ME219

Classes

→ Slot 6+7A: Wed → 11:05-12:30
Fri → 11:05-12:30
Wed → 8:30-9:25

→ Theory and tutorial will be mixed

→ Office hours → 5:00-6:00pm on Wed and Fri
mail ID → rpv@iitb.ac.in

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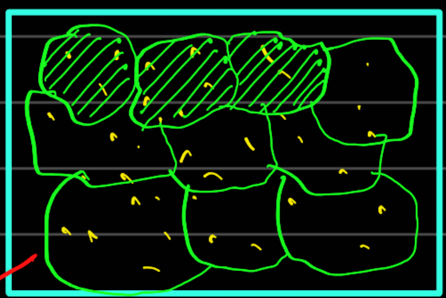
INTRO

→ Continuum approximation

→ We need to look only at avg. effects and not individual molecules
→ size of fluid particles are much larger than the dist. b/w individual molecules

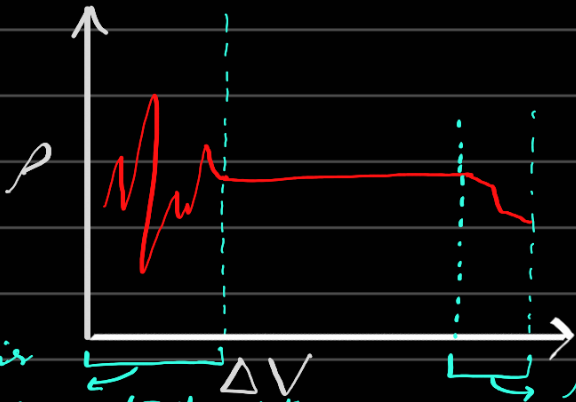


we look at these clumps of particles which are placed continuously

→ For density

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V}$$

we consider a small volume



volume is too small and causes a lot of variation

volume becomes too large

because of this variation, we use another definition of density (which uses a very small volume large enough to not show such variation)

$$\boxed{\rho = \lim_{\Delta V \to \Delta V^*} \frac{\Delta m}{\Delta V}}$$

ΔV^* → of the order $10^{-9}\,mm^3$ (mean free path)

→ Continuum approximation implies fluid is infinitely divisible

→ It also permits the 'no-slip' condition at the surface

these small molecules interact with surface and come to equilibrium with the surface

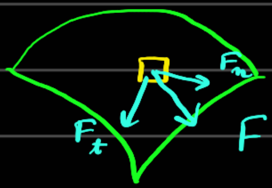
(it never slips close to the surface and has velocity same as the surface, near the surface)

→ Continuum approximation is always valid for liquids but not for gases

→ We will assume continuum approximation and no-slip condition to be true throughout

Fluid Stress

→ fluid is a material which deforms continuously under the influence of a shear stress no matter how small.



→ Outward normal will be dir^n of area vector

$$\boxed{\sigma_n = \lim_{\delta A \to \delta A^*} \frac{\vec{F_n}}{\delta \vec{A}} \; ; \; \sigma_t = \lim_{\delta A \to \delta A^*} \frac{\vec{F_t}}{\delta \vec{A}}}$$

↳ tensor quantities

σ balls

→ we can further split F_t into 2 components along x, z axes

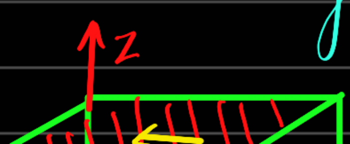
$$\Rightarrow \boxed{\sigma_n = \sigma_{yy} = \lim_{\delta A \to \delta A^*} \frac{F_n = F_y}{\delta A_y} \; ; \; \sigma_{yx} = \lim_{\delta A \to \delta A^*} \frac{F_x}{\delta A_y} \; ; \; \sigma_{yz} = \lim_{\delta A \to \delta A^*} \frac{F_z}{\delta A_y}}$$

→ In general, nomenclature

σ_{ij}

acts in this dir^n

acts on this face



stress F_1 will be $+\sigma_{yy}$

stress F_2 will be $+\sigma_{yz}$

stress F_3 will be $-\sigma_{yz}$

(sign displays the dirⁿ of $\vec{A}$ and $\vec{F}$)

stress F_4 will be $-\sigma_{zy}$

$\vec{F}$

$\sigma_{yx} = f\left(\frac{du}{dy}\right)$ (σ is funcⁿ of $\frac{du}{dy}$, u is velocity)

should not come out to be -ve

this will generate a shear stress and the upper plate will slide and continue to do so

$$\boxed{\text{Stress} \propto \text{strain rate}}$$

→ fluid statics refers to fluid that is not moving and we can imply that it isn't under shear force (converse of definition of fluid used previously)

