

# (upper) mantle rheology

**Fanny Garel**



*University of Montpellier, France*

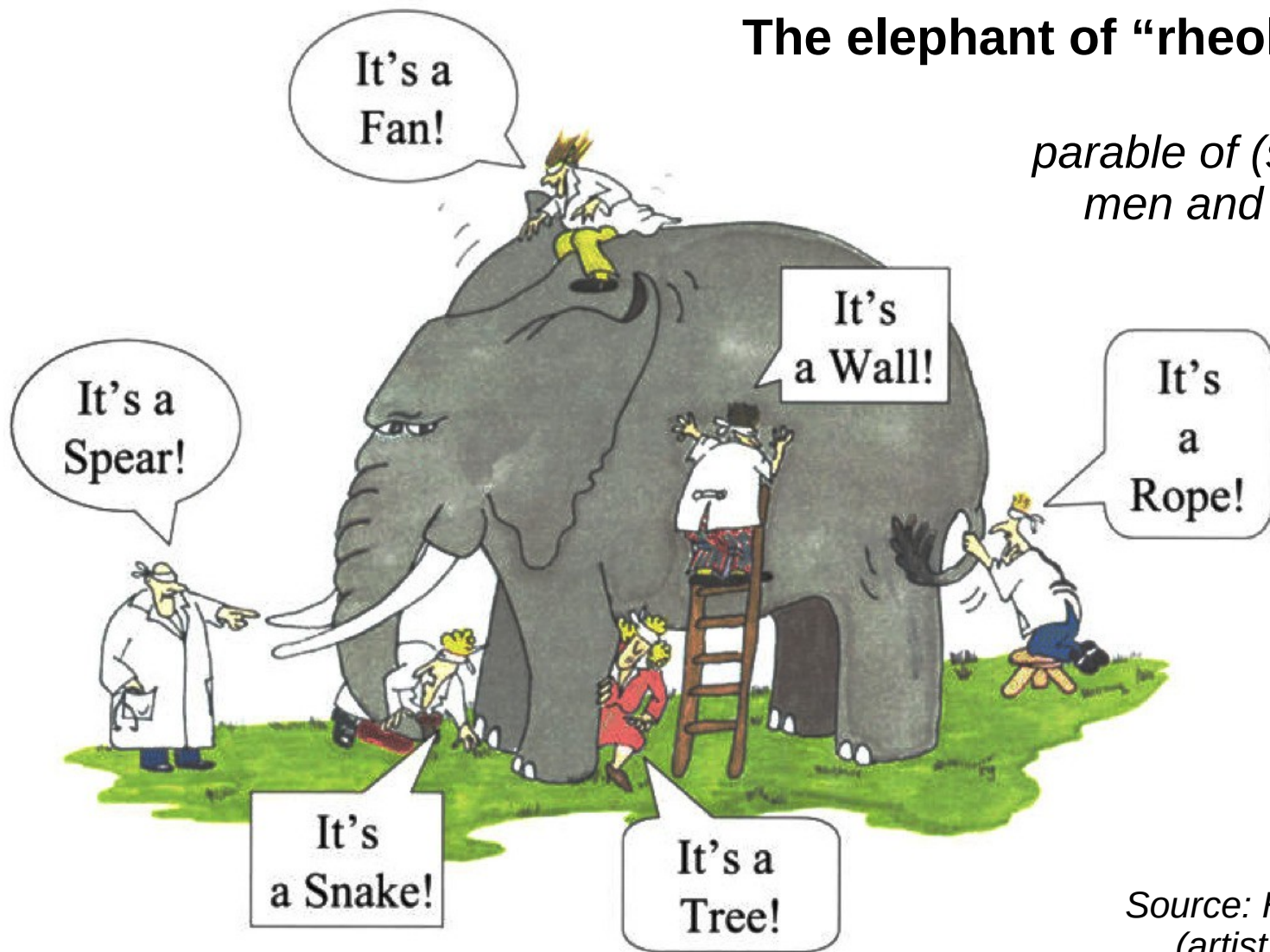
*with materials from lectures by Stéphane Labrosse, Maëlis Arnould,  
Yanick Ricard, Maylis Landeau, Catherine Thoraval*

**Deep Earth Doctoral School - Les Houches - Autumn 2022**



# The elephant of “rheology”

*parable of (scientific) blind men and an elephant*



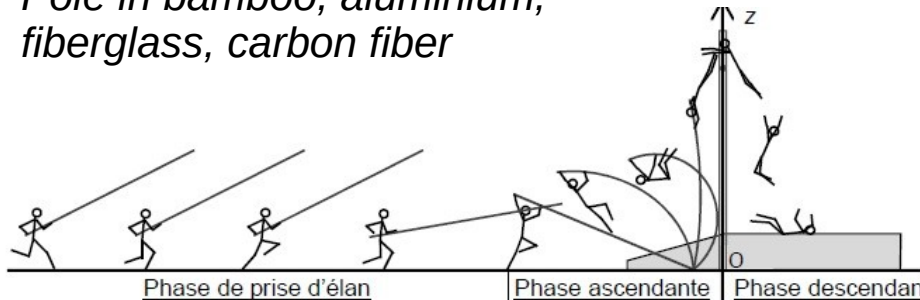
Source: Himmelfarb et al 2002  
(artist: G. Renee Guzlas)

# Various responses of solids to an applied force

Reversible deformation  
> **elastic**



*Pole in bamboo, aluminium, fiberglass, carbon fiber*



Irreversible deformation  
> **plastic**



mechanical rupture  
> **brittle**



continuous deformation  
> **ductile**



# So let's get going...?

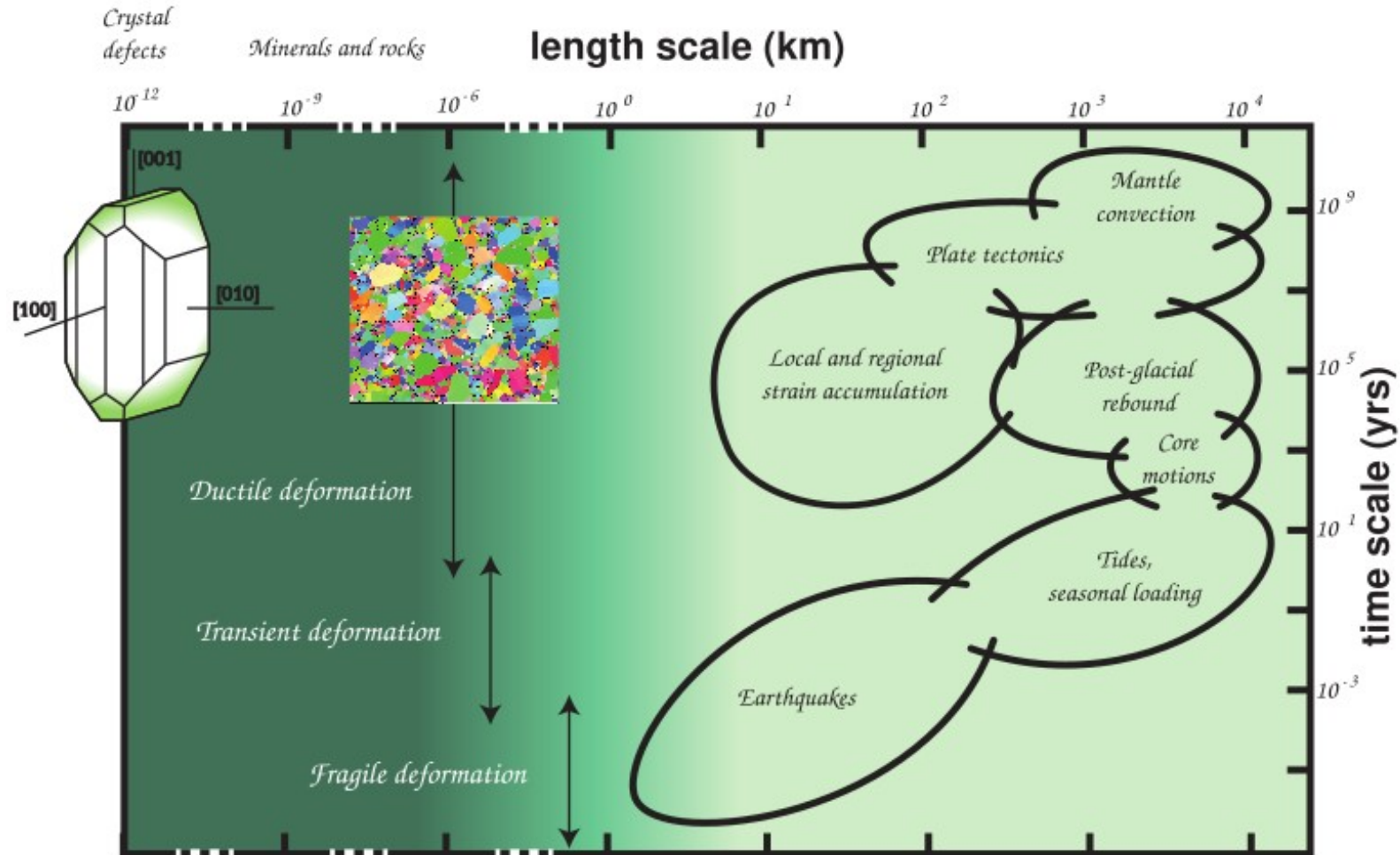
- identify & parameterize deformation processes

- + how do they coexist/interact?

- upscaling to nature

- > from crystal to plate

- > from lab rate to convection rate

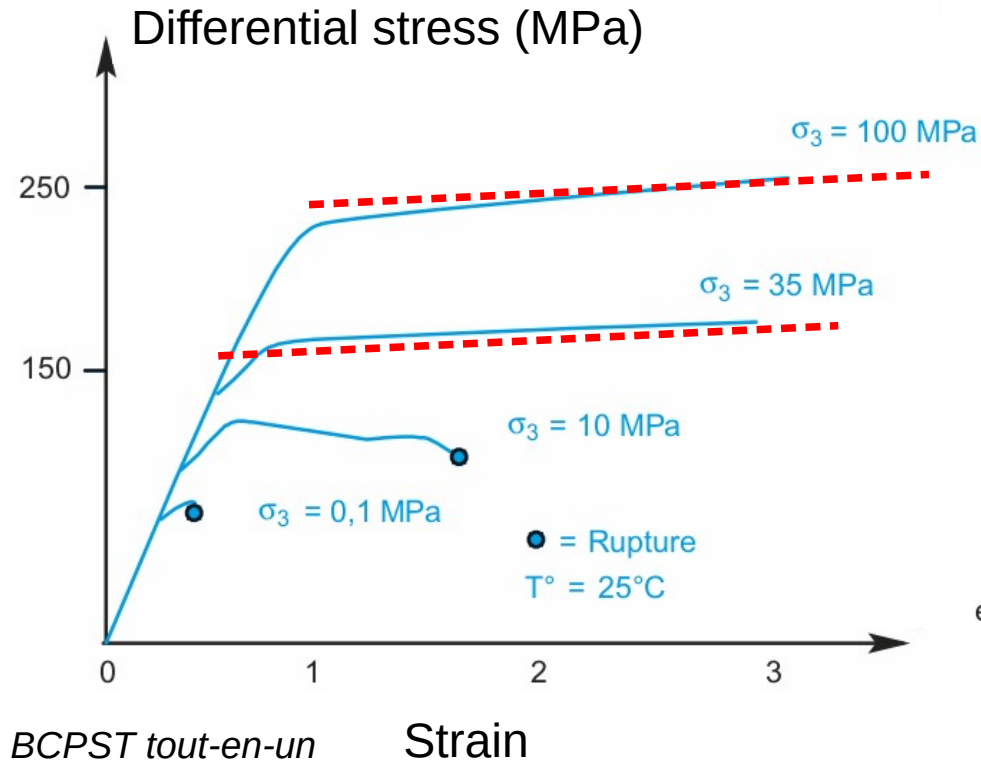
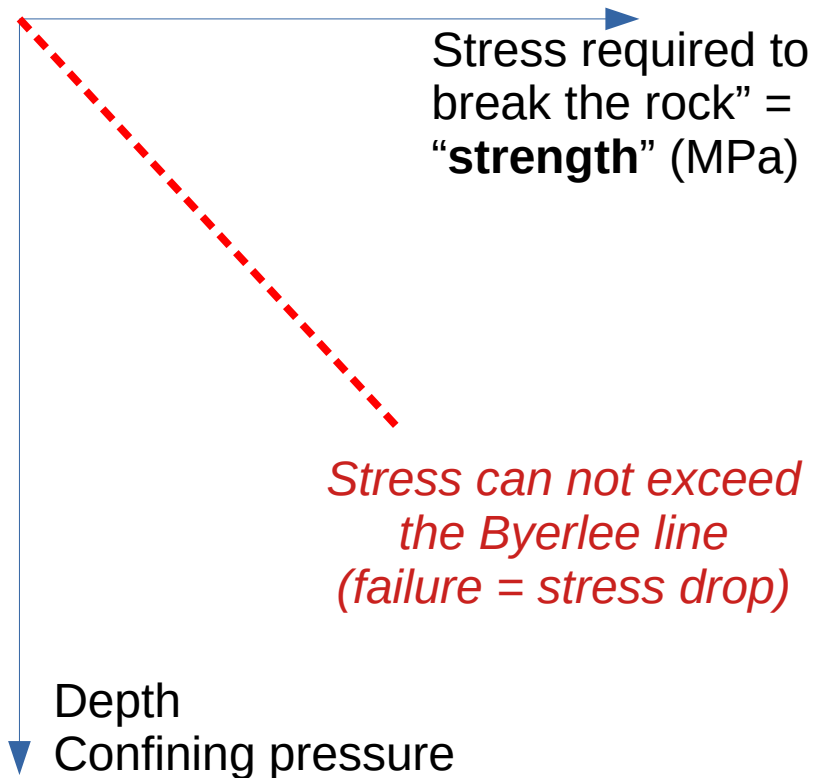


