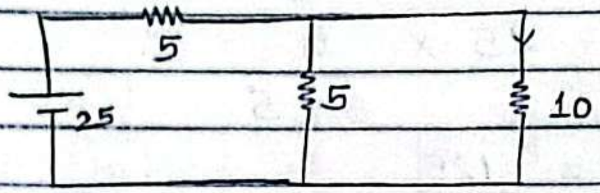
Thevenin's Theorem :

① Determine the current across 10Ω resistor using Thevenin's Theorem.

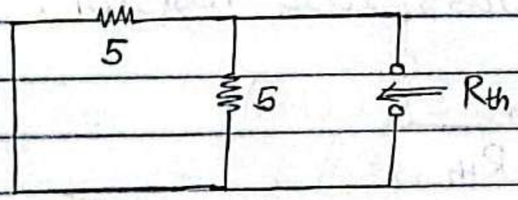


Sol: Here,

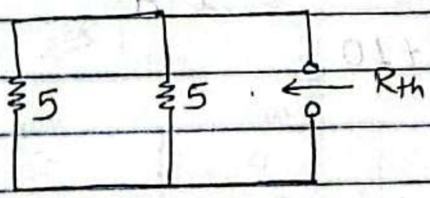
load resistance (R_L) = 10Ω

Now, to find Thevenin's eq. resistance (R_{th}):

The 10Ω resistor is open circuited and voltage source is made inactive as shown below:

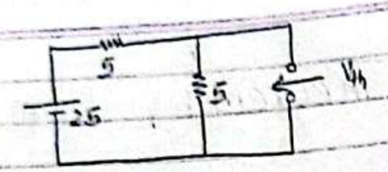


⇓



$R_{th} = 5 \parallel 5$
$= 2.5\Omega$

Now, To determine eq. voltage (V_{th}) :



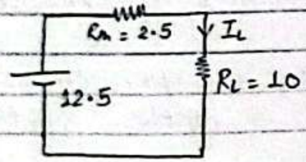
$$V_{th} = V_{5\Omega} = V \times \frac{R_1}{R_1+R_2}$$

$$= 25 \times \frac{5}{5+5}$$

$$= \frac{125}{10}$$

$$= 12.5 \text{ Volt}$$

∴ Thevenin's Equivalent circuit is :



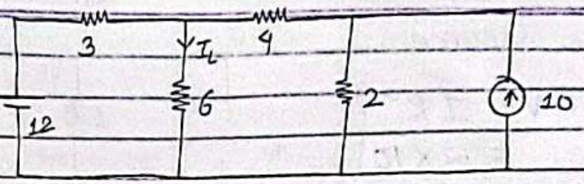
∴ current across 10Ω resistor :

$$I_L = \frac{V_{th}}{R_L + R_{th}}$$

$$= \frac{12.5}{2.5+10} = 1 \text{ A}$$

② Use Thevenin's Theorem to calculate current across 6Ω resistor.

P.T.O

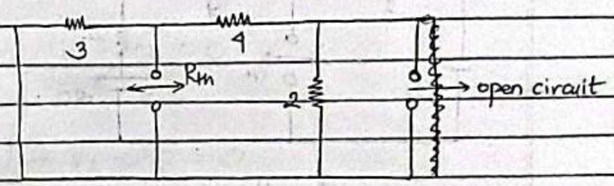


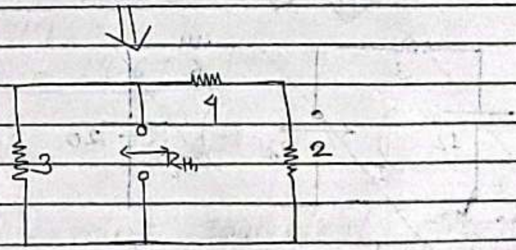
Soln :

Here,

$R_L = 6\Omega$

For R_{th} :



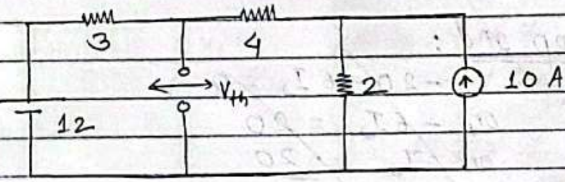


$$\therefore R_{th} = (4+2) \parallel 3$$

$$= 6 \parallel 3$$

$$= 2\Omega$$

Now, for V_{th} :



Converting the 10 A current source into

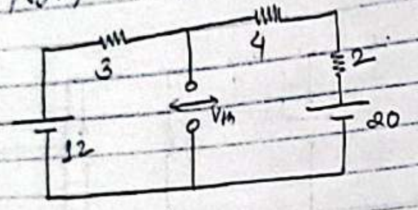
voltage source.

$\therefore V = IR$

$= 2 \times 10$

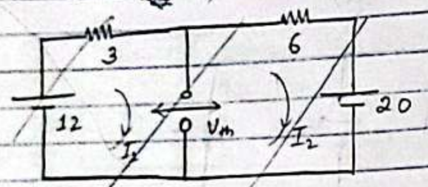
$= 20\,V$

Now, ckt becomes:



P.T.O.

$\Downarrow$



Applying KVL at:

loop 1st:

$12 - 3I_1 = 0$

or, $12 = 3I_1$

$\therefore \boxed{I_1 = 4\,A}$

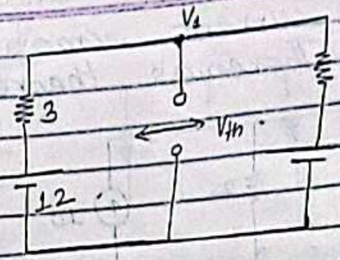
loop 2nd:

$-20 - 6I_2 = 0$

or, $-6I_2 = 20$

or, $I_2 = \frac{20}{-6}$

$\therefore \boxed{I_2 = -3.33\,A}$



Applying nodal analysis at node 1:

$$\frac{V_1 - 12}{3} + \frac{V_1 - 20}{6} = 0$$

or, $2(V_1 - 12) + V_1 - 20 = 0$

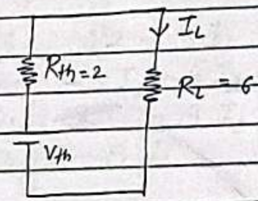
or, $2V_1 - 24 + V_1 - 20 = 0$

or, $3V_1 = 44$

$\therefore \boxed{V_1 = 14.67\,V}$

$\therefore V_1 = V_{th} = 14.67\,V$

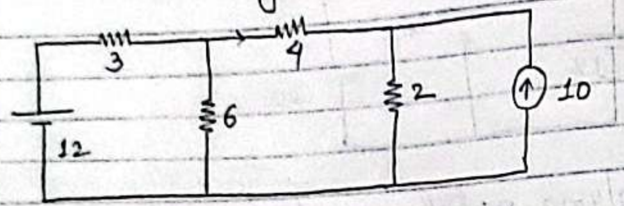
Now, Thevenin's Equivalent circuit is:



$$\therefore I_L = \frac{V_{th}}{R_L + R_{th}} = \frac{14.67}{6 + 2}$$

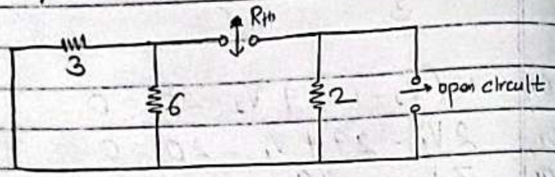
$= 1.83\,A$

(3) Determine the current across 4Ω resistor using Thevenin's theorem. (2011) (5 marks)

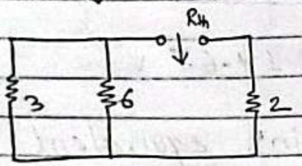


Soln: Here, $R_L = 4\Omega$

for R_{th}:

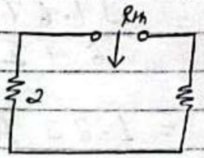


⇓



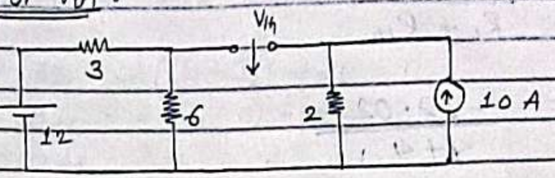
$$R_{eq} = 3 // 6 = \frac{3 \times 6}{3+6} = 2\ \Omega$$

∴ ckt becomes:



$\therefore R_{th} = 2+2 = 4\Omega$

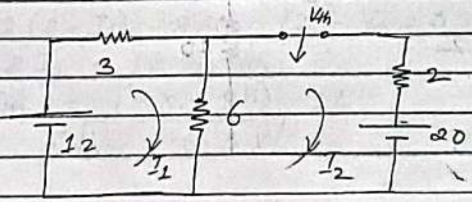
For V_{th}:



Converting 10 A current source into voltage source.

$$V = IR = 10 \times 2 = 20\ V$$

∴ ckt becomes:



Applying loop analysis at:

loop 1st: $12 - 3I_1 - 6(I_1 - I_2) = 0$

or, $-3I_1 - 6I_1 + 6I_2 = -12$

or, $-9I_1 + 6 \times 0 = -12$

or, $-9I_1 = -12$

$\therefore \boxed{I_1 = 1.33\ A}$

loop 2nd: $-6(I_2 - I_1) - V_{th} - 2I_2 - 20 = 0$

or, $-6I_2 + 6I_1 - V_{th} - 2I_2 = 20$

or, $(-6) \times 0 + 6 \times 1.33 - V_{th} - 2 \times 0 = 20$

or, $7.98 - V_{th} = 20$

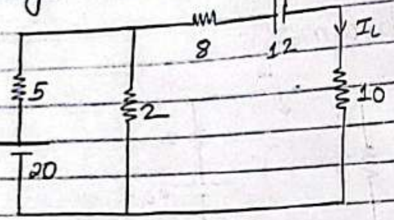
$\therefore \boxed{V_{th} = -12.02\ V}$

$$\therefore I_L = \frac{V_{th}}{R_L + R_{th}}$$

$$= \frac{-12.02}{4+4}$$

$$= -1.5 \text{ A}$$

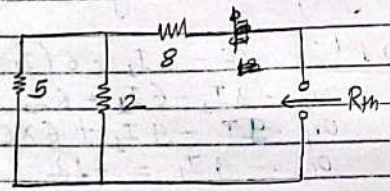
(4) Determine current across 10Ω resistor using thevenins theorem.



Soln: Here,

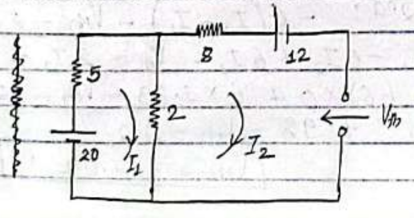
$R_L = 10\Omega$

For R_{th}:



$$R_{th} = (5 \| 2) + 8$$
$$= 9.42\ \Omega$$

For V_{th}:



Applying KVL in loop 1st:

$$-5I_1 - 2(I_1 - I_2) + 20 = 0$$

or, $-5I_1 - 2(I_1 - 0) = -20$

or, $-5I_1 - 2I_1 = -20$

or, $-7I_1 = -20$

or, $I_1 = \frac{-20}{-7}$

$$\boxed{I_1 = 2.85 \text{ A}}$$

Applying KVL in loop 2nd:

$$-2(I_2 - I_1) - 8I_2 - 12 - V_{th} = 0$$

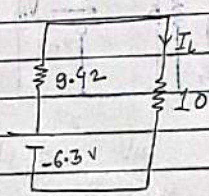
$$-2(0 - I_1) - 8 \times 0 - 12 - V_{th} = 0$$

or, $2I_1 - 12 = V_{th}$

or, $V_{th} = (2 \times 2.85) - 12$

or, $\therefore \boxed{V_{th} = -6.3 \text{ V}}$

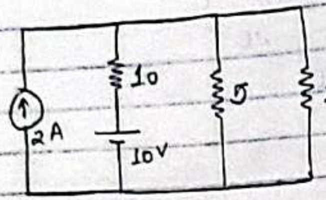
Now, Thevenin's Equivalent circuit is:



$$\therefore I_L = \frac{V_{th}}{R_L + R_{th}} = \frac{-6.3}{10 + 9.42}$$

$$= -0.32 \text{ A}$$

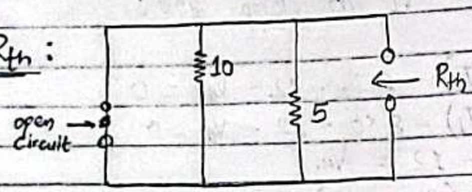
⑤ Find power loss in 1Ω resistor using Thevenin's Theorem.



Soln: Here,

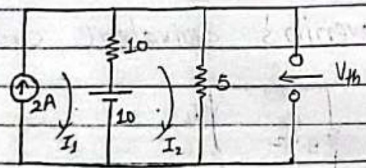
$R_L = 1\Omega$

for R_{th}:



$\therefore R_{th} = 10 \parallel 5$
$= 3.34\Omega$

for V_{th}:



$V_{th} = V_{5\Omega}$

Now,

Using mesh analysis at loop 1st:

$\boxed{I_1 = 2A}$

Using mesh analysis at loop 2nd:

$10 - 10(I_2 - I_1) - 5I_2 = 0$

or, $-10I_2 + 10I_1 - 5I_2 = -10$

or, $10I_1 - 15I_2 = -10$

or, $2I_1 - 3I_2 = \frac{-10}{5}$

or, $2I_1 - 3I_2 = -2$ $\quad [\because I_1 = 2]$

or, $2 \times 2 - 3I_2 = -2$

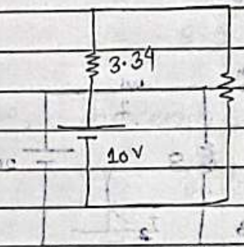
or, $-3I_2 = -2 - 4$

or, $I_2 = \frac{-6}{-3}$

$\therefore \boxed{I_2 = 2}$

$\therefore V_{th} = V_{5\Omega}$
$= I_2 \times R$
$= 2 \times 5$
$= 10\ V$

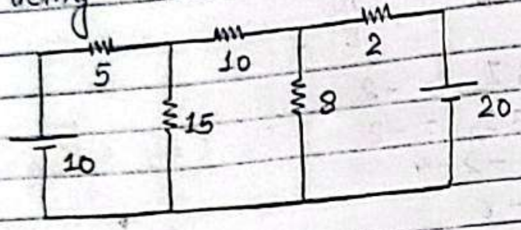
Now, Thevenin's Eq. theorem:



$\therefore I_L = \frac{V_{th}}{R_L + R_{th}} = \frac{10}{3.34 + 1}$
$= 2.3\ A$

$\therefore$ power in $1\Omega = (I_L)^2 \times R$
$= (2.3)^2 \times 1$
$= 5.3$ watt

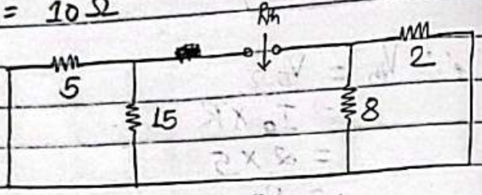
⑥ Determine current across 10Ω resistor using Thevenin's Theorem.



Soln: Here,

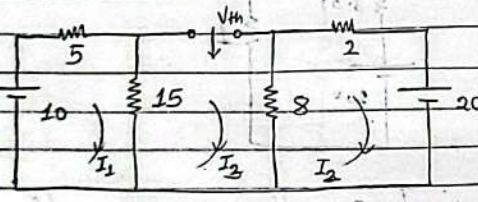
$R_L = 10\,\Omega$

For R_{th}:



$\therefore R_{th} = (5 // 15) + (8 // 2)$
$= 3.75 + 1.6$
$= 5.35\,\Omega$

For V_{th}:



Applying mesh analysis at:

loop ①: $10 - 5I_1 - 15(I_1 - I_3) = 0$

or, $-5I_1 - 15I_1 + 15I_3 = -10$

or, $-20I_1 + 15I_3 = -10$

or, $-5(4I_1 - 3I_3) = -10$

or, $4I_1 - 3I_3 = 2 \longrightarrow$ ②

or, $4I_1 - 3\times 0 = 2$

or, $I_1 = 2/4$ $\therefore \boxed{I_1 = 0.5\,A}$

loop (ii): $-15(I_3 - I_1) - V_{th} - 8(I_3 - I_2) = 0$

or, $-15I_3 + 15I_1 - V_{th} - 8I_3 + 8I_2 = 0$

or, $15I_1 - 23I_3 + 8I_2 = V_{th}$

or, $15I_1 + 8I_2 - 23I_3 = V_{th}$

or, $15\times 0.5 + 8I_2 - 23\times 0 = V_{th}$

or, $7.5 + 8I_2 = V_{th}$

or, $V_{th} = 7.5 + 8I_2 \longrightarrow$ ①

loop (iii): $-20 - 8(I_2 - I_3) - 2I_2 = 0$

or, $-20 - 8(I_2 - 0) - 2I_2 = 0$

or, $-8I_2 - 2I_2 = 20$

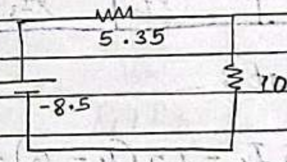
or, $-10I_2 = 20$

$\therefore \boxed{I_2 = -2\,A}$

Now, from eqn ①:

$V_{th} = 7.5 + 8\times(-2)$
$= 7.5 - 16$
$= -8.5\,V$

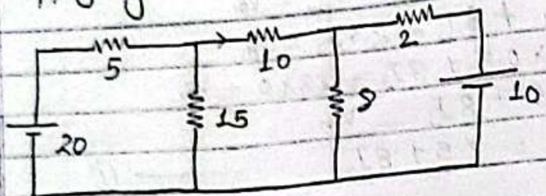
Now, Thevenin's Eq. ckt is:



$\therefore I_L = \dfrac{V_{th}}{R_L + R_{th}} = \dfrac{-8.5}{10 + 5.35}$

$\boxed{= -0.55\,A}$

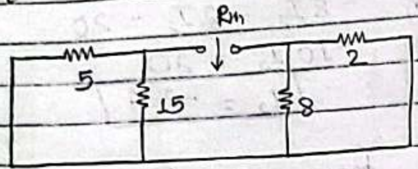
7) Calculate the current in 10Ω resistor applying Thevenin's Theorem.



Soln: Here,

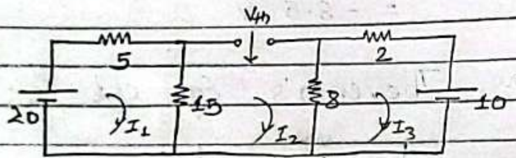
$R_L = 10\Omega$

For R_{th}:



$\therefore R_{th} = (5 // 15) + (8 // 2)$
$= 3.75 + 1.6$
$= 5.35\Omega$

For V_{th}:



Applying KVL at:

loop 1st: $20 - 5I_1 - 15(I_1 - I_2) = 0$

or, $-5I_1 - 15(I_1 - 0) = -20$

or, $-5I_1 - 15I_1 = -20$

or, $-20I_1 = -20$

$\therefore \boxed{I_1 = 1\,A}$

loop 2nd: $-15(I_2 - I_1) - V_{th} - 8(I_2 - I_3) = 0$

or, $-15(0 - I_1) - V_{th} - 8(0 - I_3) = 0$

or, $15I_1 - V_{th} + 8I_3 = 0$

or, $15\times 1 - V_{th} + 8I_3 = 0$

or, $15 + 8I_3 = V_{th}$

$\therefore V_{th} = 15 + 8I_3 \longrightarrow (A)$

loop 3rd: $-10 - 8(I_3 - I_2) - 2I_3 = 0$

or, $-8(I_3 - 0) - 2I_3 = 10$

or, $-8I_3 - 2I_3 = 10$

or, $-10I_3 = 10$

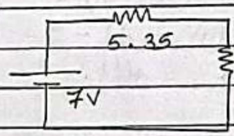
$\boxed{I_3 = -1\,A}$

$\therefore$ Equn (A) becomes:

$V_{th} = 15 + 8\times(-1)$
$= 15 - 8$

$\therefore \boxed{V_{th} = 7V}$

Now, Thevenin's Eq. ckt is:

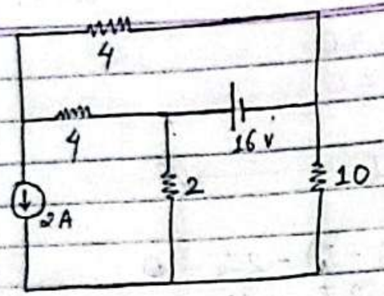


$\therefore I_L = \dfrac{V_{th}}{R_L + R_{th}} = \dfrac{7}{10 + 5.35}$

$= 0.45\,A$

8) Apply Thevenin's theorem to find the current flowing through 10Ω resistor.

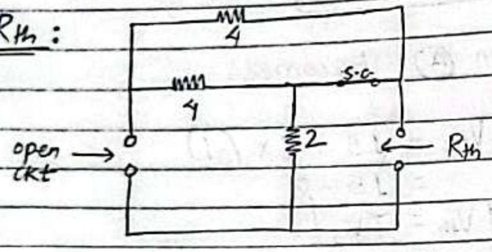
P.T.O



Soln: Here,

$R_L = 10\,\Omega$

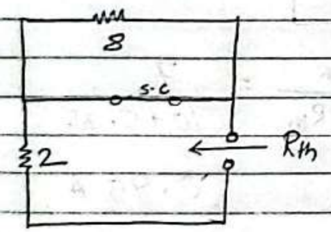
for R_{th}:



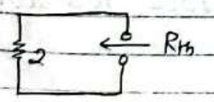
Since, 4Ω & 4Ω are in series then,

$Req = 4+4$
$= 8\,\Omega$

Now, ckt becomes:

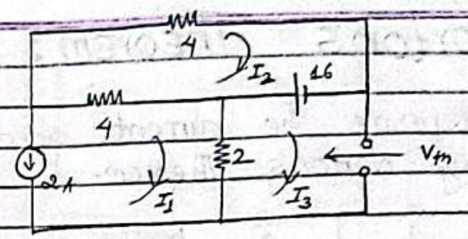


$\because$ Resistor parallel to short circuit is zero, then ckt becomes:



$\therefore R_{th} = 2\,\Omega$

for V_{th}:



Applying KVL at:

loop 1st: $I_1 = -2\,A$

loop 2nd: $-4(I_2 - I_1) - 4I_2 + 16 = 0$

or, $-4(I_2 + 2) - 4I_2 + 16 = 0$

or, $-4I_2 - 8 - 4I_2 = -16$

or, $-8I_2 = -16 + 8$

or, $-8I_2 = -8$

$\therefore \boxed{I_2 = 1\,A}$

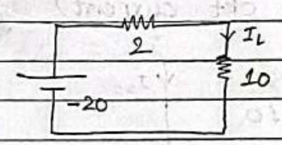
loop 3rd: $-2(I_3 - I_1) - 16 - V_{th} = 0$

or, $-2(0+2) - 16 - V_{th} = 0$

or, $-4 - 16 - V_{th} = 0$

$\therefore \boxed{V_{th} = -20\,V}$

Now, Thevenin's Eq. circuit is:



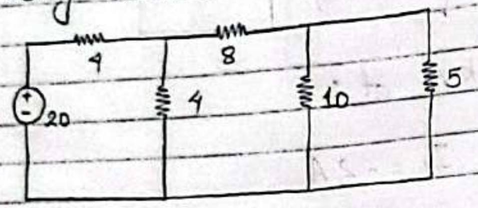
$\therefore I_L = \dfrac{V_{th}}{R_L + R_{th}} = \dfrac{-20}{2+10}$

$= -1.67\,A$

I_{sc} is equal to the current [illegible] terminal & R_0 is [illegible] open ckt terminal [illegible] circuit placed across the terminal [illegible] resistance of the n/w when viewed from these [illegible] after all voltage & current source are removed & [illegible]

Norton's Theorem:

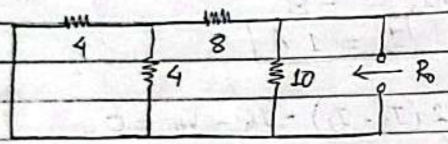
① Determine the current across 5Ω resistor using Norton's Theorem.