X——X——X——X——X

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Fluid Statics

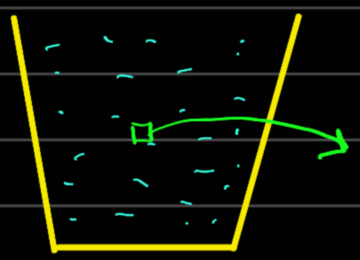
→ fluid at rest
→ no shear stresses present
→ only normal stresses

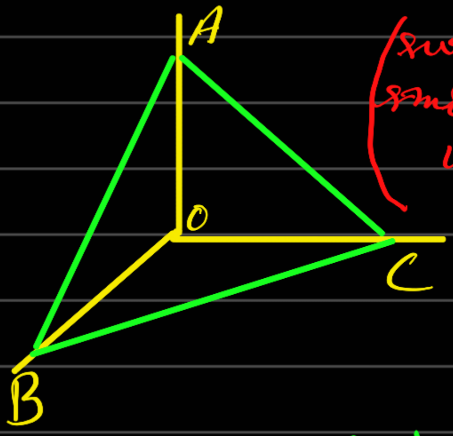
→ normal stress on any plane through a fluid element at rest is a unique value is called the Fluid pressure

$$\boxed{P = \lim_{\Delta A \to 0} \frac{\Delta F_{normal}}{\Delta A}}$$

(force due to pressure acts opp. to $\vec{A}$)

→





(surface so small, P is uniform)

$$dF_{ABC} = P_{ABC}(A_{ABC}) \rightarrow (\text{along } -\hat{A}_{ABC})$$

Consider force component in z,

$$(dF)_{z\,ABC} = -P_{ABC} A_{ABC} \cos\theta_z$$

the other pressure force in z- direction

$$dF_{OBC} = P_{OBC} A_{OBC}$$

and then the weight

$$F_w = \underbrace{\left[\frac{1}{3} A_{OBC} \Delta z\right]}_{\text{vol. of tetrahedron}} \rho g$$

Force balance gives,

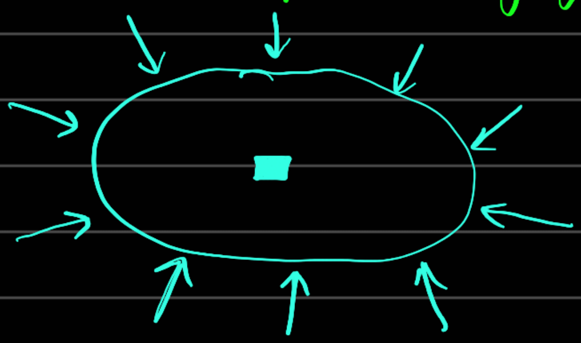
$$P_{ABC} A_{ABC} \cos\theta_z + \left[\frac{1}{3} A_{OBC} \Delta z\right] \rho g = P_{OBC} A_{OBC}$$

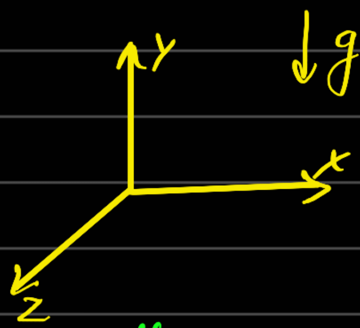
$$\Longrightarrow \boxed{P_{ABC} = P_{OBC}}$$

(as $\Delta z \to 0$ because small tetrahedron, and

$$\left(A_{ABC} \cos\theta_Z = A_{OBC} \right)$$

→ **Fundamental equation of fluid statics**





→ now consider a small vol. inside this and the force/unit mass is represented as

$$\vec{X} = X_x \hat{i} + X_y \hat{j} + X_z \hat{k}$$

(body force)

