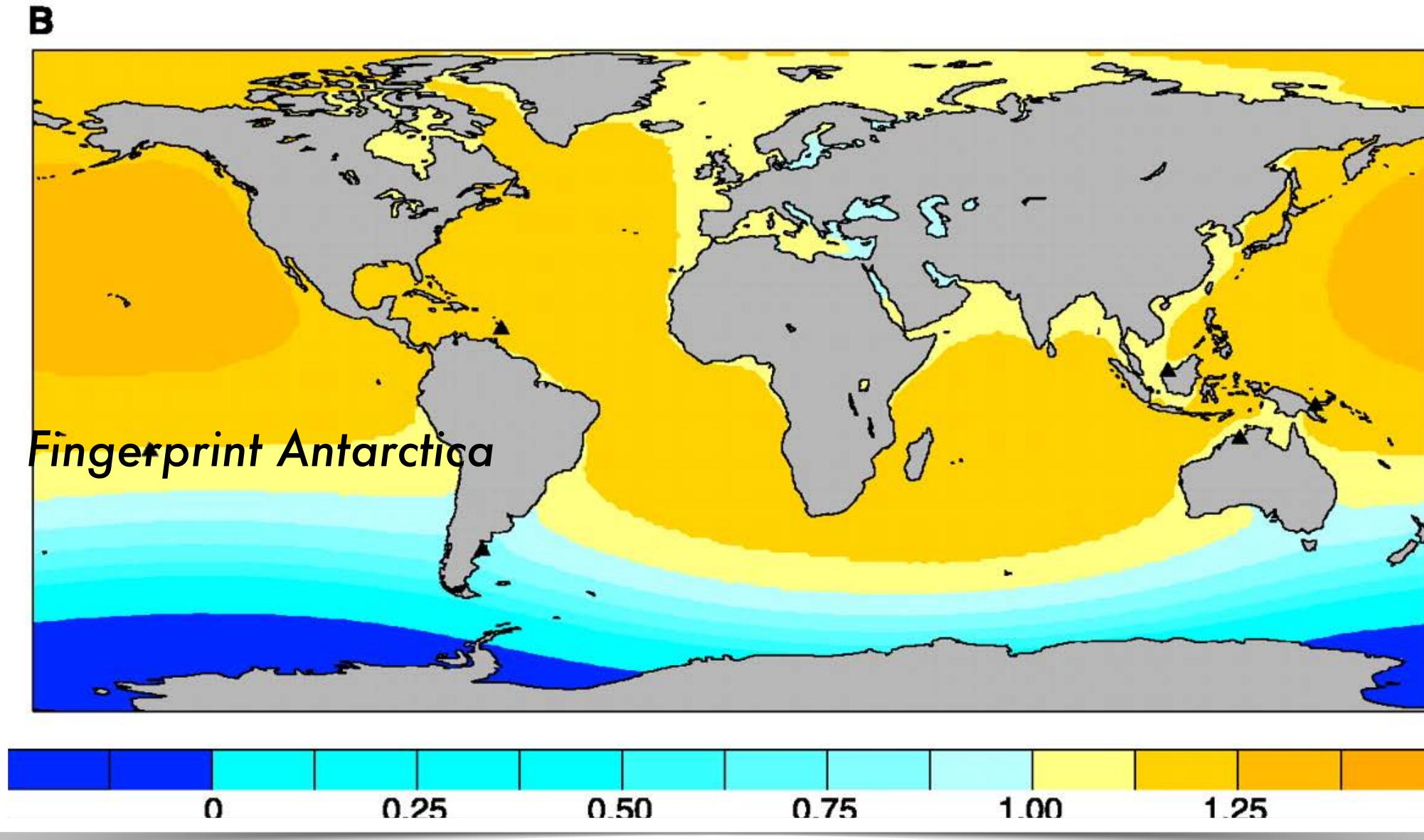
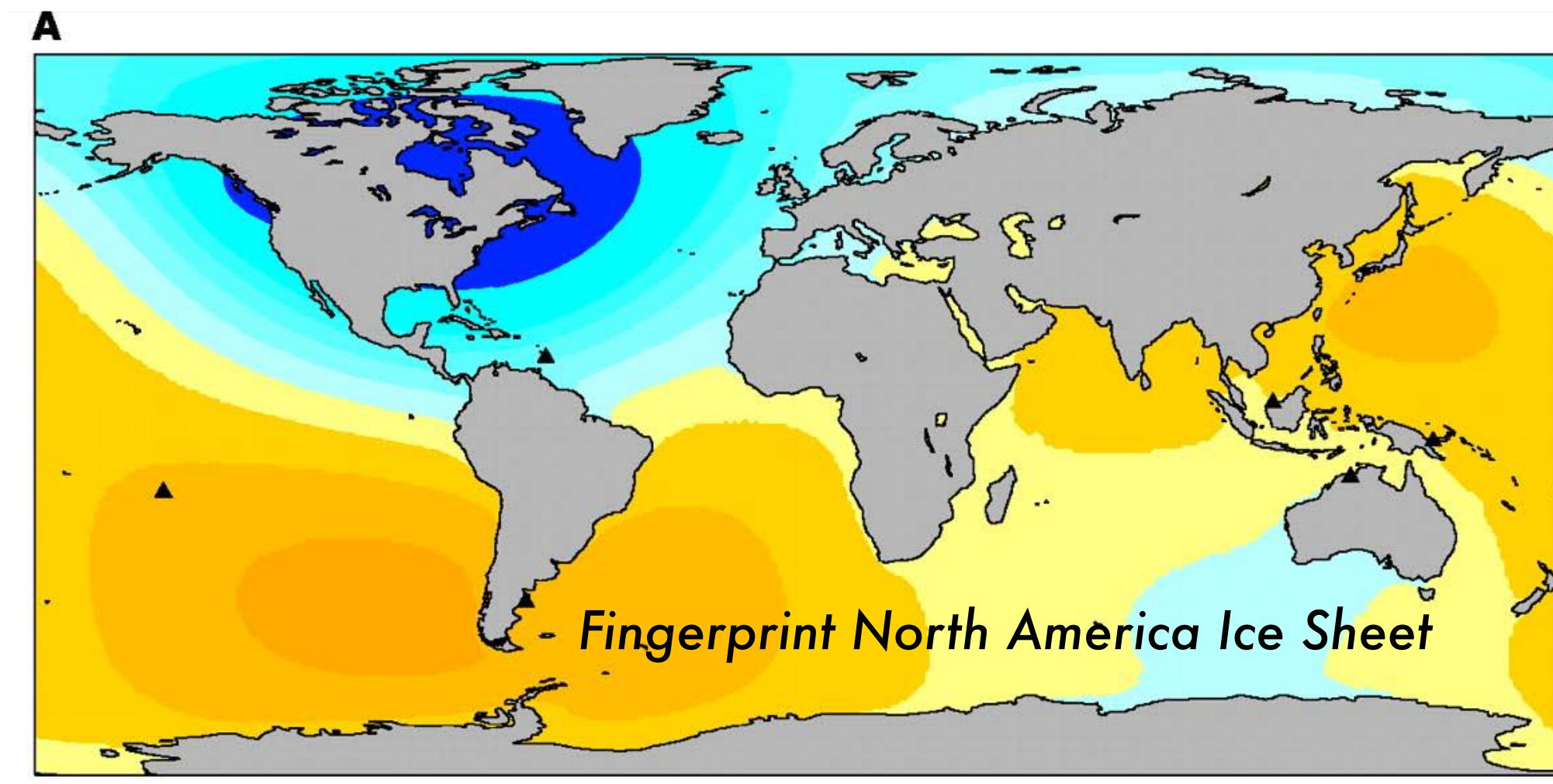


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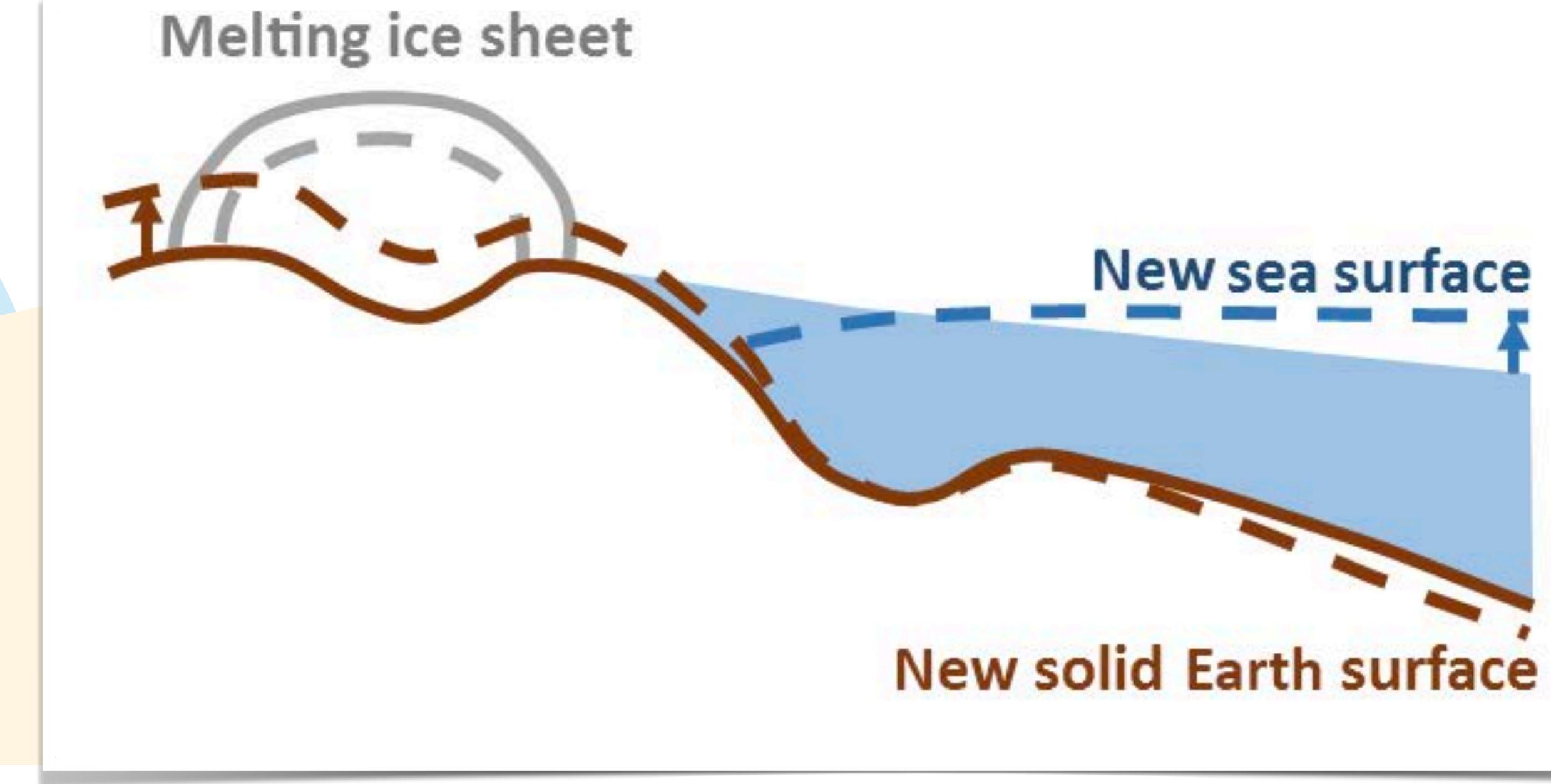


consider?

Why study GIA today?



consider?



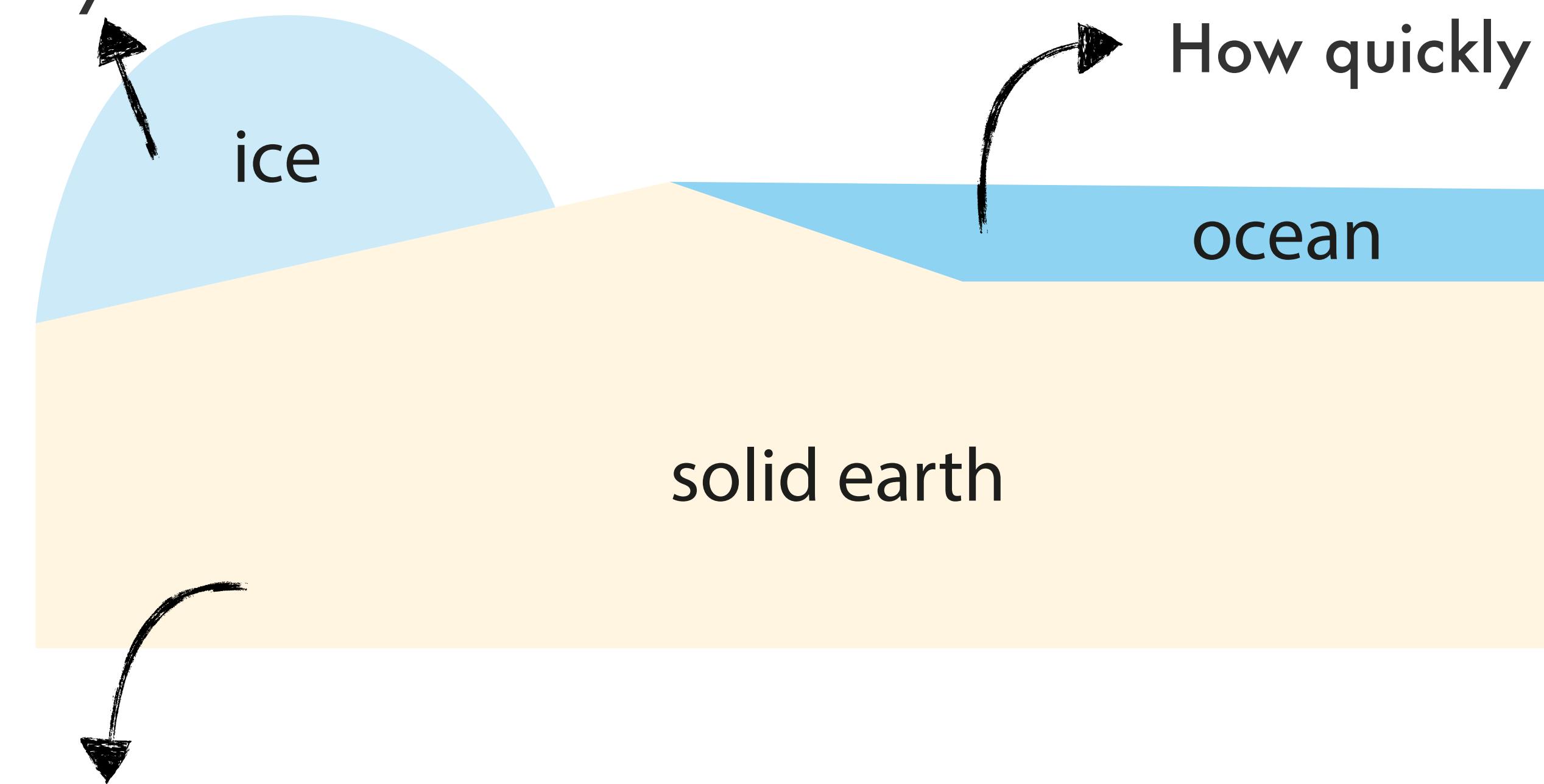
- solid Earth rebound due to ice melting
- decrease in sea surface height due to the decreased gravitational attraction of the ice sheet
- Both processes cause near-field relative sea-level fall

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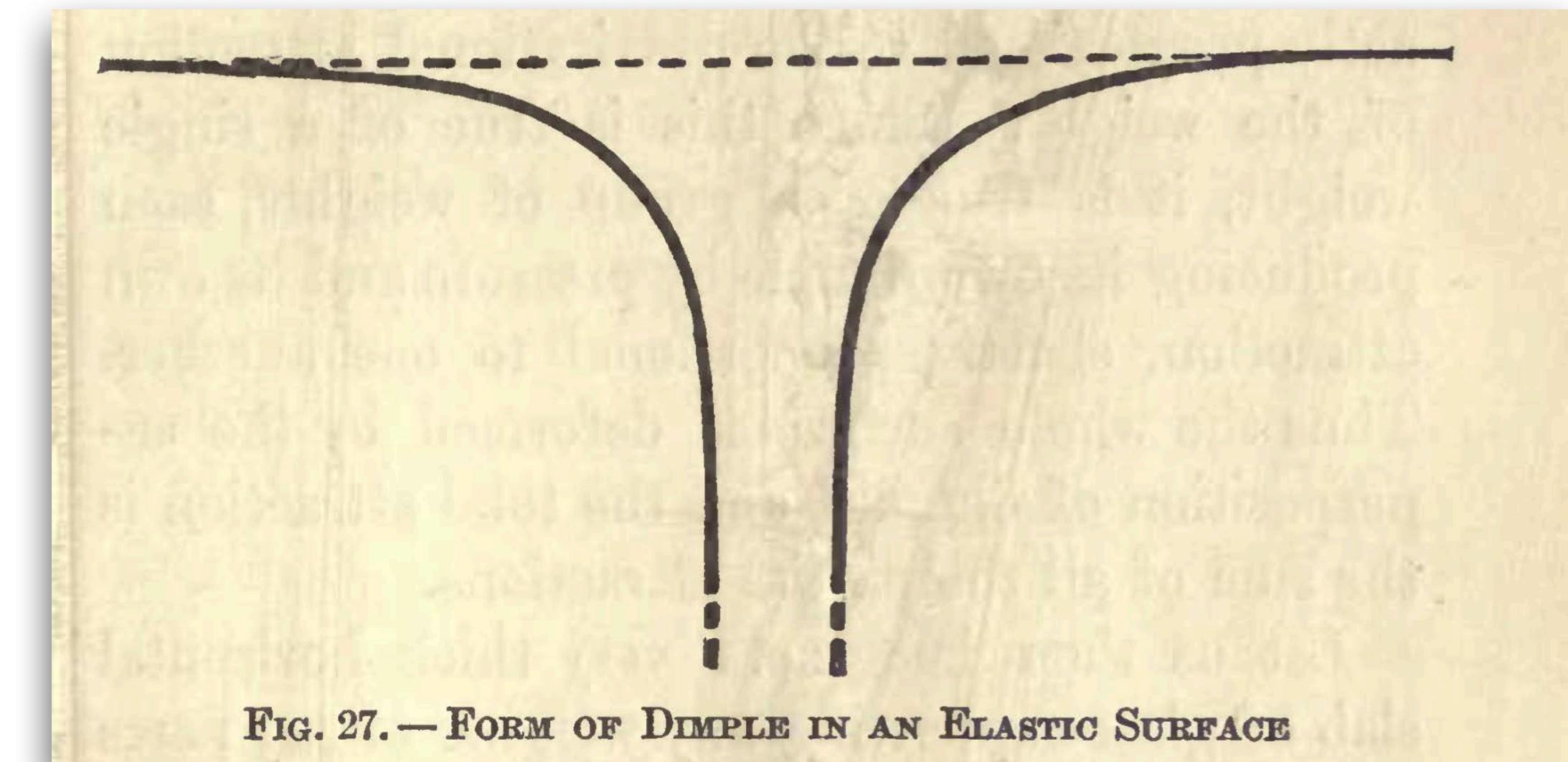
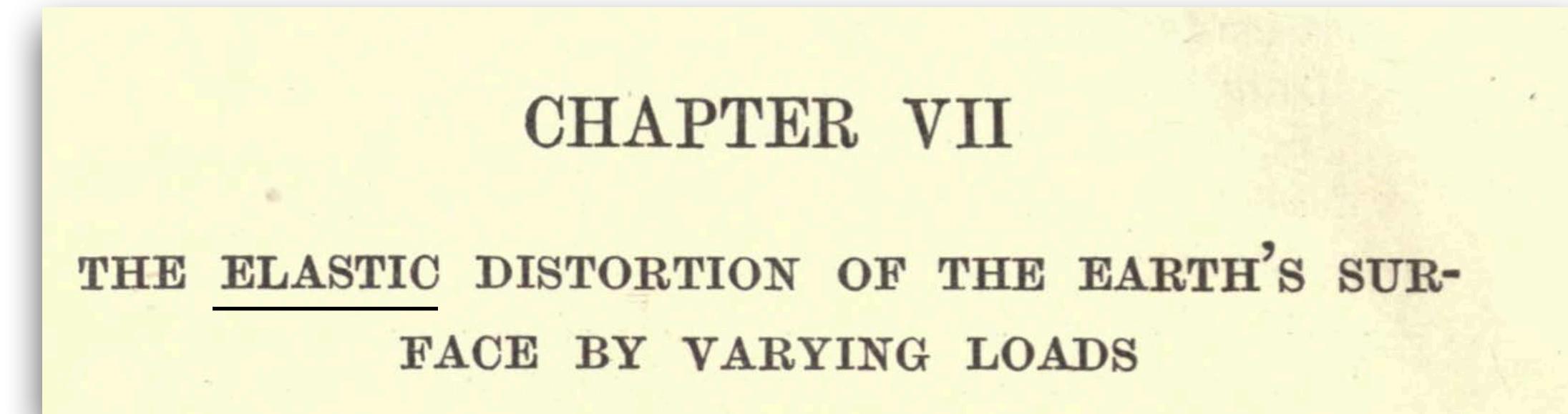
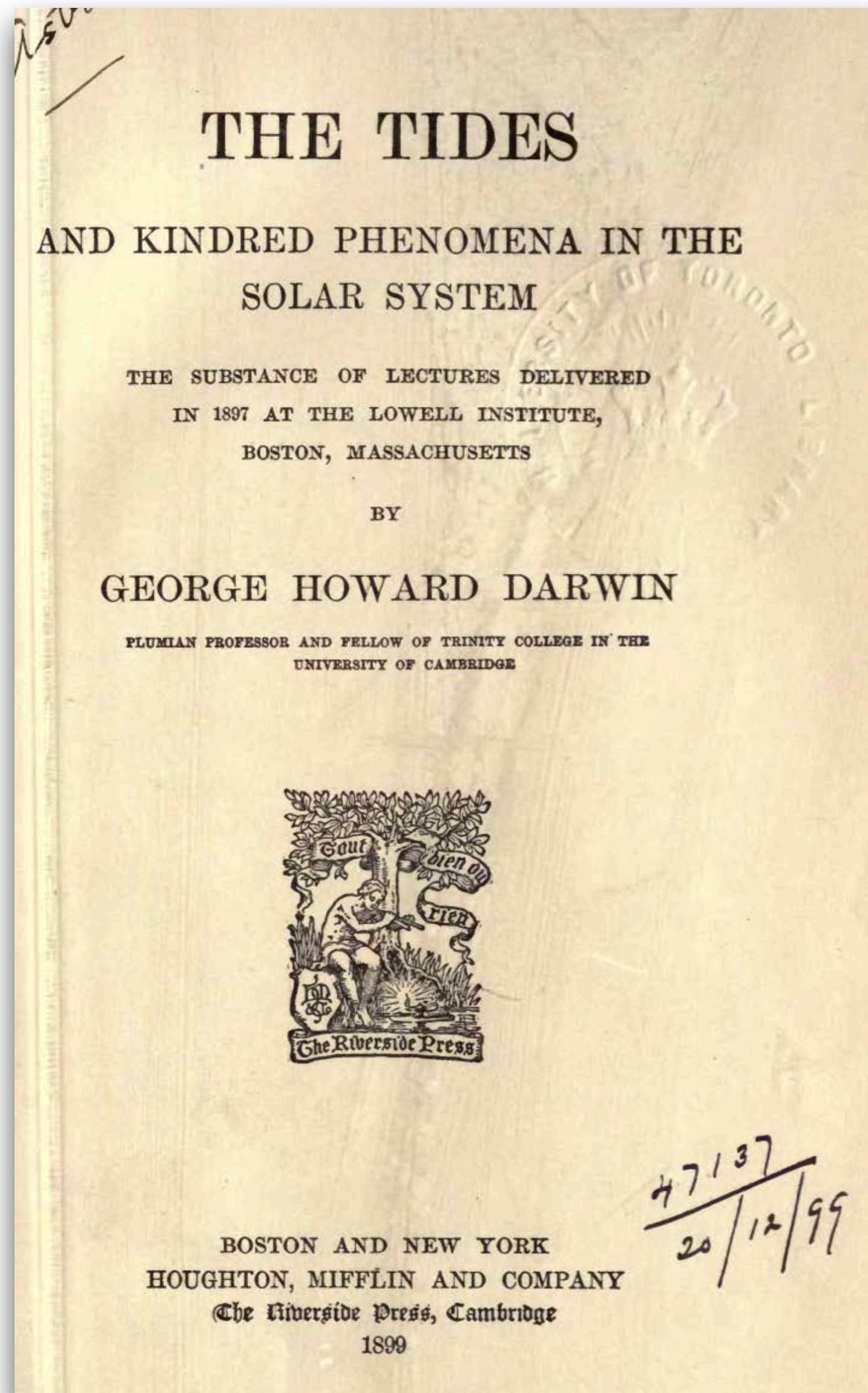
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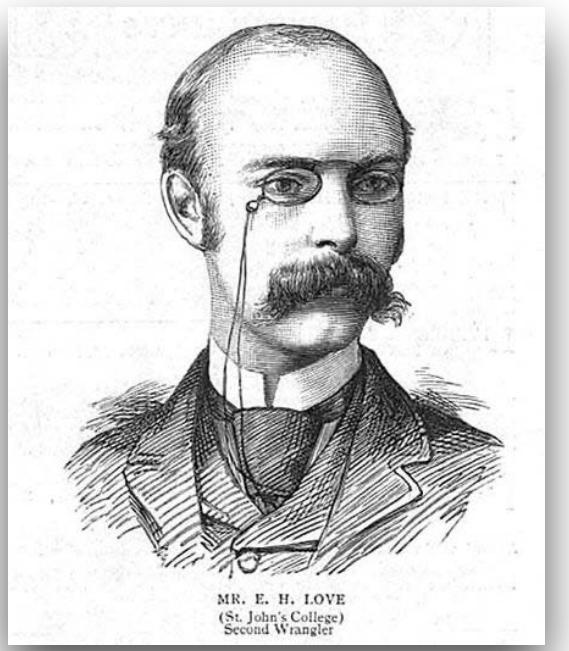
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An old story...



