Machine Learning for Large-Scale Data Analysis and Decision Making (MATH80629A)
Fall 2021

Week #4 - **Summary**

Announcement

- **Hybrid classroom**: Mondays 8:30 am - 11:30 am
  Class room: Manuvie. This classroom is located on the 1st floor of Côte-Sainte-Catherine building.
  Zoom: Zoom link.

- **Hybrid office hour**: Mondays 11:30 am - 1 pm
  Office:4.834
  Zoom: Zoom link.

- **Lab session** on week #5 (September 27)
  Lab room: Laboratoire Lachute

Today

- **Second Quiz** on Gradescope!
- Summary of Machine learning fundamental
- Q&A
- Hands-on session

Quiz 1

Login to your Gradescope account

Models for supervised learning

- (Mostly) linear models
- Focus on **classification**

1. **Non-Probabilistic Models**
   - Nearest Neighbor, Support Vector Machines (SVMs)
2. **Probabilistic Models**
   - Naive Bayes

Supervised learning

Train Data

$$
\begin{array}{c|ccccc|}
 & \text{Nb.bed.} & \text{Area} & \text{Neigh.} & \cdot & \cdot \\
\end{array}
\qquad
\text{Sell-ability}
$$

$$
\underbrace{\begin{array}{c}x_0\\x_1\\x_2\\x_3\\x_4\end{array}\begin{bmatrix}1 & 0 & 0 & 0 & 0\\1 & 100 & 1 & .2 & .5\\3 & 200 & 0 & .1 & .2\\1 & 150 & 1 & .4 & .1\\2 & 210 & 2 & .5 & 1.1\end{bmatrix}}_{X}
\quad
\underbrace{\begin{array}{c}y_0\\y_1\\y_2\\y_3\\y_4\end{array}\begin{bmatrix}1\\2\\0\\2\\1\end{bmatrix}}_{Y}
$$

Task

$$f : \mathbb{R}^n \rightarrow \{0, 1, 2\}$$

Test Data

$$
\begin{array}{c|ccccc|}
 & \text{Nb.bed.} & \text{Area} & \text{Neigh.} & \cdot & \cdot \\
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\text{Sell-ability}
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\underbrace{\begin{array}{c}x_0\\x_1\\x_2\\x_3\\x_4\end{array}\begin{bmatrix}1 & 0 & 0 & 0 & 0\\2 & 50 & 1 & .3 & .8\\1 & 100 & 1 & .5 & 1.4\\4 & 170 & 0 & .7 & .4\\1 & 120 & 3 & .9 & .5\end{bmatrix}}_{X^{new}}
\quad
\underbrace{\begin{array}{c}y_0\\y_1\\y_2\\y_3\\y_4\end{array}\begin{bmatrix}?\\?\\?\\?\\?\end{bmatrix}}_{Y^{new}}
$$

Supervised learning

Train Data

$$\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}
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\underbrace{\begin{array}{c} \text{Sell-ability} \end{array}
\begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}}_{Y}$$

Task

Models $\boxed{f}\ \mathbb{R}^n \rightarrow \{0, 1, 2\}$

Test Data

$$\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}
\underbrace{\begin{array}{ccccc} \text{Nb.bed.} & \text{Area} & \text{Neigh.} & \cdot & \cdot \\ \end{array}
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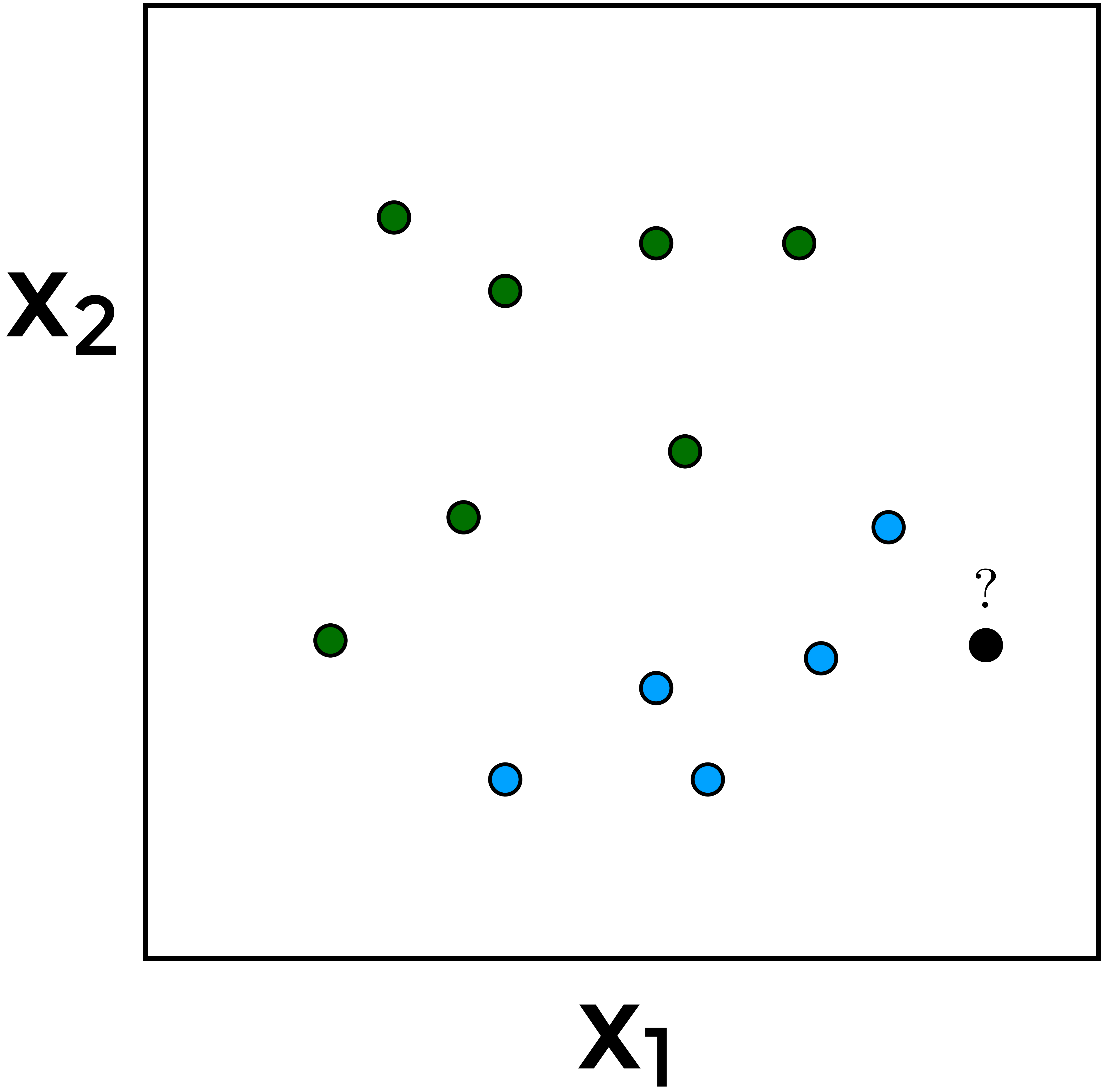
\quad
\begin{array}{c} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{array}
\underbrace{\begin{array}{c} \text{Sell-ability} \end{array}
\begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}}_{Y^{new}}$$