→ total body force,

$$d\vec{F}_B = \rho \, dV \, \vec{X}$$

$$\Rightarrow \boxed{F_B = \iiint \rho \, dV \, \vec{X}}$$

→ total surface force is

$$\boxed{F_S = -\iint p \, d\vec{A}}$$

due to direction convention

$$\Rightarrow \quad F_B + F_S = 0$$

$$\Rightarrow \quad \iiint_V \rho \vec{X} \, dV - \iint p \, d\vec{A} = 0$$

using gauss divergence theorem

$$\Rightarrow \quad \iiint_V \rho \vec{X} \, dV - \iiint_V \nabla p \, dV = 0$$

$$\Rightarrow \boxed{\rho \bar{X} = \nabla P}$$

$\vec{X}$ = body force per unit mass

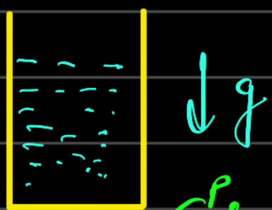
$$\Rightarrow \boxed{\begin{aligned} \rho X_x &= \frac{\partial P}{\partial x} \\ \rho X_y &= \frac{\partial P}{\partial y} \\ \rho X_z &= \frac{\partial P}{\partial z} \end{aligned}}$$

Consider the case when gravity is the only body force (which is already per unit mass)

$$\Rightarrow \vec{X} = -g\,\hat{k}$$

$$\Rightarrow \boxed{\rho g = \frac{dP}{dz}}$$

ex →



$$\int_{z}^{z_0} \rho g \, dz = \int_{P}^{P_0} dP$$

$$\Rightarrow \boxed{\rho g (z - z_0) = P - P_0}$$

→ If ρ is not constant,

ex → $\rho = \frac{P}{RT}$

$$\frac{dP}{dz} = -\frac{Pg}{RT} \Rightarrow \frac{dP}{P} = -\frac{g}{RT} dz$$

→ Gauge pressure

$$\boxed{P_{gauge} = P_{abs} - P_{atm}}$$

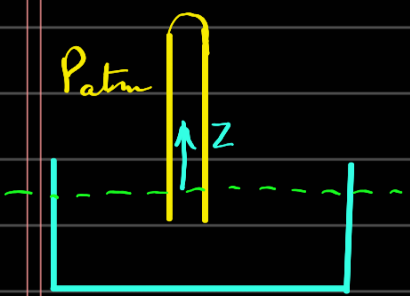
$P_{gauge} < 0 \rightarrow$ vaccuum

$P_{gauge} = 0 \rightarrow$ atmospheric

$P_{gauge} > 0 \rightarrow$ pressurized system

→ Barometer

: Atmospheric pressure measurement is usually done by barometer. Liquid usually mercury



$$P - P_0 = -\rho g(z - z_0)$$

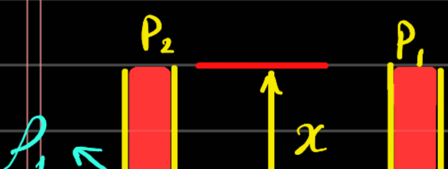
$z = z_0 = 0$; $P = P_0 = P_{atm}$

$z = z$; $P = P_{vap} \simeq 0$

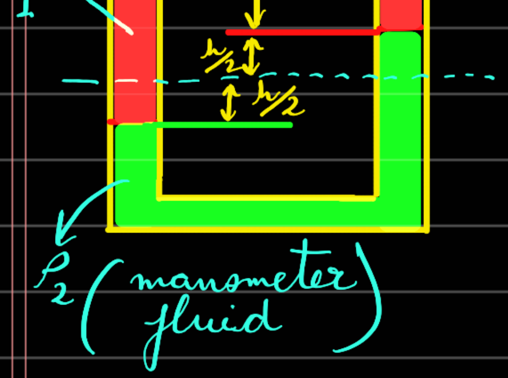
$$\boxed{P_{atm} = \rho g z}$$

→ Manometer

A manometer is a device to measure pressure, often used to measure pressure difference in flowing fluids. Need a manometric fluid.



$$P_1 + x\rho_1 g + h\rho_2 g = P_2 + x\rho_1 g + h\rho_1 g$$



$$\boxed{\begin{gathered} P_2 - P_1 = (\rho_2 - \rho_1)\, h g \\ \rho_2 > \rho_1 \end{gathered}}$$

