

## 1 Abstract

300 Words

Abstract —PLOS Computational Biology— The abstract of the paper should be succinct; it must not exceed 300 words. Authors should mention the techniques used without going into methodological detail and should summarize the most important results. While the abstract is conceptually divided into three sections (Background, Methodology/Principal Findings, and Conclusions/Significance), please do not apply these distinct headings to the abstract within the article file. Please do not include any citations and avoid specialist abbreviations. ———

What we want to achieve with this abstract: BRIAN - Introduce linear approximations of muscles for FFS and FAS MAY- Show how computationally complex it would be to get volume slices of a zonotope in 7D MAY- Show the benefits of having points MAY Discuss prediction with points and how its related to volume MAY -What is Hit-And-Run, and how does it help us solve our problem Talk about how distributions are not uniform across the dimensions, and the majority of solutions can lie in one cluster. Briefly touch on the limitations of this model and other models What does this mean in terms of neural control models made by other researchers in the past?

Things that we achieved: ? ? ?

## 2 Hit-and-Run

The Hit-and-Run algorithm used for sampling in a convex body  $K$ , was introduced by Smith in 1984 [5]. The mixing time is known to be  $\mathcal{O}^*(n^2 R^2 / r^2)$ , where  $R$  and  $r$  are the radii of the inscribed and circumscribed ball of  $K$  respectively [1, 3]. In the case of the muscles of a limb, we are interested in the polygon  $P$  that is given by the set of all possible activations  $\mathbf{a} \in \mathbb{R}^n$  that satisfy

$$\mathbf{f} = A\mathbf{a}, \mathbf{a} \in [0, 1]^n,$$

where  $\mathbf{f} \in \mathbb{R}^m$  is a fixed force vector and  $A = J^{-T} R F_m \in \mathbb{R}^{m \times n}$ .  $P$  is bounded by the unit  $n$ -cube since all variables  $a_i$ ,  $i \in [n]$  are bounded by 0 and 1 from below, above respectively.

Consider the following  $1 \times 3$  example.

$$1 = \frac{10}{3}a_1 - \frac{53}{15}a_2 + 2a_3$$

$$a_1, a_2, a_3 \in [0, 1],$$

the set of feasible activations is given by the shaded set in figure ??.

### FIGURE 1

The Hit-and-Run walk on  $P$  is defined as follows (it works analogously for any convex body).

- (a) Find a given starting point  $\mathbf{p}$  of  $P$ .
- (b) Generate a random direction through  $\mathbf{p}$  (uniformly at random over all directions).
- (c) Choose the next point of the sampling algorithm uniformly at random from the segment of the line in  $P$ .

(d) Repeat from (b) the above steps with the new point as the starting point.

**FIGURE 2:** *Figures in 2D or 3D? Better in 3D to have the overview.*

(a) *figure inner point*

(b) *figure choice of direction*

(c) *figure line segment*

(d) *figure choice of new point*

(e) *figure of distribution after some sampling*

The implementation of this algorithm is straight forward except for the choice of the random direction. How do we sample uniformly at random (u.a.r.) from all directions in  $P$ ? Suppose that  $\mathbf{q}$  is a direction in  $P$  and  $p \in P$ . Then by definition of  $P$ ,  $\mathbf{q}$  must satisfy  $\mathbf{f} = A(\mathbf{p} + \mathbf{q})$ . Since  $\mathbf{p} \in P$ , we know that  $\mathbf{f} = A\mathbf{p}$  and therefore

$$\mathbf{f} = A(\mathbf{p} + \mathbf{q}) = \mathbf{f} + A\mathbf{q}$$

and hence

$$A\mathbf{q} = 0.$$

**FIGURE 3:**  *$q$  must be parallel to the plane given by  $1 = \frac{10}{3}a_1 - \frac{53}{15}a_2 + 2a_3$ .*

We therefore need to choose directions uniformly at random from all directions in the vectorspace

$$V = \{\mathbf{q} \in \mathbb{R}^n | A\mathbf{q} = 0\}.$$

As shown by Marsaglia this can be done as follows [4].

(a) Find an orthonormal basis  $b_1, \dots, b_r \in \mathbb{R}^n$  of  $A\mathbf{q} = 0$ .

(b) Choose  $(\lambda_1, \dots, \lambda_r) \in \mathcal{N}(0, 1)^r$  (from the Gaussian distribution).

(c)  $\sum_{i=1}^r \lambda_i b_i$  is a u.a.r. direction.

A basis of a vectorspace  $V$  is a minimal set of vectors that generate  $V$ , and it is orthonormal if the vectors are pairwise orthogonal (perpendicular) and have unit length. Using basic linear algebra one can find a basis for  $V = \{A\mathbf{q} = 0\}$  and orthogonalize it with the well known Gram-Schmidt method (for details see e.g. [2]). Note that in order to get the desired u.a.r. distribution the basis needs to be orthonormal. For the limb case we can safely assume that the rows of  $A$  are linearly independent and hence the number of basis vectors is  $n - m$ .

**FIGURE 4**

(a) *Some basis*

(b) *orthonormal basis*

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## References

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- [2] T. S. Blyth E. F. Robertson. *Basic Linear Algebra*. Springer, 2002.
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- [5] R.L. Smith. Efficient monte-carlo procedures for generating points uniformly distributed over bounded regions. *Operations Res.*, 32:1296–1308, 1984.