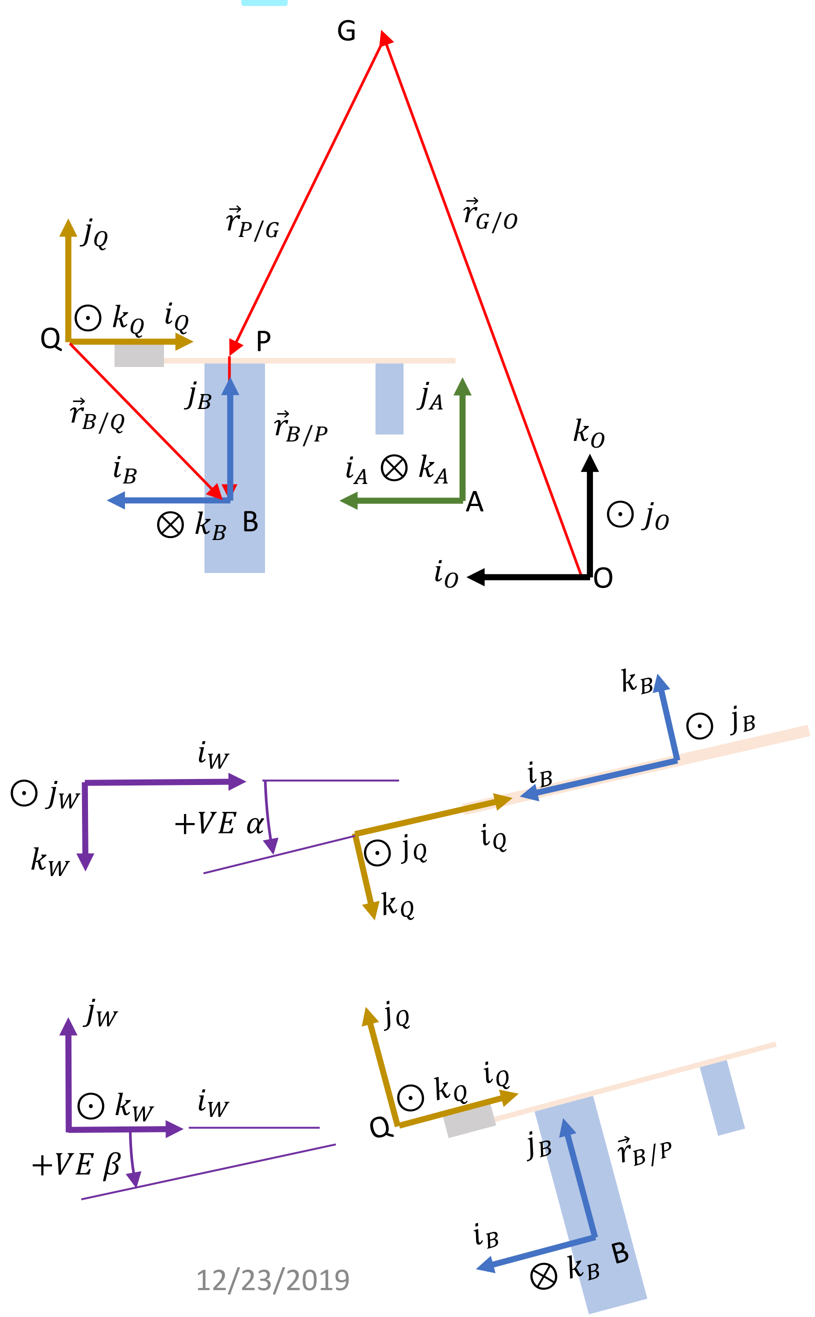
Frames:



Frames:

1. O frame, inertial
2. A frame, translates with sail, constant angle wrt O frame, still inertial
3. B frame, attached to sail at CM=B, rotates wrt O, A frames (roll, pitch, yaw)
4. Q frame, AVL body frame of the sail, fixed wrt B frame
5. W frame, relative wind frame, rotates wrt B frame (alpha, beta), relative wind along iW always

