Chapter 1: Signals, Systems and Signal Processing[i].

Signal:

We know that a signal can be a rather abstract notion, such as a flashing light on our bike (turn signal), or a referee's whistle indicating start, halt or end of football match, door bell, etc. Signal can be defined as a detectable physical quantity or impulse (as a voltage, current or magnetic field strength) by which messages or information can be transmitted.

A signal is a source of information generally a physical quantity which varies with respect to time, space, temperature like any independent variable. In other word, a *signal* is function of independent variables that carry some information.

Eg. $x(t) = 10t + 7t^2$. →Signal of one independent variable (i.e. time)

$s(x,y) = 3x + 2xy + 10y^2$. →Signal of two independent variables (i.e. x and y).

Speech, electrocardiogram (ECG), electroencephalogram (EEG), etc signals are examples of information-bearing signals that evolve as functions of a single independent variable, namely, time. An example of signal that is function of two independent variables is an image signal.

System:

A system may be defined as a physical device that performs an operation on a signal. More specifically, a system is something that can **manipulate, change, record, or transmit** signals. In Digital Signal Processing, a system may be defined by algorithm. For example, a filter used to reduce the noise and interference corrupting a desired information-bearing signal is called a system. In this case the filter performs some operation/s on the signal, which has the effect of reducing (filtering) the noise and interference from the desired information-bearing signal.

Signal Processing:

When we pass a signal through a system, as in filtering, we say that we have processed the signal. In this modern world we are surrounded by all kinds of signals in various forms. Some of the signals are natural, but most of the signals are manmade. Some signals are necessary (speech), some are pleasant (music), while many are unwanted or unnecessary in a given situation. In an engineering context, signals are carriers of information, both useful and unwanted. Therefore extracting or enhancing the useful information from a mix of conflicting information is a simplest form of signal processing. More generally, signal processing is an operation designed for extracting, enhancing, storing, and transmitting useful information. The distinction between useful and unwanted information is often subjective as well as objective. Hence signal processing tends to be application dependent.

Digital Signal Processing (DSP)

Digital Signal Processing refers to methods of filtering and analyzing time-varying signals based on the assumption that the signal amplitudes can be represented by a finite set of integers corresponding to the amplitude of the signal at a finite number of points in time. Digital Signal Processing is distinguished from other areas in computer science by the unique type of data it uses: *signals*. In most cases, these signals originate as sensory data from the real world: seismic vibrations, visual images, sound waves, etc. DSP is the mathematics, the algorithms, and the techniques used to manipulate these signals after they have been converted into a digital form. This includes a wide variety of goals, such as: enhancement of visual images, recognition and generation of speech, compression of data for storage and transmission, etc.

Digital signal processing is the study of signals in a digital representation and the processing methods of these signals. DSP includes subfields like: audio signal processing, control engineering, digital image processingand speech processing. RADAR Signal processing and communications signal processing are two other important subfields of DSP.
Since the goal of DSP is usually to measure or filter continuous real-world analog signals, the first step is usually to convert the signal from an analog to a digital form, by using an analog to digital converter. Often, the required output signal is another analog output signal, which requires a digital to analog converter.
The algorithms required for DSP are sometimes performed using specialized computers, which make use of specialized microprocessors called digital signal processors (also abbreviated *DSP*). These process signals in real time and are generally purpose-designed application-specific integrated circuits (ASICs). When flexibility and rapid development are more important than unit costs at high volume, DSP algorithms may also be implemented using field-programmable gate arrays (FPGAs).

Basic Elements of a Digital Signal Processing System:

The signals that we encounter in practice are mostly analog signals. These signals, which vary continuously in time and amplitude, are processed using electrical networks containing active and passive circuit elements. This approach is known as analog signal processing (ASP), for example, radio and television receivers.

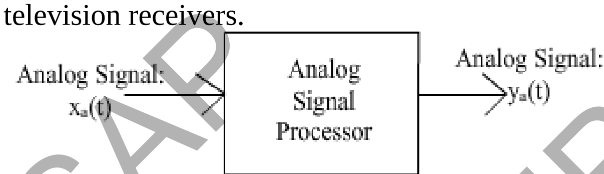

Figure 1: Analog Signal Processor.