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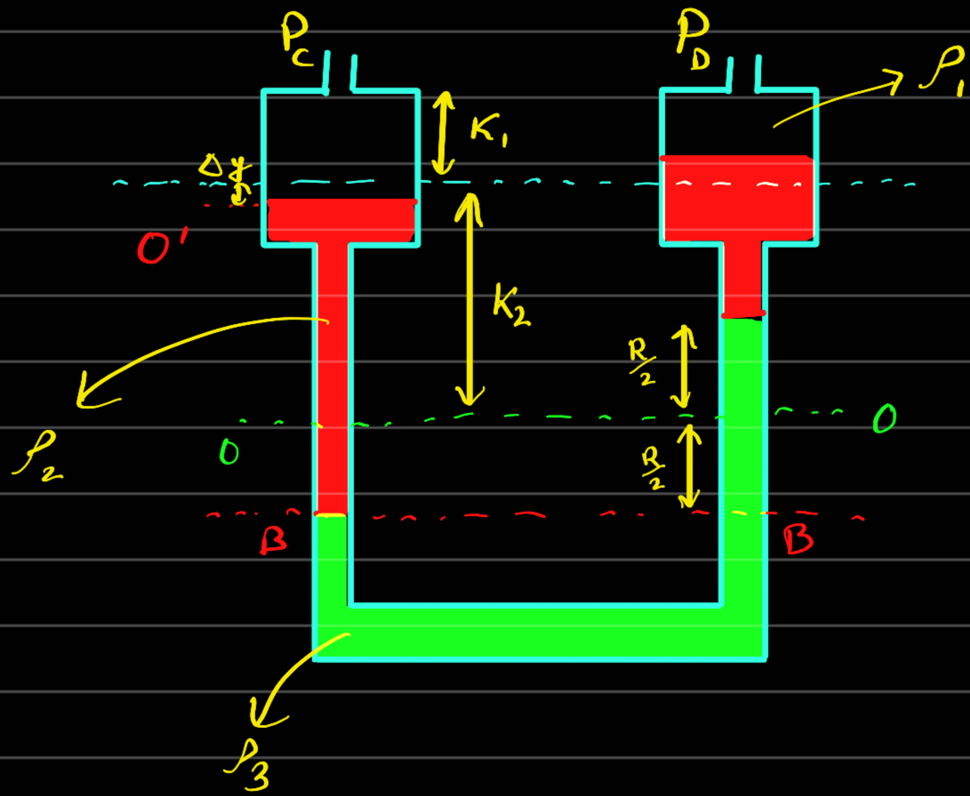
→ **Micrometer**



$$P_B = P_C + \rho_1 g (k_1 + \Delta y) + \rho_2 g \left(\frac{R}{2} + k_2 - \Delta y\right)$$

$$P_B' = P_D + \rho_1 g (k_1 - \Delta y) + \rho_2 g \left(k_2 - \frac{R}{2} + \Delta y\right) + \rho_2 g \frac{R}{2}$$

Since, $P_B = P_{B'}$

$$\Rightarrow P_C - P_D = R\left[\rho_3 g + \frac{a}{A}\rho_2 g - \rho_2 g + \frac{a}{A}\rho_1 g\right]$$

Note that → $A\Delta y = \frac{aR}{2}$

→ $P_C = P_D$ before any measurement has been done so balancing the volume change, we get

→ By design, $\frac{a}{A}$ is very small and we can ignore its contribution,

$\Rightarrow P_C - P_D = (\rho_3 - \rho_2) g R$

$\Rightarrow$ if $(\rho_3 - \rho_2)\downarrow \Rightarrow R\uparrow$

Ex →

8 cm diameter piston compresses manometer oil into an inclined 7 mm diameter tube. When a weight W is added on top of the piston, the oil rises an additional distance of 10 cm. How large is W

Sol^n: Let mass of piston be W_p

$$\Rightarrow \frac{W_P g}{\pi (4)^2} = \rho_o g x \sin 15° \quad -①$$

$\left(\rho_o = \rho_w (0.827)\right)$ given

after adding weight,

$$\frac{(W + W_P) g}{\pi (4)^2} = \rho_o g \left(x + 10 + \frac{\Delta y}{\sin 15°}\right) \sin 15°$$

