

# **Machine Learning for Large-Scale Data Analysis and Decision Making (MATH80629A) Fall 2021**

**Week #14- Summary**

# Announcement

- **Office hour:** Today (hybrid) at 11:30-1 pm & Thursday (online) 12-1:30 pm
- **Project Presentation: in-person on December 6**
- Upload your slides/poster to Gradescope due **December 6**
- **Project Report: Group & Individual due December 20**

# Today

- **Last Quiz** on Gradescope!
- Summary of Sequential decision making 2
- Q&A
- Hands on session



# Quiz 6

Login to your Gradescope account

# Brief recap

- **Markov Decision Processes (MDP)**
  - Offer a framework for sequential decision making  
 $\langle \mathbf{A}, \mathbf{S}, \mathbf{P}, \mathbf{R}, \gamma \rangle$
- **Goal:** find the **optimal policy**
  - Dynamic programming and several algorithms (e.g., VI,PI)

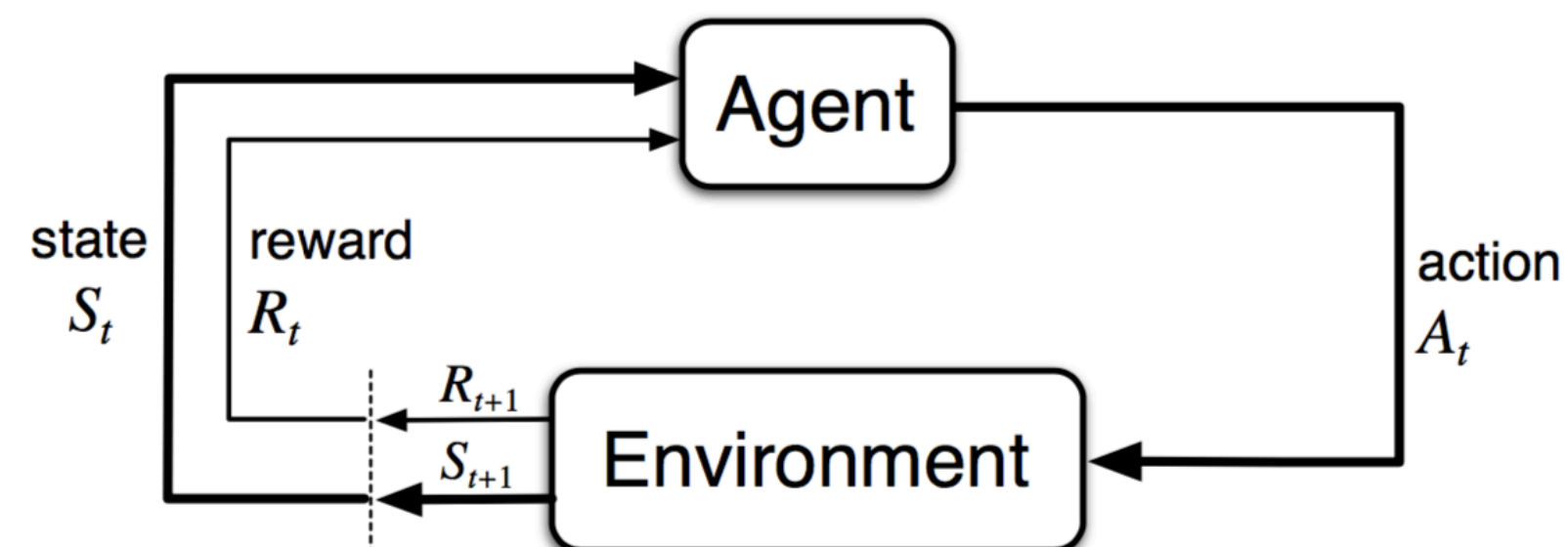


Figure 3.1, RL: An introduction

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  1. **Transition probabilities:**  $P(s' | s, a)$
  2. **Reward function:**  $R(s)$
- RL is more general
  - **In RL both are typically unknown**
  - **RL agents navigate the world to gather this information**

# Experience

## A. Supervised Learning:

- Given fixed dataset
- Goal: maximize objective on test set (population)

## B. Reinforcement Learning

- Collect data as agent interacts with the world
- Goal: maximize sum of rewards

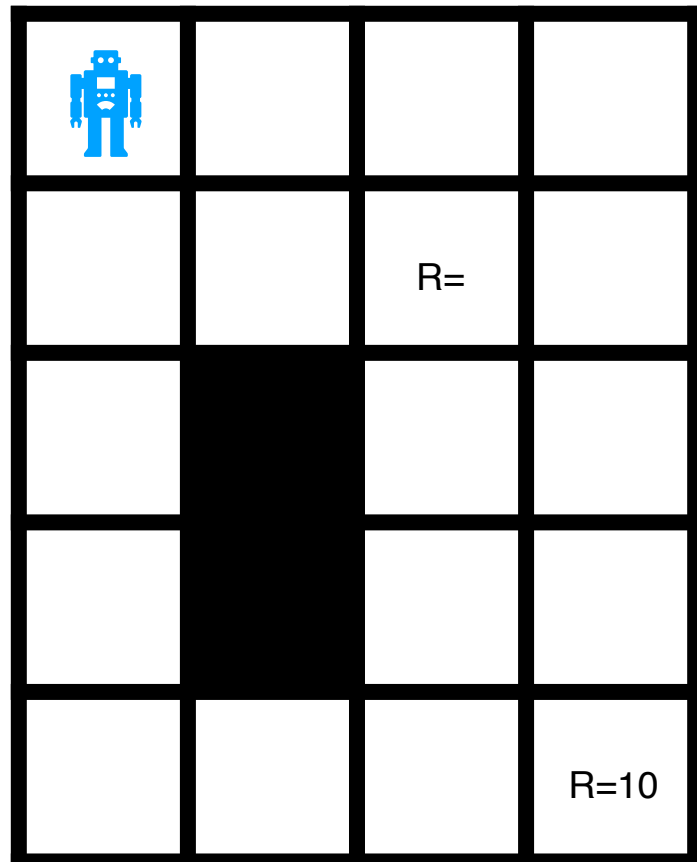
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- **Credit assignment problem:** which action(s) should be credited for obtaining a reward
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- **A small number of actions several time steps ago may be important** (test taking: study before, getting grade long after)

# Challenges of reinforcement learning

- **Credit assignment problem:** which action(s) should be credited for obtaining a reward
- **A series of actions** (getting coffee from cafeteria)
- **A small number of actions several time steps ago may be important** (test taking: study before, getting grade long after)
- **Exploration/Exploitation tradeoff:** As agent interacts should it exploit its current knowledge (*exploitation*) or seek out additional information (*exploration*)



# Application of RL

- **Robotics**
- **Video games**
- **Financial trading** is the buying and selling of financial assets.
- **Medical treatment/intervention** means the management and care of a patient to combat disease or disorder.
- A **self-driving car** is a vehicle that is capable of sensing its environment and moving safely with little or no human input.
- **Personalized tutoring** is an educational approach that aims to customize learning for each student's strengths, needs, skills, and interests.
- **Feed generation** is an automated platform that helps retailers build product feeds



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- **Input:** an environment
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- **Output:** an optimal policy
- **In practice:** need a simulator or a real environment for your agent to interact

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## **2. Model-free**

- Learns an optimal policy without explicitly learning transitions

 $\pi$

# Prediction vs. control

1. **Prediction:** evaluate a given policy
2. **Control:** Learn a policy
  - Sometimes also called
    - **passive** (prediction)
    - **active** (control)



# Monte Carlo Methods

# MC for Prediction: First-visit Monte Carlo

- Given a fixed policy (**prediction**)
- Calculate the value function  $V(s)$  for each state

## First-visit MC prediction, for estimating $V \approx v_\pi$

Initialize:

$\pi \leftarrow$  policy to be evaluated  
 $V \leftarrow$  an arbitrary state-value function  
 $Returns(s) \leftarrow$  an empty list, for all  $s \in \mathcal{S}$

Repeat forever:

Generate an episode using  $\pi$   
For each state  $s$  appearing in the episode:  
     $G \leftarrow$  the return that follows the first occurrence of  $s$   
    Append  $G$  to  $Returns(s)$   
     $V(s) \leftarrow \text{average}(Returns(s))$

[Sutton & Barto,  
RL Book, Ch 5]

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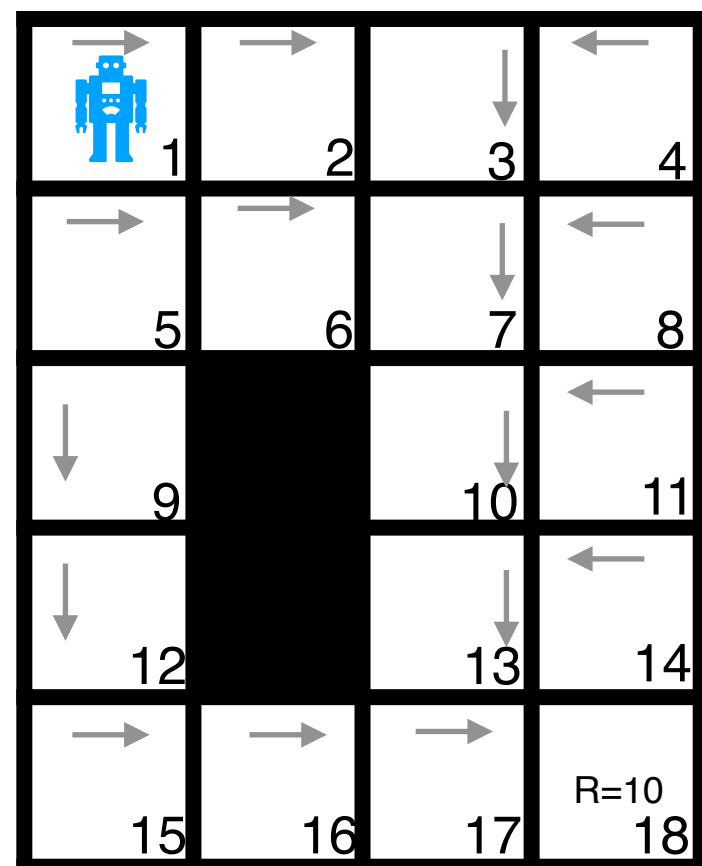
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$$V(\mathbf{s}_t) = \max_{\mathbf{a}_t} \left\{ R(\mathbf{s}_t) + \gamma \sum_{\mathbf{s}_{t+1}} P(\mathbf{s}_{t+1} \mid \mathbf{s}_t, \mathbf{a}_t) V(\mathbf{s}_{t+1}) \right\}$$