They can also be processed using digital hardware containing adders, multipliers, and logic elements or using special-purpose microprocessors. However, one needs to convert analog signals into a form suitable for digital hardware. This form of the signal is called a digital signal. It takes of the finite number of values at specific instances in time, and hence it can be represented by binary numbers, or bits. The processing of digital signals is called DSP: in block diagram form it is represented by

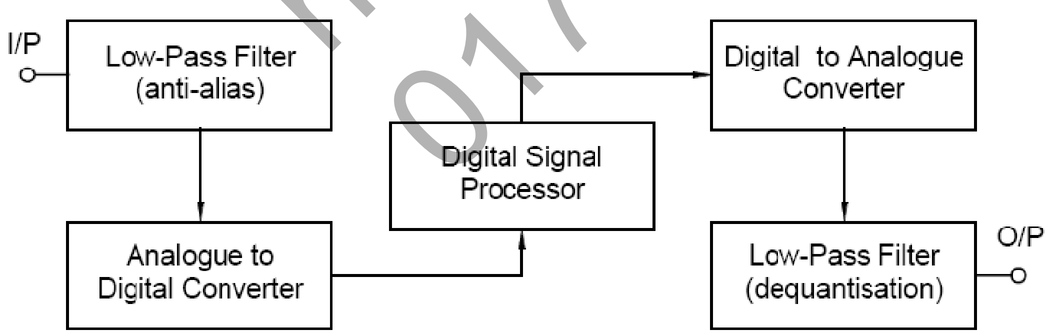

Figure 2: Basic Elements of Digital Signal Processing.

The input to and output from the systems are analog in nature. The various block elements are discussed below:

Low-Pass Filter (anti-alias): This is a prefilter or an antialiasing filter, which conditions the analog signal to prevent aliasing.

Analog to Digital Converter (ADC): ADC produces a stream of binary numbers from analog signals. It has three sub-elements as sampling, quantization and encoding.

Digital Signal Processor: This is the heart of DSP and can represent a general-purpose computer or a special-purpose processor, or digital hardware, and so on. The basic elements of DSP are adder and multiplier.

Digital to Analog Converter (DAC): This is the inverse operation to the ADC, which produces a staircase waveform from a sequence of binary numbers, a first step towards producing an analog signal.

Low-Pass Filter (dequantisation): This is a postfilter to smooth out staircase waveform into the desired analog signal. It is simply a low pass filter which smooth out (or interpolates) the sequences obtained from DAC.

Performance or these systems is usually limited by the performance (*i.e.* speed, resolution and linearity) of the analog-to-digital converter. However, when using a DSP we should never forget two facts:

- If information was not present in the sampled signal to start with, no amount of digital manipulation will extract it.
- Real signals come with noise.

Advantages of DSP over ASP:

A major drawback of ASP is its limited scope for performing complicated signal processing applications. This translates into nonflexibility in processing and complexity in system designs. All of these generally lead to expensive products. On the other hand, using DSP approach, it is possible to convert an inexpensive personal computer into a powerful signal processor. Some important advantages of DSP are:

1. Systems using the DSP approach can be developed using software running on a general purpose computer. Therefore DSP is relatively convenient to develop and test, and the software is portable.
2. DSP operations are based solely on additions and multiplications, leading to extremely stable processing capability – for example, stability independent of temperature.
3. Digital signals are easily stored on magnetic or other storable media.
4. The digital signals become transportable and can be processed off-line in a remote laboratory.
5. DSP operations can easily be modified in real time, often by simple programming changes, or by reloading of registers.
6. It is difficult to perform precise mathematical operations on signals in analog form but these same operations can be routinely implemented on a digital computer using software.
7. DSP has lower cost due to VLSI technology, which reduces costs of memories, gates, microprocessors, and so forth.

The principal disadvantage of DSP is the speed of operation of analog to digital converters and DSP, especially at very high frequencies and wide bandwidth require fast-sampling rate A/D converters and fast digital signal processors. Primarily due to the above advantages, DSP is now becoming a first choice in many technologies and applications, such as consumer electronics, communications, wireless telephones, and medical imaging.