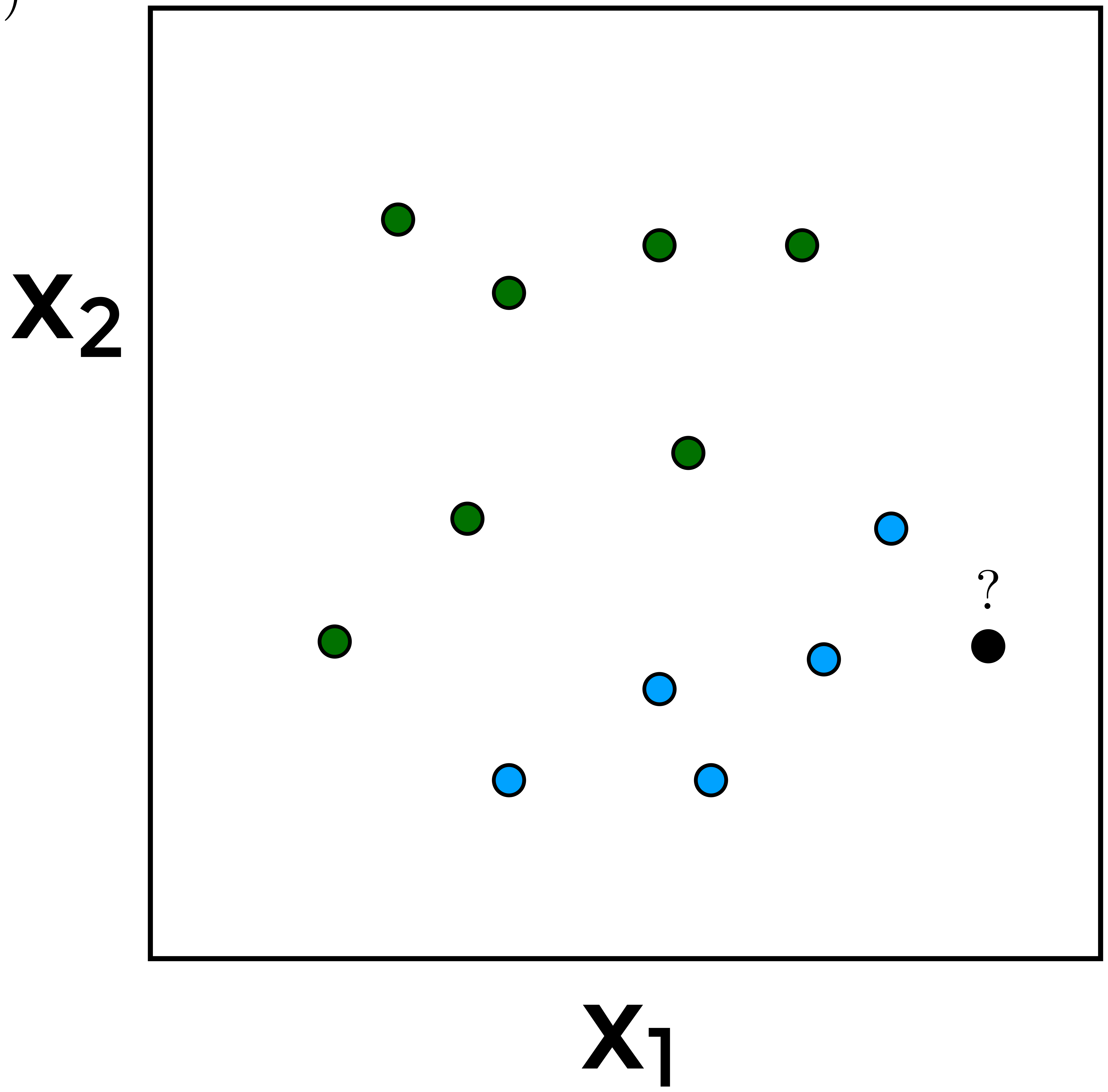


- **1-NN**

Instance classified according to its nearest neighbor

$$i' = \arg\min_i \mathsf{dist}(x_i, x_j)$$

$$y_j = y_{i'}$$



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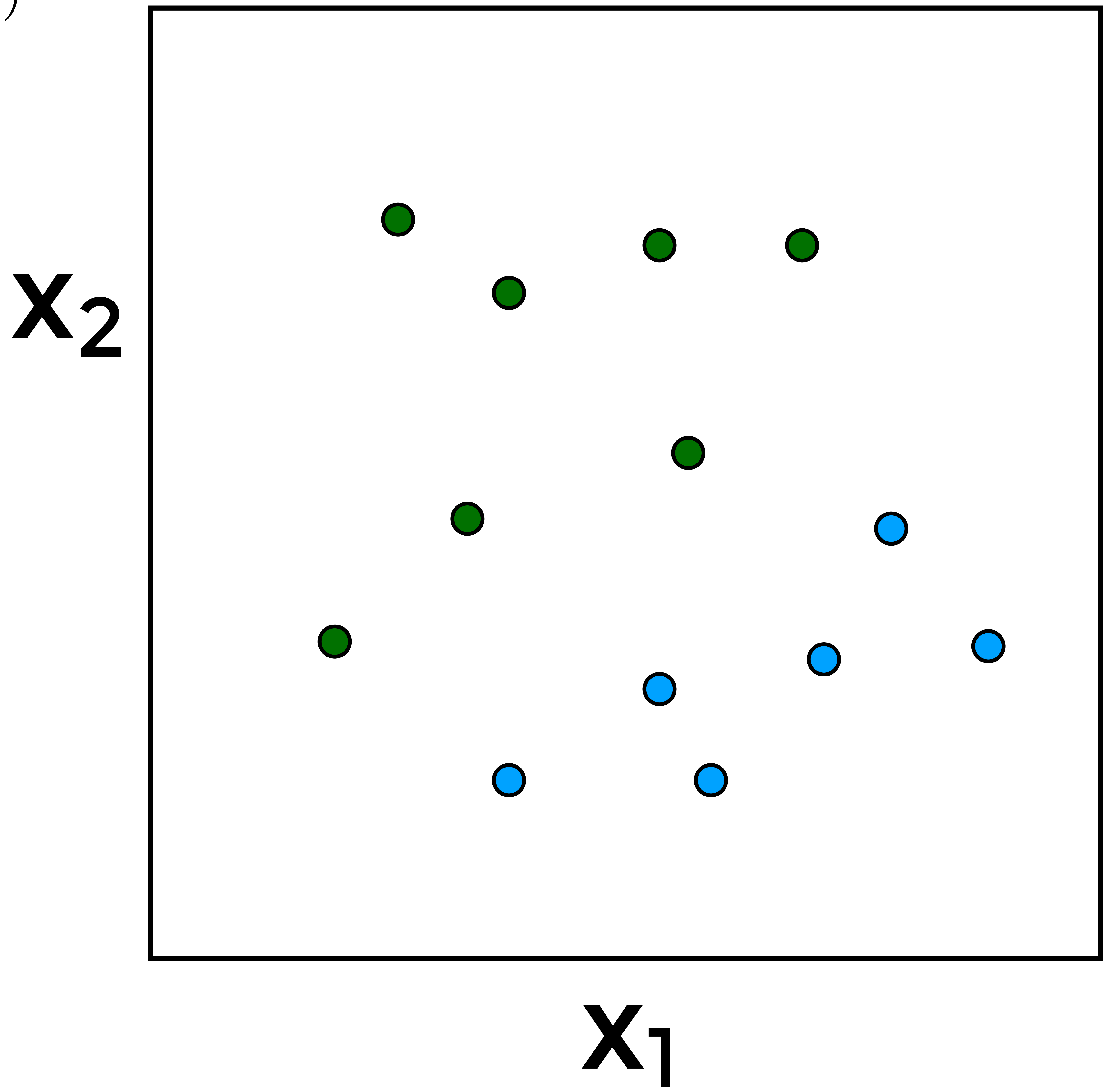

