OREGON STATE UNIVERSITY

Project Group 21

CS 325: Project 3

Colin Bradford   Charles Jenkins   Albert Le

May 25, 2015

1 PROBLEM 1

Part A:
i. Linear Problem Formulation

Let each route be represented as variables of the form "SourceDestination." For example a route from Plant 1 to Warehouse 1 would be "p1w1."

Objective function:

Minimize:

10 p1w1 + 15 p1w2 + 11 p2w1 + 8 p2w2 + 13 p3w1 + 8 p3w2 + 9 p3w3 + 14 p4w2 + 8 p4w3 + 5 w1r1 + 6 w1r2 + 7 w1r3 + 10 w1r4 + 12 w2r3 + 8 w2r4 + 10 w2r5 + 14 w2r6 + 14 w3r4 + 12 w3r5 + 12 w3r6 + 6 w3r7

Subject To:

Supply Constraints:
p1w1 + p1w2 = 150
p2w1 + p2w2 = 450
p3w1 + p3w2 + p3w3 = 250
p4w2 + p4w3 = 150

Demand Constraints:
w1r1 = 100
w1r2 = 150
w1r3 + w2r3 = 100
w1r4 + w2r4 + w3r4 = 200
w2r5 + w3r5 = 200
w2r6 + w3r6 = 150
w3r7 = 100

Balancing Constraints:
p1w1 + p2w1 + p3w1 - w1r1 - w1r2 - w1r3 - w1r4 <= 0
p1w2 + p2w2 + p3w2 + p4w2 - w2r3 - w2r4 - w2r5 - w2r6 <= 0
p3w3 + p4w3 - w3r4 - w3r5 - w3r6 - w3r7 <= 0

Non-negativity Constraints:
p1w1, p1w2, p2w1, p2w2, p3w1, p3w2, p3w3, p4w2, p4w3, w1r1, w1r2, w1r3, w1r4,
w2r3, w2r4, w2r5, w2r6, w3r4, w3r5, w3r6, w3r7 >= 0

ii. LINDO Code and Output

```
MIN 10 p1w1 + 15 p1w2 + 11 p2w1 + 8 p2w2 + 13 p3w1 + 8 p3w2 + 9 p3w3
  + 14 p4w2 + 8 p4w3 + 5 w1r1 + 6 w1r2 + 7 w1r3 + 10 w1r4 + 12 w2r3
  + 8 w2r4 + 10 w2r5 + 14 w2r6 + 14 w3r4 + 12 w3r5 + 12 w3r6 + 6 w3r7
ST
   p1w1 + p1w2 = 150
   p2w1 + p2w2 = 450
   p3w1 + p3w2 + p3w3 = 250
   p4w2 + p4w3 = 150
   w1r1 = 100
   w1r2 = 150
   w1r3 + w2r3 = 100
   w1r4 + w2r4 + w3r4 = 200
   w2r5 + w3r5 = 200
   w2r6 + w3r6 = 150
   w3r7 = 100
   p1w1 + p2w1 + p3w1 - w1r1 - w1r2 - w1r3 - w1r4 <= 0
   p1w2 + p2w2 + p3w2 + p4w2 - w2r3 - w2r4 - w2r5 - w2r6 <= 0
   p3w3 + p4w3 - w3r4 - w3r5 - w3r6 - w3r7 <= 0
   p1w1 > 0
   p1w2 > 0
   p2w1 > 0
   p2w2 > 0
   p3w1 > 0
   p3w2 > 0
```

```
    p3w3 > 0
    p4w2 > 0
    p4w3 > 0
    w1r1 > 0
    w1r2 > 0
    w1r3 > 0
    w1r4 > 0
    w2r3 > 0
    w2r4 > 0
    w2r5 > 0
    w2r6 > 0
    w3r4 > 0
    w3r5 > 0
    w3r6 > 0
    w3r7 > 0
END


    LP OPTIMUM FOUND AT STEP      13

         OBJECTIVE FUNCTION VALUE

        1)      17100.00

    VARIABLE          VALUE         REDUCED COST
        P1W1      150.000000           0.000000
        P1W2        0.000000           8.000000
        P2W1      200.000000           0.000000
        P2W2      250.000000           0.000000
        P3W1        0.000000           2.000000
        P3W2      150.000000           0.000000
        P3W3      100.000000           0.000000
        P4W2        0.000000           7.000000
        P4W3      150.000000           0.000000
        W1R1      100.000000           0.000000
        W1R2      150.000000           0.000000
        W1R3      100.000000           0.000000
        W1R4        0.000000           5.000000
        W2R3        0.000000           2.000000
        W2R4      200.000000           0.000000
        W2R5      200.000000           0.000000
        W2R6        0.000000           1.000000
        W3R4        0.000000           7.000000
        W3R5        0.000000           3.000000
```