# Example: grid world



- Start state is top-left (start of episode)
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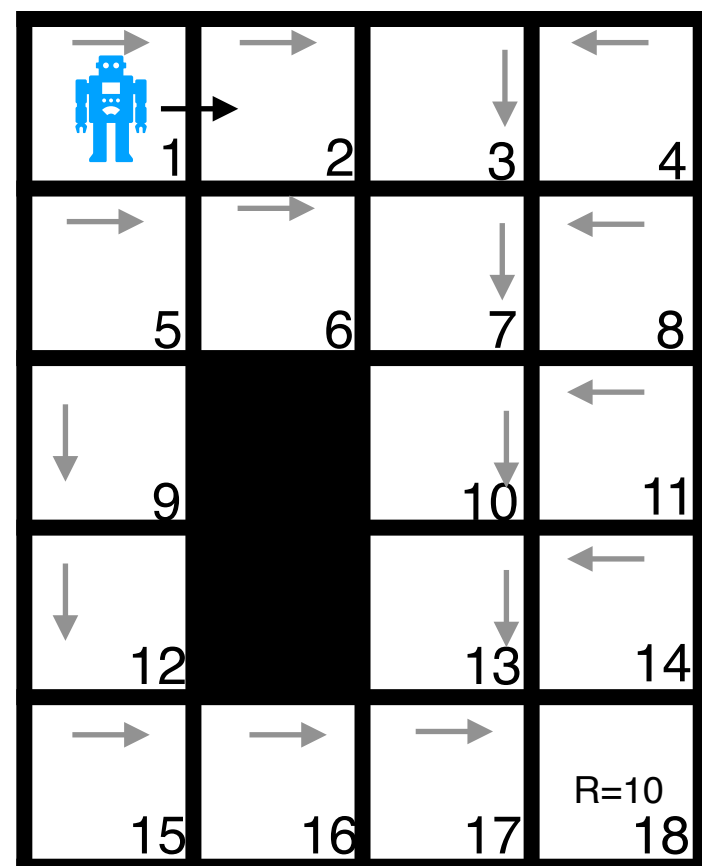
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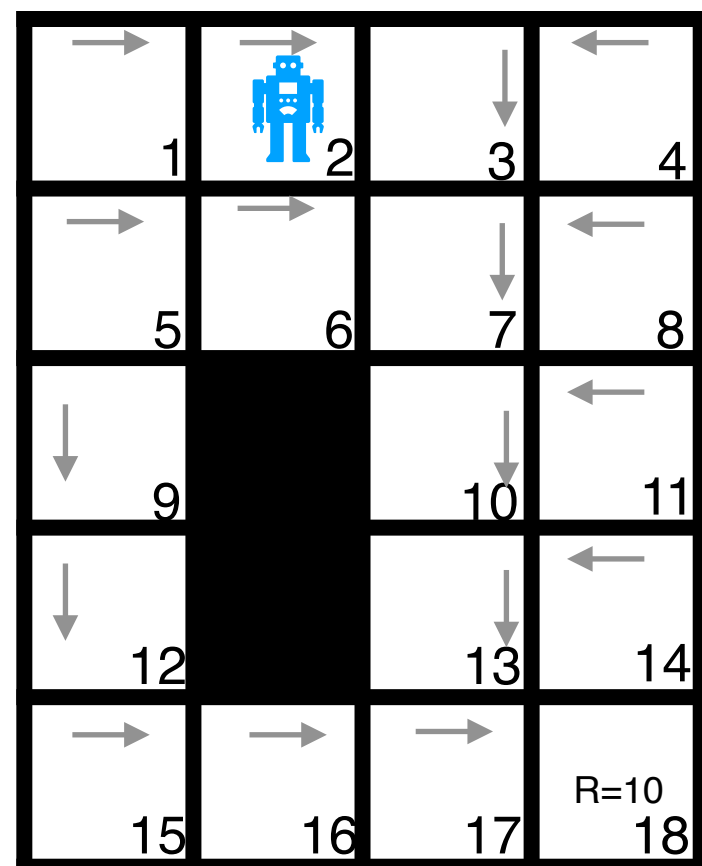
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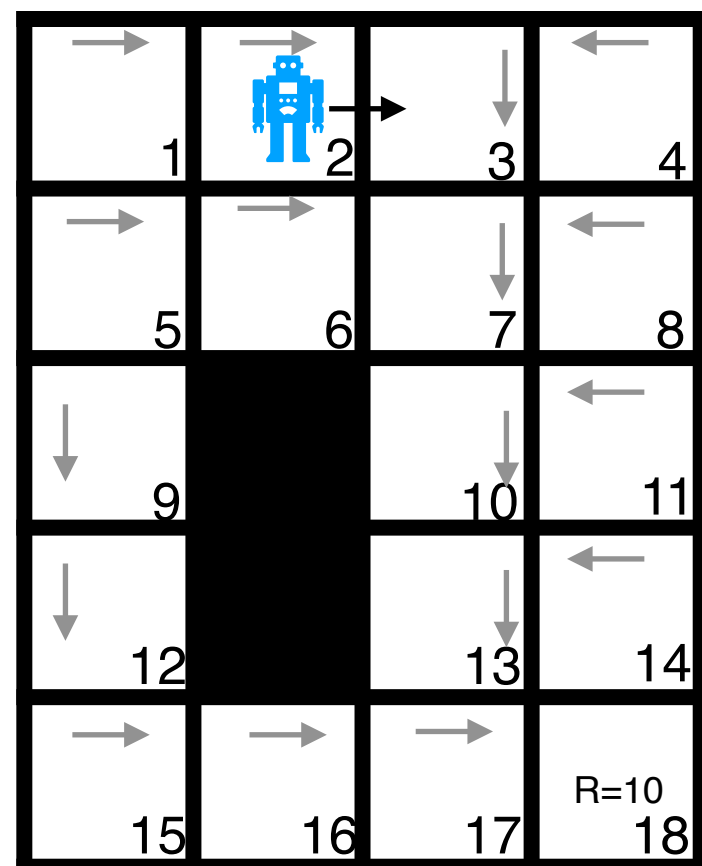
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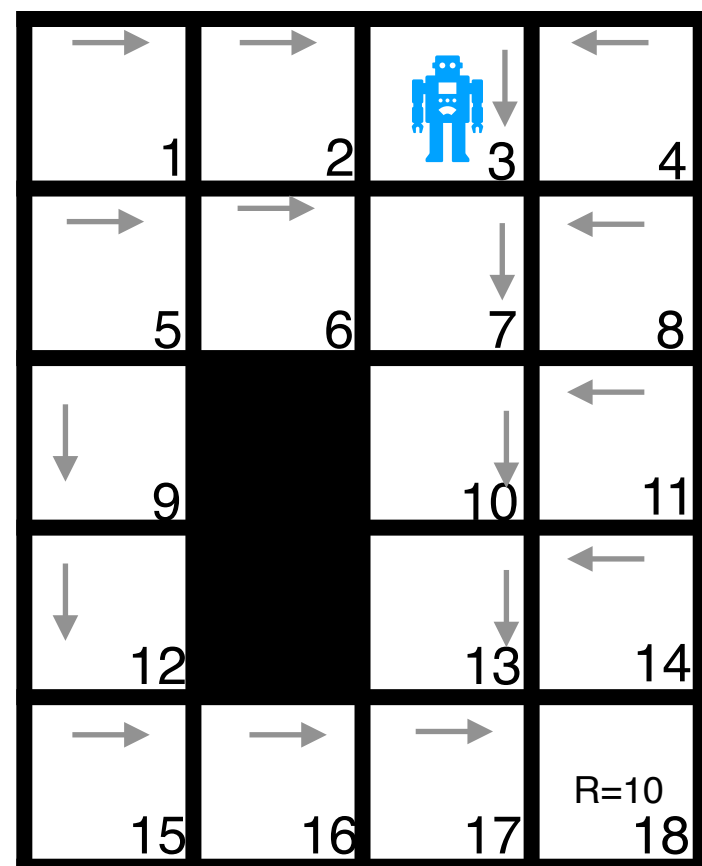
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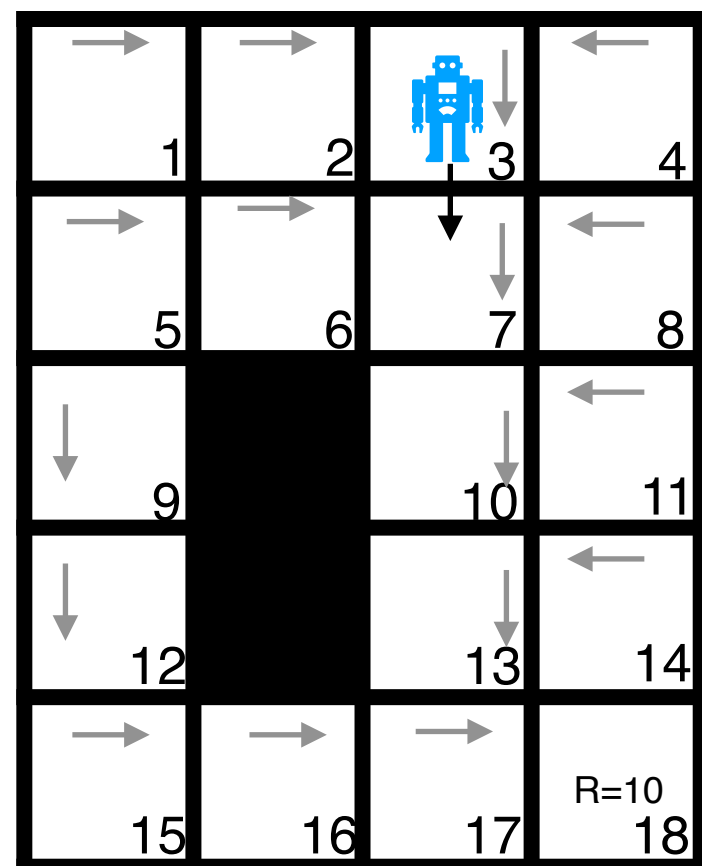
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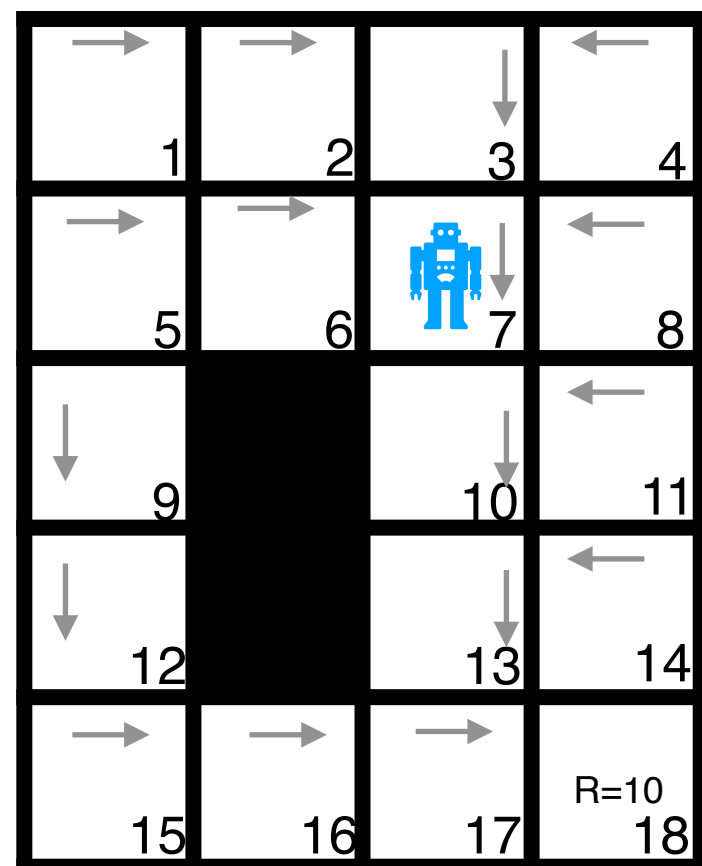
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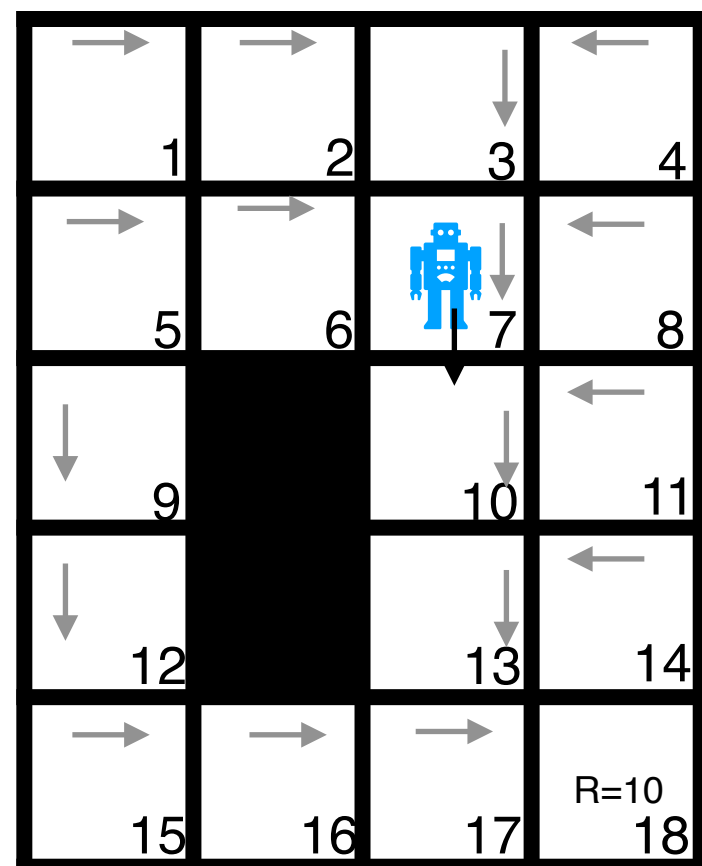