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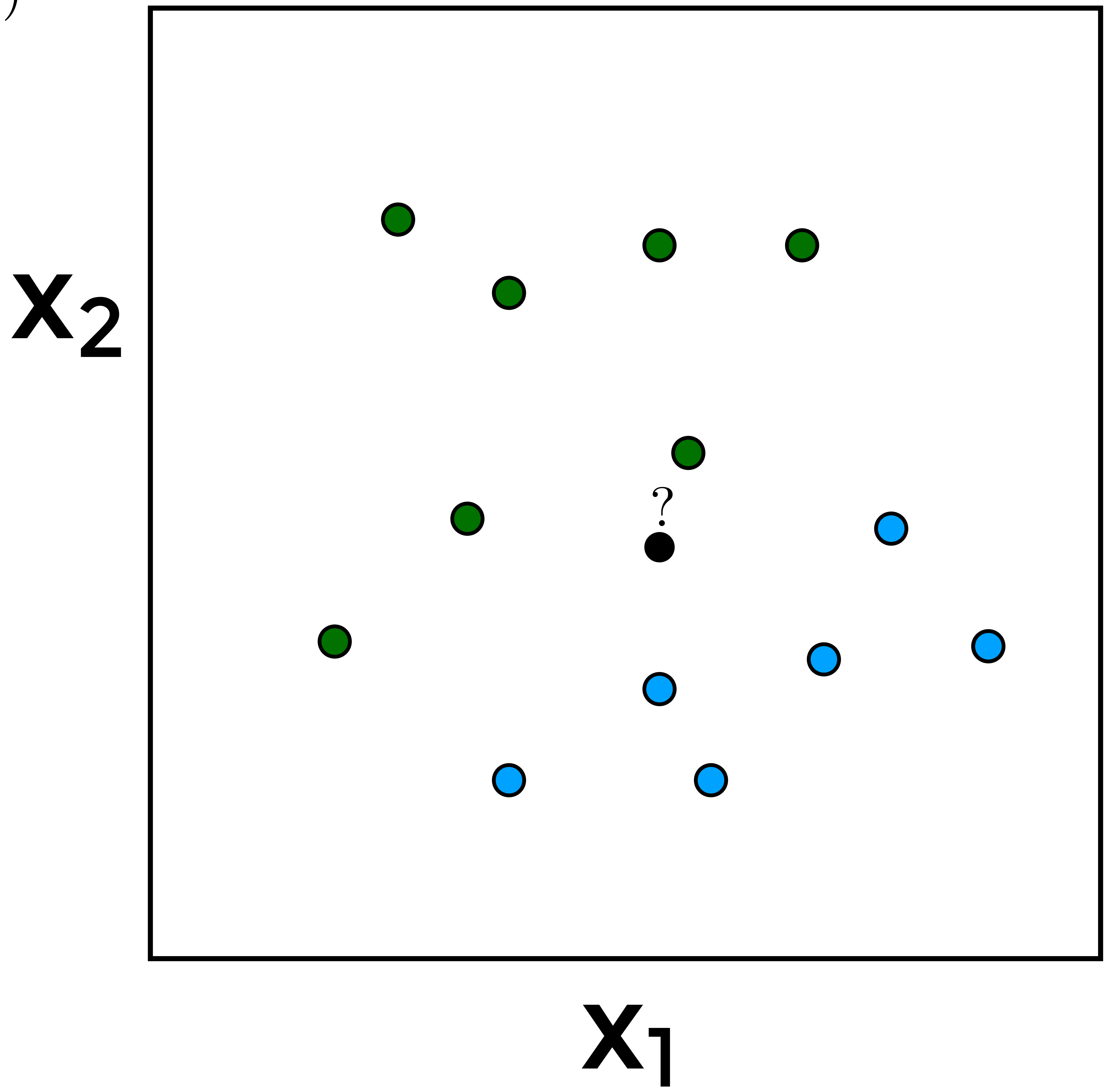


- **1-NN**

Instance classified according to its nearest neighbor

- **K-NN**

Instance classified according to the majority of its K nearest neighbors

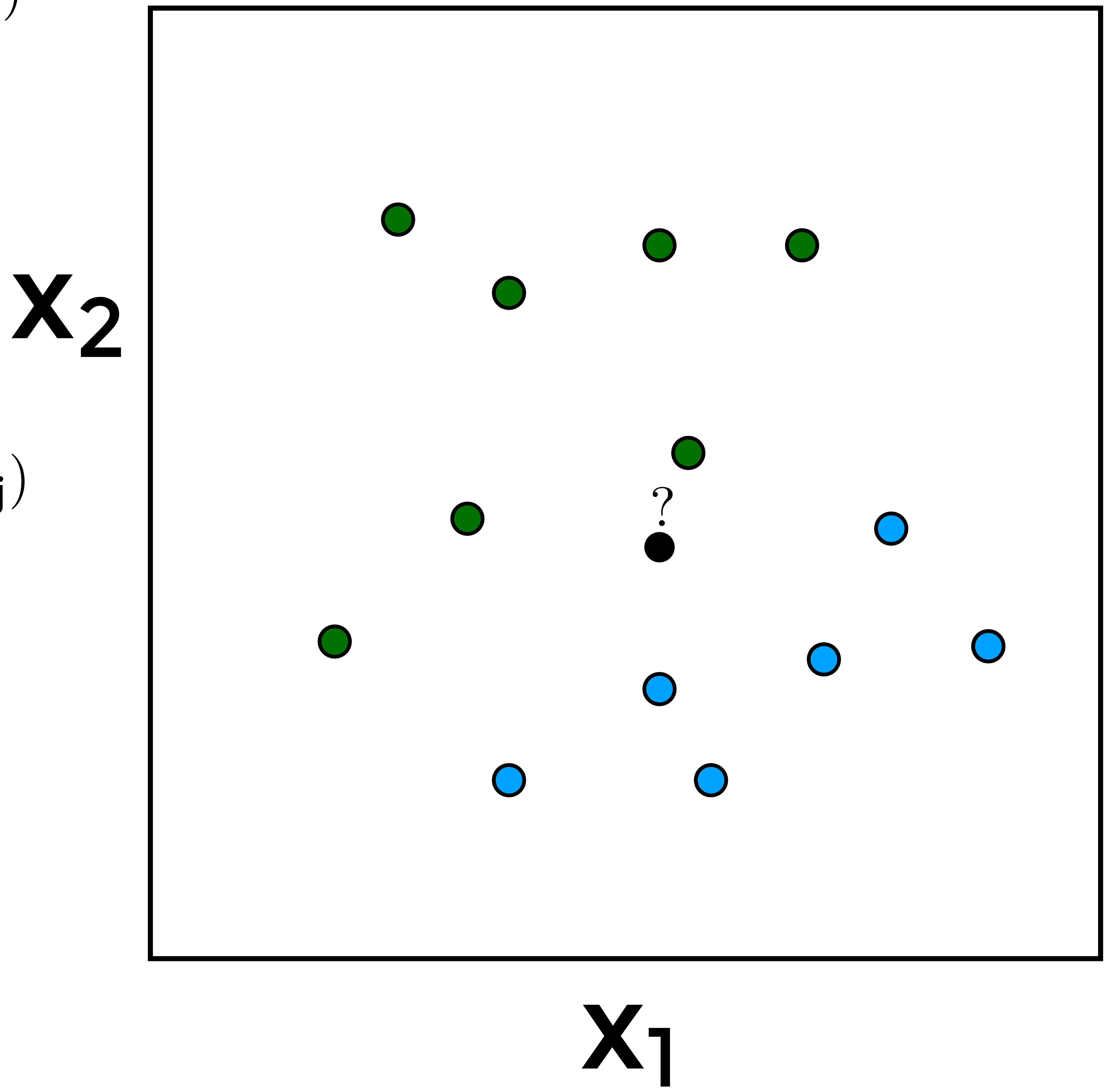
$$\mathsf{i}' = \arg\min_{\mathsf{i}} \mathsf{dist}(\mathsf{x_i}, \mathsf{x_j})$$

$$\mathsf{y_j} = \mathsf{y_{i'}}$$

$$\mathsf{k} = 5 \text{ (assumption)}$$

$$\mathsf{i} = \arg\mathrm{sort}_{\mathsf{i}}\, \mathsf{dist}(\mathsf{x_i}, \mathsf{x_j})$$