A (not so) old story...

Augustus Edward
Hough Love

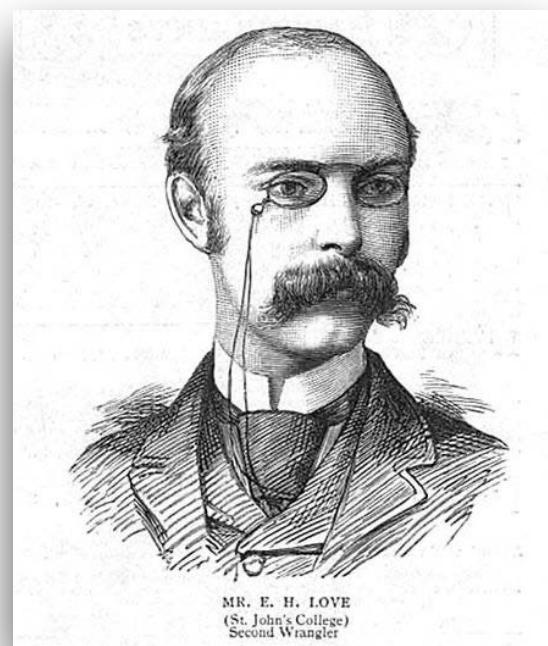


1911

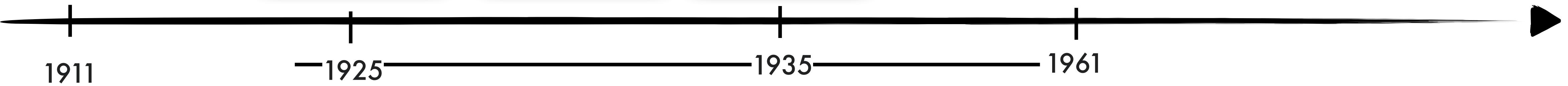
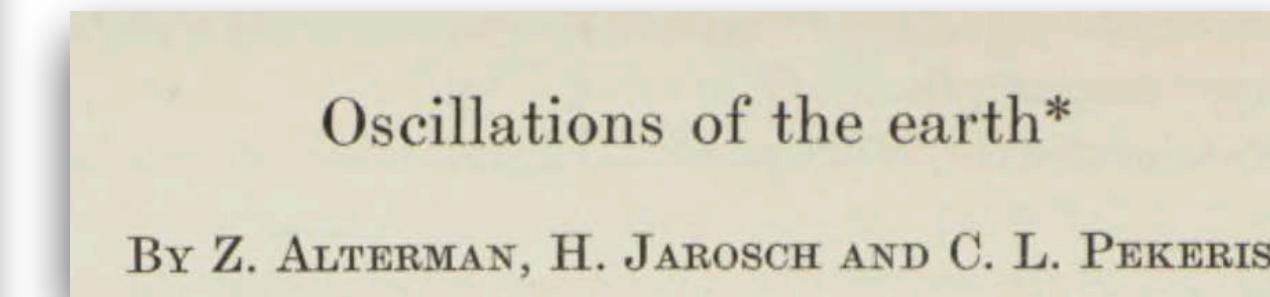
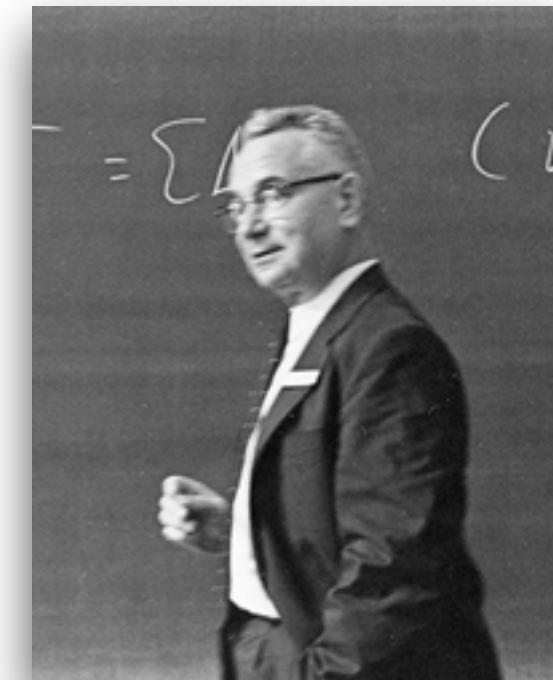
- Deformation of the spherical homogeneous elastic Earth
- Introduction of the Love numbers

A (not so) old story...

Augustus Edward
Hough Love



Chaim Lieb Pekeris Louis B. Slichter Norman Haskell



The era of seismologists

- Deformation of the spherical homogeneous elastic Earth
- Introduction of the Love numbers
- Free oscillations of the Earth
- GIA derived viscosity of the mantle using a half-space model

— 10^{21} Pa.s —

THE VISCOSITY OF THE ASTHENOSPHERE.

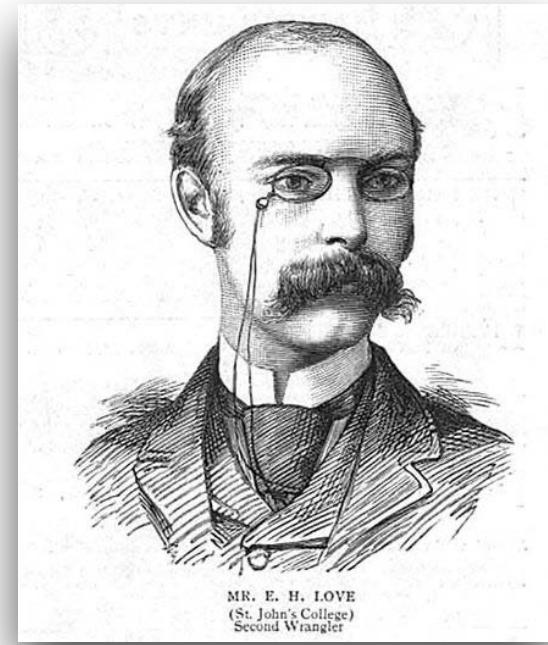
NORMAN A. HASKELL.

ABSTRACT.

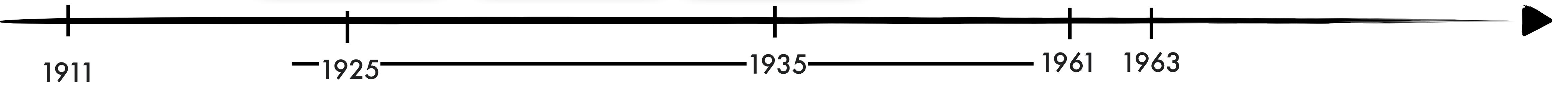
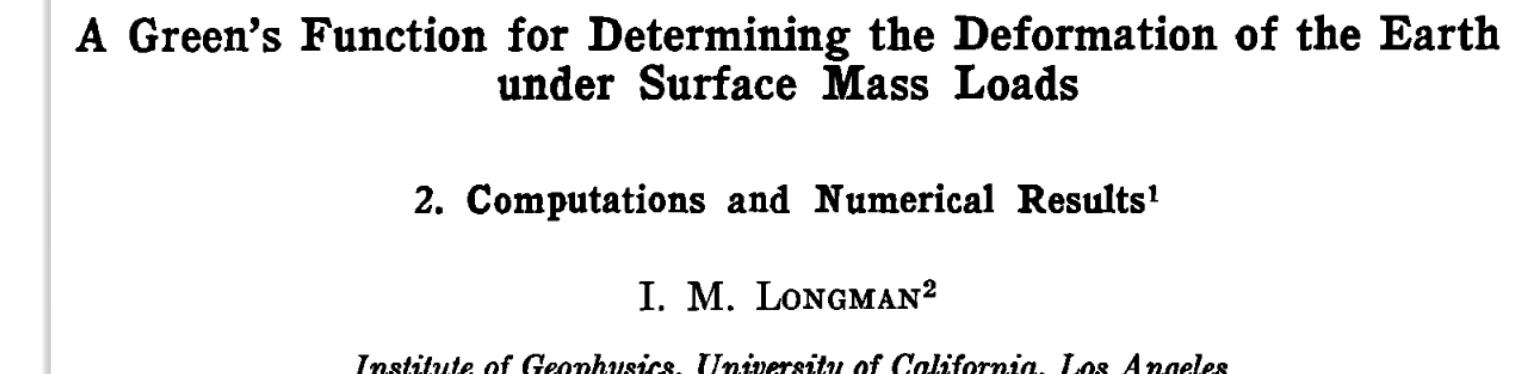
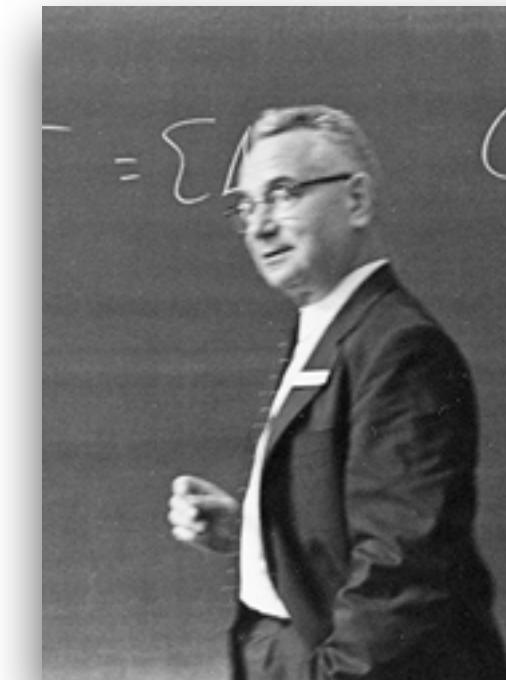
A method of determining the mean viscosity of the asthenosphere from the rate of uplift of the earth's crust after the melting of the last Pleistocene ice sheets is outlined. The kinematic viscosity indicated is 2.9×10^{21} c.g.s. units, implying a relaxation time of the order of 260 years. The distribution of displacements beneath a subsiding load is found to have the general character of those postulated in Daly's "down-punching" hypothesis. The time required for the attainment of isostatic equilibrium varies inversely with the width of the load, and for a load having a span of 2000 km. is of the order of 18,000 years. A formula which may be used in estimating the order of magnitude of the time involved in other types of viscous yielding is set up. When applied to the lateral compression of a mobile zone of the earth's crust, it indicates a time of the order of a few hundred thousand years for the duration of an orogenic episode, agreeing with an estimate of Bucher's.

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The era of seismologists

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- GIA derived viscosity of the mantle using a half-space model
- Deformation of an elastic spherical layered Earth under surface loading

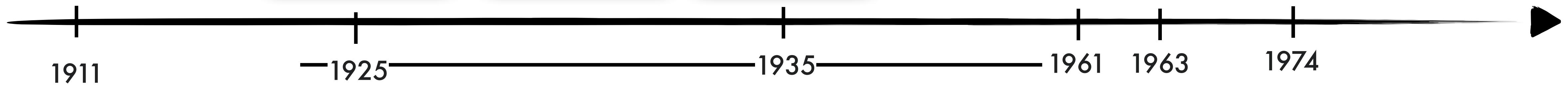
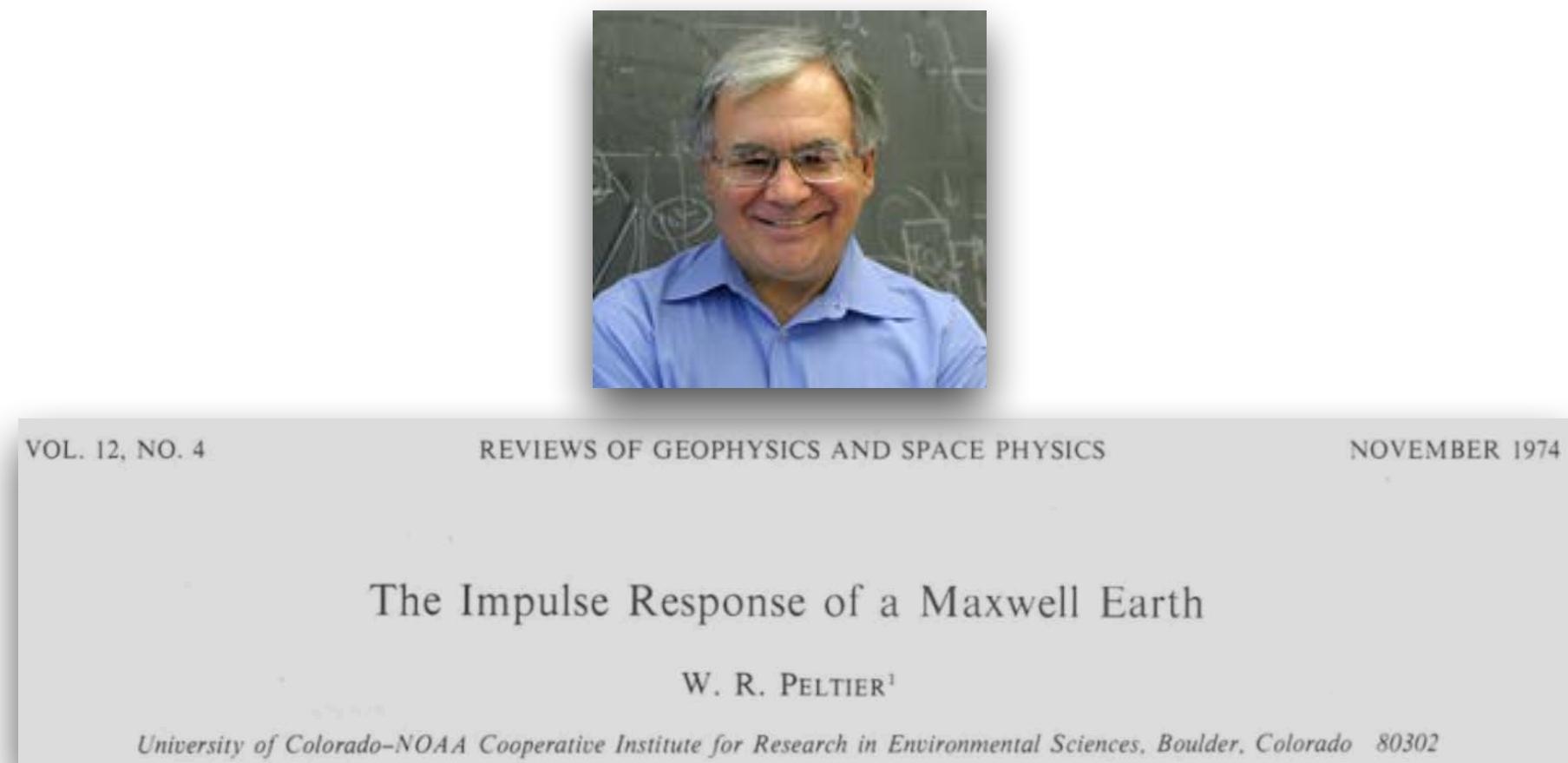
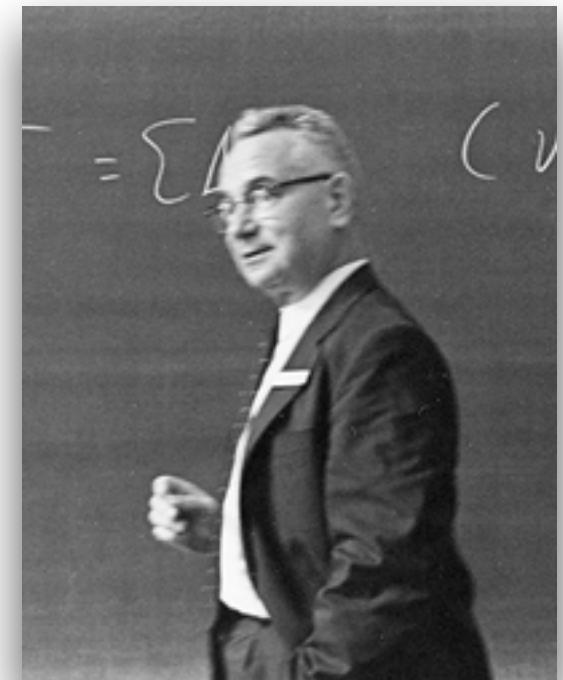
— 10^{21} Pa.s —

A (not so) old story...