Modified from K. Lambeck (1988)

Rheology = **deformation** of material under a mechanical stress

→ *approached through laboratory experiments of crystal, aggregate or rock deformation*

- Byerlee's "law" of friction and strength profile
- Creep (++strain without additionnal stress) > **"yield stress"**



# For fluid and solid: viscous flow

*Pitch drop exp. : 9 drops in 84 years !*



*Credit : John Mainstone*

→ **if you wait long enough,  
everything flows !**

*dimensionless Debora number  
“the mountains flowed before the Lord”*

Viscosity = **measure** of material resistance  
to flow/deformation over time (> strain rate)

= resulting from “internal” friction

- *dynamic viscosity  $\eta$  or  $\mu$  in Pa.s*

- *kinematic viscosity  $\nu$  in  $m^2/s$   
“momentum diffusivity”*



# For fluid and solid: viscous flow

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## **Viscosity – orders of magnitude**

Water  $\sim 10^{-3}$  Pa.s,

Oil  $\sim 10^{-2}$  Pa.s

Maple syrup  $\sim 1$  Pa.s

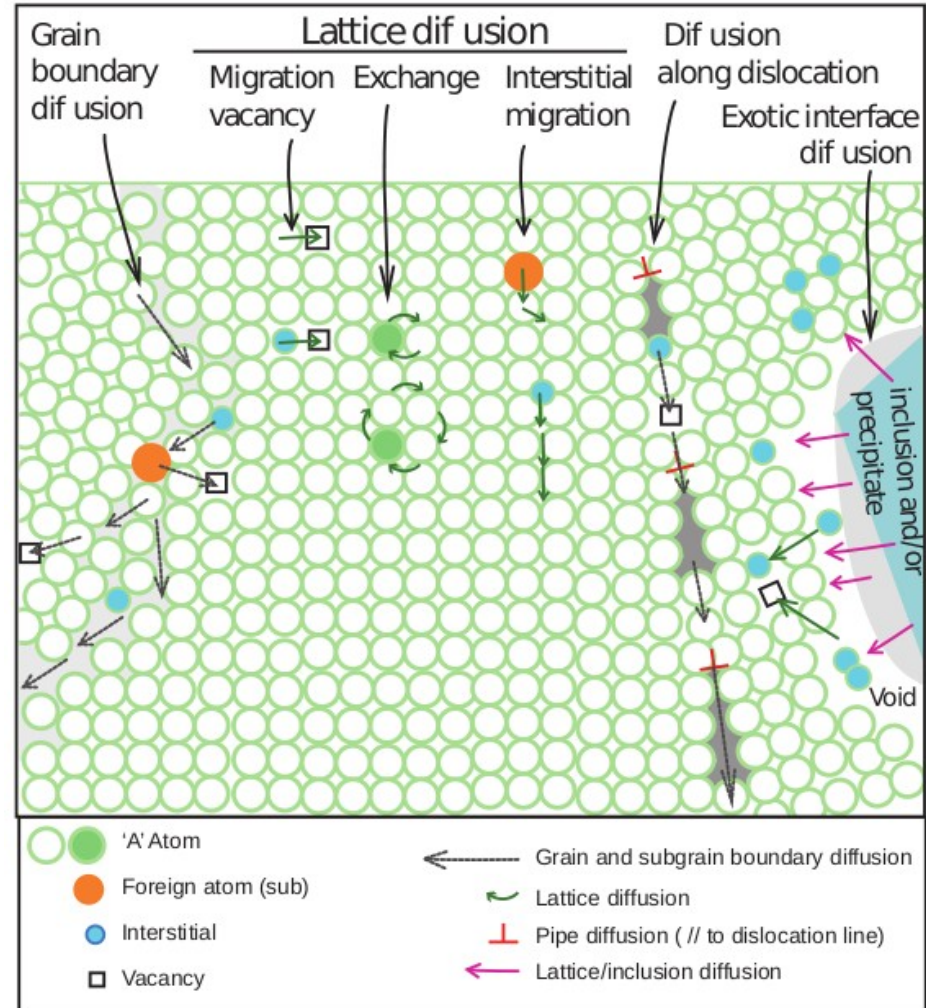
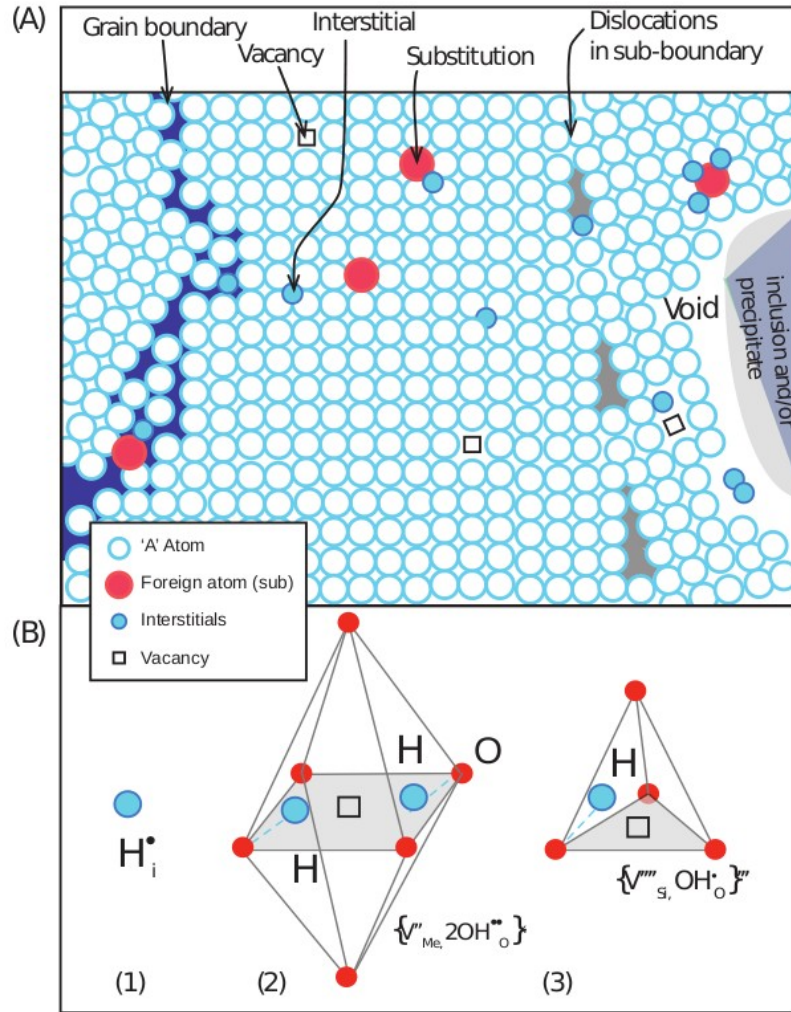
Peanut butter  $\sim 10^2$  Pa.s

Pea  $\sim 10^{11}$  Pa.s

Salt diapir  $\sim 10^{18}$  Pa.s

Rock  $\sim 10^{19-28}$  Pa.s

# How do rocks flow ? > deformation by defects motion at micro-scale



Courtesy of S. Demouchy



# Diffusion creep flow' law

Arrhenius-type law  
(thermally-activated)

$$\dot{\epsilon} = A_{\text{diff}} \sigma_d \exp \left( - \frac{E_a + PV_a}{RT} \right)$$

