Oregon State University

Project Group 21

CS 325: Project 3

Colin Bradford Charles Jenkins Albert Le

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1 Problem 1

Part A:
i. Linear Problem Formulation

Let each route be represented as variables of the form "SourceDestination." For example a route from Plant 1 to Warehouse 1 would be "p1w1."

Objective function:

Minimize:

10 p1w1 + 15 p1w2 + 11 p2w1 + 8 p2w2 + 13 p3w1 + 8 p3w2 + 9 p3w3 + 14 p4w2 + 8 p4w3 + 5 w1r1 + 6 w1r2 + 7 w1r3 + 10 w1r4 + 12 w2r3 + 8 w2r4 + 10 w2r5 + 14 w2r6 + 14 w3r4 + 12 w3r5 + 12 w3r6 + 6 w3r7

Subject To:

Supply Constraints:
p1w1 + p1w2 = 150
p2w1 + p2w2 = 450
p3w1 + p3w2 + p3w3 = 250
p4w2 + p4w3 = 150

Demand Constraints:
w1r1 = 100
w1r2 = 150
w1r3 + w2r3 = 100
w1r4 + w2r4 + w3r4 = 200
w2r5 + w3r5 = 200
w2r6 + w3r6 = 150
w3r7 = 100

Balancing Constraints:
p1w1 + p2w1 + p3w1 - w1r1 - w1r2 - w1r3 - w1r4 <= 0
p1w2 + p2w2 + p3w2 + p4w2 - w2r3 - w2r4 - w2r5 - w2r6 <= 0
p3w3 + p4w3 - w3r4 - w3r5 - w3r6 - w3r7 <= 0

Non-negativity Constraints:
p1w1, p1w2, p2w1, p2w2, p3w1, p3w2, p3w3, p4w2, p4w3, w1r1, w1r2, w1r3, w1r4, w2r3, w2r4, w2r5, w2r6, w3r4, w3r5, w3r6, w3r7 >= 0

ii. LINDO Code and Output

```
MIN 10 p1w1 + 15 p1w2 + 11 p2w1 + 8 p2w2 + 13 p3w1 + 8 p3w2 + 9 p3w3
  + 14 p4w2 + 8 p4w3 + 5 w1r1 + 6 w1r2 + 7 w1r3 + 10 w1r4 + 12 w2r3
  + 8 w2r4 + 10 w2r5 + 14 w2r6 + 14 w3r4 + 12 w3r5 + 12 w3r6 + 6 w3r7
ST
   p1w1 + p1w2 = 150
   p2w1 + p2w2 = 450
   p3w1 + p3w2 + p3w3 = 250
   p4w2 + p4w3 = 150
   w1r1 = 100
   w1r2 = 150
   w1r3 + w2r3 = 100
   w1r4 + w2r4 + w3r4 = 200
   w2r5 + w3r5 = 200
   w2r6 + w3r6 = 150
   w3r7 = 100
   p1w1 + p2w1 + p3w1 - w1r1 - w1r2 - w1r3 - w1r4 <= 0
   p1w2 + p2w2 + p3w2 + p4w2 - w2r3 - w2r4 - w2r5 - w2r6 <= 0
   p3w3 + p4w3 - w3r4 - w3r5 - w3r6 - w3r7 <= 0
   p1w1 > 0
   p1w2 > 0
   p2w1 > 0
   p2w2 > 0
   p3w1 > 0
   p3w2 > 0
```

```
   p3w3 > 0
   p4w2 > 0
   p4w3 > 0
   w1r1 > 0
   w1r2 > 0
   w1r3 > 0
   w1r4 > 0
   w2r3 > 0
   w2r4 > 0
   w2r5 > 0
   w2r6 > 0
   w3r4 > 0
   w3r5 > 0
   w3r6 > 0
   w3r7 > 0
END


 LP OPTIMUM FOUND AT STEP      13

        OBJECTIVE FUNCTION VALUE

        1)      17100.00

  VARIABLE        VALUE          REDUCED COST
      P1W1       150.000000          0.000000
      P1W2         0.000000          8.000000
      P2W1       200.000000          0.000000
      P2W2       250.000000          0.000000
      P3W1         0.000000          2.000000
      P3W2       150.000000          0.000000
      P3W3       100.000000          0.000000
      P4W2         0.000000          7.000000
      P4W3       150.000000          0.000000
      W1R1       100.000000          0.000000
      W1R2       150.000000          0.000000
      W1R3       100.000000          0.000000
      W1R4         0.000000          5.000000
      W2R3         0.000000          2.000000
      W2R4       200.000000          0.000000
      W2R5       200.000000          0.000000
      W2R6         0.000000          1.000000
      W3R4         0.000000          7.000000
      W3R5         0.000000          3.000000
```