$$\mathsf{y_j} = \mathsf{majority}(\mathsf{i}_{:5})$$



- **1-NN**

Instance classified according to its nearest neighbor

- **K-NN**

Instance classified according to the majority of its K nearest neighbors

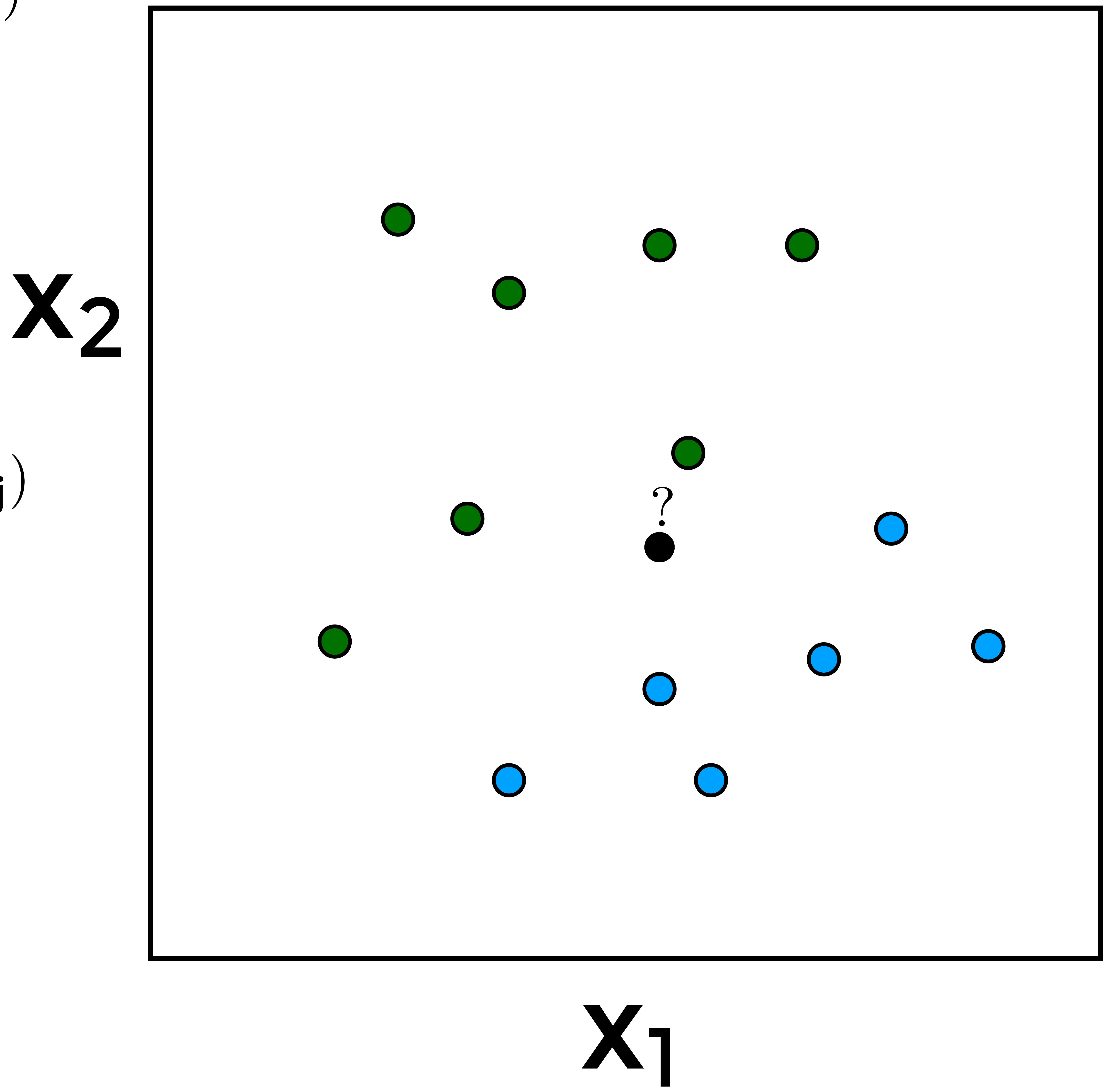
$$i' = \arg\min_i \mathsf{dist}(x_i, x_j)$$

$$y_j = y_{i'}$$

$$k = 5 \text{ (assumption)}$$

$$i = \arg\operatorname{sort}_i \mathsf{dist}(x_i, x_j)$$

$$y_j = \text{majority}(i_{:5})$$



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- **K-NN**

Instance classified according to the majority of its K nearest neighbors

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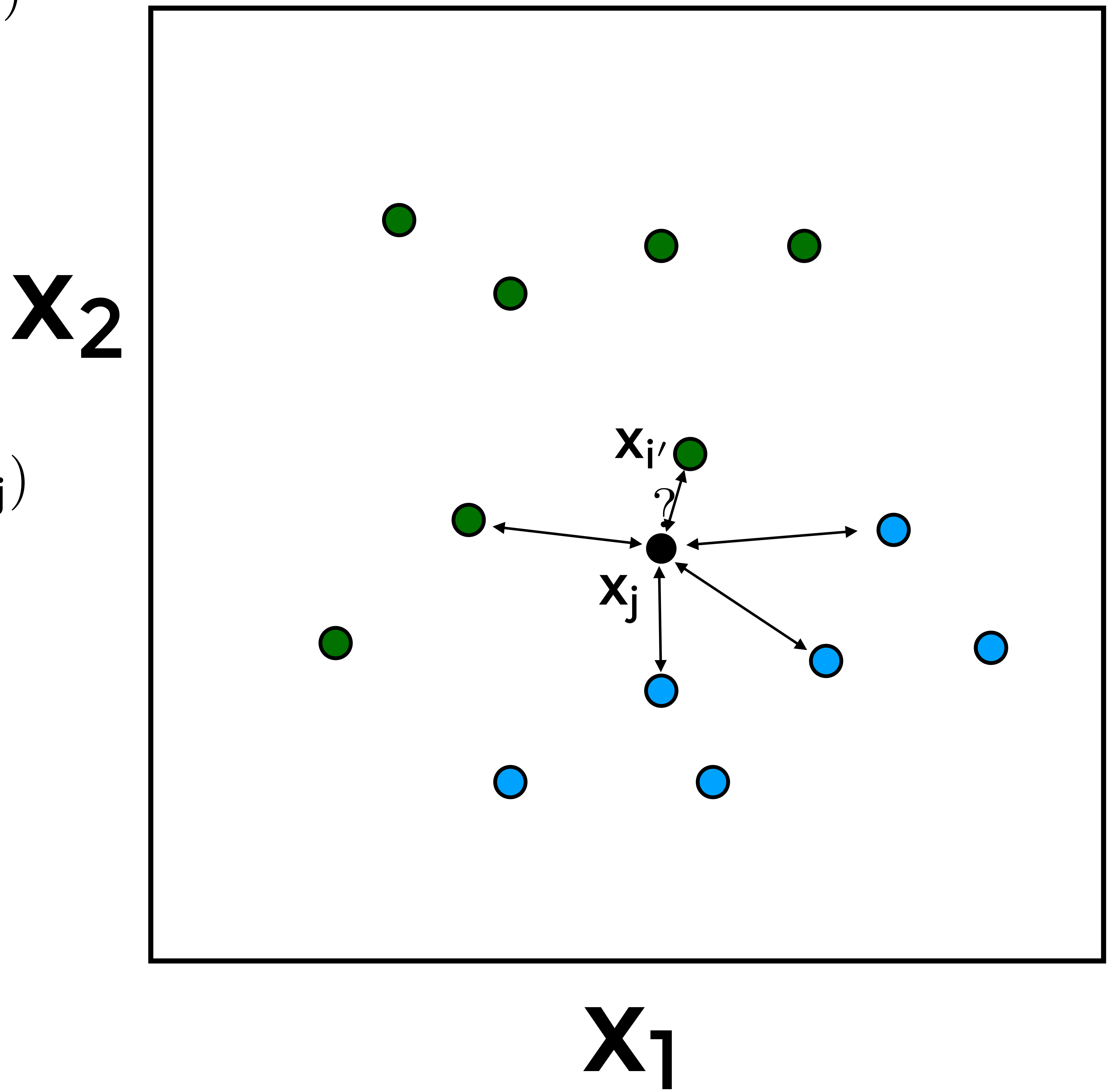