Positions:

$$\{\vec{r}_{G/O}\}_O = \begin{Bmatrix} 0 \\ L_1 \\ 0 \end{Bmatrix}$$

$$\{\vec{r}_{P/B}\}_B = \begin{Bmatrix} x_{PB} \\ y_{PB} \\ 0 \end{Bmatrix}$$

$$\{\vec{r}_{Q/B}\}_B = \begin{Bmatrix} x_{QB} \\ y_{QB} \\ 0 \end{Bmatrix}$$

$$\{\vec{r}_{B/O}\}_B = \begin{Bmatrix} x_{BO} \\ y_{BO} \\ z_{BO} \end{Bmatrix}$$

$$\vec{r}_{B/O} \stackrel{\text{def}}{=} \vec{r}_{G/O} + \vec{r}_{P/G} + \vec{r}_{B/P}$$

$$\therefore \vec{r}_{G/P} = \vec{r}_{G/O} + \vec{r}_{B/P} - \vec{r}_{B/O}$$

Angles:

$${}^{A}[C]^{O} = [R(\pi/2)]_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(\pi/2) & s(\pi/2) \\ 0 & -s(\pi/2) & c(\pi/2) \end{bmatrix}$$

$${}^{Q}[C]^{B} = [R(\pi)]_Y = \begin{bmatrix} c(\pi) & 0 & -s(\pi) \\ 0 & 1 & 0 \\ s(\pi) & 0 & c(\pi) \end{bmatrix}$$

$${}^{B}[C]^{A} = [R\phi]_X [R\psi]_Z [R\theta]_Y$$

$$[R\phi]_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(\phi) & s(\phi) \\ 0 & -s(\phi) & c(\phi) \end{bmatrix}, [R\psi]_Z = \begin{bmatrix} c(\psi) & s(\psi) & 0 \\ -s(\psi) & c(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[R\theta]_Y = \begin{bmatrix} c(\theta) & 0 & -s(\theta) \\ 0 & 1 & 0 \\ s(\theta) & 0 & c(\theta) \end{bmatrix}$$

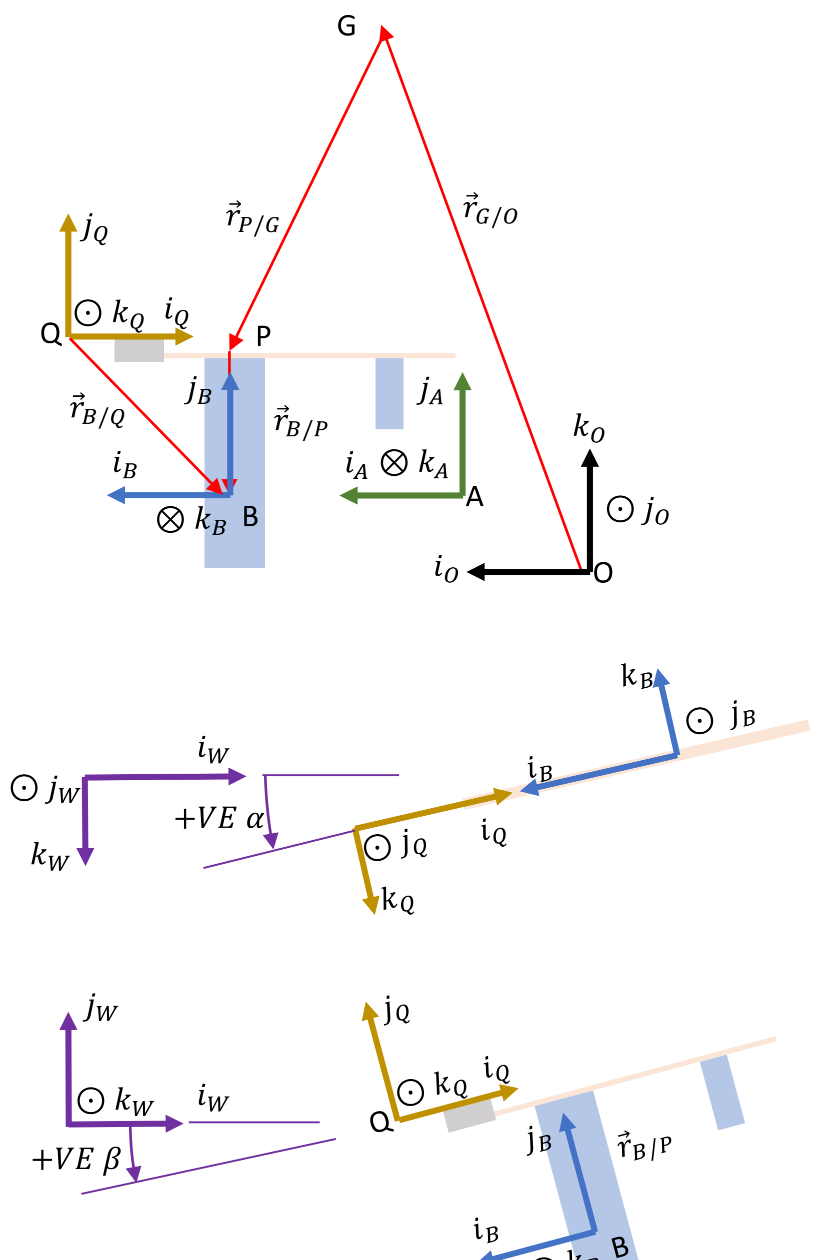
Goal:

Derive EOMs for Sail (for B wrt O)

Constants:

$L_1, x_{PB}, y_{PB}, x_{QB}, y_{QB}$

Frames:



12/23/2019

Velocities:

$${}^{O}\vec{v}_{B/O} = {}^{B}\vec{v}_{B/O} + {}^{O}\vec{\omega}^{B} \times \vec{r}_{B/O}$$

Inertia:

$$\{\tilde{I}_{CM}\}_Q = \begin{bmatrix} I_{xx} & I_{xy} & 0 \\ I_{xy} & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

NOTE: This is an awkward frame to express I_CM in. Suggest obtaining the moments of inertia in B fame directly about CM.

$$\{\tilde{I}_{CM}\}_B = {}^{B}[C]^{Q}\{\tilde{I}_{CM}\}_Q\, {}^{Q}[C]^{B}$$

Angular Velocities:

$${}^{B}[C]^{O} = {}^{B}[C]^{A}\, {}^{A}[C]^{O}$$

$${}^{O}\vec{\omega}^{B} \cdot i_B \stackrel{\text{def}}{=} \{0 \quad 0 \quad 1\}\, {}^{B}[C]^{O}\, {}^{O}[\dot{C}]^{B} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$${}^{O}\vec{\omega}^{B} \cdot j_B \stackrel{\text{def}}{=} \{1 \quad 0 \quad 0\}\, {}^{B}[C]^{O}\, {}^{O}[\dot{C}]^{B} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

$${}^{O}\vec{\omega}^{B} \cdot k_B \stackrel{\text{def}}{=} \{0 \quad 1 \quad 0\}\, {}^{B}[C]^{O}\, {}^{O}[\dot{C}]^{B} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

Angular momentum:

$${}^{O}_{F}\vec{h}_R = \vec{r}_{CM/R} \times m\,{}^{O}\vec{v}_{H/F} + \vec{r}_{H/R} \times m\,{}^{O}\vec{v}_{CM/H} + {}^{U}_{H}\vec{h}_H + \{\tilde{I}_H\}_U \cdot {}^{O}\vec{\omega}^{U}$$

$${}^{O}_{O}\vec{h}_B = \overset{0}{\cancel{\vec{r}_{B/B}}} \times m\,{}^{O}\vec{v}_{B/O} + \overset{0}{\cancel{\vec{r}_{B/B}}} \times m\,{}^{O}\vec{v}_{B/B} + \underset{\text{Rigid body}}{\overset{0}{\cancel{{}^{B}_{B}\vec{h}_B}}} + \{\tilde{I}_B\}_B \cdot {}^{O}\vec{\omega}^{B}$$

$${}^{O}_{O}\vec{h}_B = \{\tilde{I}_B\}_B \cdot {}^{O}\vec{\omega}^{B}$$

Choose:
O = IRF frame = O
F = Point in IRF = O
R = Point of torques = B
CM = Center of mass = B
H = Inertia point = B
U = Body frame = B