3

```
W3R6          150.000000              0.000000
W3R7          100.000000              0.000000


   ROW     SLACK OR SURPLUS      DUAL PRICES
    2)          0.000000         -10.000000
    3)          0.000000         -11.000000
    4)          0.000000         -11.000000
    5)          0.000000         -10.000000
    6)          0.000000          -5.000000
    7)          0.000000          -6.000000
    8)          0.000000          -7.000000
    9)          0.000000          -5.000000
   10)          0.000000          -7.000000
   11)          0.000000         -10.000000
   12)          0.000000          -4.000000
   13)          0.000000           0.000000
   14)          0.000000           3.000000
   15)          0.000000           2.000000
   16)        150.000000           0.000000
   17)          0.000000           0.000000
   18)        200.000000           0.000000
   19)        250.000000           0.000000
   20)          0.000000           0.000000
   21)        150.000000           0.000000
   22)        100.000000           0.000000
   23)          0.000000           0.000000
   24)        150.000000           0.000000
   25)        100.000000           0.000000
   26)        150.000000           0.000000
   27)        100.000000           0.000000
   28)          0.000000           0.000000
   29)          0.000000           0.000000
   30)        200.000000           0.000000
   31)        200.000000           0.000000
   32)          0.000000           0.000000
   33)          0.000000           0.000000
   34)          0.000000           0.000000
   35)        150.000000           0.000000
   36)        100.000000           0.000000

 NO. ITERATIONS=       13
```

iii. Optimal Shipping Routes and Minimum Cost

The optimal shipping routes and quantity of refrigerators per route are:

```
Route          Refrigerators
    P1W1            150
    P2W1            200
    P2W2            250
    P3W2            150
    P3W3            100
    P4W3            150
    W1R1            100
    W1R2            150
    W1R3            100
    W2R4            200
    W2R5            200
    W3R6            150
    W3R7            100
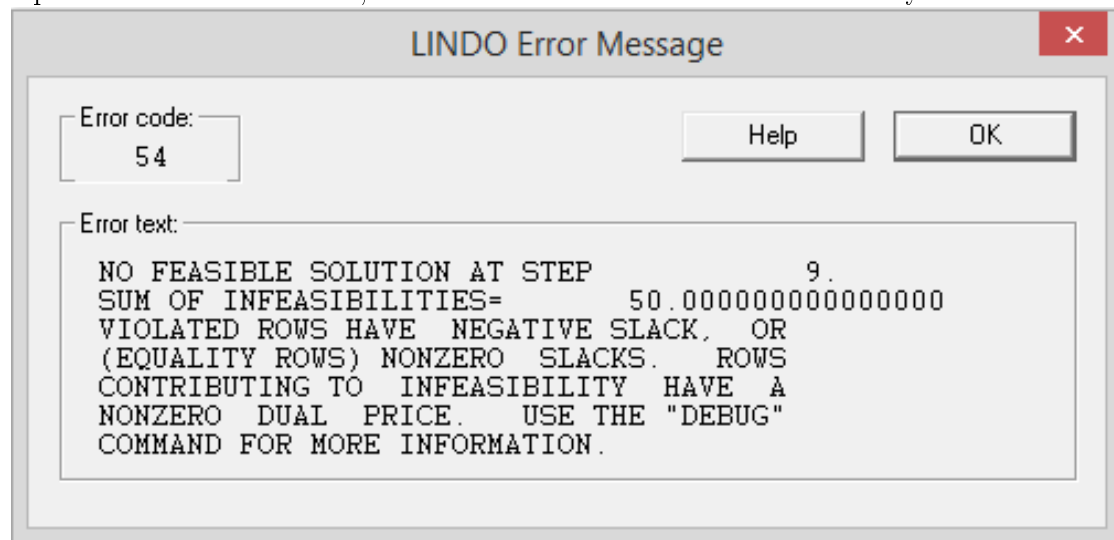```

The optimal minimum cost is $17,100

**Part B:**
To remove Warehouse 2 and all its associated routes, we remove all variables in our equation that contain "w2" in it. The resulting LINDO code is as follows:

```
MIN 10 p1w1 + 11 p2w1 + 13 p3w1 + 9 p3w3 + 8 p4w3 + 5 w1r1 + 6 w1r2
    + 7 w1r3 + 10 w1r4 + 14 w3r4 + 12 w3r5 + 12 w3r6 + 6 w3r7
ST
    p1w1 = 150
    p2w1 = 450
    p3w1 + p3w3 = 250
    p4w3 = 150
    w1r1 = 100
    w1r2 = 150
    w1r3 = 100
    w1r4 + w3r4 = 200
    w3r5 = 200
    w3r6 = 150
    w3r7 = 100
    p1w1 + p2w1 + p3w1 - w1r1 - w1r2 - w1r3 - w1r4 <= 0
    p3w3 + p4w3 - w3r4 - w3r5 - w3r6 - w3r7 <= 0
    p1w1 > 0
    p2w1 > 0
    p3w1 > 0
```