W3R6	150.000000	0.000000
W3R7	100.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-10.000000
3)	0.000000	-11.000000
4)	0.000000	-11.000000
5)	0.000000	-10.000000
6)	0.000000	-5.000000
7)	0.000000	-6.000000
8)	0.000000	-7.000000
9)	0.000000	-5.000000
10)	0.000000	-7.000000
11)	0.000000	-10.000000
12)	0.000000	-4.000000
13)	0.000000	0.000000
14)	0.000000	3.000000
15)	0.000000	2.000000
16)	150.000000	0.000000
17)	0.000000	0.000000
18)	200.000000	0.000000
19)	250.000000	0.000000
20)	0.000000	0.000000
21)	150.000000	0.000000
22)	100.000000	0.000000
23)	0.000000	0.000000
24)	150.000000	0.000000
25)	100.000000	0.000000
26)	150.000000	0.000000
27)	100.000000	0.000000
28)	0.000000	0.000000
29)	0.000000	0.000000
30)	200.000000	0.000000
31)	200.000000	0.000000
32)	0.000000	0.000000
33)	0.000000	0.000000
34)	0.000000	0.000000
35)	150.000000	0.000000
36)	100.000000	0.000000

NO. ITERATIONS= 13

iii. Optimal Shipping Routes and Minimum Cost

The optimal shipping routes and quantity of refrigerators per route are:

Route	Refrigerators
P1W1	150
P2W1	200
P2W2	250
P3W2	150
P3W3	100
P4W3	150
W1R1	100
W1R2	150
W1R3	100
W2R4	200
W2R5	200
W3R6	150
W3R7	100