*strain rate* points to  $\dot{\epsilon}$

*pre-factor* points to  $A_{\text{diff}}$

*stress* points to  $\sigma_d$

*activation energy* points to  $E_a$

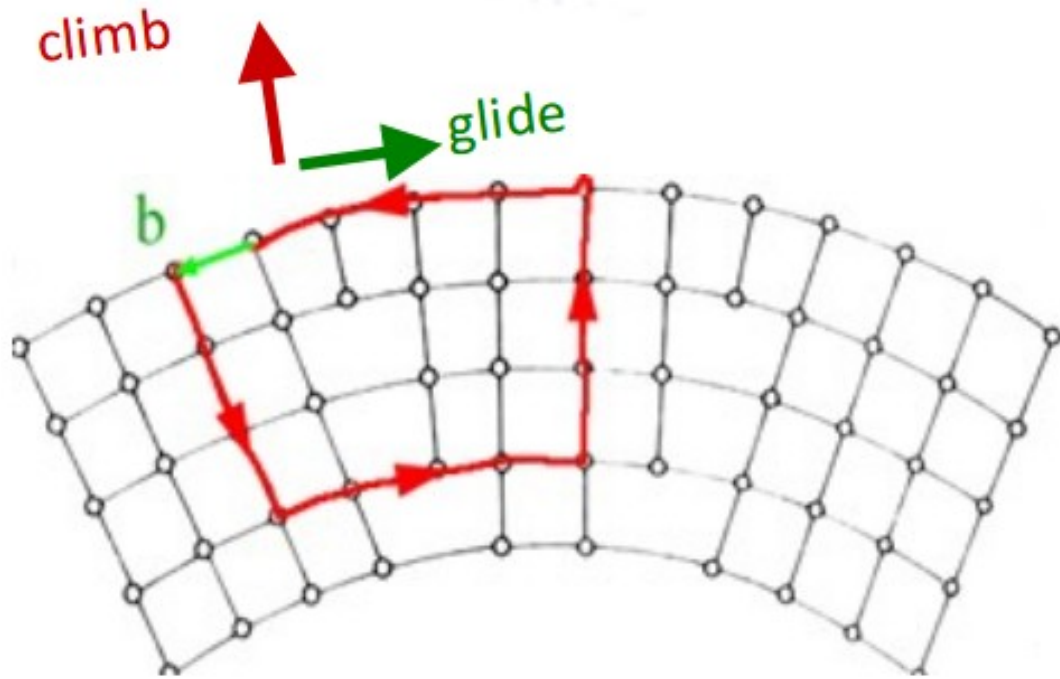
*lithostatic pressure* points to  $P$

*activation volume* points to  $V_a$

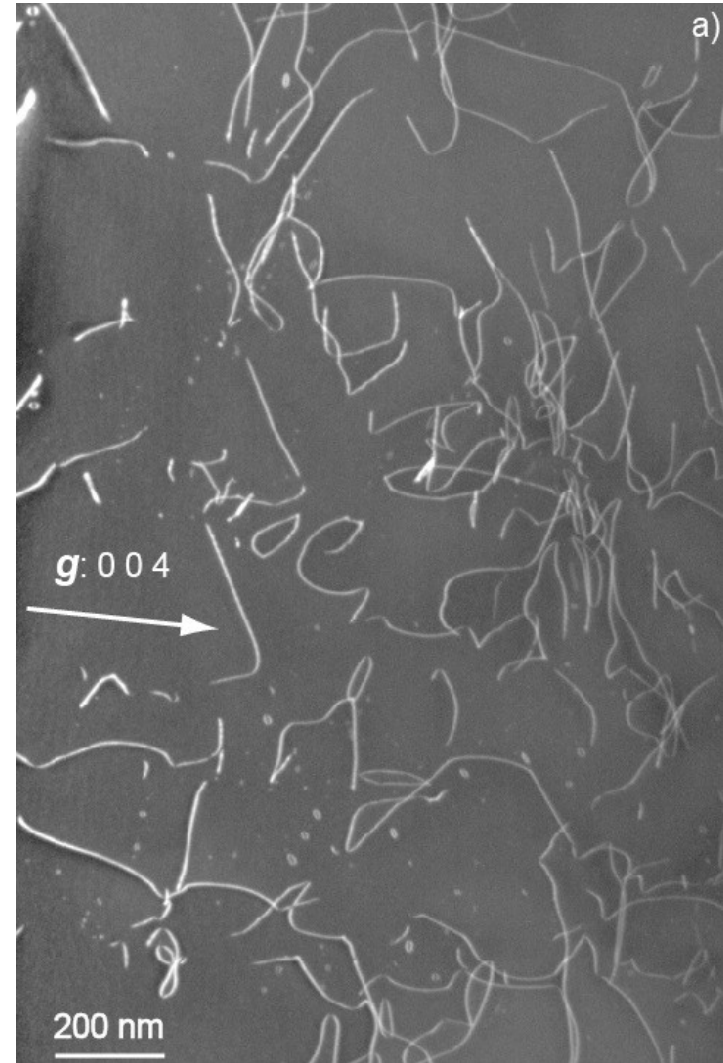
*temperature* points to  $T$

Dependencies other than P and T : **grain size, mineralogy, chemistry/water**

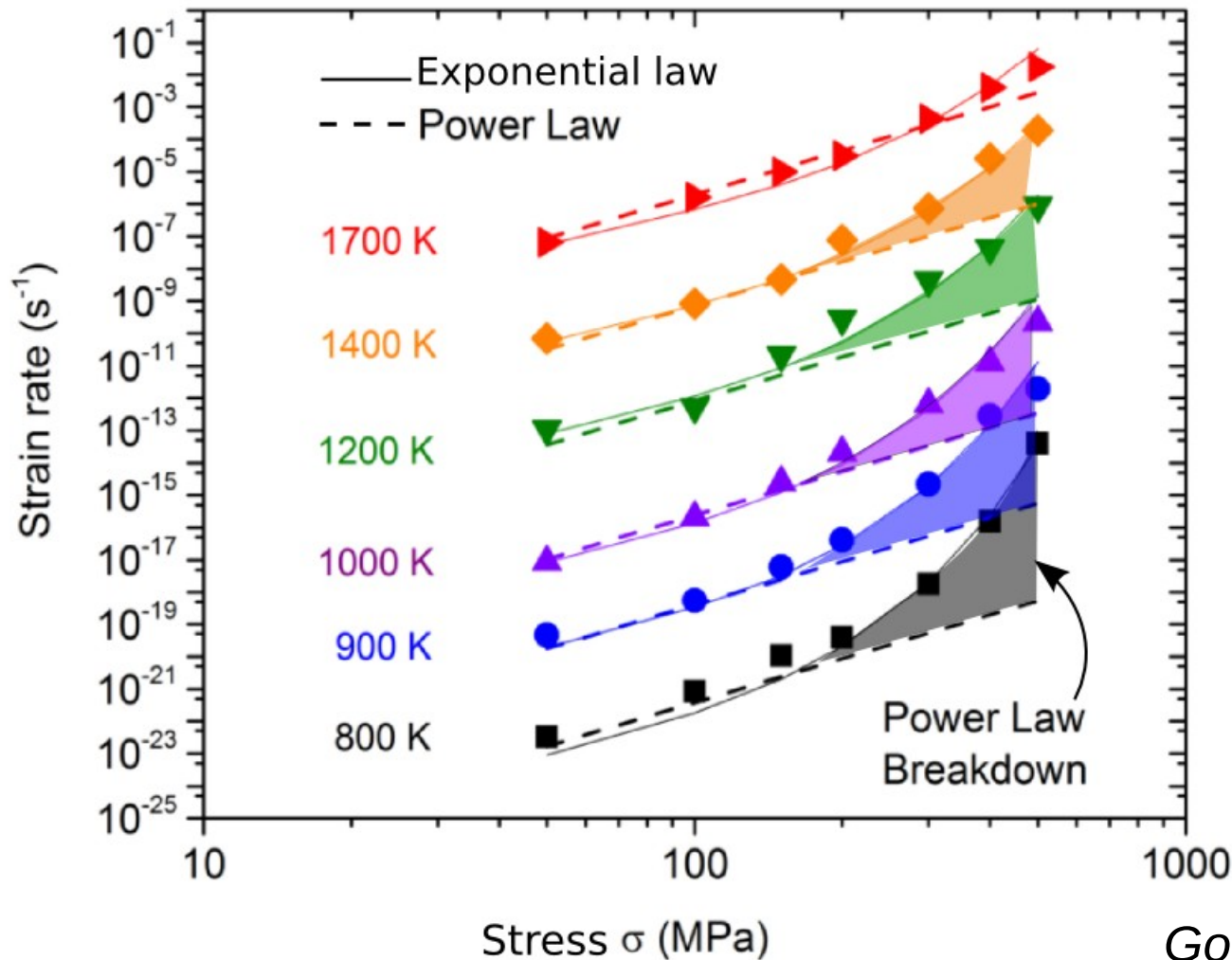
# Deformation by defects motion at micro-scale



Dislocations  
in olivine from TEM  
(*Mussi et al., 2015*)



## 2.5-D dislocation dynamics (DD) *numerical experiments* on olivine

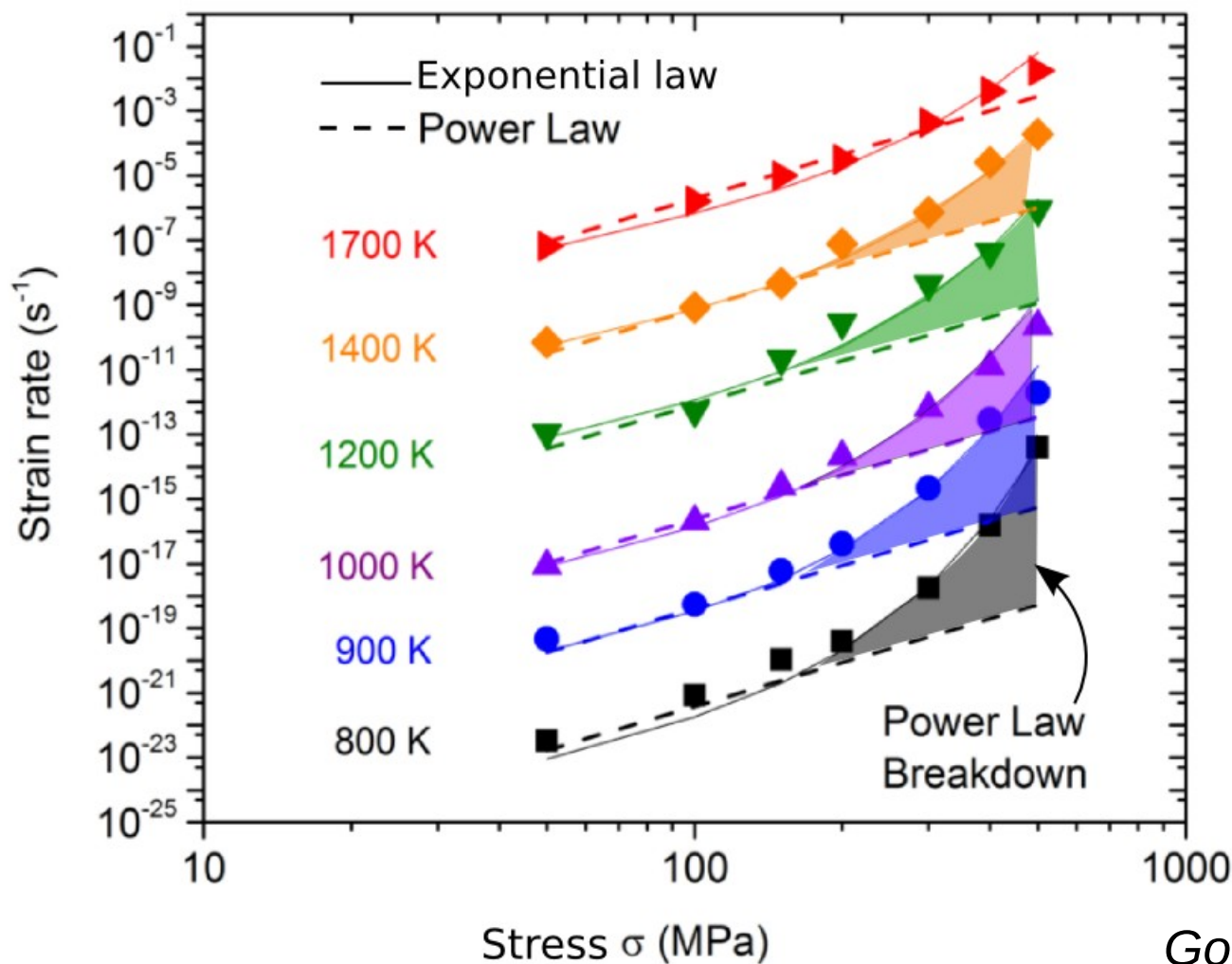


→ continuity of dislocation dynamics (glide + climb) from 800 to 1700 K and from low to high stresses

*Gouriet et al., EPSL, 2019*



## 2.5-D dislocation dynamics (DD) *numerical experiments* on olivine



- **Power-law** between strain rate and stress  
valid at low stress and high T

- **power-law breakdown**  
at low T, high stress  
(deforming plates and slabs)

*Gouriet et al., EPSL, 2019*

## 2.5-D dislocation dynamics (DD) *numerical experiments* on olivine

$$\dot{\epsilon} = A_{\text{disl}} \sigma_d^n \exp \left( -\frac{E_a + PV_a}{RT} \right)$$

- **Power-law** between strain rate and stress  
valid at low stress and high T

$$\dot{\epsilon} = A \sigma^n \exp \left[ -\frac{E_a}{RT} \left( 1 - \left( \frac{\sigma}{\sigma_P} \right)^p \right)^q \right]$$

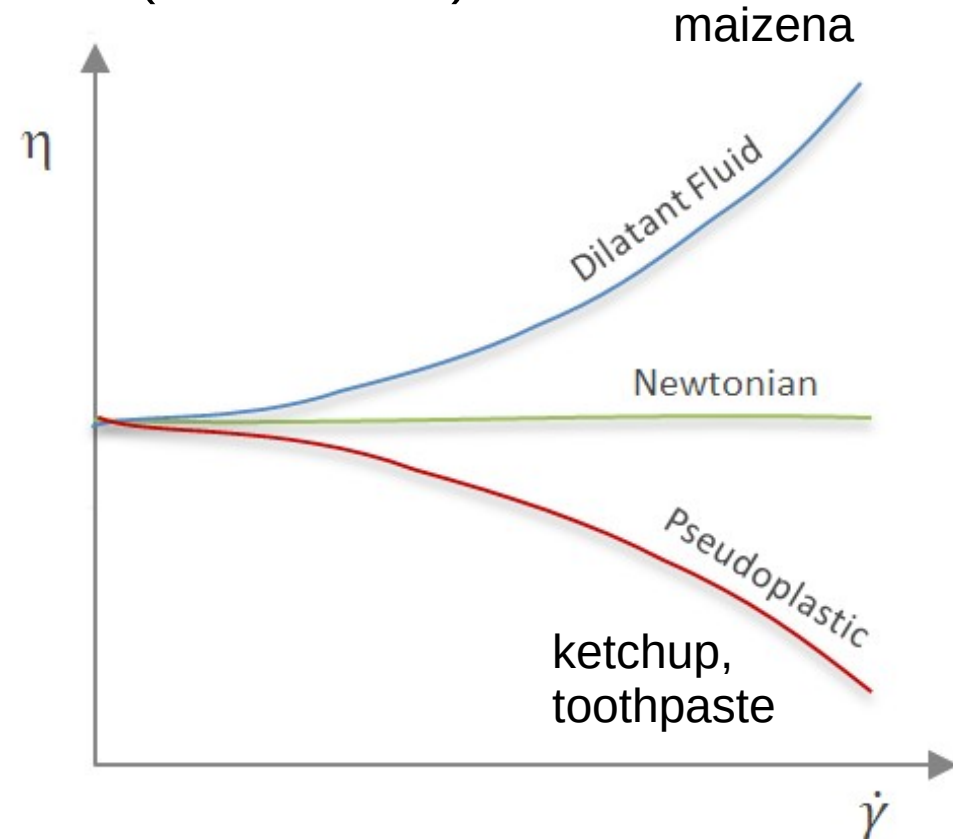
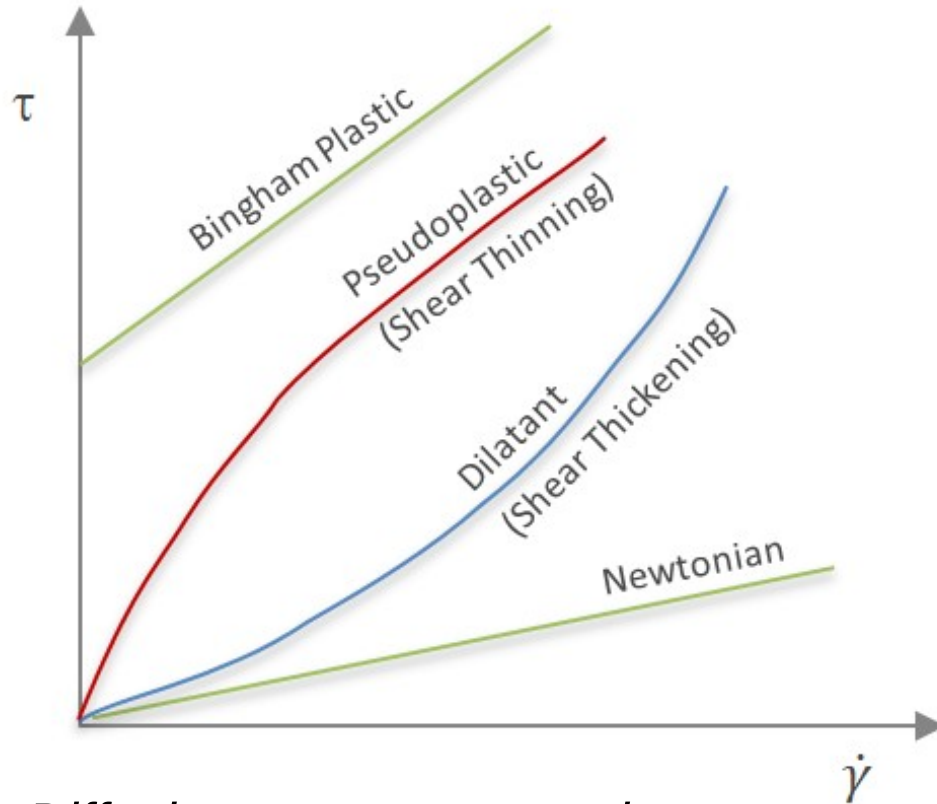
- **power-law breakdown**

at low T, high stress  
(deforming plates and slabs)

*Frost and Ashby, 1982*

# From flow “laws” to viscosity

At first order : viscosity = stress / (2 strain rate)