```
      p3w3 > 0
      p4w3 > 0
      w1r1 > 0
      w1r2 > 0
      w1r3 > 0
      w1r4 > 0
      w3r4 > 0
      w3r5 > 0
      w3r6 > 0
      w3r7 > 0
END
```

Upon execution of the code, we are met with these solution infeasibility notices:

LINDO Error Message

Error code:
54

Help   OK

Error text:
```
NO FEASIBLE SOLUTION AT STEP              9.
SUM OF INFEASIBILITIES=        50.000000000000000
VIOLATED ROWS HAVE  NEGATIVE SLACK,  OR
(EQUALITY ROWS) NONZERO  SLACKS.   ROWS
CONTRIBUTING TO  INFEASIBILITY  HAVE  A
NONZERO  DUAL  PRICE.   USE THE "DEBUG"
COMMAND FOR MORE INFORMATION.
```

LINDO Solver Status

Optimizer Status

Status:	Infeasible
Iterations:	4
Infeasibility:	50
Objective:	18800
Best IP:	N/A
IP Bound:	N/A
Branches:	N/A
Elapsed Time:	00:00:18

Update Interval: 1

Interrupt Solver   Close

There is no feasible solution here because Warehouse 3's total possible supply (400) is not capable of supporting the combined demand of R4, R5, R6, and R7 (650). On the other hand, Warehouse 1 is in fact capable of supporting R1, R2, R3, and R4 with its 850 supply to 550 demand ratio, but that still does not solve the larger supply chain deficit and feasibility issue. Overall, we need Warehouse 2 to supplement the supply to

R3, R4, R5, and R6 and sufficiently "spread the load" on the warehouses.

Part C:

To simulate the limitation of 100 shipments (inbound and outbound) on Warehouse 2, we add two new constraints to our code for Part A. The constraints limit the inbound and outbound shipments to being less than or equal to 100.

LINDO Code and Output:

```
MIN 10 p1w1 + 15 p1w2 + 11 p2w1 + 8 p2w2 + 13 p3w1 + 8 p3w2 + 9 p3w3
  + 14 p4w2 + 8 p4w3 + 5 w1r1 + 6 w1r2 + 7 w1r3 + 10 w1r4 + 12 w2r3
  + 8 w2r4 + 10 w2r5 + 14 w2r6 + 14 w3r4 + 12 w3r5 + 12 w3r6 + 6 w3r7
ST
   p1w1 + p1w2 = 150
   p2w1 + p2w2 = 450
   p3w1 + p3w2 + p3w3 = 250
   p4w2 + p4w3 = 150
   w1r1 = 100
   w1r2 = 150
   w1r3 + w2r3 = 100
   w1r4 + w2r4 + w3r4 = 200
   w2r5 + w3r5 = 200
   w2r6 + w3r6 = 150
   w3r7 = 100
   p1w1 + p2w1 + p3w1 - w1r1 - w1r2 - w1r3 - w1r4 <= 0
   p1w2 + p2w2 + p3w2 + p4w2 - w2r3 - w2r4 - w2r5 - w2r6 <= 0
   p3w3 + p4w3 - w3r4 - w3r5 - w3r6 - w3r7 <= 0
   p1w1 > 0
   p1w2 > 0
   p2w1 > 0
   p2w2 > 0
   p3w1 > 0
   p3w2 > 0
   p3w3 > 0
   p4w2 > 0
   p4w3 > 0
   w1r1 > 0
   w1r2 > 0
   w1r3 > 0
   w1r4 > 0
   w2r3 > 0
   w2r4 > 0
   w2r5 > 0
```

```
   w2r6 > 0
   w3r4 > 0
   w3r5 > 0
   w3r6 > 0
   w3r7 > 0
   p1w2 + p2w2 + p3w2 + p4w2 <= 100
   w2r3 + w2r4 + w2r5 + w2r6 <= 100
END


 LP OPTIMUM FOUND AT STEP        7

         OBJECTIVE FUNCTION VALUE

       1)      18300.00

   VARIABLE         VALUE          REDUCED COST
       P1W1      150.000000           0.000000
       P1W2        0.000000           8.000000
       P2W1      350.000000           0.000000
       P2W2      100.000000           0.000000
       P3W1        0.000000           4.000000
       P3W2        0.000000           2.000000
       P3W3      250.000000           0.000000
       P4W2        0.000000           9.000000
       P4W3      150.000000           0.000000
       W1R1      100.000000           0.000000
       W1R2      150.000000           0.000000
       W1R3      100.000000           0.000000
       W1R4      150.000000           0.000000
       W2R3        0.000000           7.000000
       W2R4       50.000000           0.000000
       W2R5       50.000000           0.000000
       W2R6        0.000000           4.000000
       W3R4        0.000000           4.000000
       W3R5      150.000000           0.000000
       W3R6      150.000000           0.000000
       W3R7      100.000000           0.000000