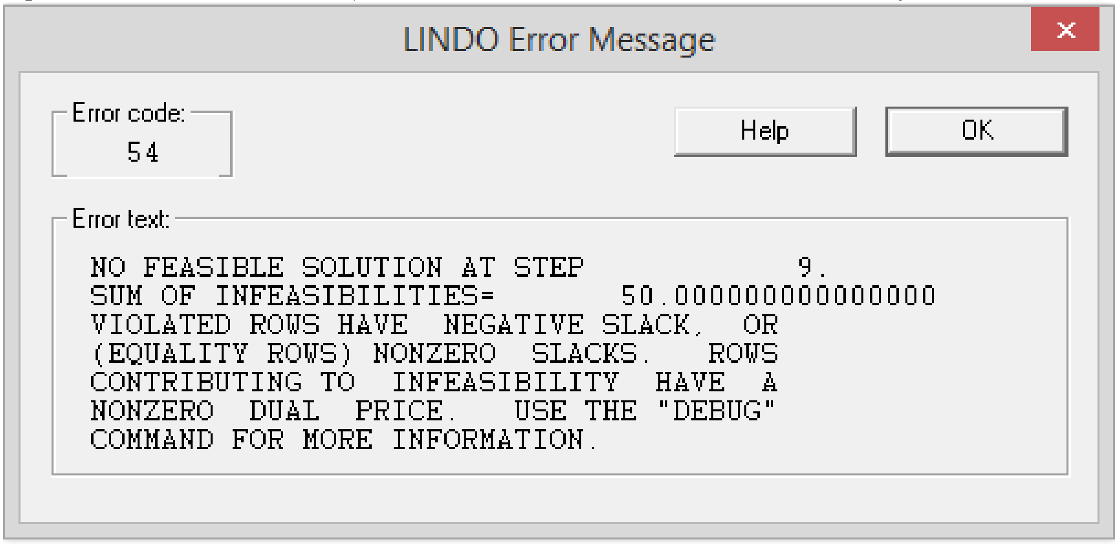
The optimal minimum cost is $17,100

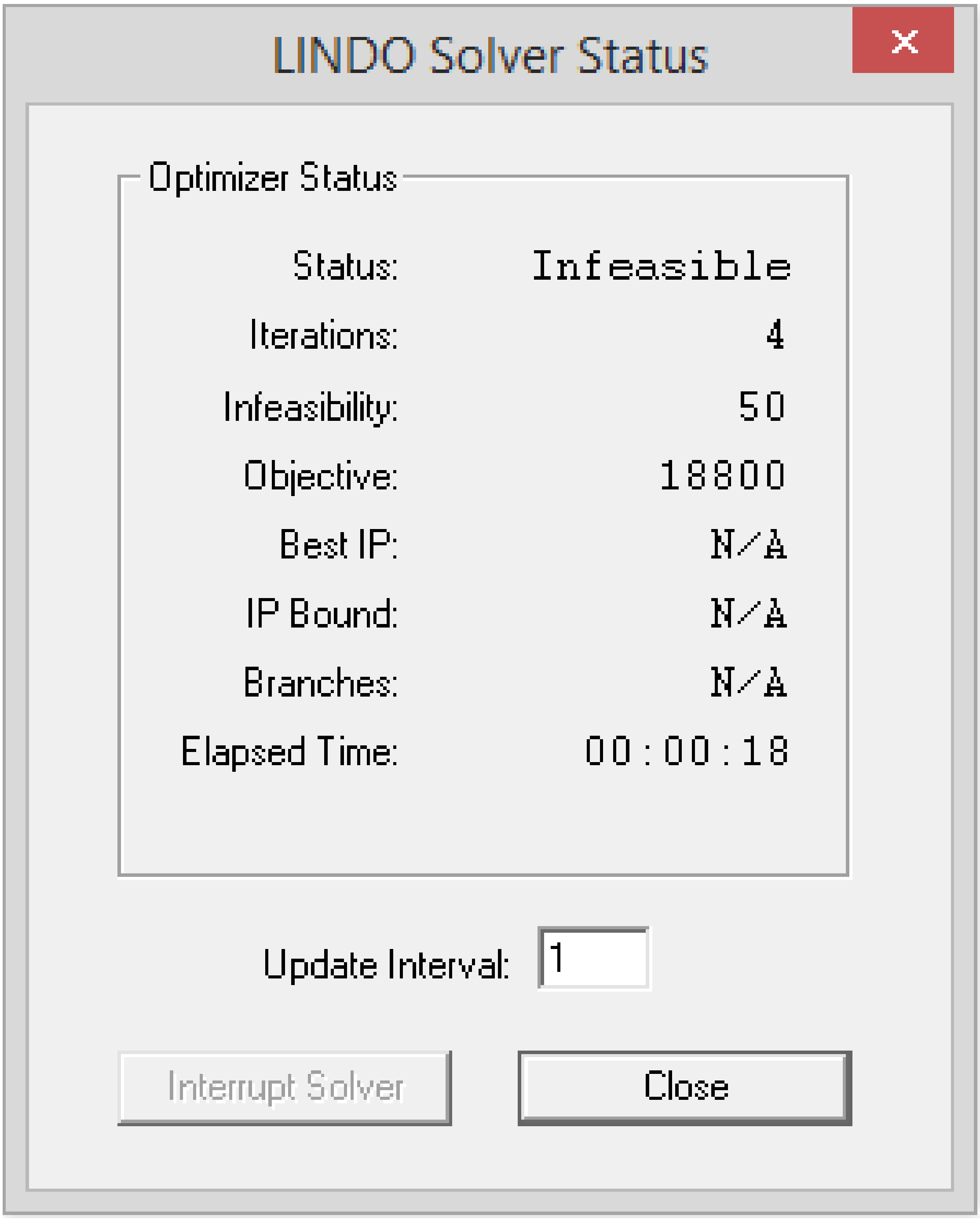
Part B:
To remove Warehouse 2 and all its associated routes, we remove all variables in our equation that contain "w2" in it. The resulting LINDO code is as follows:

```
MIN 10 p1w1 + 11 p2w1 + 13 p3w1 + 9 p3w3 + 8 p4w3 + 5 w1r1 + 6 w1r2
   + 7 w1r3 + 10 w1r4 + 14 w3r4 + 12 w3r5 + 12 w3r6 + 6 w3r7
ST
   p1w1 = 150
   p2w1 = 450
   p3w1 + p3w3 = 250
   p4w3 = 150
   w1r1 = 100
   w1r2 = 150
   w1r3 = 100
   w1r4 + w3r4 = 200
   w3r5 = 200
   w3r6 = 150
   w3r7 = 100
   p1w1 + p2w1 + p3w1 - w1r1 - w1r2 - w1r3 - w1r4 <= 0
   p3w3 + p4w3 - w3r4 - w3r5 - w3r6 - w3r7 <= 0
   p1w1 > 0
   p2w1 > 0
   p3w1 > 0
```

```
    p3w3 > 0
    p4w3 > 0
    w1r1 > 0
    w1r2 > 0
    w1r3 > 0
    w1r4 > 0
    w3r4 > 0
    w3r5 > 0
    w3r6 > 0
    w3r7 > 0
END
```

Upon execution of the code, we are met with these solution infeasibility notices:





There is no feasible solution here because Warehouse 3's total possible supply (400) is not capable of supporting the combined demand of R4, R5, R6, and R7 (650). On the other hand, Warehouse 1 is in fact capable of supporting R1, R2, R3, and R4 with its 850 supply to 550 demand ratio, but that still does not solve the larger supply chain deficit and feasibility issue. Overall, we need Warehouse 2 to supplement the supply to

R3, R4, R5, and R6 and sufficiently "spread the load" on the warehouses.

