Structure of the set of feasible neural commands for complex motor tasks

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***Abstract*— The brain must select its control strategies among an infinite set of possibilities, thereby researches believe that it must be solving an optimization problem. While this set of feasible solutions is infinite and lies in high dimensions, it is bounded by kinematic, neuromuscular, and anatomical constraints, within which the brain must select optimal solutions. That is, the seat of feasible activations is well structured. However, to date there is not method to describe and quantify the structure of these high-dimensional solution spaces, other than bounding boxes or dimensionality reduction algorithms that do not capture its full structure. We present a novel approach based on the well-known Hit-and-Run algorithm in computational geometry to extract the structure of the feasible activations that produce 50% of maximal fingertip force. We use a realistic model of a human index finger with 7 muscles, 4DOF, and 4 output dimensions. For a given force vector at the endpoint, the feasible activation space is a 3D convex polytope, embedded in the 7D unit cube. It is known that explicitly computing the volume of this polytope can become too computationally complex in many instances. However, our algorithm was able to produce $1{,}000{,}000$ random points in the feasible activation space, which converged to the uniform distribution. The computed distribution of activation across each muscle shed light onto the structure of these solution spaces—rather than simply exploring their maximal and minimal values. Although this paper presents a 7 dimensional case of the index finger, our methods extend to systems with up to at least 40 muscles. This will allow our motor control community to understand the distributions of feasible muscle activations, which will provide important contextual information into the learning, optimization and adaptation of motor patterns in future research.**

I. INTRODUCTION

Muscle redundancy is the term used to describe the underdetermined nature of neural control of musculature. The classical notion of muscle redundancy proposes that, faced with an infinite number of possible muscle activation patterns for a given task, the nervous system optimizes in some fashion to select one solution. Here, each of N muscles represents a dimension of control on an end effector, and at any moment of a task, a muscle activation pattern exists as a point in $\mathbb{R}^N$, with a level of activation for every muscle involved [18]. Thus researchers often seek to infer the optimization approach and the cost functions the nervous system utilizes to select effective points in activation space to produce natural behavior [2], [12], [13], [16], [4], [7].

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Implicit in these optimization procedures is the notion that there exists a well structured set of feasible solutions. Thus several of us have focused on describing and understanding those high-dimensional subspaces embedded in $\mathbb{R}^N$ [10], [11], [15], [18], [8].

For the case of submaximal and static force production with a limb, the muscle redundancy problem is phrased in computational geometry: find the structure of the set of all feasible muscle activations, given the limb mechanics and the task constraints [1], [18], [17], [8]. We aim to explore what the solution space looks like, and uncover the structure of the feasible activation space for a given static force task. If each muscle's maximal activation is normalized to 1, the constraint $\mathbf{a} \in [0,1]^n$ describes that the feasible activation space lies in the n-dimensional unit cube (also called the n-cube); the activation space for the index finger is within the unit 7-cube.

A. *High dimensionality difficulties*

Consider a model of a static fingertip force, with 7 muscles arculating the index finger's 4 Degrees of Freedom. Assuming independent control of each muscle (non-synergistic model), each muscle has a unique force vector at the endpoint; the end effector has 7 unique vectors it can linearly combine to generate any vector of static force. This yields a unit 7-cube in charge of producing a 4-dimensional output wrench. On order to uncover the structure and relationship of these spaces, we cannot visualize all dimensions simultaneously as we could with a simple 3-muscle model.

This convex polytope is called the *feasible activation set*. To date, the structure of this high-dimensional polytope is inferred by its bounding box [10], [15], [8]. But the bounding box of a convex polytope will always overestimate its volume, and lose the details of its shape. Empirical dimensionality-reduction methods have also been used to calculate a basis vectors for such subspaces [3], [5], [9]. But those basis vectors only provide a description of the dimension, orientation, and aspect ratio of the polytope, but not of its boundaries or internal structure.

Here we present a novel application of the well-known Hit-and-Run algorithm [14] to describe the internal structure of these high-dimensional feasible activation sets. The input to our hit-and-run procedure requires a task force, along with the system's endpoint Jacobian, maximal tendon forces, and a moment arm matrix.

We applied our approach to two separate musculoskeletal models: 1. A fabricated schematic system, which we designed to have three muscles articulating one DOF, and one dimension of output force. 2. A realistic model, with seven muscles articulating four DOFs, and four dimensional output force [18]. With this, below are the key ideas and findings we present with this paper:

- For some muscles, we found that the bounding box exceptionally misconstrues the actual shape of the feasible activation space.
- Hit-and-run sampling of the solution space is computationally tractable; fewer than ten thousand points uncovered the shape of the distributions.
- Our approach provides a more granular context to the space within which the central nervous system optimizes.
- We apply six different cost functions (post-hoc) to all solutions, thereby providing spatial context to where 'optimal' solutions lie within the space.
- We designed an interactive parallel coordinates platform for visualizing and manipulating constraints to the solution space, such as muscle dysfunction, muscle hyperactivity, as well as constraining the upper and lower bounds for six different cost functions. We can compare cost functions side-by-side and view subsets of the dataset after applying cost function constraints.