- **1-NN**

Instance classified according to its nearest neighbor

- **K-NN**

Instance classified according to the majority of its K nearest neighbors

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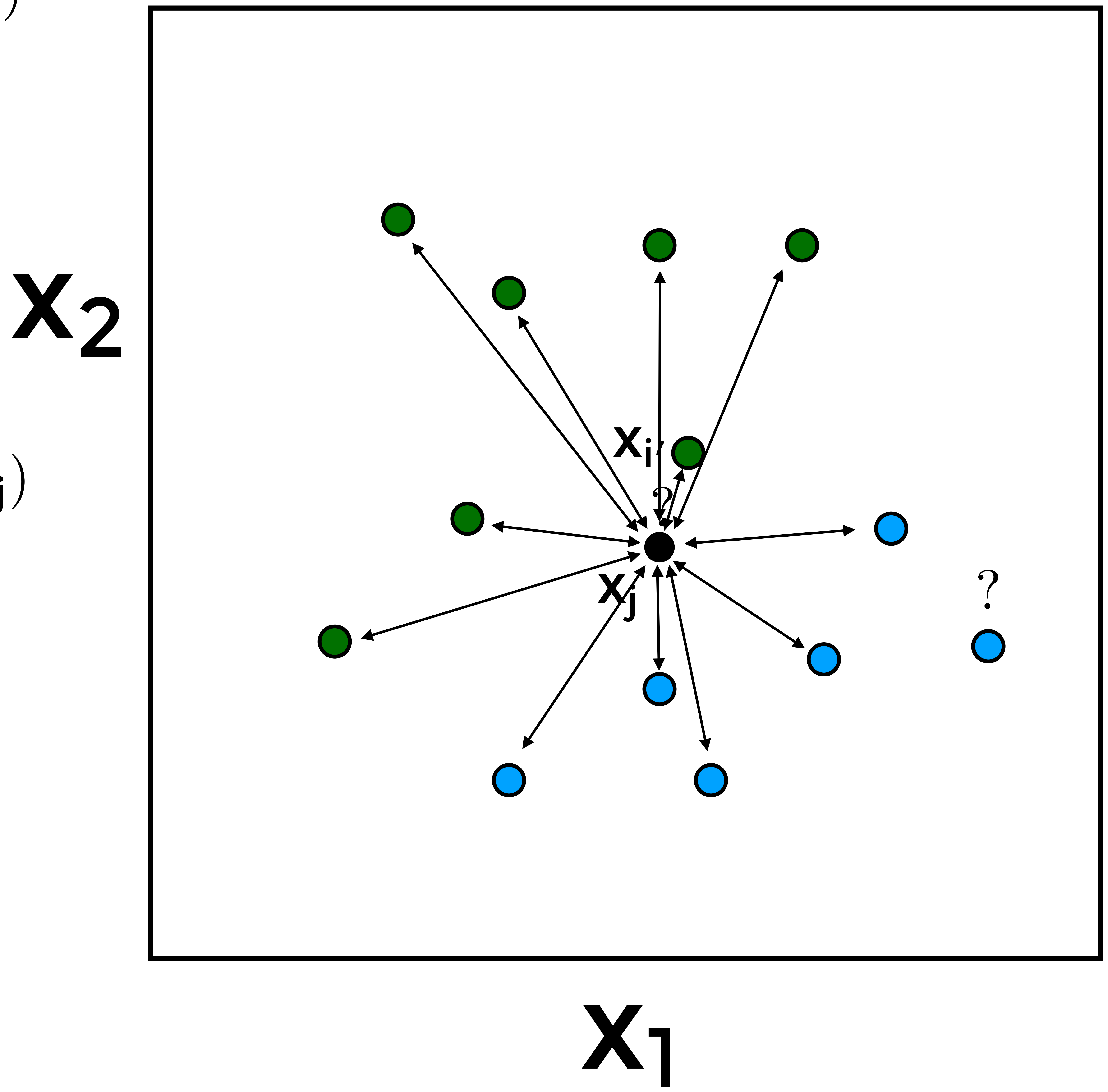


- **1-NN**

Instance classified according to its nearest neighbor

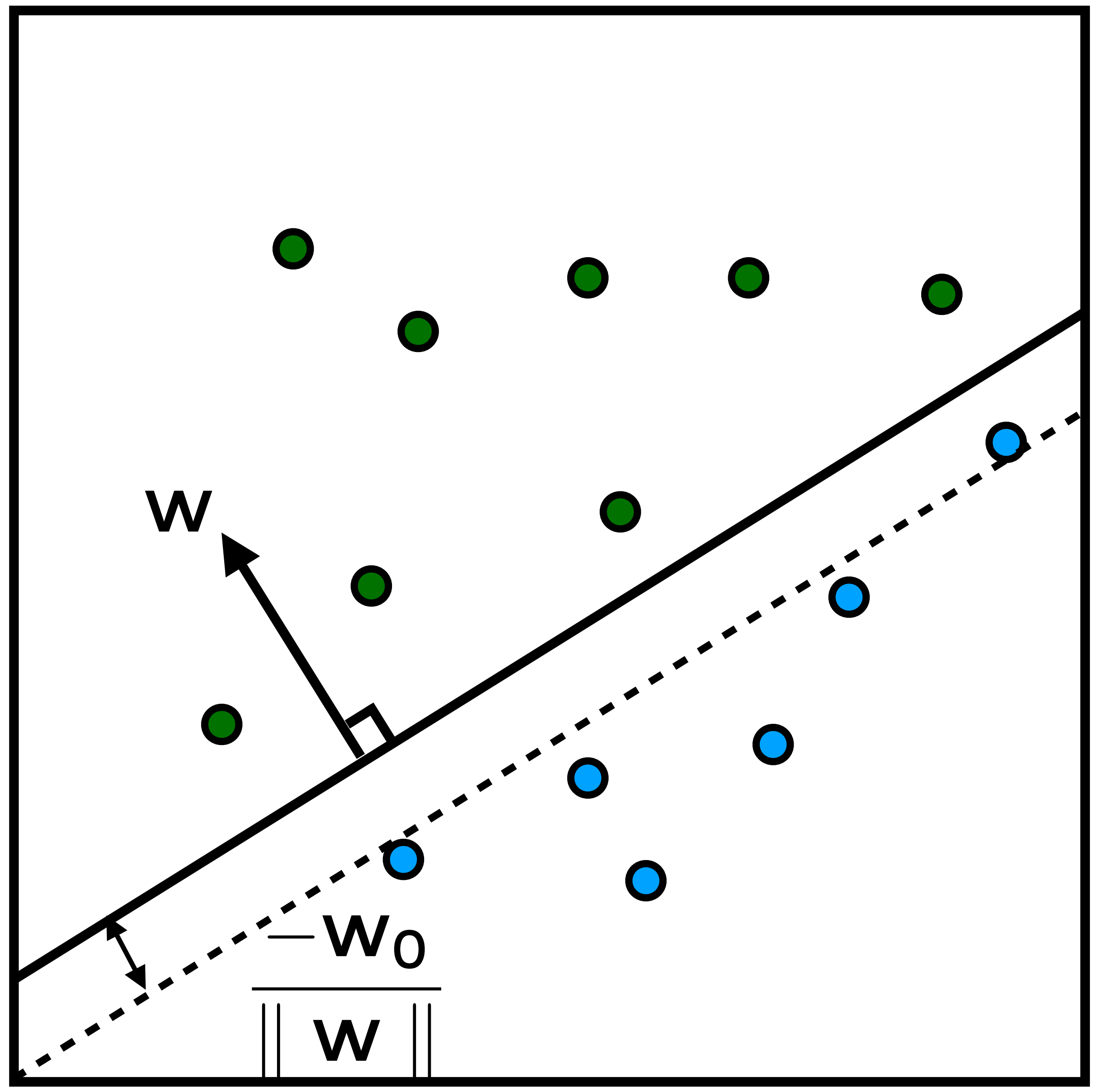
- **K-NN**

Instance classified according to the majority of its K nearest neighbors

- **weighted-NN**

Instance classified according to all neighbors. The contribution of each neighbor is weighted by its distance.

Linear Classification



decision boundary: $y(x) = 0$
take two points on the boundary: x_a, x_b

then: $w^\top x_a + w_0 = w^\top x_b + w_0$
$\implies w^\top (x_a - x_b) = 0$
$\implies$ w is perpendicular to the decision boundary
w represents the orientation of the decision boundary