Sol: Here,

$R_L = 5\Omega$

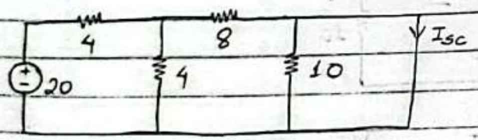
For R_0: (open ckt eq resistance)



$\therefore R_0 = [(4 \| 4) + 8] \| 10$

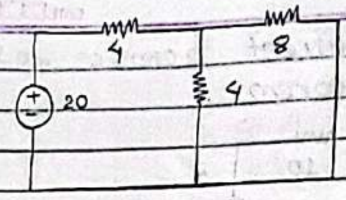
$= 5\Omega$

For I_{sc}: (short ckt current)



Since, 10Ω becomes zero due to parallel to short circuit.

Now, ckt becomes:



$\therefore R_{eq} = (8 \| 4) + 4$

$= 6.67\ \Omega$

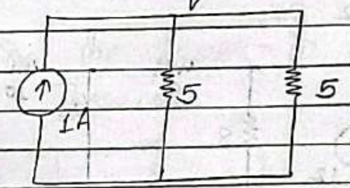
$\therefore I = \dfrac{20}{6.67} = 3\ A$

Since, I_{sc} = Current across 8Ω resistor ($I_{8\Omega}$)

$\therefore I_{sc} = 3 \times \dfrac{4}{8+4}$

$= 1\ A$

Now, Norton's Eq. ckt is:

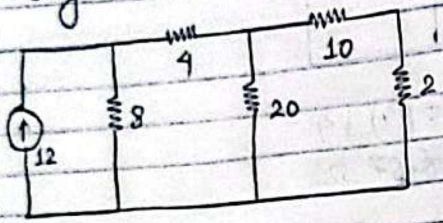


$\therefore I_L = I \times \dfrac{R_2}{R_1 + R_2}$

$= 1 \times \dfrac{5}{5+5}$

$= 0.5\ A$

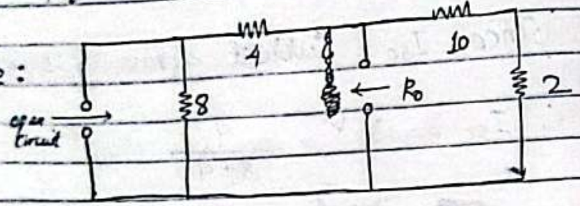
② Determine the current across 20Ω resistor using Norton's Theorem.



Soln: Here,

$R_L = 20\Omega$

for R_0:

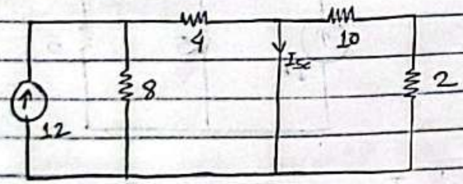


$\therefore R_0 = \cancel{(8\|4)+(1}\cdot(8+4)\,\|\,(10+2)$

$= 12 \,\|\, 12$

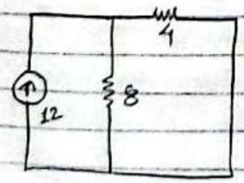
$= 6\ \Omega$

For I_{sc}:



Since, 10Ω & 2Ω becomes zero due to parallel to short circuit.

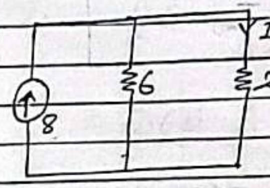
Then, ckt becomes:



$\therefore I_{sc} = I_{4\Omega} = 12 \times \dfrac{8}{8+4}$

$= 8\ A$

Now, Norton's Eq. ckt:

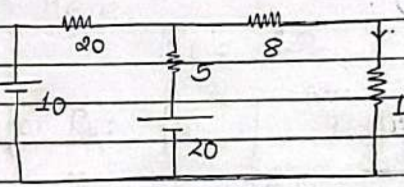


$\therefore I_L = I \times \dfrac{R_2}{R_1+R_2}$

$= 8 \times \dfrac{6}{6+20}$

$= 1.85\ A$

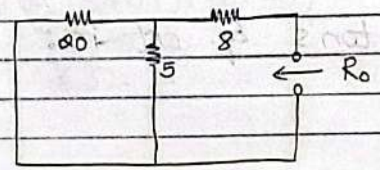
③ Determine the power in 15Ω resistor using Norton's Theorem.



Soln: Here,

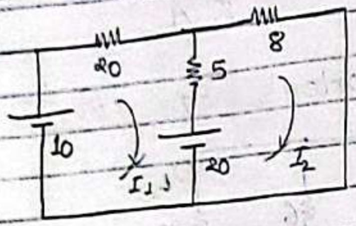
$R_L = 15\Omega$

For R_0:



$R_0 = (20 \parallel 5) + 8$
$= 12\,\Omega$

for I_{sc}:



Applying KVL at:

loop 1st: $10 - 20I_1 - 5(I_1 - I_2) - 20 = 0$

or, $-20I_1 - 5I_1 + 5I_2 = 20 - 10$

or, $-25I_1 + 5I_2 = 10$

or, $-5(5I_1 - I_2) = 10$

or, $5I_1 - I_2 = -2 \longrightarrow$ (i)

loop 2nd: ~~$20 - 5I_2 - 8I_2 = 0$~~

or, $-5(I_2 - I_1) - 8I_2 + 20 = 0$

or, $-5I_2 + 5I_1 - 8I_2 = -20$

or, $5I_1 - 13I_2 = -20 \longrightarrow$ (ii)

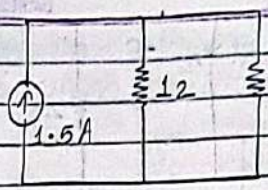
Solving (i) & (ii):

$$\begin{aligned} \cancel{5I_1} - I_2 &= -2 \\ \cancel{5I_1} - 13I_2 &= -20 \\ - \quad + \quad & \quad + \\ \hline 12I_2 &= 18 \end{aligned}$$

$\therefore \boxed{I_2 = 1.5\text{ A}}$

Now, Norton's Eq. ckt is:

P.T.O



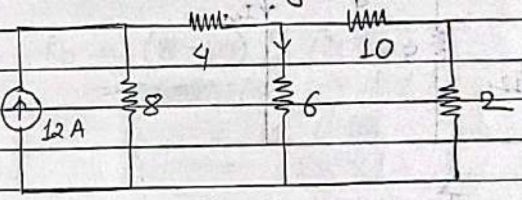
$\therefore I_L = 1.5 \times \dfrac{12}{12+15}$

$= 0.65$ A

$\therefore P_{15\Omega} = (0.65)^2 \times 15$
$= 6.33$ watt

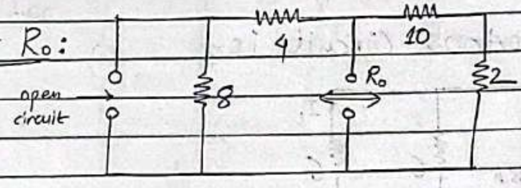
(4) Determine the current across 6Ω resistor in the given circuit:

2006 (8 marks)



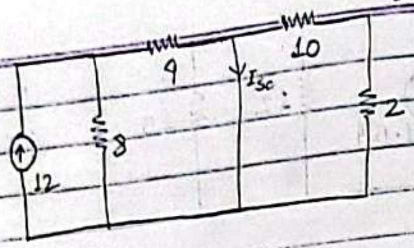
Soln: Here,
$R_L = 6\Omega$

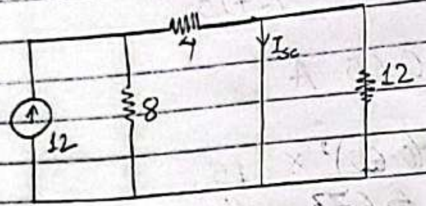
For R_0:



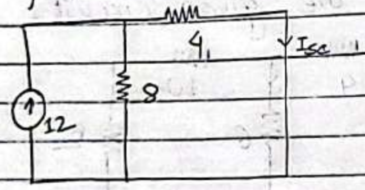
$\therefore R_0 = (8+4) \parallel (10+2)$
$= 12 \parallel 12$
$= 6\,\Omega$

For I_{sc}:





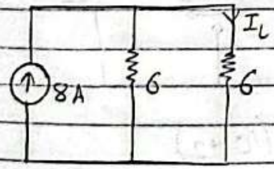
$\because$ 12Ω resistor is parallel to short circuit so it is zero. Therefore, ckt becomes:



$\therefore I_{sc} = I_{4\Omega} = 12 \times \dfrac{8}{4+8}$

$= 8\ A$

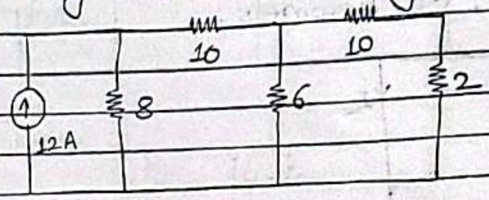
Now, Norton's Circuit is:



$\therefore I_L = 8 \times \dfrac{6}{6+6} = 4\ A$

(5) Determine the power in 6Ω resistor in the given circuit using Norton's Theorem.

2007 (8 marks)

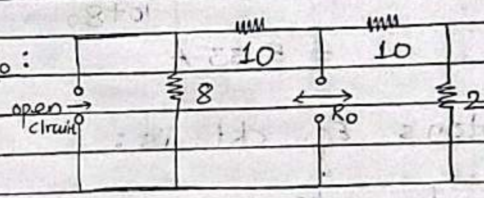


Soln.

Here,

$R_L = 6\Omega$

For R_o:



$R_o = (8+10) \,//\, (10+2)$

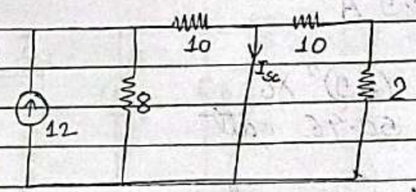
$= (8+10) \,//\, 12$

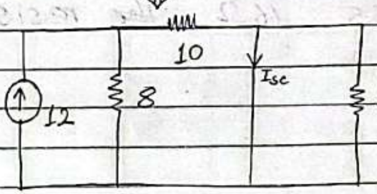
$= 18 \,//\, 12$

$= \dfrac{18 \times 12}{18+12}$

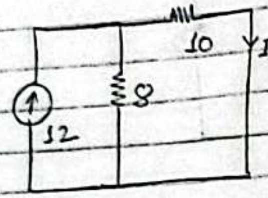
$= 7.2\ \Omega$

For I_{sc}:





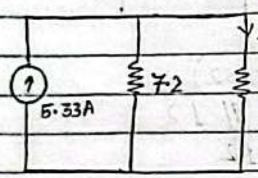
$\because$ Short circuit & 12Ω resistor are parallel
So it is zero.
Then, ckt becomes:



$\therefore I_{sc} = I_{10\Omega} = 12 \times \frac{8}{10+8}$

$= 5.33\ A$

Now, Norton's Eq. ckt is:



$\therefore I_L = 5.33 \times \frac{7.2}{7.2+6}$

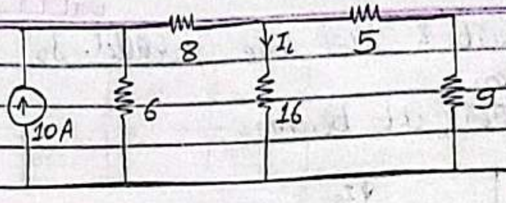
$= 2.9\ A$

$\therefore P_{6\Omega} = (2.9)^2 \times 6$
$= 50.46$ watt

⑥ Using Norton's theorem, calculate the voltage across 16Ω ~~the~~ resistor.

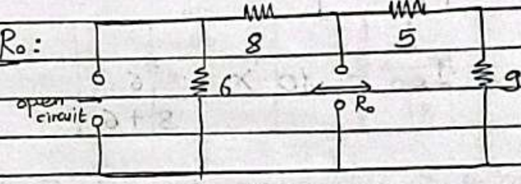
(2008) (Spring) 8 marks

P.T.O

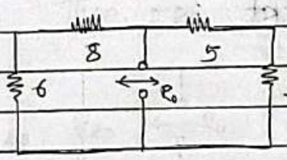


Soln: Here,
$R_L = 16\Omega$

For R_o:

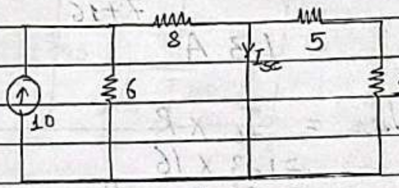


⇓

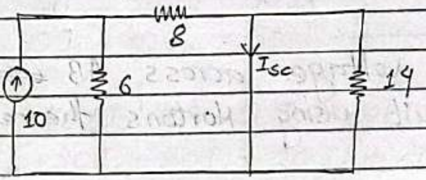


$\therefore R_o = (6+8) \parallel (5+9)$
$= 14 \parallel 14$
$= 7\ \Omega$

Now, For I_{sc}:

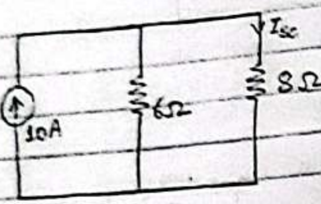


⇓



∵ Short circuit & 14Ω are parallel. So 14Ω becomes zero.

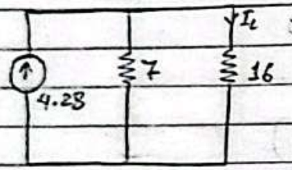
Then, ckt becomes:



$\therefore I_{sc} = I_{8\Omega} = 10 \times \frac{6}{8+6}$

$= 4.28$ A

Now, Norton's Eq. ckt is:



$\therefore I_L = 4.28 \times \frac{7}{7+16}$

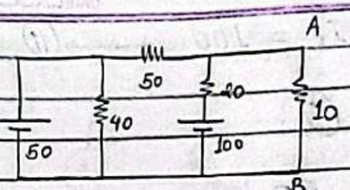
$= 1.3$ A

$\therefore V_{16\Omega} = I_L \times R$

$= 1.3 \times 16$

$= 20.8$ volt

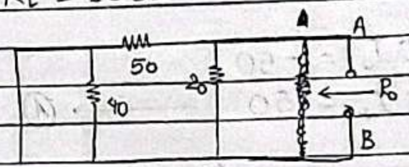
⑦ find voltage across AB ~~~~ in the given circuit using Norton's theorem.



Soln: Here,

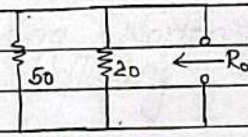
$R_L = 10\Omega$

For R_o:



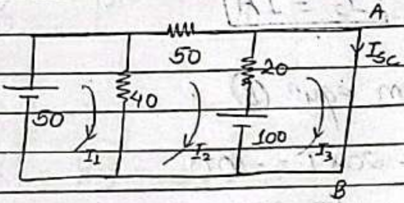
∵ 40Ω becomes zero due to parallel to short circuit.

Then, ckt becomes:



$\therefore R_o = 20 \| 50 = 14.28\ \Omega$

For I_{sc}:



At loop 1st: $50 - 40(I_1 - I_2) = 0$

or, $50 = 40 I_1 - 40 I_2$

or, $40 I_1 - 40 I_2 = 50 \longrightarrow (1)$

At loop 2nd: $-40(I_2 - I_1) - 50 I_2 - 20(I_2 - I_3) - 100 = 0$

or, $-40 I_2 + 40 I_1 - 50 I_2 - 20 I_2 + 20 I_3 = 100$

or, $40I_1 - 110I_2 + 20I_3 = 100 \longrightarrow$ (ii)

Solving equ^n (i) & (ii):

$$40I_1 - 40I_2 = 50$$
$$20I_3 + 40I_1 - 110I_2 = 100$$
$$- \quad - \quad + \quad -$$

$-20I_3 + 70I_2 = -50 \longrightarrow$ (A)

or, $20I_3 - 70I_2 = 50$

At loop 3rd: $100 - 20(I_3 - I_2) = 0$

or, $100 - 20I_3 + 20I_2 = 0$

or, $-20I_3 + 20I_2 = -100 \longrightarrow$ (B)

Solving equ^n (A) & (B)

$$20I_3 - 70I_2 = 50$$
$$-20I_3 + 20I_2 = -100$$
$$-50I_2 = -50$$

$\therefore \boxed{I_2 = 1A}$

New, from equ^n (B):

$-20I_3 + 20 \times 1 = -100$

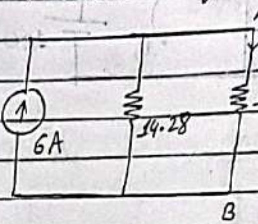
Or, $-20I_3 = -100 - 20$

Or, $-20I_3 = -120$

$\therefore \boxed{I_3 = 6A}$

$\therefore \boxed{I_{sc} = 6A}$

Now, Norton's Eq. ckt is:



$$I_L = 6 \times \frac{14.28}{10 + 14.28} = 3.52\ A$$

$\therefore$ Voltage across 10Ω is:

$V_{10\Omega} = 3.52 \times 10$

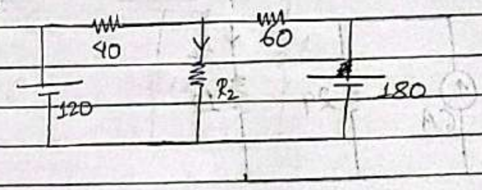
$= 35.2$ Volt Ans

(8) Using Norton's theorem, find current flowing in R_2 when

(i) $R_2 = 12$

(ii) $R_2 = 24$

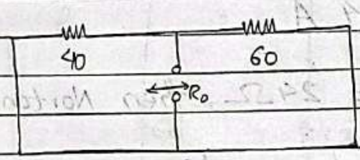
(iii) $R_2 = 36$



Soln:

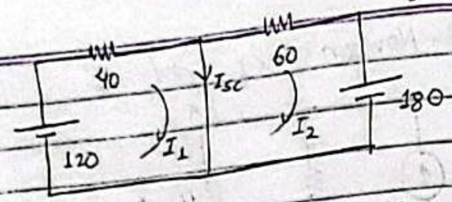
(i) Here, $R_L = R_2 = 12\Omega$.

For R_0:



$$R_0 = 60 \| 40 = 24\Omega$$

For I_{sc}:



Applying KVL at:

loop 1st: $120 - 40I_1 = 0$

or, $\frac{120}{40} = I_1$

$\therefore \boxed{I_1 = 3A}$

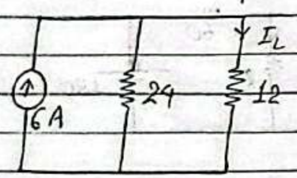
loop 2nd: $-60I_2 - 180 = 0$

or, $-180 = 60I_2$

$\therefore \boxed{I_2 = -3A}$

$\therefore I_{sc} = I_1 - I_2 = 3 - (-3) = 6A$

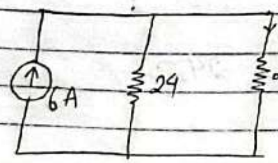
Now, Norton's Eq. ckt is:



$\therefore I_L = 6 \times \frac{24}{12+24}$

$= 4A$

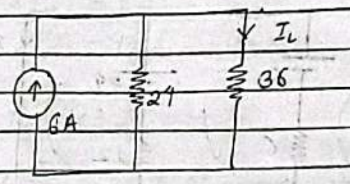
(ii) When $R_L = 24\Omega$, then Norton's ckt is:



$\therefore I_L = 6 \times \frac{24}{24+24}$

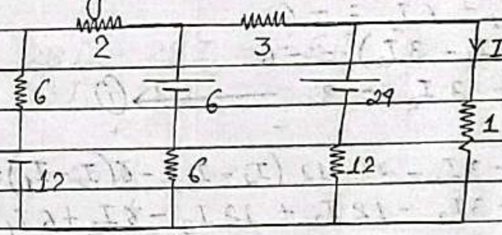
$= 3A$

(iii) When $R_L = 36$, then Norton's Eq. ckt is:



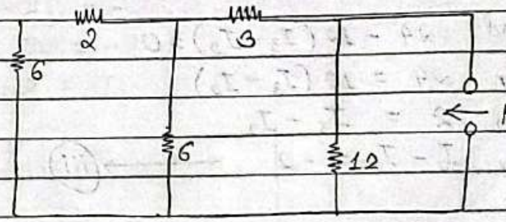
$\therefore I_L = 6 \times \frac{24}{24+36} = 2.4A$

(9) Determine current across 1Ω resistor using Norton's Theorem in the given ckt.



Soln: Here, $R_L = 1\Omega$

For R_o:



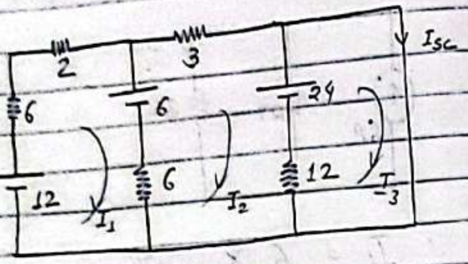
$\therefore R_o = [\{(6+2) \| 6\} + 3] \| 12$

$= [(8 \| 6) + 3] \| 12$

$= (3.42 + 3) \| 12$

$= 6.42 \| 12 = 4.18\ \Omega$

For I_{sc}:



Applying KVL at:

loop 1st: $12 - 6I_1 - 2I_1 - 6(I_1 - I_2) - 6 = 0$

or, $-8I_1 - 6I_1 + 6I_2 = -12 + 6$

or, $-14I_1 + 6I_2 = -6$

or, $-2(7I_1 - 3I_2) = -6$

or, $7I_1 - 3I_2 = 3 \longrightarrow$ (i)

loop 2nd: $6 - 3I_2 - 24 - 12(I_2 - I_3) - 6(I_2 - I_1) = 0$

or, $-18 - 3I_2 - 12I_2 + 12I_3 - 6I_2 + 6I_1 = 0$

or, $6I_1 - 21I_2 + 12I_3 = 18$

or, $3(2I_1 - 7I_2 + 4I_3) = 18$

or, $2I_1 - 7I_2 + 4I_3 = 6 \longrightarrow$ (ii)

loop 3rd: $24 - 12(I_3 - I_2) = 0$

or, $24 = 12(I_3 - I_2)$

or, $2 = I_3 - I_2$

or, $I_2 - I_3 = -2 \longrightarrow$ (iii)

Solving equn (i) & (ii)

$7I_1 - 3I_2 = 3 \longrightarrow$ (i)

$4I_3 + 2I_1 - 7I_2 = 6 \longrightarrow$ (ii)

Multiplying (i) by 2 & (ii) by 7 & solving them:

$14I_1 - 6I_2 = 6$

$28I_3 + 14I_1 - 49I_2 = 42$

$-\quad +\quad -$

$-28I_3 + 43I_2 = -36$

$43I_2 - 28I_3 = -36 \longrightarrow$ (iv)

Solving (iv) & (iii):

$[I_2 - I_3 = -2 \longrightarrow \text{(iii)}] \times 28$

$43I_2 - 28I_3 = -36 \longrightarrow$ (iv)

$28I_2 - \cancel{28I_3} = -56$

$43I_2 - \cancel{28I_3} = -36$

$-\quad +\quad +$

$-15I_2 = -20$

$\therefore \boxed{I_2 = 1.33A}$

From equ^n (iii):

$I_2 - I_3 = -2$

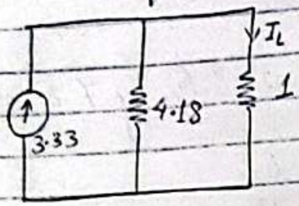
or, $1.33 - I_3 = -2$

or, $1.33 + 2 = I_3$

$\therefore \boxed{I_3 = 3.33 A}$

$\because I_{sc} = I_3 = 3.33 A$

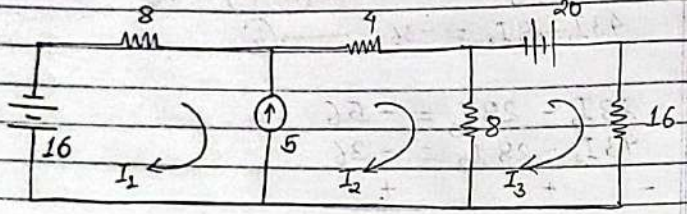
Now, Norton's Eq. ckt is:



$\therefore I_L = 3.33 \times \frac{4.18}{4.18+1}$

$= 2.68$ A

(10) Use the concept of supermesh to find I_1, I_2 and I_3.