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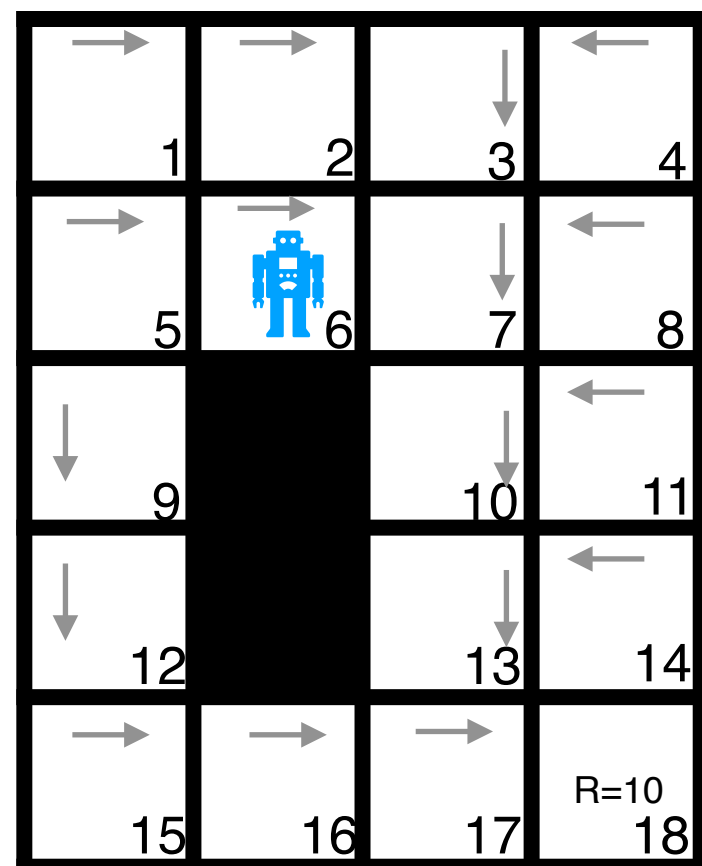
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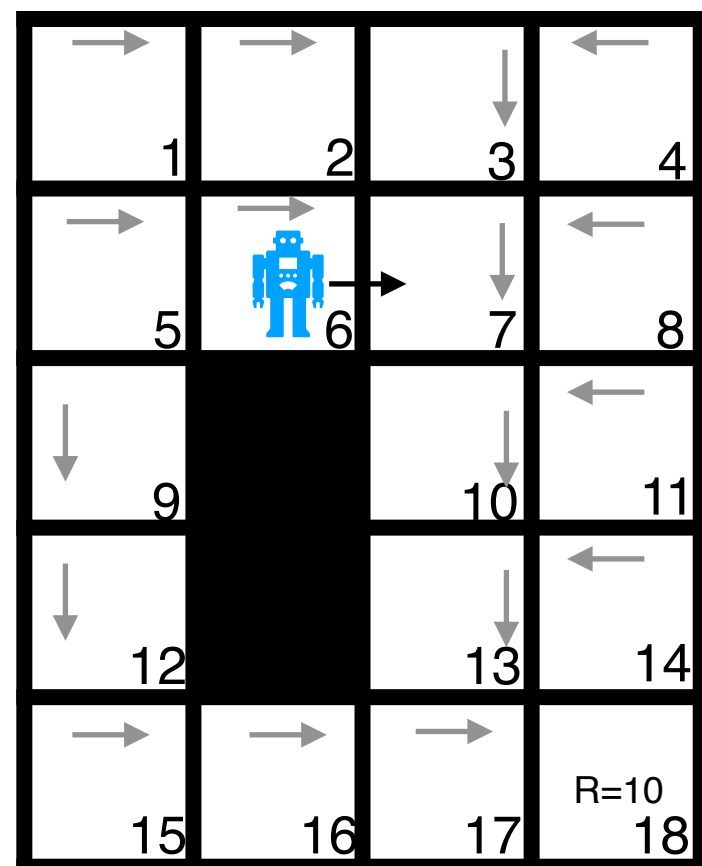
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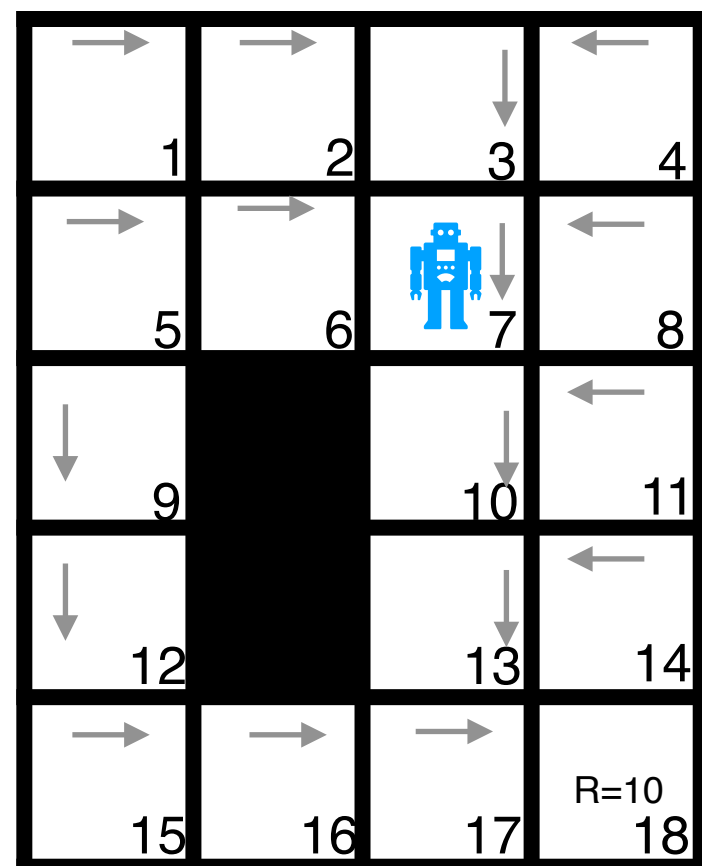
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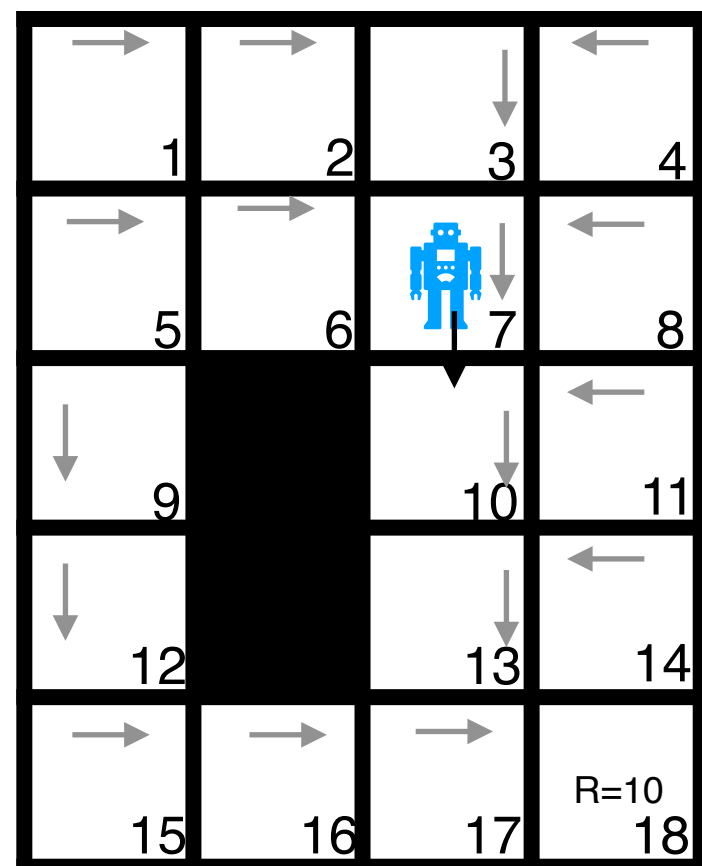
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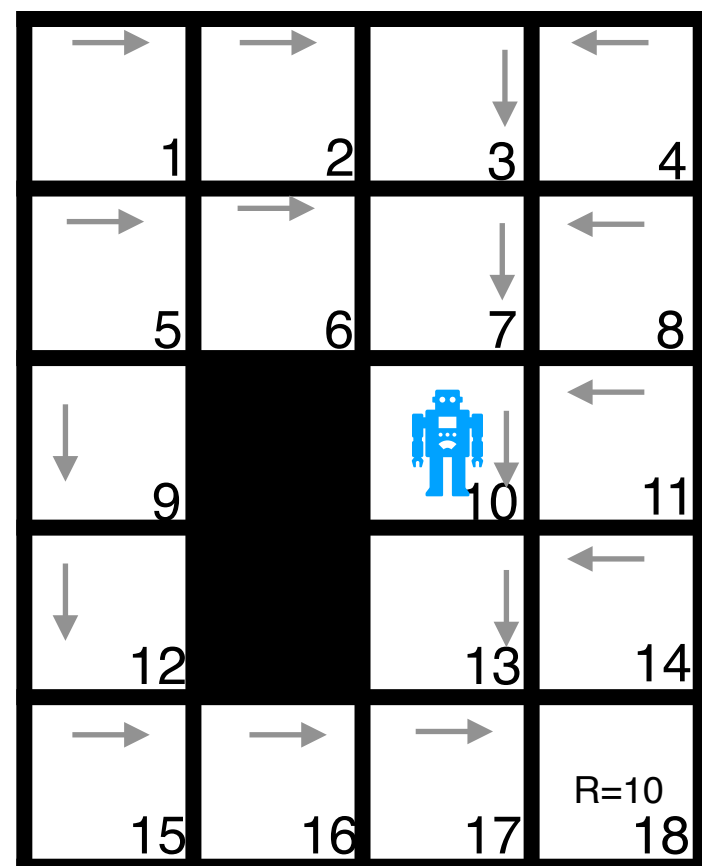
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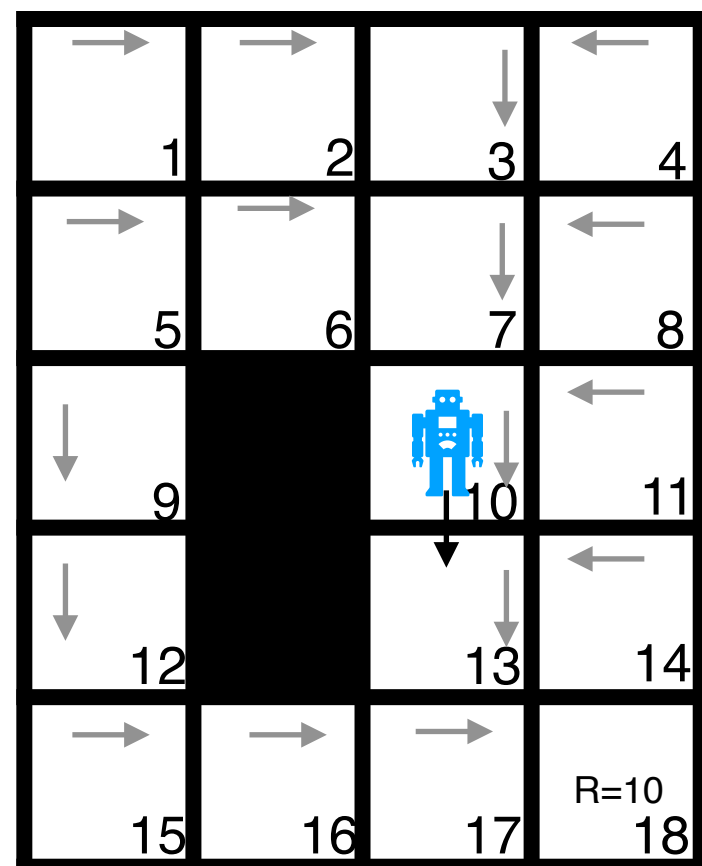
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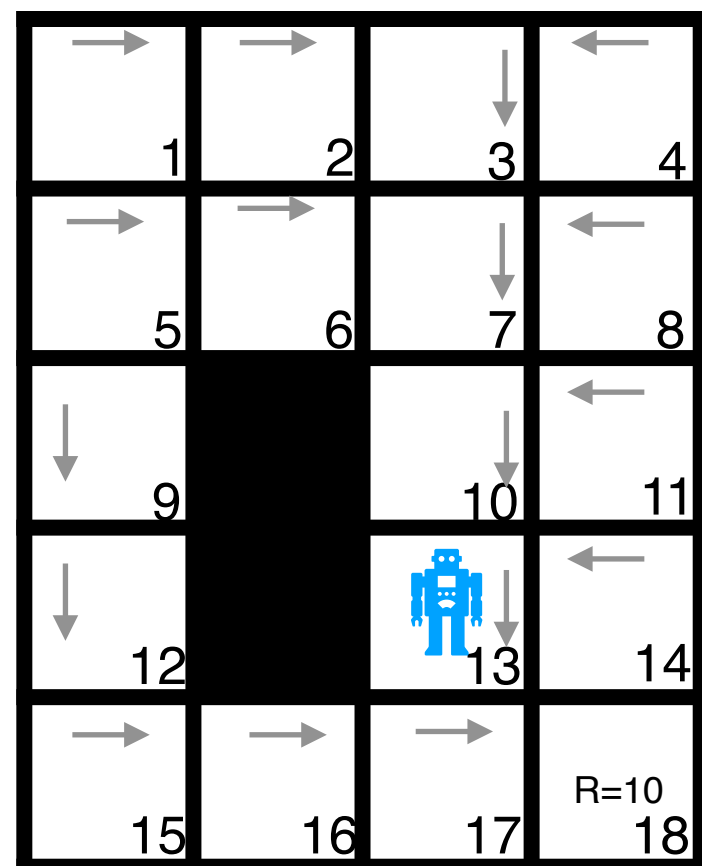
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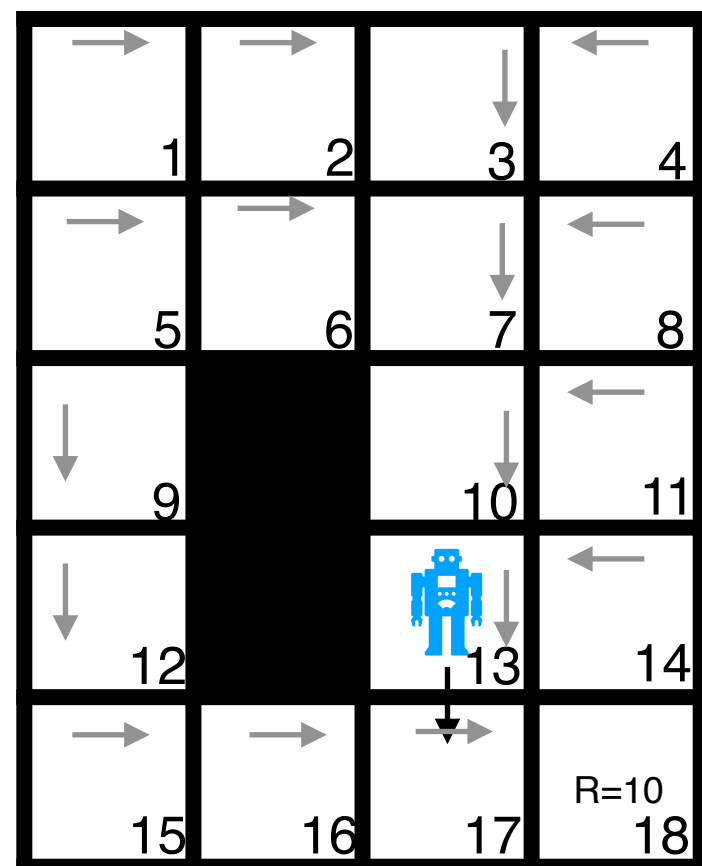