Volume balance,

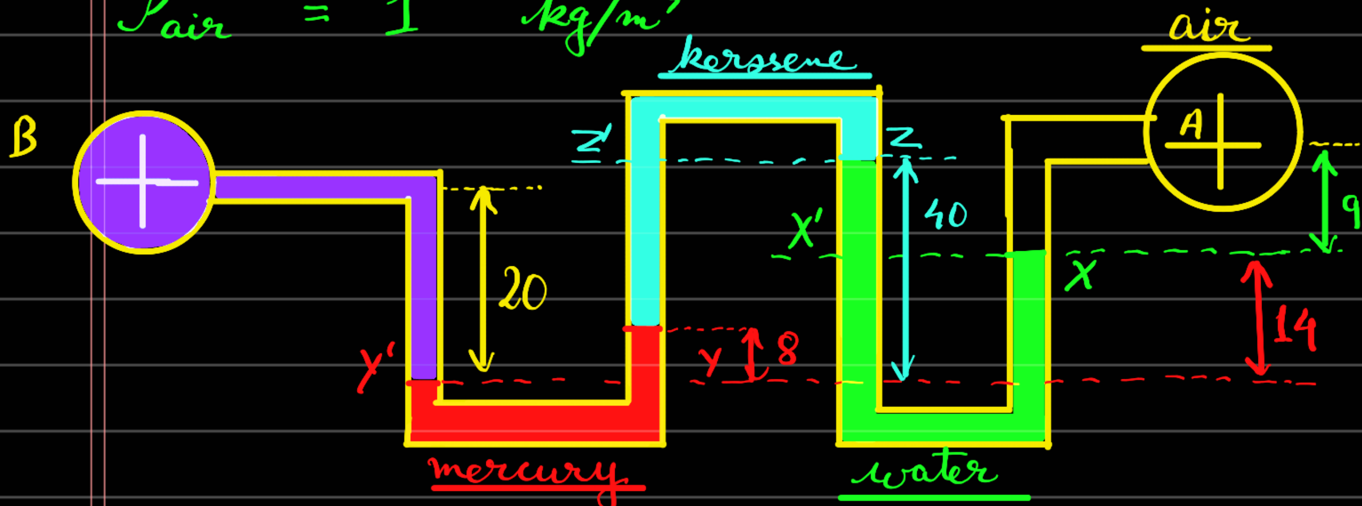
$$a \Delta y = A(10)$$

substitute to solve

→ Example-2

Determine pressure diff. b/w A and B

$\rho_{benzene} = 881$
$\rho_{mercury} = 13600$
$\rho_{kerosine} = 804$
$\rho_{water} = 1000$
$\rho_{air} = 1 \quad kg/m^3$



$P_{z'} = P_z , \ P_{x'} = P_x$

$= P_{air} + [9\times10^{-2}]\rho_{air}\, g + [14\times10^{-2}]\rho_{water}\, g$

$P_{y}' = P_B + \rho_{benz}\, g\,(20\times10^{-2})$

$P_y = P_z' + \rho_{ker}\, g\left(32\times10^{-2}\right) + \rho_{mercury}\, g\,(8\times10^{-2})$

Pressure balance at X and X'

$\Rightarrow P_A + (9\times10^{-2})(1)(g) = P_z + (40-14)\times10^{-2}\times1000g$