Soln: Supermesh equation:

$I_2 - I_1 = 5$ ---------- (i)

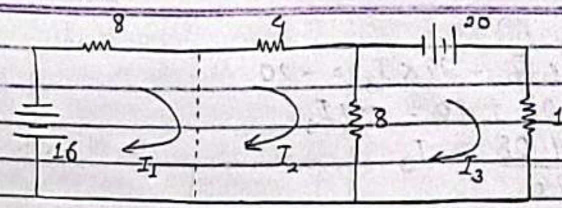
At loop 3rd: (KVL apply)

$-8(I_3 - I_2) + 20 - 16I_3 = 0$

or, $-8I_3 + 8I_2 - 16I_3 = -20$

or, $8I_2 - 24I_3 = -20$ ---------- (ii)

Applying KVL at supermesh:



$-16 - 8I_1 - 4I_2 - 8(I_2 - I_3) = 0$

or, $-8I_1 - 4I_2 - 8I_2 + 8I_3 = 16$

or, $-8I_1 - 12I_2 + 8I_3 = 16$ ---------- (iii)

Solving equn (i) & (iii):

$I_2 - I_1 = 5$] $\times 8 \Rightarrow -8I_1 + 8I_2 = 40$

$-8I_1 - 12I_2 + 8I_3 = 16 \Rightarrow -8I_1 - 12I_2 + 8I_3 = 16$

$+ \quad + \quad - \quad -$

$20I_2 - 8I_3 = 24$ ---------- (iv)

Solving (ii) & (iv):

$8I_2 - 24I_3 = -20 \Rightarrow 8I_2 - 24I_3 = -20$

$20I_2 - 8I_3 = 24$] $\times 3 \Rightarrow 60I_2 - 24I_3 = 72$

$- \quad + \quad -$

$-52I_2 = -92$

$I_2 = \frac{-92}{-52}$

$\therefore I_2 = 1.76$ A

Now,

from equn (i):

$1.76 - I_1 = 5$

$1.76 - 5 = I_1$

$\therefore I_1 = -3.24$ A

from equn (ii):

$8 \times 1.76 - 24 \times I_3 = -20$

or, $14.08 + 20 = 24 I_3$

or, $\frac{34.08}{24} = I_3$

$\therefore I_3 = 1.42$ A

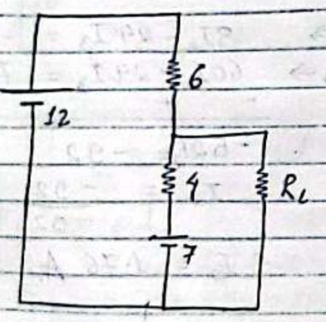
Hence, $I_1 = 1.76$ A

$I_2 = -3.29$ A

$I_3 = 1.42$ A

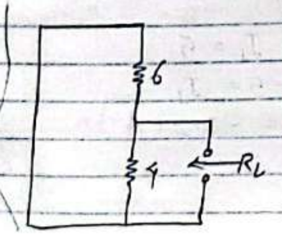
Maximum Power Transfer Theorem:

(1) Find the value of R_L so that it transfer maximum power and also find the maximum power.



Soln:

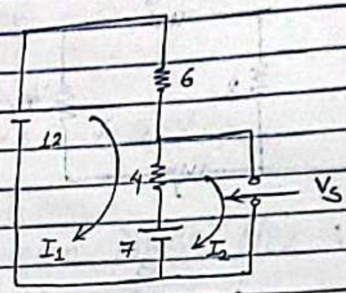
for R_L:



$\therefore R_L = 6 \| 4$

$= 2.4 \Omega$

for V_S:



At loop (I):

$12 - 6I_1 - 4(I_1 - I_2) - 7 = 0$

or, $12 - 6I_1 - 4I_1 - 7 = 0$

or, $5 - 10 I_1 = 0$

or, $5 = 10 I_1$

$\therefore \boxed{I_1 = 0.5 \text{ A}}$

At loop (II):

$-4(I_2 - I_1) - V_S + 7 = 0$

or, $4 I_1 + 7 = V_S$

or, $4 \times 0.5 + 7 = V_S$

$\therefore \boxed{V_S = 9 \text{ V}}$

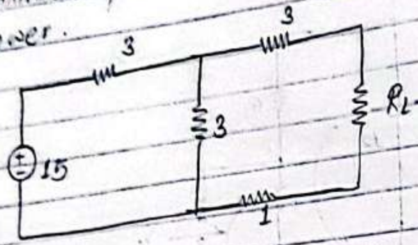
Maximum power, $P_L = \frac{V_S^2}{4 R_L}$

$= \frac{(9)^2}{4 \times 2.4}$

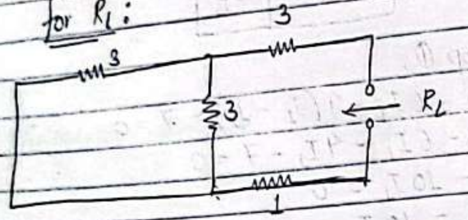
$= \frac{81}{9.6}$

$= 8.43$ watt

(2) Find the value of R_L so that it transfers maximum power and also find the maximum power.

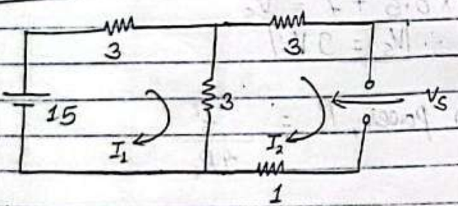


Soln: for R_L:



$$R_L = (3 \parallel 3) + 3 + 1$$
$$= 1.5 + 4$$
$$= 5.5\ \Omega$$

for V_s:



Applying KVL at loop 1st:

$$15 - 3I_1 - 3(I_1 - I_2) = 0$$
or, $15 - 3I_1 - 3I_1 = 0$
or, $15 = 6I_1$
$\therefore I_1 = 2.5\ A$

Applying KVL at loop 2nd:

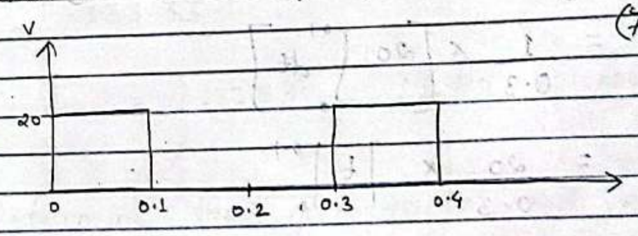
$$-3(I_2 - I_1) - 3I_2 - V_S - I_2 = 0$$
or, $3I_1 - 0 - V_S - 0 = 0$
or, $3I_1 = V_S$
or, $3 \times 2.5 = V_S$
$\therefore V_S = 7.5\ V$

Now,

$$\text{Maximum Power } (P_L) = \frac{V_S^2}{4R_L}$$
$$= \frac{(7.5)^2}{4 \times 5.5}$$
$$= 2.55 \text{ watt}$$

Single Phase AC Circuit Analysis:

(1) Determine the average value and effective value of the square voltage wave as shown below: [2006/2009 (spring)] (7 marks)



P.T.O

$V_m = V(t) = 20$

Time period of given wave $(T) = 0.3$

Now, Average value of voltage is given by:

$$V_{av} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{0.3}\int_0^{0.3} v(t)\,dt$$

$$= \frac{1}{0.3}\left[\int_0^{0.1} v_1(t)\,dt + \int_{0.1}^{0.2} v_2(t)\,dt + \int_{0.2}^{0.3} v_3(t)\,dt\right]$$

From above wave form, we have:

$v_1(t) = 20$

$v_2(t) = 0$

$v_3(t) = 0$

$$\therefore V_{av} = \frac{1}{0.3}\left[\int_0^{0.1} 20\,dt + 0 + 0\right]$$

$$= \frac{1}{0.3} \times \left[20\int_0^{0.1} dt\right]$$

$$= \frac{20}{0.3} \times \left|t\right|_0^{0.1}$$

$$= \frac{20}{0.3} \times (0.1 - 0) = 6.67\ V$$

Now,

$$V_{rms}^2 = \frac{1}{T}\int_0^T V^2(t)\,dt$$

$$= \frac{1}{0.3}\int_0^{0.3} V^2(t)\,dt$$

$$= \frac{1}{0.3}\left[\int_0^{0.1} V_1^2(t)\,dt + \int_{0.1}^{0.2} V_2^2(t)\,dt + \int_{0.2}^{0.3} V_3^2(t)\,dt\right]$$

$$= \frac{1}{0.3}\left[\int_0^{0.1} (20)^2\,dt + 0 + 0\right]$$

$$= \frac{1}{0.3} \times \left[400\int_0^{0.1} dt\right]$$

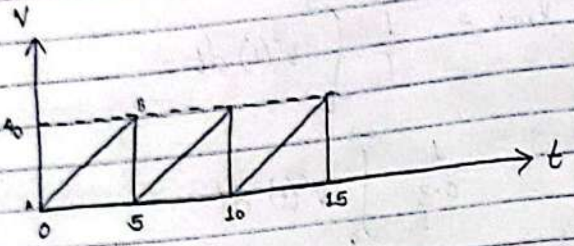
$$= \frac{400}{0.3} \times \left|t\right|_0^{0.1}$$

$$= \frac{400}{0.3} \times (0.1 - 0)$$

$$= 133.33$$

$\therefore V_{rms} = \sqrt{133.33} = 11.54\ V$ Ans

(2) Determine the form factor and peak factor of the given saw-tooth wave form:



Sol^n: Here,
Time period of the given waveform $(T) = 5$

Now, Average value of voltage is:

$$V_{av} = \frac{1}{T}\int_0^T v(t)\,dt$$

or, $V_{av} = \frac{1}{5}\int_0^5 v(t)\,dt$ ——— (i)

$v(t)$ at $(0-5)$ is: $A(0,0)$, $B(5,40)$

Using two point formula:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

or, $y - 0 = \frac{40-0}{5-0}(x-0)$

or, $y = 8(x-0)$

or, $y = 8x$

Since, $y = V$ & $x = t$

$\therefore v(t) = 8t$

putting value of $v(t)$ in equn (i):

$$V_{av} = \frac{1}{5}\int_0^5 8t\,dt$$

$$= \frac{8}{5} \times \int_0^5 t\,dt$$

$$= \frac{8}{5} \times \left|\frac{t^2}{2}\right|_0^5$$

$$= \frac{8}{5} \times \frac{1}{2} \times (25-0)$$

$\therefore V_{av} = 20\ V$

Now, $V_{rms}^2 = \frac{1}{T}\int_0^T V^2(t)\,dt$

$$= \frac{1}{5}\int_0^5 (8t)^2\,dt$$

$$= \frac{1}{5}\int_0^5 64t^2\,dt$$

$$= \frac{64}{5} \times \left|\frac{t^3}{3}\right|_0^5$$

$$= \frac{64}{5} \times \frac{1}{3}(125-0)$$

$V_{rms}^2 = 533.33 \Rightarrow V_{rms} = 23.09\ V$

Now, form factor $(K_f) = \frac{\text{Rms value.}}{\text{Average value}}$

$= \frac{23.09}{20}$

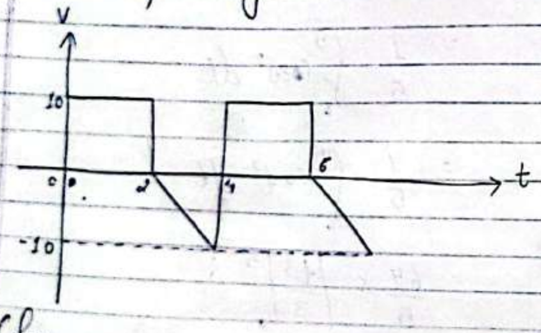
$= 1.15$

And, peak fact $(K_a) = \frac{\text{Maximum value}}{\text{Rms value}}$

$= \frac{40}{23.09}$

$= 1.73$

③ Determine the average and effective value of the given saw-tooth wave form:



Soln:

Time period of given wave form $(T) = 4$

Now, Average value of voltage is given by:

$V_{av} = \frac{1}{T}\int_0^T v(t)\,dt$

$= \frac{1}{4}\int_0^4 v(t)\,dt$

$= \frac{1}{4}\left[\int_0^2 v_1(t)\,dt + \int_2^4 v_2(t)\,dt\right]$ ----- (i)

From above waveform, we have:

$V_1(t) = \cancel{20}\ 10$

$V_2(t)$ at $(2-4)$ = $A(2,0)$, $B(4,-10)$

Using two point formula:

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

or, $y - 0 = \frac{-10-0}{4-2}\cdot(x-2)$

or, $y = -5(x-2)$

or, $y = -5x + 10$

Since, $y = v(t)$ & $x = t$.

$\therefore\ V_2(t) = -5t + 10$

Substituting the value of $v_1(t)$ & $v_2(t)$ in eqn (i):

$$V_{av} = \frac{1}{4}\left[\int_0^2 10\,dt + \int_2^4 (-5t+10)\,dt\right]$$

$$= \frac{1}{4}\left[10\int_0^2 dt + -5\int_2^4 t\,dt + 10\int_2^4 dt\right]$$

$$= \frac{1}{4}\left[10\,\Big|t\Big|_0^2 - 5\,\Big|\frac{t^2}{2}\Big|_2^4 + 10\,\Big|t\Big|_2^4\right]$$

$$= \frac{1}{4}\left[10\times(2-0) - \frac{5}{2}(16-4) + 10(4-2)\right]$$

$$= \frac{1}{4}\left[20 - 30 + 20\right]$$

$$= \frac{1}{4}\times 10$$

$$\therefore V_{av} = 2.5\ V$$

Now, $V_{rms}^2 = \frac{1}{T}\int_0^T V^2(t)\,dt$

$$= \frac{1}{4}\int_0^4 V^2(t)\,dt$$

P.T.O

$$= \frac{1}{4}\left[\int_0^2 V_1^2(t)\,dt + \int_2^4 V_2^2(t)\,dt\right]$$

$$= \frac{1}{4}\left[\int_0^2 (10)^2\,dt + \int_2^4 (-5t+10)^2\,dt\right]$$

$$= \frac{1}{4}\left[100\int_0^2 dt + \int_2^4 (5t-10)^2\,dt\right]$$

$$= \frac{1}{4}\left[100\times\Big|t\Big|_0^2 + \int_2^4 (25t^2 - 100t + 100)\,dt\right]$$

$$= \frac{1}{4}\left[100\times(2-0) + 25\int_2^4 t^2\,dt - 100\int_2^4 t\,dt + 100\int_2^4 dt\right]$$

$$= \frac{1}{4}\left[200 + 25\times\Big|\frac{t^3}{3}\Big|_2^4 - 100\times\Big|\frac{t^2}{2}\Big|_2^4 + 100\Big|t\Big|\right.$$

$$= \frac{1}{4}\left[200 + \frac{25\times(64-8)}{3} - \frac{100}{2}\times(16-4) + 100\times(4-2)\right.$$

$$= \frac{1}{4}\left[200 + \frac{1400}{3} - 600 + 200\right]$$

$$= \frac{1}{4}\left\{\cancel{200} - \cancel{\frac{1400}{3}}\right\}$$

$$\cancel{= \frac{1}{4} \times \frac{400}{3}} = \frac{800}{4 \times 3}$$

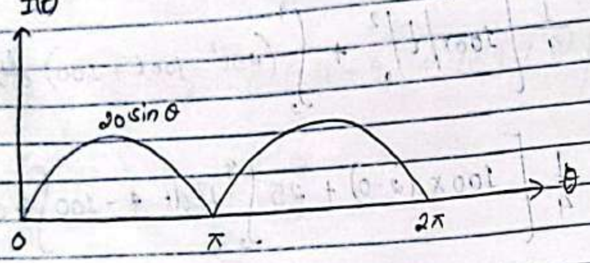
$$V_{rms}^2 = 66.67$$

$$\cancel{V_{rms}^2 = 33.33}$$

$$\therefore V_{rms} = 8.16 \text{ V}$$

$$\cancel{\therefore V_{rms} = 5.77 \text{ V}}$$

(4) Determine average & rms value of the given waveform.



Soln: Here,

Time period (T) = π

$I = 20 \sin\theta$

Now, Average value of current is:

$$I_{av} = \frac{1}{T}\int_0^T I(t)\,dt$$

$$= \frac{1}{\pi}\int_0^\pi 20\sin\theta \, d\theta$$

$$= \frac{20}{\pi}\int_0^\pi \sin\theta \, d\theta$$

$$= \frac{20}{\pi}\Big[-\cos\theta\Big]_0^\pi$$

$$= \frac{-20}{\pi}(\cos\pi - \cos 0)$$

$$= \frac{-20}{\pi} \times (-1-1)$$

$$= \frac{40}{\pi} = 12.73 \text{ A}$$

Now, $I_{rms}^2 = \frac{1}{T}\int_0^T I^2(t)\,dt$

$$= \frac{1}{\pi}\int_0^\pi (20\sin\theta)^2 d\theta$$

$$= \frac{1}{\pi} \times 400 \int_0^\pi \sin^2\theta \, d\theta$$

$$= \frac{400}{\pi} \times \frac{1}{2}\int_0^\pi 2\sin^2\theta \, d\theta$$

$$= \frac{200}{\pi}\int_0^\pi (1 - \cos 2\theta)\, d\theta$$

$$= \frac{200}{\pi}\left[\int_0^\pi d\theta - \int_0^\pi \cos 2\theta \, d\theta\right]$$

$$= \frac{200}{\pi}\left[\Big[\theta\Big]_0^\pi - \Big[\frac{\sin 2\theta}{2}\Big]_0^\pi\right]$$

$$= \frac{200}{\pi}\left[(\pi-0) - \frac{1}{2}(\sin 2\pi - \sin 0)\right]$$

$$= \frac{200}{\pi}\left[\pi - \frac{1}{2}\times 0\right]$$

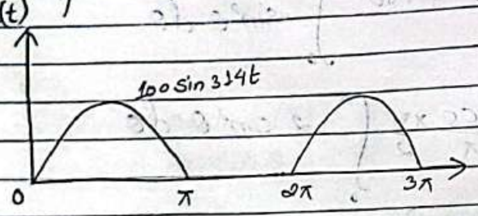
$$= \frac{200 \times \pi}{\pi}$$

$$= 200$$

$$I_{rms} = \sqrt{200}$$

$$= 14.14$$

(5) Determine the form factor of the given waveform.



Soln: Time period $(T) = 2\pi$

$$I(t) = 100\sin 314t \quad (\text{let, } 314t = \theta)$$

$$= 100\sin\theta$$

Now,

Averarge current is given by:

$$I_{av} = \frac{1}{T}\int_0^T I(t)\,dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} I(t)\,dt$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} I_1(t)\,dt + \int_{\pi}^{2\pi} I_2(t)\,dt\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} 100\sin\theta\,d\theta + 0\right]$$

$$= \frac{1}{2\pi} \times 100 \int_0^{\pi} \sin\theta\,d\theta$$

$$= \frac{1}{2\pi} \times 100 \left|-\cos\theta\right|_0^{\pi}$$

$$= \frac{-50}{\pi} \times (\cos\pi - \cos 0)$$

$$= \frac{-50}{\pi} \times (-1 - 1)$$

$$= \frac{100}{\pi}$$

$$= 31.83 \text{ A}$$

Now,

$$I_{rms}^2 = \frac{1}{T}\int_0^T I^2(t)\,dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} I^2(t)\,dt$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} I_1^2(t)\,dt + \int_{\pi}^{2\pi} I_2^2(t)\,dt\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi}(100\sin\theta)^2\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right]$$

$$= \frac{1}{2\pi} \times 10000 \int_0^{\pi} \sin^2\theta\,d\theta$$

$$= \frac{5000}{\pi}\int_0^{\pi} \frac{1}{2} \times 2\sin^2\theta\,d\theta$$

$$= \frac{2500}{\pi}\int_0^{\pi} 2\sin^2\theta\,d\theta.$$

$$= \frac{2500}{\pi}\int_0^{\pi}(1-\cos 2\theta)\,d\theta$$

$$= \frac{2500}{\pi}\left[\int_0^{\pi} d\theta - \int_0^{\pi}\cos 2\theta\,d\theta\right]$$

$$= \frac{2500}{\pi}\left[\left|\theta\right|_0^{\pi} - \left|\frac{\sin 2\theta}{2}\right|_0^{\pi}\right]$$

$$= \frac{2500}{\pi}\left[(\pi - 0) - \frac{1}{2}(\sin 2\pi - \sin 0)\right]$$

$$= \frac{2500}{\pi} \times \left[\pi - \frac{1}{2} \times 0\right]$$

$$= \frac{2500 \times \pi}{\pi}$$

$$I_{rms}^2 = 2500$$

$$I_{rms} = \sqrt{2500}$$

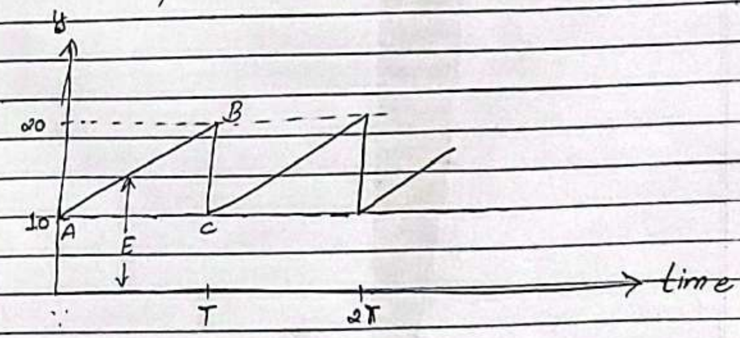
$$= 50$$

$\therefore$ form factor $(k_f) = \dfrac{\text{Rms value}}{\text{average value}}$

$$= \frac{50}{31.83}$$

$$= 1.5$$

(6) Determine the rms and average value of the waveform shown below:



Soln:

Here,

Time period = T

$$\therefore y_{av} = \frac{1}{T}\int_0^T y\,dt \quad \text{------ (1)}$$

The value of y at 0 - T can be obtained

by $A(0,10)$ & $B(T,20)$

Now, $y - 10 = \frac{20-10}{T-0}(x-0)$

or, $y - 10 = \frac{10x}{T}$

or, $y = \frac{10x}{T} + 10$

$\because x = t$ then, $y(t) = \frac{10t}{T} + 10$

from eqn (i)

$$y_{av} = \frac{1}{T}\int_0^T \left(\frac{10t}{T} + 10\right) dt$$

$$= \frac{1}{T}\left[\int_0^T \frac{10t}{T}\, dt + \int_0^T 10\, dt\right]$$

$$= \frac{1}{T}\left[\frac{10}{T}\int_0^T t\, dt + 10\int_0^T dt\right]$$

$$= \frac{1}{T}\left[\frac{10}{T}\left|\frac{t^2}{2}\right|_0^T + 10\left|t\right|_0^T\right]$$

$$= \frac{1}{T}\left[\frac{10}{2T}(T^2 - 0) + 10(T-0)\right]$$

$$= \frac{1}{T}\left[\frac{5}{T} \times T^2 + 10T\right]$$

$$= \frac{1}{T}\left[5T + 10T\right]$$

$$= \frac{1}{T} \times 15T$$

$$= 15$$

Now, $y_{rms}^2 = \frac{1}{T}\int_0^T y^2(t)\, dt$

$$= \frac{1}{T}\int_0^T \left(\frac{10t}{T} + 10\right)^2 dt$$

$$= \frac{1}{T}\int_0^T \left(\frac{100t^2}{T^2} + 2 \times \frac{10t}{T} \times 10 + 100\right) dt$$

$$= \frac{1}{T}\left[\int_0^T \frac{100t^2}{T^2} dt + \int_0^T \frac{200t}{T} dt + \int_0^T 100\, dt\right]$$

$$= \frac{1}{T}\left[\frac{100}{T^2}\int_0^T t^2\, dt + \frac{200}{T}\int_0^T t\, dt + 100\int_0^T dt\right]$$

$$= \frac{1}{T}\left[\frac{100}{T^2} \times \left|\frac{t^3}{3}\right|_0^T + \frac{200}{T}\left|\frac{t^2}{2}\right|_0^T + 100\left|t\right|_0^T\right]$$

$$= \frac{1}{T}\left[\frac{100}{T^2} \times \frac{1}{3}(T^3 - 0) + \frac{200 \times 1}{T \cdot 2}(T^2 - 0) + 100 \times T\right]$$

$$= \frac{1}{T}\left[\frac{100}{T^2}\times\frac{1}{3}\times T^3 + \frac{200}{T}\times\frac{1}{2}\times T^2 + 100\times T\right]$$

$$= \frac{1}{T}\left[\frac{100T}{3} + 100T + 100T\right]$$

$$= \frac{1}{T}\left[\frac{100T}{3} + 200T\right]$$

$$= \frac{1}{T}\times\frac{700T}{3}$$

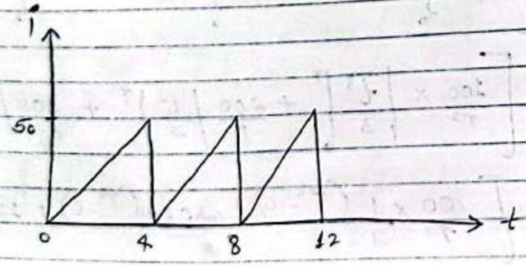
$$= \frac{700}{3}$$

$$y_{rms}^2 = 233.33$$

$$y_{rms} = \sqrt{233.33}$$

$$\therefore y_{rms} = 15.27$$

(7) find the form-factor of the wave form given below:



Soln: Here,

Time period $(T) = 4$

Now,

Average current is given by:

$$I_{av} = \frac{1}{T}\int_0^T I(t)\,dt \quad \text{(i)}$$

$$= \frac{1}{4}\int_0^4 I(t)\,dt \quad \text{------(i)}$$

$I(t)$ at $0-4$ is obtained by $A(0,0)$ & $B(4,50)$

Now, $y-0 = \frac{50-0}{4-0}(x-0)$

or, $y = \frac{50x}{4}$

$$\therefore I(t) = \frac{50t}{4}$$

Putting value of $I(t)$ in equn (i):

$$I_{av} = \frac{1}{4}\int_0^4 \frac{50t}{4}\,dt$$

$$= \frac{1}{4}\times\frac{50}{4}\int_0^4 t\,dt$$

$$= \frac{50}{16}\left[\frac{t^2}{2}\right]_0^4$$

$= \frac{50}{16} \times \frac{1}{2} (4^2 - 0)$

$= 25$ A

Now,

$I_{rms}^2 = \frac{1}{T} \int_0^T I^2(t)\, dt$

$= \frac{1}{4} \int_0^4 \left(\frac{50t}{4}\right)^2 dt$

$= \frac{2500}{4 \times 16} \int_0^4 t^2\, dt$

$= \frac{2500}{64} \times \left| \frac{t^3}{3} \right|_0^4$

$= \frac{2500}{64 \times 3} \times (4^3 - 0)$

$= \frac{2500 \times 64}{64 \times 3}$

$= 833.33$

$I_{rms} = \sqrt{833.33}$

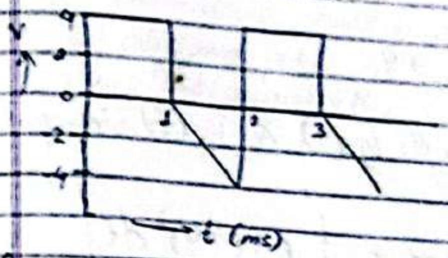
$= 28.86$ A

Now, form factor $(K_f) = \frac{\text{RMS value}}{\text{Average value}}$

$= \frac{28.86}{25} = 1.15$

⑤ Calculate the rms and average value of the voltage wave shown below:

(2010, spring) 8 marks



Soln:

Here,

Time period (T) = 2

Now,

Average value of voltage is given by:

$V_{av} = \frac{1}{T} \int_0^T v(t)\, dt$

$= \frac{1}{2} \int_0^2 v(t)\, dt$

$= \frac{1}{2} \left[\int_0^1 v_1(t)\, dt + \int_1^2 v_2(t)\, dt \right] \quad \cdots (i)$

From above given waveform:

$V_1(t) = 4$

$V_2(t)$ at 1-2 is obtained from A(1,0) & B(2,-

Then,

$y - 0 = \frac{-4-0}{2-1} (x-1)$

or $y = -4(x-1)$

or $y = -4x + 4$

$\therefore V_2(t) = -4t + 4$

putting value of $V_1(t)$ & $V_2(t)$ in eqn (1)

$$V_{rms}^2 = \frac{1}{2}\left[\int_0^1 (4)^2 dt + \int_1^2 (-4t+4)^2 dt\right]$$

$$= \frac{1}{2}\left[16\int_0^1 dt + \int_1^2 (16t^2 - 32t + 16)\,dt\right]$$

$$= \frac{1}{2}\left[16\,|t|_0^1 + \int_1^2 16t^2 dt - \int_1^2 32t\,dt + \int_1^2 16\,dt\right]$$

$$= \frac{1}{2}\left[16\times(1-0) + 16\left|\frac{t^3}{3}\right|_1^2 - 32\left|\frac{t^2}{2}\right|_1^2 + 16|t|_1^2\right]$$

$$= \frac{1}{2}\left[16 + \frac{16}{3}\times(8-1) - \frac{32}{2}\times(4-1) + 16(2-1)\right]$$

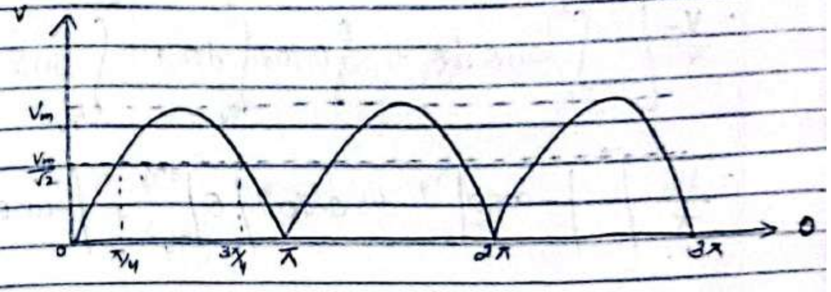
$$= \frac{1}{2}\left[16 + \frac{112}{3} - 48 + 16\right]$$

$$= \frac{1}{2}\left[32 - 48 + \frac{112}{3}\right]$$

$$= \frac{1}{2}\left(\frac{112-48}{3}\right) = \frac{64}{2\times 3} = 10.67$$

$V_{rms} = \sqrt{10.67} = 3.26$ volt

(3) A full-wave rectified sinusoidal voltage is clipped at $1/\sqrt{2}$ of its maximum value. Calculate the average and rms values of such a voltage.



Soln: Here,

Time period (T) = π

The voltage at different intervals are:

$0 < \theta < \pi/4$: $V_m \sin\theta$

$\pi/4 < \theta < 3\pi/4$: $V_m/\sqrt{2} = 0.707\,V_m$

$3\pi/4 < \theta < \pi$: $V = V_m \sin\theta$

Now,

Average voltage is given by:

$$V_{av} = \frac{1}{T}\int_0^T v(\theta)\,d\theta$$

$$= \frac{1}{\pi}\int_0^\pi v(\theta)\,d\theta$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/4} v_1(\theta)\,d\theta + \int_{\pi/4}^{3\pi/4} v_2(\theta)\,d\theta + \int_{3\pi/4}^{\pi} v_3(\theta)\,d\theta\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/4} V_m \sin\theta\,d\theta + \int_{\pi/4}^{3\pi/4} 0.707\,V_m\,d\theta + \int_{3\pi/4}^{\pi} V_m \sin\theta\,d\theta\right]$$

$$= \frac{V_m}{\pi}\left[\int_0^{\pi/4} \sin\theta\,d\theta + 0.707\int_{\pi/4}^{3\pi/4} d\theta + \int_{3\pi/4}^{\pi} \sin\theta\,d\theta\right]$$

$$= \frac{V_m}{\pi}\left[\left|-\cos\theta\right|_0^{\pi/4} + 0.707\left|\theta\right|_{\pi/4}^{3\pi/4} + \left|-\cos\theta\right|_{3\pi/4}^{\pi}\right]$$

$$= \frac{V_m}{\pi}\left[\left(-\cos\frac{\pi}{4} + \cos 0\right) + 0.707\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + \left(-\cos\pi + \cos\frac{3\pi}{4}\right)\right]$$

$$= \frac{V_m}{\pi}\left[\left(-\frac{1}{\sqrt{2}} + 1\right) + \left(0.707 \times \frac{\pi}{2}\right) + \left(1 - \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{V_m}{\pi}\left[1 + 1 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + \frac{0.707\pi}{2}\right]$$

$$= \frac{V_m}{\pi}\left[2 - \frac{2}{\sqrt{2}} + \frac{0.707\pi}{2}\right]$$

$$= \frac{V_m}{\pi}\left[2 - \sqrt{2} + \frac{0.707\pi}{2}\right]$$

$$= \frac{V_m}{\pi}\left(0.585 + 1.11\right)$$

$$= \frac{V_m \times 1.696}{\pi}$$

$$= 0.54\,V_m$$

New,

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(\theta)\,d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} v^2(\theta)\,d\theta$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/4} V_m^2 \sin^2\theta\,d\theta + \int_{\pi/4}^{3\pi/4} (0.707)^2 V_m^2\,d\theta + \int_{3\pi/4}^{\pi} V_m^2 \sin^2\theta\,d\theta\right]$$

$$= \frac{V_m^2}{\pi}\left[\int_0^{\pi/4} \sin^2\theta\,d\theta + 0.499\int_{\pi/4}^{3\pi/4} d\theta + \int_{3\pi/4}^{\pi} \sin^2\theta\,d\theta\right]$$

$$= \frac{V_m^2}{\pi}\left[\frac{1}{2}\int_0^{\pi/4} 2\sin^2\theta\,d\theta + 0.499\int_{\pi/4}^{3\pi/4} d\theta + \frac{1}{2}\int_{3\pi/4}^{\pi} 2\sin^2\theta\,d\theta\right]$$

$$= \frac{V_m^2}{\pi}\left[\frac{1}{2}\int_0^{\pi/4} (1 - \cos 2\theta)\,d\theta + 0.499\left|\theta\right|_{\pi/4}^{3\pi/4} + \frac{1}{2}\int_{3\pi/4}^{\pi} (1 - \cos 2\theta)\,d\theta\right]$$

$$= \frac{V_m^2}{\pi}\left[\frac{1}{2}\left(\int_0^{\pi/4} d\theta - \int_0^{\pi/4}\cos 2\theta\, d\theta\right) + \left(0.499 \times \frac{\pi}{2}\right)\left(\frac{3\pi}{4}-\frac{\pi}{4}\right) + \frac{1}{2}\left(\int_{3\pi/4}^{\pi} d\theta - \int_{3\pi/4}^{\pi}\cos 2\theta\, d\theta\right)\right]$$

$$= \frac{V_m^2}{\pi}\left[\frac{1}{2}\left(\left|\theta\right|_0^{\pi/4} - \left|\frac{\sin 2\theta}{2}\right|_0^{\pi/4}\right) + \left(0.499 \times \frac{\pi}{2}\right) + \frac{1}{2}\left(\left|\theta\right|_{3\pi/4}^{\pi} - \left|\frac{\sin 2\theta}{2}\right|_{3\pi/4}^{\pi}\right)\right]$$

$$= \frac{V_m^2}{\pi}\left[\frac{1}{2}\left(\left(\frac{\pi}{4}-0\right) - \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin 0\right)\right) + \frac{0.499\pi}{2} + \frac{1}{2}\left(\left(\pi - \frac{3\pi}{4}\right) - \frac{1}{2}\left(\sin 2\pi - \sin\frac{3\pi}{2}\right)\right)\right]$$

$$= \frac{V_m^2}{\pi}\left[\frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2}(1-0)\right] + \frac{0.499\pi}{2} + \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2}(0+1)\right]\right]$$

$$= \frac{V_m^2}{\pi}\left[\frac{1}{2}\times\left(\frac{\pi}{4}-\frac{1}{2}\right) + \frac{0.499\pi}{2} + \frac{1}{2}\times\left(\frac{\pi}{4}-\frac{1}{2}\right)\right]$$

$$= \frac{V_m^2}{\pi}\left[\frac{0.285}{2} + \frac{0.499\pi}{2} + \frac{0.285}{2}\right]$$

$$= \frac{V_m^2}{\pi} \times 1.06$$

$$= 0.33 V_m^2$$

$$\therefore V_{rms} = \sqrt{0.33 V_m^2}$$

$$= 0.58 V_m \quad \text{Ans}$$

(15) The resistance 20Ω, capacitor $150\mu F$, inductor $0.2H$ are connected in series with $230V, 50Hz$ supply calculate:

(i) Inductance (X_L)
(ii) Capacitance (X_C)
(iii) Impedence (Z)
(iv) Current (I)

Sol: Here,

$R = 20\Omega$
$C = 150\mu F = 150 \times 10^{-6} F$
$L = 0.2H$
$f = 50Hz$

Now,

Inductance $(X_L) = 2\pi f L$
$= 2\pi \times 50 \times 0.2$
$= 62.83$

capacitance $(X_C) = \frac{1}{2\pi f C}$
$= \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$
$= 21.22$

Impedence (z) = $R + j(X_L - X_C)$

$= 20 + j(62.83 - 21.22)$

$= 20 + j\,41.61$ ------ (i)

$I = \frac{V}{z} = \frac{230}{20 + j\,41.61}$

Now, $|z| = \sqrt{(20)^2 + (j\,41.61)^2}$

$= \sqrt{(20)^2 + (41.61)^2}$

$= 46.16$

$\theta = \tan^{-1}\left(\frac{41.61}{20}\right) = 64.3$

$\therefore z = 46.16 \angle 64.3$ (rectangular)

From equn (1):

$I = \frac{230 \angle 0}{46.16 \angle 64.3}$

$= 4.98 \angle -64.3$

Note: When $Z_1 = r_1 \angle \theta_1$

$Z_2 = r_2 \angle \theta_2$

(i) $Z_1 + Z_2 = r_1 \angle \theta_1 + r_2 \angle \theta_2$

$= (a_1 + jb_1) + (a_2 + jb_2)$

$= (a_1 + a_2) + j(b_1 + b_2)$

(ii) $Z_1 \times Z_2 = r_1 \angle \theta_1 \times r_2 \angle \theta_2$

$= (r_1 \times r_2) \angle (\theta_1 + \theta_2)$

(iii) $\frac{Z_1}{Z_2} = \frac{r_1 \angle \theta_1}{r_2 \angle \theta_2} = \frac{r_1}{r_2} \angle (\theta_1 - \theta_2)$

(11) The resistance 20Ω, inductor 0.2 H and capacitor 150 μF are connected in series with a supply voltage of 230 V, 50 Hz.

Determine (i) Inductive ~~capreactance~~ reactance
(ii) Capacitive reactance
(iii) Impedance
(iv) Admitance
(v) Current
(vi) Power factor
(vii) Active power.
(viii) Reactive power

[8 marks]

Sol: Here,

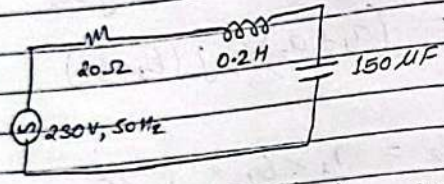
$R = 20\Omega$

$L = 0.2\Omega$

$C = 150\ \mu F = 150 \times 10^{-6}\ \mu$

$V = 230\ V$

$f = 50H_z$



(i) Inductive Reactance $(X_L) = 2\pi f L$

$= 2\pi \times 50 \times 0.2$

$= 62.83$

(ii) Capacitive Reactance $(X_C) = \dfrac{1}{2\pi f C}$

$= \dfrac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$

$= 21.22$

(iii) Impedance $(Z) = R + j(X_L - X_C)$

$= 20 + j(62.83 - 21.22)$

$= 20 + j41.61$

$= 46.16 \angle 64.3\Omega$

(iv) Admitance $(Y) = \dfrac{1}{Z}$

$= \dfrac{1}{46.16 \angle 64.3}$

$= 0.02 \angle -64.3$

(v) Current $(I) = \dfrac{V}{Z}$

$= \dfrac{230 \angle 0}{46.16 \angle 64.3}$

$= 4.98 \angle -64.3\ A$

(vi) Power factor $= \cos\phi$

$= \cos(\theta_v - \theta_i)$

$= \cos(0 - (-64.3))$

$= \cos 64.3$

$= 0.43$

(vii) Active power $= VI\cos\phi$

$= 230 \times 4.98 \times 0.43$

$= 492.5$ watt

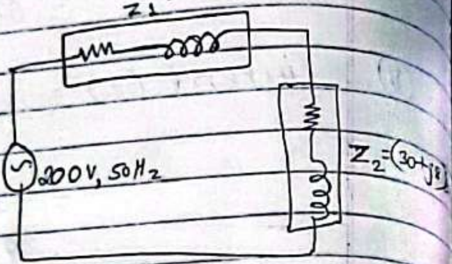
(viii) Reactive power $= VI\sin\phi$

$= 230 \times 4.98 \times \sin(64.3)$

$= 1032.09$ VAR

$= 1032$ KVAR

(12) An impedance $(20+j5)\Omega$ & $(30+j8)\Omega$ are connected in series across a 200V, 50Hz supply. Calculate: Current, active power, apparant power & reactive power and voltage across Z_2 & Z_1.



Sol^n: $V = 200V$

$f = 50Hz$

$Z_1 = (20+j5)\Omega$

$Z_2 = (30+j8)\Omega$

Now,

Total impedance $(Z) = Z_1 + Z_2$

$= (20+j5) + (30+j8)$

$= 20 + j5 + 30 + j8$

$= 20 + 30 + j(5+8)$

$= 50 + j13$

$= 51.66 \angle 14.57$

$\therefore$ Current $(I) = \frac{V}{Z}$

$= \frac{200}{51.66 \angle 14.57}$

$= 3.87 \angle -14.57$ A

Power factor $= \cos\phi$

$= \cos(\theta_v - \theta_I)$

$= \cos(0 - (-14.57))$

$= \cos 14.57$ ~~0.96~~ $= 0.96$

Active power $= VI\cos\phi$

$= 200 \times 3.87 \times$ ~~0.96~~ $\cos 14.57$

$= 749.1$ watt

Reactive power $= VI\sin\phi$

$= 200 \times 3.87 \times \sin 14.57$

$= 194.7$ VAR

Apparant power $= VI$

$= 200 \times 3.87$

$= 774$ volt Ampere

$\therefore$ Voltage across Z_2 is given by:

$V_{Z_2} = I \cdot Z_2$

$= (3.87 \angle -14.57) \times (30 + j8)$

$= (3.87 \angle -14.57) \times (31.04 \angle 14.93)$

$= (3.87 \times 31.04) \angle (-14.57 + 14.93)$

$= 120.12 \angle 0.36$ Volt **Ans**

$\therefore$ Voltage across Z_1 is given by:

$V_{Z_1} = I \cdot Z_1$

$= (3.87 \angle -14.57) \times (20 + j5)$

$= (3.87 \angle -14.57) \times (20.61 \angle 14.03)$

$= (3.87 \times 20.61) \angle (-14.57 + 14.03)$

$= 79.76 \angle -0.54$ volt **Ans**

(13) An impedance of $(6-j8)\Omega$ is connected in parallel with $(8+j6)\Omega$. The impedance are fed from a 220V, 50Hz supply. Find the current through each branch, ~~total~~ total impedance, total current, total admitance, active power, reactive power and power factor of the given circuit.

Sol^n:

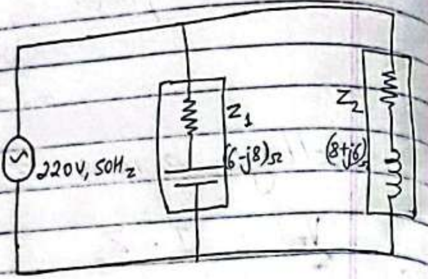
Here,

$z_1 = (6-j8)\Omega$

$z_2 = (8+j6)\Omega$

$V = 220$ V

$f = 50Hz$



Now,

Branch current:

$$I_1 = \frac{V}{z_1} = \frac{220}{6-j8}$$

$$= \frac{220}{10\angle -53.13}$$

$$= 22\angle 53.13 \text{ A}$$

$$I_2 = \frac{V}{z_2} = \frac{220}{8+j6}$$

$$= \frac{220}{10\angle 36.86}$$

$$= 22\angle -36.86 \text{ A}$$

Total Current $(I) = $

Total impedance $(z) = \dfrac{z_1 \cdot z_2}{z_1+z_2}$

$$= \frac{(6-j8)\times(8+j6)}{(6-j8)+(8+j6)}$$

$$= \frac{(10\angle -53.13)\times(10\angle 36.86)}{6-j8+8+j6}$$

$$= \frac{(10\angle -53.13)\times(10\angle 36.86)}{14-j2}$$

$$= \frac{(10\angle -53.13)\times(10\angle 36.86)}{14.14\angle -8.13}$$

$$= \frac{(10\times 10)\angle(-53.13+36.86)}{14.14\angle -8.13}$$

$$= \frac{100\angle -16.27}{14.14\angle -8.13}$$

$$= \frac{100}{14.14}\angle(-16.27+8.13)$$

$$= 7.07\angle -8.14\ \Omega$$

Total ~~impedance~~ admittance $(Y) = \dfrac{1}{z}$

$$= \frac{1}{7.07\angle -8.14}$$

$$= 0.14\angle 8.14$$

Total current (I) $= \frac{V}{Z}$

$= \frac{220}{7.07\angle -8.14}$

$= 31.1 \angle 8.14$

Power factor $= \cos\phi$

$= \cos(\theta_v - \theta_i)$

$= \cos(0 - 8.14)$

$= \cos(-8.14)$

$= 0.98$

Active power $= VI\cos\phi$

$= 220 \times 31.1 \times \cos(-8.14)$

$= 6773.06$ watt

Reactive power $= VI\sin\phi$

$= 220 \times 31.1 \times \sin(-8.14)$

$= -968.77$ VAR

(14) An impedances of $(3-j8)$ and $(2+j15)$ are connected in parallel. The circuit is fed from a 230 V, 50 Hz supply. Find the current through each branch, total circuit current, impedance, p.f, active power, reactive power & apparant power?
(2009 fall)

P.T.O

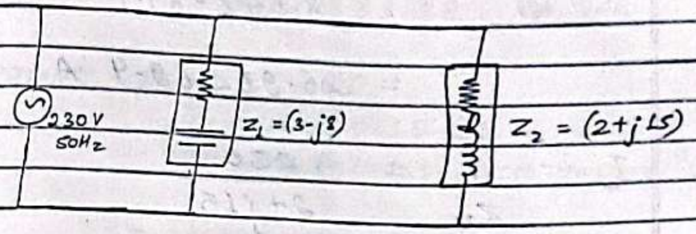
Sol^n: Here,

$Z_1 = (3-j8)$

$Z_2 = (2+j15)$

$V = 230$ V

$f = 50$ Hz



Total impedance (Z) $= \frac{Z_1 \times Z_2}{Z_1 + Z_2}$

$= \frac{(3-j8) \times (2+j15)}{3-j8 + 2+j15}$

$= \frac{(8.54\angle -69.4) \times (15.13\angle 82.4)}{5+j7}$

$= \frac{(8.54 \times 15.13)\angle(-69.4+82.4)}{8.6\angle 54.4}$

$= \frac{129.2\angle 13}{8.6\angle 54.5}$

$= \frac{129.2}{8.6}\angle(13-54.5)$

$= 15.02\angle -41.5$

Branch Current:

$$I_1 = \frac{V}{Z_1} = \frac{230}{3-j5}$$

$$= \frac{230}{5.54\angle -69.4}$$

$$= 26.93\angle 69.4 \text{ A}$$

$$I_2 = \frac{V}{Z_2} = \frac{230}{2+j15}$$

$$= \frac{230}{\cancel{2+}\ 15.13\angle 82.4}$$

$$= 15.2\angle -82.4 \text{ A}$$

$$\text{Total Current } (I) = \frac{V}{Z}$$

$$= \frac{230}{15.02\angle -41.5}$$

$$= 15.31\angle 41.5 \text{ A}$$

$$\text{Power factor } (p.f) = \cos\phi$$

$$= \cos(\theta_v - \theta_i)$$

$$= \cos(0 - 41.5)$$

$$= 0.74$$

$$\text{Active power} = VI\cos\phi$$

$$= 230 \times 15.31 \times \cos(-41.5)$$

$$= 2637.29 \text{ watt}$$

$$\text{Reactive power} = VI\sin\phi$$

$$= 230 \times 15.31 \times \cancel{\cos}\ \sin(-41.5)$$

$$= -2333.28\ \text{VAR}$$

$$\text{Apparant power} = VI$$

$$= 230 \times 15.31$$

$$= 352.13 \text{ volt ampere}$$

(15) (2008 Fall 8 marks) Two impedance of $(10+j5)\Omega$ & $(20+j30)\Omega$ are connected in series across a 200V, 50Hz supply. Find current, active power, apparant power and reactive power factor.