Part C:

To simulate the limitation of 100 shipments (inbound and outbound) on Warehouse 2, we add two new constraints to our code for Part A. The constraints limit the inbound and outbound shipments to being less than or equal to 100.

LINDO Code and Output:

```
MIN 10 p1w1 + 15 p1w2 + 11 p2w1 + 8 p2w2 + 13 p3w1 + 8 p3w2 + 9 p3w3
  + 14 p4w2 + 8 p4w3 + 5 w1r1 + 6 w1r2 + 7 w1r3 + 10 w1r4 + 12 w2r3
  + 8 w2r4 + 10 w2r5 + 14 w2r6 + 14 w3r4 + 12 w3r5 + 12 w3r6 + 6 w3r7
ST
   p1w1 + p1w2 = 150
   p2w1 + p2w2 = 450
   p3w1 + p3w2 + p3w3 = 250
   p4w2 + p4w3 = 150
   w1r1 = 100
   w1r2 = 150
   w1r3 + w2r3 = 100
   w1r4 + w2r4 + w3r4 = 200
   w2r5 + w3r5 = 200
   w2r6 + w3r6 = 150
   w3r7 = 100
   p1w1 + p2w1 + p3w1 - w1r1 - w1r2 - w1r3 - w1r4 <= 0
   p1w2 + p2w2 + p3w2 + p4w2 - w2r3 - w2r4 - w2r5 - w2r6 <= 0
   p3w3 + p4w3 - w3r4 - w3r5 - w3r6 - w3r7 <= 0
   p1w1 > 0
   p1w2 > 0
   p2w1 > 0
   p2w2 > 0
   p3w1 > 0
   p3w2 > 0
   p3w3 > 0
   p4w2 > 0
   p4w3 > 0
   w1r1 > 0
   w1r2 > 0
   w1r3 > 0
   w1r4 > 0
   w2r3 > 0
   w2r4 > 0
   w2r5 > 0
```

```
   w2r6 > 0
   w3r4 > 0
   w3r5 > 0
   w3r6 > 0
   w3r7 > 0
   p1w2 + p2w2 + p3w2 + p4w2 <= 100
   w2r3 + w2r4 + w2r5 + w2r6 <= 100
END


 LP OPTIMUM FOUND AT STEP      7

        OBJECTIVE FUNCTION VALUE

        1)      18300.00

  VARIABLE        VALUE          REDUCED COST
      P1W1       150.000000          0.000000
      P1W2         0.000000          8.000000
      P2W1       350.000000          0.000000
      P2W2       100.000000          0.000000
      P3W1         0.000000          4.000000
      P3W2         0.000000          2.000000
      P3W3       250.000000          0.000000
      P4W2         0.000000          9.000000
      P4W3       150.000000          0.000000
      W1R1       100.000000          0.000000
      W1R2       150.000000          0.000000
      W1R3       100.000000          0.000000
      W1R4       150.000000          0.000000
      W2R3         0.000000          7.000000
      W2R4        50.000000          0.000000
      W2R5        50.000000          0.000000
      W2R6         0.000000          4.000000
      W3R4         0.000000          4.000000
      W3R5       150.000000          0.000000
      W3R6       150.000000          0.000000
      W3R7       100.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000        -20.000000
        3)         0.000000        -21.000000
        4)         0.000000        -19.000000
```

```
       5)          0.000000         -18.000000
       6)          0.000000           5.000000
       7)          0.000000           4.000000
       8)          0.000000           3.000000
       9)          0.000000           0.000000
      10)          0.000000          -2.000000
      11)          0.000000          -2.000000
      12)          0.000000           4.000000
      13)          0.000000          10.000000
      14)          0.000000          13.000000
      15)          0.000000          10.000000
      16)        150.000000           0.000000
      17)          0.000000           0.000000
      18)        350.000000           0.000000
      19)        100.000000           0.000000
      20)          0.000000           0.000000
      21)          0.000000           0.000000
      22)        250.000000           0.000000
      23)          0.000000           0.000000
      24)        150.000000           0.000000
      25)        100.000000           0.000000
      26)        150.000000           0.000000
      27)        100.000000           0.000000
      28)        150.000000           0.000000
      29)          0.000000           0.000000
      30)         50.000000           0.000000
      31)         50.000000           0.000000
      32)          0.000000           0.000000
      33)          0.000000           0.000000
      34)        150.000000           0.000000
      35)        150.000000           0.000000
      36)        100.000000           0.000000
      37)          0.000000           0.000000
      38)          0.000000           5.000000

 NO. ITERATIONS=       7
```

We see that this is a feasible solution, though it raises our optimal cost from \$17,100 to \$18,300.