Augustus Edward
Hough Love



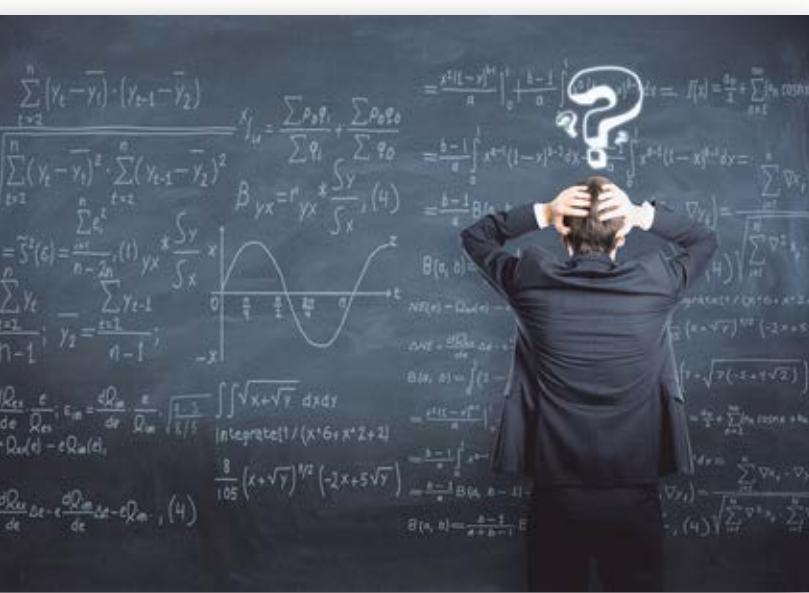
Chaim Lieb Pekeris Louis B. Slichter Norman Haskell



The era of seismologists

- Deformation of the spherical homogeneous elastic Earth
- Introduction of the Love numbers
- Free oscillations of the Earth
- GIA derived viscosity of the mantle using a half-space model
- 10^{21} Pa.s —
- Deformation of an elastic spherical layered Earth under surface loading
- Deformation of a viscoelastic spherical layered Earth under surface loading

OK... now some equations



- ▶ The Love h and k , and Shida l numbers (Love, 1911; Shida and Matsuyama, 1912) characterize the Earth's response to a potential (ex: the external tidal potential)
- ▶ For describing the response to surface load, another type of Love numbers, h' , l' and k' , were introduced: the loading Love numbers (Longman, 1963; Farrell, 1972)
- ▶ Load Love numbers are computed by integrating the equations of motion, the **stress-strain relationship** and Poisson equation inside the Earth, from the center to the surface using a spherical Earth model

OK... now some equations

► Linearization of motion and Poisson equation

$$\nabla \cdot \tau - \nabla(\rho g s \cdot \mathbf{e}_r) - \rho \nabla \phi + g \nabla \cdot (\rho s \mathbf{e}_r) = 0, \quad \begin{aligned} \tau &\text{ is the incremental stress tensor} \\ \rho \text{ and } g &\text{ are the unperturbed density and gravitational acceleration} \\ \mathbf{s} &\text{ is the displacement vector} \\ \phi &\text{ is the perturbed gravitational potential} \\ \mathbf{e}_r &\text{ denotes the unit vector of the vertical component.} \end{aligned}$$
$$\nabla^2 \phi = -4\pi G \nabla \cdot (\rho s)$$

► Supposing a spherically symmetric **elastic** Earth model, solutions of linearised equations become:

$$s = \sum_{n=0}^{\infty} \left(U_n(r) P_n(\cos \psi) \mathbf{e}_r + V_n(r) \frac{\partial P_n(\cos \psi)}{\partial \psi} \mathbf{e}_v \right), \quad \begin{aligned} \text{where } U_n, V_n \text{ and } \Phi_n &\text{ are transformed variables indicating vertical displacements,} \\ &\text{tangential displacements and potential, respectively. } \mathbf{e}_v &\text{ denotes the unit vector of} \\ &\text{the horizontal component.} \end{aligned}$$
$$\phi = \sum_{n=0}^{\infty} \Phi_n(r) P_n(\cos \psi),$$

OK... now some equations

► Linearization of motion and Poisson equation

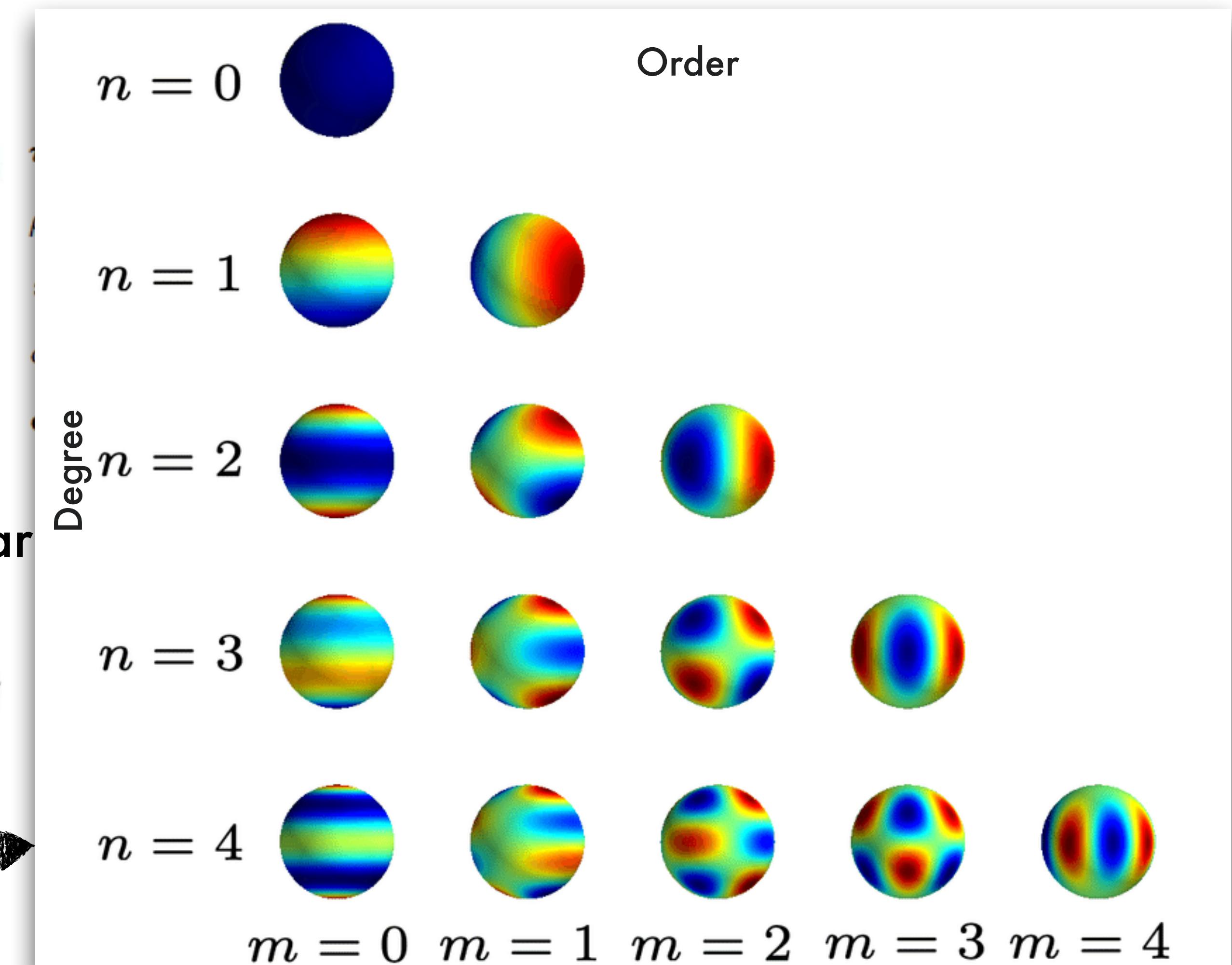
$$\nabla \cdot \tau - \nabla(\rho g \mathbf{s} \cdot \mathbf{e}_r) - \rho \nabla \phi + g \nabla \cdot (\rho s \mathbf{e}_r) = 0,$$

$$\nabla^2 \phi = -4\pi G \nabla \cdot (\rho s)$$

► Supposing a spherically symmetric **elastic** Earth

$$\mathbf{s} = \sum_{n=0}^{\infty} \left(U_n(r) P_n(\cos \psi) \mathbf{e}_r + V_n(r) \frac{\partial P_n(\cos \psi)}{\partial \psi} \mathbf{e}_v \right),$$

$$\phi = \sum_{n=0}^{\infty} \Phi_n(r) P_n(\cos \psi),$$



OK... now some equations

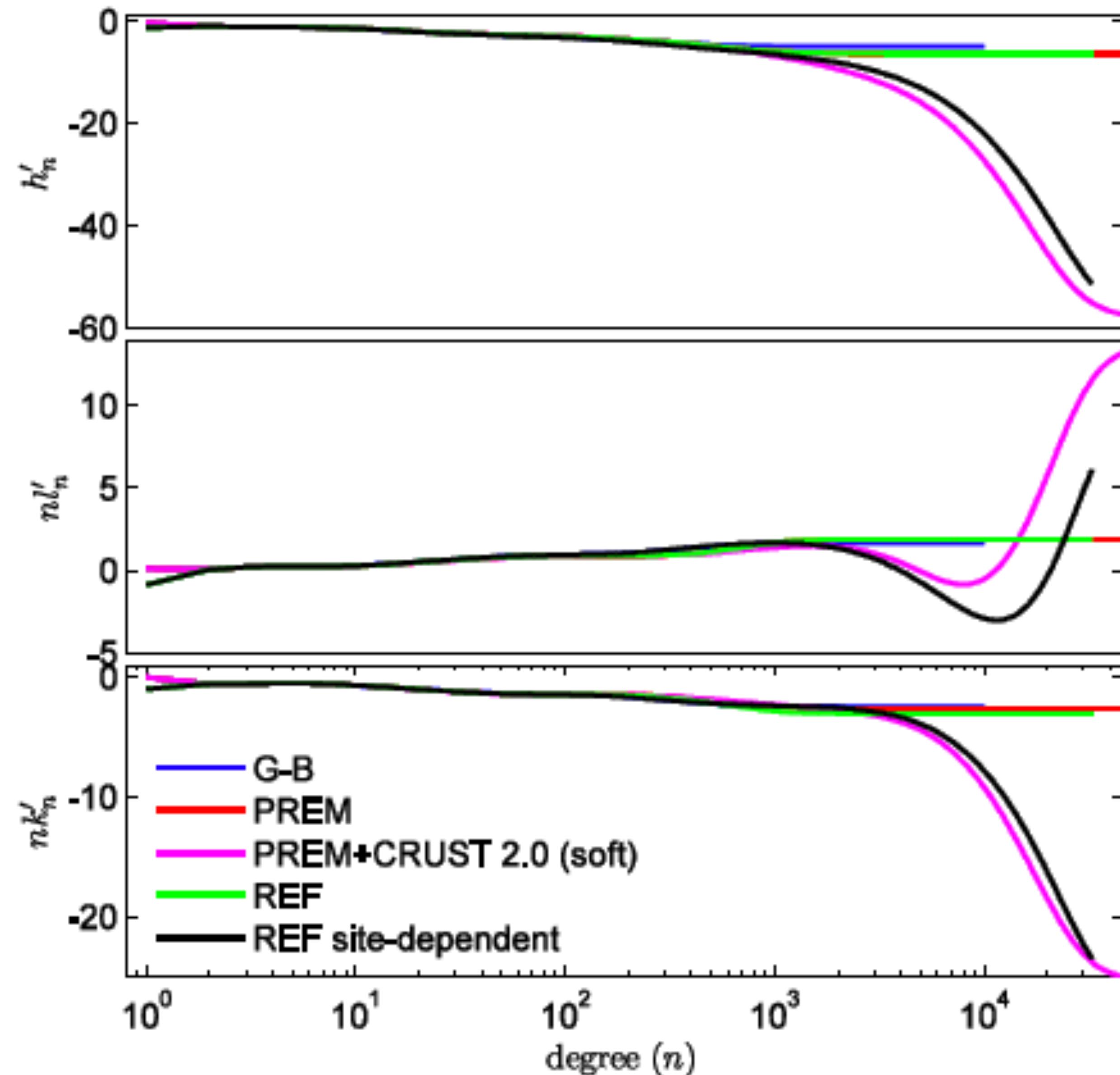
- With a bit of work and some simplifications,

$$\begin{bmatrix} U_n(r) \\ V_n(r) \\ \Phi_n(r) \end{bmatrix} = W(r) \begin{bmatrix} h'_n(r)/g \\ l'_n(r)/g \\ k'_n(r) \end{bmatrix}$$

where $W(r)$ stands for the potential induced by the point mass

- Solving and integrating this equation from the inner Earth to the surface gives the set of loading Love numbers which are based on the given Earth model.

- 1)Gutenberg-Bullen (Farrell, 1972);
- 2)PREM (Dziewonski and Anderson, 1981);
- 3)a modified PREM with crustal structures adapting from CRUST 2.0 model (Wang et al., 2012)
- 4)REF (Kustowski et al., 2008)
- 5)REF model with a site-dependent setting (Gegout, 2013).



Predict elastic displacement from surface load

$$\bar{U}(\bar{r}, S, Z, \rho) = \int_{\Omega'} G(|\bar{r}' - r|, S) \rho(\bar{r}') Z(\bar{r}') d\Omega'$$

Predict elastic displacement from surface load

$$\bar{U}(\bar{r}, \underline{S}, Z, \rho) = \int_{\Omega'} G(|\bar{r}' - r|, \underline{S}) \rho(\bar{r}') Z(\bar{r}') d\Omega'$$

Load Green's function

Load height at load point r'

Load density at load point r'

Integration over Earth's surface

Spherically symmetric structure

Response at observation point r

Predict elastic displacement from surface load

Earth's rheology

$$\bar{U}(\bar{r}, \underline{S}, Z, \rho) = \int_{\Omega'} G(|\bar{r}' - r|, \underline{S}) \rho(\bar{r}') Z(\bar{r}') d\Omega'$$

Load Green's function

Load height at load point \bar{r}'

Load density at load point \bar{r}'

Integration over Earth's surface

Response at observation point r

Spherically symmetric structure

Predict viscoelastic displacement from surface load?

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REVIEWS OF GEOPHYSICS AND SPACE PHYSICS

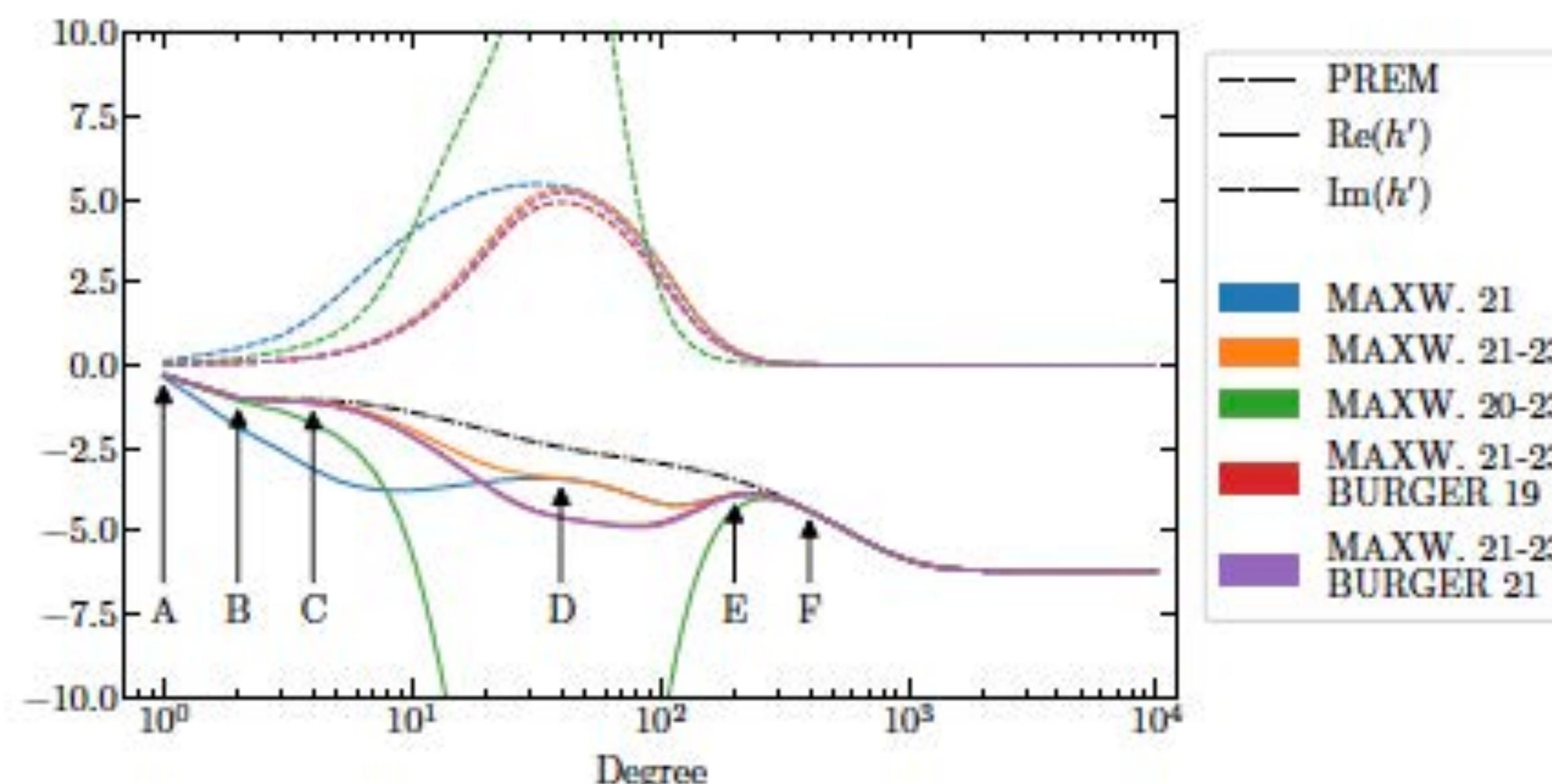
NOVEMBER 1974

The Impulse Response of a Maxwell Earth

W. R. PELTIER¹

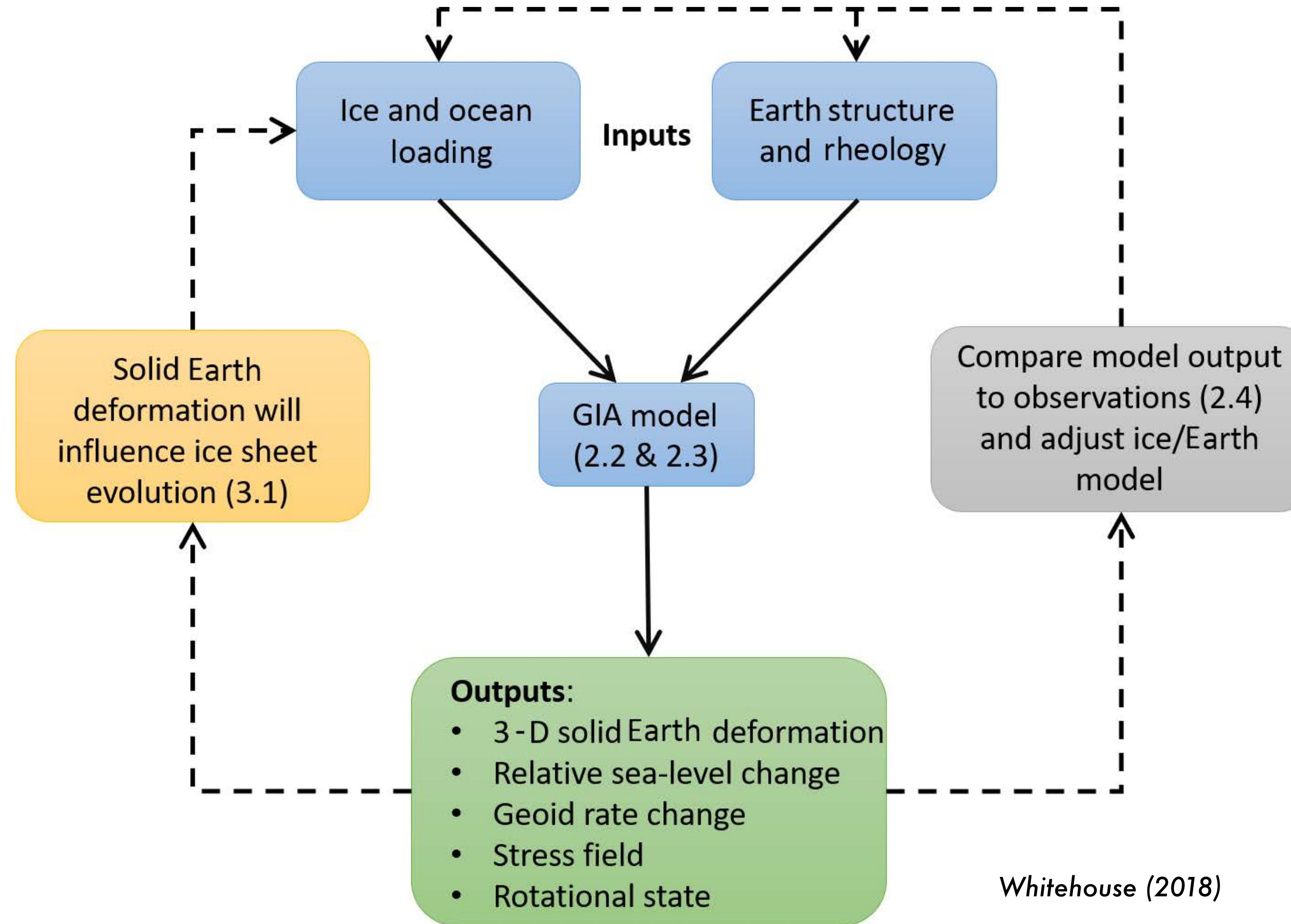
University of Colorado-NOAA Cooperative Institute for Research in Environmental Sciences, Boulder, Colorado 80302

Correspondance principle in the Laplace (or Fourier) domain



GIA theory ready to go !

An iterative scheme



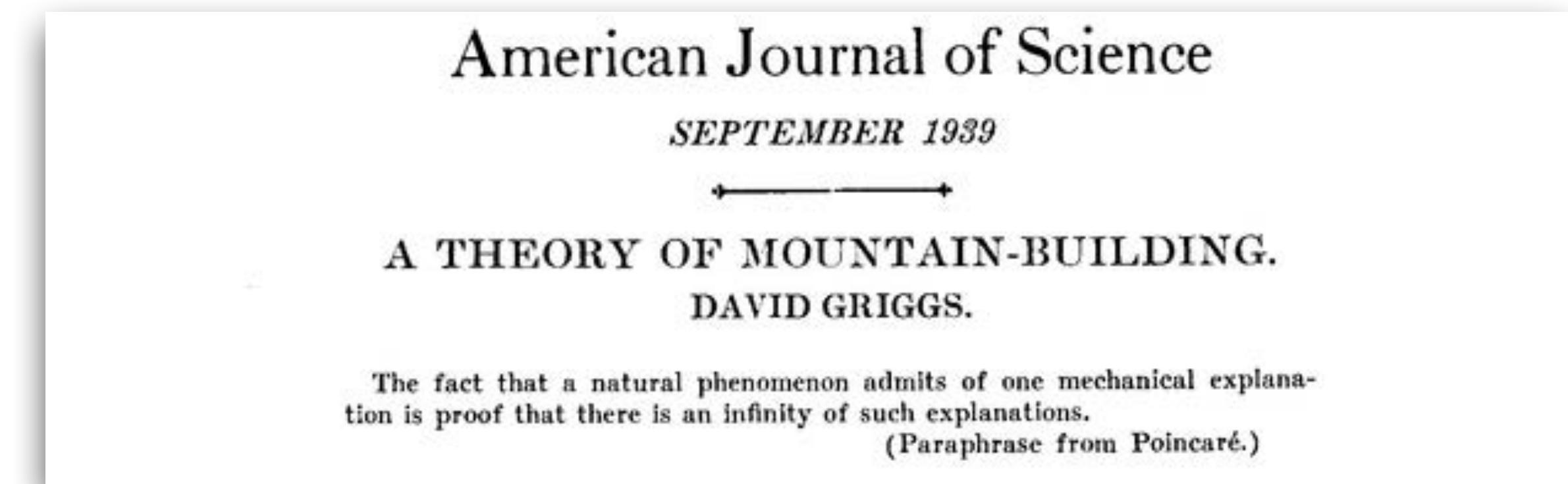
But what to use for Earth rheology?