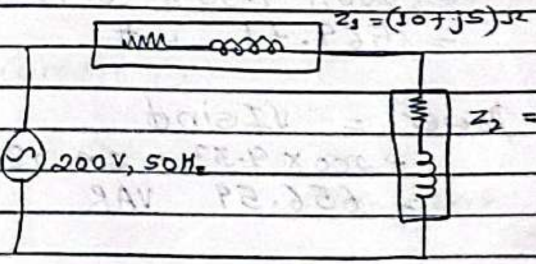
Soln: Here,

$Z_1 = (10+j5)\Omega$

$Z_2 = (20+j30)\Omega$

$V = 200\text{V}$

$f = 50\text{Hz}$



Total Impedence $(z) = z_1 + z_2$

$= (10+j5) + (20+j30)$

$= 10+20+j5+j30$

$= 30+j35$

$= 46.09 \angle 49.3$

$\therefore$ Current $(I) = \dfrac{V}{Z}$

$= \dfrac{200}{46.09 \angle 49.3}$

$= 4.33 \angle -49.3$

~~Active Power = VI~~

Power factor $= \cos\phi$

$= \cos(\theta_v - \theta_i)$

$= \cos(0 - (-49.3))$

$= \cos 49.3$

$= 0.65$

Active power $= VI\cos\phi$

$= 200 \times 4.33 \times \cos 49.3$

$= 564.71$ watt

Reactive power $= VI\sin\phi$

$= 200 \times 4.33 \times \sin 49.3$

$= 656.54$ VAR

Apparant power $= VI$

$= 200 \times 4.33$

$= 866$ volt ampere

(16) (2008 Spring 4 marks) Two impedance of $(20+j5)\Omega$ and $(30+j8)\Omega$ are connected in series across a 200V, 50Hz supply. Find current, active power, apparent power and reactive power & power factor.

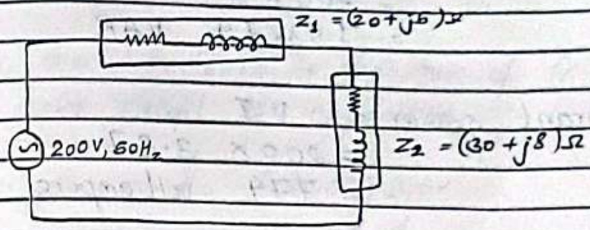
Soln: Here,

$Z_1 = (20+j5)\Omega$

$Z_2 = (30+j8)\Omega$

$V = 200V$

$f = 50\,Hz$



Total Impedence $(z) = Z_1 + Z_2$

$= (20+j5) + (30+j8)$

$= 20+30+j5+j8$

$= 50+j13$

$= 51.66 \angle 14.5$

$\therefore$ Current $(I) = \dfrac{V}{Z}$

$= \dfrac{200}{51.66 \angle 14.5}$

$= 3.87 \angle -14.5$

Power factor (p.f) = $\cos\phi$

$= \cos(\theta_v - \theta_i)$

$= \cos(0 - (-14.5))$

$= \cos 14.5$

$= 0.96$

Active power = $VI\cos\phi$

$= 200 \times 3.87 \times \cos 14.5$

$= 749.34$ watt

Reactive power = $VI\sin\phi$

$= 200 \times 3.87 \times \sin 14.5$

$= 193.79$ VAR

Apparant power = VI

$= 200 \times 3.87$

$= 774$ volt ampere

(17) An AC voltage of $(80+j60)V$ applied to a circuit result current of $(4+j10)A$. (2010 spring 8 marks)

Find:

(i) The impedance of the circuit stating whether it is inductive or capacitive

(ii) Power consumed

(iii) Phase angle

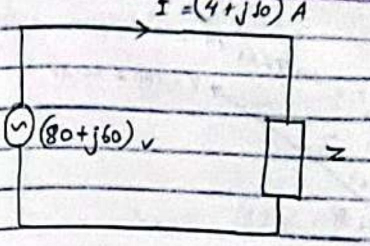
(iv) Power factor

(v) Draw a phasor diagram

Soln: Here,

$V = (80+j60)\,V = 100\angle 36.86\ V$

$I = (4+j10)\,A = 10.77\angle 68.19\ A$



(i) $Z = \dfrac{V}{I} = \dfrac{100\angle 36.86}{10.77\angle 68.19}$

$= 9.28\angle -31.33$

$= 7.93 - j4.82$

Since, the impedance is the form of $(R - jX_c)$. So the given circuit is capacitive.

(ii) power consumed = Active power

$= VI\cos\phi$

$= 100 \times 10.77 \times \cos(36.86 - 68.19)$

$= 100 \times 10.77 \times \cos(-31.33)$

$= 919.95$ watt

(iii) phase angle: $\phi = \theta_v - \theta_i$

$= 36.86 - 68.19$

$= -31.33$

(iv) power factor = $\cos\phi$

$= \cos(-31.33)$

$= 0.85$

+ve → lag
-ve → lead

(V) Phasor diagram

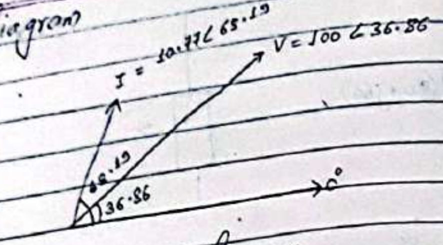
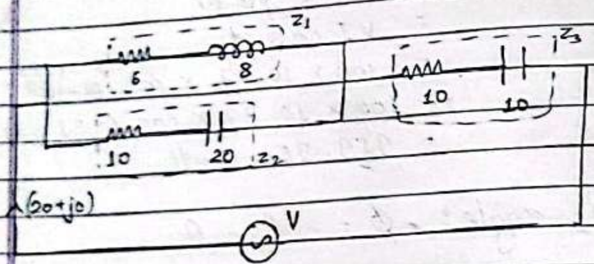


fig: phasor diagram

(15) In the circuit given below total current $I = (20 + j0)$ A.
Calculate: (i) Branch Current
(ii) Voltage V, V_1 & V_2
(iii) Power factor
(iv) Active & Reactive power
(v) Draw a phasor diagram.



Sol: Here,

$Z_1 = (6 + j8)\Omega = 10 \angle 53.13$

$Z_2 = (10 - j20)\Omega = 22.36 \angle -63.43$

$Z_3 = (10 - j10)\Omega = 14.14 \angle -45$

$I = (20 + j10)\,A = 20\,A \angle 0$

Total Impedance $(Z) = (Z_1 \| Z_2) + Z_3$

$$= \frac{Z_1 \times Z_2}{Z_1 + Z_2} + Z_3$$

$$= \frac{(10 \angle 53.13) \times (22.36 \angle -63.43)}{6 + j8 + 10 - j20} + (10 - j10)$$

$$= \frac{(10 \times 22.36) \angle (53.13 - 63.43)}{16 - j12} + (10 - j10)$$

$$= \frac{223.6 \angle -10.3}{20 \angle -36.86} + (10 - j10)$$

$$= 11.18 \angle 26.56 + (10 - j10)$$

$$= 10 + j5 + 10 - j10$$

$$= 20 - j5$$

$$= 20.61 \angle -14.03$$

Now,

(ii) $V = I \times Z$
$= (20 \angle 0) \times (20.61 \angle -14.03)$
$= 412.2 \angle -14.03$

$V_1 = V_{Z_1 Z_2} = I \times (Z_1 \| Z_2)$

$= (20 \angle 0) \times (11.18 \angle 26.56)$

$= 223.6 \angle 26.56$

$V_2 = V_{Z_3} = I \times Z_3$

$= (20\angle 0) \times (14.14\angle -45)$

$= 282.8\angle -45$

(i) Branch Currents:

$I_{Z_1} = \dfrac{V_1}{Z_1} = \dfrac{223.6\angle 26.56}{10\angle 53.13}$

$= 22.36\angle -26.57$

$I_{Z_2} = \dfrac{V_1}{Z_2} = \dfrac{223.6\angle 26.56}{22.36\angle -63.43}$

$= 10\angle 89.99$

$I_{Z_3} = \dfrac{V_2}{Z_3} = \dfrac{282.8\angle -45}{14.14\angle -45}$

$= 20\angle 0$

(iii) power factor (p.f) $= \cos\phi$

$= \cos(\theta_v - \theta_i)$

$= \cos(-14.03 - 0)$

$= \cos(-14.03)$

$= 0.97$

(iv) Active power $= VI\cos\phi$

$= 412.2 \times 20 \times \cos(-19.03)$

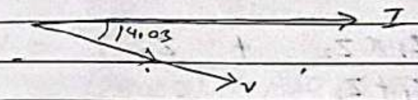
$= 7998.07$ watt.

Reactive power $= VI\sin\phi$

$= 412.2 \times 20 \times \sin(-19.03)$

$= -1998.5$ VAR

(v) phasor diagram



(19) In the circuit given below total current $I = (25 + j0)$ A. calculate

(i) Branch current

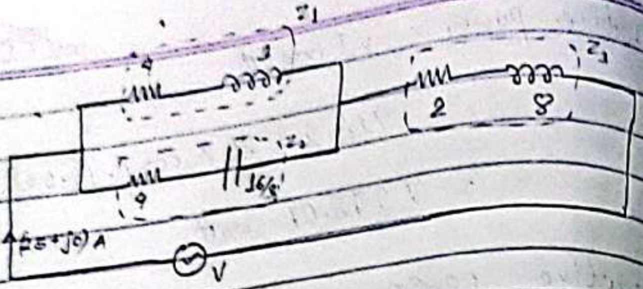
(ii) Voltage V_1, V_2, V

(iii) power factor

(iv) Active & Reactive Power

(v) total power in the circuit & power in each branch.

(vi) phasor diagram



Sol: Here,

$Z_1 = (4+j3)\Omega = 5\angle 36.87$

$Z_2 = \left(4 - j\frac{16}{3}\right)\Omega = 6.67\angle -53.13$

$Z_3 = (2+j8)\Omega = 8.24\angle 76$

$I = (25+j0)\,A = 25\angle 0$

Now, Total impedance is:

$Z = (Z_1 \| Z_2) + Z_3$

$= \frac{Z_1 \times Z_2}{Z_1 + Z_2} + Z_3$

$= \frac{(5\angle 36.87)\times(6.67\angle -53.13)}{4+j3+4-j\frac{16}{3}} + (2+j8)$

$= \frac{(5\times 6.67)\angle(36.87-53.13)}{8-j\frac{7}{3}} + (2+j8)$

$= \frac{33.35\angle -16.26}{8.33\angle -16.26} + (2+j8)$

$= 4\angle 0 + (2+j8)$

$= 4+j0+2+j8$

$= 6+j8$

$= 10\angle 53.13$

Now,

(ii) $V_{Z_1Z_2} = V_1 = I \times (Z_1 \| Z_2)$

$= (25\angle 0)\times(4\angle 0)$

$= 100\angle 0$

And, $V_{Z_3} = V_2 = I \times Z_3$

$= (25\angle 0)\times(8.24\angle 76)$

$= 206\angle 76$

And, $V = IZ$

$= (25\angle 0)\times(10\angle 53.13)$

$= 250\angle 53.13$

(i) Branch Currents:

$I_{Z_1} = \frac{V_1}{Z_1} = \frac{100\angle 0}{5\angle 36.87}$

$= 20\angle -36.87$

$I_{Z_2} = \frac{V_1}{Z_2} = \frac{100\angle 0}{6.67\angle -53.13} = 15\angle 53.13$

$$I_{z_3} = \frac{V_2}{Z_3} = \frac{206\angle 76}{8.24\angle 76}$$
$$= 25\angle 0$$

Now,

(iii) Power factor (p.f) $= \cos\phi$
$= \cos(\theta_v - \theta_i)$
$= \cos(53.13 - 0)$
$= \cos(53.13)$
$= 0.6$

(iv) Active power $= VI\cos\phi$
$= 250 \times 25 \times \cos(53.13)$
$= 3750$ watt

Reactive power $= VI\sin\phi$
$= 250 \times 25 \times \sin(53.13)$
$= 5000$ VAR

(v) Total power in the circuit is:

$P = I^2 Z$
$= I \times IZ$
$= I \times V$
$= (25\angle 0) \times (250\angle 53.13)$
$= 6250\angle 53.13$
$= 3750 + j5000$

$P_{z_1} = I_{z_1} \times V_{z_1}$
$= (20\angle -36.87) \times (100\angle 0)$
$= 2000\angle -36.87 = 1600 - j1200$

$P_{z_2} = I_{z_2} \times V_1$
$= (15\angle 53.13) \times (100\angle 0)$
$= 1500\angle 53.13$
$= 900 + j1200$

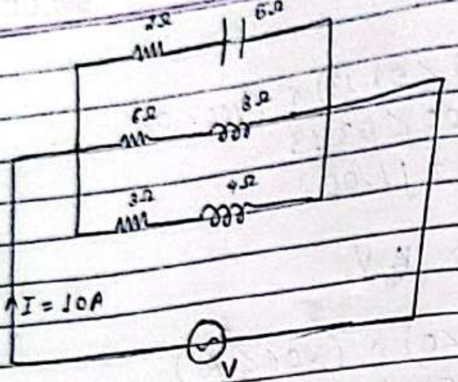
$P_{z_3} = I_{z_3} \times V_2$
$= (25\angle 0) \times (206\angle 76)$
$= 5150\angle 76$
$= 1246 + j4997$

(vi) phasor diagram

(20) A total current of 10 A flows through the parallel combination of three impandance: $(2-j5)\Omega$, $(6+j3)\Omega$ and $(3+j4)\Omega$. Calculate
(i) the current flowing through each branch
(ii) p.f
(iii) voltage flowing through the circuit (V).

P.T.O

Soln.



Here,

$Z_1 = (2 - j5)\,\Omega = 5.38 \angle -68.19$

$Z_2 = (6 + j3)\,\Omega = 6.7 \angle 26.56$

$Z_3 = (3 + j4)\,\Omega = 5 \angle 53.13$

$I = 10\text{ A} = (10 + j0)\text{A} = 10 \angle 0$

Now, Total Impedance $(Z) = \dfrac{Z_1 \cdot Z_2 \cdot Z_3}{Z_1 Z_2 + Z_2 \cdot Z_3 + Z_3 \cdot Z_1}$

where,

$Z_1 Z_2 = (5.38 \angle -68.19) \times (6.7 \angle 26.56)$
$= 36.04 \angle -41.63$
$= 27 - j24$

$Z_2 Z_3 = (6.7 \angle 26.56) \times (5 \angle 53.13)$
$= 33.5 \angle 79.69$
$= 6 + j33$

$Z_3 \cdot Z_1 = (5 \angle 53.13) \times (5.38 \angle -68.19)$
$= 26.9 \angle -15.06$
$= 26 - j7$

$Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1 = 27 - j24 + 6 + j33 + 26 - j7$
$= 27 + 6 + 26 - j24 - j7 + j33$
$= 59 + j2$
$= 59.03 \angle 1.94$

$Z_1 \cdot Z_2 \cdot Z_3 = (Z_1 \cdot Z_2) \cdot Z_3$
$= (36.04 \angle -41.63) \times (5 \angle 53.13)$
$= 180.2 \angle 11.5$
$= 176.5 + j35.92$

$\therefore Z = \dfrac{180.2 \angle 11.5}{59.03 \angle 1.94}$

$= 3.05 \angle 9.56$
$= 3 + j0.5$

Now, (iii) $V = I \times Z$
$= (10 \angle 0) \times (3.05 \angle 9.56)$
$= 30.5 \angle 9.56$

(i) Current through each branch:

$I_1 = I_{Z_1} = I \times \dfrac{Z_2 Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$

$= (10 \angle 0) \times \dfrac{(33.5 \angle 79.69)}{(59.03 \angle 1.94)}$

$= (10 \angle 0) \times (0.567 \angle 77.75)$
$= 5.67 \angle 77.75$
$= 1.2 + j5.5$

$$I_2 = I_{Z_2} = I \times \frac{Z_1 Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

$$= (10\angle 0) \times \frac{26.9\angle -15.06}{59.03\angle 1.94}$$

$$= (10\angle 0) \times (0.455\angle -17)$$
$$= 4.55\angle -17$$
$$= 4.3 - j1.33$$

$$I_3 = I_{Z_3} = I \times \frac{Z_1 Z_2}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

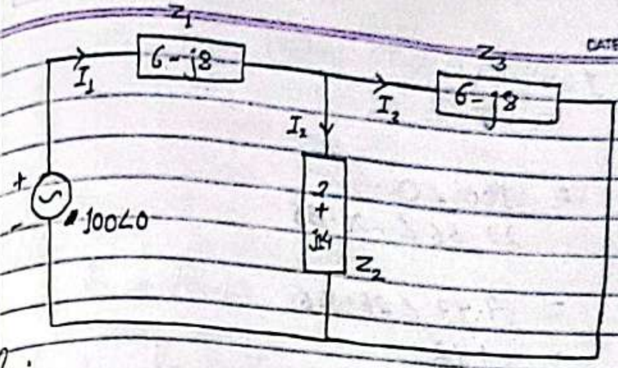
$$= (10\angle 0) \times \frac{(36.04\angle -41.63)}{(59.03\angle 1.94)}$$

$$= (10\angle 0) \times (0.61\angle -43.57)$$
$$= 6.1\angle -43.57$$
$$= 4.41 - j4.2$$

(ii) $p.f = \cos\phi$
$= \cos(\theta_v - \theta_i)$
$= \cos(9.56 - 0)$
$= 0.986$

(21) Determine the average power delivered to each of the three boxed network in the given circuit.

P.T.O



Soln:

Here,

$Z_1 = 6 - j8 = 10\angle -53.13$
$Z_2 = 2 + j14 = 14.14\angle 81.87$
$Z_3 = 6 - j8 = 10\angle -53.13$
$V = 100 + j0 = 100\angle 0$

Now, Total Impedance:

$$Z = Z_1 + (Z_2 \| Z_3)$$

$$= (6 - j8) + \frac{(14.14\angle 81.87) \times (10\angle -53.13)}{2 + j14 + 6 - j8}$$

$$= (6 - j8) + \frac{141.4\angle 28.74}{8 + j6}$$

$$= (6 - j8) + \frac{141.4\angle 28.74}{10\angle 36.86}$$

$$= 6 - j8 + 14.14\angle -8.12$$
$$= 6 - j8 + 14 - j2$$
$$= 20 - j10$$
$$= 22.36\angle -26.56$$

$$\therefore I = \frac{V}{Z}$$

$$= \frac{100 \angle 0}{22.36 \angle -26.56}$$

$$= 4.47 \angle 26.56$$

$$= 4 + j2$$

$$\therefore I = I_1 = 4.47 \angle 26.56$$

Now,

$$V_{Z_1} = I Z_1$$

$$= (4.47 \angle 26.56) \times (10 \angle -53.13)$$

$$= 44.7 \angle -26.57$$

$$= 40 - j20$$

$$V_{Z_2 Z_3} = I \times (Z_2 \parallel Z_3)$$

$$= (4.47 \angle 26.56) \times (14.14 \angle -8.12)$$

$$= 63.2 \angle 18.49$$

$$= 60 + j20$$

Now,

$$I_2 = \frac{V_{Z_2 Z_3}}{Z_2} = \frac{63.2 \angle 18.49}{14.14 \angle 81.87}$$

$$= 4.47 \angle -63.49$$

$$I_3 = \frac{V_{Z_2 Z_3}}{Z_3} = \frac{63.2 \angle 18.49}{10 \angle -53.13}$$

$$= 6.32 \angle 71.57$$

Now,

P_1 = Power across Z_1:

$$P_1 = V_{Z_1} \times I_1 \times \cos \phi_1$$

$$= 44.7 \times 4.47 \times \cos(\theta_v - \theta_i)$$

$$= 44.7 \times 4.47 \times \cos(-26.57 - 26.56)$$

$$= 44.7 \times 4.47 \times \cos(-53.13)$$

$$= 120 \text{ watt}$$

$\therefore$ Power across Z_2:

$$P_2 = V_{Z_2 Z_3} \times I_2 \times \cos \phi_2$$

$$= 44.7 \times 4.47 \times \cos(18.49 + 63.49)$$

$$= 40 \text{ watt}$$

$\therefore$ Power across Z_3:

$$P_3 = V_{Z_2 Z_3} \times I_3 \times \cos \phi_3$$

$$= 44.7 \times 4.47 \times \cos(18.49 - 71.57)$$

$$= 240 \text{ watt}$$

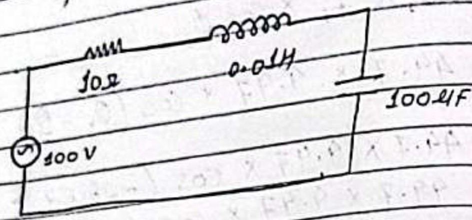
$\therefore$ Total power in the circuit is:

$$P = P_1 + P_2 + P_3 = 120 + 40 + 240 = 400 \text{ watt}$$

Resonant frequency:

(22) A series R-L-C circuit has 10Ω resistance, $0.01\,H$ inductance & $100\,\mu F$ capacitance. A 100V supply with a frequency of 50 Hz is applied to the circuit. Find the resonant frequency, band width, Q-factor, total current, resonance current and voltage across inductor. [2009, fall]

(2009, fall)
(2004, fall)



Soln:

Here,

$R = 10\Omega$

$L = 0.01\,H$

$C = 100\,\mu F = 100\times10^{-6}\,F$

$V = 100\,V$

$f = 50\,Hz$

Now, $X_L = 2\pi f L = 2\pi\times 50\times 0.01$

$= 3.14$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi\times50\times100\times10^{-6}}$

$= 31.83$

$\therefore Z = R + j(X_L - X_C)$

$= 10 + j(3.14 - 31.83)$

$= 10 - j\,28.69$

$= 30.38\angle -70.78$

remember & Bandwidth(BW) $= \dfrac{\text{Resonant frequency}(f_0)}{\text{quality factor}(Q)} = f_0/Q$

$\therefore$ Total current $(I) = \dfrac{V}{Z}$

$= \dfrac{100\angle 0}{30.38\angle -70.78}$

$= 3.29\angle 70.78\ A$

Resonance Current $(I_0) = \dfrac{V}{R}$

$= \dfrac{100\angle 0}{30.38\angle}$

$= \dfrac{100}{10}$

$= 10\ A$

voltage across inductor $(V_L) = I\times X_L$

$= 3.29\times 3.14$

$= 10.33$ volt

Resonance frequency $(f_0) = \dfrac{1}{2\pi\sqrt{LC}}$

$= \dfrac{1}{2\pi\sqrt{0.01\times100\times10^{-6}}}$

$= 159.15\ Hz$

Q-factor $= \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

$= \dfrac{1}{10}\times\sqrt{\dfrac{0.01}{100\times10^{-6}}}$

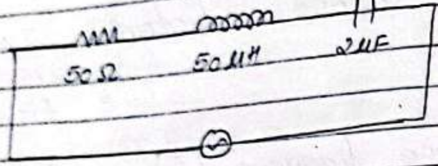
$= 1$

Band width $(\Delta\omega) = \frac{R}{L}$

$= \frac{10}{0.01}$

$= 1000$

(23) For given circuit determine
(i) Q-factor
(ii) the new value of capacitor required for resonance at same frequency if inductance is doubled.
(iii) new Q-factor.