*Diffusion creep > newtonian*

*Dislocation creep > non-newtonian*



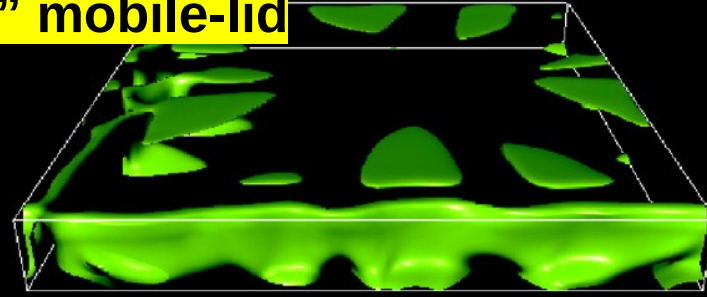
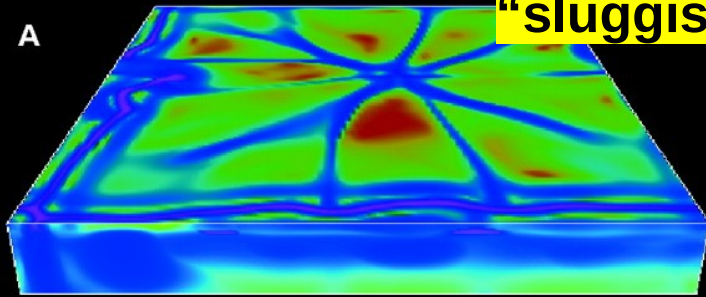


increasing yield strength  
↓

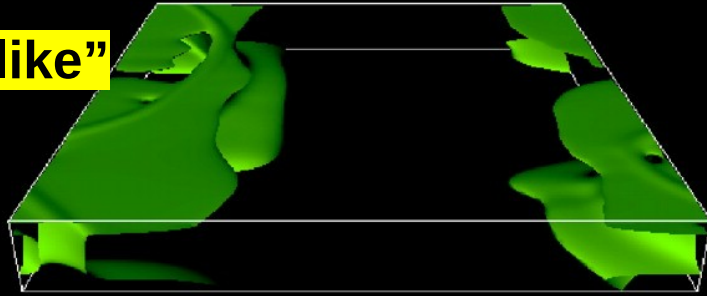
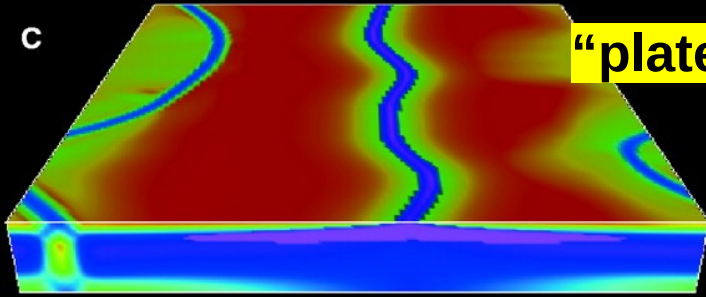
log(Viscosity)

cold temperature (downwellings)

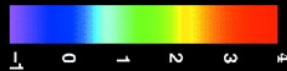
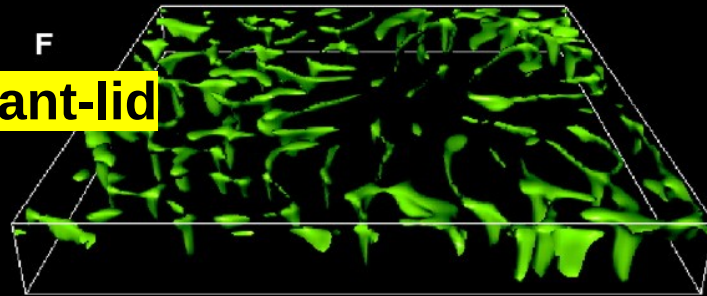
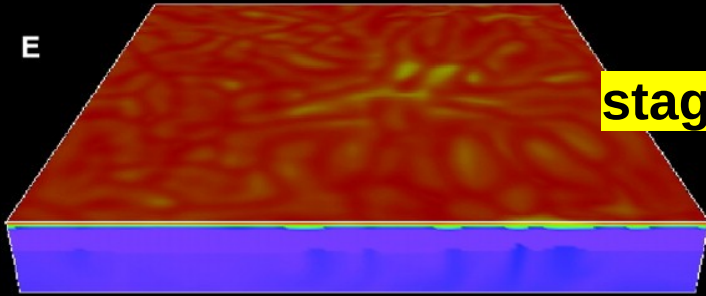
**“sluggish” mobile-lid**



**“plate-like”**



**stagnant-lid**

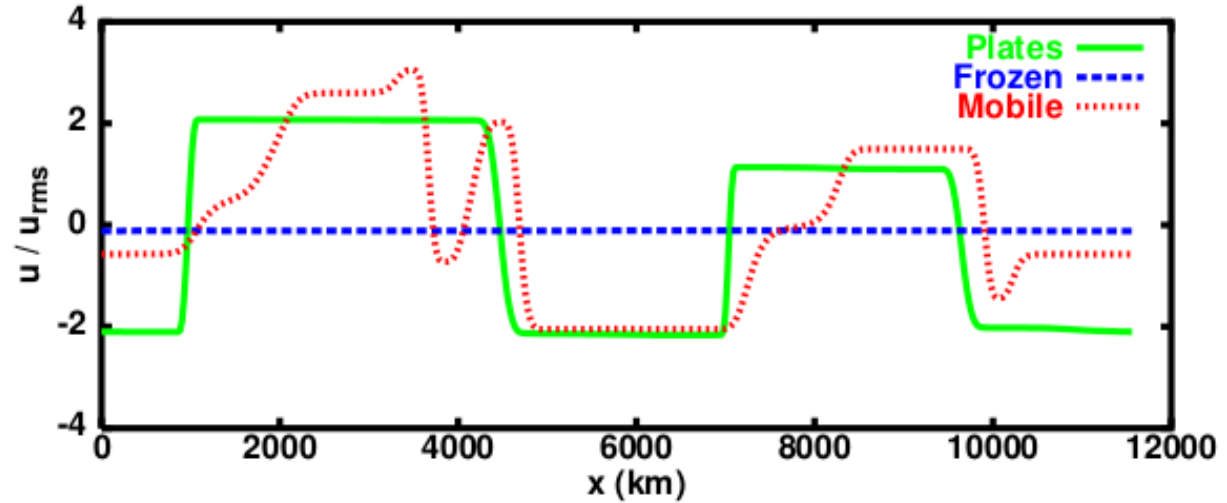


**yield strength =**  
mechanical  
resistance of the  
cold lithospheric  
boundary layer

*Tackley  
Science  
2000*

# Importance of viscosity parameterization

localizing  
deformation at  
plate boundary



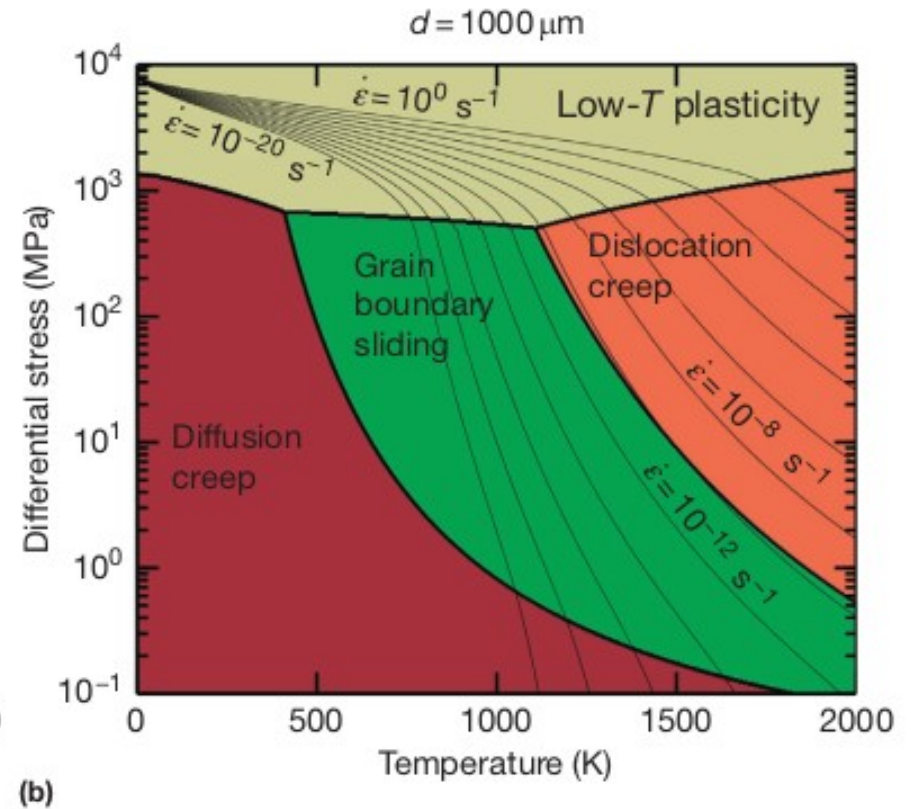
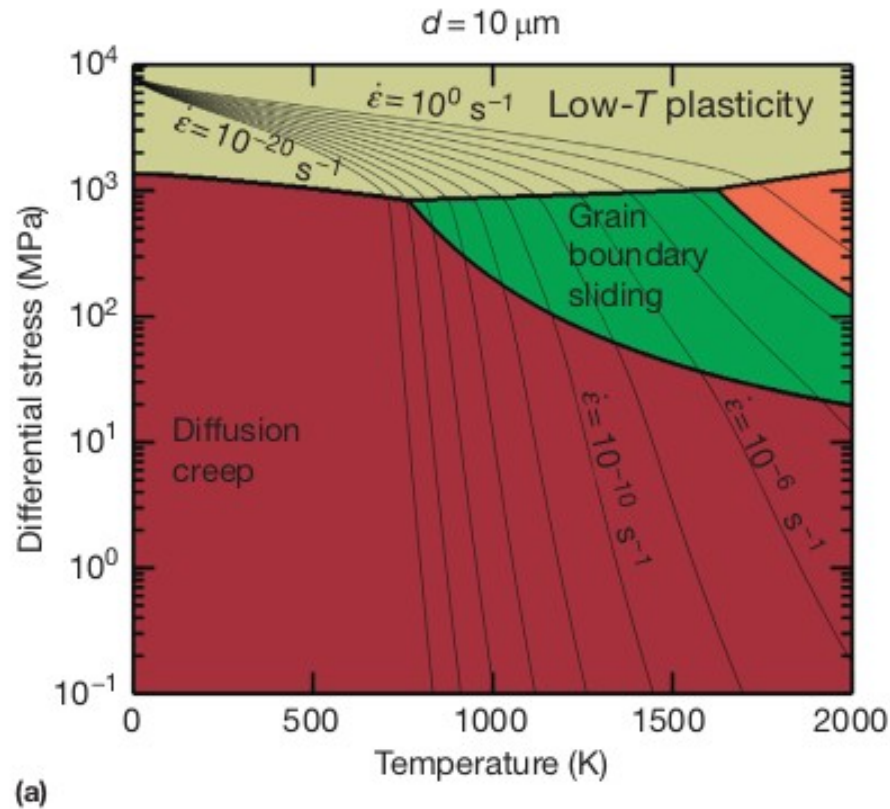
$\eta^* / \sigma_y$ (MPa)	Inf	150	100	80	60	50	30	10
1	●	●	◐	◐	◐	●	●	●
100	●	●	●	◐	◐	◐	●	●
300	●	●	●	■	■	◐	●	●
500	●	●	◐	■	■	◐	●	●
1000	●	●	■	■	■	◐	●	●
10000	●	■	■	■	◐	◐	●	●

● Frozen   
 ■ Plates   
 ● Mobile

lithosphere mobility as a  
function of **yield stress**  $\sigma_y$   
and viscosity contrast  $\eta^*$  for a  
**sub-plate low-viscosity layer**

Richards  $G^3$  2001

# Deformation partitioning (minimal viscosity)



Kohlstedt & Hansen, Treatise on Geophysics, 2015

*... but probably  
coexisting mechanisms rather  
than bimodal partitioning*



# From flow “laws” to viscosity

Several coexisting deformation mechanisms → addition of strain rates ?

$$\dot{\epsilon}_{\text{tot}} = \dot{\epsilon}_{\text{el}} + \dot{\epsilon}_{\text{plastic}}$$

$$\dot{\epsilon}_{\text{plastic}} = \dot{\epsilon}_{\text{brittle}} + \dot{\epsilon}_{\text{ductile}}$$

$$\dot{\epsilon}_{\text{ductile}} = \dot{\epsilon}_{\text{disl}} + \dot{\epsilon}_{\text{diff}}$$