       ROW    SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -20.000000
        3)         0.000000         -21.000000
        4)         0.000000         -19.000000
```

```
          5)        0.000000        -18.000000
          6)        0.000000          5.000000
          7)        0.000000          4.000000
          8)        0.000000          3.000000
          9)        0.000000          0.000000
         10)        0.000000         -2.000000
         11)        0.000000         -2.000000
         12)        0.000000          4.000000
         13)        0.000000         10.000000
         14)        0.000000         13.000000
         15)        0.000000         10.000000
         16)      150.000000          0.000000
         17)        0.000000          0.000000
         18)      350.000000          0.000000
         19)      100.000000          0.000000
         20)        0.000000          0.000000
         21)        0.000000          0.000000
         22)      250.000000          0.000000
         23)        0.000000          0.000000
         24)      150.000000          0.000000
         25)      100.000000          0.000000
         26)      150.000000          0.000000
         27)      100.000000          0.000000
         28)      150.000000          0.000000
         29)        0.000000          0.000000
         30)       50.000000          0.000000
         31)       50.000000          0.000000
         32)        0.000000          0.000000
         33)        0.000000          0.000000
         34)      150.000000          0.000000
         35)      150.000000          0.000000
         36)      100.000000          0.000000
         37)        0.000000          0.000000
         38)        0.000000          5.000000

   NO. ITERATIONS=        7
```

We see that this is a feasible solution, though it raises our optimal cost from $17,100 to $18,300.

The optimal shipping routes and quantity of refrigerators per route are:

```
  Route       Refrigerators
     P1W1          150
     P2W1          350
     P2W2          100
```

```
P3W3          250
P4W3          150
W1R1          100
W1R2          150
W1R3          100
W1R4          150
W2R4           50
W2R5           50
W3R5          150
W3R6          150
W3R7          100
```

Part D:
For some cost c_{ab} and some route x_{ab} where their subscripts represent the route from a to b, the objective function is:
$Min(\sum_p \sum_w c_{pw} x_{pw} + \sum_w \sum_r c_{wr} x_{wr})$

$s.t.$
(Supply Constraint) For each source node p, where s is supply: $\sum_w x_{pw} = s_p$
(Demand Constraint) For each destination node r, where d is demand: $\sum_w x_{wr} = d_r$
(Balancing Constraint) For each intermediate node w: $\sum_p x_{pw} - \sum_r x_{wr} <= 0$
(Non-negativity Constraint) For all nodes p and r: $x_{pw}, x_{wr} >= 0$

2 PROBLEM 2

Part A:
i. Linear Problem Formulation

Let each ingredient be represented as variables using the first two letters of the ingredient's name. For example spinach is "sp."

Objective function:

Minimize:

21 to + 16 le + 40 sp + 41 ca + 41 su + 120 sm + 164 ch + 884 oi

Subject To:

Nutrition Constraints:
0.85 to + 1.62 le + 2.86 sp + 0.93 ca + 23.4 su + 16 sm + 9 ch >= 15
0.33 to + 0.20 le + 0.39 sp + 0.24 ca + 48.7 su + 5 sm + 2.6 ch + 100 oi >= 2

0.33 to + 0.20 le + 0.39 sp + 0.24 ca + 48.7 su + 5 sm + 2.6 ch + 100 oi <= 8
4.64 to + 2.37 le + 3.63 sp + 9.58 ca + 15 su + 3 sm + 27 ch >= 4
9 to + 28 le + 65 sp + 69 ca + 3.8 su + 120 sm + 78 ch <= 200
le + sp - .4le - .4sp - .4to - .4ca - .4su - .4sm - .4ch - .4oi >= 0

Non-negativity Constraints:
le, sp, to, ca, su, sm, ch, oi >= 0

ii. LINDO Code and Output

```
MIN 21 to + 16 le + 40 sp + 41 ca + 41 su + 120 sm + 164 ch + 884 oi
ST
    0.85 to + 1.62 le + 2.86 sp + 0.93 ca + 23.4 su + 16 sm + 9 ch >= 15
    0.33 to + 0.20 le + 0.39 sp + 0.24 ca + 48.7 su + 5 sm + 2.6 ch + 100 oi >=
    0.33 to + 0.20 le + 0.39 sp + 0.24 ca + 48.7 su + 5 sm + 2.6 ch + 100 oi <=
    4.64 to + 2.37 le + 3.63 sp + 9.58 ca + 15 su + 3 sm + 27 ch >= 4
    9 to + 28 le + 65 sp + 69 ca + 3.8 su + 120 sm + 78 ch <= 200
    le + sp − .4le − .4sp − .4to − .4ca − .4su − .4sm − .4ch − .4oi >= 0
    le > 0
    sp > 0
    to > 0
    ca > 0
    su > 0
    sm > 0
    ch > 0
    oi > 0
END