NOTE: $\tilde{I}_B = {}^{B}[C]^{Q}\{\tilde{I}_{CM}\}_Q\, {}^{Q}[C]^{B}$ and thus the torques will have angle dots due to the derivative. Need to bookkeep the angle rates separately. Alternate statement without time derivatives of angles is below:

$${}^{O}_{F}\vec{h}_R = \overset{0}{\cancel{\vec{r}_{B/B}}} \times m\,{}^{O}\vec{v}_{B/O} + \overset{0}{\cancel{\vec{r}_{B/B}}} \times m\,{}^{O}\vec{v}_{B/B} + \underset{\text{Rigid body}}{\overset{0}{\cancel{{}^{B}_{B}\vec{h}_B}}} + \{\tilde{I}_B\}_Q \cdot {}^{O}\vec{\omega}^{Q}$$

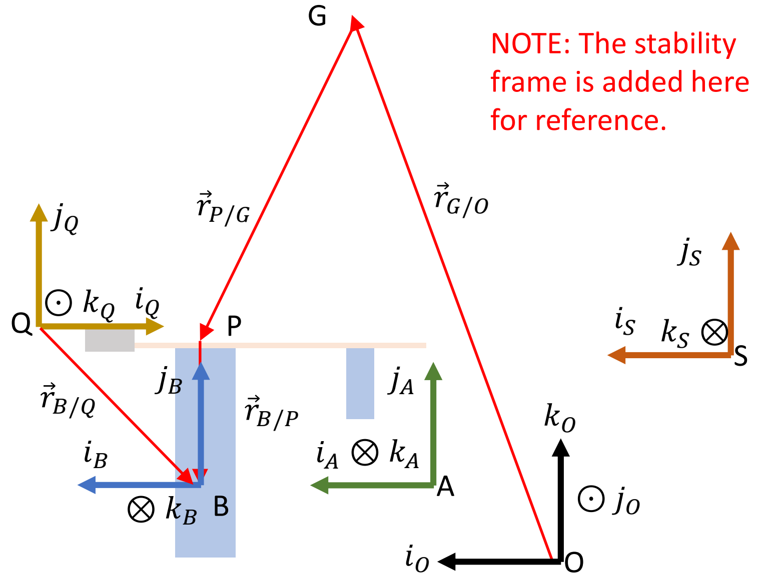
$${}^{O}_{F}\vec{h}_R = \{\tilde{I}_B\}_Q \cdot {}^{O}\vec{\omega}^{Q}$$

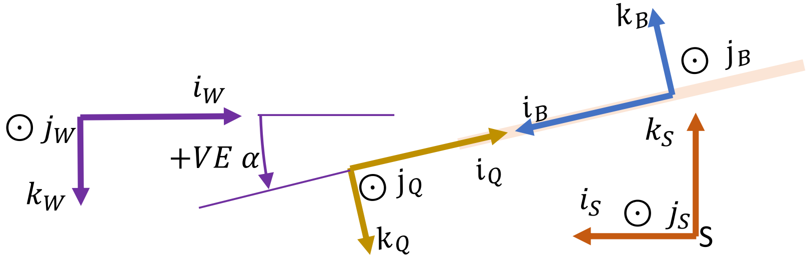
NOTE: This vector is now in the Q frame and will need to be rotated to the B frame for Implementation into the EOMs. Rotation back AFTER the derivatives to torques will be a safe operation.