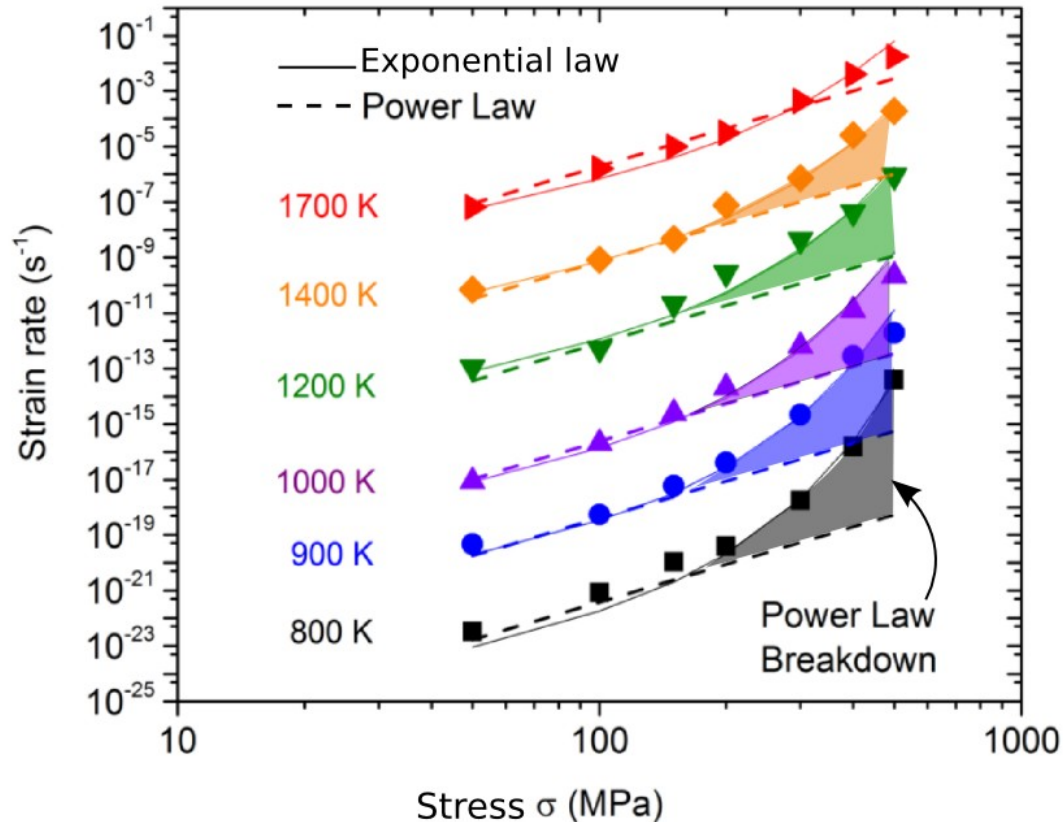
# From flow “laws” to viscosity

Several coexisting deformation mechanisms → addition of strain rates ?

$$\begin{aligned}\mu_{\text{eff}} &= \frac{\tau_{II}}{2\dot{\varepsilon}_{II}} \\ \frac{1}{\mu_{\text{eff}}} &= \frac{2\dot{\varepsilon}_{II}}{\tau_{II}} \\ &= \frac{2\dot{\varepsilon}_{\text{disl}}}{\tau_{II}} + \frac{2\dot{\varepsilon}_{\text{diff}}}{\tau_{II}} \\ &= \frac{1}{\mu_{\text{disl}}} + \frac{1}{\mu_{\text{diff}}} \\ \mu_{\text{eff}} &= \left( \frac{1}{\mu_{\text{disl}}} + \frac{1}{\mu_{\text{diff}}} \right)^{-1}\end{aligned}$$

**pseudo-  
harmonic mean  
for viscosity**

# Reconciling experimental micro-scale rheology and large-scale mantle viscosity (through numerical modelling)



→ semi-empirical exponential fit

$$\dot{\epsilon} = A\sigma^n \exp \left[ -\frac{E_a}{RT} \left( 1 - \left( \frac{\sigma}{\sigma_P} \right)^p \right)^q \right]$$

*Frost and Ashby, 1982*

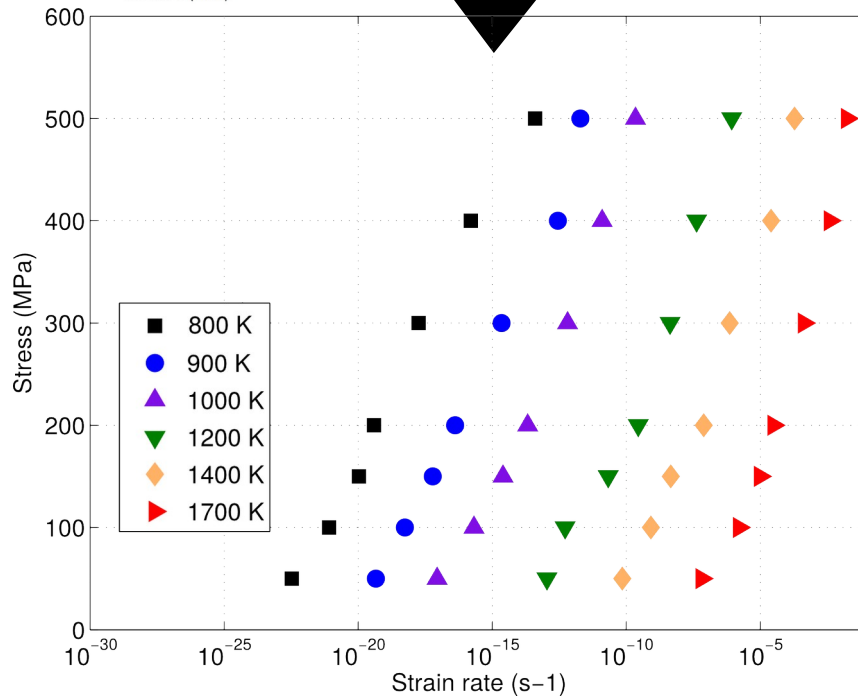
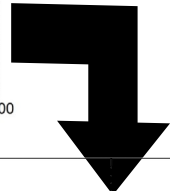
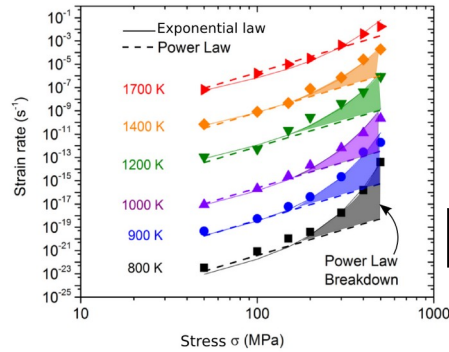
*not modeler-friendly...*

→ need to express stress  
as a function of strain rate  
for **viscosity calculation**

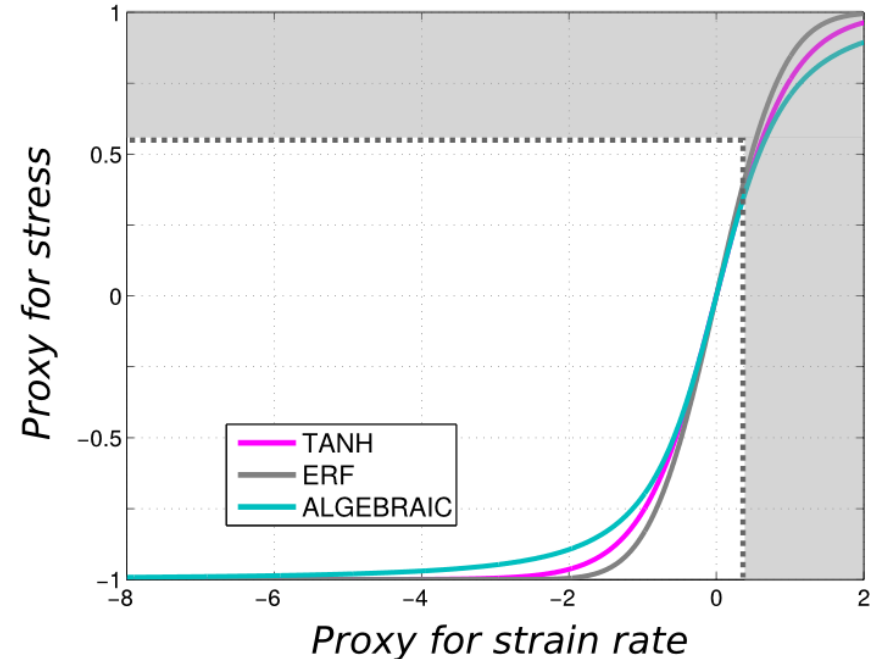
*Gouriet et al., EPSL, 2019*



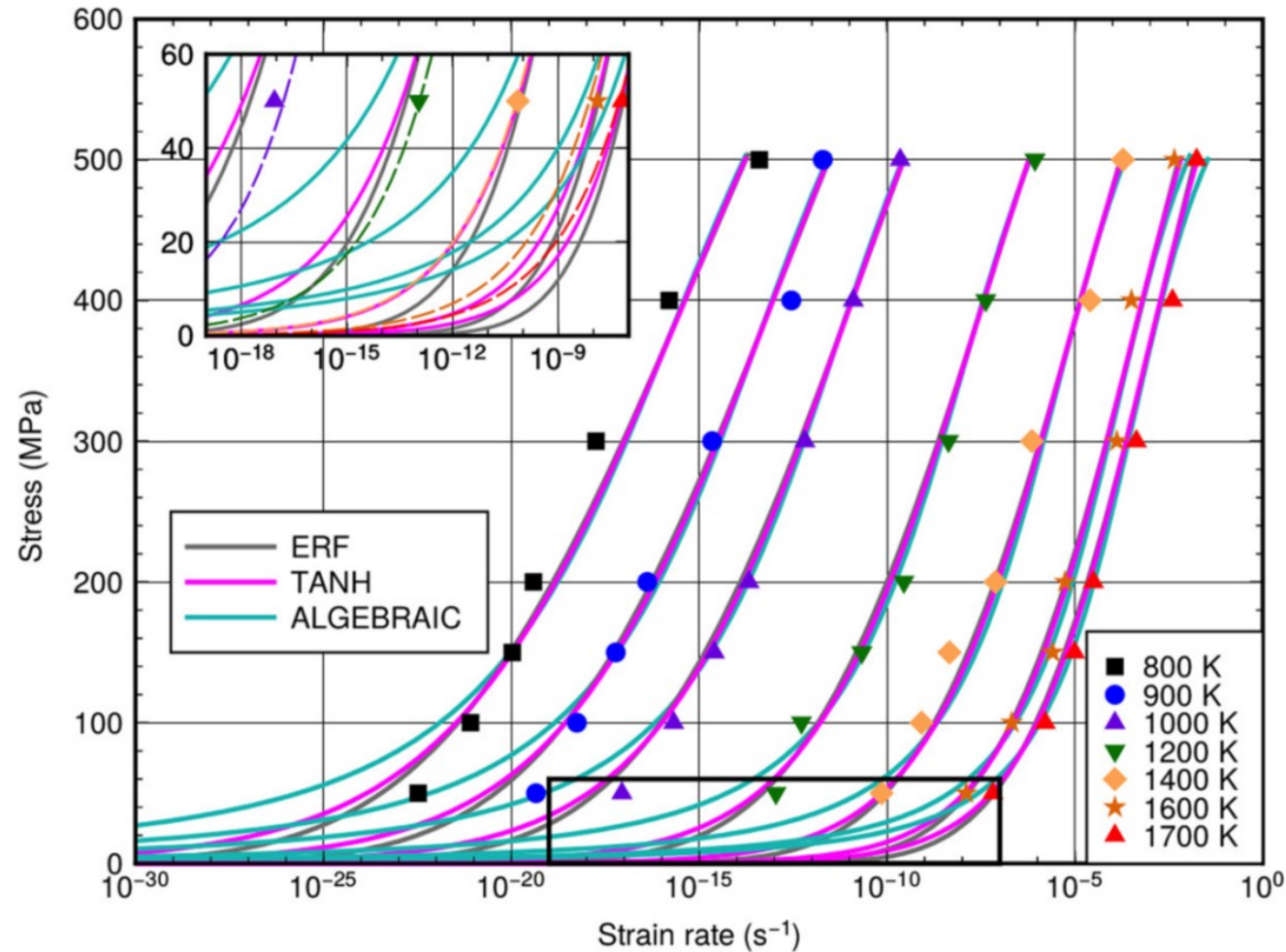
# Reconciling experimental micro-scale rheology and large-scale mantle viscosity (through numerical modelling)



3  $\neq$  sigmoid functions to fit both high-T and low-T data



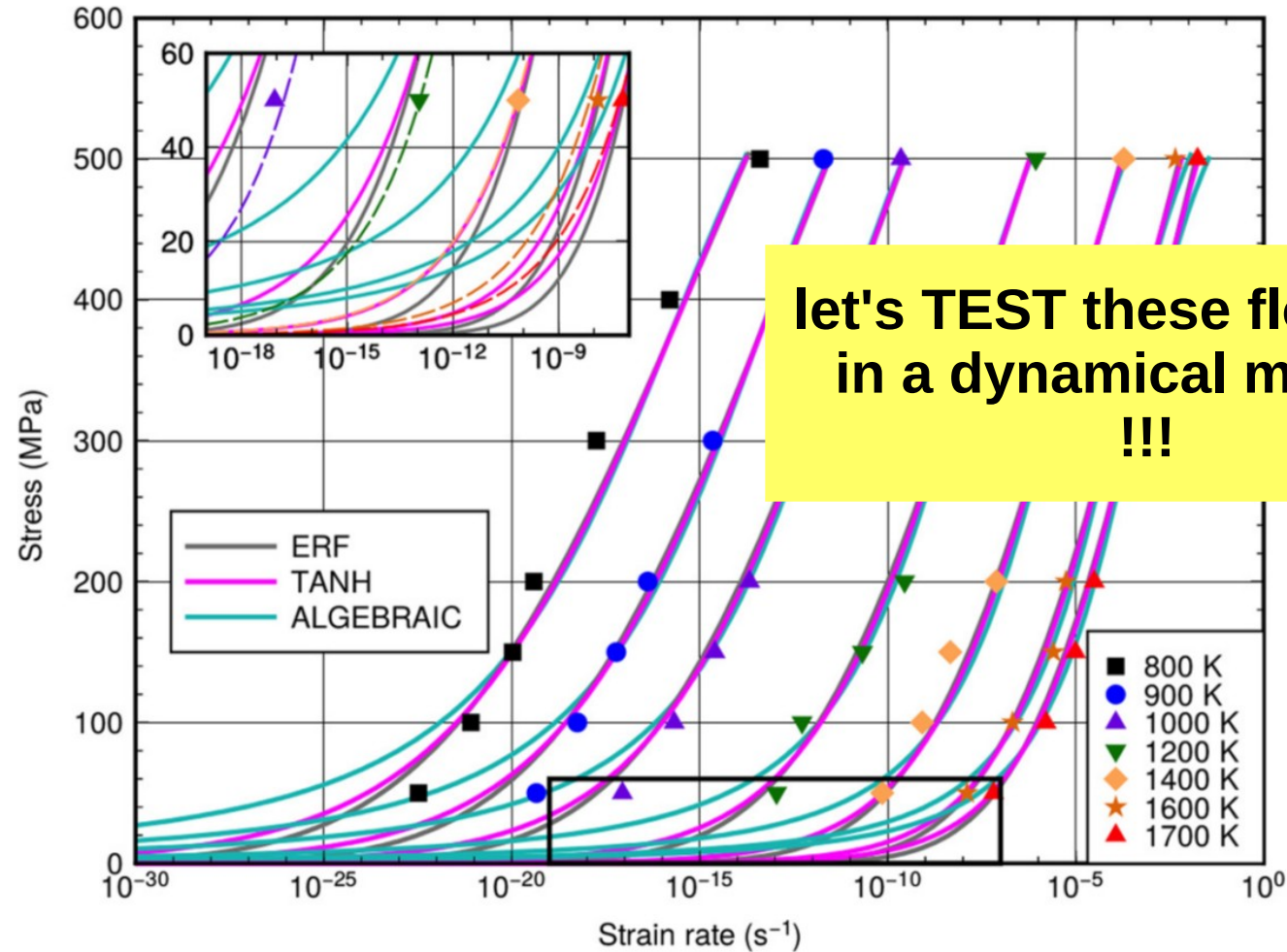
### 3 parameterizations for dislocation creep rheology... *fitting equally well the data !*