 LP OPTIMUM FOUND AT STEP        8

        OBJECTIVE FUNCTION VALUE

        1)      103.3371

  VARIABLE         VALUE          REDUCED COST
        TO         0.000000         16.520000
        LE         0.561163          0.000000
        SP         0.000000         13.916369
        CA         0.000000         35.535095
        SU         0.084189          0.000000
        SM         0.757556          0.000000
        CH         0.000000         96.583527
        OI         0.000000       1212.744385
```

```
ROW     SLACK OR SURPLUS        DUAL PRICES
 2)          0.000000            -8.632957
 3)          6.000000             0.000000
 4)          0.000000             3.269654
 5)          0.865455             0.000000
 6)         93.060745             0.000000
 7)          0.000000            -4.447570
 8)          0.561163             0.000000
 9)          0.000000             0.000000
10)          0.000000             0.000000
11)          0.000000             0.000000
12)          0.084189             0.000000
13)          0.757556             0.000000
14)          0.000000             0.000000
15)          0.000000             0.000000


NO. ITERATIONS=         8
```

The minimum amount of calories a salad can contain while meeting the nutrition constraints is 103.3371 calories.
The optimal ingredient mix is:

```
Ingredient    Amount
     LE        0.561163
     SU        0.084189
     SM        0.757556
```

iii. The cost of this salad is: $0.561163*.75+0.084189*.45+0.757556*2.15 = \$2.0875 \approx$ $\$2.09$
Part B:
i.
ii.
iii.
Part C:
i.
ii.
iii.

3 PROBLEM 3

Part A:

i. General Problem Statement

Objective:

$Min \sum_{i=1}^{n} b_i$, for some b_i representing $|y_i - (a_1 x_i + a_0)|$ for each i = 1...n

s.t.

For each i = 1...n, $b_i - (y_i - (a_1 x_i + a_0)) >= 0$

For each i = 1...n, $b_i + (y_i - (a_1 x_i + a_0)) >= 0$

ii. LINDO Code and Output

```
MIN b1 + b2 + b3 + b4 + b5 + b6 + b7
ST
   b1 + 1 a1 + a0 >= 5
   b1 - 1 a1 - a0 >= -5
   b2 + 1 a1 + a0 >= 3
   b2 - 1 a1 - a0 >= -3
   b3 + 2 a1 + a0 >= 13
   b3 - 2 a1 - a0 >= -13
   b4 + 3 a1 + a0 >= 8
   b4 - 3 a1 - a0 >= -8
   b5 + 4 a1 + a0 >= 10
   b5 - 4 a1 - a0 >= -10
   b6 + 5 a1 + a0 >= 14
   b6 - 5 a1 - a0 >= -14
   b7 + 6 a1 + a0 >= 18
   b7 - 6 a1 - a0 >= -18
   b1 > 0
   b2 > 0
   b3 > 0
   b4 > 0
   b5 > 0
   b6 > 0
   b7 > 0
END

 LP OPTIMUM FOUND AT STEP        7

          OBJECTIVE FUNCTION VALUE

        1)      12.25000

  VARIABLE         VALUE          REDUCED COST
        B1        2.000000            0.000000
        B2        0.000000            0.250000
```

14

```
        B3           7.250000                0.000000
        B4           0.500000                0.000000
        B5           1.250000                0.000000
        B6           0.000000                0.750000
        B7           1.250000                0.000000
        A1           2.750000                0.000000
        A0           0.250000                0.000000


      ROW     SLACK OR SURPLUS      DUAL PRICES
       2)          0.000000           -1.000000
       3)          4.000000            0.000000
       4)          0.000000            0.000000
       5)          0.000000           -0.750000
       6)          0.000000           -1.000000
       7)         14.500000            0.000000
       8)          1.000000            0.000000
       9)          0.000000           -1.000000
      10)          2.500000            0.000000
      11)          0.000000           -1.000000
      12)          0.000000            0.000000
      13)          0.000000           -0.250000
      14)          0.000000           -1.000000
      15)          2.500000            0.000000
      16)          2.000000            0.000000
      17)          0.000000            0.000000
      18)          7.250000            0.000000
      19)          0.500000            0.000000
      20)          1.250000            0.000000
      21)          0.000000            0.000000
      22)          1.250000            0.000000