David T. Griggs

(1911-1974)

The father of rock deformation studies



& Creep of rocks, *Journal of Geology*, 1939

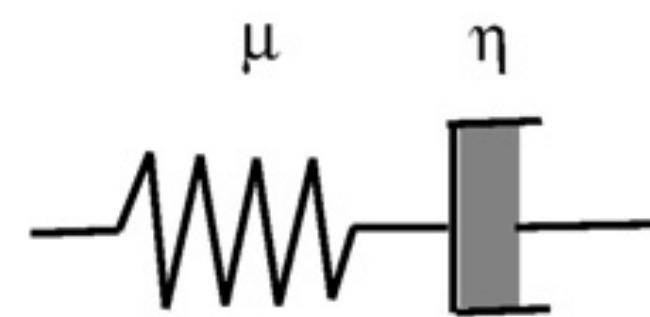
— based on Haskell mantle viscosity study, and on experimental work,
he advocates that we need a better theory for rheology —

In the 1960s, it is understood that elastic/density properties of the Earth's mantle can be well explained by using olivine laboratory derived rheologies

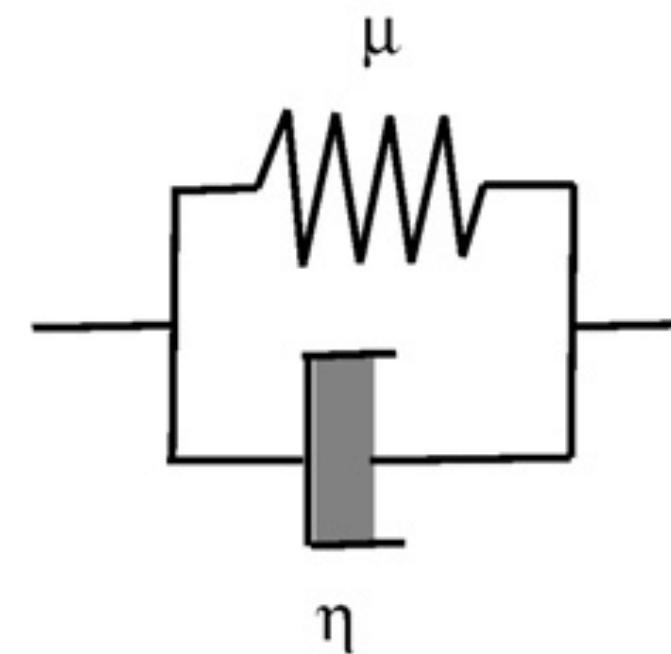


Linking the short and long term behaviours

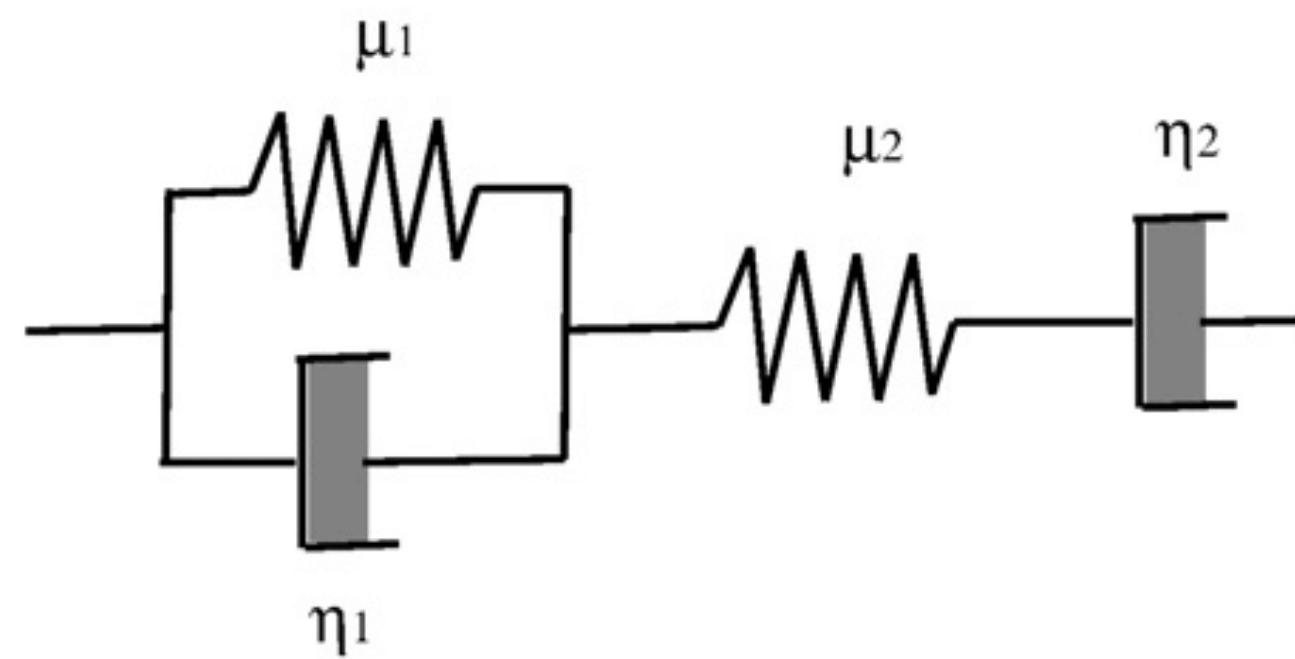
(a) Maxwell model



(b) Kelvin-Voigt model

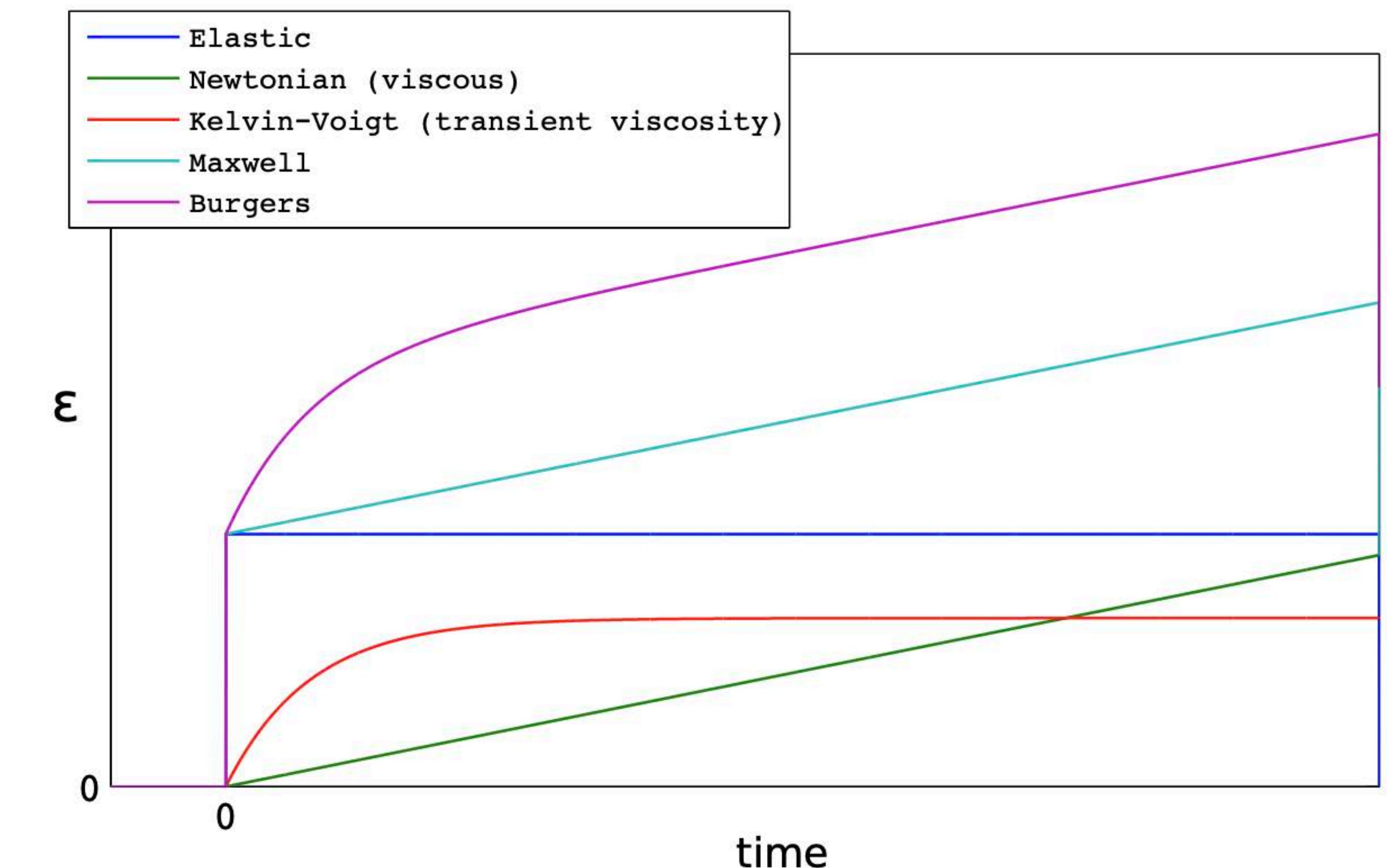


(c) Burgers model



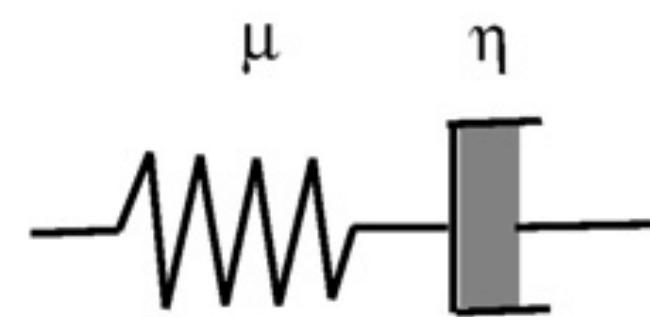
CREEP FUNCTIONS

*Elastic (seismic constraints) to viscoelastic
(postseismic) to VISCOelastic (GIA)*

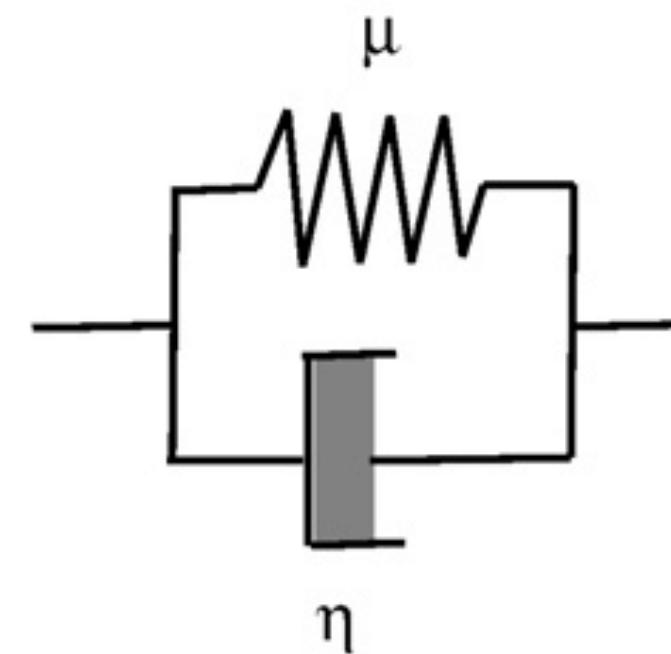


Linking the short and long term behaviours

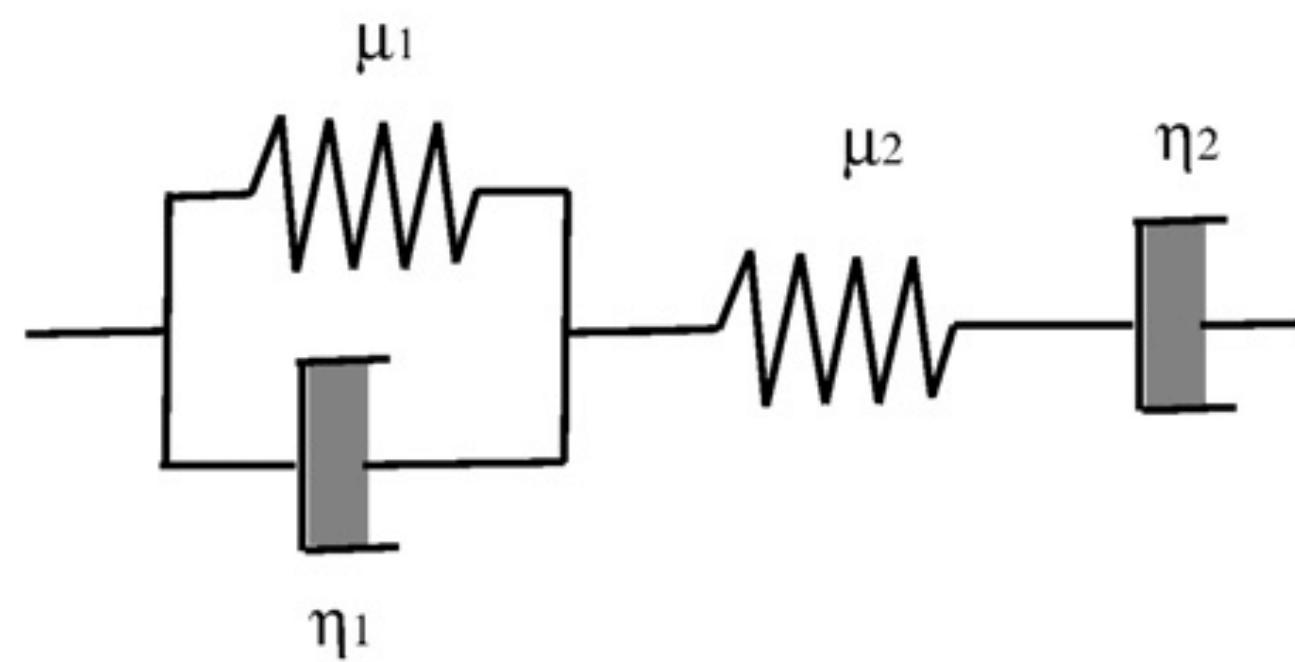
(a) Maxwell model



(b) Kelvin-Voigt model

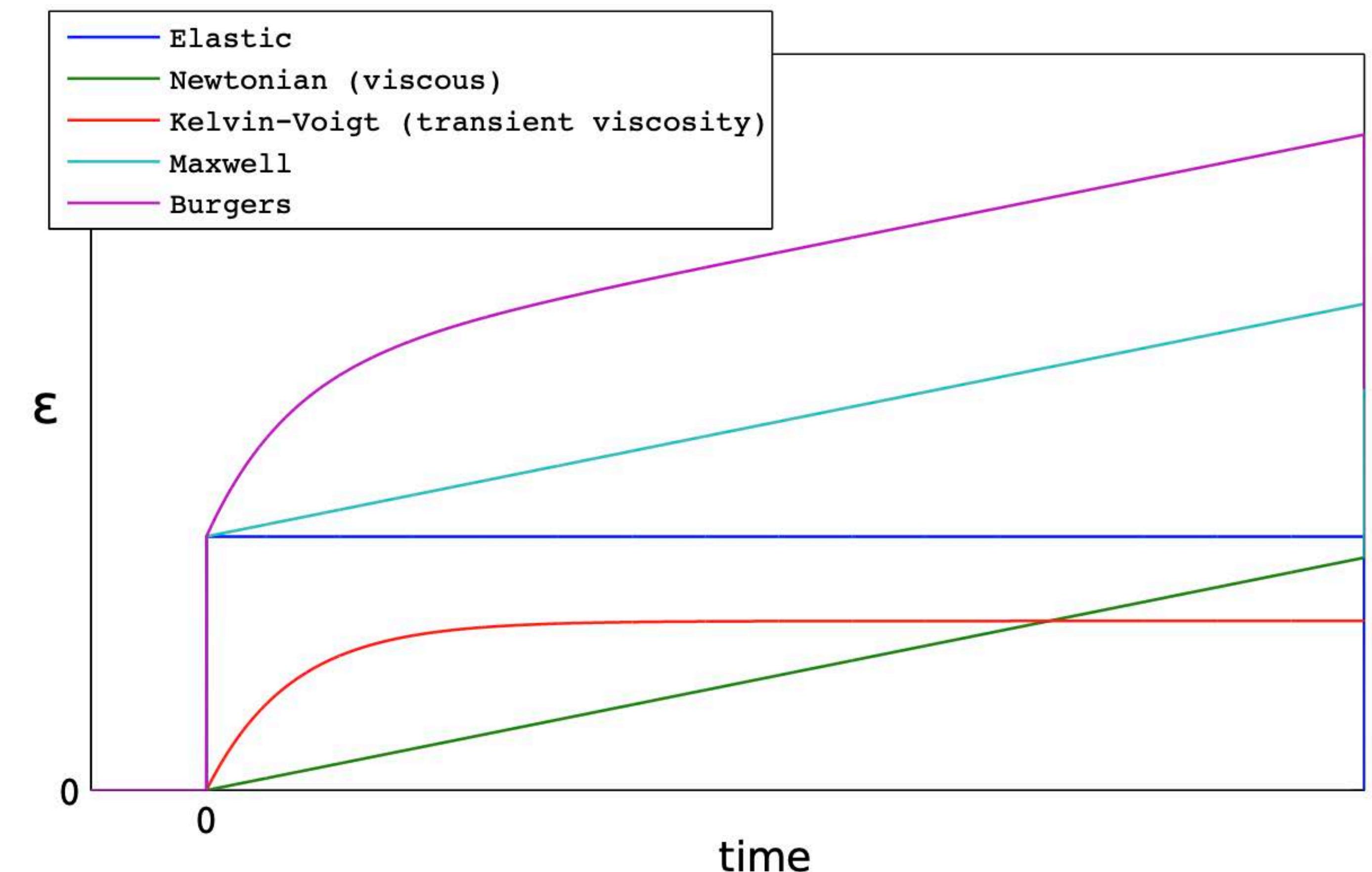


(c) Burgers model



CREEP FUNCTIONS

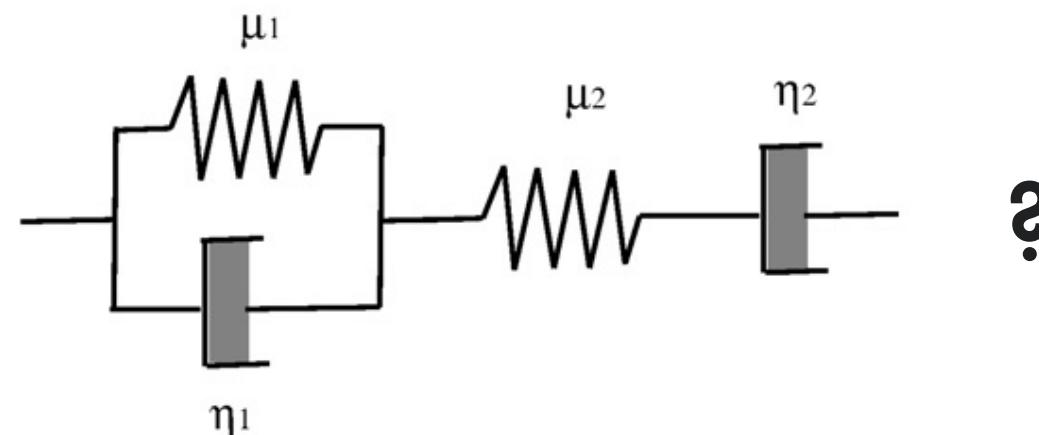
*Elastic (seismic constraints) to viscoelastic
(postseismic) to VISCOelastic (GIA)*



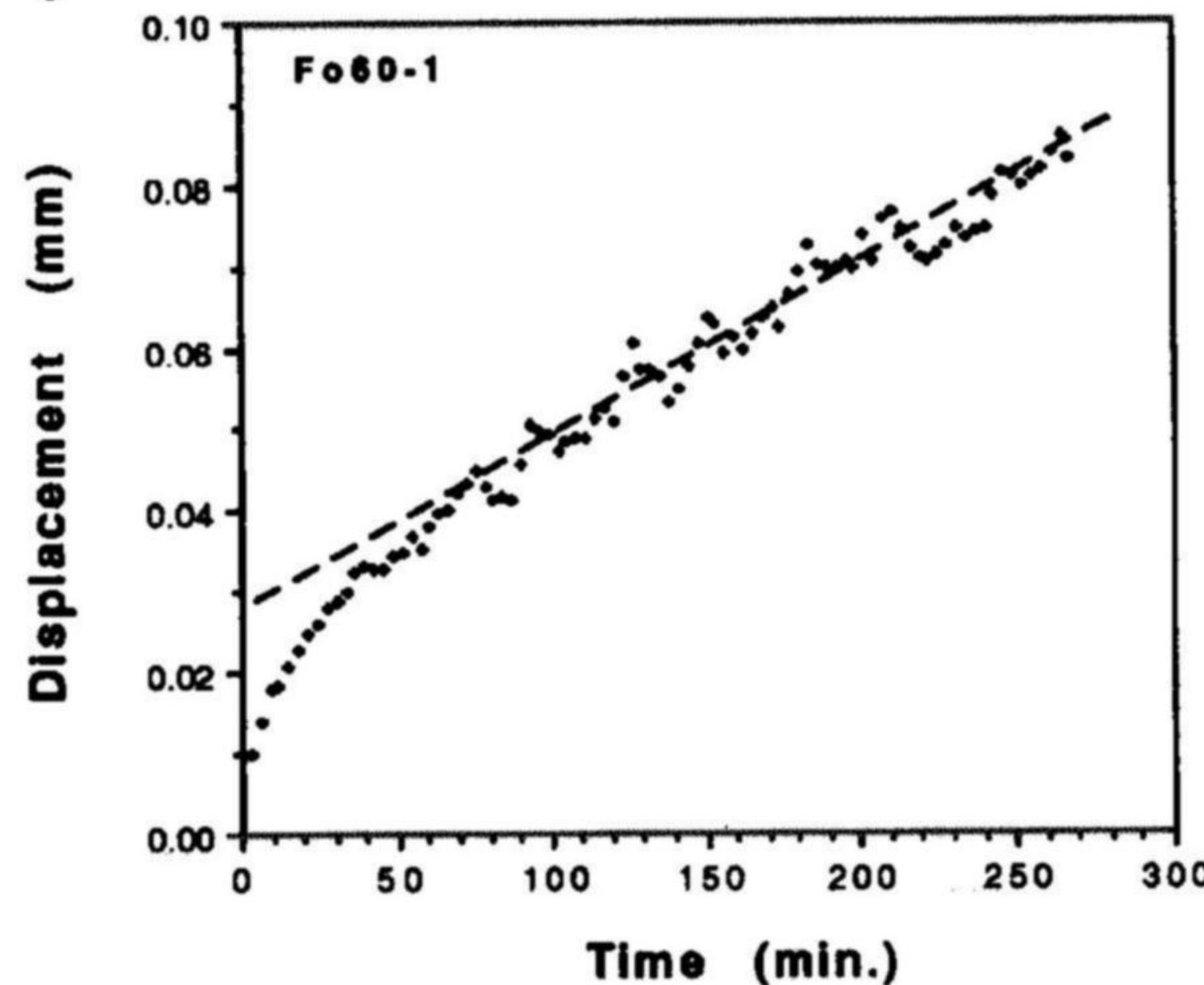
A Burgers rheology to fit observations... but why?