II. METHODS

A. construction of the finger model

$$F_o = (123.0, 219.0, 23.52, 91.74, 21.6, 124.8, 129.6)$$

$$JR = \begin{pmatrix} -0.08941 & -0.0447 & -0.009249 & 0.03669 & 0.1421 & 0.2087 & -0.2138 \\ -0.04689 & -0.1496 & 0.052 & 0.052 & 0.0248 & 0.0 & 0.0248 \\ 0.06472 & 0.001953 & -0.1518 & -0.1518 & 0.2919 & 0.0568 & 0.2067 \\ 0.003081 & -0.002352 & -0.0001649 & -0.0001649 & -0.0004483 & 0.0001578 & -0.000685 \end{pmatrix}$$

$task_x = (1.0, 0.0, 0.0, 0.0)$ $task_y = (0.0, 1.0, 0.0, 0.0)$ Palmar force is $task_z = (0.0, 0.0, 1.0, 0.0)$ $task_xy = (1.0, 1.0, 0.0, 0.0)$

B. Hit-and-Run algorithm

Hit-and-Run is a method used to uniformly sample a convex [14], and as the set of all feasible activations is defined by the mechanics of the limb and the constraints of the task (described in II-C). We decided to use Hit-and-run as a way to sample muscle activation solutions. In the case of a schematic tendon-driven limb with three muscles, the feasible activation space is the unit cube (as muscles can only be activated positively from 0 to a maximal normalized value of 1). As explained in [17], when task constraints are introduced to the system, the feasible activation set is further reduced; in this context, a task is a static force vector produced at the endpoint of the limb, which is represented as a set of inequality constraints. Thus if this simple limb meets all constraints, the feasible activation set of the polygon P contains all feasible activation $\mathbf{a} \in \mathbb{R}^n$ that satisfy

$$\mathbf{f} = A\mathbf{a}, \mathbf{a} \in [0,1]^n,$$

where $\mathbf{f} \in \mathbb{R}^m$ is a fixed force vector and $A = J^{-T}RF_o \in \mathbb{R}^{m \times n}$—the matrices of the Jacobian of the limb, the moment arms of the tendons, and the strengths of the muscles, respectively [18], [17]. P is bounded by the unit n-cube since all variables a_i, $i \in [n]$ are bounded by 0 and 1 from below, above respectively. Consider the following 1×3 fabricated example, where the task is a 1N unidimensional force.

$$1 = \frac{10}{3}a_1 - \frac{53}{15}a_2 + 2a_3$$
$$a_1, a_2, a_3 \in [0,1],$$

the set of feasible activations is given by the shaded set in Figure II-B.

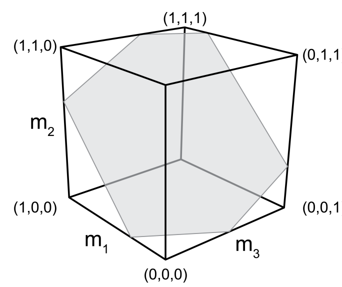


Fig. 1. The feasible activation set for a three-muscle system meeting one functional constraint is a polygon in $\mathbb{R}^3$. Note that muscle activations are assumed to be bounded between 0 and 1.

The Hit-and-Run walk on P is defined as follows (it works analogously for any convex body).

1) Inner Point: Find a given starting point $\mathbf{p}$ of P (Figure 2(a)) .
2) Direction: Generate a random direction from $\mathbf{p}$ (uniformly at random over all directions) (Figure 2(b)).
3) Endpoints: Find the intersection points of the random direction with the edges of the polytope (Figure 2(c)).
4) New Point: Pick a random distance along the line formed by the endpoints (Figure 2(d)).
5) Repeat from (a) the above steps with the new point as the starting point .