   NO. ITERATIONS=        7
```

The sum of the absolute deviations was 12.25.

iii. Graph

Least Squares vs Least Absolute Deviations
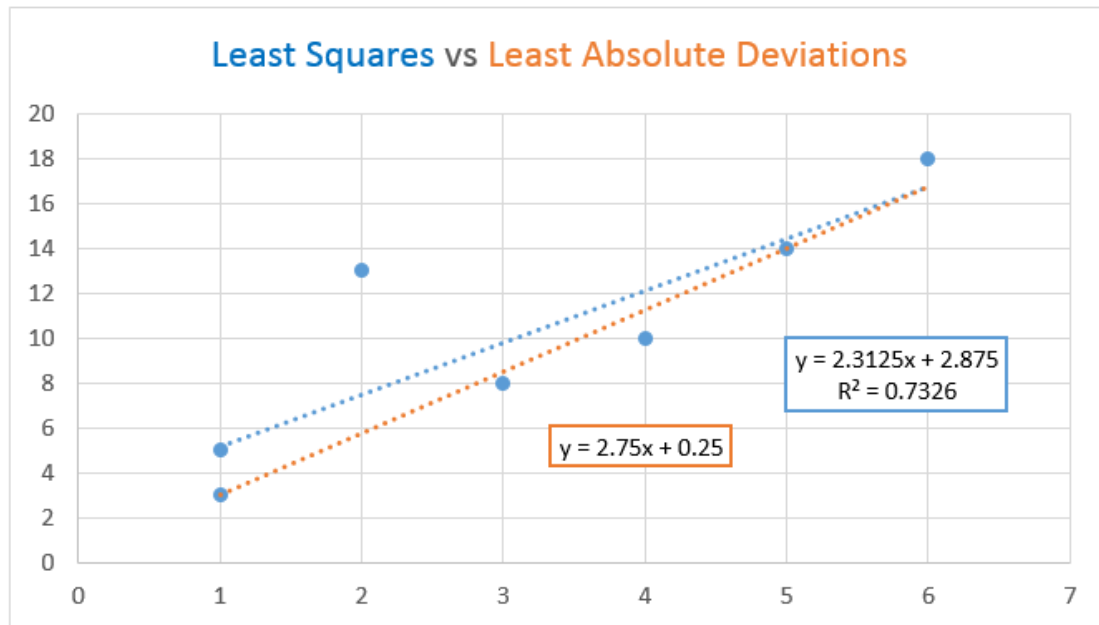
The LAD approach creates a line that has a steeper slope and lesser y-intercept than the least squares line. The line more tightly follows the path of the majority of the data points and seems to be less affected by the outlier at x = 2.

Part B:
i. General Problem Statement
Objective:
Min b, for some b representing $Max|y_i - (a_1x_i + a_0)|$
s.t.
For each i = 1...n, $b + a_1x_i + a_0 >= y_i$
For each i = 1...n, $b - a_1x_i - a_0 >= -y_i$

ii. LINDO Code and Output

```
MIN b
ST
    b + 1 a1 + a0 >= 5
    b - 1 a1 - a0 >= -5
    b + 1 a1 + a0 >= 3
    b - 1 a1 - a0 >= -3
    b + 2 a1 + a0 >= 13
    b - 2 a1 - a0 >= -13
    b + 3 a1 + a0 >= 8
    b - 3 a1 - a0 >= -8
    b + 4 a1 + a0 >= 10
```

16

```
   b - 4 a1 - a0 >= -10
   b + 5 a1 + a0 >= 14
   b - 5 a1 - a0 >= -14
   b + 6 a1 + a0 >= 18
   b - 6 a1 - a0 >= -18
   b > 0
END

 LP OPTIMUM FOUND AT STEP        3

          OBJECTIVE FUNCTION VALUE

      1)      3.833333

   VARIABLE         VALUE        REDUCED COST
        B          3.833333          0.000000
        A1         2.333333          0.000000
        A0         4.500000          0.000000


      ROW    SLACK OR SURPLUS      DUAL PRICES
       2)         5.666667          0.000000
       3)         2.000000          0.000000
       4)         7.666667          0.000000
       5)         0.000000         -0.333333
       6)         0.000000         -0.500000
       7)         7.666667          0.000000
       8)         7.333333          0.000000
       9)         0.333333          0.000000
      10)         7.666667          0.000000
      11)         0.000000         -0.166667
      12)         6.000000          0.000000
      13)         1.666667          0.000000
      14)         4.333333          0.000000
      15)         3.333333          0.000000
      16)         3.833333          0.000000