CREEP EXPERIMENT

on a *mixture* of olivine and pyroxenes



(a)

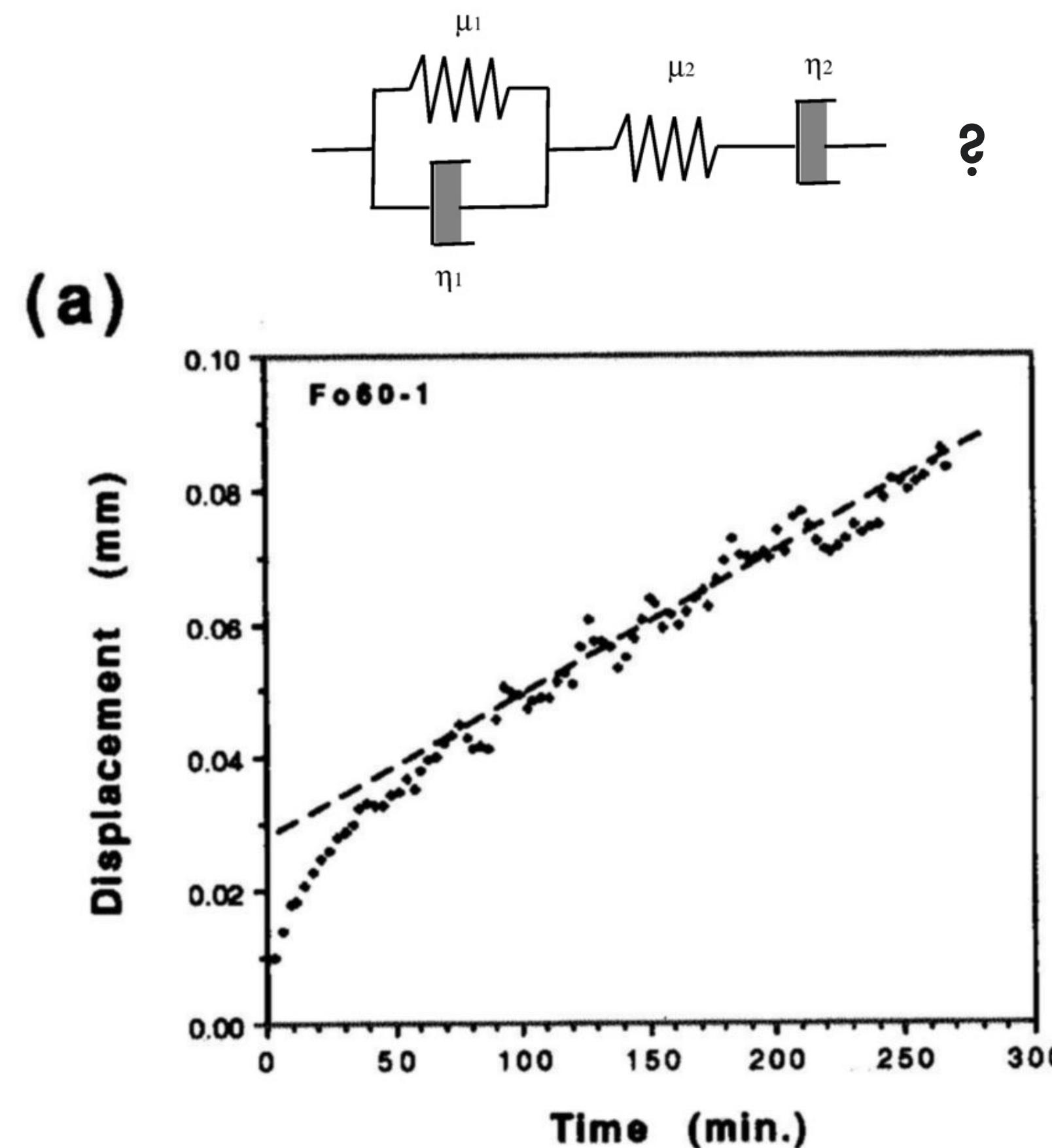


!

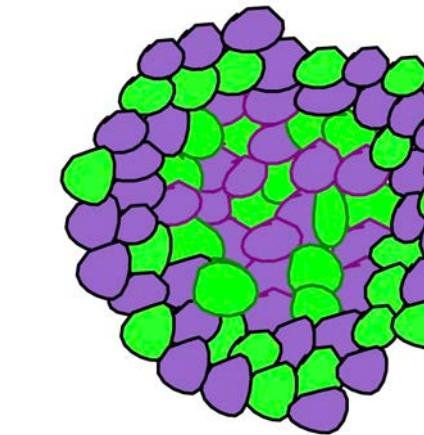
A Burgers rheology to fit observations... but why?

CREEP EXPERIMENT

on a *mixture of olivine and pyroxenes*



Self-consistent homogenization model
(elastic or viscoelastic parameters)



$$1 = \frac{f_1}{1 + \frac{3(K_1 - K)}{3K + 4\mu}} + \frac{f_2}{1 + \frac{3(K_2 - K)}{3K + 4\mu}}$$

A mixture of two minerals
with different viscosities

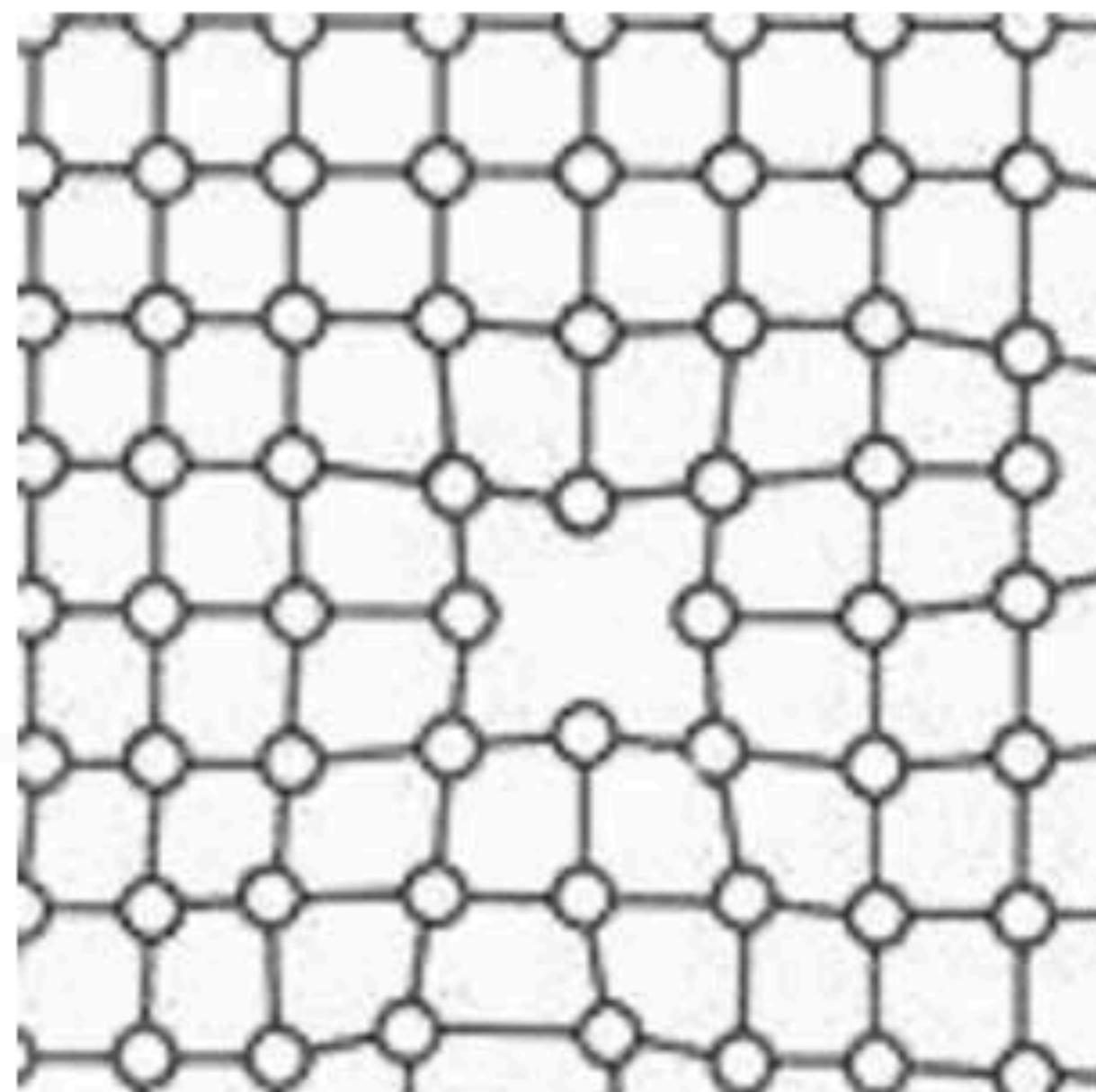
$$1 = \frac{f_1}{1 + \frac{6(K+2\mu)}{5(3K+4\mu)} \frac{(\mu_1 - \mu)}{\mu}} + \frac{f_2}{1 + \frac{6(K+2\mu)}{5(3K+4\mu)} \frac{(\mu_2 - \mu)}{\mu}}$$

The mixture of two Maxwell materials behaves like a Burgers model !

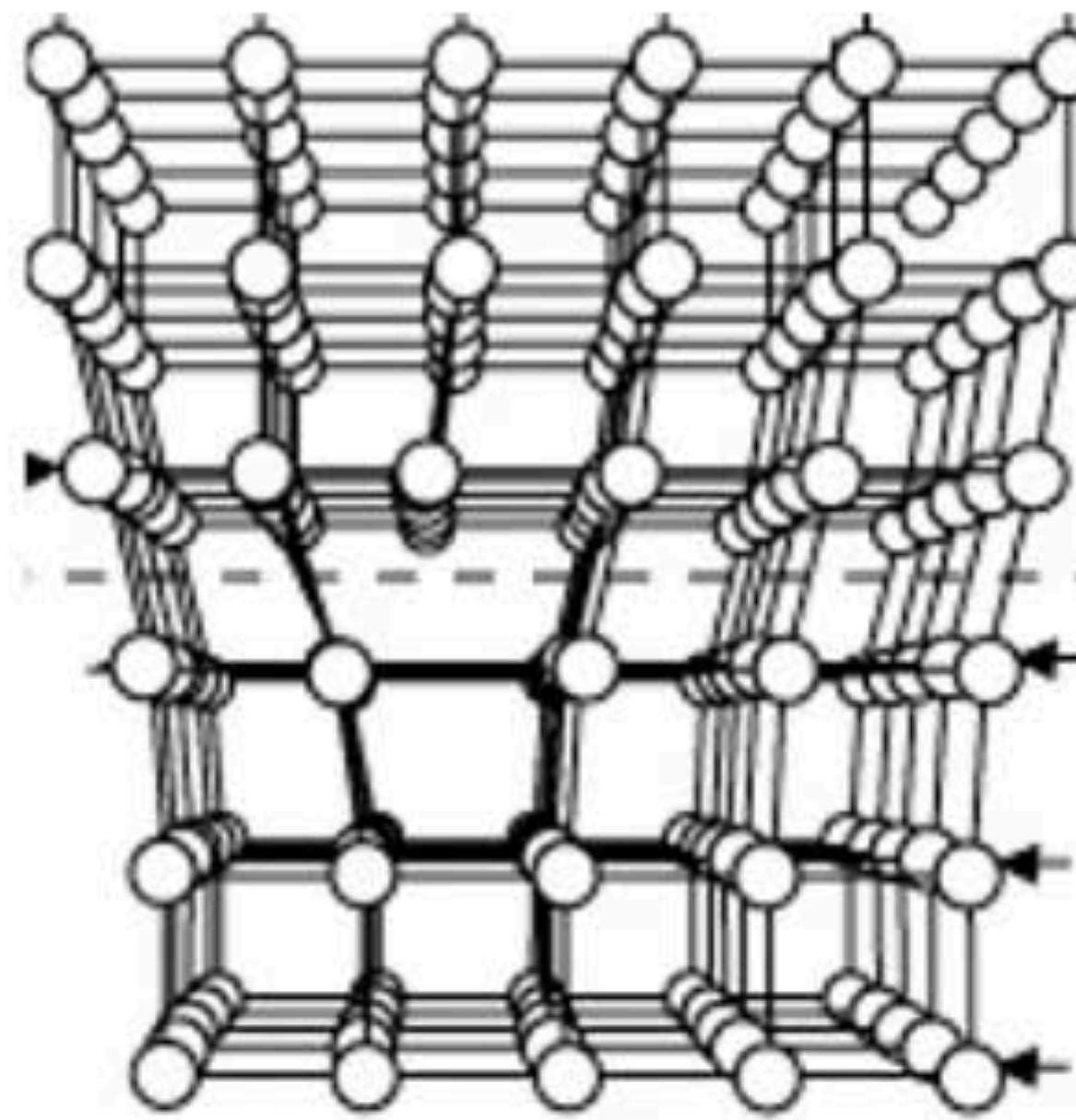
What is a rock and how does it deform?

What is a rock and how does it deform?

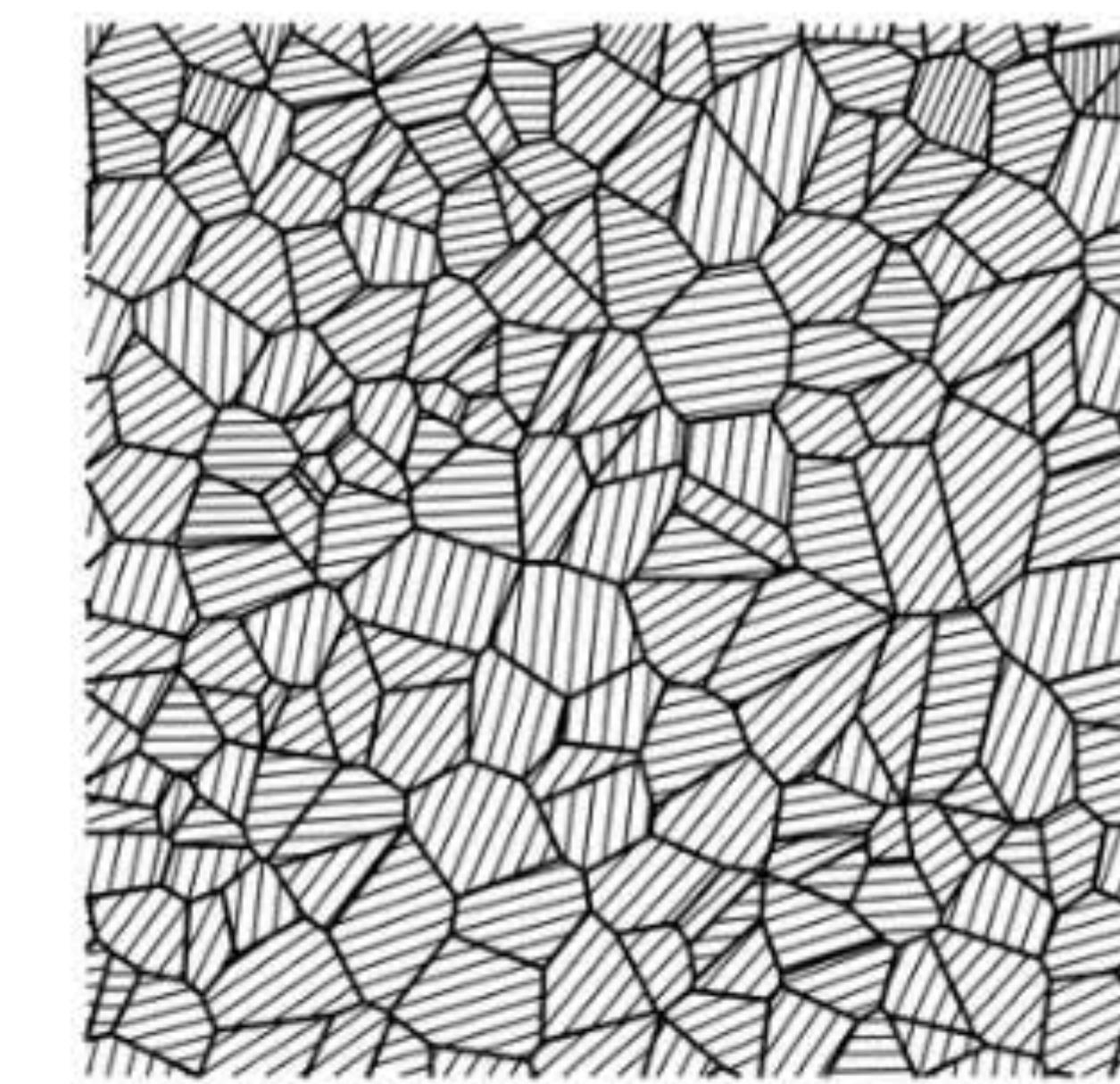
Vacancies



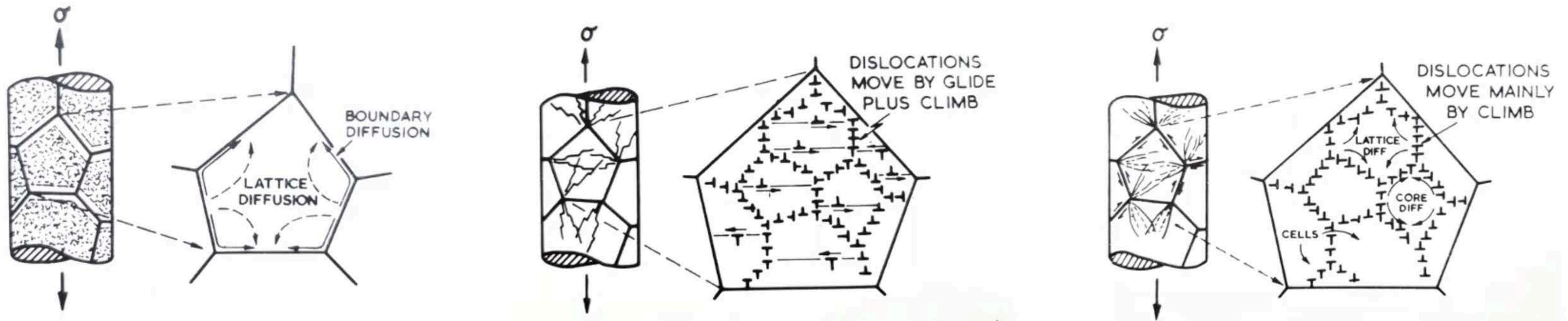
Dislocations



Grain Boundaries



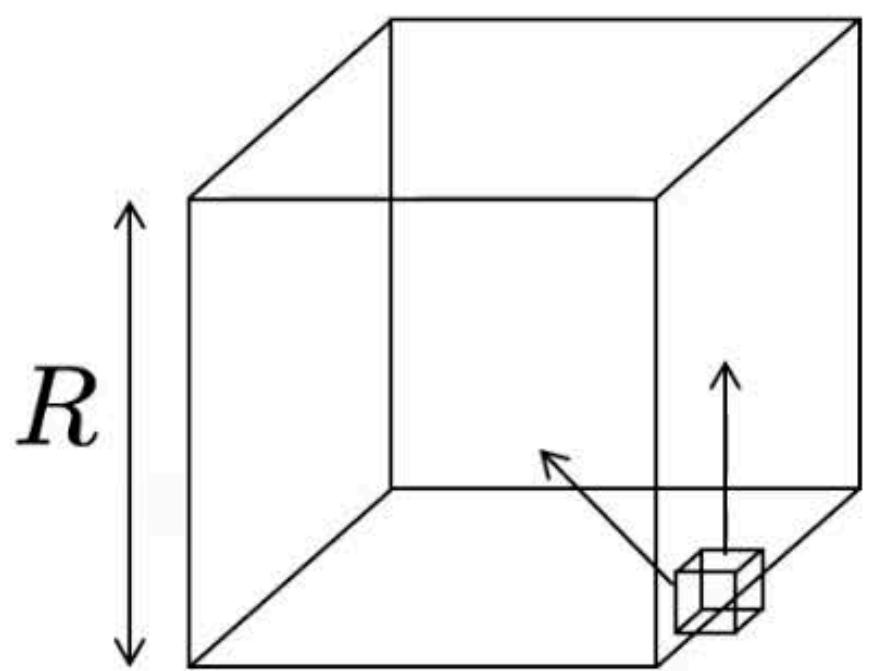
Micromechanisms of rock creep in the mantle



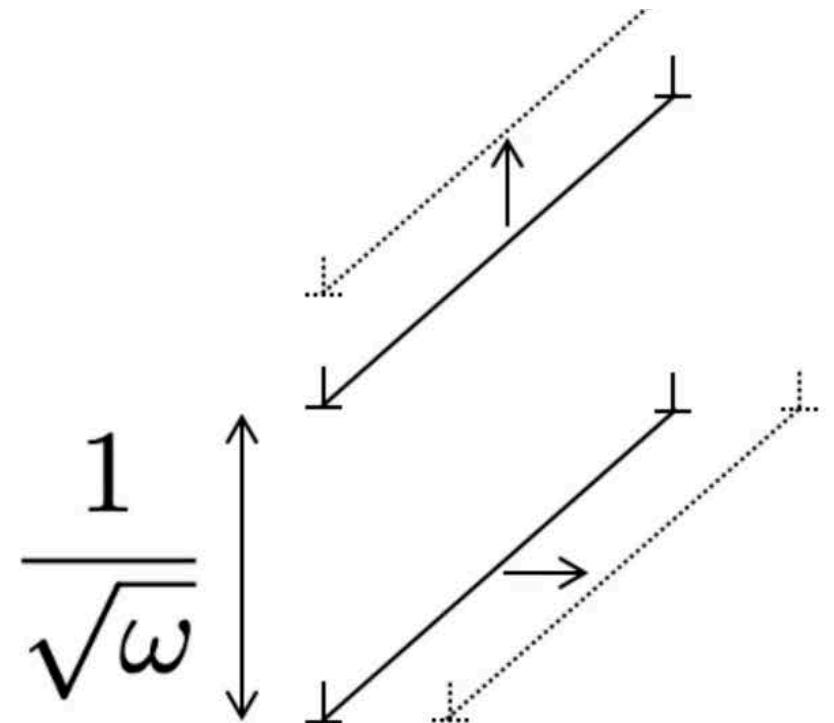
(Frost and Ashby, 1982)

Micophysical Theory Prediction

Diffusion creep: variation in vacancy concentrations due to stress drives vacancy fluxes between grain boundaries