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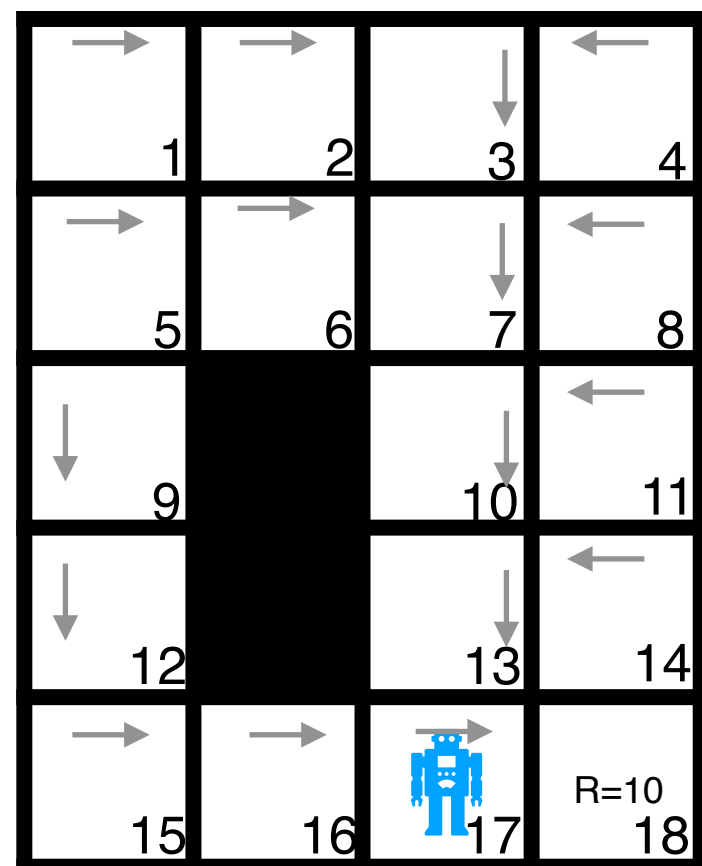
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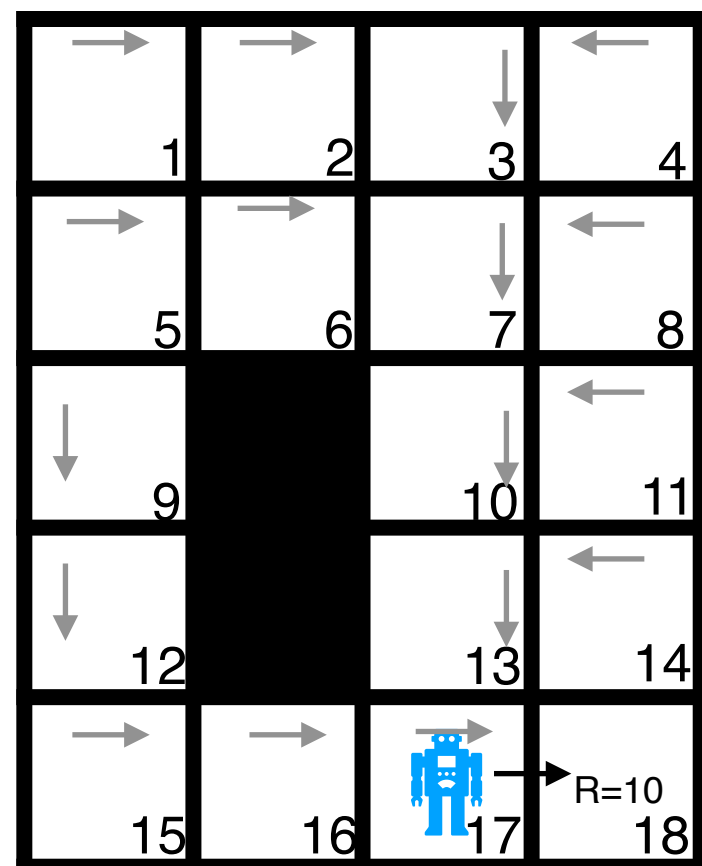
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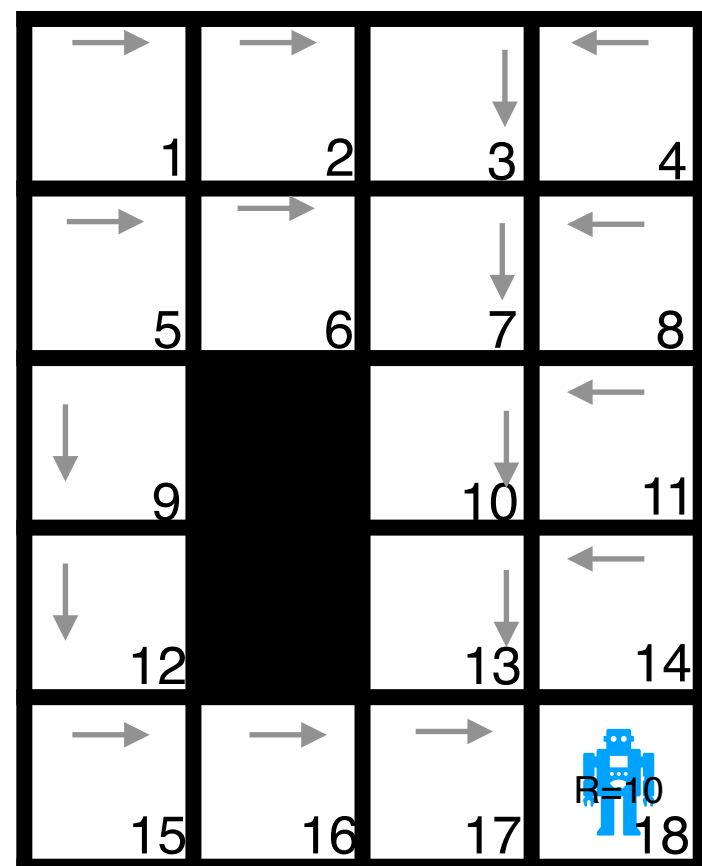
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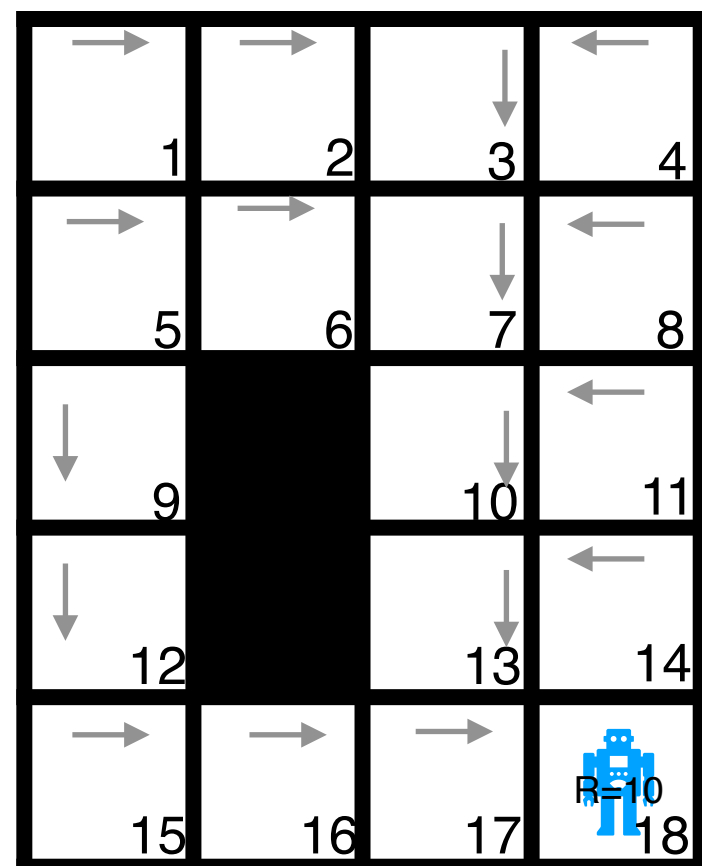
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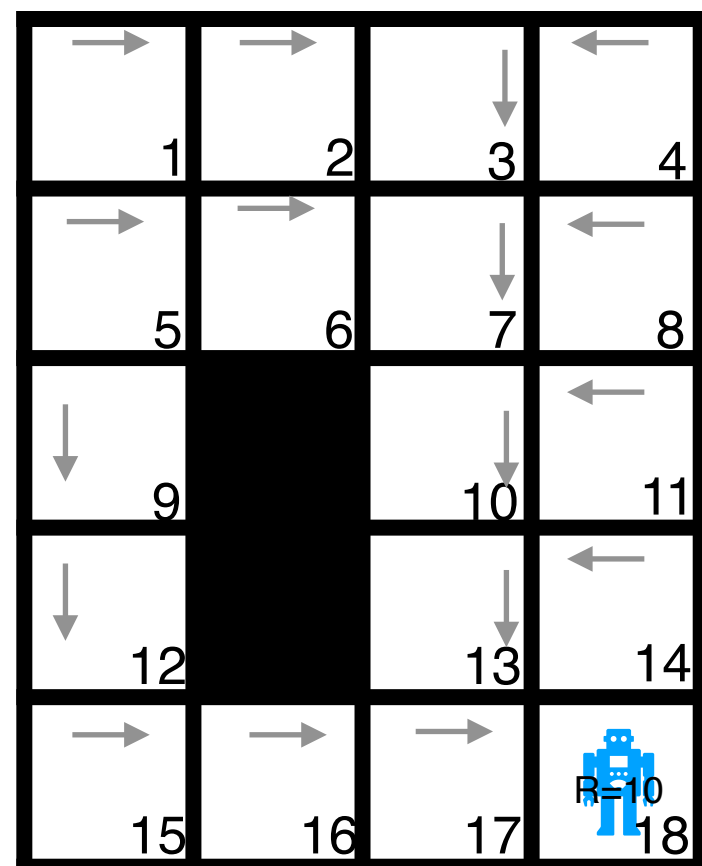
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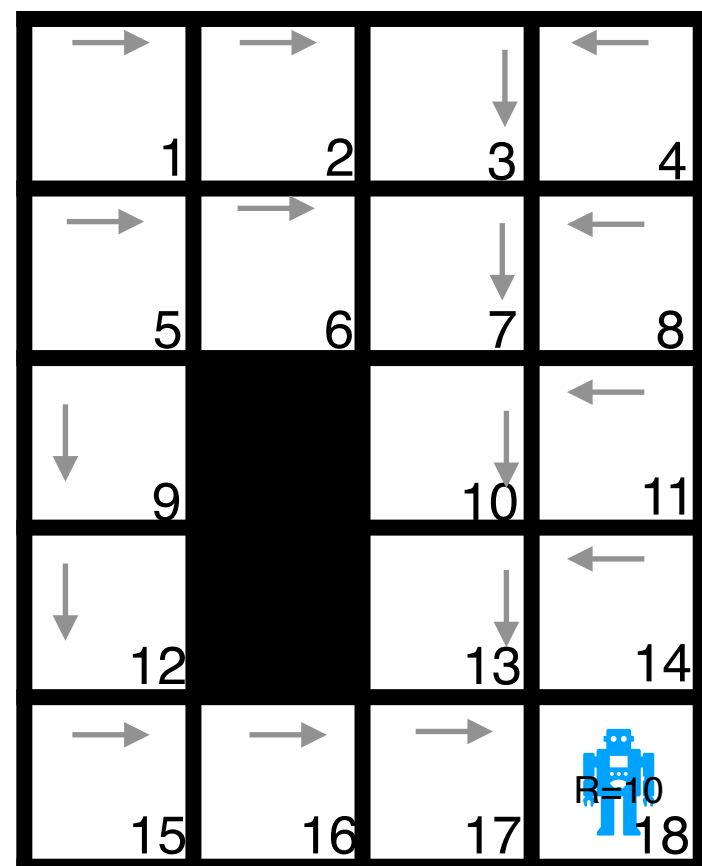
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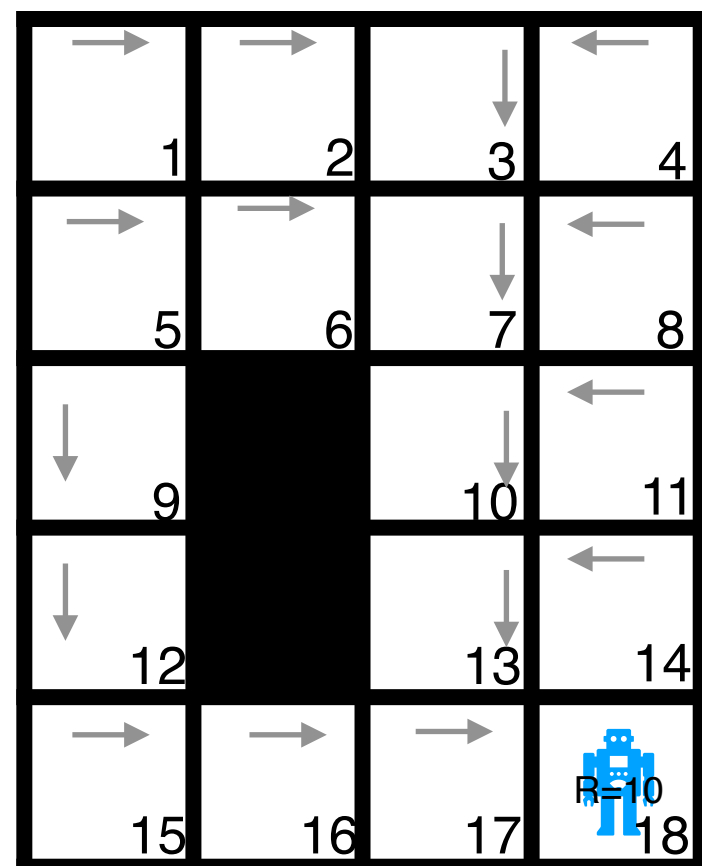
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For state 7:

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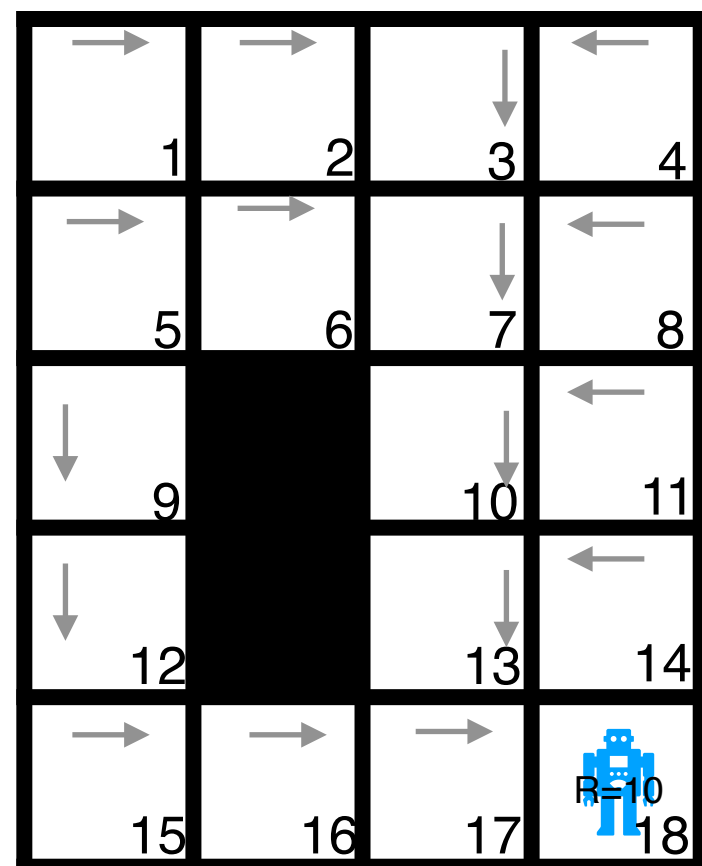
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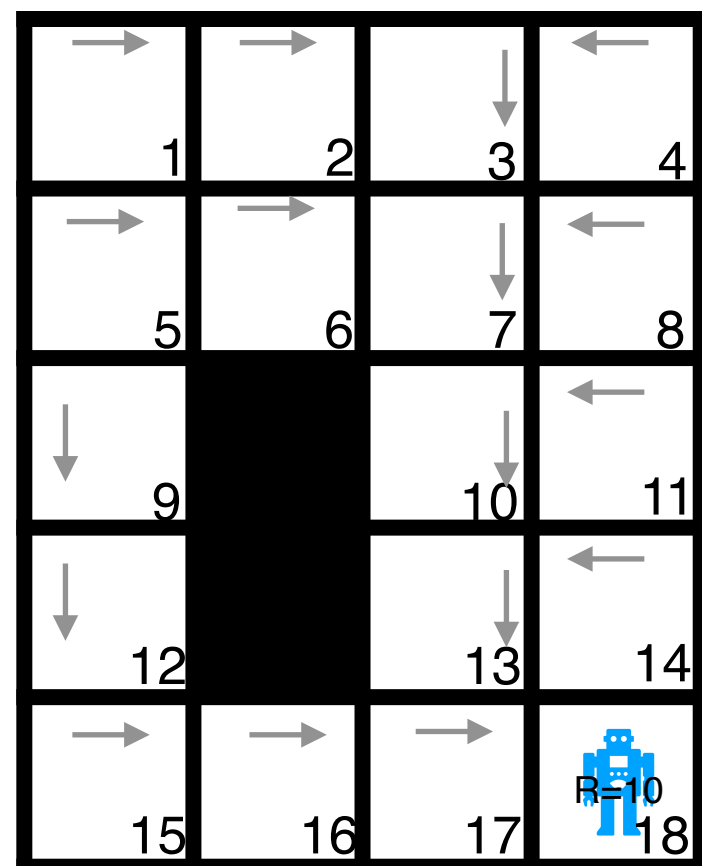
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$$V(7) = \gamma^6 * 10$$