 NO. ITERATIONS=        3
```
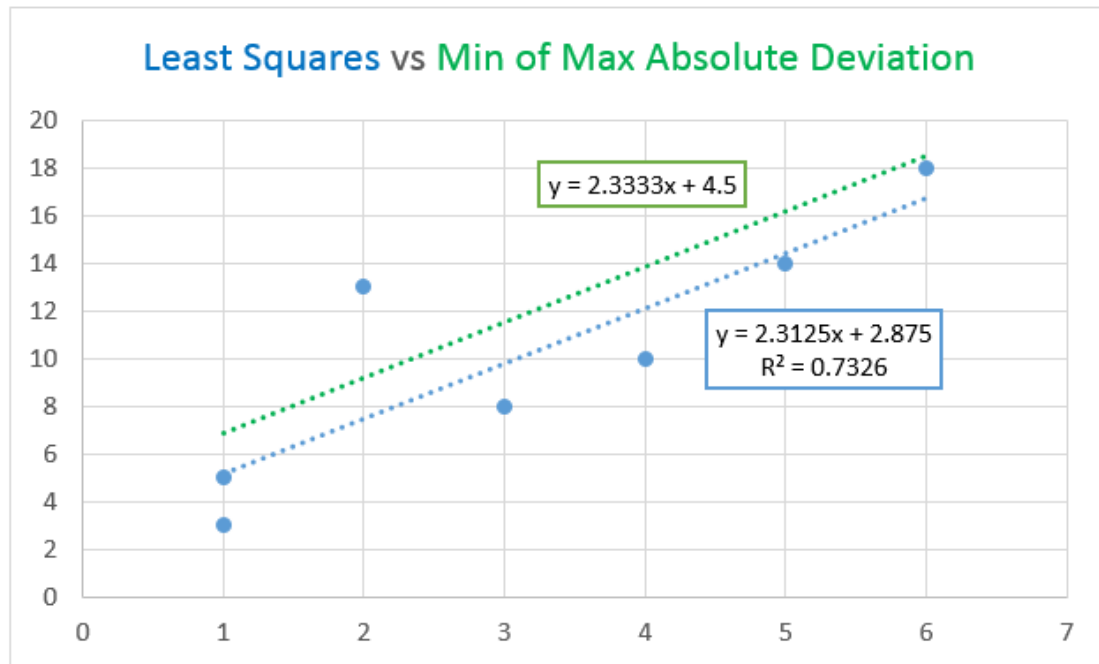
The min of the max absolute deviations was 3.833333.

iii. Graph

The MMAD approach gives a line that appears to be more affected by the outlier at x = 2 with a much higher y-intercept than the LAD and least squares approaches. It's slope however, is only slightly steeper than the least squares line.

iv. Same Line Data Set
Yes, the data set (x,y) = [(1,2), (2,4), (3,6)] computes the same line for all three methods.
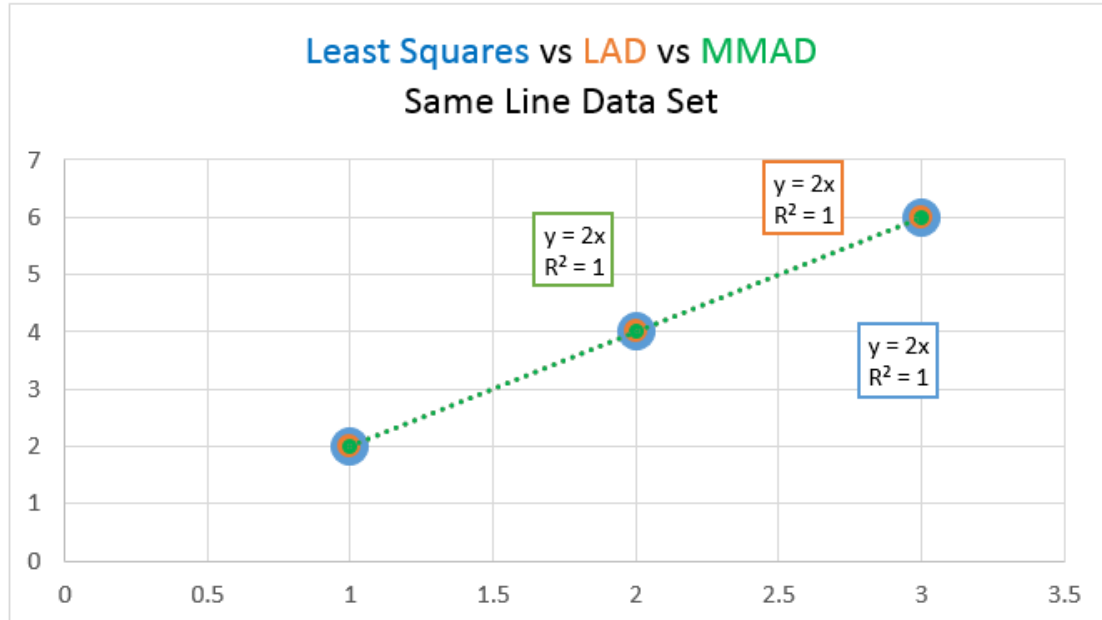
```
LAD Method:

MIN b1 + b2 + b3
ST
    b1 + 1 a1 + a0 >= 2
    b1 - 1 a1 - a0 >= -2
    b2 + 2 a1 + a0 >= 4
    b2 - 2 a1 - a0 >= -4
    b3 + 3 a1 + a0 >= 6
    b3 - 3 a1 - a0 >= -6
    b1 > 0
    b2 > 0
    b3 > 0
END