Dislocation creep: stress generates and moves dislocations by climb and glide

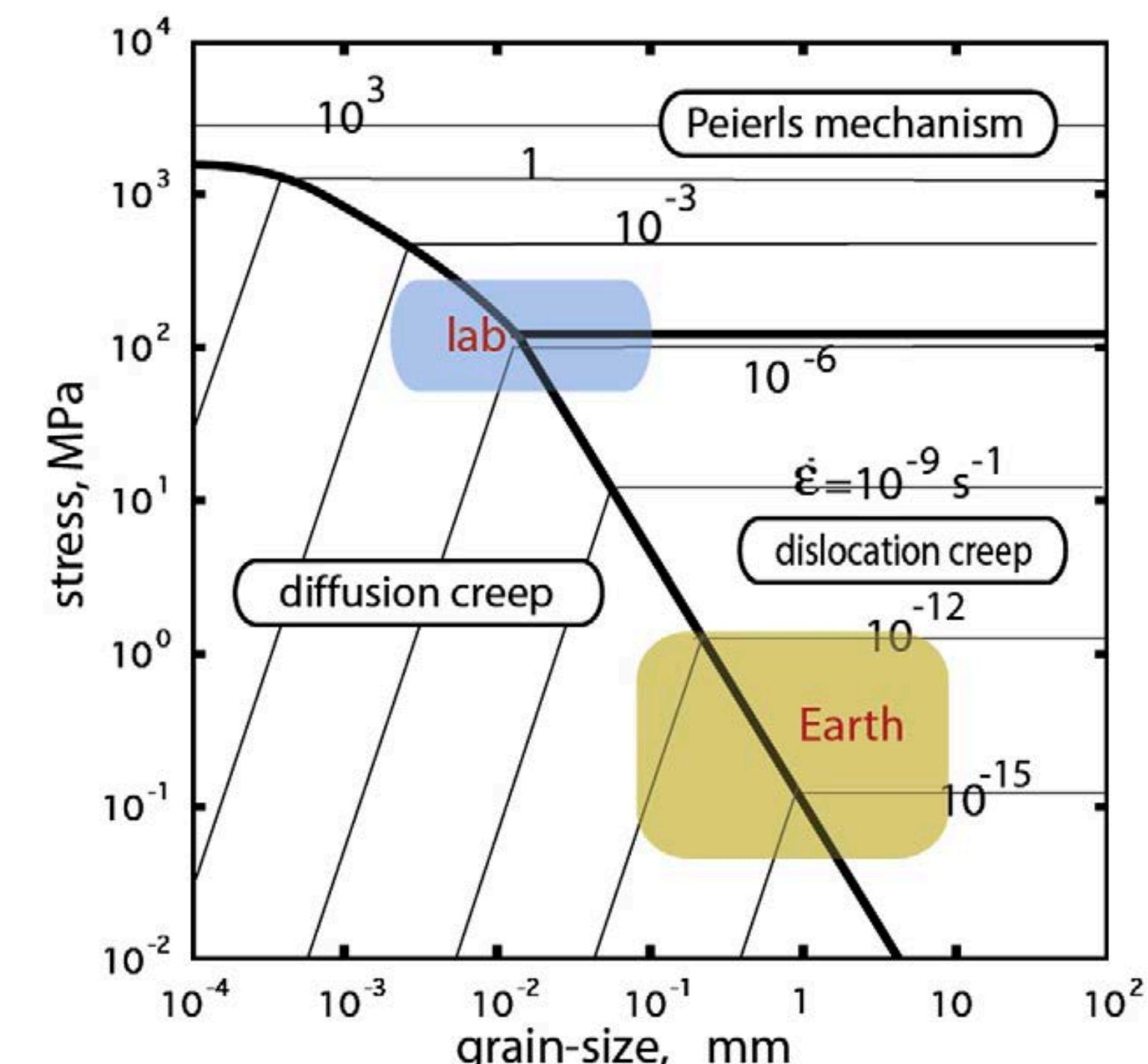


$$\dot{\epsilon}_{diff} \sim \frac{\sigma}{R^2}, \frac{\sigma}{R^3}$$

$$\dot{\epsilon}_{diff} = \frac{B}{R^m} \sigma$$

$$\dot{\epsilon}_{disl} \sim \omega v \sim \sigma^n$$

$$\dot{\epsilon}_{disl} = A \sigma^n$$

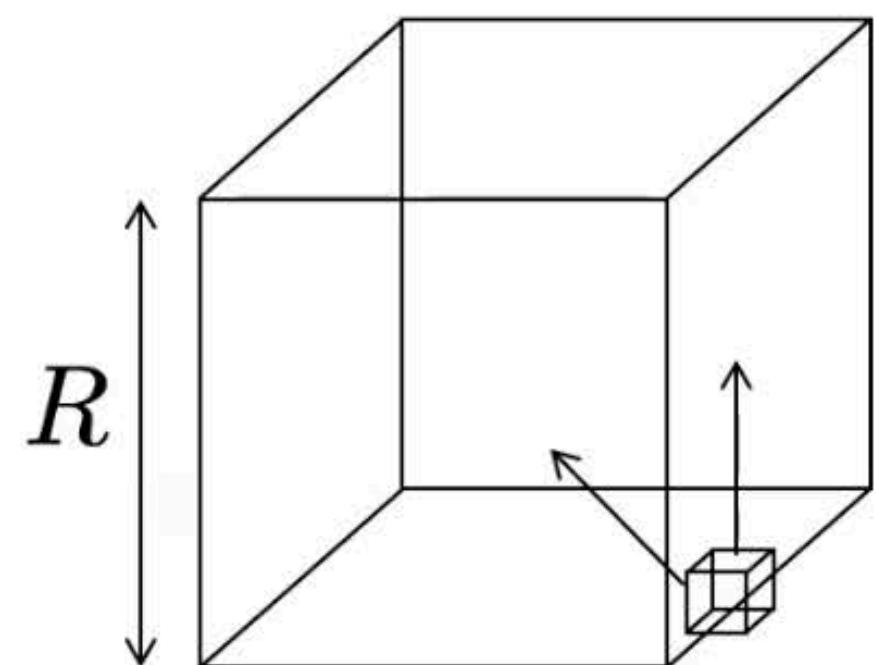


Karato (2010)

Micophysical Theory Prediction

At low GIA strain rate, newtonian rheology?

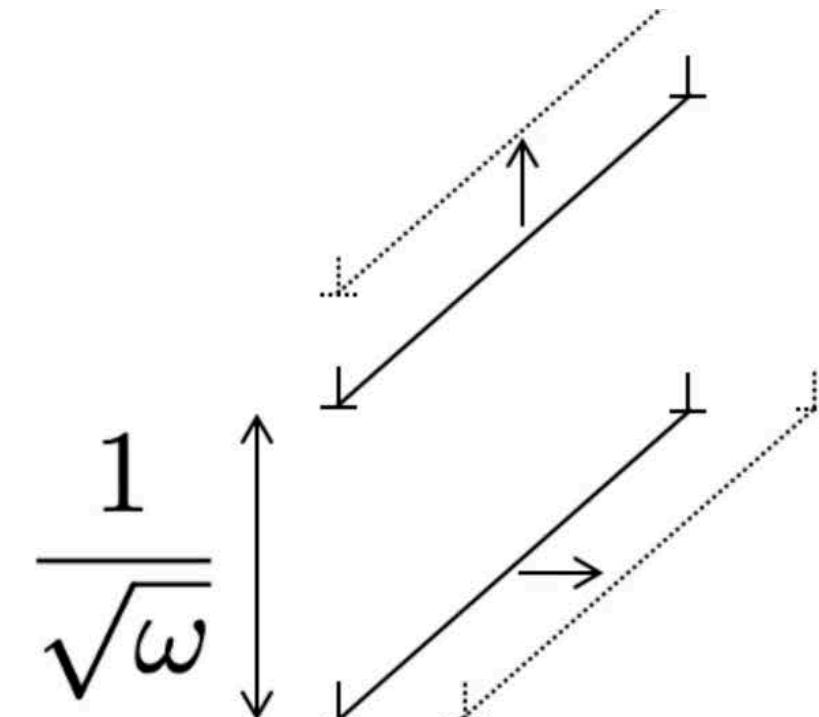
Diffusion creep: variation in vacancy concentrations due to stress drives vacancy fluxes between grain boundaries



$$\dot{\epsilon}_{diff} \sim \frac{\sigma}{R^2}, \frac{\sigma}{R^3}$$

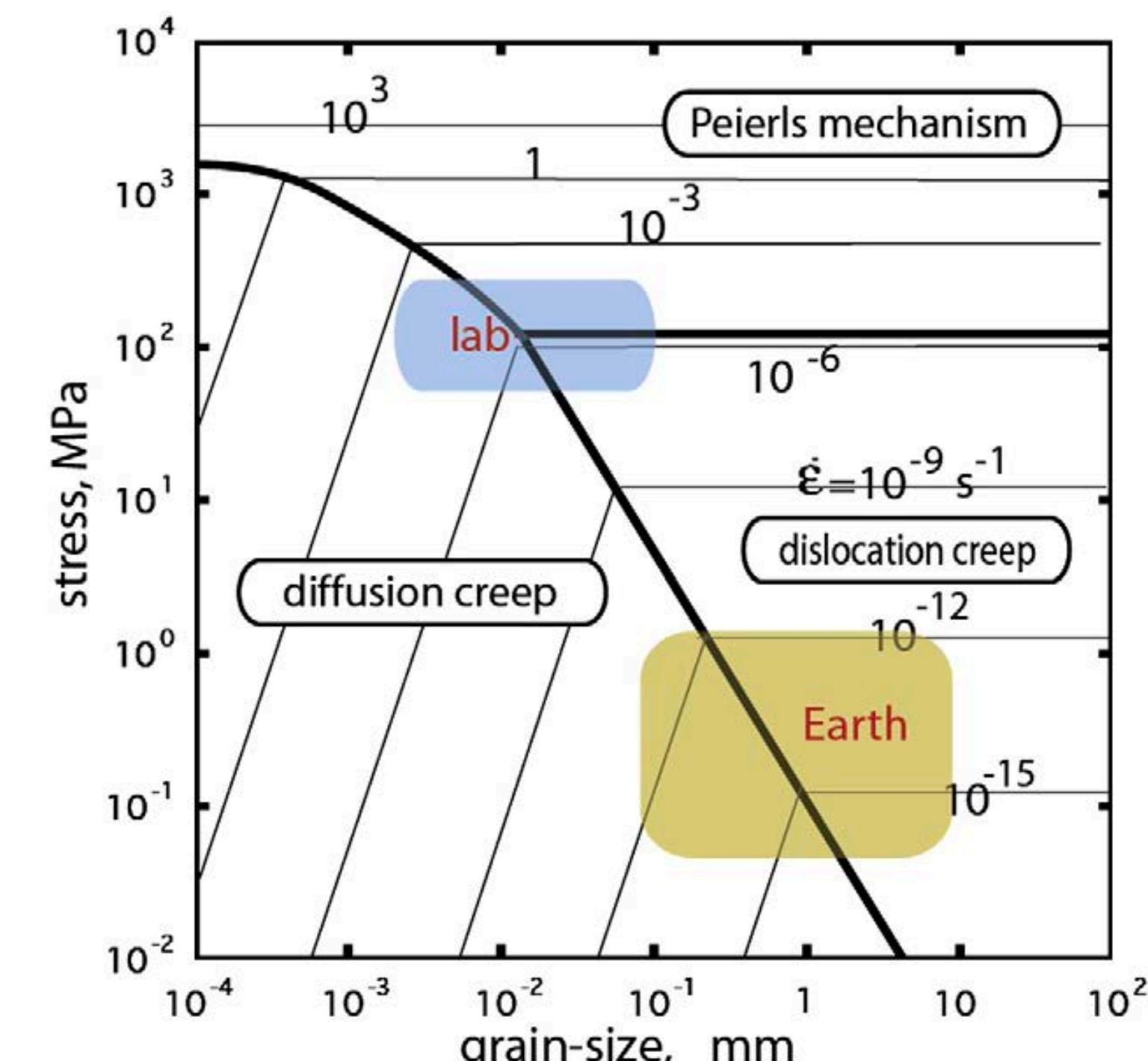
$$\dot{\epsilon}_{diff} = \frac{B}{R^m} \sigma$$

Dislocation creep: stress generates and moves dislocations by climb and glide



$$\dot{\epsilon}_{disl} \sim \omega v \sim \sigma^n$$

$$\dot{\epsilon}_{disl} = A \sigma^n$$



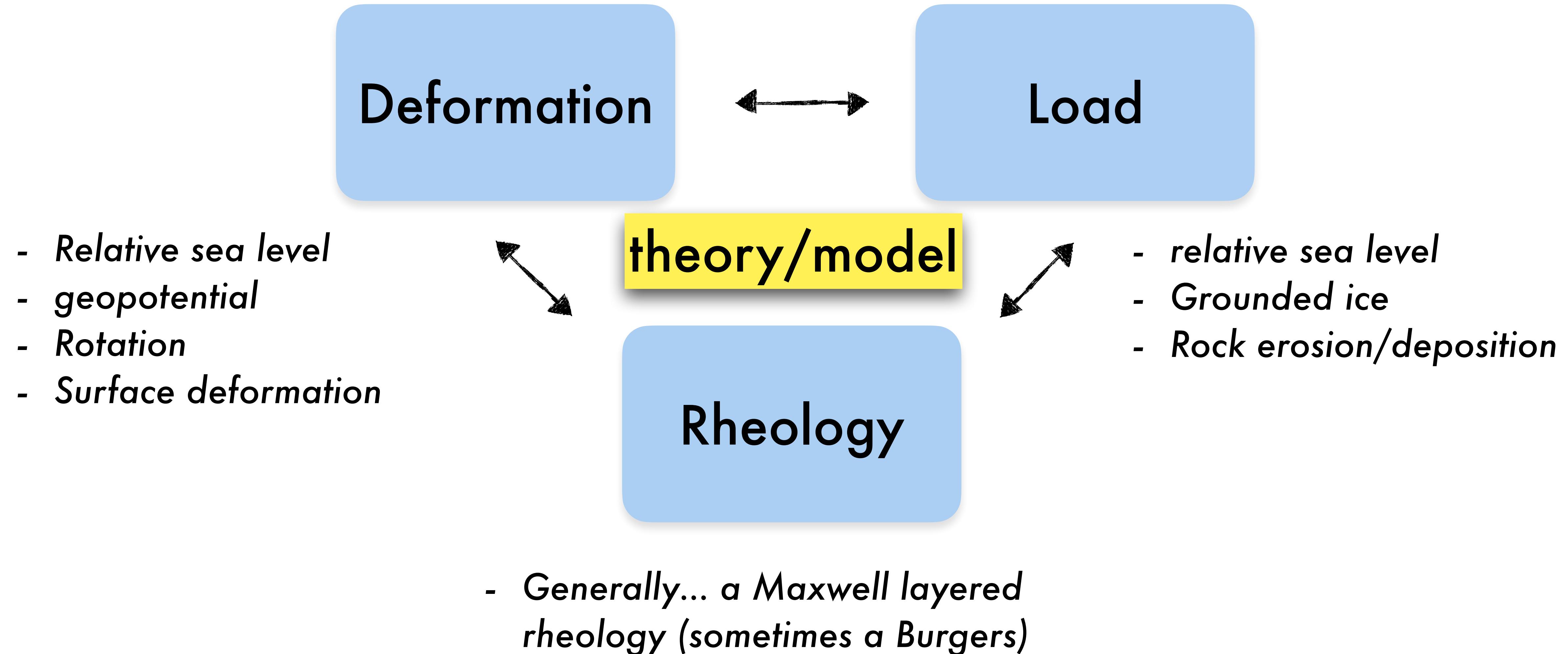
Karato (2010)

But these processes are still poorly studied under low strain rate in the laboratory... please help?

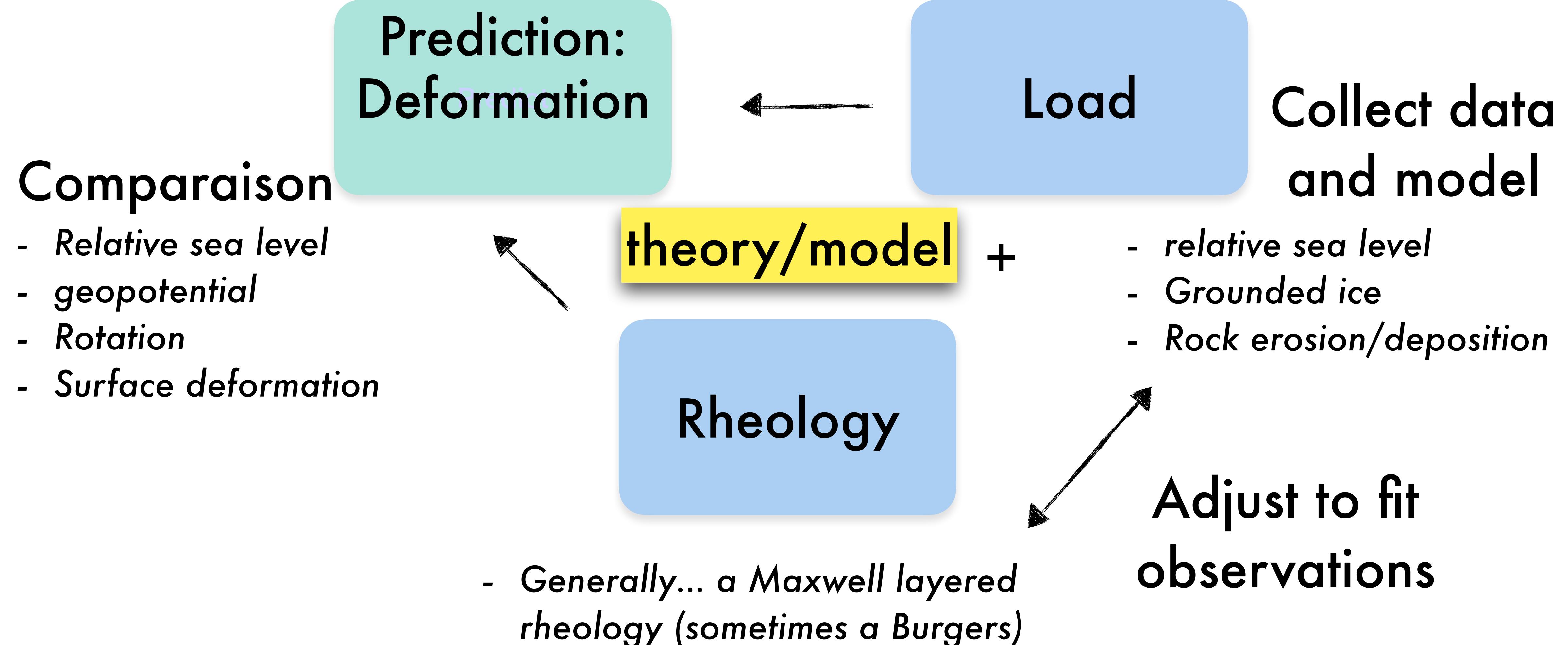
The (almost) perfect case of Glacial Isostatic Adjustment (GIA)

1. What is GIA and why study it?
2. Theory: general concepts, modelling and rheology
3. Observations : constraining ice history and monitoring deformation

Forward modelling of Glacial Isostatic Adjustment (GIA)



Forward modelling of Glacial Isostatic Adjustment (GIA)



What observations?

What observations?

Relative sea level

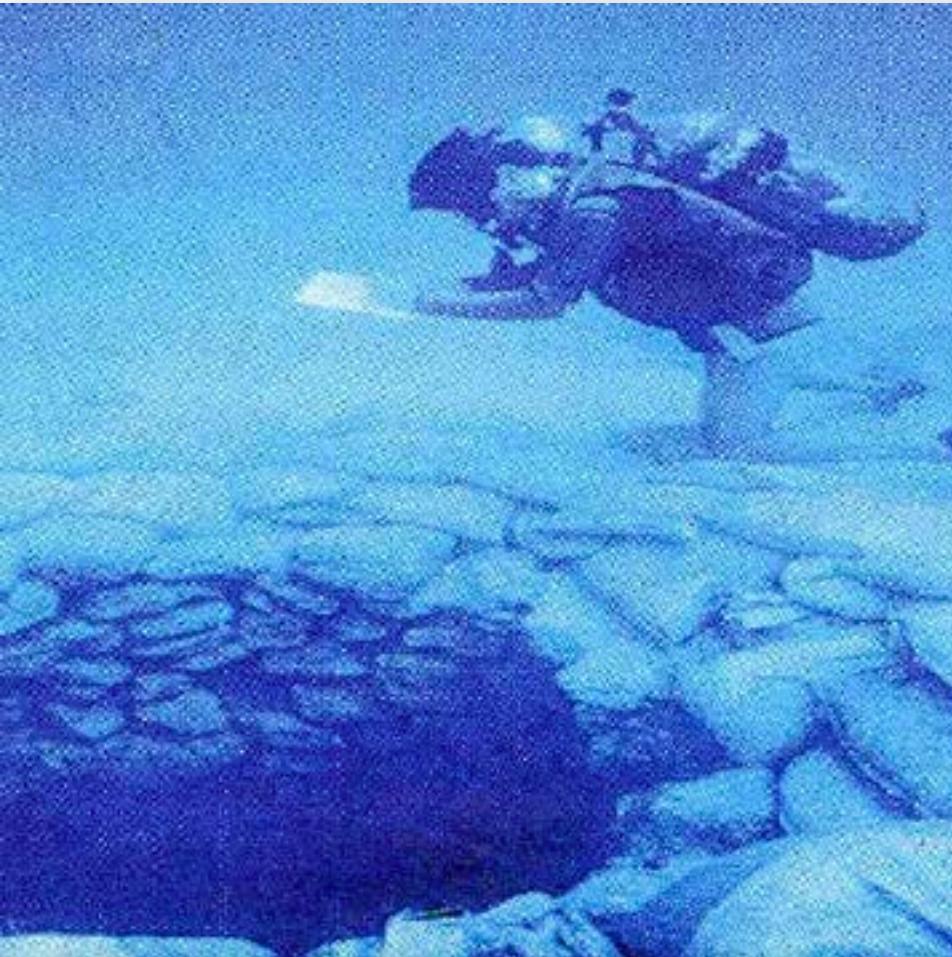


Bronze-Age carvings
from former coastal
dwelling site now
inland , SW Sweden

Ice core



Submerged well
from coast of Israel



corals

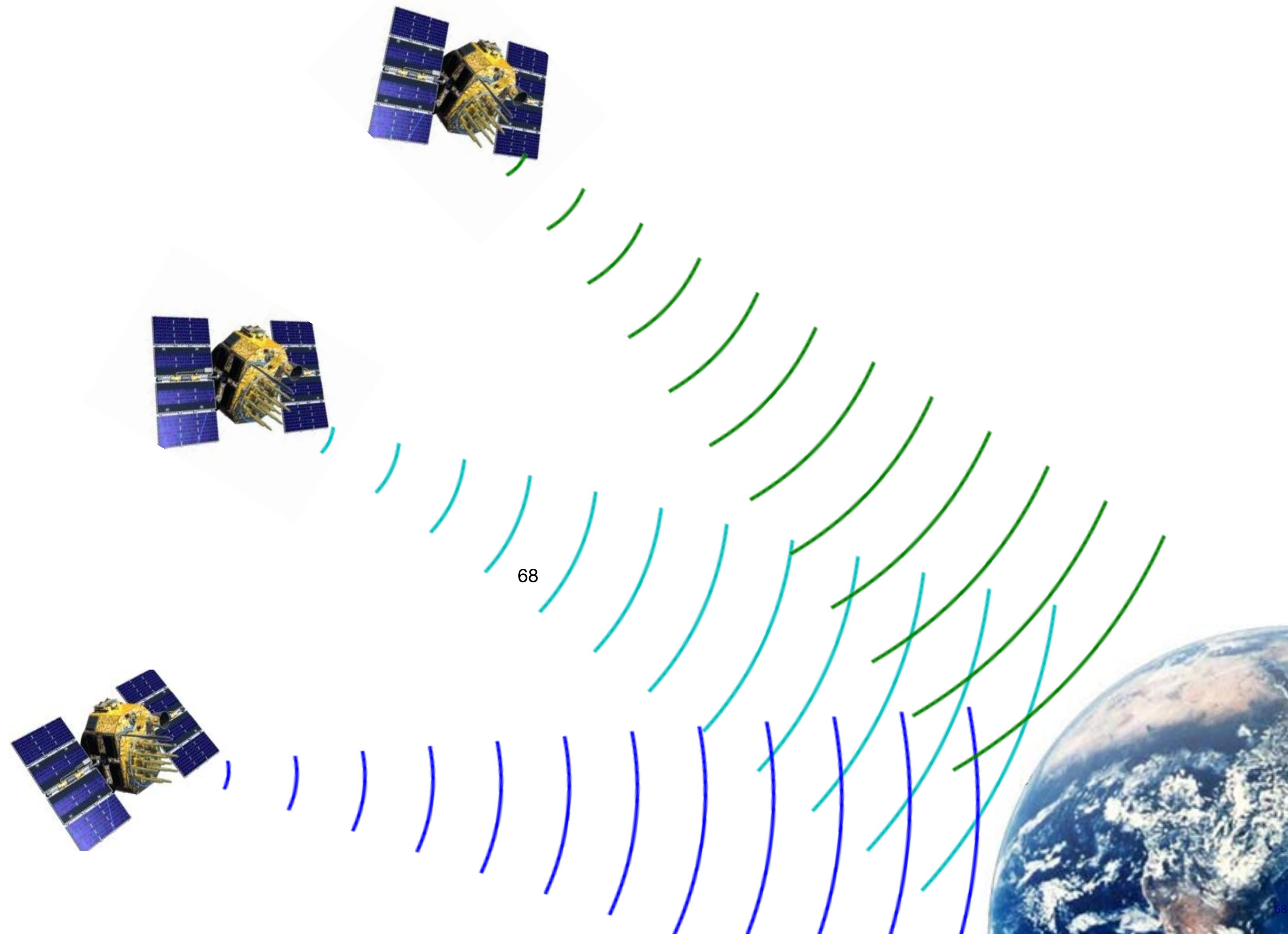


Uplifted beaches

What observations?