Garel et al.,  
EPSL, 2020



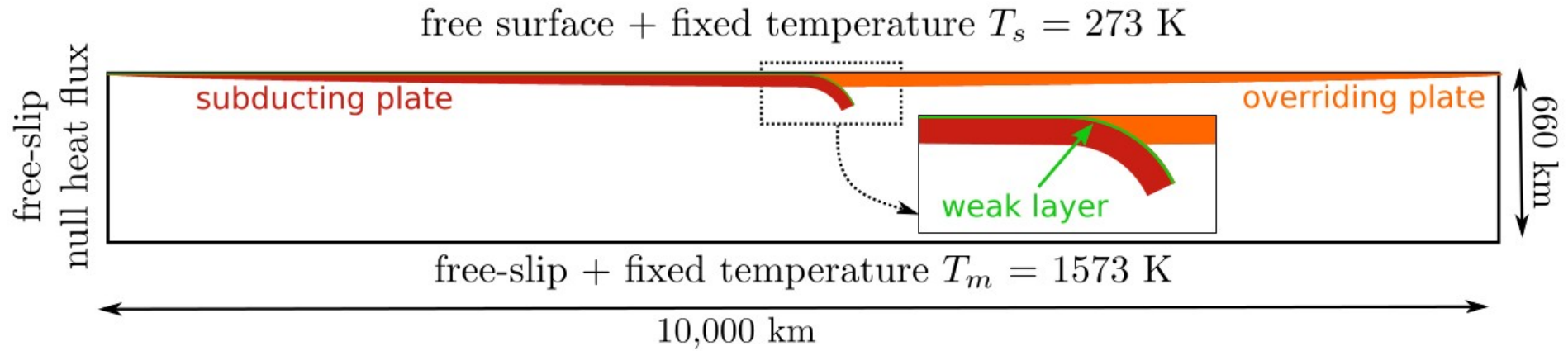
### 3 parameterizations for dislocation creep rheology... *fitting equally well the data !*



*Garel et al.,  
EPSL, 2020*



# Free subduction model



## Composite rheology (addition of strain rates)

- pseudo-brittle / yield stress
- diffusion creep

$$\frac{1}{\mu} = \frac{1}{\mu_{diff}} + \frac{1}{\mu_{disl}} + \frac{1}{\mu_{br}}$$

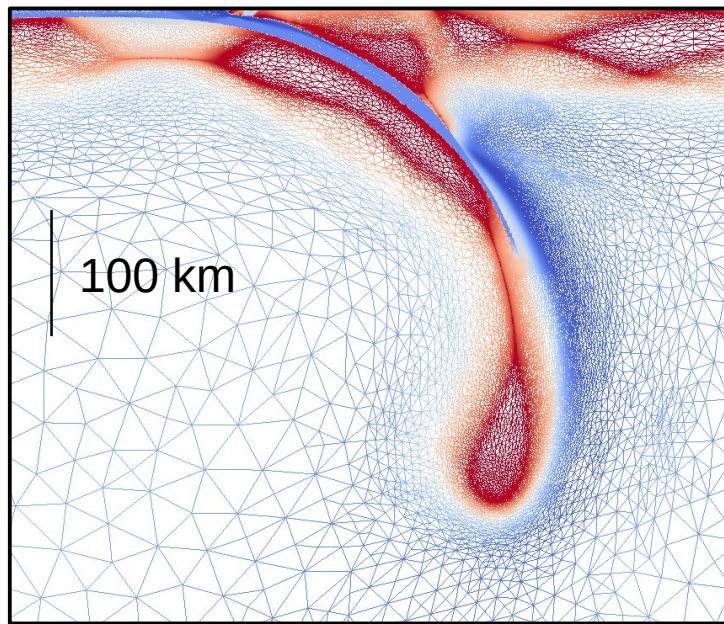
- 1 single parameterization for dislocation creep  
(continuous from low to high temperature)

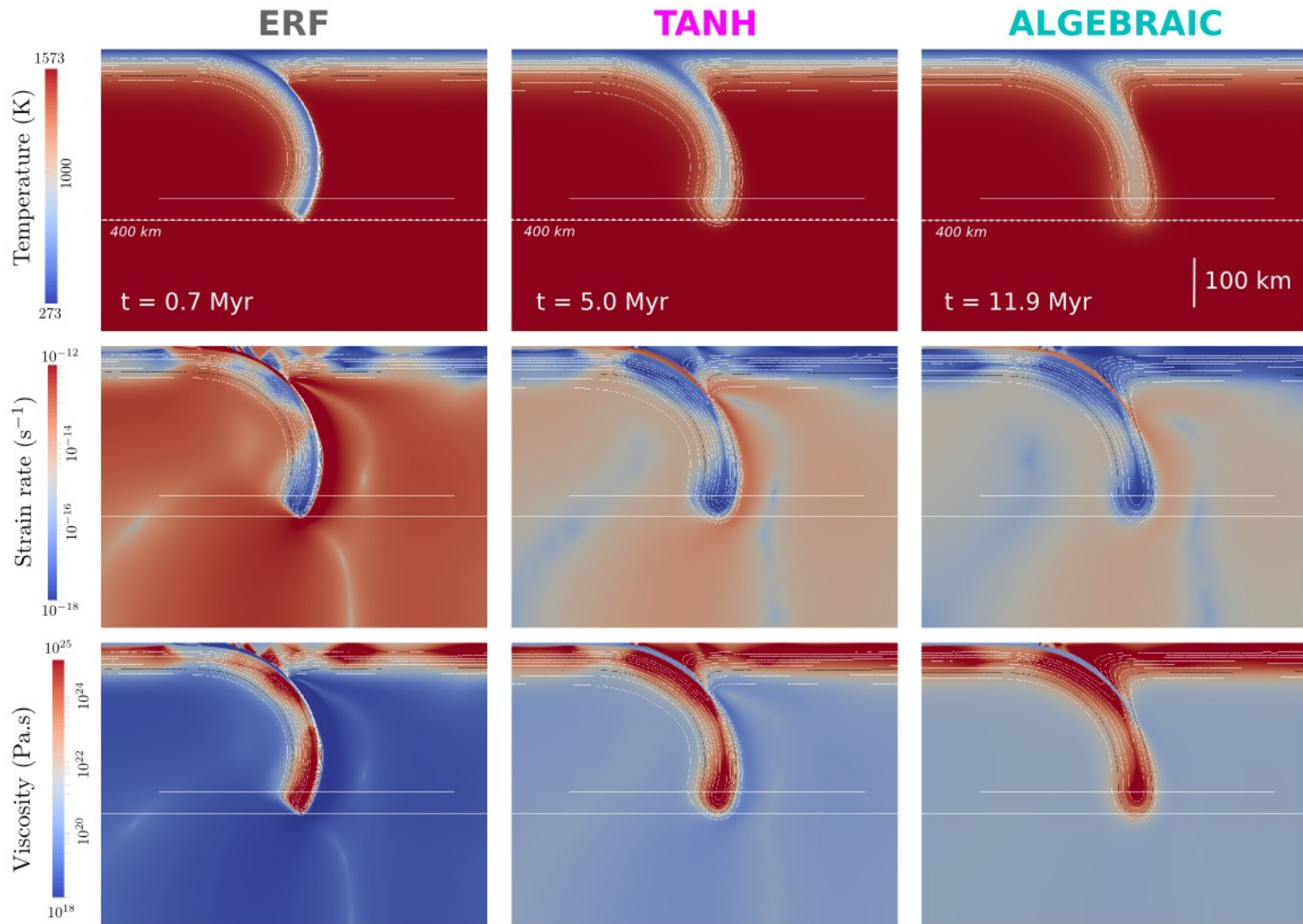
~~BEFORE : 2 dislocation creeps (high-T and Peierls low-T)~~



*Davies et al., G3, 2011*  
*Kramer et al., PEPI, 2012*

- **finite-element**, parallel-running code **solving conservation equations**
- developed by the **AMCG group** (Imperial College London)
- automatic **adaptive meshing** depending on spatial variations of temperature, velocity, viscosity...



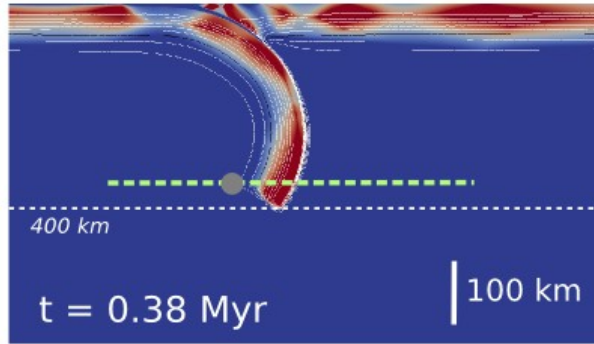


**VERY**  
contrasted  
subduction  
dynamics

*Garel et al.,  
EPSL, 2020*



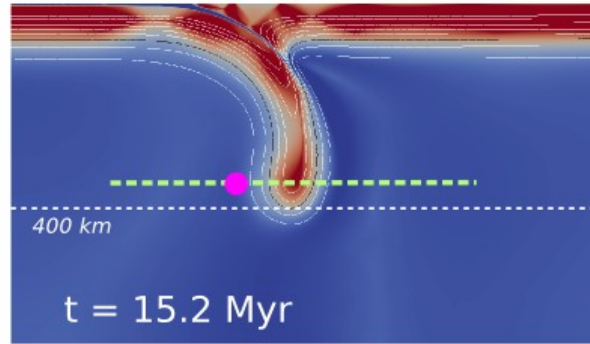
## ERF



### Sinking ++

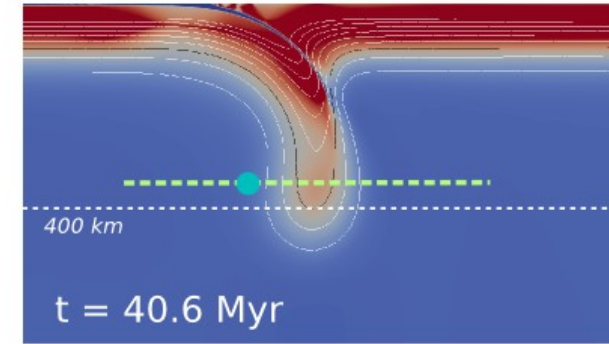
- very weak asthenosphere
- high asth. strain rates
- very fast subduction
- cold, thin slab