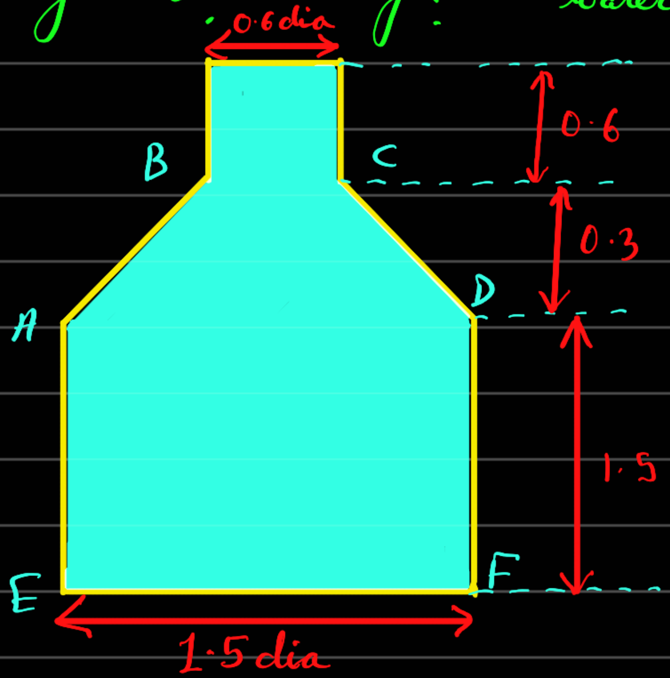
Now, using $P_z = P_z'$, $P_y = P_y'$

we get, $P_A + (9\times10^{-2})g = (26\times10^{-2})(1000g) + P_B + \rho_{benz}\, g(20\times10^{-2}) - \rho_{ker}\, g(32\times10^{-2}) - \rho_{mercury}\, g(8\times10^{-2})$

$\Rightarrow \boxed{P_B - P_A = 8919 \ Pa}$

→ Example - 3

A container has a circular cross-section. Determine the upward force on the cone frustum? What is the downward force on the plane EF. Is this force equal to the weight of the fluid? Why? $\rho_{water} = 998\ kg/m^3$



$$P_{EF} = (0.6+0.3+1.5)(998)(9.81) = 23496.9\ N/m^2$$

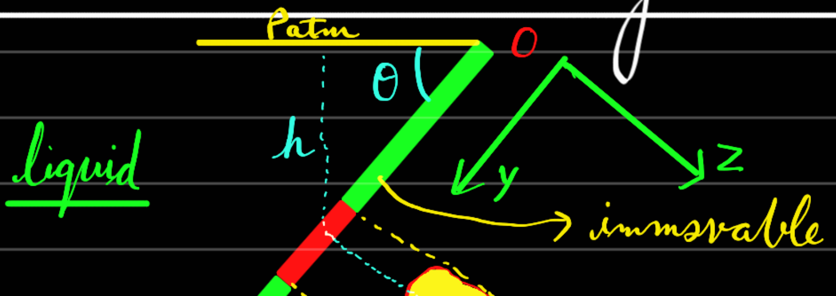
$$\Rightarrow F_{EF} = 41522.5N$$

$$W = \left(\frac{\pi}{4\times3}\left((1.5)^2(0.5) - (0.6)^2(0.2)\right)\right) + \frac{\pi}{4}\left(1.5^2\,1.5 + (0.6)^2(0.6)\right)$$
$$\times\ 998 \times 9.81 = \underline{\underline{30311.36N}}$$

$$F_{on\ frustum} = \underbrace{F_{EF}}_{} - W = \underline{\underline{11211.1N}}$$

↳ total weight of fluid considering a complete cylinder

Forces on Submerged Areas



$$dF = \rho g\, h\, dA$$

$$dF = [\rho g\, y \sin\theta + P_{atm}]\,dA - (P_{atm} \cdot dA)$$

$$F = \int dF = \int \rho g\, \underbrace{y\, dA}\, \sin\theta$$

$$\int y\, dA = y_c A$$

$$\boxed{F = [\rho g \sin\theta]\, A y_c}$$

→ Point of application of force

We calculate moment about O

$$d\tau = dF\, y$$

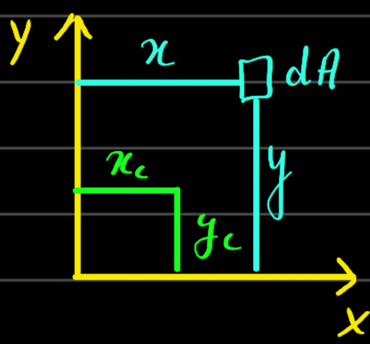
$$= \rho g \sin\theta\, y^2 dA$$

$$\tau = \rho g \sin\theta \underbrace{\iint y^2 dA}_{A y_c^2 + I_{x_c x_c}} = F \cdot y_0$$

F → net force

y_0 → point of application

$A y_c^2 + \underbrace{I_{x_c x_c}}$ → moment of inertia about centroidal axis



$$y_c = \frac{\iint y\, dA}{A}$$

$$x_c = \frac{\iint x\, dA}{A}$$

x_c, y_c are the co-ords of centroid

Area moment of inertia

(parallel axis theorem)