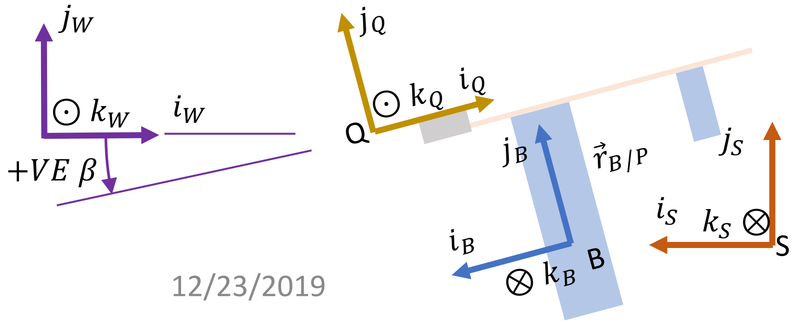
Choose:
O = IRF frame = O
F = Point in IRF = O
R = Point of torques = B
CM = Center of mass = B
H = Inertia point = B
U = Body frame = Q

Frames:

NOTE: The stability frame is added here for reference.







Accelerations:

$$^{O}\vec{a}_{B/O} = {}^{B}\vec{a}_{B/O} + 2\,{}^{O}\vec{\omega}^{B} \times {}^{B}\vec{v}_{B/O} + {}^{O}\vec{\alpha}^{B} \times \vec{r}_{B/O} + {}^{O}\vec{\omega}^{B} \times \left({}^{O}\vec{\omega}^{B} \times \vec{r}_{B/O}\right)$$

Forces:

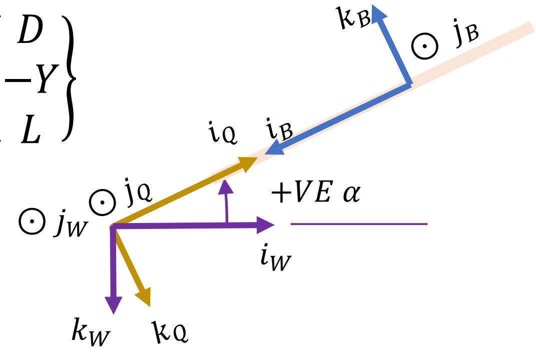
$$\sum \vec{F} = m_T\,{}^{O}\vec{a}_{B/O} = \vec{T} + \vec{F}_{aero} + \vec{W}$$

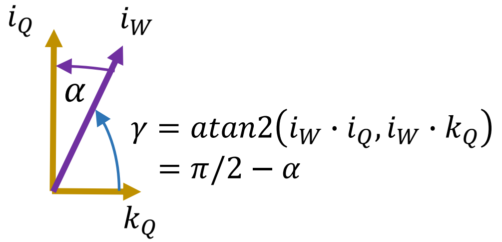
NOTE: Remember that the wind frame W is aligned such that the relative wind is always along iW.

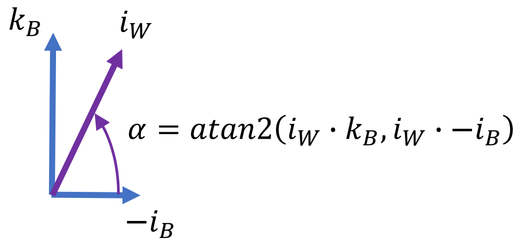
$$\vec{T} = \left[k\left(\|\vec{r}_{G/P}\| - L_2\right) + c\frac{d}{dt}\left(\|\vec{r}_{G/P}\| - L_2\right)\right]\vec{r}_{G/P}/\|\vec{r}_{G/P}\|$$

$$\left\{\vec{F}_{aero}\right\}_S = \begin{Bmatrix} -D \\ -Y \\ -L \end{Bmatrix} \rightarrow \left\{\vec{F}_{aero}\right\}_W = \begin{Bmatrix} D \\ -Y \\ L \end{Bmatrix}$$