The optimal shipping routes and quantity of refrigerators per route are:

```
 Route      Refrigerators
     P1W1       150
     P2W1       350
     P2W2       100
```

P3W3	250
P4W3	150
W1R1	100
W1R2	150
W1R3	100
W1R4	150
W2R4	50
W2R5	50
W3R5	150
W3R6	150
W3R7	100

Part D:
For some cost c_{ab} and some route x_{ab} where their subscripts represent the route from a to b, the objective function is:
$Min(\sum_p \sum_w c_{pw}x_{pw} + \sum_w \sum_r c_{wr}x_{wr})$

$s.t.$
(Supply Constraint) For each source node p, where s is supply: $\sum_w x_{pw} = s_p$
(Demand Constraint) For each destination node r, where d is demand: $\sum_w x_{wr} = d_r$
(Balancing Constraint) For each intermediate node w: $\sum_p x_{pw} - \sum_r x_{wr} <= 0$
(Non-negativity Constraint) For all nodes p and r: $x_{pw}, x_{wr} >= 0$

2 Problem 2

Part A:
i.
ii.
iii.
Part B:
i.
ii.
iii.
Part C:
i.
ii.
iii.

3 Problem 3

Part A:

i. General Problem Statement
Objective:
$Min \sum_{i=1}^{n} b_i$, for some b_i representing $|y_i - (a_1 x_i + a_0)|$ for each i = 1...n
s.t.
For each i = 1...n, $b_i - (y_i - (a_1 x_i + a_0)) >= 0$
For each i = 1...n, $b_i + (y_i - (a_1 x_i + a_0)) >= 0$

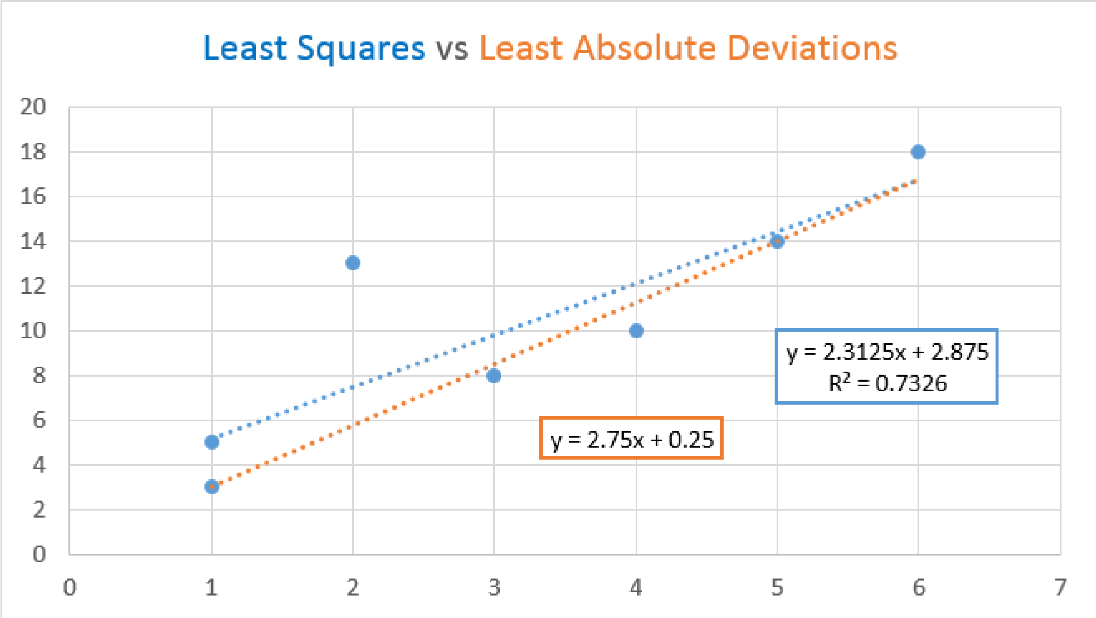
ii. LINDO Code and Output

```
MIN b1 + b2 + b3 + b4 + b5 + b6 + b7
ST
   b1 + 1 a1 + a0 >= 5
   b1 - 1 a1 - a0 >= -5
   b2 + 1 a1 + a0 >= 3
   b2 - 1 a1 - a0 >= -3
   b3 + 2 a1 + a0 >= 13
   b3 - 2 a1 - a0 >= -13
   b4 + 3 a1 + a0 >= 8
   b4 - 3 a1 - a0 >= -8
   b5 + 4 a1 + a0 >= 10
   b5 - 4 a1 - a0 >= -10
   b6 + 5 a1 + a0 >= 14
   b6 - 5 a1 - a0 >= -14
   b7 + 6 a1 + a0 >= 18
   b7 - 6 a1 - a0 >= -18
   b1 > 0
   b2 > 0
   b3 > 0
   b4 > 0
   b5 > 0
   b6 > 0
   b7 > 0
END

 LP OPTIMUM FOUND AT STEP       7

        OBJECTIVE FUNCTION VALUE

        1)      12.25000

  VARIABLE        VALUE          REDUCED COST
        B1         2.000000          0.000000
        B2         0.000000          0.250000
```

```
       B3          7.250000             0.000000
       B4          0.500000             0.000000
       B5          1.250000             0.000000
       B6          0.000000             0.750000
       B7          1.250000             0.000000
       A1          2.750000             0.000000
       A0          0.250000             0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)          0.000000            -1.000000
       3)          4.000000             0.000000
       4)          0.000000             0.000000
       5)          0.000000            -0.750000
       6)          0.000000            -1.000000
       7)         14.500000             0.000000
       8)          1.000000             0.000000
       9)          0.000000            -1.000000
      10)          2.500000             0.000000
      11)          0.000000            -1.000000
      12)          0.000000             0.000000
      13)          0.000000            -0.250000
      14)          0.000000            -1.000000
      15)          2.500000             0.000000
      16)          2.000000             0.000000
      17)          0.000000             0.000000
      18)          7.250000             0.000000
      19)          0.500000             0.000000
      20)          1.250000             0.000000
      21)          0.000000             0.000000
      22)          1.250000             0.000000

 NO. ITERATIONS=       7
```