→ $\iint y^2 dA = I_{xx} = I_{x_c x_c} + A y_c^2$

$I_{x_c x_c}$ ↓ area moment of inertia about x_c axis

area moment of inertia is similar

its mass moment of inertia and centroid is similar to COM

$I_{xy} = \iint xy\,dA$

$\Rightarrow \tau = \rho g \sin\theta \iint y^2\,dA = F \cdot y_o$

$\Rightarrow \rho g \sin\theta\left[A y_c^2 + I_{x_c x_c}\right] = (A y_c \rho g \sin\theta)\, y_o$

$$\boxed{\Rightarrow y_o = y_c + \frac{I_{x_c x_c}}{A y_c}}$$

Similarly in x- direction,

$$x_o = x_c + \frac{I_{x_c y_c}}{A y_c}$$

$$x_o = \frac{\iint y x\,dA}{\iint y\,dA}$$

$$I_{xy} = \iint xy\,dA$$

$$= \underbrace{I_{x_c y_c}}_{\text{zero for a body with line of symmetry as } x \text{ or } y} + A x_c y_c$$

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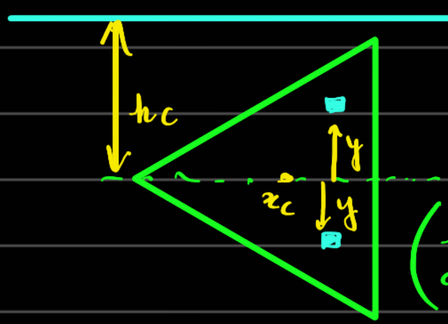


(point of application)

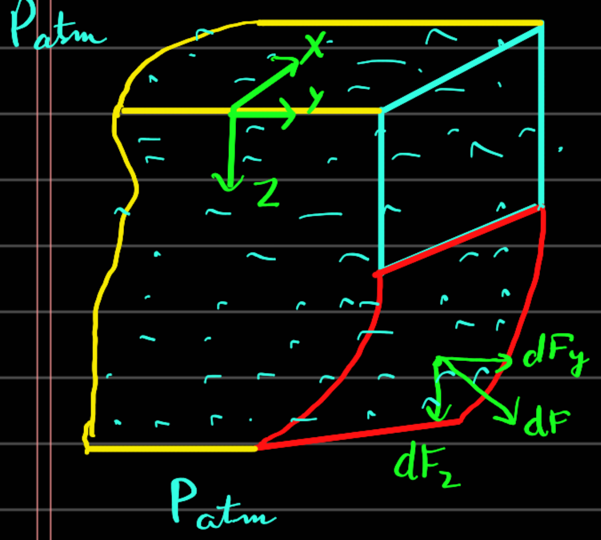
$$x_y = \frac{\left(\iint \rho g (h_c - y) x + \iint \rho g (h_c + y) x\right)\frac{dA}{2}}{\rho g h_c \iint dA}$$

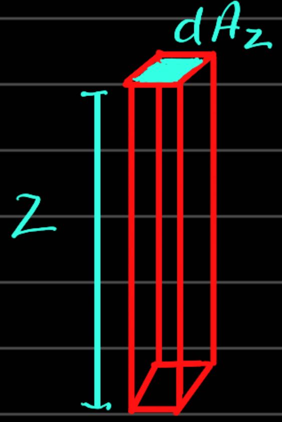
$$= \frac{\rho g h_c \iint x\,dA}{\rho g h_c \iint dA}$$

→ which is nothing but the centroid.

Forces on Curved Surface

→ Consider a surface that is submerged under a liquid. Blue outline is immovable, red is movable





$$dF = P\,dA = [\rho g z]\,dA$$

$$dF_y = [dA_y]\,\rho g z$$

$$dF_z = [dA_z]\,\rho g z$$

we know that $\rho g z\, dA_z = \rho g\, dV$

$$\Rightarrow F_y = \iint dF_y = \iint \rho g z\, dA_y = \rho g z_c A_y$$

z_c → centroid of projected area of blue and red region

$$F_z = \iint dF_z = \iint \rho g\, dV = \rho g V$$

→ point of application of F_y: $\iiint [\rho g z\, dA]\, z = \{\rho g z_c A_y\}\, z_p$