$$\vec{W} = -mgk_O$$



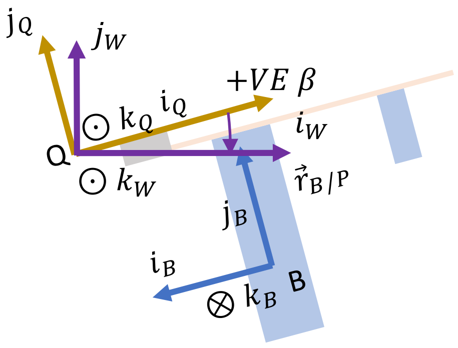


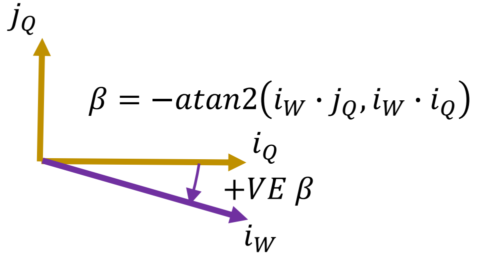


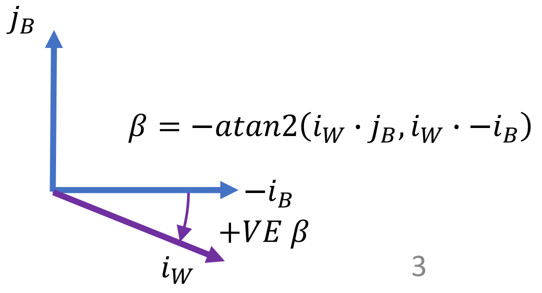
Relative wind:

$$\vec{v}_{rel} = \vec{v}_{inertial} - {}^{O}\vec{v}_{B/O}$$
$$= {}^{Q}[C]^{O}\{\vec{v}_{iner}\}_O - {}^{Q}[C]^{B}\left\{{}^{O}\vec{v}_{B/O}\right\}_B$$

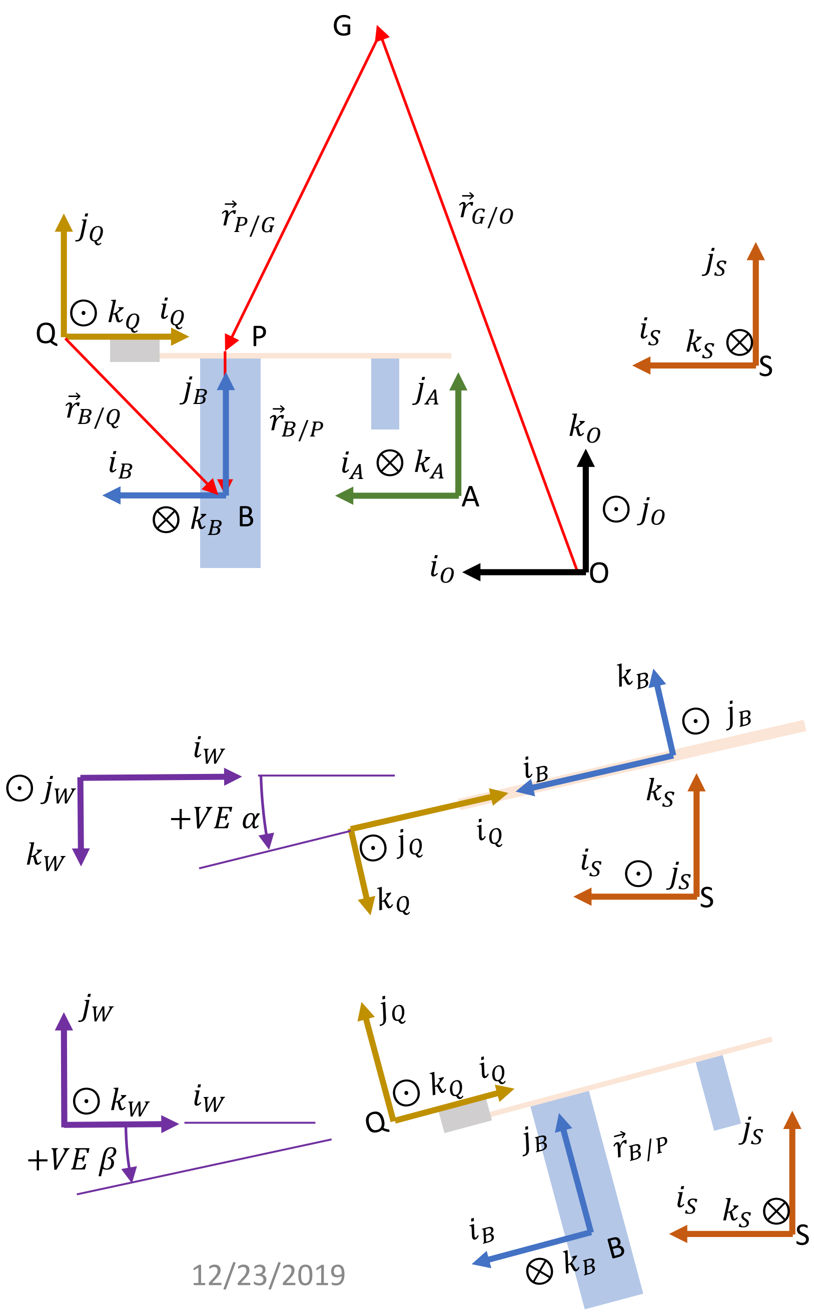
NOTE: α and β can be found using the B frame instead of the Q frame, see alternate definitions.