## TANH



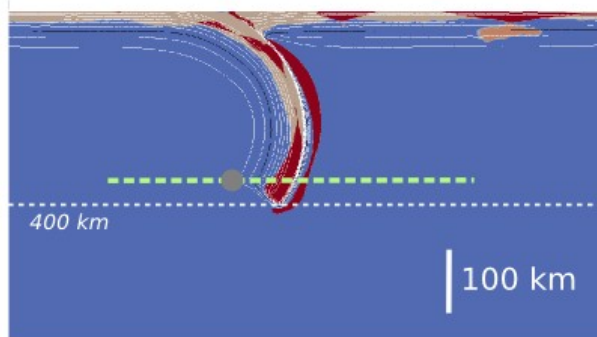
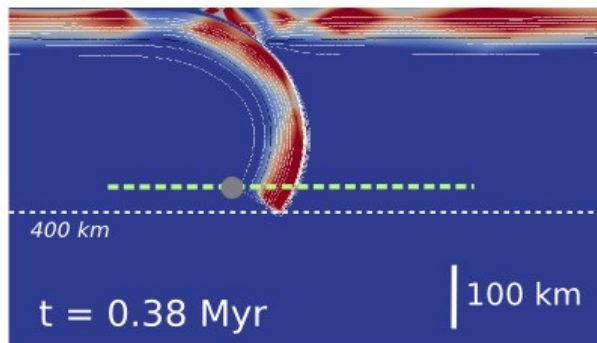
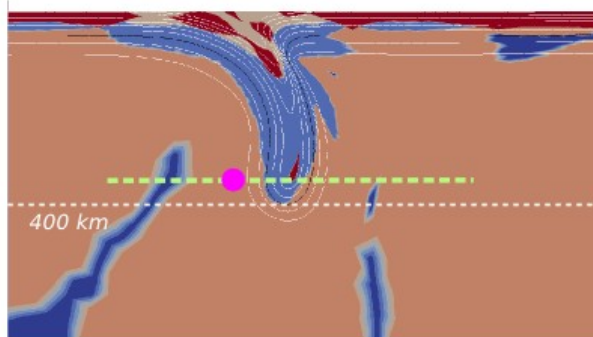
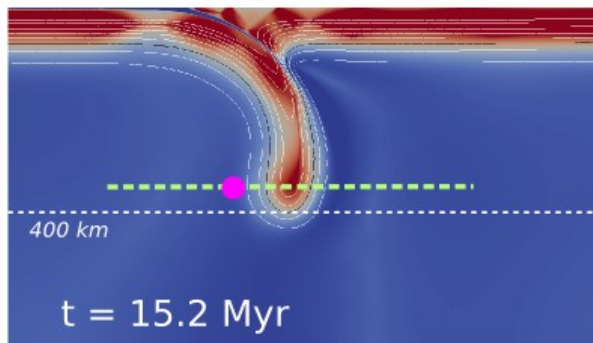
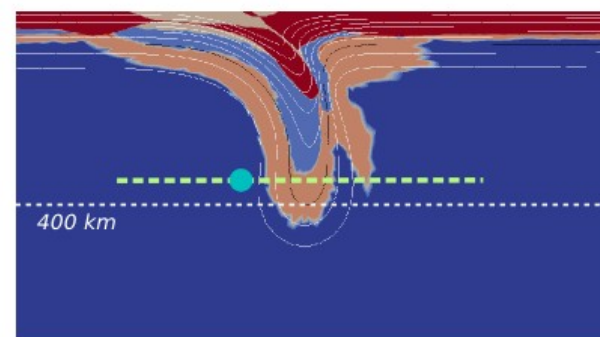
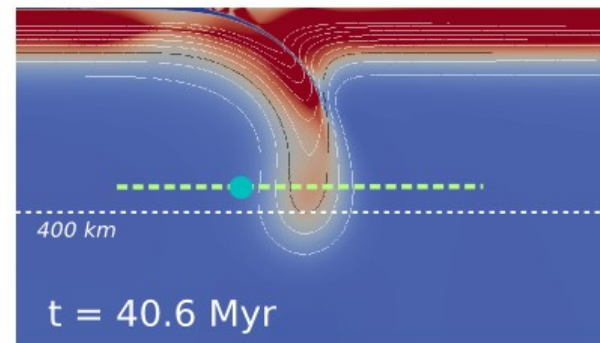
### Diffusive AND sinking

## ALGEBRAIC



### Thermal diffusion++

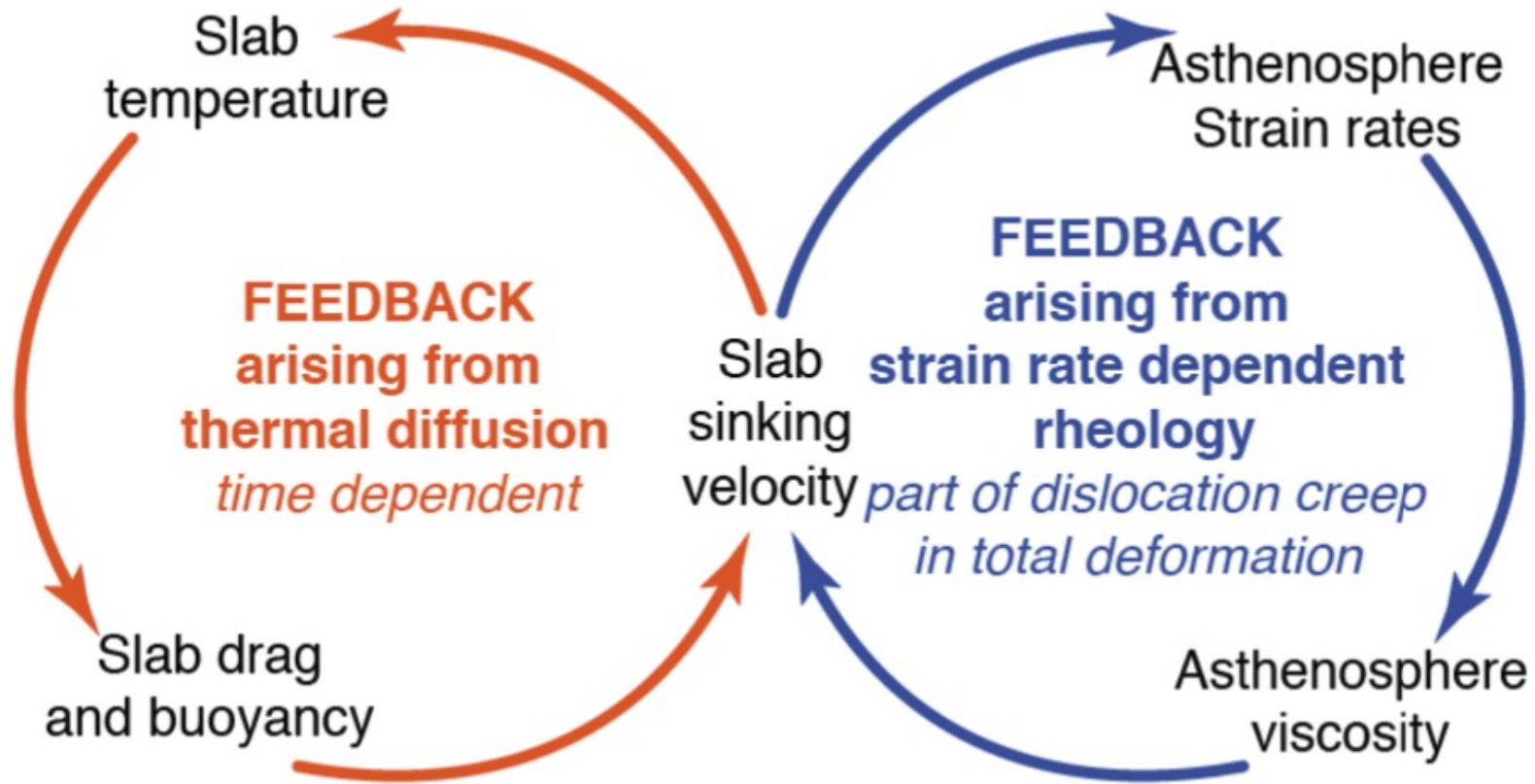
- high-viscosity asth.
- low asth. strain rates
- very slow subduction
- hot and wide slab

**ERF****TANH****ALGEBRAIC****Deformation mechanisms**

**partitioning btn viscosity depending on strain rate**














(disl. creep,  $n > 1$ ) **or not** (diff. creep,  $n=1$ )

*Garel et al., EPSL, 2020*

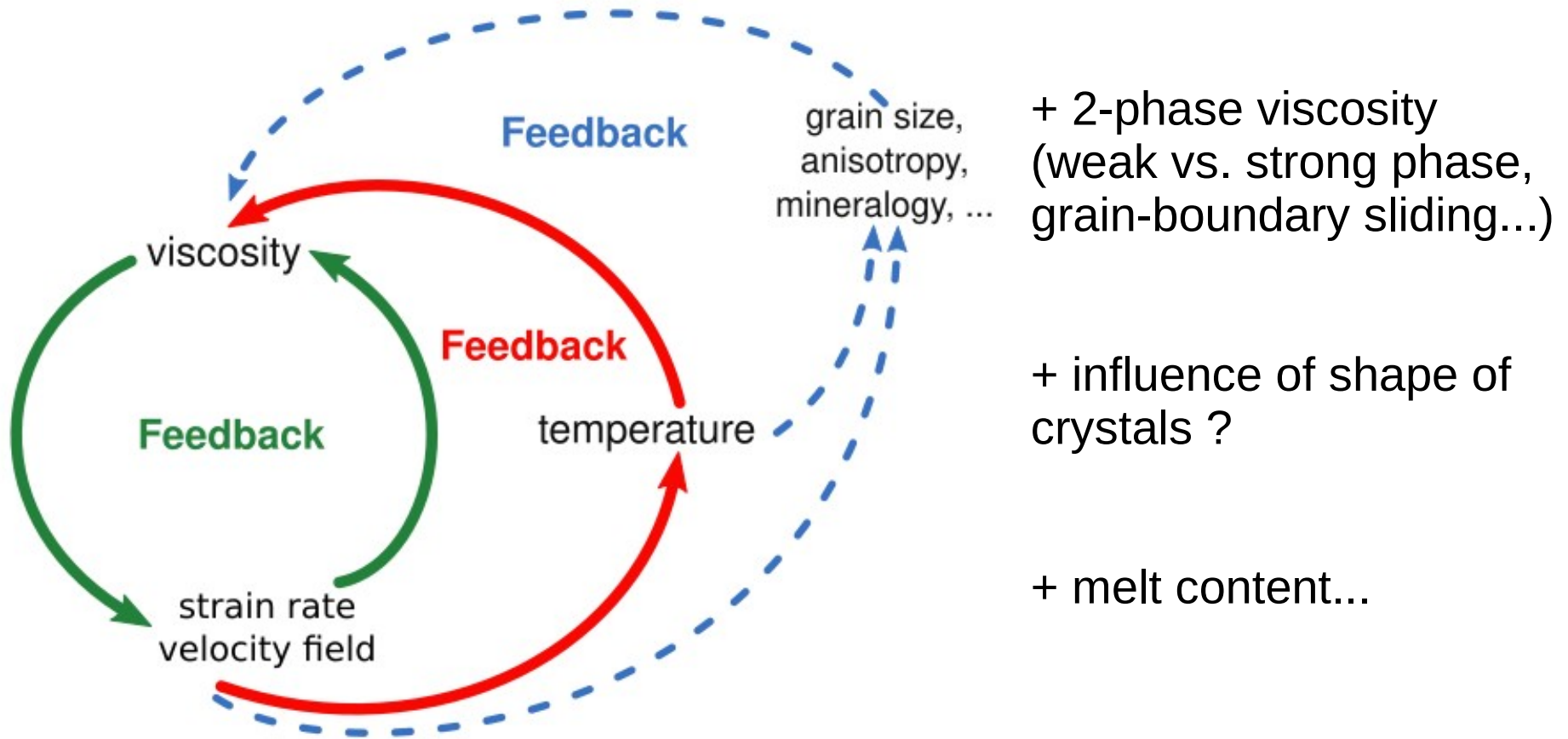




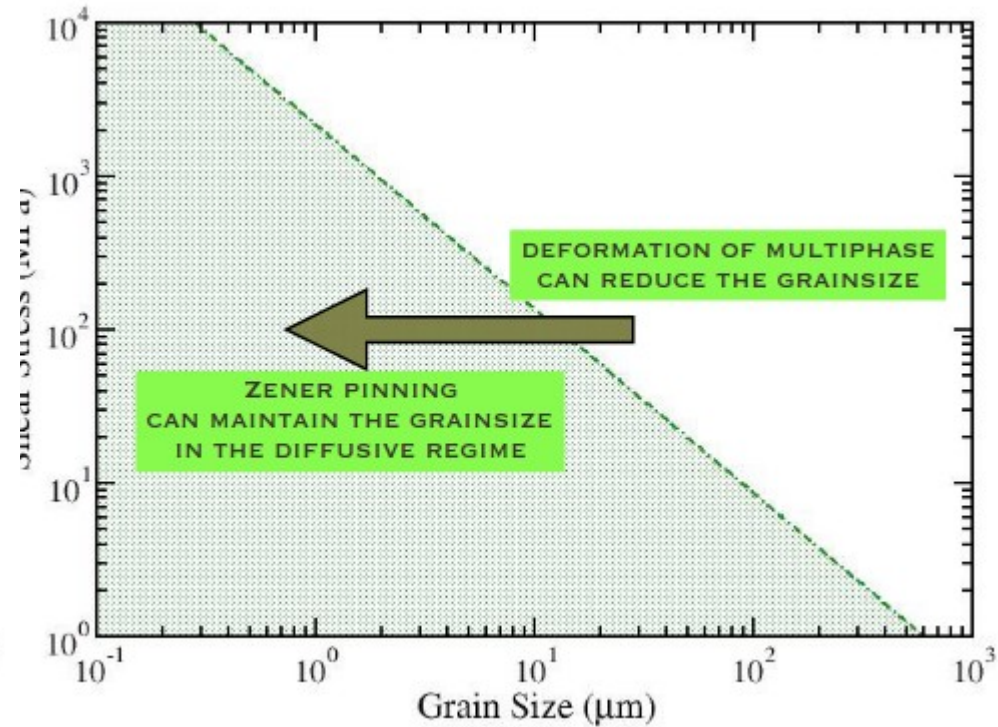
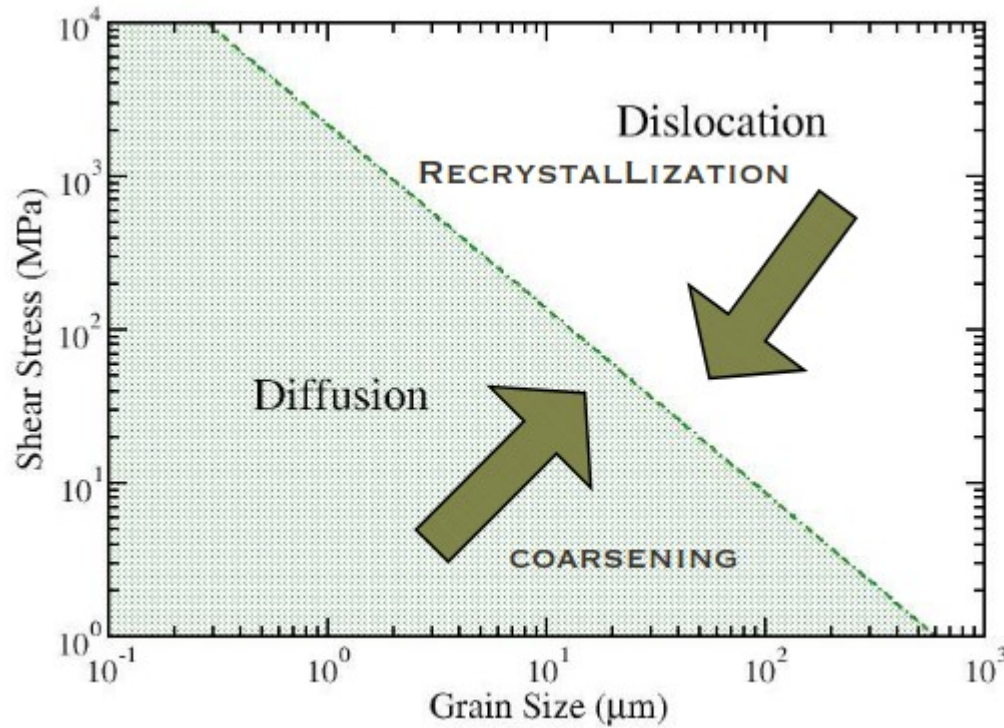
# Comparison with geological time-scales observables