Soln: Here,

$R = 50\,\Omega$

$L = 50\,\mu H = 50 \times 10^{-6}\,H$

$C = 2\,\mu F = 2 \times 10^{-6}\,F$

(i) Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$

$= \frac{1}{50}\sqrt{\frac{50 \times 10^{-6}}{2 \times 10^{-6}}}$

$= 0.1$

(ii) Let, initial inductor be L_1 then final inductor be L_2. Similarly, initial capactor be C_1 & final capacitor be C_2.

Acc^n to the question:

$L_2 = 2L_1$

At resonance: $f = f_0$

$f = \frac{1}{2\pi\sqrt{L_1 C_1}}$

But, when inductance is doubled then $L_2 = 2L_1$, so we get:

$\therefore \frac{1}{2\pi\sqrt{L_1 C_1}} = \frac{1}{2\pi\sqrt{L_2 C_2}}$

Or, $\frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{2L_1 C_2}}$

Or, $\frac{1}{L_1 C_1} = \frac{1}{2 \cdot L_1 C_2}$

Or, $\frac{1}{C_1} = \frac{1}{2C_2}$

Or, $C_2 = \frac{C_1}{2}$

Or, $C_2 = \frac{2 \times 10^{-6}}{2}$ $\therefore \boxed{C_2 = 1 \times 10^{-6}\,F}$

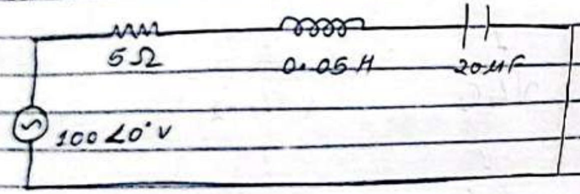
(iii) New Q-factor $= \frac{1}{R}\sqrt{\frac{L_2}{C_2}}$

$= \frac{1}{R}\sqrt{\frac{2L_1}{C_2}}$

$= \frac{1}{50}\sqrt{\frac{2\times 50\times 10^{-6}}{1\times 10^{-6}}}$

$= 0.2$

(24) In the series circuit shown, the applied voltage is 100V of variable frequency. If the frequency is varied till the resonance occurs in the circuit than find the current, resonant frequency, bandwidth, quality factor and the voltage across the inductors.



Soln: Here,

$R = 5\Omega$

$L = 0.05 H$

$C = 20 \mu F = 20\times 10^{-6} F$

$V = 100\angle 0^\circ V$

$f = 50 Hz$

Now,

$X_L = 2\pi f L = 2\pi \times 50 \times 0.05$

$= 15.71$

$X_C = \frac{1}{2\pi f C}$

$= \frac{1}{2\pi \times 50 \times 20\times 10^{-6}}$

$= 159.15$

$\therefore Z = R + j(X_L - X_C)$

$= 5 + j(15.71 - 159.15)$

$= 5 - j143.44$

$= 143.52 \angle -88$

$\therefore$ Total current $(I) = \frac{V}{Z}$

$= \frac{100\angle 0}{143.52\angle -88}$

$= 0.69\angle 88$ A

$\therefore$ Resonance current $(I_0) = \frac{V}{R}$

$= \frac{100}{5}$

$= 20 A$

Resonant frequency $(f_0) = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi\sqrt{0.05\times 20\times 10^{-6}}}$

$= 159.15 Hz$

Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$

$= \frac{1}{5}\sqrt{\frac{0.05}{20\times10^{-6}}}$

$= 10$

Bandwidth $(\Delta\omega) = \frac{R}{L}$

$= \frac{5}{0.05} = 100$

Voltage across inductors $(V_L) = I \times X_L$

$= 0.69 \times 15.71$

$= 10.8$ V

(2) An AC series circuit has a resistance of 10Ω and an inductance of 0.2H and capacitance of $60\mu F$. If the applied voltage is 200 V, calculate resonant frequency and power at resonance.

2006, Spring 5 marks

Soln: Here,

$R = 10\Omega$

$L = 0.02$ H

$C = 60\mu F = 60\times10^{-6}$ F

$V = 200$ V

Now,

Resonant frequency $(f_0) = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi\sqrt{0.2\times60\times10^{-6}}} = 45.94$ Hz

Resonance current $(I_0) = \frac{V}{R}$

$= \frac{200}{10}$

$= 20$ A

$\therefore$ Power at resonance $(P) = I_0^2 R$

$= (20)^2 \times 10$

$= 4000$ watt

Polyphase AC Circuit Analysis (3-ϕ)

Star/Delta Connected System:

(1) A balanced star connected node of $(10+j5)\Omega$ per phase is connected in a balanced 3-ϕ supply of 400V, 50Hz. Determine:

(i) Line current

(ii) Phase voltage

(iii) Power factor

(iv) Active power

Soln: Here,

$V_L = 400$ V

$f = 50$ Hz

$Z = (10+j5)\Omega$

$= 11.18 \angle 29.5$

In Star(Y) connected system:

$I_L = I_{ph}$ &

And, $V_L = \sqrt{3}\, V_{ph}$

or, $V_{ph} = \frac{V_L}{\sqrt{3}}$

or, $V_{ph} = \frac{400}{\sqrt{3}}$

$\therefore \boxed{V_{ph} = 230.94}$ → phase voltage

Now,

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94\angle 0}{11.18\angle 29.5}$

$\therefore \boxed{I_{ph} = 20.65\angle -29.5}$

$\therefore I_L = I_{ph}$

$\therefore \boxed{I_L = 20.65\angle -29.5}$ → line current

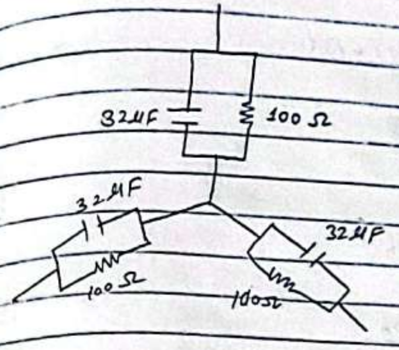
Power factor $= \cos\phi$
$= \cos(\theta_v - \theta_i)$
$= \cos(0 - (-29.5))$
$= \cos(29.5)$
$= 0.89$

$\therefore$ Active power $= \sqrt{3}\, V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 20.65 \times \cos(29.5)$
$= 12.7$ kW

(2) Each phase of a star connected system consist of a resistance of 100 Ω in parallel with a capacitor of 32 μF.

Calculate : (i) line current
(ii) Power consumed
(iii) Total kVA (Apparent power)
(iv) Power factor when connected to a 415 V, 50 Hz 3 ϕ supply.



Soln Here,

$R = 100\,\Omega$

$C = 32\,\mu F = 32\times10^{-6}$ F

Now, $Z = R \,//\, (-jX_C)$

Where, $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\times 50\times 32\times10^{-6}}$

$= 99.47\,\Omega$

$\therefore Z = R \,//\, (-jX_C)$

$= 100 \,//\, (-j\,99.47)$

$= \frac{100\times(-j\,99.47)}{100 - j\,99.47}$

$= \frac{-j\,9947}{100 - j\,99.47}$

$$= \frac{9947\angle -\cancel{100}\ -90}{141.04\angle -\cancel{99.83}\ -44.84}$$

$$Z = 70.52\angle -\cancel{50.17}\ -45.16$$

Now,

(i) line current:

$\cancel{I_L} = \frac{\cancel{V_L}}{\cancel{Z}}$

$$I_L = I_{ph}$$

$$= \frac{V_{ph}}{Z_{ph}}$$

$$= \frac{V_L/\sqrt{3}}{Z_{ph}} \quad \left[\because V_{ph} = \frac{V_L}{\sqrt{3}}\right]$$

$$= \frac{V_L}{\sqrt{3}\,Z_{ph}}$$

$$= \frac{415}{\sqrt{3}\times(70.52\angle -50.17)}$$

$$= \frac{239.60}{70.52\angle -50.17}$$

$$\therefore I_L = 3.39\angle 50.17 \ \ A$$

(ii) Power consumed $= \sqrt{3}\,V_L\,I_L\cos\phi$

$$= \sqrt{3}\times 415\times 3.39\times\cos(0-50.17)$$

$$= \sqrt{3}\times 415\times 3.39\times\cos(50.17)$$

$$= 1.71 \ \ kW$$

(iii) Total KVA $= \sqrt{3}\,V_L\,I_L$

$$= \sqrt{3}\times 415\times 3.39$$

$$= 2436.73 \ VA$$

$$= 2.43 \ KVA$$

(5) ~~When~~ ~~the~~

P.T.O

value of ϕ ... +ve lag, -ve lead

(3) When the balance impedance are connected to Δ (delta) across 500v, 50Hz supply the line current drawn is 20A at a power factor of 0.3 lagging. Calculate the impedance & state whether its capacitive or inductive.

Sol: Here,

$V_L = 500\,V$

$f = 50\,Hz$

$I_1 = 20\,A$

$p.f = 0.3$ lagging

$Z = ?$

For Δ system:

$V_l = V_{ph} = 500\,V$

& $I_l = \sqrt{3}\, I_{ph}$

$\therefore I_{ph} = \frac{I_L}{\sqrt{3}}$

Now, We know that;

$I_{ph} = \frac{V_{ph}}{Z}$

or, $\frac{I_L}{\sqrt{3}} = \frac{500}{Z}$

or, $\frac{20}{\sqrt{3}} = \frac{500}{Z}$

$I_{ph} = 20/\sqrt{3}$

$\therefore \boxed{I_{ph} = 11.54\,A}$

We have:

$Z = \frac{V_{ph}}{I_{ph}}$

$= \frac{500}{11.54}$

$= 43.32\,\Omega$

$Z = \frac{500 \times \sqrt{3}}{20}$

$R/Z = \cos\phi$: power factor

$\frac{X_L}{Z} = \sin\phi$ = reactive factor

$\therefore \boxed{Z_{ph} = Z = 43.3\,\Omega}$

New, $p.f = 0.3$

or, $\cos\phi = 0.3$

or, $\phi = \cos^{-1}(0.3)$

$\therefore \phi = 72.54$

Since, p.f is lagging, ~~therefore~~ So $\angle\phi$ is positive and Z is inductive.

Now, Resistance (R) $= Z_{ph}\cos\phi$

$= 43.3 \times 0.3$

$= 12.99\,\Omega$

Inductance $(X_L) = Z_{ph}\sin\phi$

$= 43.3 \times \sin(72.54)$

$= 39.33\,\Omega$

$\therefore$ Required Impedance $(Z) = R + jX_L$

$= 12.99 + j39.33$

$= (41.41 \angle 79.69)\,\Omega$

(4) Three equal impedance of $z = 10\angle 30°$ are connected in star/Wye across 400v 3ϕ supply. What is the power taken from the supply if the same impedance are now connected in delta across same supply. Compare the results.

(2008, spring)
8 marks

P.T.O

Soln: Here,

$Z = 10\angle 30$

$V_L = 400\ V$

Star Connection:

In this case; we have:

$I_L = I_{ph}$ And, $V_L = \sqrt{3}\, V_{ph}$

$\Rightarrow V_{ph} = \frac{V_L}{\sqrt{3}}$

or, $V_{ph} = \frac{400}{\sqrt{3}}$

$\therefore \boxed{V_{ph} = 230.94\ V}$

Now, we know:

$I_{ph} = \frac{V_{ph}}{Z}$

$= \frac{230.94}{10\angle 30}$

$= 23.09\angle -30$

$\therefore \boxed{I_{ph} = I_L = 23.09\angle -30}$

$\therefore P = \sqrt{3}\, I_L V_L \cos\phi$

$= \sqrt{3} \times 23.09 \times 400 \times \cos(\theta_v - \theta_i)$

$= \sqrt{3} \times 23.09 \times 400 \times \cos(0+30)$

$= \sqrt{3} \times 23.09 \times 400 \times \cos(30)$

$= 13854\ W$

Delta Connection:

In this case; we have:

$V_L = V_{ph} = 400\ V$

And, $I_L = \sqrt{3}\, I_{ph}$

$= \sqrt{3} \times \frac{V_{ph}}{Z}$ $[\because V = IZ,\ I = V/Z]$

$= \sqrt{3} \times \frac{400}{10\angle 30}$

$= \sqrt{3} \times 40\angle -30$

$= 69.28\angle -30$

$\therefore$ Power $(P) = \sqrt{3}\, V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 69.28 \times \cos(0+30)$

$= \sqrt{3} \times 400 \times 69.28 \times \cos(30)$

$= 41568\ W$

(5) Three equal impedance having resistance of 8Ω and inductance of 6Ω are connected in series across a three phase 400V, 50Hz supply. If it is connected to (i) Star (Y) (ii) Delta (Δ)

2078 Fall 8 marks

Determine: (i) Phase Current

(ii) Line current

(iii) Power consumed in each case

Soln: Here,

$Z = (8 + j6)\ \Omega$ $\quad Z = R + jX_L = R + j2\pi fL$

$= 10\angle 36.87$

$V_L = 400\ V$

$f = 50\ Hz$

(i) Star:
In this case:

$V_L = \sqrt{3} V_{ph}$

or, $V_{ph} = \frac{V_L}{\sqrt{3}}$

or, $V_{ph} = \frac{400}{\sqrt{3}}$

$\therefore \boxed{V_{ph} = 230.94 \text{ V}}$

And, $I_L = I_{ph} = \frac{V_{ph}}{Z}$

$= \frac{230.94}{10\angle 40.96}$

$\therefore \boxed{I_{ph} = I_L = 23.09\angle -40.96}$

Now,
Power consumed (P) $= \sqrt{3} V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 23.09 \times \cos(0+40.96)$

$= \sqrt{3} \times 400 \times 23.09 \times \cos(40.96)$

$= 12.79$ KW

(ii) Delta:
In this case:

$V_L = V_{ph} = 400$ V

And, $I_L = \sqrt{3} I_{ph}$

$= \sqrt{3} \times \frac{V_{ph}}{Z}$

$= \sqrt{3} \times \frac{400}{10\angle 40.96}$

$= \sqrt{3} \times 40\angle -40.96$

$= 69.28\angle -40.96$

$\therefore \boxed{I_L = 69.28\angle -40.96}$

And, $I_{ph} = \frac{V_{ph}}{Z}$

$= \frac{400}{10\angle 40.96}$

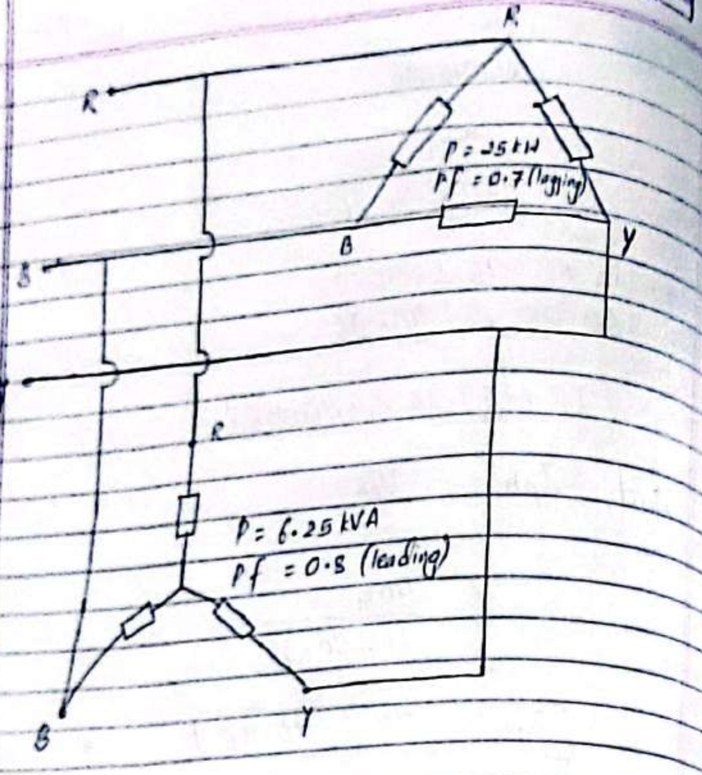
$\boxed{I_{ph} = 40\angle -40.96}$

Now, power consumed (P) $= \sqrt{3} V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 69.28 \times \cos(40.96)$

$= 38.40$ kW

(8) A 3-ϕ 381 V, 50 Hz power line has two loads connected to it. The first is Δ connected and draws 25 kW power at 0.7 p.f lagging, the second is star connected and draws 6.25 KVA at 0.8 p.f leading. What is the total line current and combine power factor (p.f).



Sol. Here,

$V_L = 381\,V$

$f = 50\,Hz$

(i) Delta (Δ) System:

power consumed $= 25\,kW$

or, $\sqrt{3}\, V_L I_L \cos\phi = 25 \times 10^3$

or, $I_L = \dfrac{25 \times 10^3}{\sqrt{3} V_L \cos\phi}$

or, $I_L = \dfrac{25 \times 10^3}{\sqrt{3} \times 381 \times 0.7}$

$\therefore \boxed{I_L = 54.12\ A}$

Now, $p.f = \cos\phi$

or, $0.7 = \cos\phi$

or, $\phi = \cos^{-1}(0.7)$

$\therefore \boxed{\phi = 45.57}$ (lagging so it is +ve)

$\because \phi = \theta_v - \theta_i$

or, $45.57 = 0 - \theta_i$

or, $45.57 = -\theta_i$

$\therefore \boxed{\theta_i = -45.57}$

$\therefore (I_L)_\Delta = 54.12 \angle -45.57$

(ii) Star (Y) system:

Power consumed $= 6.25\,kVA$ (apparent power)

or, $\sqrt{3}\, V_L I_L = 6.25 \times 10^3\, VA$

or, $I_L = \dfrac{6.25 \times 10^3}{\sqrt{3}\, V_L}$

or, $I_L = \dfrac{6.25 \times 10^3}{\sqrt{3} \times 381}$

$\therefore \boxed{I_L = 9.47 A}$

pf = 0.8 leading means ϕ = -Ve which also means capacitive ckt.
so, $pf = \cos(-\phi)$
or, $0.8 = \cos(-\phi)$
or, $-\phi = \cos^{-1}(0.8)$
$\therefore \phi = -36.87$
(leading so it is -ve)

Now, $p.f = \cos\phi$

or, $0.8 = \cos\phi$

or, $\phi = \cos^{-1}(0.8)$

$\therefore \boxed{\phi = -40.96}$

$\because \phi = \theta_v - \theta_i$

or, $-40.96 = -\theta_i$ $\quad \therefore \boxed{\theta_i = 40.96}$

$(I_L)_Y = 9.47\angle 36.87$

Hence, $(I_L)_Y = 9.47\angle -40.96$

$\therefore$ Total current $(I_L) = (I_L)_\Delta + (I_L)_Y$

$= (54.12\angle -45.57) + (9.47\angle ...)$

$= (40.83 - j35.51) + (7.57 + j5.63)$

$= 48.4 - j29.83$

$= 56.85\angle -35.16$

Combined power factor :

$\cos\phi = \cos(\theta_v - \theta_i)$

$= \cos(0 - (-35.16))$

$= \cos(35.16)$

$= 0.85$

Electric Machines

Transformer :

(1) 25 kVA transformer has 500 turns on primary winding & 40 turns on secondary winding. The primary winding is connected with 3.3 kV. Calculate

(i) Primary & secondary current

(ii) Secondary emf

(iii) Maximum emf

(iv) core loss ⇒ due to hysteresis & eddy currents = constant quantity

copper loss ⇒ due to the currents flowing in the primary & secondary windings ⇒ has two components ① primary winding copper loss ② S.W.C.L.

Voltage Regulation :

The way in which the secondary terminal voltage varies with the load depends on the load current, the internal impedance and the load power factor. The change in secondary terminal voltage from no-load to full load at any particular load is termed regulation. It is usually expressed as percentage of the rated no load terminal voltage.

$$\text{Voltage regulation} = \frac{\text{no-load output voltage} - \text{full-load output voltage}}{\text{full-load output voltage}}$$

$$\% = \frac{V_2(NL) - V_2(FL)}{V_2(FL)} \times 100\%$$

Transformer Efficiency

The efficiency of a transformer is given by,

$$\text{Efficiency } (\eta) = \frac{P_o}{P_i} \times 100\%$$

$$(\eta) = \frac{V_2 I_2 \cos\theta_2}{V_1 I_1 \cos\theta_1} \times 100\%$$

Since $P_i = P_o + \text{losses}$

$$\therefore \eta = \frac{P_o}{P_o + \text{losses}} \times 100\%$$

or, $$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + I_1^2 R_1 + I_2^2 R_2 + P_c}$$

or, $E_2 = \frac{3.3 \times 10^3 \times 40}{...}$ $\therefore \boxed{E_2 = 264}$

(2)

Q. 25 KVA transformer has 500 turns on primary winding & 40 turns on secondary winding. The primary winding is connected with 3.3KV. Calculate

(i) primary and secondary current

(ii) secondary emf

(iii) maximum emf

Soln Given:

Apparent power $(S) = 25\,kVA = 25 \times 10^3\,VA$

primary voltage $(E_1) = 3.3\,KV = 3.3 \times 10^3\,V$

No. of turns in primary side $(N_1) = 500$

" " " secondary side $(N_2) = 40$

primary current $(I_1) = ?$

Secondary " $(I_2) = ?$

secondary emf $(E_2) = ?$

maximum flux $(\phi_{max}) = ?$

We know,

$$S = E_1 I_1$$

$$or,\ 25 \times 10^3 = E_1 \times I_1$$

$$\therefore I_1 = \frac{25 \times 10^3}{3.3 \times 10^3}$$

$$I_1 = 7.58A$$

$$\therefore \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

$$or,\ I_2 = \frac{N_1 \times I_1}{N_2}$$

$$= \frac{500 \times 7.58}{40}$$

$$= 94.69\,A$$

Again, the relationship between voltages & turns is:

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$or,\ \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\therefore E_2 = \frac{3.3 \times 10^3 \times 40}{500}$$

$$= 264\,V$$

For maximum emf:

The emf equation of transformer is,

$$E_1 = 4.44 f N_1 \phi_{max}$$

$$or,\ \phi_{max} = \frac{E_1}{4.44 \times 50 \times 500} \quad \left[\because f = 50Hz\right]$$

$$= \frac{3.3 \times 10^3}{4.44 \times 50 \times 500} = 0.029\,Wb$$

$$or,\ E_2 = \frac{3.3 \times 10^3 \times 40}{500} \quad \therefore \boxed{E_2 = 264}$$

No② A 30KVA 2400/120 V load, 50Hz transformer has a high voltage winding resistance of 0.1Ω and leakage reactance of 0.22Ω. The low voltage winding resistance is 0.035Ω and leakage reactance of ~~0.012~~ 0.012Ω. Find the equivalent winding resistance, reactance and impedance referred to high voltage side and low voltage side.