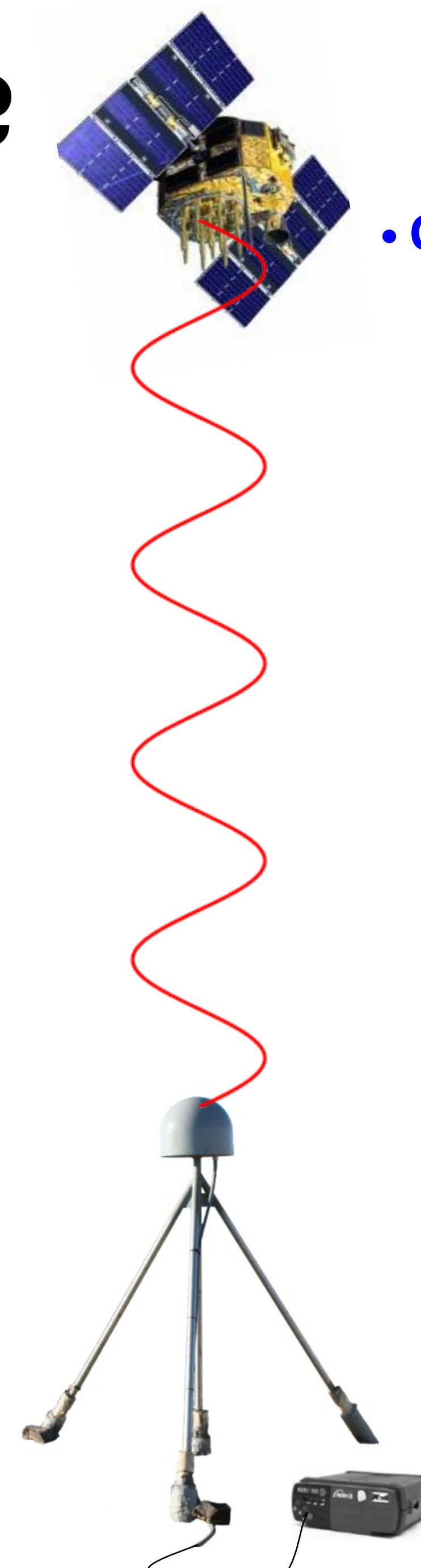
Land uplift

GNSS principle

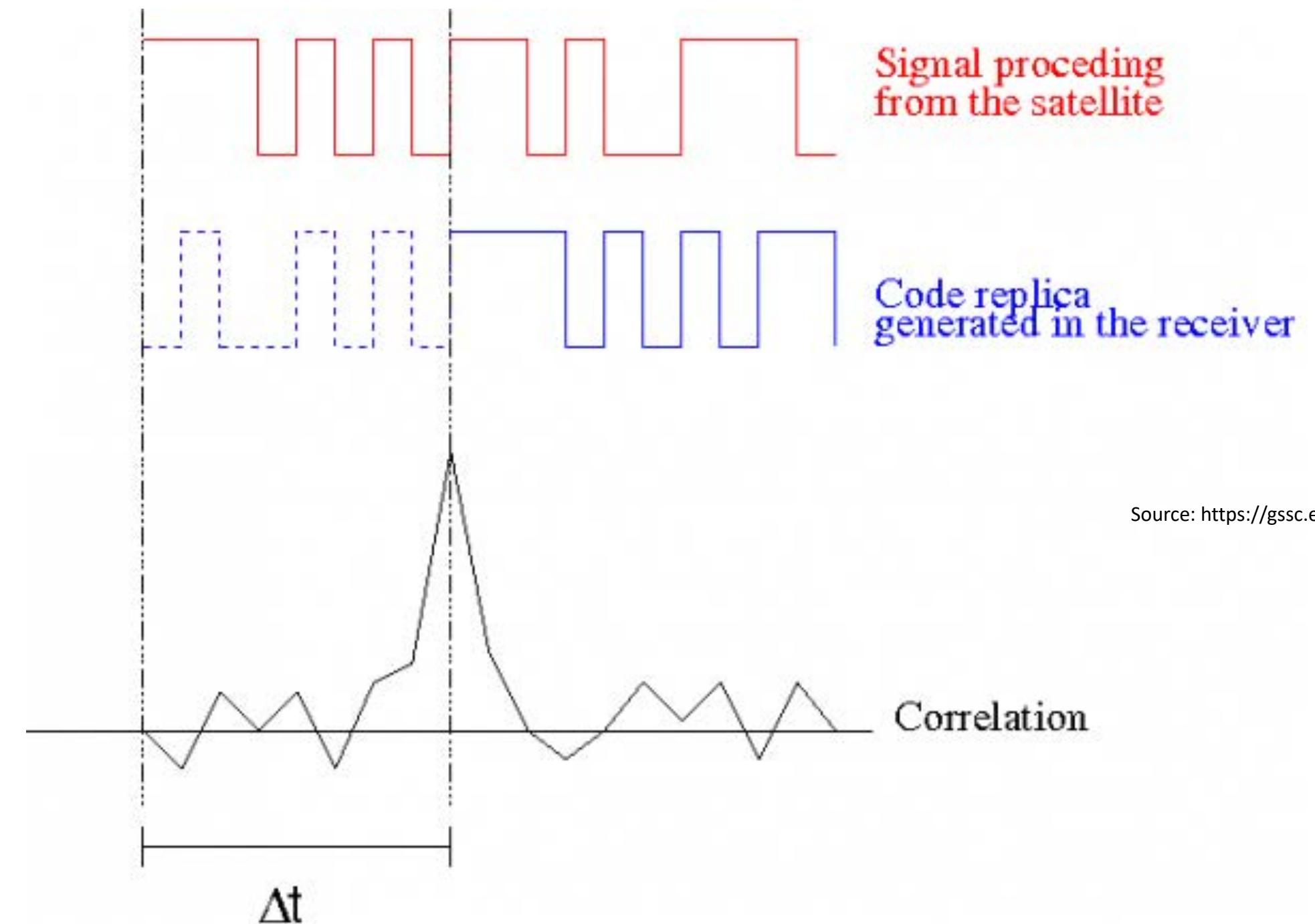


What observations?

Land uplift



- **Code measurement:**
 - Based on pseudo-random code that modulates the signal

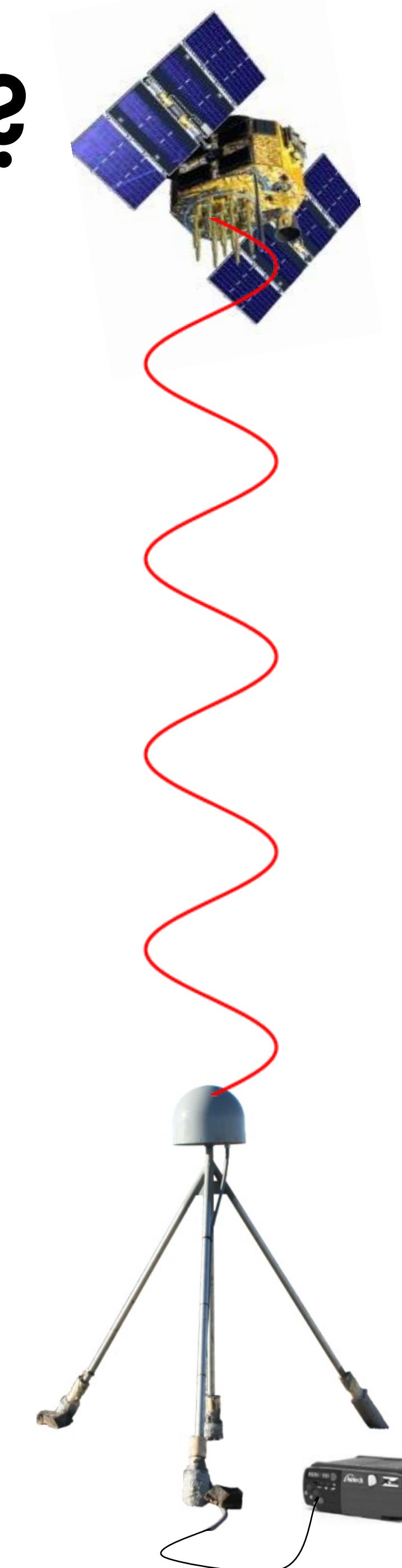


Source: <https://gssc.esa.int/>

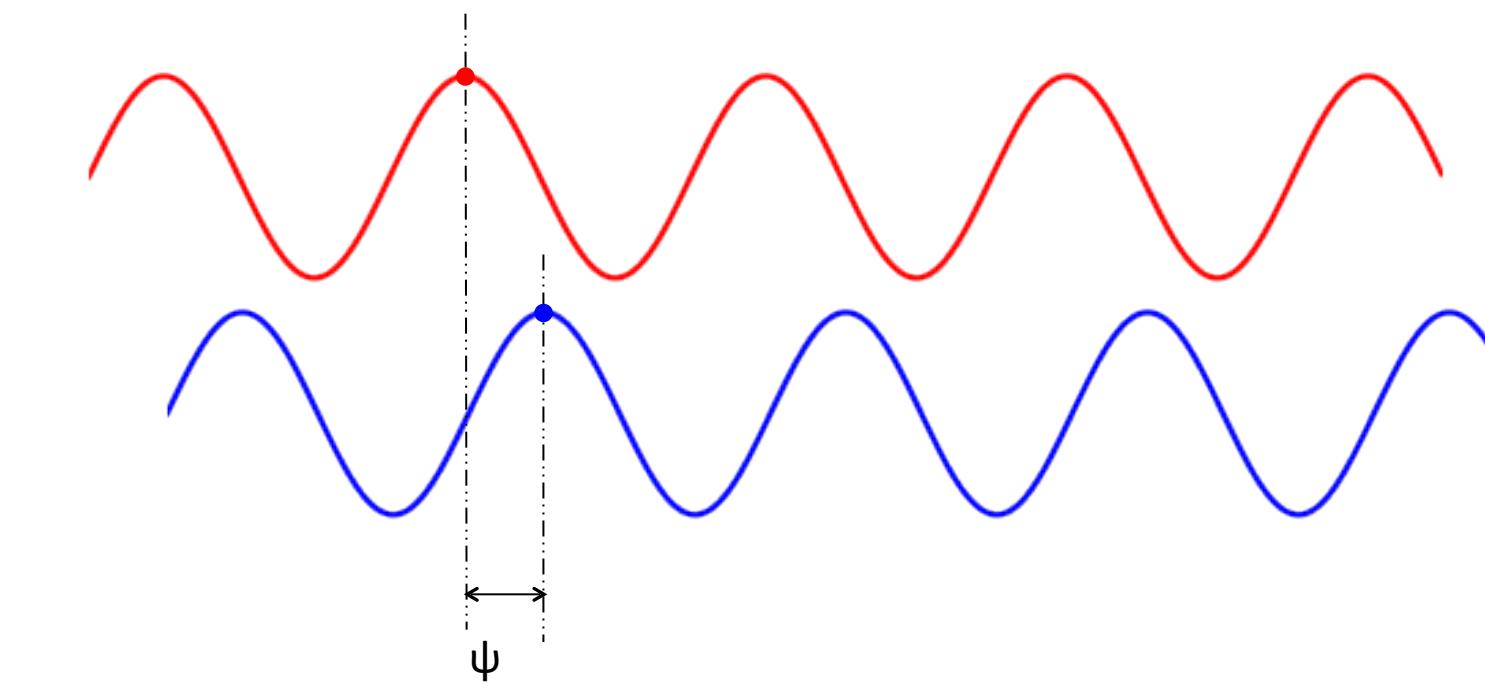
- Propagation time, up to synchronization errors
 - Precision: $\approx 1 \text{ ns} (\approx 30 \text{ cm})$

What observations?

Land uplift



- Phase measurement:

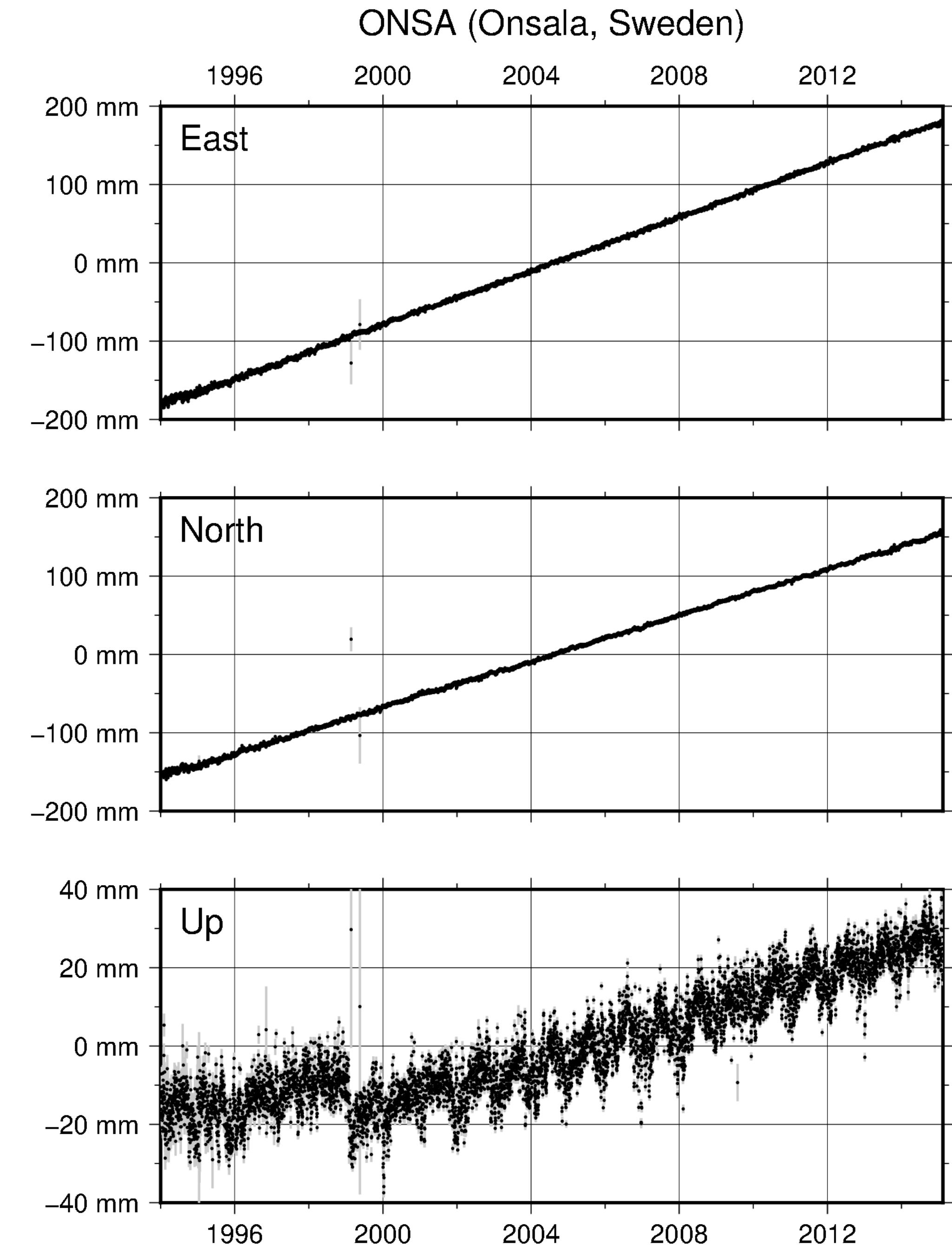


Source: <https://gssc.esa.int/>

–Precision: ≈ 0.01 cycle (≈ 2 mm)

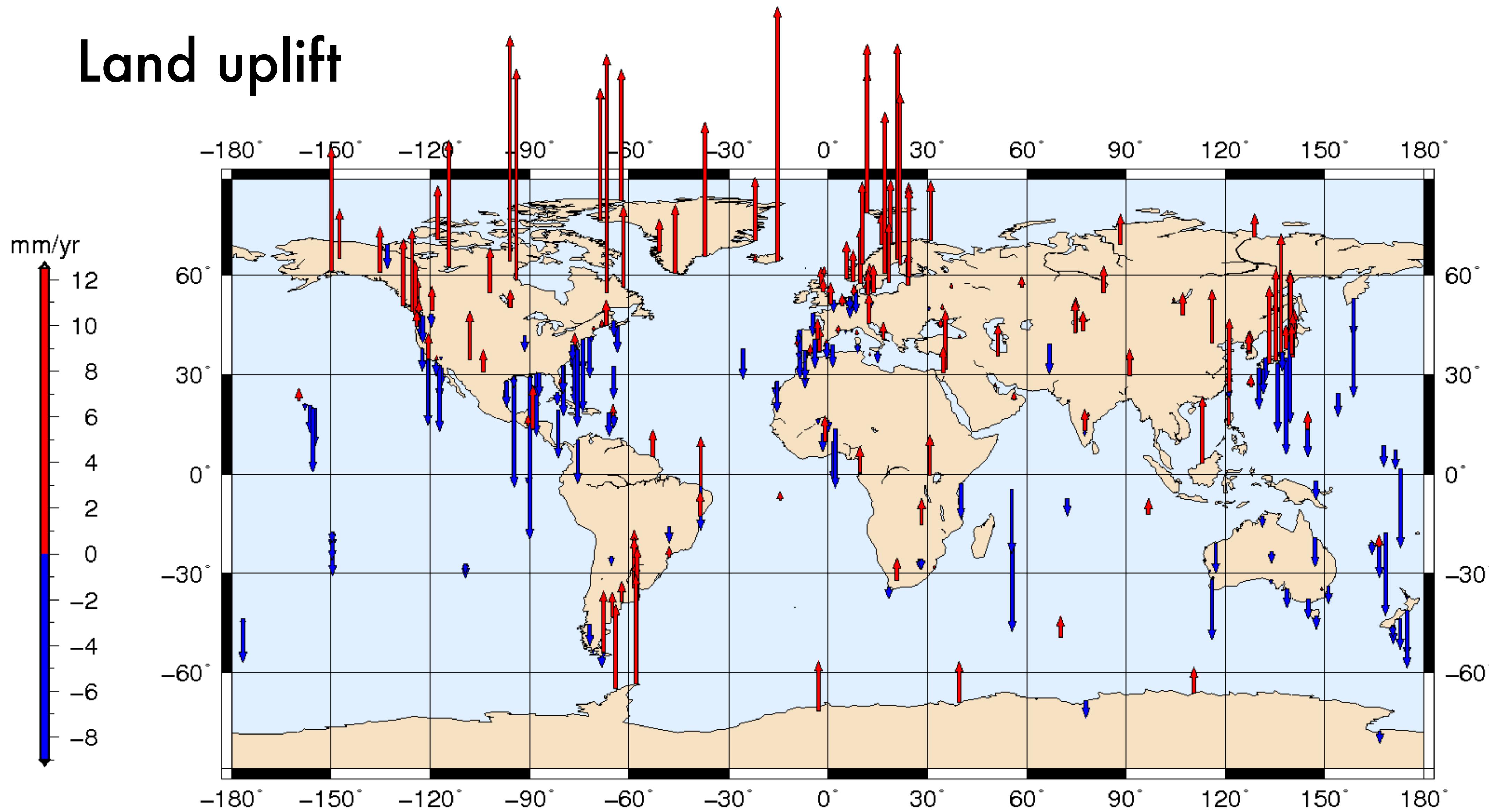
What observations?