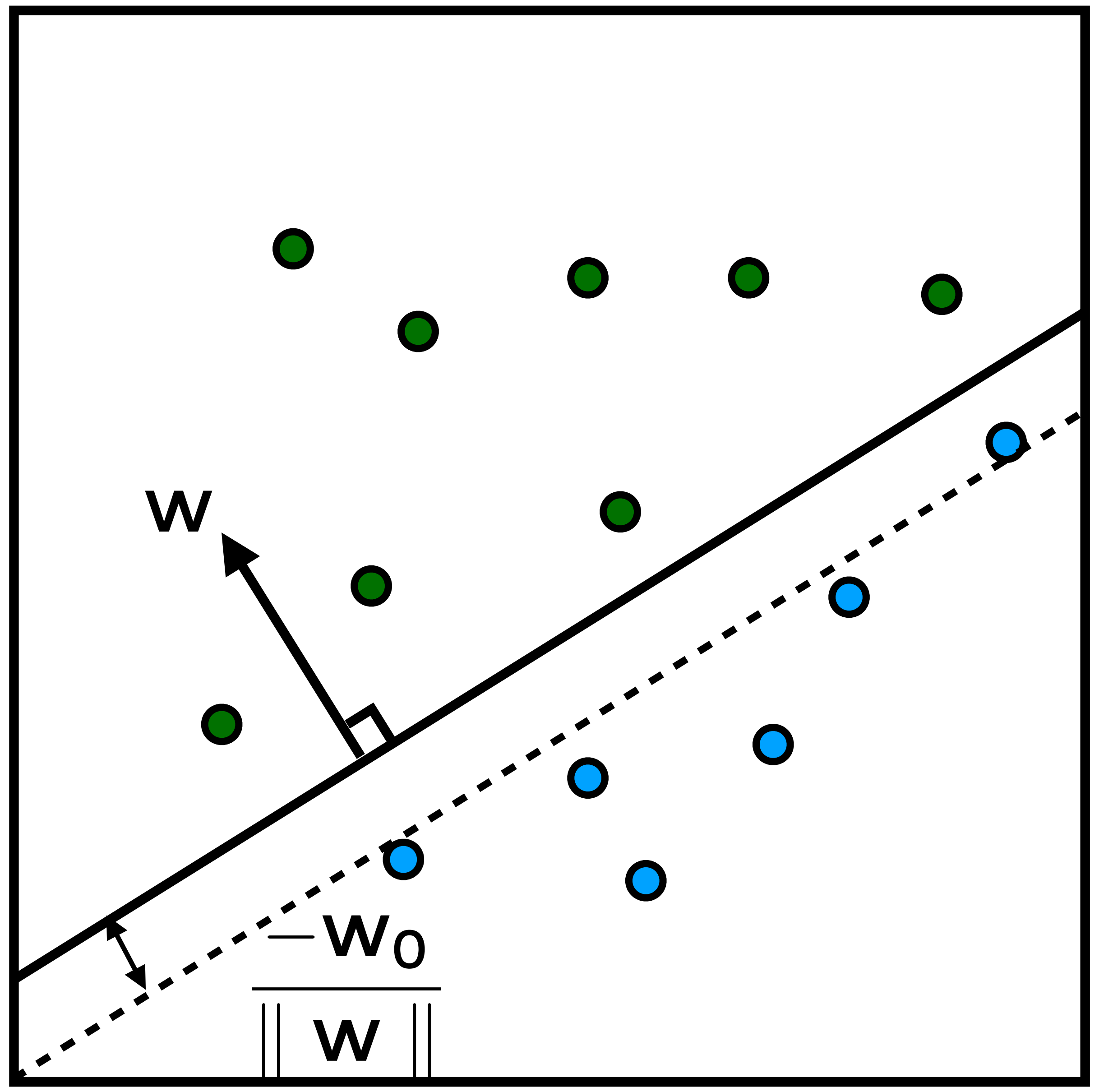
$$y(x) = w^\top x + w_0$$

Decision

$(w^\top x + w_0) > 0 \implies$ ● (green)

$(w^\top x + w_0) < 0 \implies$ ● (blue)

Linear Classification



$$y(x) = w^\top x + w_0$$

Decision

$$(w^\top x + w_0) > 0 \Longrightarrow$$

$$(w^\top x + w_0) < 0 \Longrightarrow$$

w_0 is a scalar

you can think of it like an intercept

take x' as the closest point on the decision boundary to the origin

$x' = \beta w$

$$\Longrightarrow y(x') = w^\top x' + w_0$$

$$\Longrightarrow y(x') = w^\top (\beta w) + w_0$$

$$\Longrightarrow 0 = \beta \| w \|^2 + w_0$$

$$\Longrightarrow \beta = \frac{-w_0}{\| w \|^2}$$

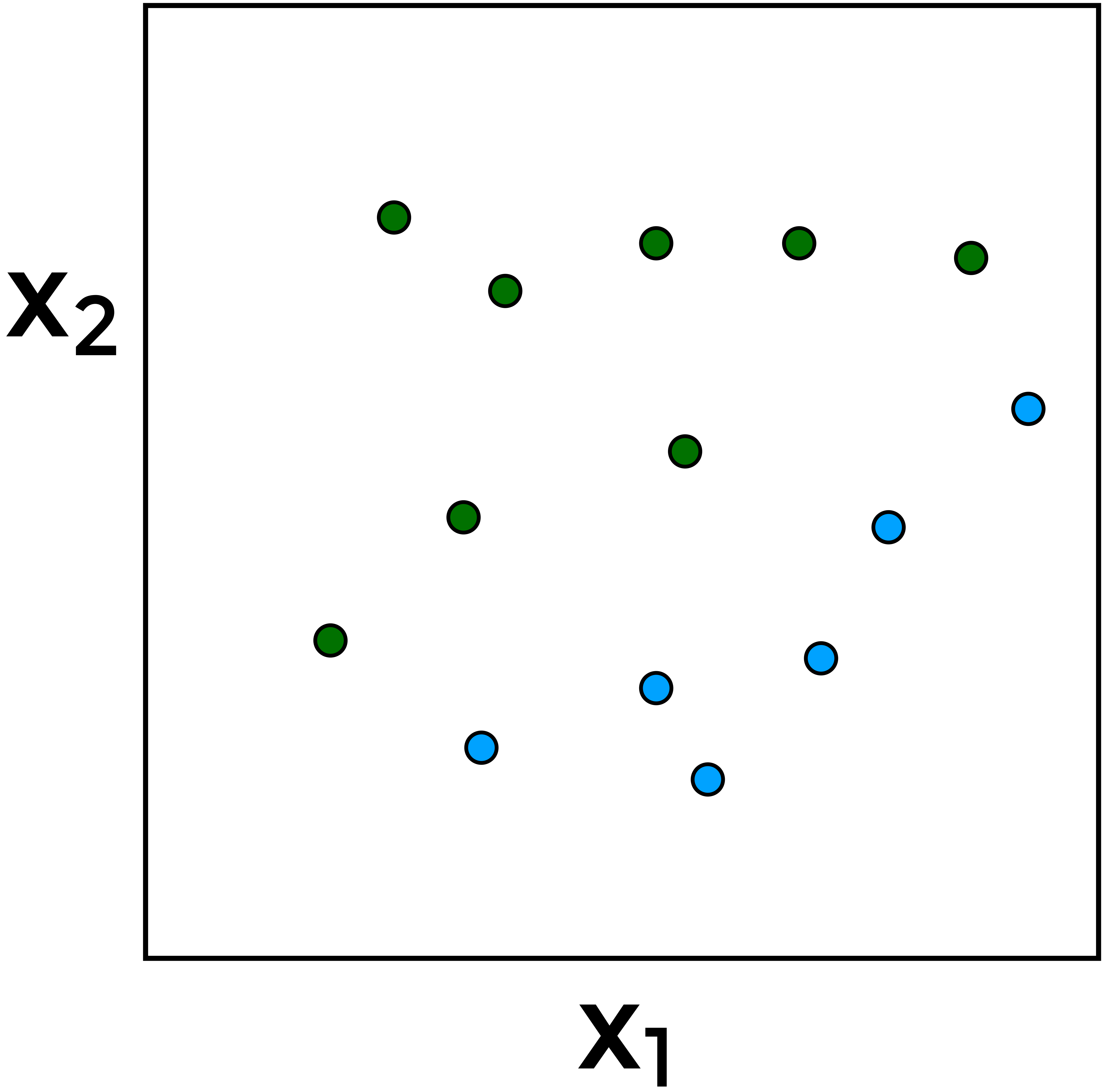
Then you know that the distance from the origin to x' is:

$$\| x' \| = \| \beta w \|$$

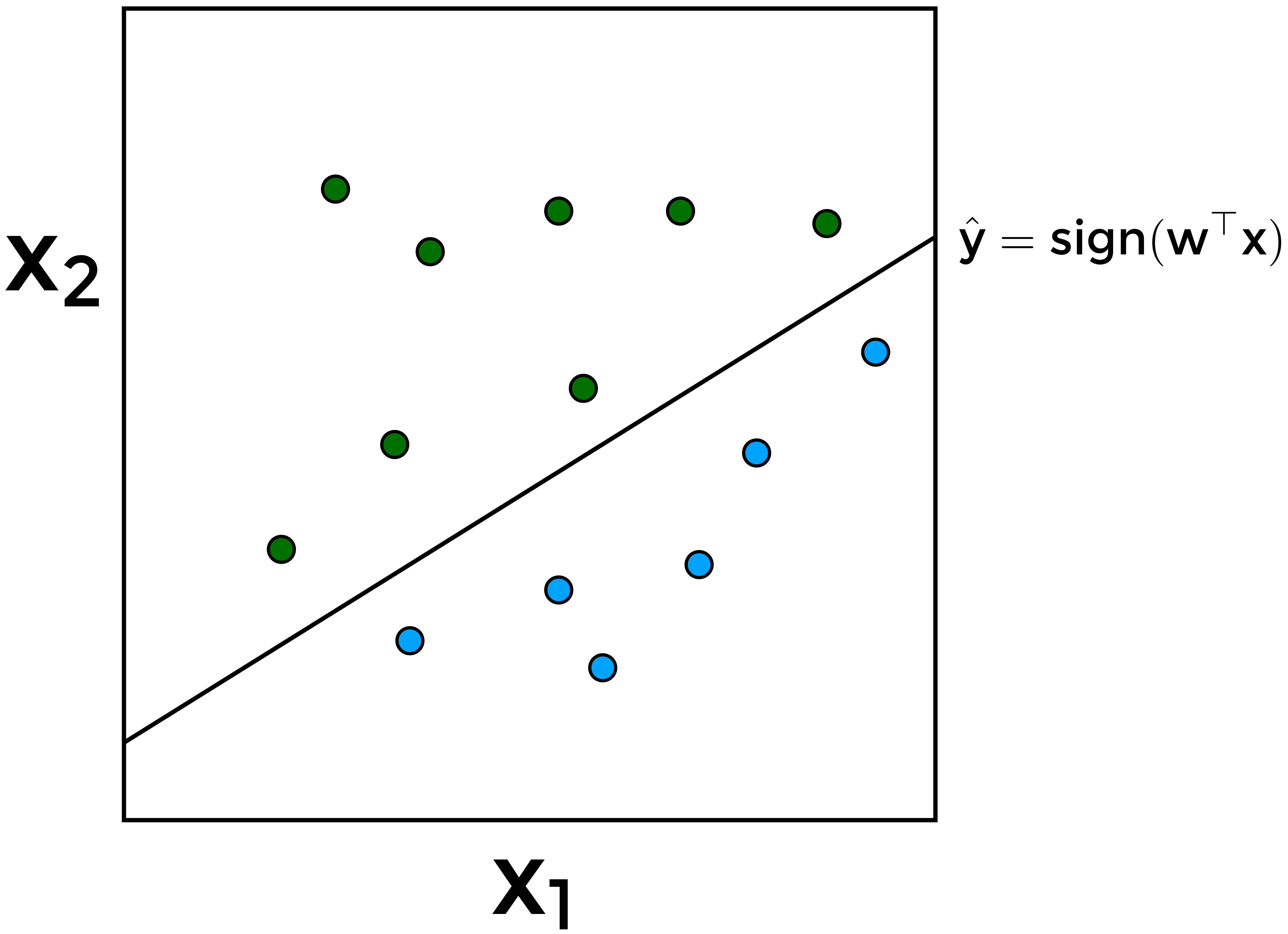
$$\Longrightarrow \| x' \| = \beta \| w \|$$

$$\Longrightarrow \| x' \| = \frac{-w_0}{\| w \|^2} \| w \|$$

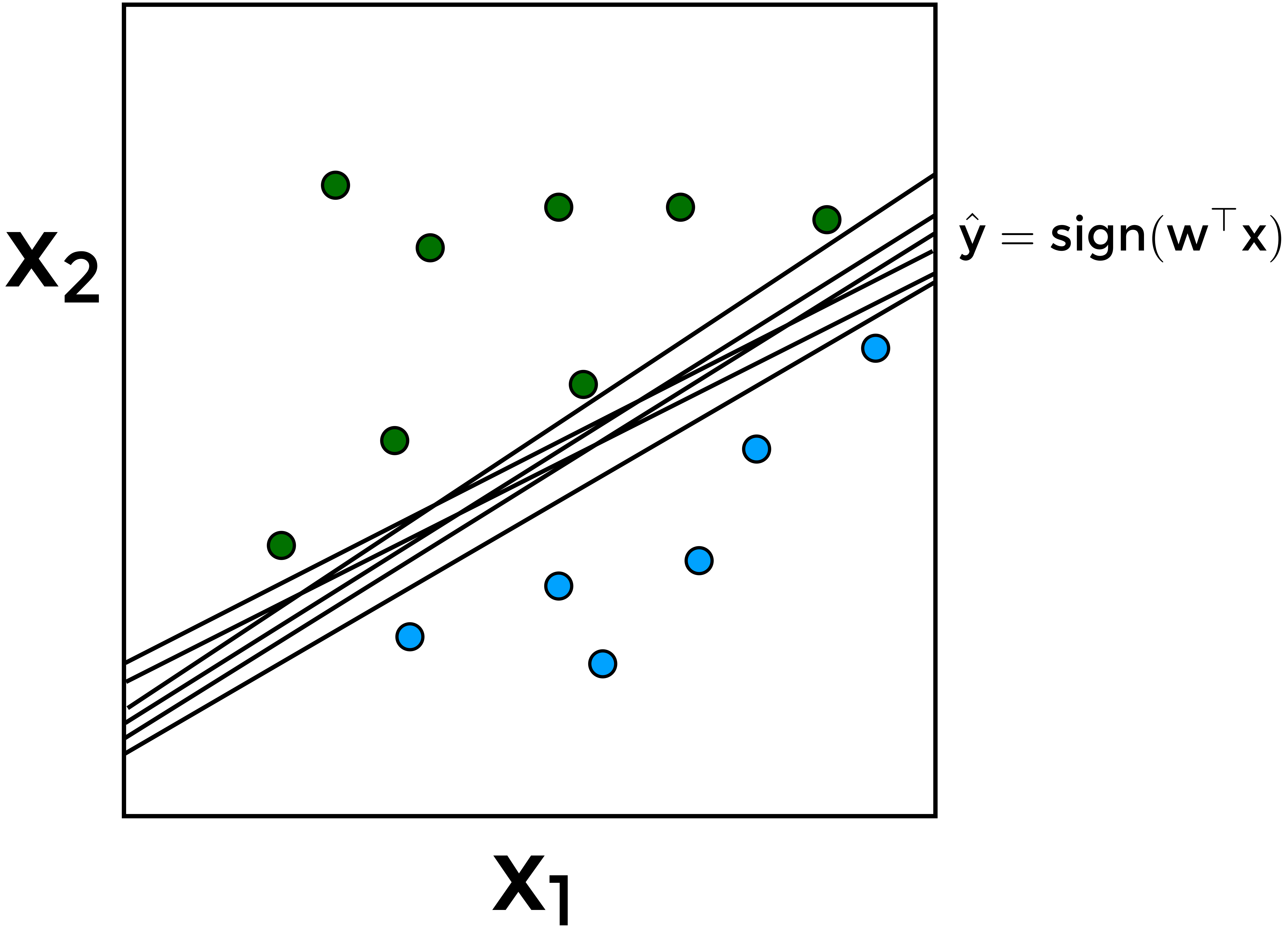
$$\Longrightarrow \| x' \| = \frac{-w_0}{\| w \|}$$