Soln: Given,

Apparent power (S) = 30KVA

primary/secondary (voltages) = 2400/120V (E_1/E_2)

frequency (f) = 50Hz

primary resistance $(R_1) = 0.1\Omega$

" reactance $(X_1) = 0.22\Omega$

Secondary resistance $(R_2) = 0.035\Omega$

" reactance $(X_2) = 0.012\Omega$

To calculate:

In high voltage/primary side:

Equivalent resistance $(R_{eq}) = ?$

" reactance $(X_{eq}) = ?$

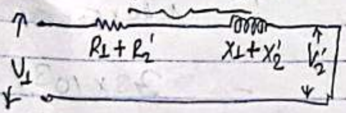
" impedance $(Z_{eq}) = ?$

Now,

transformation ratio $(K) = E_2/E_1 = \dfrac{120}{2400}$

$= 0.05$

The equivalent ckt of transformer referred to primary



first

$\therefore R_{eq} = R_1 + R_2' = ?$

$X_{eq} = X_1 + X_2' = ?$

$Z_{eq} = R_{eq} + jX_{eq} = ?$

first,

$R_2' = R_2/K^2$ (transferred from secondary to p

$= \dfrac{0.035}{0.05^2}$

$= 14\Omega$

$\therefore R_{eq} = 0.1 + 14$

$= 14.1\Omega$

Illy,

$X_{eq} = X_1 + X_2'$

$= 0.012 + X_2/K^2 = 0.22 + 0.012/0.05^2$

$= \cancel{0.012 +}$

$= 5.02\Omega$

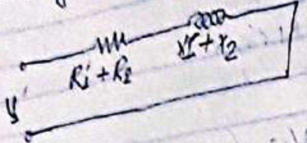
$Z_{eq} = R_{eq} + jX_{eq}$

$= 14.1 + j5.02$

$= 14.96\angle 19.5$

or, $E_2 = \dfrac{3.3\times10^{-3}\times40}{500}$ $\therefore \boxed{E_2 = 264}$

the equivalent circuit of transformer referred to Secondary/low side:



In this case,

$R_{eq} = R_1' + R_2$

$= R_1 \times K^2 + R_2$

$= 0.1 \times 0.05^2 + 0.035$

$= 0.03525\,\Omega$

$||^{ly}$,

$X_{eq} = X_1' + X_2$

$= X_1 \times K^2 + X_2$

$= 0.22 \times 0.05^2 + 0.012$

$= 0.01255\,\Omega$

Equivalent impedance referred to secondary is

$\hat{Z}_{eq} = R_{eq} + jX_{eq}$

$= 0.03525 + j\,0.01255$

$= 0.037\angle 19.5$

③ A single phase transformer has 1000 turns on primary and 200 turns on secondary. The no load current is 3A at a power factor of 0.2 lagging. Calculate the primary current when the secondary current is 280A at power factor of 0.8 lagging.

Soln. Given

$N_1 = 1000$ = primary turns

Secondary turns $(N_2) = 200$

no-load current $(I_0) = 3A$

power factor at primary $(\cos\phi_0) = 0.2$

or $\cos\phi_0 = 0.2$

$\phi_0 = \cos^{-1}(0.2)$

$= 78.46$

$\therefore\ I_0 = 3\angle 78.46$

Again

Given, Secondary current $(I_2) = 280A$

Secondary power factor $(\cos\phi_2) = 0.8$ lagging

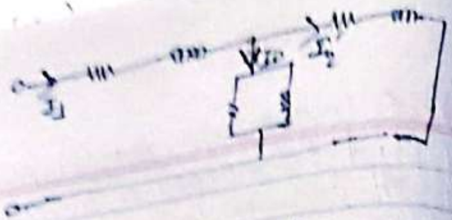
or $\cos\phi_2 = 0.8$

$\therefore\ \phi_2 = \cos^{-1}(0.8)$

$= 36.87$

$\therefore\ I_2 = 280\angle 36.87$

or, $E_2 = \frac{3.3 \times 10^3 \times 40}{500}$ $\therefore E_2 = 264$



Again, to calculate primary current.

First,

$I_2' = I_2/k$ $\qquad I_2' = I_2 K$ $\qquad \left[\therefore N_1 I_2' = N_2 I_2 \quad \therefore I_2' = k \cdot I_2 \right.$

Where,

$$k = \frac{N_2}{N_1} = \frac{200}{\frac{1000}{5}} = 0.2$$

$\therefore I_2' = I_2 K$

$= 280 \angle 36.87 \times 0.2$

$= 56 \angle 36.87$

We know,

$I_1 = I_0 + I_2'$

$= 56 \angle 36.87 + 3 \angle 78.46$

$= 58.27 \angle 38.82$ A

or, $E_2 = \dfrac{3.3 \times 10^3 \times 40}{500}$ $\quad \therefore \boxed{E_2 = 264}$

Q. The primary and secondary winding of 500 kVA, 1-φ transformer have resistance of 0.4 Ω and 0.0015 Ω respectively. The primary and secondary voltages are 6000V and 400V respectively and iron loss is 3.2 kW. Calculate the efficiency on
(i) full load
(ii) half load assuming the p.f of the load to be 0.8

Soln

Given,

Apparent power $(S) = 500 kVA = 500\times10^3 VA$
primary resistance $(R_1) = 0.4 \Omega$
Secondary " $(R_2) = 0.0015 \Omega$
primary voltage $(E_1) = 6000V$
secondary " $(E_2) = 400V$
iron loss (or core loss) $(P_c) = 3.2 kW = 3.2\times$

$\eta = ?$ for full load ckt of
$\eta = ?$ " half load " of transformer
power factor (p.f) $= \cos\phi = 0.8$

We know,

$$\text{efficiency} (\eta) = \frac{\text{output voltage}}{\text{output voltage} + \text{losses}} \times 100$$

$$= \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + I_1^2 R_1 + I_2^2 R_2 + P_c}$$

where,
$\cos\phi = 0.8$

$S = E_1 I_1$

or $500\times10^3 = 6000 \times I_1$

$\therefore I_1 = 83.33A$

& Also,
$S = I_2 E_2$
$\frac{500\times10^3}{400} = I_2$
$\therefore I_2 = 1250A$

(i) $$\therefore \eta = \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + I_1^2 R_1 + I_2^2 R_2 + P_c} \times 100\%$$

$$= \frac{400\times1250\times0.8}{400\times1250\times0.8 + 83.33^2\times0.4 + 1250^2\times0.0015 + 3.2\times10^3} \times 100$$

$= 0.979\times100\%$

$\eta = 97.96\%$ Ans

(ii) for half load, then

$$S = \frac{500 kVA}{2} = 250 kVA = 250\times10^3 VA$$

Again,

$250\times10^3 = E_1 I_1$

$$\therefore I_1 = \frac{250\times10^3}{6000} = 41.67A \quad \& \quad I_2 = \frac{250\times10^3}{400} = 1000$$

$$\therefore \eta = \frac{E_2 I_2 \cos\phi}{E_2 I_2 \cos\phi + I_1^2 R_1 + I_2^2 R_2 + P_c} \times 100\%$$

$$= \frac{400\times1000\times0.8}{400\times1000\times0.8 + 41.67^2\times0.4 + 1000^2\times0.0015 + 3.2\times10^3} \times 100$$

$= 95.411\%$

or, $E_2 = \frac{3.3\times10^3\times40}{500}$ $\therefore \boxed{E_2 = 264}$

Here,

Apparent power = 25 kVA = 25×10^3 VA

Primary voltage (E_1 or V_1) = 3.3 kVA = 3.3×10^3 VA

No. of turns in primary (N_1) = 500

No. of turns in Secondary (N_2) = 40

Primary current (I_1) = ?

Secondary current (I_2) = ?

Secondary emf (E_2) = ?

Maximum emf (ϕ_{max}) = ?

Now,

$$\because A.P = V_1 I_1$$

$$\text{or, } 25 \times 10^3 = 3.3 \times 10^3 \times I_1$$

$$\text{or, } I_1 = \frac{25 \times 10^3}{3.3 \times 10^3}$$

$$\therefore \boxed{I_1 = 7.58}$$

$$\because \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$\text{or, } I_2 = \frac{I_1 N_1}{N_2}$$

$$\text{or, } I_2 = \frac{7.58 \times 500}{40}$$

$$\therefore \boxed{I_2 = 94.75}$$

Again,

$$\because \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$\text{or, } E_2 = \frac{E_1 N_2}{N_1}$$

$$\text{or, } E_2 = \frac{3.3 \times 10^3 \times 40}{500} \quad \therefore \boxed{E_2 = 264}$$

emf equation of transformer is:

$$E = 4.44 f N_1 \phi_{max}$$

or, $$\phi_{max} = \frac{E_1}{4.44 f N_1}$$

or, $$\phi_{max} = \frac{3.3 \times 10^{3}}{4.44 \times 50 \times 500}$$

$$\therefore \boxed{\phi_{max} = 0.029 \text{ Tesla}}$$

(c) A 30 kVA 2400/120 V load, 50 Hz transformer has a high voltage winding resistance of 0.1 Ω and leakage resistance of 0.22 Ω. The low voltage winding resistance is 0.035 Ω and leakage reactance 0.012 Ω. Find the equivalent winding resistance, reactance and impedance referred to high voltage side and low voltage side.

Soln: Here,

primary Resistance $(R_1) = 0.1\ \Omega$

Primary Reactance $(X_1) = 0.22\ \Omega$

secondary Resistance $(R_2) = 0.035\ \Omega$

Secondary Reactance $(X_2) = 0.012\ \Omega$

Eq. Winding Resistance referred to primary $(R_1') = ?$

Eq. Winding Reactance referred to primary $(X_1') = ?$

Eq. Winding Impedance referred to primary $(Z_1') = ?$

Eq. winding Resistance referred to secondary $(R_2') = ?$

Eq. winding Reactance referred to secondary $(X_2') = ?$

Eq. winding Impedance referred to secondary $(Z_2') = ?$

Now, Voltage Transformation ratio:

$$k = \frac{E_2}{E_1} = \frac{120}{2400} = 0.05$$

Now, Eq. Resistance referred to primary:

$$R_1' = R_1 + R_2'$$
$$= 0.1 + \frac{R_2}{k^2}$$
$$= 0.1 + \frac{0.035}{(0.05)^2}$$
$$= 14.1\ \Omega$$

Eq. reactance referred to Primary:

$$X_1' = X_1 + X_2'$$
$$= X_1 + \frac{X_2}{k^2}$$
$$= 0.22 + \frac{0.012}{(0.05)^2}$$
$$= 5.02\ \Omega$$

P.T.O

Eq. Impedance referred to primary:

$$\cancel{Z_1' = R_1'^2 + j X_1'^2}$$
$$= \cancel{(14.1)^2 + j(5.02)^2}$$
$$= \cancel{198.81 + j25.2004} = \cancel{224.01 \angle 0}$$

$$\sqrt{R_1^2 + X_1^2}$$
$$= \sqrt{(14.1)^2 + (5.02)^2}$$
$$= \sqrt{224.01}$$

Eq. Impedance referred to primary:

$Z_1' = R_1' + jX_1'$

$= 14.1 + j5.02$

$= 14.96 \angle 19.5$

Now,

Eq. resistance referred to secondary:

$R_2' = R_2 + R_1'$

$= R_2 + R_1 \times k^2$

$= 0.035 + 0.1 \times (0.05)^2$

$= 0.03525\ \Omega$

Eq. reactance referred to secondary:

$X_2' = X_2 + X_1'$

$\cancel{= X_2 + \frac{X_1}{k^2}}$

$= X_2 + X_1 \times k^2$

$= 0.012 + 0.22 \times (0.05)^2$

$= 0.01255\ \Omega$

Eq. Impedance referred to secondary:

$Z_2' = R_2' + jX_2'$

$= 0.03525 + j0.01255$

$= 0.037 \angle 19.5$

Q. A 50kVA 4400/200V load transformer has the parameters $R_1 = 3.45\Omega$, $R_2 = 0.009\Omega$, $X_1 = 5.2\Omega$, $X_2 = 0.015\Omega$.

Calculate the transformer's

i) Eq. resistance as referred to primary (R_1') and secondary (R_2').

ii) Eq. ~~resista~~ reactance as referred to primary (X_1') & secondary (X_2').

iii) Eq. Impedance as referred to primary (Z_1') & secondary (Z_2').

Sol: Here,

$R_1 = 3.45\Omega$

$X_1 = 5.2\Omega$

$R_2 = 0.009\Omega$

$X_2 = 0.015\Omega$

Now, voltage Transformation ratio:

$$K = \frac{E_2}{E_1} = \frac{200}{4400} = 0.045$$

Now,

Eq. resistance as referred to primary:

$R_1' = R_1 + R_2'$

$= R_1 + \frac{R_2}{K^2}$

$= 3.45 + \frac{0.009}{(0.045)^2}$

$= 7.89\ \Omega$

Eq. reactance as referred to primary:

$$X_1' = X_1 + X_2'$$
$$= X_1 + \frac{X_2}{k^2}$$
$$= 5.2 + \frac{0.015}{(0.045)^2}$$
$$= 12.6\ \Omega$$

Eq. Impedance referred to primary:

$$Z_1' = R_1' + jX_1'$$
$$= 7.89 + j\,12.6$$
$$= 14.86 \angle 57.9$$

Eq. resistance as referred to secondary:

$$R_2' = R_2 + R_1'$$
$$= R_2 + R_1 \times k^2$$
$$= 0.009 + 3.45 \times (0.045)^2$$
$$= 15.98 \times 10^{-3}\ \Omega$$

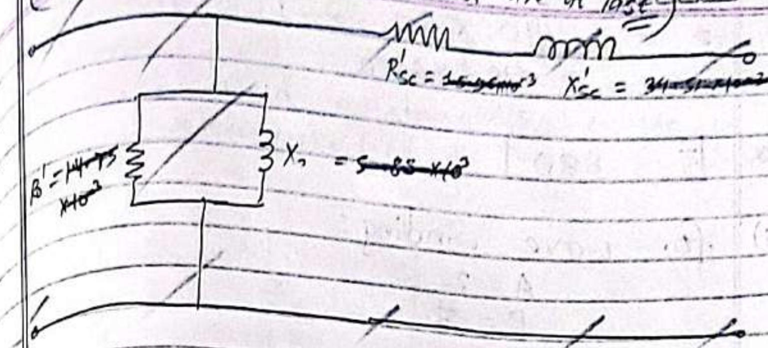
Eq. reactance as referred to secondary:

$$X_2' = X_2 + X_1'$$
$$= X_2 + X_1 \times k^2$$
$$= 0.015 + 5.2 \times (0.045)^2$$
$$= 25.53 \times 10^{-3}\ \Omega$$

Eq. Impedance referred to secondary:

$$Z_2' = R_2' + j\,X_2' = (15.98 \times 10^{-3}) + j\,(25.53 \times 10^{-3})$$
$$= 0.03 \angle 57.95$$

(Numericals of S.C & O.C test are at last)



Emf equation of dc machine:

(8) A 8 pole dc generator running at 1200 rpm with a flux of 25 mwb per pole generates 440V. Calculate the no. of conductor if the armature is (i) lap winding/wounded (ii) wave winding/ "

Soln Here,

No. of magnetic pole (p) = 8

speed of armature (N) = 1200 rpm

magnetic flux per pole (ϕ) = 25 mwb $= 25 \times 10^{-3}$ wb

Back emf (E_b) = 440 V

No. of conductor in armature (Z) = ?

(i) For lap winding:

$A = P = 8$

$$\therefore E_b = \frac{Z\phi N}{60} \times \frac{P}{A}$$

or, $$440 = \frac{Z \times 25 \times 10^{-3} \times 1200}{60} \times \frac{8}{8}$$

$$\therefore Z = \frac{440 \times 60}{25\times10^{-3}\times1200} \times \frac{1}{1}$$

$$\therefore \boxed{Z = 880}$$

(ii) for wave winding:

$A = 2$

$P = 8$

$$\therefore E_b = \frac{Z\phi N}{60} \times \frac{P}{A}$$

$$\text{or, } 440 = \frac{Z \times 25\times10^{-3}\times1200}{60} \times \frac{8}{2}$$

$$\text{or, } Z = \frac{440\times60\times2}{25\times10^{-3}\times1200\times8}$$

$$\therefore \boxed{Z = 220}$$

Long shunt Compound Generator:

(1) A long shunt compound generator delivers 60A to the load at 220V. calculate the emf generated by the armature given that armature winding resistance 0.04Ω, series field resistance 0.06Ω & shunt field winding resistance 110Ω.

Sol: Here,

load current $(I_L) = 60A$

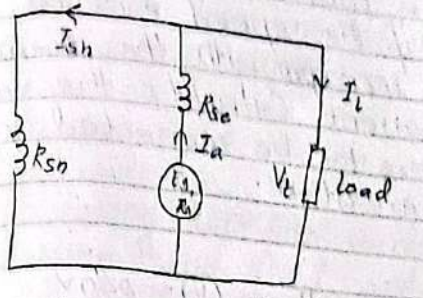
Terminal voltage $(V_t) = 220V$

Armature resistance $(R_a) = 0.04\Omega$

series field resistance $(R_{se}) = 0.06\Omega$

Shunt field resistance $(R_{sh}) = 110\Omega$

Emf generated $(E) = ?$



we know, for long shunt compound generator emf induced is:

$$E = V_t + I_aR_a + I_aR_{se} \quad \text{---(1)}$$

$$\left(\because V = E - I_aR_a - I_aR_{se} \Rightarrow V + I_aR_a + I_aR_{se} = E\right)$$

$$I_a = I_{sh} + I_L$$

$$= \frac{V_L}{R_{sh}} + 60$$

$$= \frac{220}{110} + 60$$

$$= 62\ A$$

$$\therefore E = 220 + (62\times0.04) + (62\times0.06)$$

$$= 226.2\ V$$

Ans

Speed control of dc motor:

(10) A 220V shunt motor runs at 1450 rpm at full load with an armature current of 11A. The total resistance of armature is 0.6 Ω. If the speed is to be reduced to 1000 rpm with the same armature current, calculate the value of resistance to be connected in series with the armature.

Soln: Here,

supplied voltage $(V) = 220V$

full load speed $(N_1) = 1450$ rpm

Initial Armature current $(I_a) = 11$ A

Armature resistance $(R_a) = 0.6\,\Omega$

load speed $(N_2) = 1000$ rpm

final Armature current $(I_{a_2}) = 11$ A

Variable Resistance $(R_v) = ?$

Now, we have:

$$\frac{N_2}{N_1} = \frac{V - I_{a_2}(R_a + R_v)}{V - I_{a_1} R_a}$$

or, $$\frac{1000}{1450} = \frac{220 - 11(0.6 + R_v)}{220 - 11 \times 0.6}$$

or, $$\frac{1000 \times 213.4}{1450} = 220 - 6.6 - 11R_v$$

or, $$\frac{213400}{1450} - 220 + 6.6 = -11R_v$$

or, $$\frac{-96030}{1450} \times \frac{1}{-11} = R_v$$

$$\boxed{R_v = 6.02\,\Omega}$$

(11) A 220 V dc has no load speed of 800 rpm and armature current of 2A. Calculate the resistance required to insert in series with the field winding resistance so that the motor may run at the speed of 1000 rpm taking the line current of 30A. The armature and field winding resistance are 0.4 Ω & 165 Ω respectively.

Soln: Here,

Supplied voltage $(V) = 220V$

no load speed $(N_1) = 800$ rpm

Armature current $(I_a) = 2A$

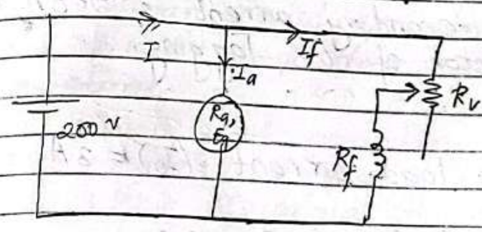
full load speed $(N_2) = 1000$ rpm

line current $(I) = 30$ A

Armature resistance $(R_a) = 0.4\,\Omega$

field winding resistance $(R_f) = 165\,\Omega$

Resistance to insert in series to increase the speed $(R_v) = ?$



$$\boxed{I_1 = I_a + I_f}$$

$$\boxed{I_f = \frac{V}{R_v + R_f}}$$

We have:

$I = I_a + I_f$

or, $30 = 2 + I_f$

or, $I_f = 30 - 2$

$\therefore \boxed{I_f = 28A}$

Again, $I_f = \frac{V}{R_v + R_f}$

or, $28 = \frac{220}{R_v + 165}$

or, $28 R_v + 4620 = 220$

or, $R_v = \frac{220 - 4620}{28}$

$\therefore \boxed{R_v = -157.14\,\Omega}$

Transformer:

(12) A single phase transformer has 1000 turns on primary and 200 turns on secondary. The no load current is 3A at a power factor of 0.2 lagging. Calculate the primary current when the secondary current is 280A at power factor of 0.8 lagging.

Soln: Here,

No load current $(I_0) = 3A$

P.f at primary $\cos\phi_0 = 0.2$

or, $\phi_0 = \cos^{-1}(0.2)$

$\therefore \phi_0 = 78.46$

$\therefore I_0 = 3\angle 78.46$

Now,

$I_2 = 280\,A$

P.f at secondary $\cos\phi_2 = 0.8$

or, $\phi_2 = \cos^{-1}(0.8)$

$\therefore \phi_2 = 36.87$

$\therefore I_2 = 280\angle 36.87$ (V_2 is taken as reference)

Now,

$I_2' = I_2 \times K \angle 36.87$

when $K = \frac{N_2}{N_1} = \frac{200}{1000} = 0.2$

$\therefore I_2' = (280 \times 0.2)\angle 36.87$

$= 56\angle 36.87$

Now, $I_1 = I_0 + I_2'$

$= 56\angle 36.87 + 3\angle 78.46$

$= 58.27\angle 38.82$ Ans

(13) The primary and secondary winding of a 500 kVA, 1-ϕ transformer have resistance of $0.4\,\Omega$ and $0.0015\,\Omega$ respectively. The primary and secondary voltages are 6000 V and 400 V respectively and iron loss is 3.2 kW. Calculate the efficiency on

(i) full load

(ii) half load assuming the p.f of the load to be 0.8.

Soln: Here,

$S = 500\ kVA = 500 \times 10^3\ VA$

primary voltage $(V_1) = 6000\ V$

Secondary voltage $(V_2) = 400\ V$

$R_1 = 0.4\ \Omega$ & $R_2 = 0.0015\ \Omega$

We know that:

Apparent power $(S) = V_1 I_1 = V_2 I_2$

$\therefore I_2 = \frac{S}{V_2} = \frac{500 \times 10^3}{400} = 1250\ A$

Now, $k = \frac{V_2}{V_1} = \frac{400}{6000} = 0.067$

Now, Total resistance referred to secondary side:

$R_2' = R_2 + R_1'$

$= R_2 + R_1 \times K^2$

$= 0.0015 + 0.4 \times (0.067)^2$

$= 0.0032956$

$= 3.2956 \times 10^{-3}\ \Omega$

full load copper loss $(W_{cu}) = I_2^2 R_2'$

$= (1250)^2 \times 3.2956 \times 10^{-3}$

$= 5149.375\ W$

$= 5.149\ kW$

Iron loss $(W_{fe}) = 3.2\ kW$

(i) full load output at 0.8 pf ie $\cos\phi = 0.8$

$= S \times \cos\phi$

$= 500 \times 0.8$

$= 400\ kW$

efficiency $(\eta) = \frac{\text{output}}{\text{output} + \text{losses}} \times 100\%$

$= \frac{400}{400 + 3.2 + 5.149} \times 100\%$

$= 97.95\ \%$

(ii) Output at half load at 0.8 pf ie $\cos\phi = 0.8$

$= \frac{1}{2} \times S \times \cos\phi$

$= \frac{1}{2} \times 500 \times 0.8 = 200\ kW$

~~efficiency~~

copper loss at half load $= \left(\frac{1}{2}\right)^2 W_{cu}$

$= \left(\frac{1}{2}\right)^2 \times 5.149$

$= 1.28725\ kW$

efficiency $(\eta) = \frac{\text{output}}{\text{output} + \text{iron loss} + \text{copper loss}} \times 100\%$

$= \frac{200}{200 + 3.2 + 1.28725} \times 100\%$

$= 97.81\ \%$

Voltage Regulation:

The way in which the secondary terminal voltage varies with the load depends on the load current the internal impedance and the load power factor. The change in secondary terminal voltage from no load to full load at any particular load is termed regulation. It is usually expressed as a percentage of the rated no load terminal voltage.