Frames:



Moments:

$$ {}^{O}_{O}\vec{h}_B = \{\tilde{I}_B\}_B \cdot {}^{O}\vec{\omega}^{B} $$

$$ \frac{{}^{O}d}{dt}\left({}^{O}_{O}\vec{h}_B\right) = \frac{{}^{B}d}{dt}\left(\{\tilde{I}_B\}_B \cdot {}^{O}\vec{\omega}^{B}\right) + {}^{O}\vec{\omega}^{B} \times \left(\{\tilde{I}_B\}_B \cdot {}^{O}\vec{\omega}^{B}\right) = \sum \tau_B $$

$$ \sum \tau_B = \vec{M}_{aero} + \vec{M}_T $$

$$ \vec{M}_{aero} = \begin{Bmatrix} l \\ m \\ n \end{Bmatrix}_S = \begin{Bmatrix} -l \\ m \\ -n \end{Bmatrix}_W $$

$$ \vec{M}_T = \vec{r}_{P/B} \times \vec{T} $$

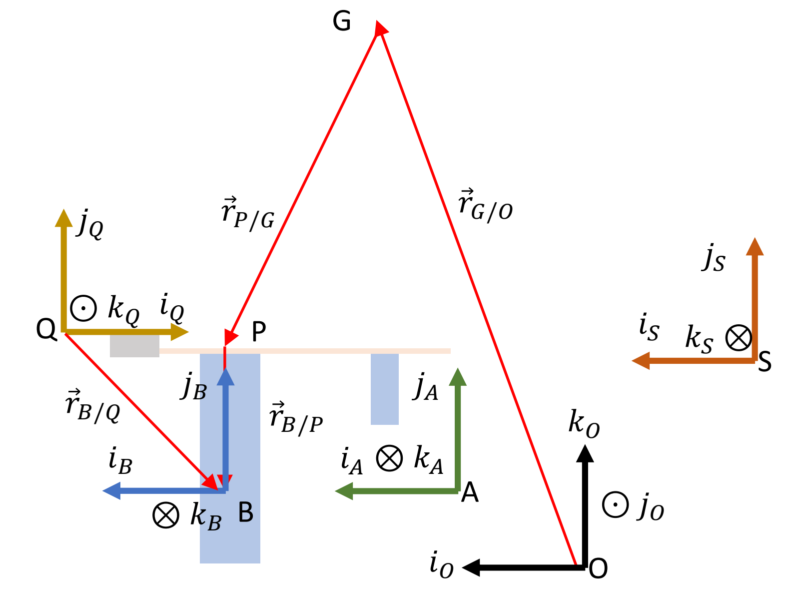
NOTE: Using the alternate frame for angular momentum, the equations for torques would be (assuming preference for B frame):