The sum of the absolute deviations was 12.25.

iii. Graph



The LAD approach creates a line that has a steeper slope and lesser y-intercept than the least squares line. The line more tightly follows the path of the majority of the data points and seems to be less affected by the outlier at x = 2.

Part B:

i. General Problem Statement

Objective:

Min b, for some b representing $Max|y_i - (a_1x_i + a_0)|$

s.t.

For each i = 1...n, $b + a_1x_i + a_0 >= y_i$

For each i = 1...n, $b - a_1x_i - a_0 >= -y_i$

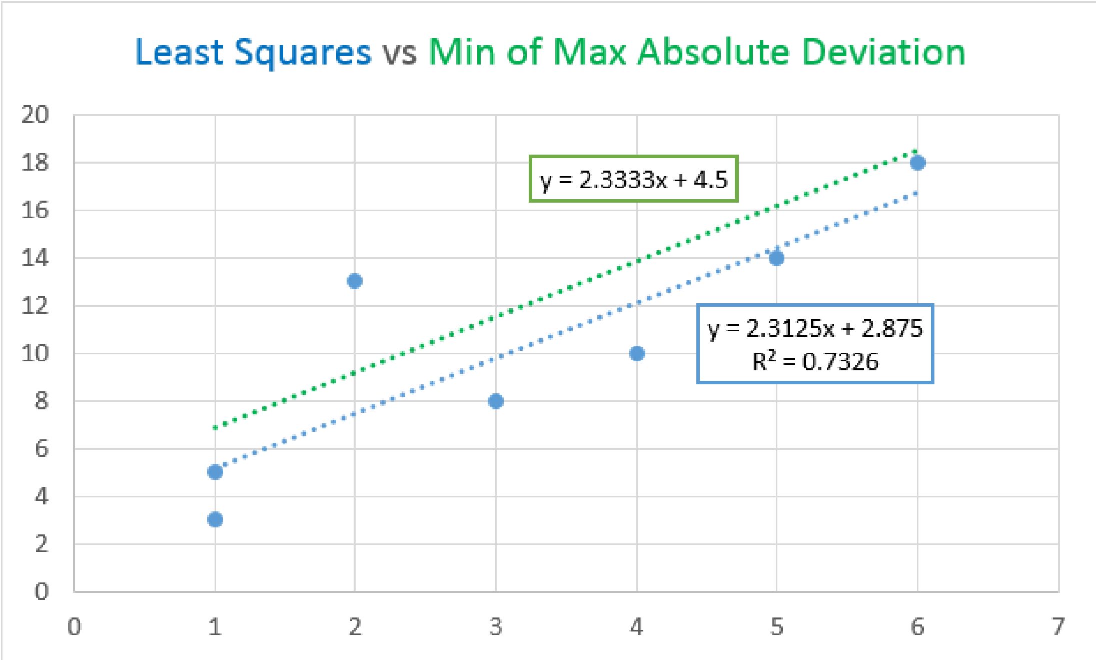
ii. LINDO Code and Output

```
MIN b
ST
   b + 1 a1 + a0 >= 5
   b - 1 a1 - a0 >= -5
   b + 1 a1 + a0 >= 3
   b - 1 a1 - a0 >= -3
   b + 2 a1 + a0 >= 13
   b - 2 a1 - a0 >= -13
   b + 3 a1 + a0 >= 8
   b - 3 a1 - a0 >= -8
   b + 4 a1 + a0 >= 10
```

```
   b - 4 a1 - a0 >= -10
   b + 5 a1 + a0 >= 14
   b - 5 a1 - a0 >= -14
   b + 6 a1 + a0 >= 18
   b - 6 a1 - a0 >= -18
   b > 0
END

 LP OPTIMUM FOUND AT STEP      3

        OBJECTIVE FUNCTION VALUE

        1)      3.833333

  VARIABLE        VALUE          REDUCED COST
         B         3.833333           0.000000
        A1         2.333333           0.000000
        A0         4.500000           0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         5.666667           0.000000
        3)         2.000000           0.000000
        4)         7.666667           0.000000
        5)         0.000000          -0.333333
        6)         0.000000          -0.500000
        7)         7.666667           0.000000
        8)         7.333333           0.000000
        9)         0.333333           0.000000
       10)         7.666667           0.000000
       11)         0.000000          -0.166667
       12)         6.000000           0.000000
       13)         1.666667           0.000000
       14)         4.333333           0.000000
       15)         3.333333           0.000000
       16)         3.833333           0.000000

 NO. ITERATIONS=       3
```

The min of the max absolute deviations was 3.833333.

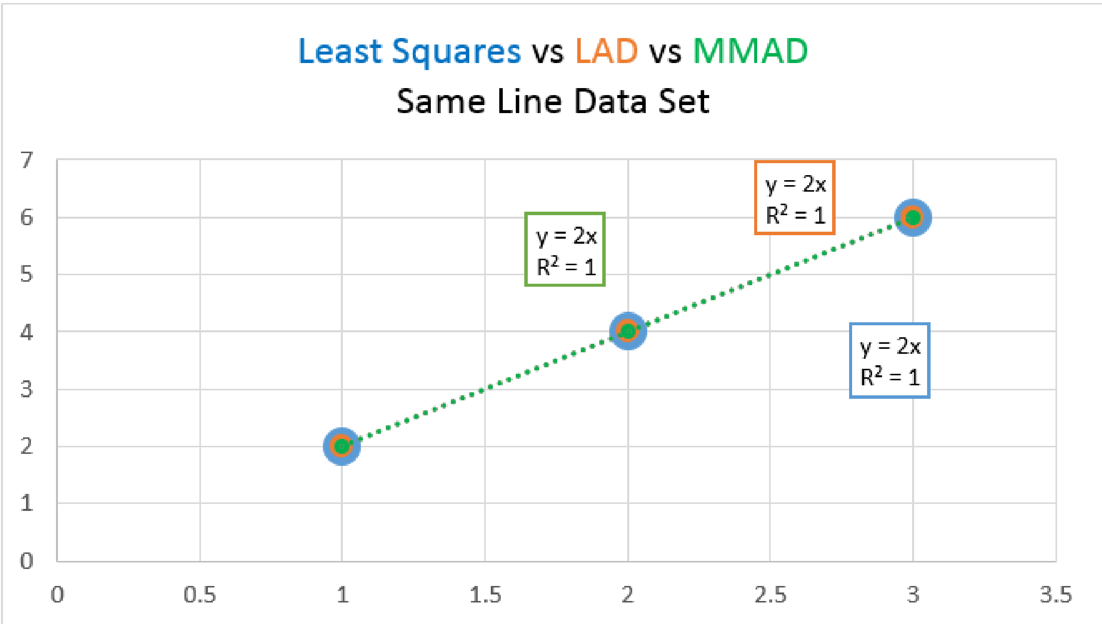
iii. Graph



The MMAD approach gives a line that appears to be more affected by the outlier at x = 2 with a much higher y-intercept than the LAD and least squares approaches. It's slope however, is only slightly steeper than the least squares line.

iv. Same Line Data Set

Yes, the data set (x,y) = [(1,2), (2,4), (3,6)] computes the same line for all three methods.