To find a starting point in

$$\mathbf{f} = A\mathbf{a}, \mathbf{a} \in [0,1]^n,$$

we only need to find a feasible activation vector. For the Hit-and-Run algorithm to mix faster, we want the starting point to be centrally located within the polytope. We use the following standard trick with slack variables ε_i.

$$\begin{array}{lrcl} \text{maximize} & \sum_{i=1}^{n} \varepsilon_i & & \\ \text{subject to} & \mathbf{f} & = & A\mathbf{a} \\ & a_i & \in & [\varepsilon_i, 1-\varepsilon_i], \quad \forall i \in \{1,\ldots,n\} \\ & \varepsilon_i & \geq & 0, \quad \forall i \in \{1,\ldots,n\}. \end{array} \tag{1}$$

How many points are necessary to reach a uniform distribution across the polytope? For convex polygons in higher dimensions c. 40, experimental results suggest that $\mathcal{O}(n)$ steps of the Hit-and-Run algorithm are sufficient. In particular Emiris and Fisikopoulos paper suggest that $(10 + 10\frac{n}{)}n$

steps are enough to converge upon the uniform distribution [6]. In the index finger model we executed the Hit-and-Run algorithm $1,000,000$ times.

C. Realistic index finger model

We used our published model in [18] to find matrix $A \in \mathbb{R}^{4\times 7}$, where $\mathbf{a} \in \mathbb{R}^7$ and the four degrees of freedom were ad-abduction, flexion-extension at the metacarpophalangeal joint, and flexion-extension at the proximal and distal interphalangeal joints. The force direction we simulated is in the palmar direction in the posture shown in Figure 3.

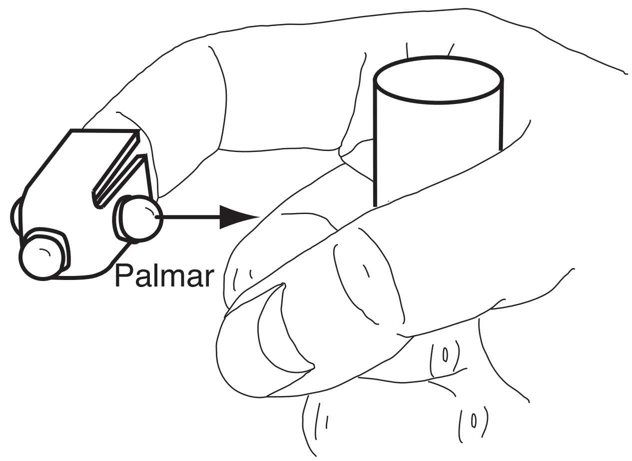


Fig. 3. The index finger model simulated 50% of maximal force production in the palmar direction. Adapted from [18].