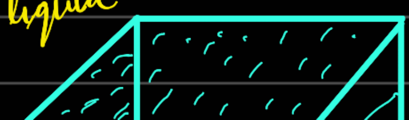
similarly for x

→ point of application of F_z: $\iint [\rho g z\, dA]\, x = \{\rho g z_c A_y\}\, x_p$

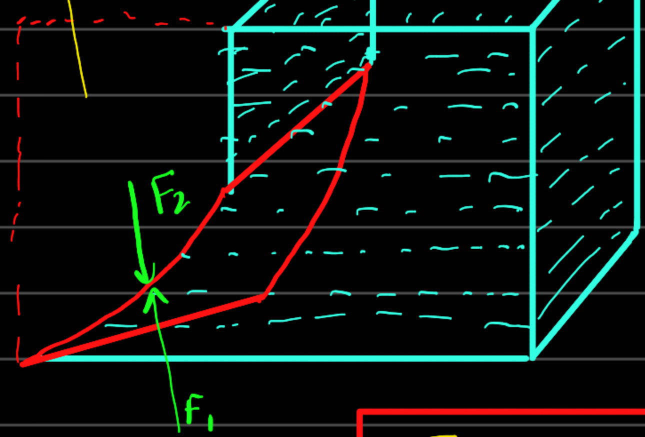
similarly for y

→ Now consider liquid on the other side of the curved surface

fill with liquid



F_z due to weight of new liquid



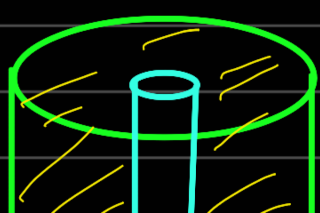
$$\boxed{F_1 = F_2}$$

→ Example - 1

- A hemispherical dome of 2m radius with a cylindrical extension of 3cm diameter, shown in the figure weighs 30 kN and is filled with water and attached to the floor by six equally spaced bolts. What is the force in each bolt required to hold down the dome ?

Sol^n: We consider a cylinder around the original shape and fill it with liquid. So the weight of the newly added liquid will be the one exerting force.

So, $$F = \rho\left[\pi R^2 H - \underbrace{\frac{4}{3}\pi \frac{R^3}{2} - \pi r^2 h_1}_{\text{subtracting weight of original liquid}}\right] g$$



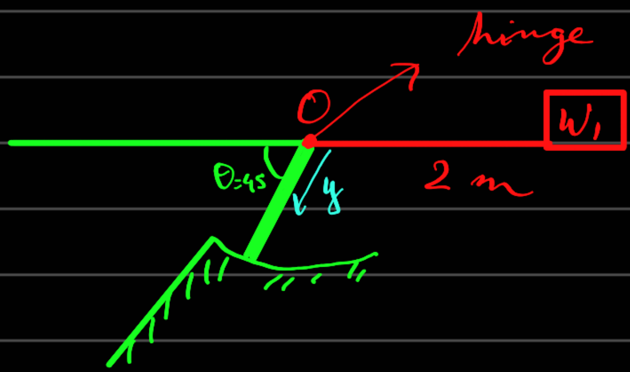
newly added liquid will have same effect as

body is in equilibrium

→ Example-2

- The gate shown in the figure is in equilibrium. Compute 'W1' the weight of counter weight per meter width by neglecting the weight of gate

Sol^n:



$$\tau_0 = 0$$

$$\tau_{\text{on gate}} = \int d\tau = \int_0^2 (\rho g\, y \sin 45)(dy\, w)(y)$$

$$= (\rho g\, w \sin 45)\left(\frac{8}{3}\right)$$

$$\tau_{\text{block}} = -2 \times W_1 g$$

using $\tau_0 = 0$

$$\Rightarrow (\rho \cancel{g}\, W \sin 45)\left(\frac{\overset{4}{\cancel{8}}}{3}\right) = \cancel{2} W_1 \cancel{g}$$

$$\boxed{W_1 = \frac{4 \rho g W}{3} \sin 45}$$

→ Example-3

- Calculate the force 'F' required to hold the gate(green) in position if R=0.46m. Gate width normal to the plane of paper=1m

Sol^n:

P_0 at top border, using the manometer,

$$(\rho_{manometer}\, g R) + P_{atm} = P_0 + \rho_{oil}\, g(0.6) + \rho_w\, g(1.2)$$

this will give P_0

Later we will use ΔP as $\rho g h + P_0 - P_{atm}$