LAD Method:

```
MIN b1 + b2 + b3
ST
   b1 + 1 a1 + a0 >= 2
   b1 - 1 a1 - a0 >= -2
   b2 + 2 a1 + a0 >= 4
   b2 - 2 a1 - a0 >= -4
   b3 + 3 a1 + a0 >= 6
   b3 - 3 a1 - a0 >= -6
   b1 > 0
   b2 > 0
   b3 > 0
END

 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)     0.0000000E+00

  VARIABLE        VALUE          REDUCED COST
        B1         0.000000          1.000000
        B2         0.000000          1.000000
        B3         0.000000          1.000000
        A1         2.000000          0.000000
        A0         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.000000
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          0.000000
        6)         0.000000          0.000000
        7)         0.000000          0.000000
        8)         0.000000          0.000000
        9)         0.000000          0.000000
       10)         0.000000          0.000000

 NO. ITERATIONS=       1
```

MMAD Method:

```
MIN b
ST
   b + 1 a1 + a0 >= 2
   b - 1 a1 - a0 >= -2
   b + 2 a1 + a0 >= 4
   b - 2 a1 - a0 >= -4
   b + 3 a1 + a0 >= 6
   b - 3 a1 - a0 >= -6
   b > 0
END

 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)     0.0000000E+00

  VARIABLE        VALUE          REDUCED COST
         B         0.000000          1.000000
        A1         2.000000          0.000000
        A0         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.000000
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          0.000000
        6)         0.000000          0.000000
        7)         0.000000          0.000000
        8)         0.000000          0.000000

 NO. ITERATIONS=       1

Least Squares Line was produced in Excel (see graph above) rather than LINDO.
```

Part C: Objective:
$Min \sum_{i=1}^{n} b_i$, for some b_i representing $|y_i - (a_2 x_{2i} + a_1 x_{1i} + a_0)|$ for each i = 1...n
s.t.
For each i = 1...n, $b_i - (y_i - (a_2 x_{2i} + a_1 x_{1i} + a_0)) >= 0$
For each i = 1...n, $b_i + (y_i - (a_2 x_{2i} + a_1 x_{1i} + a_0)) >= 0$

```
MIN b1 + b2 + b3 + b4 + b5 + b6
ST
   b1 + 1 a2 + 1 a1 + a0  >= 5
   b1 - 1 a2 - 1 a1 - a0 >= -5
   b2 + 2 a2 + 1 a1 + a0  >= 9
   b2 - 2 a2 - 1 a1 - a0 >= -9
   b3 + 2 a2 + 2 a1 + a0  >= 12
   b3 - 2 a2 - 2 a1 - a0 >= -12
   b4 + 1 a2 + 0 a1 + a0  >= 3
   b4 - 1 a2 - 0 a1 - a0 >= -3
   b5 + 0 a2 + 0 a1 + a0  >= 0
   b5 - 0 a2 - 0 a1 - a0 >= 0
   b6 + 3 a2 + 1 a1 + a0  >= 11
   b6 - 3 a2 - 1 a1 - a0 >= -11
   b1 > 0
   b2 > 0
   b3 > 0
   b4 > 0
   b5 > 0
   b6 > 0
END

 LP OPTIMUM FOUND AT STEP      8

        OBJECTIVE FUNCTION VALUE

        1)      2.000000

  VARIABLE        VALUE          REDUCED COST
        B1         1.000000          0.000000
        B2         0.000000          0.000000
        B3         0.000000          0.500000
        B4         0.000000          0.000000
        B5         0.000000          0.500000
        B6         1.000000          0.000000
        A2         3.000000          0.000000
        A1         3.000000          0.000000
        A0         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         2.000000          0.000000
        3)         0.000000         -1.000000
        4)         0.000000         -1.000000
```

```
        5)          0.000000          0.000000
        6)          0.000000         -0.500000
        7)          0.000000          0.000000
        8)          0.000000         -1.000000
        9)          0.000000          0.000000
       10)          0.000000          0.000000
       11)          0.000000         -0.500000
       12)          2.000000          0.000000
       13)          0.000000         -1.000000
       14)          1.000000          0.000000
       15)          0.000000          0.000000
       16)          0.000000          0.000000
       17)          0.000000          0.000000
       18)          0.000000          0.000000
       19)          1.000000          0.000000

 NO. ITERATIONS=        8
```

The sum of the absolute deviations was 2.
The regression model is: $y = 3x_2 + 3x_1 + 0$