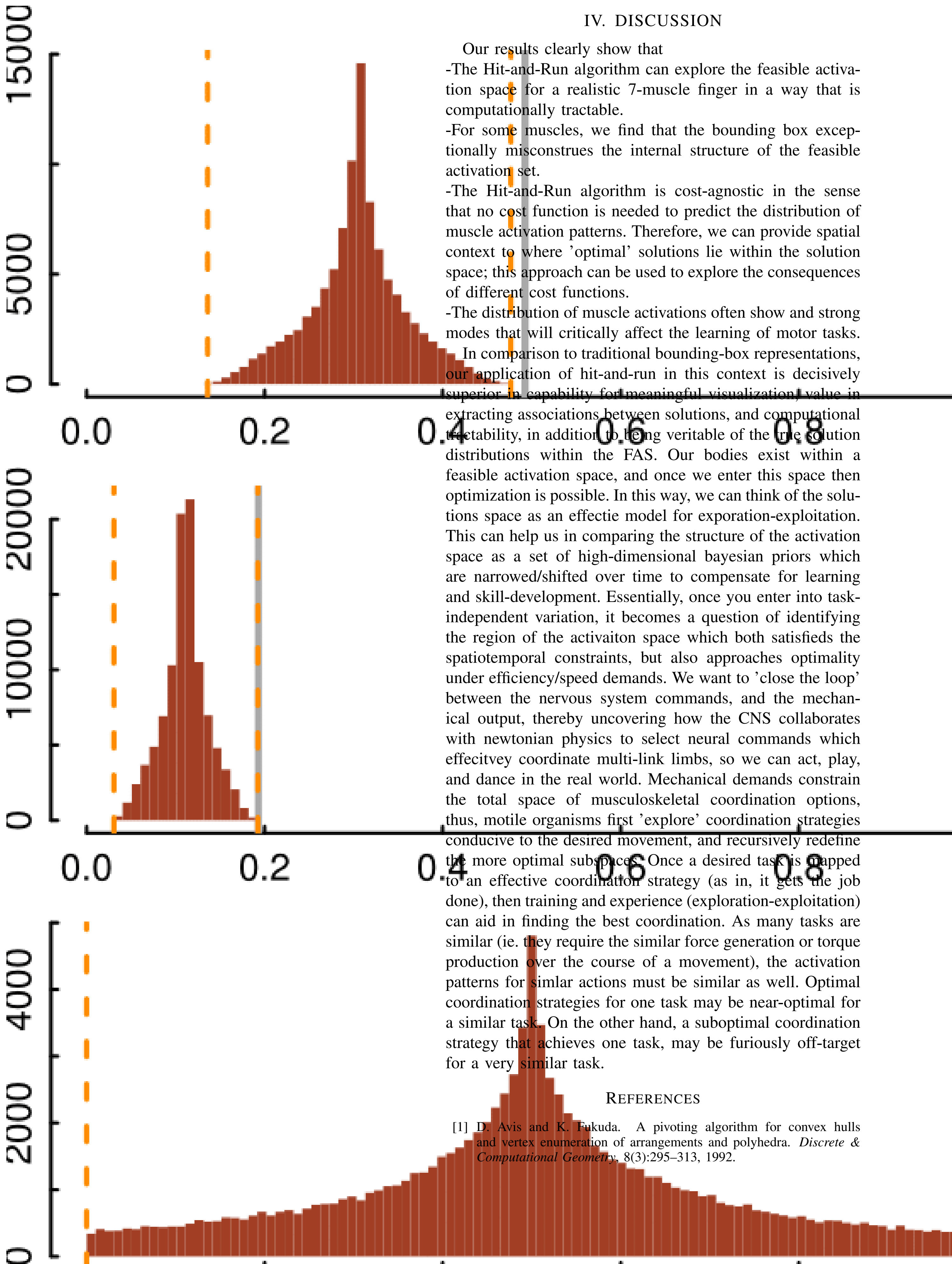
III. RESULTS

Using Hit-and-Run to sample feasible activation sets, Figure 4 shows the distributions of activation solutions for a palmar submaximal force.resulting from $1,000,000$ solutions computed with Hit-and-Run sampling. This is the first time (to our knowledge) that the internal structure of the feasible activation set has been visualized for a sub-maximal force.

Notice also that the lower and upper bounds of the activations (i.e., the dashed lines that indicate their bounding box), are uniquely uninformative of the actual density of distribution of feasible activations. Note also that the activation needed for the maximal force output (thick gray line) is very often not the mode of the activations at 50% of output.

Results: Projection onto a given muscle dimension Simple histograms at 80Activation progression-march (3) x y z Parallel coordinates with cost Full Parallel Coordinates with cost Lower Middle Upper constraint by cost Yes you could put activation constraints directly in the A matrix, instead of bounds between 0 and 1. There is no advantage to adding activation constraints beforehand in the A matrix, as sampling is uniform- as long as the resulting dataset is large enough for your purpose. You could also put l1 and weighted l1 cost bounds as constraints in the A matrix. Cannot put higher order cost functions such as l2,l3 or weighted l2,l3.

Talk about slopes in Parallel if they



IV. DISCUSSION

Our results clearly show that

-The Hit-and-Run algorithm can explore the feasible activation space for a realistic 7-muscle finger in a way that is computationally tractable.

-For some muscles, we find that the bounding box exceptionally misconstrues the internal structure of the feasible activation set.

-The Hit-and-Run algorithm is cost-agnostic in the sense that no cost function is needed to predict the distribution of muscle activation patterns. Therefore, we can provide spatial context to where 'optimal' solutions lie within the solution space; this approach can be used to explore the consequences of different cost functions.

-The distribution of muscle activations often show and strong modes that will critically affect the learning of motor tasks.

In comparison to traditional bounding-box representations, our application of hit-and-run in this context is decisively superior in capability for meaningful visualization, value in extracting associations between solutions, and computational tractability, in addition to being veritable of the true solution distributions within the FAS. Our bodies exist within a feasible activation space, and once we enter this space then optimization is possible. In this way, we can think of the solutions space as an effectie model for exporation-exploitation. This can help us in comparing the structure of the activation space as a set of high-dimensional bayesian priors which are narrowed/shifted over time to compensate for learning and skill-development. Essentially, once you enter into task-independent variation, it becomes a question of identifying the region of the activaiton space which both satisfieds the spatiotemporal constraints, but also approaches optimality under efficiency/speed demands. We want to 'close the loop' between the nervous system commands, and the mechanical output, thereby uncovering how the CNS collaborates with newtonian physics to select neural commands which effecitvey coordinate multi-link limbs, so we can act, play, and dance in the real world. Mechanical demands constrain the total space of musculoskeletal coordination options, thus, motile organisms first 'explore' coordination strategies conducive to the desired movement, and recursively redefine the more optimal subspaces. Once a desired task is mapped to an effective coordination strategy (as in, it gets the job done), then training and experience (exploration-exploitation) can aid in finding the best coordination. As many tasks are similar (ie. they require the similar force generation or torque production over the course of a movement), the activation patterns for simlar actions must be similar as well. Optimal coordination strategies for one task may be near-optimal for a similar task. On the other hand, a suboptimal coordination strategy that achieves one task, may be furiously off-target for a very similar task.

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