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- When state transitions are unknown what can we do?
- **$Q(s,a)$**  the value function of a **(state,action) pair**

$$\pi^*(s) = \arg \max_a \{ Q^*(s, a) \} \quad \forall s$$



# MC for Control: Monte Carlo ES

## Monte Carlo ES (Exploring Starts), for estimating $\pi \approx \pi_*$

Initialize, for all  $s \in \mathcal{S}$ ,  $a \in \mathcal{A}(s)$ :

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$Q(s, a) \leftarrow \text{average}(Returns(s, a))$

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[Sutton & Barto,  
RL Book, Ch.5]

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- Strong reasons to believe that it converges to the optimal policy
- “Exploring starts” requirement may be unrealistic



# Monte Carlo without exploring starts (on policy)

## On-policy first-visit MC control (for $\epsilon$ -soft policies), estimates $\pi \approx \pi_*$

Initialize, for all  $s \in \mathcal{S}$ ,  $a \in \mathcal{A}(s)$ :

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Repeat forever:

- (a) Generate an episode using  $\pi$
- (b) For each pair  $s, a$  appearing in the episode:
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  - Append  $G$  to  $Returns(s, a)$
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- (c) For each  $s$  in the episode:
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- Policy value cannot decrease

$$V_{\pi'}(s) \geq V_{\pi}(s), \forall s \in \mathcal{S}$$

$\pi$  : policy at current step

$\pi'$  : policy at next step

# Monte-Carlo methods summary

- Allow a policy to be learned through interactions
  - **(Does not learn transitions)**
- States are effectively treated as being independent
  - Focus on a subset of states (e.g., states for which playing optimally is of particular importance)
- Episodic (with or without exploring starts)

# Temporal Difference



# Temporal Difference (TD) Learning

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$$\mathbf{V}'(\mathbf{s}_t) = \mathbf{V}(\mathbf{s}_t) + \alpha [\mathbf{G}_t - \mathbf{V}(\mathbf{s}_t)]$$

Observed return :

Step size

$$\mathbf{G}_t = \sum_t^T \gamma^t \mathbf{R}(\mathbf{s}_t)$$

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- TD(0)

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$$V'(s_t) = V(s_t) + \alpha G_t - V(s_t)$$

Observed return :  $G_t$

Step size :  $\alpha$

$$G_t = \sum_t^T \gamma^t R(s_t)$$

- TD(0)

- updates “instantly”

$$V'(s_t) = V(s_t) + \alpha \underbrace{[R(s_t) + \gamma V(s_{t+1})]}_{\approx G_t} - V(s_t)$$

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$Returns(s) \leftarrow$  an empty list, for all  $s \in \mathcal{S}$

Repeat forever:

Generate an episode using  $\pi$

For each state  $s$  appearing in the episode:

$G \leftarrow$  the return that follows the first occurrence of  $s$

Append  $G$  to  $Returns(s)$

$V(s) \leftarrow \text{average}(Returns(s))$



# TD(0) for prediction VS TD for control

## Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size  $\alpha \in (0, 1]$ , small  $\varepsilon > 0$

Initialize  $Q(s, a)$ , for all  $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$ , arbitrarily except that  $Q(\text{terminal}, \cdot) = 0$

Loop for each episode:

Initialize  $S$

Choose  $A$  from  $S$  using policy derived from  $Q$  (e.g.,  $\varepsilon$ -greedy)

Loop for each step of episode:

Take action  $A$ , observe  $R, S'$

Choose  $A'$  from  $S'$  using policy derived from  $Q$  (e.g.,  $\varepsilon$ -greedy)

$Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma Q(S', A') - Q(S, A)]$

$S \leftarrow S'; A \leftarrow A';$

until  $S$  is terminal

## Tabular TD(0) for estimating $v_\pi$

Input: the policy  $\pi$  to be evaluated

Initialize  $V(s)$  arbitrarily (e.g.,  $V(s) = 0$ , for all  $s \in \mathcal{S}^+$ )

Repeat (for each episode):

Initialize  $S$

Repeat (for each step of episode):

$A \leftarrow$  action given by  $\pi$  for  $S$

Take action  $A$ , observe  $R, S'$

$V(S) \leftarrow V(S) + \alpha [R + \gamma V(S') - V(S)]$

$S \leftarrow S'$

until  $S$  is terminal

[Sutton & Barto,  
RL Book, Ch.6]

# Q-learning for control

Q-learning (off-policy TD control) for estimating  $\pi \approx \pi_*$

Initialize  $Q(s, a)$ , for all  $s \in \mathcal{S}, a \in \mathcal{A}(s)$ , arbitrarily, and  $Q(\text{terminal-state}, \cdot) = 0$

Repeat (for each episode):

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$\epsilon$ -greedy policy

$$\mathbf{a} = \begin{cases} \arg \max_a Q(\mathbf{a}, \mathbf{s}) & \text{with probability } 1 - \epsilon, \\ \text{random } \mathbf{a} & \text{with probability } \epsilon. \end{cases}$$

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[Sutton & Barto,  
RL Book, Ch.6]

- Converges to  $Q^*$  as long as all  $(s,a)$  pairs continue to be updated and with minor constraints on learning rate

# Comparing TD and MC

- MC requires going through full episodes before updating the value function. Episodic.
- Converges to the optimal solution
- TD updates each  $V(s)$  after each transition. Online.
- Converges to the optimal solution (some conditions on  $\alpha$ )
- Empirically TD methods tend to converge faster

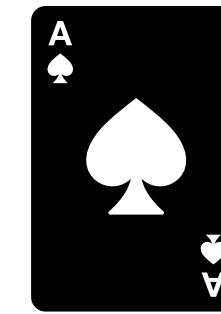


# Practical difficulties

- **Compared to supervised learning** setting up an RL problem is often **harder**
  - Need an **environment** (or at least a simulator)
- **Rewards**
  - In some domains it's clear (e.g., in games)
  - In others it's much more subtle (e.g., you want to please a human)

**Hand on Session  
+  
Extra material  
(Some will be used for this week's exercises)**

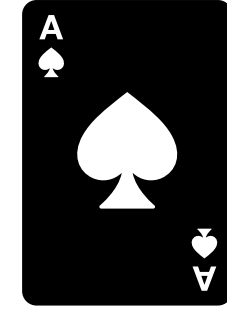
# Black Jack



The most widely played casino banking game in the world, also known as Twenty-One.



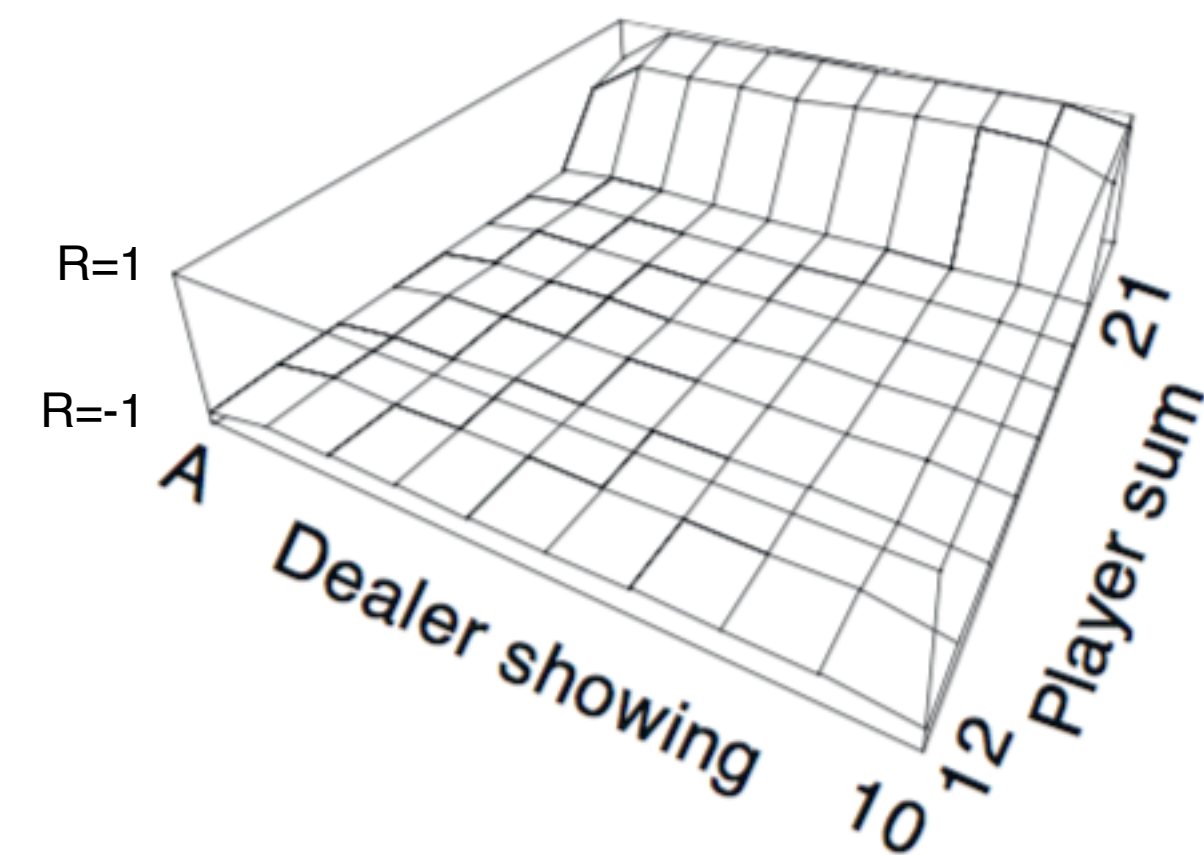
# Black Jack



- **Episode: one hand**
- **States: Sum of player's cards, dealer's card, usable ace**
- **Actions: {Stay, Hit}**
- **Rewards: {Win +1, Tie 0, Loose -1}**
- **A few other assumptions: infinite deck**