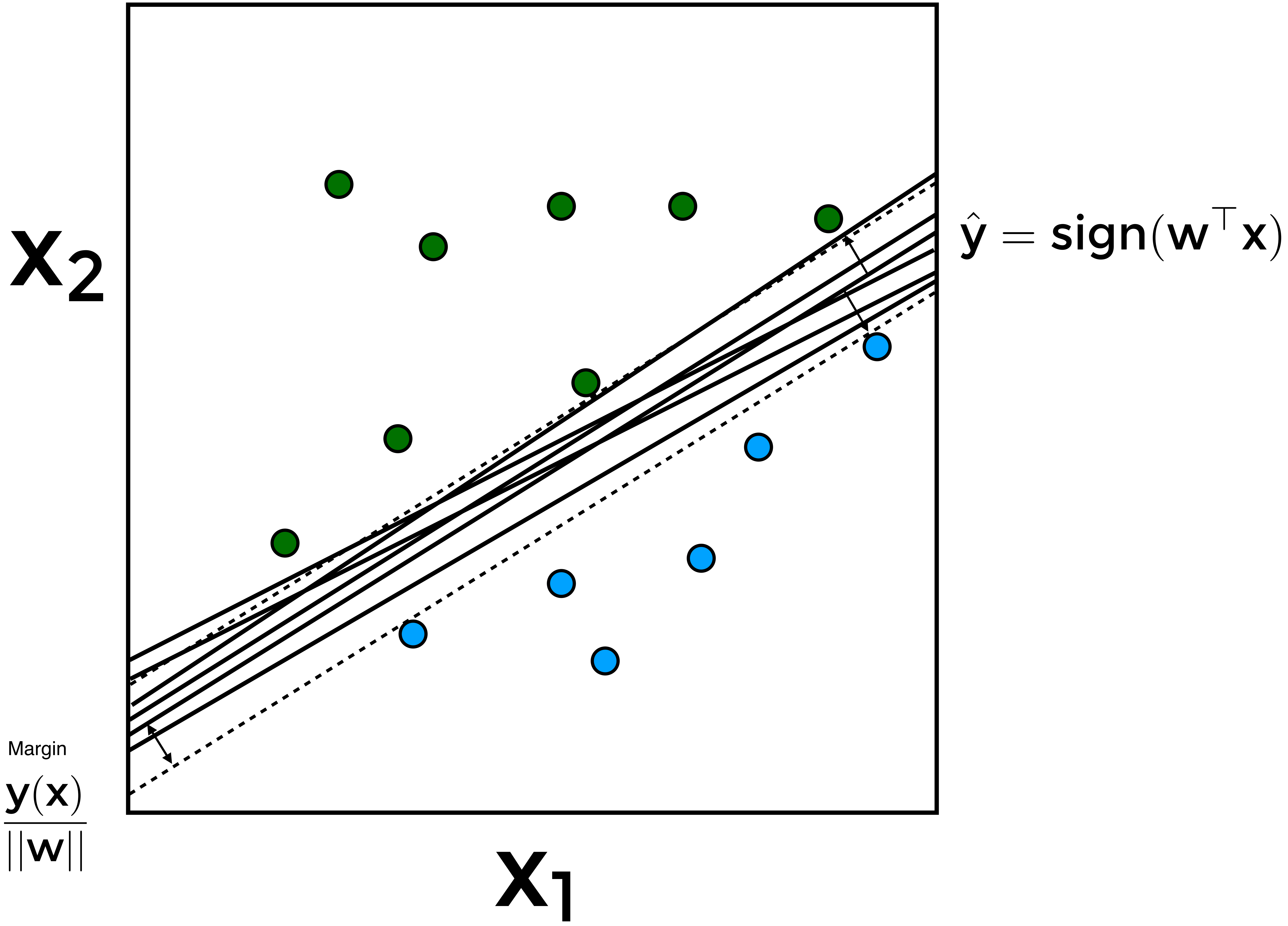
x_2
x_1



x_2
$\hat{y} = \text{sign}(w^\top x)$
x_1


x_2
$\hat{y} = \text{sign}(w^\top x)$
x_1

The objective is to find the separating boundary that maximizes the margin



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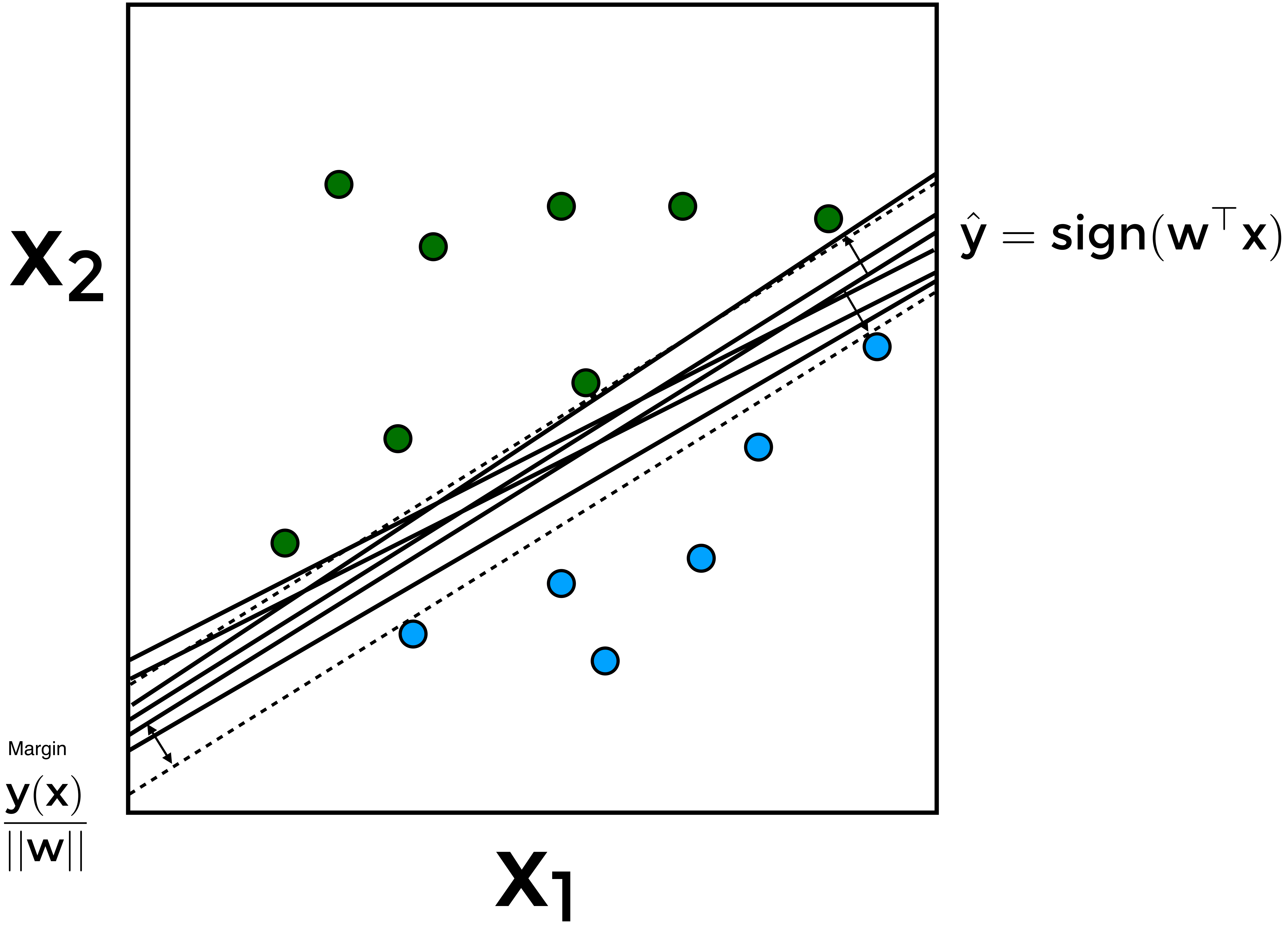
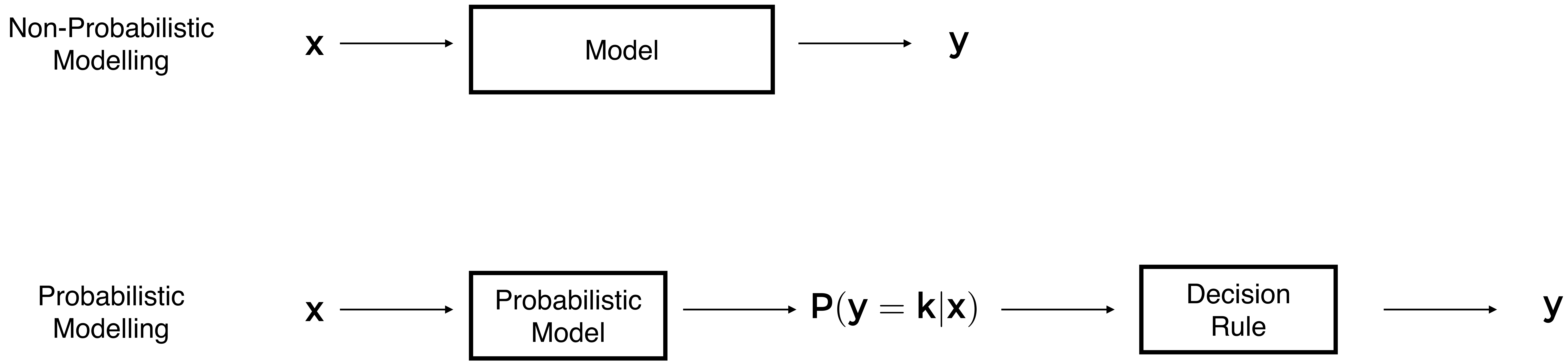
The objective is to find the separating boundary that maximizes the margin



Probabilistic Models for Classification

Probabilistic Models separate Decision and Inference



Probabilistic models

1. **Model the conditional directly:**

$$P(y = k|x)$$

2. **Model the joint (or the prior and the class conditionals):**

$\boxed{\text{Bayes' Theorem}}$

$$\underbrace{P(y = k|x)}_{\text{posterior}} \propto \underbrace{P(y = k, x)}_{\text{joint}}$$

$$= \underbrace{P(x \mid y = k)}_{\text{class conditional densities}} \underbrace{P(y = k)}_{\text{class prior}}$$

Probabilistic Modelling

Often inteertwinned

1. **Posit a model: P(X, Y)**
   - How the data is generated
2. **Parametrize the distributions: P(X, Y | Parameters)**
3. **Set the objective (e.g., MLE)**
4. **Learn the parameters of the model:**
   - E.g., Naive Bayes: learn the parameters of the class conditional P(X | Y) and of the prior P(Y)
5. **Use the model (e.g., for predictions)**