$\therefore$ percentage regulation $= \dfrac{\text{Terminal voltage on no-load} - \text{terminal voltage on load}}{\text{Terminal voltage on no load}}$

for lagging p.f:

$$\% \text{ Regulation} = \frac{I_2 R_2' \cos\phi + I_2 X_2' \sin\phi}{E_2} \times 100\%$$

$$= \left(\frac{I_2 R_2' \cos\phi}{E_2} + \frac{I_2 X_2'}{E_2} \sin\phi \right) \times 100\%$$

$$= \frac{I_2 R_2'}{E_2} \times 100 \times \cos\phi + \frac{I_2 X_2'}{E_2} \times 100 \times \sin\phi$$

$$= \% R \cos\phi + \% X \sin\phi$$

for leading p.f:

$$\% \text{ Regulation} = \left(\frac{I_2 R_2' \cos\phi - I_2 X_2' \sin\phi}{E_2} \right) \times 100\%$$

(15) The primary and secondary windings of 40 kVA, 6600/250 V, 1-ϕ transformer have resistance of 10Ω and 0.02Ω respectively. The total leakage reactance is 35Ω as referred to the primary winding. Find the full load regulation at p.f of 0.8 lagging.

Sol: Here,

$S = 40 \text{ kVA} = 40 \times 10^3 \text{ VA}$

$R_1 = 10\Omega$

$R_2 = 0.02\Omega$

$V_1 = 6600 \text{ V}$

$V_2 = 250 \text{ V}$

$X_1' = 35\Omega$

Now, $K = \dfrac{V_2}{V_1}$

or, $k = \dfrac{250}{6600}$

$\therefore \boxed{k = 0.0378}$

$I_2 = \dfrac{S}{V_2}$

or, $I_2 = \dfrac{40 \times 10^3}{250}$

$\therefore \boxed{I_2 = 160 \text{ A}}$

Now, Eq. resistance referred to secondary:

$R_2' = R_2 + R_1'$

$= R_2 + R_1 \times k^2$

$= 0.02 + 10 \times (0.0378)^2$

$= 34.28 \times 10^{-3}\ \Omega$

Eq. reactance referred to secondary:

$X_2' = X_2 + X_1'$
$= 0 + X_1 \times k^2$
$= 35 \times (0.0378)^2$
$= 50.0094 \times 10^{-3}\ \Omega$

Power factor = 0.8
$\cos\phi = 0.8$

$\sin\phi = \sqrt{1-\cos^2\phi} = \sqrt{1-(0.8)^2} = 0.6$

full load regulation is given by:

$= \frac{I_2 R_2' \cos\phi + I_2 X_2' \sin\phi}{E_2} \times 100\%$

$= \frac{(160 \times 34.28 \times 10^{-3} \times 0.8) + (160 \times 50.0094 \times 10^{-3} \times 0.6)}{250} \times 100$

$= 3.67\ \%$

All day efficiency:

(15) A 400kVA, distribution transformer has full load iron loss of 2.5kW of copper loss of 3.5kW. During a day, its load cycle per 24 hours is:

6 hrs 300 kW at 0.8 p.f
4 hrs 1000 kW at 0.9 p.f

Determine its all day efficiency.

Sol: Given:

$P_{Fe} = 2.5\ kW$
$P_{Cu} = 3.5\ kW$
400 kVA

∵ Iron losses is constant for 24 hrs. so energy spent due to iron (Fe) losses for 24 hrs is:

$P_{Fe} = 2.5 \times 24$ hrs
$= 60$ kWh

Total energy output in a day from give load cycle is:

Energy output = 300 × 6 hrs + 200 × 10 hrs + 100 × 4 hrs
= 4200 kWh

To calculate energy spent due to copper loss:

(i) load 1 of 300 kW at p.f 0.8 i.e $\cos\phi = 0.8$

∴ kVA supplid $= \frac{kW}{\cos\phi}$

$= \frac{300}{0.8} = 375$ kVA

∴ η = load kVA / kVA rating

$= \frac{375}{400} = 0.9375$

Copper losses are proportional to square of kVA ratio i.e η^2

$\therefore$ load 1 $P_{cu} = \eta^2 \times (P_{cu})$.

$= (0.9375)^2 \times 3.5$

$= 3.076$ kW

Energy spent $= 3.076 \times 6$ hrs

$= 18.457$ kWh

(ii) load 2 of 200 kW at $\cos\phi = 0.7$

kVA supplied $= \frac{kW}{\cos\phi}$

$= \frac{200}{0.7}$

$= 285.71$ kVA

$\eta = \frac{\text{load kVA}}{\text{kVA rating}}$

$= \frac{285.71}{400}$

$= 0.7142$

$\therefore$ load 2 $P_{cu} = \eta^2 \times (P_{cu})$

$= (0.7142)^2 \times 3.5$

$= 1.785$ kW

$\therefore$ Energy spent $= 1.785 \times 10$

$= 17.85$ kWh

(iii) load 3 of 100 kW at $\cos\phi = 0.9$

kVA supplid $= \frac{kW}{\cos\phi} = \frac{100}{0.9} = 111.111$ kVA

$\eta = \frac{\text{load kVA}}{\text{kVA rating}}$

$= \frac{111.111}{400}$

$= 0.2778$

$\therefore$ load 3 $P_{cu} = \eta^2 \times P_{cu}$

$= (0.2778)^2 \times 3.5$

$= 0.2701$ kW

Energy spent $= 0.2701 \times 4$

$= 1.0804$ kWh

(iv) No load hence negligible copper losses.

$\therefore$ Total energy spent = Energy spent due to (Iron loss + Copper loss)

$= 60 + 18.457 + 17.85 + 1.0804$

$= 97.3874$ kWh

and Total output = 4200 kWh

$\therefore$ All day efficiency(η) $= \frac{\text{Total energy for 24hrs}}{\text{Total output for 24 hrs + Total energy spend in 24hrs}} \times 100\%$

$$= \frac{4200}{4200 + 97.3874} \times 100\%$$

$= 97.73\%$ Ans

Open Circuit and Short Circuit Test:

① The no load current of 1-ϕ transformer is 5A at a powerfactor of 0.3. When supplied at 230V, 50Hz. The no. of turns in primary winding 200. Calculate:
(i) max value of flux
(ii) Iron loss of transformer
(iii) Magnetizing current

Soln: Here,
No load current $(I_0) = 5A$
p.f $\cos\phi = 0.3$
primary voltage $(V_1) = 230V$
frequency $(f) = 50\,Hz$
No. of turns in primary winding $(N_1) = 200$
max value of flux $\phi_{max} = ?$
Iron loss $= ?$
Magnetizing current $(I_\mu) = ?$

Now, we know that:

$$E = 4.44 f N_1 \phi_{max}$$

or, $\phi_{max} = \frac{E}{4.44 f N_1}$

or, $\phi_{max} = \frac{230}{4.44 \times 50 \times 200}$

$\therefore \phi_{max} = 5.18 \times 10^{-3}$ Tesla

Now, ~~$W_0 = V_1 I_0 \cos\phi = 230 \times 5 \times 0.2 = 345$ watt~~

Iron loss $(I_w) = I_0 \cos\phi$
$= 5 \times 0.3 = 1.5A$

~~$I_\mu = I_0\cos\phi = 5 \times 0.3 = 1.5A$~~

~~$I_\mu = I_0 \sin\phi = 5 \times \sin$~~

$\because I_0^2 = I_w^2 + I_\mu^2$
$I_0^2 - I_w^2 = I_\mu^2$
$I_\mu^2 = 25 - 2.25$
$\therefore I_\mu = 4.76A$

② Calculate the open circuit parameters having 200/400V, 50Hz transformer with the given open circuit parameters.

O.C Test : 200V 0.7A 70W on L.V side

Soln: Here,
open circuit voltage $(V_1) = 200V$
No load current $(I_0) = 0.7A$
power $(W_0) = 70W$

Now, $W_0 = V_1 I_0 \cos\phi$
or, $70 = 200 \times 0.7 \times \cos\phi$
or, $\cos\phi = \frac{70}{200 \times 0.7}$
or, $\cos\phi = 1/2$
$\therefore \boxed{\phi = 60}$

$I_w = I_0 \cos\phi$
$= 0.7 \times \cos 60 = 0.35A$

$I_\mu / I_m = I_0 \sin\phi$
$= 0.7 \times \sin 60 = 0.61 A$

$R_0 = \frac{V_1}{I_\omega} = \frac{200}{0.35} = 571.42\ \Omega$

$X_0 = \frac{V_1}{I_\mu} = \frac{200}{0.61} = 327.86\ \Omega$

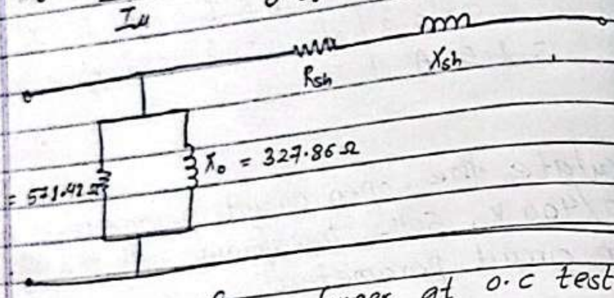


fig: Eq. circuit of transfomer at o.c test.

Note:

Open Circuit Parameters:

$\boxed{W_0 = V_1 I_0 \cos\phi}$ $\boxed{I_0 = \sqrt{I_\mu^2 + I_w^2}}$

Given:
V_1 (o.c voltage)
I_0 (no load current)
W_0 (power)

$I_\omega = I_0 \cos\phi$

$I_\mu / I_m = I_0 \sin\phi$

$R_0 = \frac{V_1}{I_\omega}$

$X_0 = \frac{V_1}{I_m}$ or $\frac{V_1}{I_\mu}$

Short Circuit Parameters:

Given:
V_{sc} (S.C voltage)
I_{sc} (S.C current)
W_{sc} (S.C power)

$R_{sc} = \frac{W_{sc}}{I_{sc}^2}$

$Z_{sc} = \frac{V_{sc}}{I_{sc}}$

$X_{sc} = \sqrt{Z_{sc}^2 - R_{sc}^2}$

(3) following data were obtained from the test of 30kVA, 2000/110 v, 50Hz 1ϕ transformer.

O.C Test : 110 V 10.2 A 350 W (L.V side)
S.C Test : 150 V 10 A 500 W (H.V side)

Calculate and draw the equivalent circuit referred to primary side.

Sol: For O.C Test (L.V side):

O.C voltage $(V_1) = 110$ v
No load current $(I_0) = 10.2$ A
Power $(W_0) = 350$ W

$\because W_0 = V_1 I_0 \cos\phi$

or $\cos\phi = \frac{W_0}{V_1 I_0}$

$\therefore \cos\phi = \frac{350}{110 \times 10.2}$

$\boxed{\phi = 71.82}$

$I_w = I_0 \cos\phi$
$= 10.2 \times \cos(71.82)$
$= 3.18\ A$

$I_\mu / I_m = I_0 \sin\phi$
$= 10.2 \times \sin(71.82)$
$= 9.69\ A$

$R_0 = \frac{V_1}{I_w} = \frac{110}{3.18} = 34.59\ \Omega$

$X_0 = \frac{V_1}{I_\mu} = \frac{110}{9.69} = 11.35\ \Omega$

For S.C Test (H.V side):

secondary voltage (V_{sc}) = 150 V
W_{sc} = 500 W
I_{sc} = 10 A

Now, $R_{sc} = \frac{W_{sc}}{I_{sc}^2} = \frac{500}{(10)^2} = 5\ \Omega$

$Z_{sc} = \frac{V_{sc}}{I_{sc}} = \frac{150}{10} = 15\ \Omega$

$X_{sc} = \sqrt{Z_{sc}^2 - R_{sc}^2}$
$= \sqrt{(15)^2 - (5)^2} = 14.14\ \Omega$

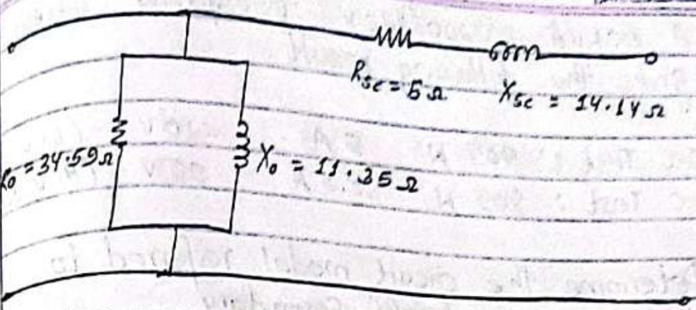


fig: Eq. circuit of transformormer

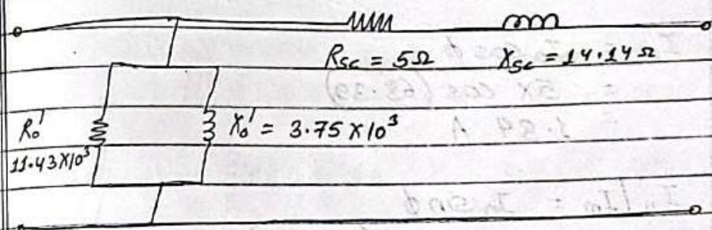
Now, Circuit parameters referred to Primary side (L.V side - H.V side):

Voltage Transformation ratio:

$K = \frac{E_2}{E_1} = \frac{110}{2000} = 0.055$

$R_0' = \frac{R_0}{K^2} = \frac{34.59}{(0.055)^2} = 11.43 \times 10^3$

$X_0' = \frac{X_0}{K^2} = \frac{11.35}{(0.055)^2} = 3.75 \times 10^3$



Eq. circuit of transformer referred to primary side

(4) A 50kVA, 2200/220V transformer when tested gives the following result:

O.C Test : 405 W 5 A 220 V (L.V side)
S.C Test : 805 W 20.2 A 95 V (H.V side)

Determine the circuit modal referred to (i) primary and (ii) secondary

Sol. Here,

For O.C Test (L.V side):

$V_1 = 220$ V
$I_0 = 5$ A
$W_0 = 405$ W

Now, $W_0 = V_1 I_0 \cos\phi$

or, $\cos\phi = \frac{W_0}{V_1 I_0}$

or, $\cos\phi = \frac{405}{220 \times 5}$

or, $\phi = \cos^{-1}\left(\frac{405}{2200 \times 5}\right)$

$\boxed{\therefore \phi = 68.39}$

$I_w = I_0 \cos\phi$
$= 5 \times \cos(68.39)$
$= 1.84$ A

$I_\mu / I_m = I_0 \sin\phi$
$= 5 \times \sin(68.39)$
$= 4.64$

$R_0 = \frac{V_1}{I_w} = \frac{220}{1.84} = 119.56\ \Omega$

$X_0 = \frac{V_1}{I_\mu} = \frac{220}{4.64} = 47.41\ \Omega$

For S.C Test (H.V side):

$W_{sc} = 805$ W
$I_{sc} = 20.2$ A
$V_{sc} = 95$ V

Now, $R_{sc} = \frac{W_{sc}}{I_{sc}^2} = \frac{805}{(20.2)^2} = 1.97$

$Z_{sc} = \frac{V_{sc}}{I_{sc}} = \frac{95}{20.2} = 4.702$.

$X_{sc} = \sqrt{Z_{sc}^2 - R_{sc}^2}$

$= \sqrt{(4.702)^2 - (1.97)^2}$

$= 4.26\ \Omega$

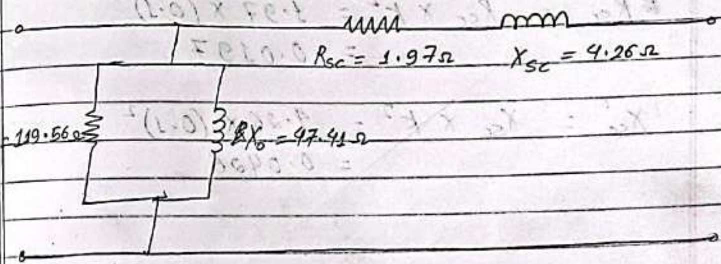


fig: Equivalent circuit of transformer

(i) When circuit parameters referred to primary (l.v side) - H.V side)

$$k = \frac{E_2}{E_1} = \frac{220}{2200} = 0.1$$

$$R_0' = \frac{R_0}{k^2} = \frac{119.56}{(0.1)^2} = 11956\ \Omega$$

$$X_0' = \frac{X_0}{k^2} = \frac{47.41}{(0.1)^2} = 4741\ \Omega$$

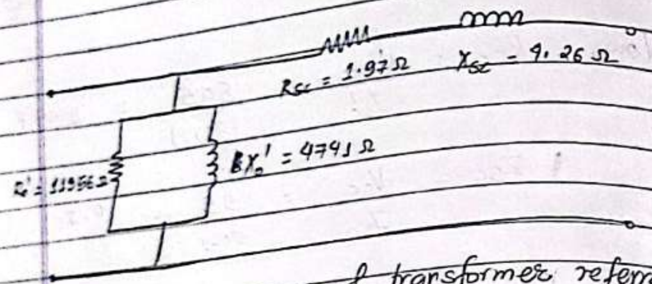


fig: Equivalent circuit of transformer referred to primary.

(ii) When circuit parameters referred to secondary (H.V side) - L.V side)

$$R_{sc}' = R_{sc} \times k^2 = 1.97 \times (0.1)^2$$
$$= 0.0197$$

$$X_{sc}' = X_{sc} \times k^2 = 4.26 \times (0.1)^2$$
$$= 0.0426$$

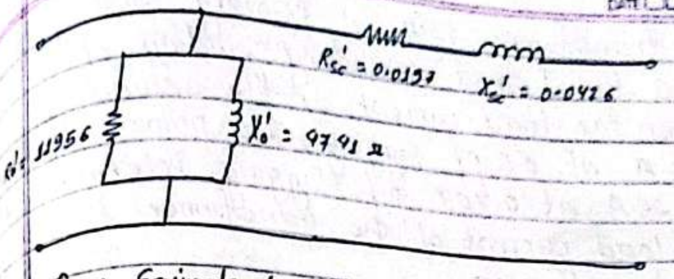


fig: Equivalent circuit of transformer referred to secondary.

Transformer:

① A transformer has a primary winding of 800 turns and secondary winding of 200 turns. When the load current on the secondary is 80 A at 0.8 pf lagging. The primary current is 25A at 0.707 p.f lagging. Determine the no load current of the transformer. Draw the vector diagram.

Sol: Here,

$N_1 = 800$

$N_2 = 200$

$I_2 = 80$ A

pf at secondary, $\cos\phi_2 = 0.8$

$\therefore \phi_2 = 36.86$

$\therefore \boxed{I_2 = 80\angle 36.86 \text{ A}}$

$I_1 = 25$ A

p.f at primary, $\cos\phi_1 = 0.707$

$\phi_1 = 45$

$\therefore \boxed{I_1 = 25\angle 45 \text{ A}}$

$\because I_1 = I_0 + I_2'$

$\Rightarrow I_0 = I_1 - I_2'$

Now, $K = \frac{N_2}{N_1} = \frac{200}{800} = 0.25$

$\therefore I_2' = 80 \times 0.25 \angle 36.86$

$= 20\angle 36.86$

$\therefore I_0 = I_1 - I_2'$

$= (25\angle 45) - (20\angle 36.86)$

$= (17.67 + j\,17.67) - (16 + j12)$

$= 1.67 + j\,5.67$

$= 5.91\angle 73.5$ A Ans

vector diagram:

② A 12 KVA transformer having primary voltage of 2000 V at 50Hz has $N_1 = 180$, $N_2 = 30$. Find (i) the full load primary & secondary current (ii) The no-load secondary induced emf (iii) the maximum flux in the core.

Sol: Here,

$N_1 = 180$

$N_2 = 30$

$E_1 = V_1 = 2000$ V

$AP = 12$ KVA $= 12 \times 10^3$ VA

$f = 50$ Hz

(i) $I_1 = ?$ & $I_2 = ?$

(ii) secondary emf $(E_2) = ?$

(iii) maximum flux $(\phi_{max}) = ?$

Now, $\because AP = 12 \times 10^3$

or, $V_1 I_1 = 12 \times 10^3$

or, $2000 \times I_1 = 12 \times 10^3$

or, $I_1 = \frac{12 \times 10^3}{2000}$

$\boxed{I_1 = 6A}$

$\because \frac{I_1}{I_2} = \frac{N_2}{N_1}$

or, $I_2 = \frac{I_1 N_1}{N_2}$

or, $I_2 = \frac{6 \times 180}{30}$

$\therefore \boxed{I_2 = 36\ A}$

~~Now, emf per turn of primary side(E_1) = $\frac{V_1}{N_1}$~~

Now, we know that:

$= \cancel{\frac{2000}{180}}$

$\frac{E_1}{E_2} = \frac{N_1}{N_2}$

$= \cancel{11.11}$

or, $E_2 = \frac{E_1 N_2}{N_1}$

or, $E_2 = \frac{2000 \times 30}{180}$

$\therefore \boxed{E_2 = 333.33\text{V}}$

Now, Emf equn of transformer is:

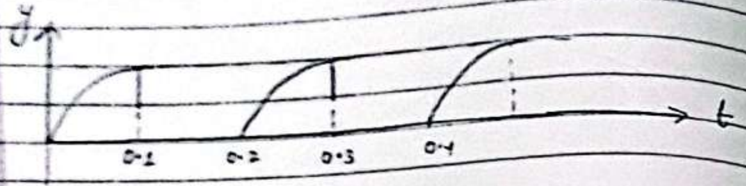
$$E_1 = 4.44 f N_1 \phi_{max}$$

or, $\phi_{max} = \frac{E_1}{4.44 f N_1}$

or, $\phi_{max} = \frac{2000}{4.44 \times 50 \times 180}$

$\therefore \boxed{\phi_{max} = 0.05\ \text{Tesla}}$

Q. Determine the peak factor and form factor of the function shown below if it is given that: for $0 < t < 0.1$, $y = 10(1 - e^{-100t})$
$0.1 < t < 0.2$, $y = 10e^{-50(t-0.1)}$



Sol:

Time period $(T) = 0.2$

$$y_{av} = \frac{1}{0.2}\int_0^{0.2} y(t)\,dt$$

$$= \frac{1}{0.2}\left[\int_0^{0.1} y_1(t)\,dt + \int_{0.1}^{0.2} y_2(t)\,dt\right]$$

$$= \frac{1}{0.2}\left[\int_0^{0.1} 10(1-e^{-100t})\,dt + \int_{0.1}^{0.2} 10e^{-50(t-0.1)}\,dt\right]$$

$$= \frac{10}{0.2}\left[\int_0^{0.1}(1-e^{-100t})\,dt + \int_{0.1}^{0.2} e^{-50(t-0.1)}\,dt\right]$$

$$= 50\left[\left|t - \frac{e^{-100t}}{-100}\right|_0^{0.1} + \int_{0.1}^{0.2}(e^{-50t+5})\,dt\right]$$