Land uplift



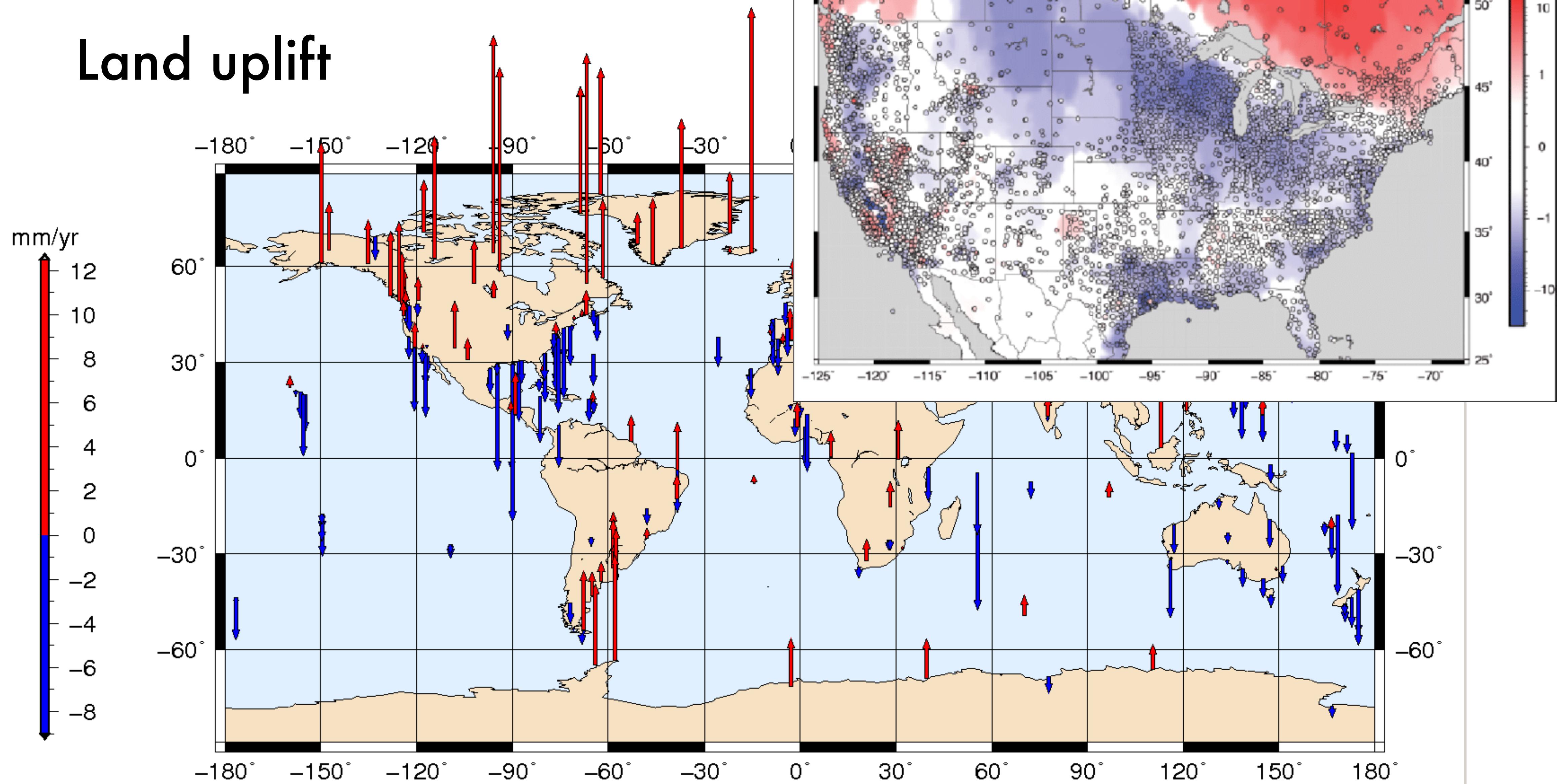
What observations?

Land uplift



What observations?

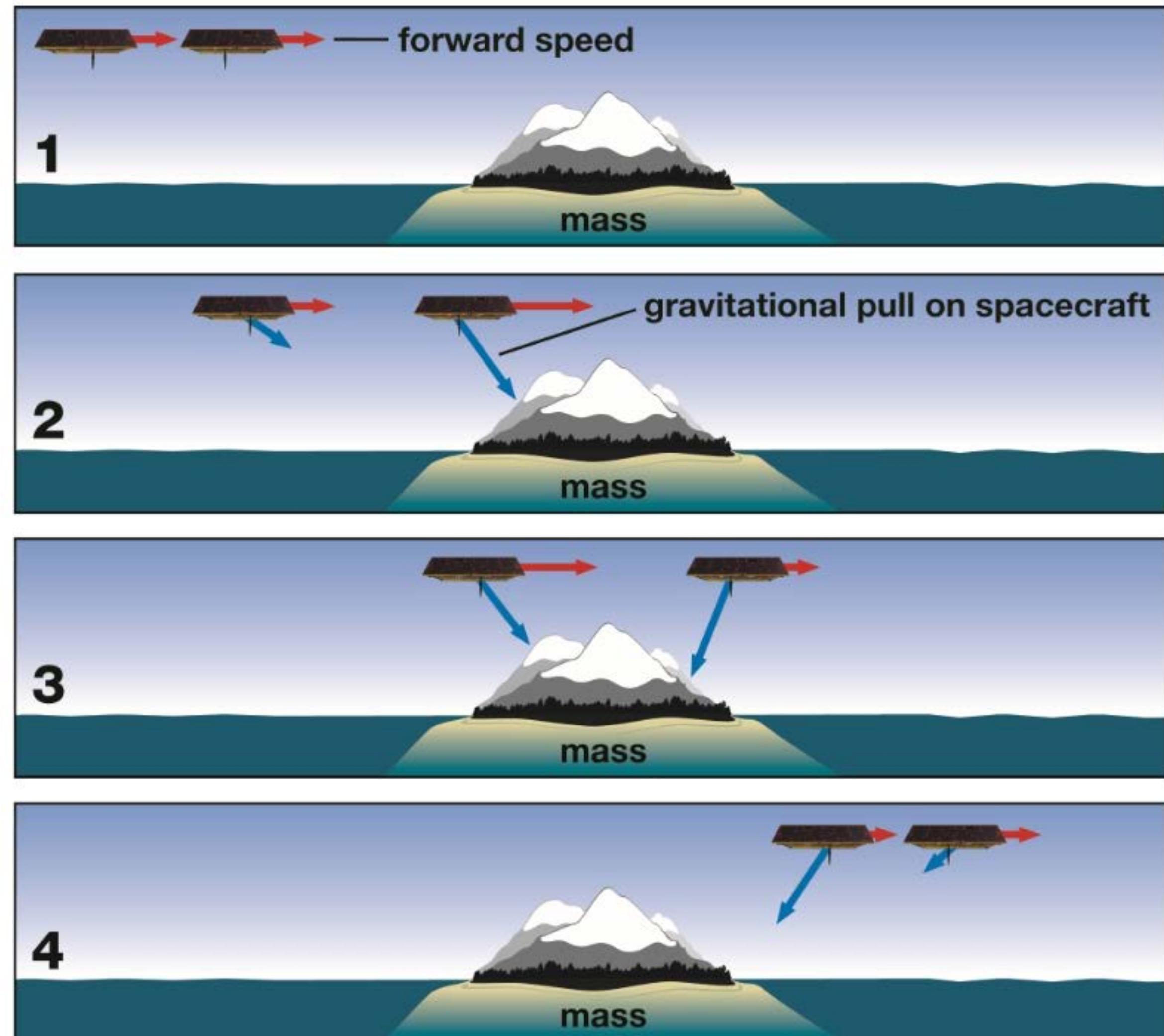
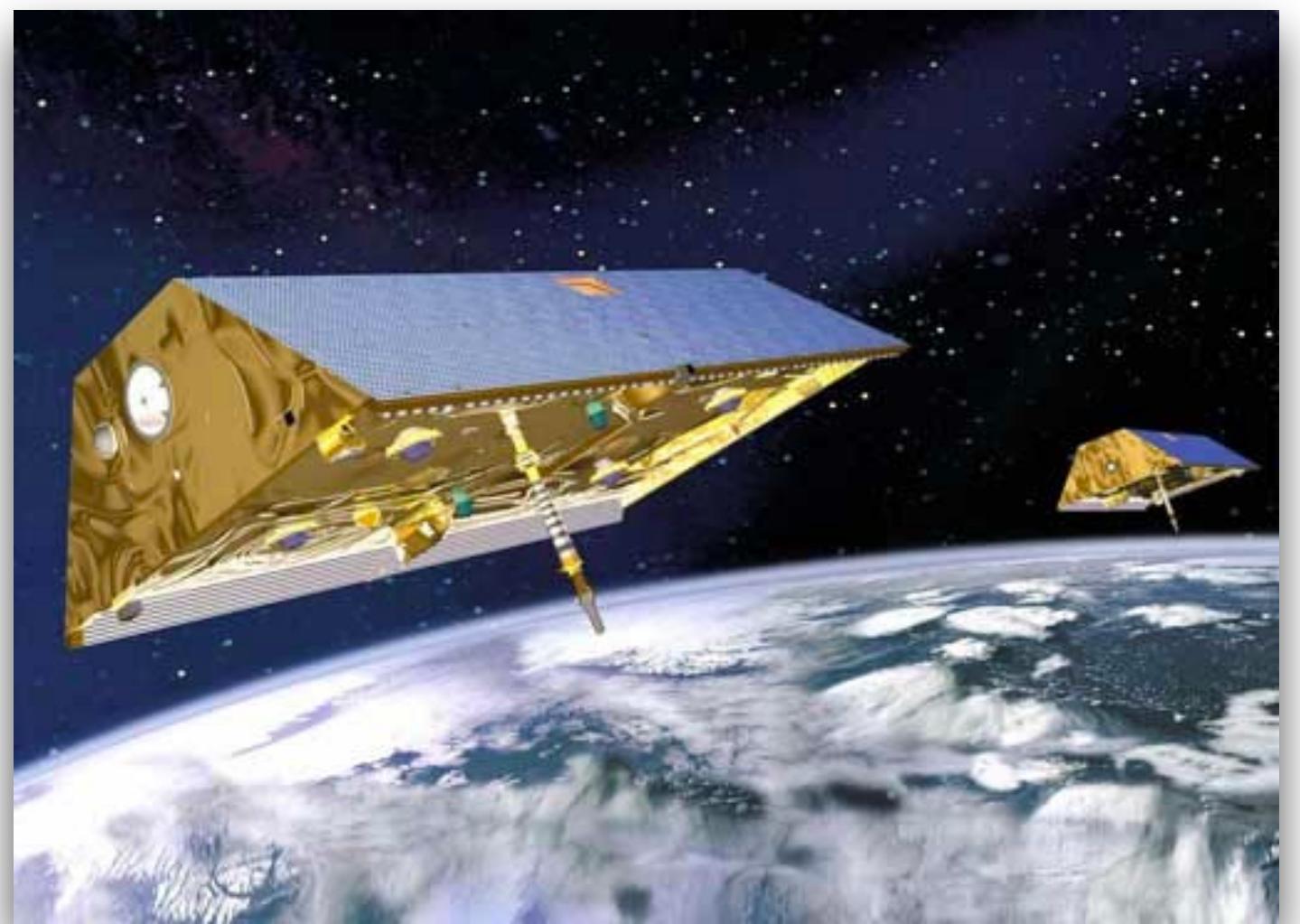
Land uplift



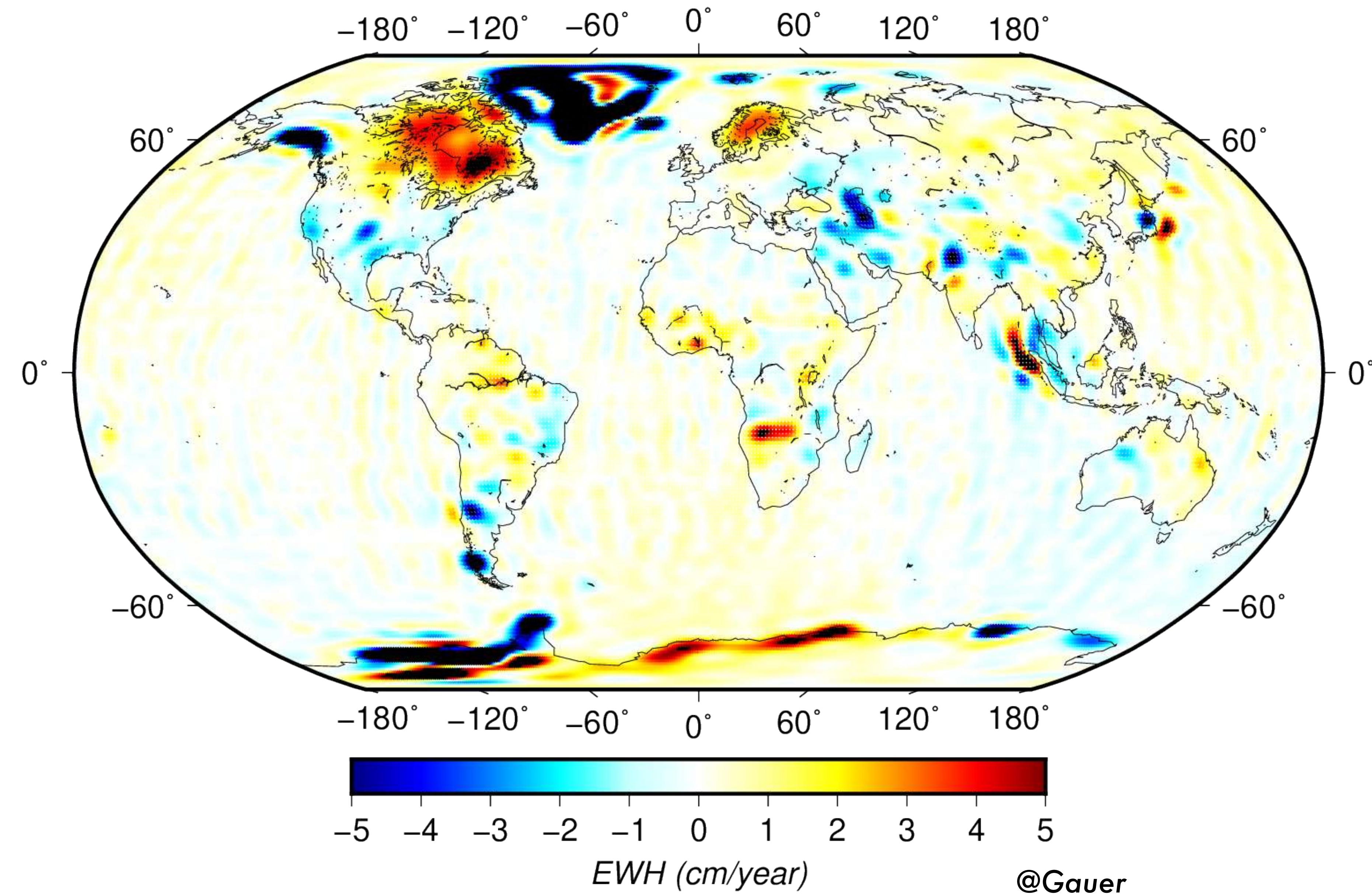
What observations?

Gravity data

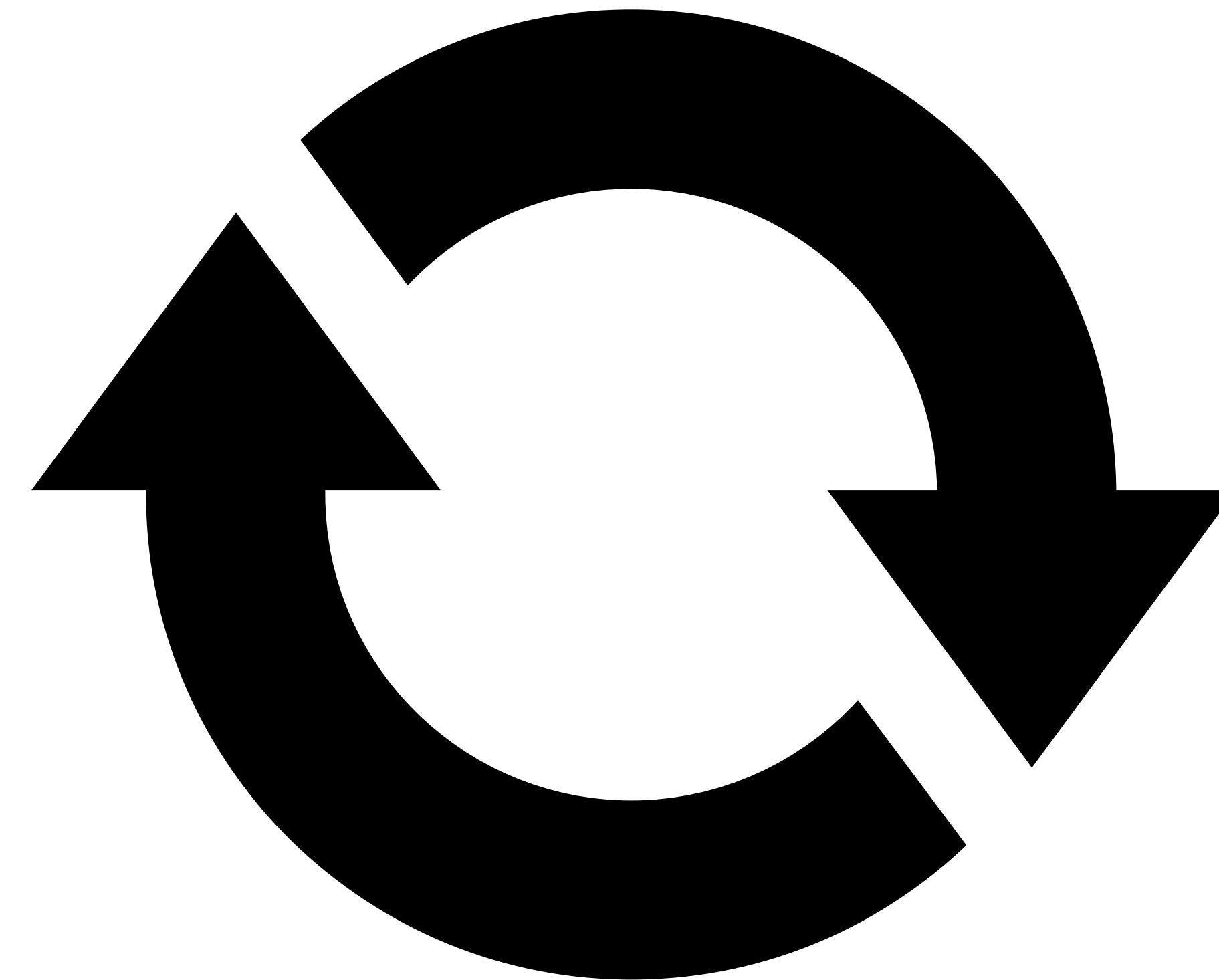
Gravity and Recovery Climate Experiment (GRACE)



GRACE 2003-2015



Solving GIA + Sea level equation until data are best fitted



Iteration model - data comparaison
or inversion

And finally...

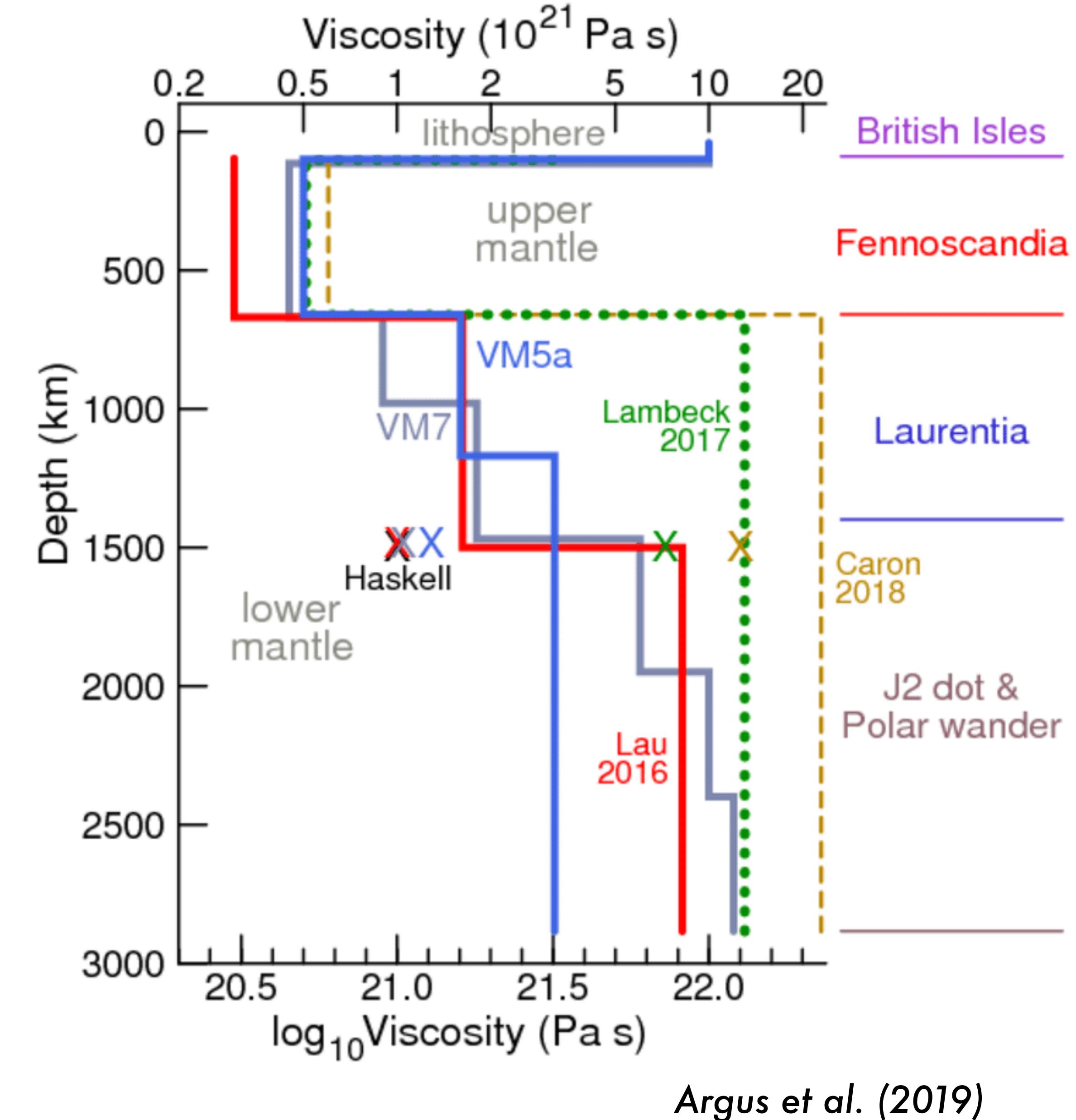
Outstanding questions?

- large uncertainty on the total volume of ice melt since LGM
- Average mantle viscosity of 10^{21} Pa.s what is the upper/lower mantle contrast?

Norman Haskell 1935



- Simple rheologies : lateral heterogeneities, Burgers vs Maxwell?



Take home message

- 1) Looking at the **surface loads** is interesting to **probe the Earth rheology, ice history, better understand present day melting and improve sea level predictions in a changing climate**
- 2) It's a *messy problem* (but nice equations) with some poorly constrained parameters (ideas?)
- 3) Linking **large scale and laboratory derived rheological laws** is not easy
- 4) The story of the Earth's mantle viscosity is probably not over...

What's next?

