

$$\sum_s \underbrace{f(s) P_{\theta}(s)}^{E_{\theta}[f(s)]} = \sum_s f(s) [\underbrace{P_{\theta}} + \underbrace{P_{\theta'} - P_{\theta}}]$$

$$= \sum f(s) P_{\theta'} + \sum f(s) [P_{\theta} - P_{\theta'}]$$

$$\leq \sum f(s) P_{\theta'} + \sum \underbrace{|f(s)|} |P_{\theta} - P_{\theta'}|$$

$$\leq \underbrace{\sum f(s) P_{\theta'}}_{E_{\theta'}[f(s)]} + \max_s |f(s)| \sum |P_{\theta} - P_{\theta'}|$$

$$\underbrace{E_{\theta'}[f(s)] + \max_s |f(s)| \cdot \|P_{\theta} - P_{\theta'}\|}_{\text{Total Error Bound}}$$