- Evaluates the policy that hits except when the sum of the cards is 20 or 21

No  
usable  
ace

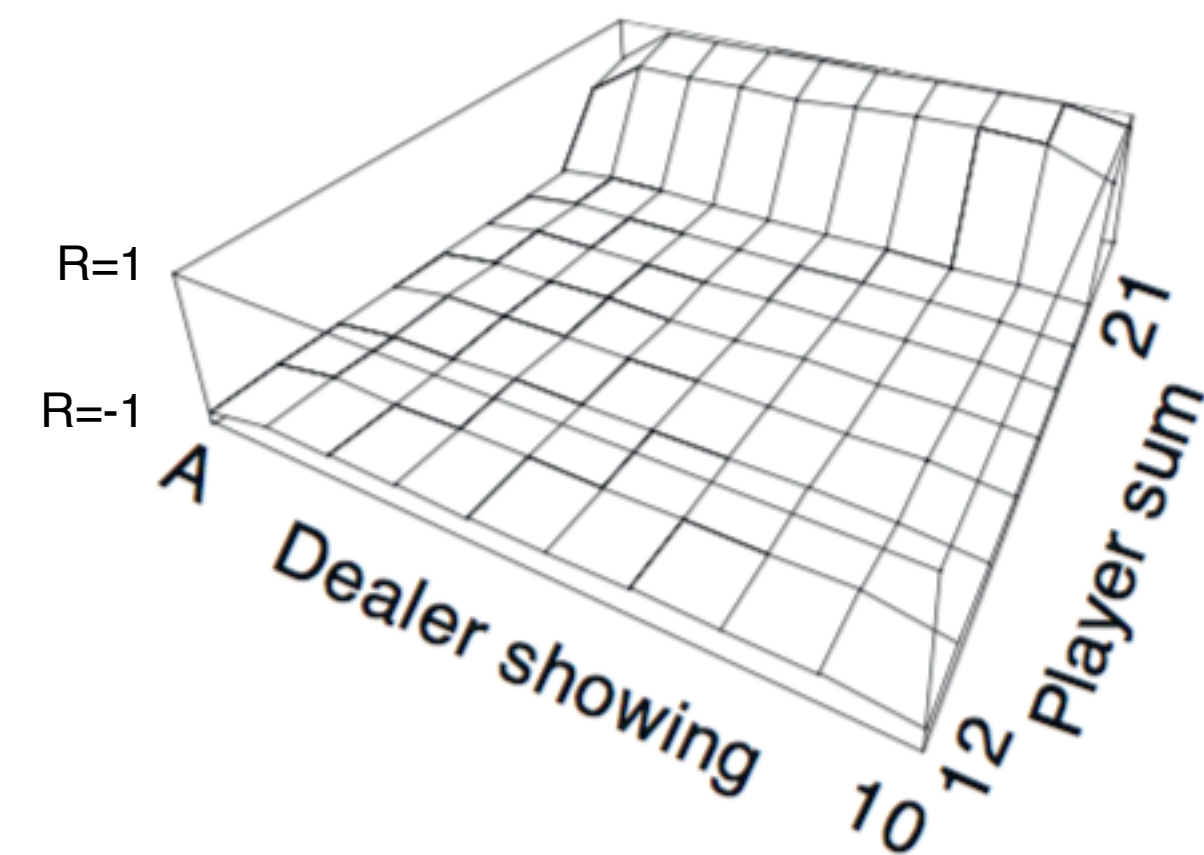


[Figure 5.1, Sutton & Barto]

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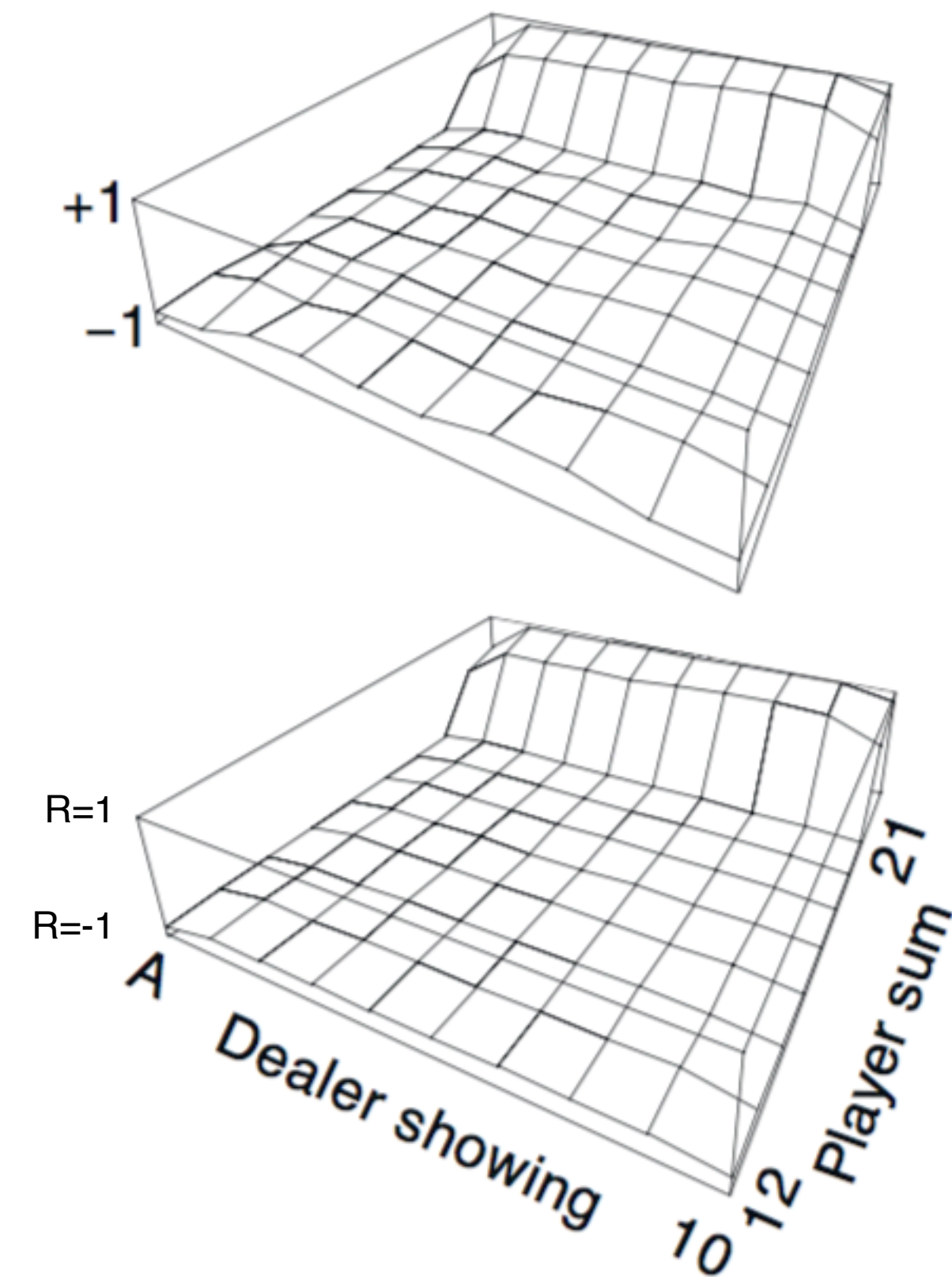
[Figure 5.1, Sutton & Barto]



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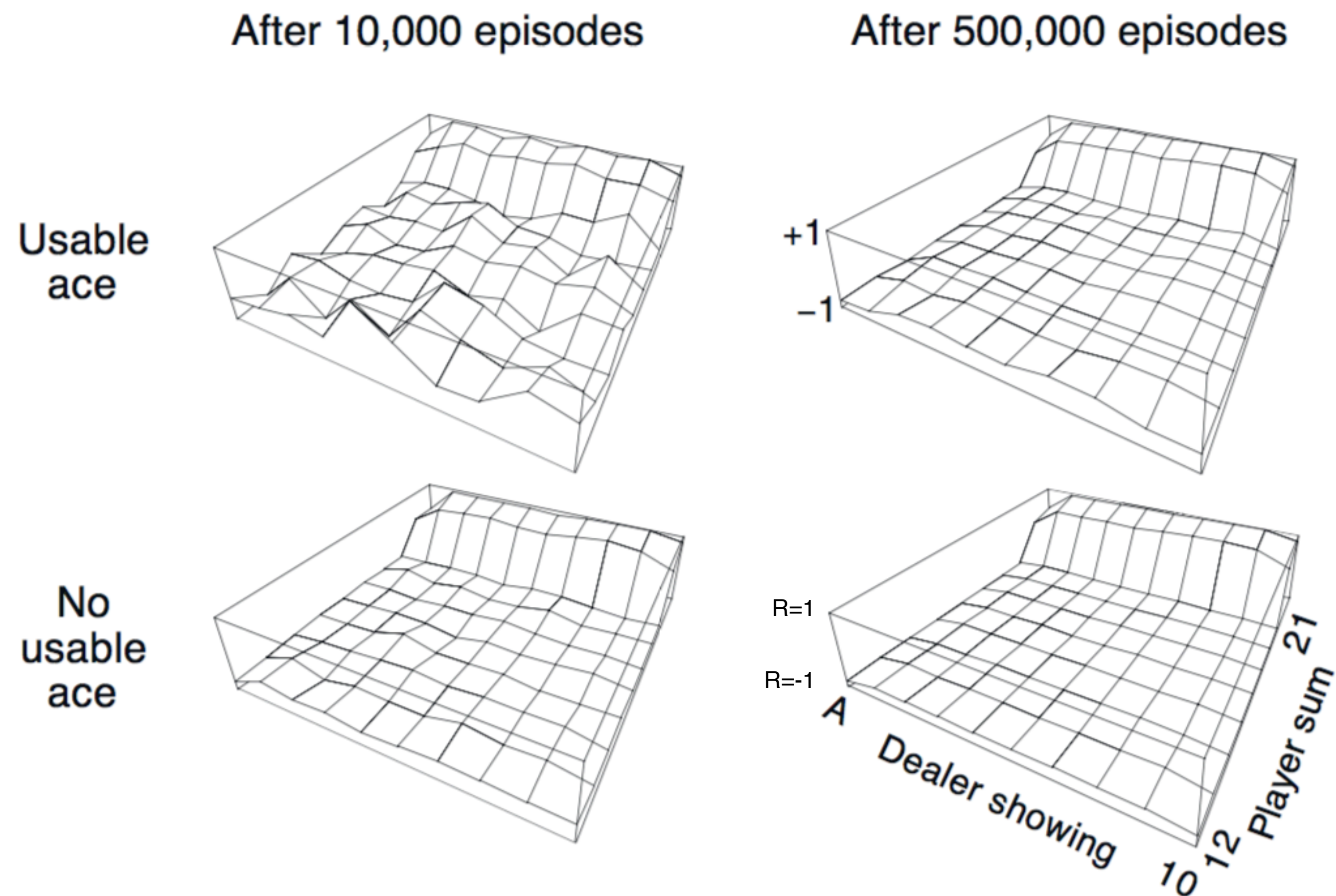
Usable  
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[Figure 5.1, Sutton & Barto]