 LP OPTIMUM FOUND AT STEP        1

         OBJECTIVE FUNCTION VALUE

        1)     0.0000000E+00

   VARIABLE          VALUE        REDUCED COST
        B1         0.000000          1.000000
        B2         0.000000          1.000000
        B3         0.000000          1.000000
        A1         2.000000          0.000000
        A0         0.000000          0.000000


     ROW    SLACK OR SURPLUS    DUAL PRICES
      2)         0.000000          0.000000
      3)         0.000000          0.000000
      4)         0.000000          0.000000
      5)         0.000000          0.000000
      6)         0.000000          0.000000
      7)         0.000000          0.000000
      8)         0.000000          0.000000
      9)         0.000000          0.000000
     10)         0.000000          0.000000

  NO. ITERATIONS=        1

MMAD Method:
```

```
MIN b
ST
    b + 1 a1 + a0 >= 2
    b - 1 a1 - a0 >= -2
    b + 2 a1 + a0 >= 4
    b - 2 a1 - a0 >= -4
    b + 3 a1 + a0 >= 6
    b - 3 a1 - a0 >= -6
    b > 0
END


 LP OPTIMUM FOUND AT STEP        1


        OBJECTIVE FUNCTION VALUE


        1)      0.0000000E+00


    VARIABLE         VALUE          REDUCED COST
           B         0.000000          1.000000
          A1         2.000000          0.000000
          A0         0.000000          0.000000



        ROW    SLACK OR SURPLUS     DUAL PRICES
         2)         0.000000          0.000000
         3)         0.000000          0.000000
         4)         0.000000          0.000000
         5)         0.000000          0.000000
         6)         0.000000          0.000000
         7)         0.000000          0.000000
         8)         0.000000          0.000000


    NO. ITERATIONS=        1
```

Least Squares Line was produced in Excel (see graph above) rather than LINDO.

Part C: Objective:
$Min \sum_{i=1}^{n} b_i$, for some b_i representing $|y_i - (a_2 x_{2i} + a_1 x_{1i} + a_0)|$ for each i = 1...n
s.t.
For each i = 1...n, $b_i - (y_i - (a_2 x_{2i} + a_1 x_{1i} + a_0)) >= 0$
For each i = 1...n, $b_i + (y_i - (a_2 x_{2i} + a_1 x_{1i} + a_0)) >= 0$

```
MIN b1 + b2 + b3 + b4 + b5 + b6
ST
   b1 + 1 a2 + 1 a1 + a0  >= 5
   b1 - 1 a2 - 1 a1 - a0 >= -5
   b2 + 2 a2 + 1 a1 + a0  >= 9
   b2 - 2 a2 - 1 a1 - a0 >= -9
   b3 + 2 a2 + 2 a1 + a0  >= 12
   b3 - 2 a2 - 2 a1 - a0 >= -12
   b4 + 1 a2 + 0 a1 + a0  >= 3
   b4 - 1 a2 - 0 a1 - a0 >= -3
   b5 + 0 a2 + 0 a1 + a0  >= 0
   b5 - 0 a2 - 0 a1 - a0 >= 0
   b6 + 3 a2 + 1 a1 + a0  >= 11
   b6 - 3 a2 - 1 a1 - a0 >= -11
   b1 > 0
   b2 > 0
   b3 > 0
   b4 > 0
   b5 > 0
   b6 > 0
END

 LP OPTIMUM FOUND AT STEP      8

         OBJECTIVE FUNCTION VALUE

       1)       2.000000

   VARIABLE          VALUE        REDUCED COST
         B1        1.000000          0.000000
         B2        0.000000          0.000000
         B3        0.000000          0.500000
         B4        0.000000          0.000000
         B5        0.000000          0.500000
         B6        1.000000          0.000000
         A2        3.000000          0.000000
         A1        3.000000          0.000000
         A0        0.000000          0.000000


       ROW    SLACK OR SURPLUS     DUAL PRICES
        2)        2.000000          0.000000
        3)        0.000000         -1.000000
        4)        0.000000         -1.000000
```

```
 5)          0.000000             0.000000
 6)          0.000000            -0.500000
 7)          0.000000             0.000000
 8)          0.000000            -1.000000
 9)          0.000000             0.000000
10)          0.000000             0.000000
11)          0.000000            -0.500000
12)          2.000000             0.000000
13)          0.000000            -1.000000
14)          1.000000             0.000000
15)          0.000000             0.000000
16)          0.000000             0.000000
17)          0.000000             0.000000
18)          0.000000             0.000000
19)          1.000000             0.000000

   NO. ITERATIONS=          8
```

The sum of the absolute deviations was 2.
The regression model is: $y = 3x_2 + 3x_1 + 0$