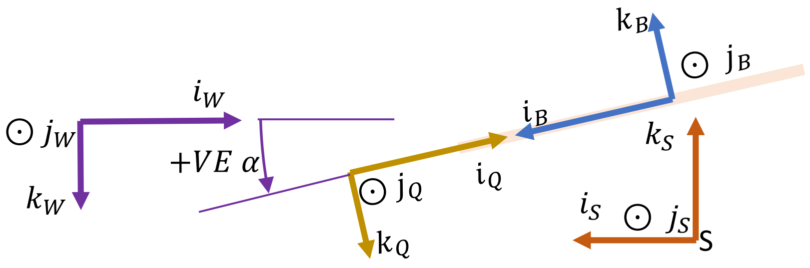
$$ {}^{O}_{O}\vec{h}_B = \{\tilde{I}_B\}_Q \cdot {}^{O}\vec{\omega}^{Q} $$

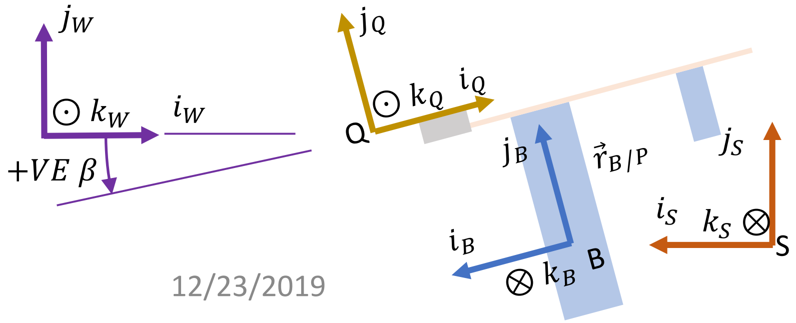
$$ \frac{{}^{O}d}{dt}\left({}^{O}_{O}\vec{h}_B\right) = \frac{{}^{Q}d}{dt}\left(\{\tilde{I}_B\}_Q \cdot {}^{O}\vec{\omega}^{Q}\right) + {}^{O}\vec{\omega}^{Q} \times \left(\{\tilde{I}_B\}_Q \cdot {}^{O}\vec{\omega}^{Q}\right) $$

$$ {}^{B}[C]^{Q}\left(\frac{{}^{Q}d}{dt}\left(\{\tilde{I}_B\}_Q \cdot {}^{O}\vec{\omega}^{Q}\right) + {}^{O}\vec{\omega}^{Q} \times \left(\{\tilde{I}_B\}_Q \cdot {}^{O}\vec{\omega}^{Q}\right)\right) = \sum \tau_B $$

Frames:







States:

$x_{BO}, y_{BO}, z_{BO}, u_{BO}, v_{BO}, w_{BO}, \theta, \phi, \psi, p, q, r$

State derivatives:

$\dot{x}_{BO}, \dot{y}_{BO}, \dot{z}_{BO}, \dot{u}_{BO}, \dot{v}_{BO}, \dot{w}_{BO}, \dot{\theta}, \dot{\phi}, \dot{\psi}, \dot{p}, \dot{q}, \dot{r}$

14 unknowns, 14 equations:

From velocity

$\dot{x}_{BO} = u_{BO}$

$\dot{y}_{BO} = v_{BO}$

$\dot{z}_{BO}, = w_{BO}$

From F=ma:

$\dot{u}_{BO}, \dot{v}_{BO}, \dot{w}_{BO}$

*From BCO*OCBd*

$\dot{\theta}, \dot{\phi}, \dot{\psi} = f(p, q, r)$

From tau = oddt ohob

$\dot{p}, \dot{q}, \dot{r}$

From v_rel

α, β