- Evaluates the policy that hits except when the sum of the cards is 20 or 21



[Figure 5.1, Sutton & Barto]

# Approximation techniques

- Methods we studied are “tabular”
- State value functions (and Q) can be approximated
  - **Linear approximation:**  $V(\mathbf{s}) = \mathbf{w}^\top \mathbf{x}(\mathbf{s})$ 
    - Coupling between states through  $\mathbf{x}(\mathbf{s})$
  - **Adapt the algorithms for this case.**
    - Updates to the value function now imply updating the weights  $\mathbf{w}$  using a gradient

# Approximation techniques (prediction)

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[Sutton & Barto,  
RL Book, Ch.9]

## Gradient Monte Carlo Algorithm for Estimating $\hat{v} \approx v_\pi$

Input: the policy  $\pi$  to be evaluated

Input: a differentiable function  $\hat{v} : \mathcal{S} \times \mathbb{R}^d \rightarrow \mathbb{R}$

Initialize value-function weights  $\mathbf{w}$  as appropriate (e.g.,  $\mathbf{w} = \mathbf{0}$ )

Repeat forever:

    Generate an episode  $S_0, A_0, R_1, S_1, A_1, \dots, R_T, S_T$  using  $\pi$

    For  $t = 0, 1, \dots, T - 1$ :

$\mathbf{w} \leftarrow \mathbf{w} + \alpha [G_t - \hat{v}(S_t, \mathbf{w})] \nabla \hat{v}(S_t, \mathbf{w})$

## First-visit MC prediction, for estimating $V \approx v_\pi$

Initialize:

$\pi \leftarrow$  policy to be evaluated

$V \leftarrow$  an arbitrary state-value function

$Returns(s) \leftarrow$  an empty list, for all  $s \in \mathcal{S}$

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$G_t$  is an unbiased estimator of  $\mathbf{v}_\pi(s_t)$

# Approximation techniques

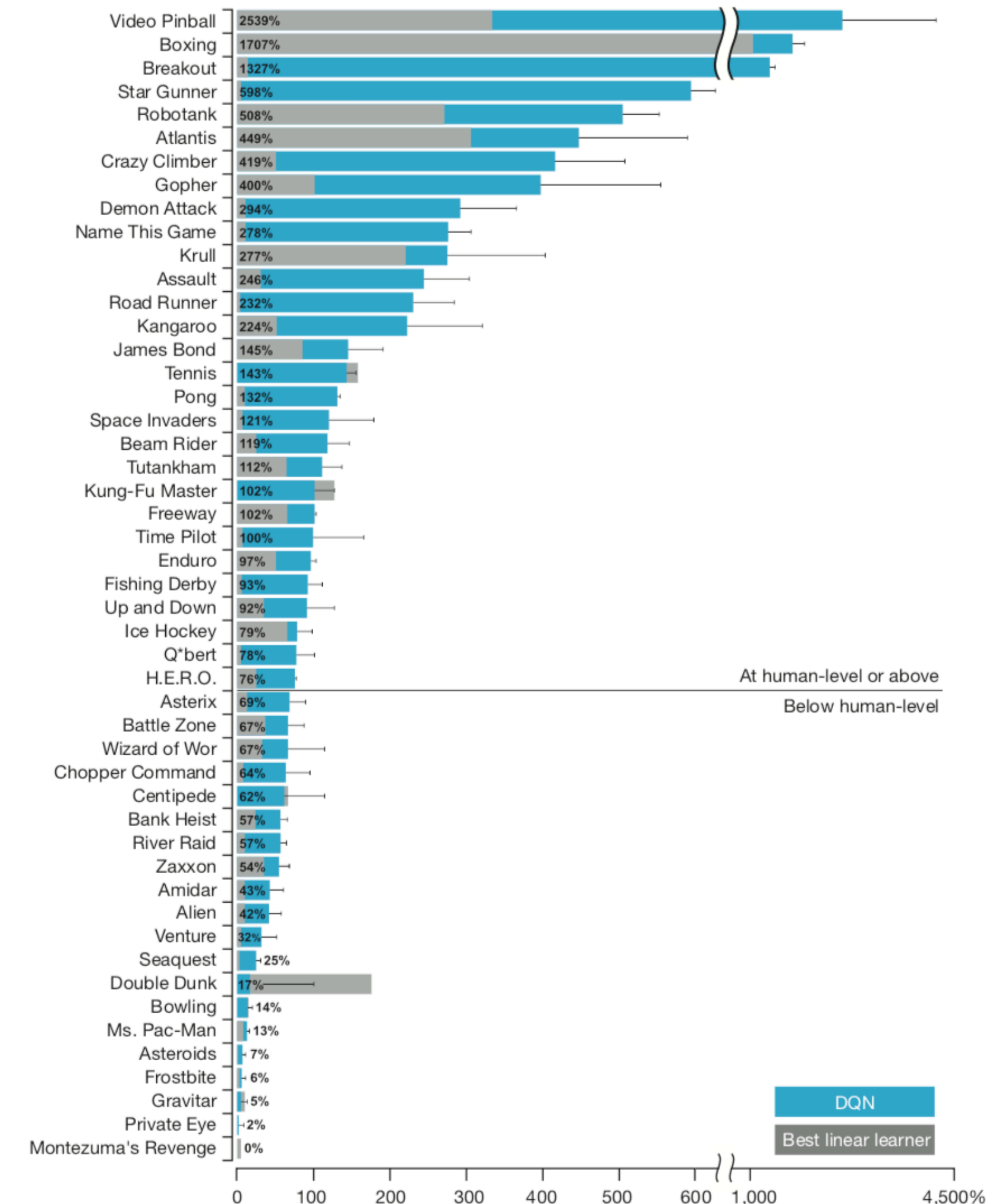
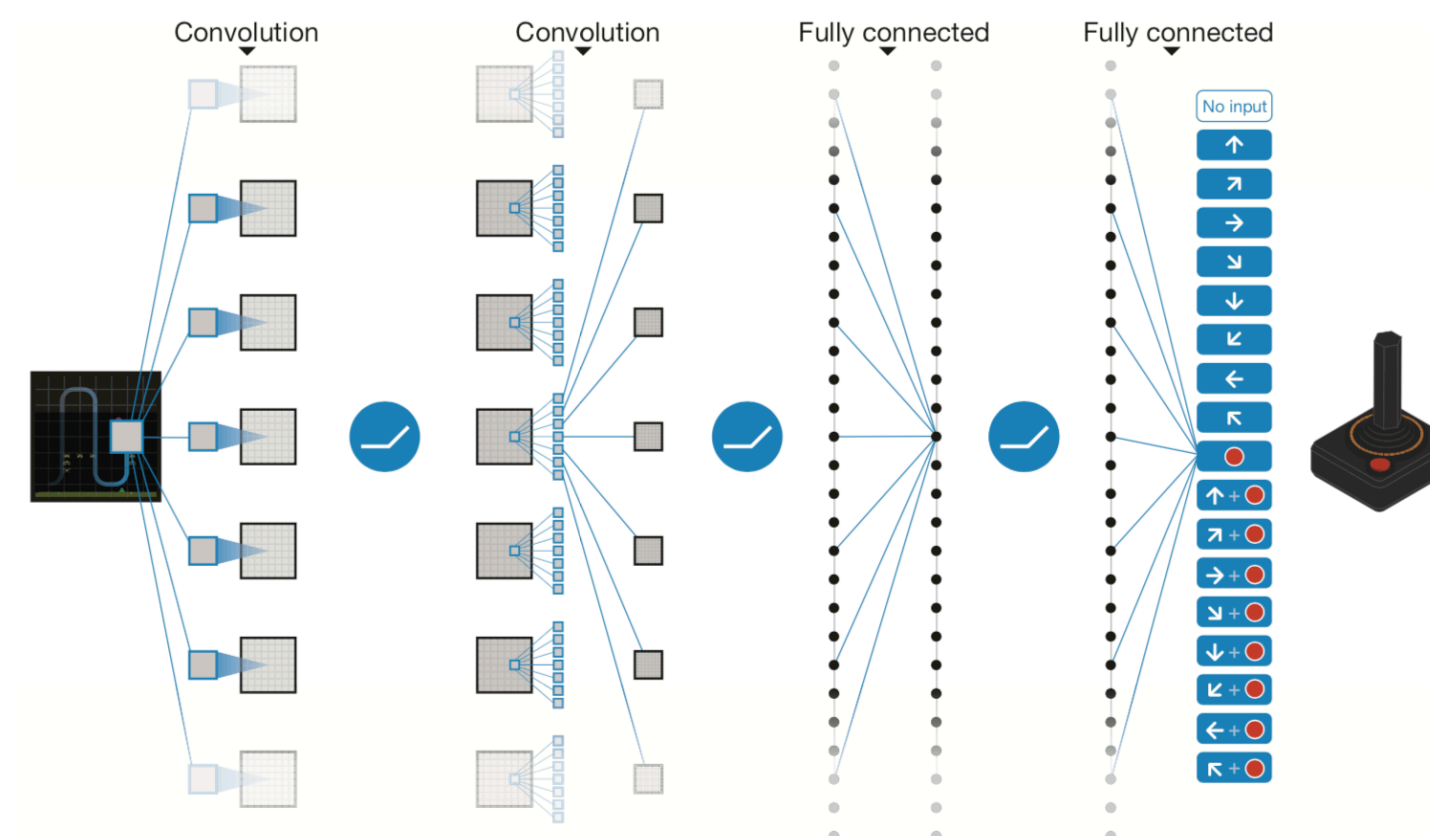
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# Approximation techniques

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# Approximation techniques

- Works both for prediction and control
- Any model can be used to approximate
- Recent work using deep neural networks yield impressive performance on computer (Atari) games



# Summary

- Today we have defined RL studied several algorithms for solving RL problems (mostly for for tabular case)
- **Main challenges**
  - Credit assignment
  - Exploration/Exploitation tradeoff
- **Algorithms**
  - Prediction
    - Monte Carlo and TD(0)
  - Control
    - Q-learning
- **Approximation algorithms can help scale reinforcement learning**