Dislocation creep parameterization	ERF	TANH	ALGEBRAIC
Subduction dynamics	Thin, cold slab in free fall in low-viscosity upper mantle  No time for thermal diffusion		Hot, diffusive slab in high-viscosity asthenosphere  No strain-rate weakening
Time for the slab to reach 400-km	~0.5 Myr	~15 Myr	~40 Myr
Mean asthenosphere viscosity <i>post-glacial rebound and geoid modelling (~10<sup>20-21</sup> Pa.s)</i>	10 <sup>18-20</sup> Pa.s 	10 <sup>20-21</sup> Pa.s 	10 <sup>20-21</sup> Pa.s 
surface velocity of the subducting plate <i>Paleomagnetism and GPS data (1-10 cm/yr)</i>	~200 cm/yr 	~1.5 cm/yr 	~0.7 cm/yr 
Slab morphology <i>tomographic imaging</i>			
Dislocation creep in the upper mantle <i>Seismic anisotropy</i>			

# Feedbacks between deformation and dynamics



# Feedbacks between deformation and dynamics



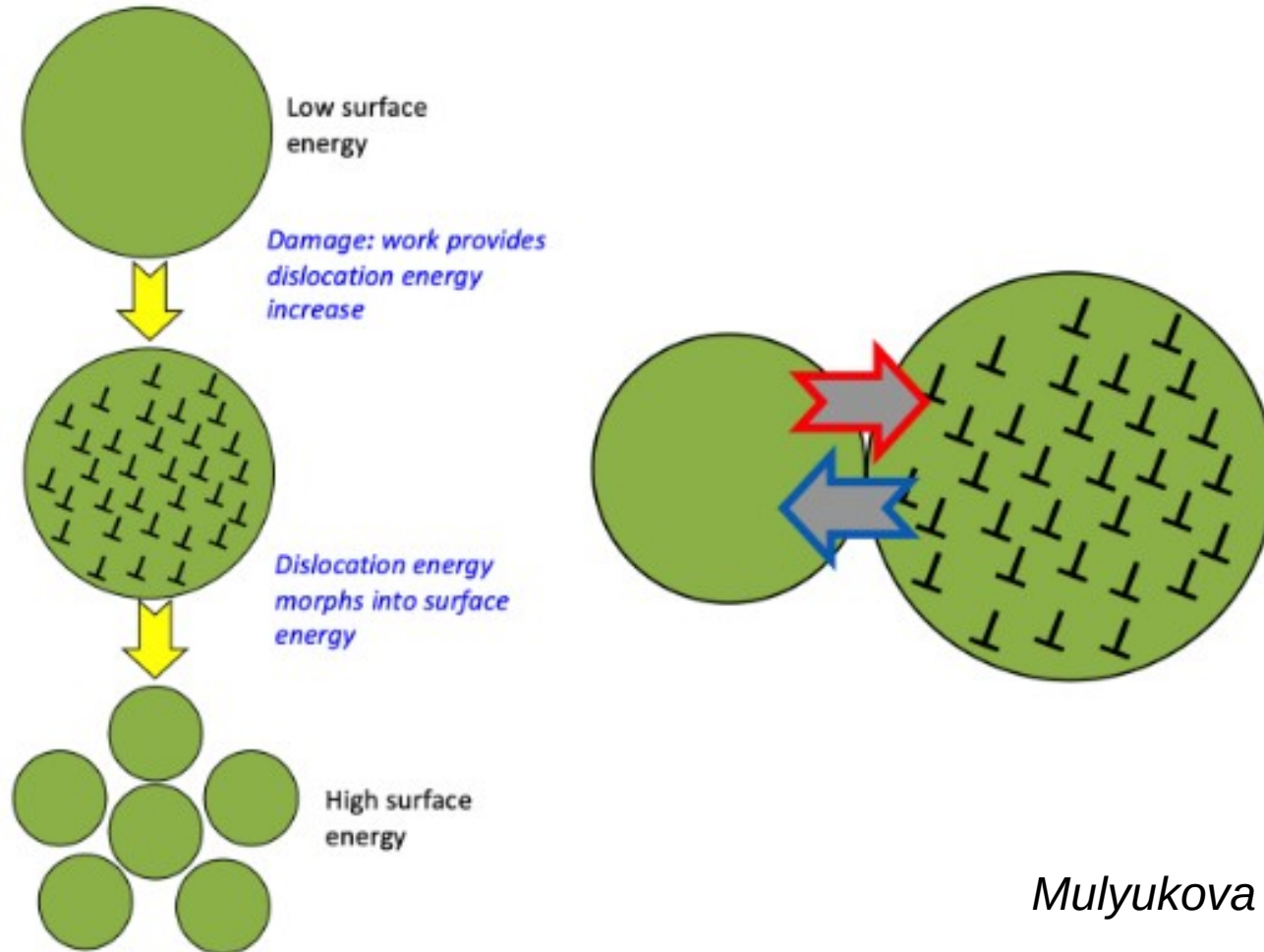
deformation-induced GS reduction + grain-size-dep viscosity (**diffusion creep**)

→ **deformation localization & plate tectonics**

Ricard + Bercovici  
2012, 2013, 2014, 2016

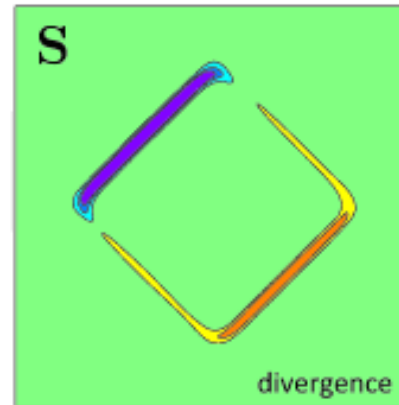
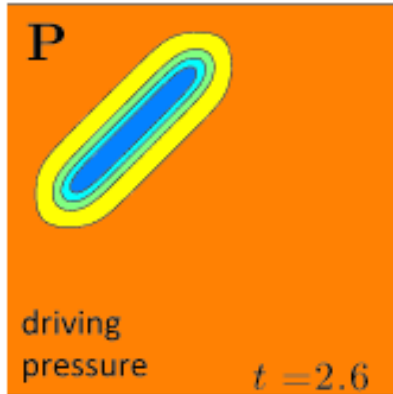
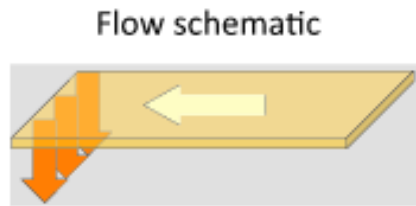


# Feedbacks between deformation and dynamics

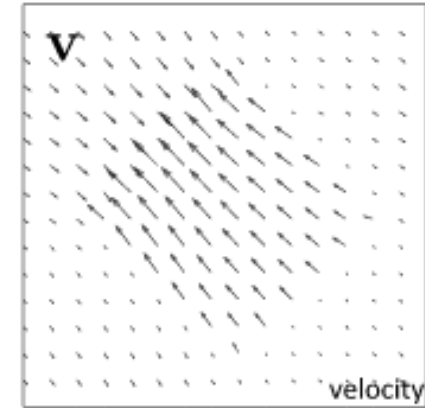


*Mulyukova & Bercovici, 2019*

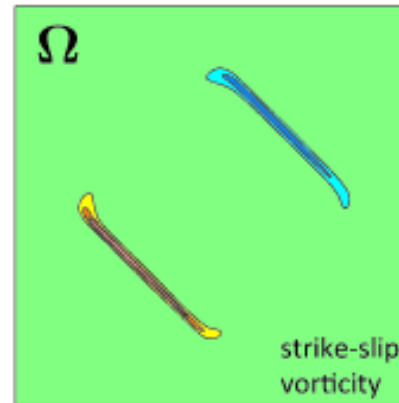
# Feedbacks between deformation and dynamics



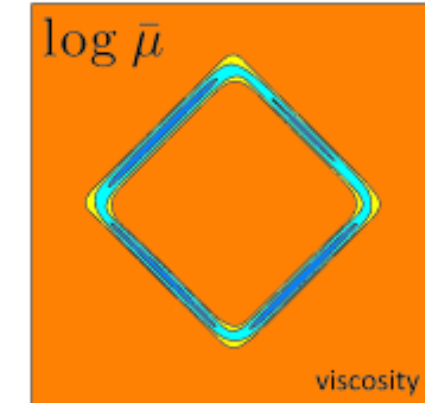
$$S_{min/max} = -1.269/0.496$$



$$v_{max} = 0.044$$



$$\Omega_{min/max} = -0.848/1.257$$

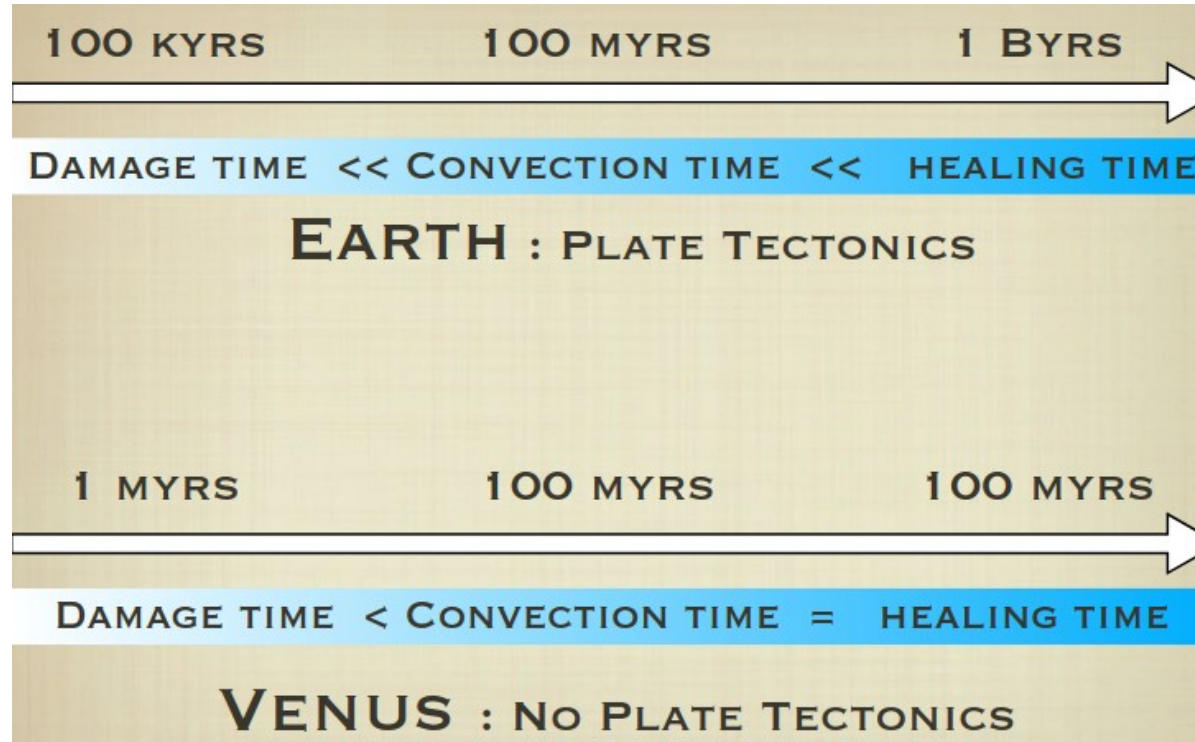


$$\mu_{min/max} = 0.0388/10.38$$

*Mulyukova & Bercovici, 2019*

Ricard +Bercovici  
2012, 2013, 2014, 2016

# Feedbacks between deformation and dynamics



deformation-induced GS reduction + grain-size-dep viscosity (**diffusion creep**)

→ **deformation localization & plate tectonics**

Ricard + Bercovici  
2012, 2013, 2014, 2016