Applications of Digital Signal Processing:

There are various application areas of digital signal processing (DSP) due to the availability of high resolution spectral analysis. It requires high speed processor to implement the Fast Fourier Transform. Some of these areas can be listed as below:

1. **Speech Processing**

Speech is a one dimensional signal. Digital processing of speech is applied to a wide range of speech problems such as speech spectrum analysis, channel vocoders, etc. DSP is applied to speech coding, speech enhancement, speech analysis and synthesis, speech recognition and speaker recognition.

2. **Image Processing**

Any two-dimensional pattern is called an image. Digital processing of images requires two-dimensional DSP tools such as Discrete Fourier Transform (DFT), Fast Fourier Transform (FFT) algorithms and z-transforms. Processing of electrical signals extracted from images by digital techniques include image formation and recording, image compression, image restoration, image reconstruction and image enhancement.

3. **Radar Signal Processing**

Rader stands for "Radio Detection and Ranging". Improvement in signal processing is possible by digital technology. Development of DSP has led to greater sophistication of radar tracking algorithms. Radar systems consist of transmit-receive antenna, digital processing system and control unit.

4. **Digital Communications**

Application of DSP in digital communication specially telecommunications comprises of digital transmission using pulse code modulation (PCM), digital switching using Time Division Multiplexing (TDM), echo control and digital tape-recorders. DSP in telecommunication systems are found to be cost effective due to availability of medium and large scale digital ICs. These ICs have desirable properties such as small size, low cost, low power, immunity to noise and reliability.

5. **Spectral Analysis**

Frequency-domain analysis is easily and effectively possible in digital signal processing using fast Fourier transform (FFT) algorithms. These algorithms reduce computational complexity and also reduce the computational time.

6. **Sonar Signal Processign**

Sonar stands for "Sound Navigation and Ranging". Sonar is used to determine the range, velocity and direction of targets that are remote from the observer. Sonar uses sound waves at lower frequencies to detect objects under water.

7. Aviation
8. Astronomy
9. Telecommunication networks
10. Satellite communication
11. Microprocessor systems
12. Industrial noise control.

Types of Signals:

1. **Continuous time Vs. Discrete-time signal**

   As the names suggest, this classification is determined by whether or not the time axis (x-axis) is discrete (countable) or continuous. Continuous-time signals are represented by x(t) where t denotes continuous-time and discrete-time signals or sequences are represented by x[n] where n is an integer denotes discrete-time.

2. **Continuous value Vs. Discrete-value Signal**

   In discrete-value signals, there are finite values in y-axis. For example if we look for values between 0 to 1 V, there are infinite numbers of values (Continuous value). But if we fixed the number of values to say 10 (0, 0.1, 0.2, …) or say 5 (0, 0.2, 0.4, 0.6, …) then there are finite numbers of values in y-axis, known as discrete-value signal.

3. **Periodic Vs. Non-Periodic Signal**
   Periodic signals repeat with some period T, while aperiodic or nonperiodic signals do not. We can define a periodic function through the following mathematical expression, where t can be any number and T is a positive constant:
   $$\boldsymbol{x(t) = x(t+T)}$$
   Where T is the fundamental period (the smallest value of T that still allows above equation to be true.
   For discrete time signal
   $$x[n] = x[n+N]$$
   Here, N is the time period which is an integer. That mean the signal repeats after N samples.
4. **Causal Vs. Anticausal Vs. Noncausal**
   Causal signals are signals that are zero for all negative time, while anticausal are signals that are zero for all positive time. Noncausal signals are signals that have nonzero values in both positive and negative time.
5. **Even Vs. Odd**
   An even signal is any signal x(t) such that x(t) = x(-t). Even signals can be easily spotted as they are symmetric around the vertical axis. An odd signal, on the other hand, is a signal that satisfy x(t) = -x(-t) (Also known as Anti-symmetric signal).
   Using the definitions of even and odd signals, we can show that any signal can be written as a combination of an even and odd signal. That is, every signal has an odd-even decomposition.
   $$x(t) = x_e(t) + x_o(t)\ (i.e. ever\ part + odd\ part)$$
   $$x_e(t) = \frac{x(t) + x(-t)}{2}\ \&\ x_o(t) = \frac{x(t) - x(-t)}{2}$$
6. **Deterministic Vs. Random Signal**
   A deterministic signal is a signal in which each value of the signal is fixed and can be determined by a mathematical expression, rule, or table. Because of this the future values of the signal can be calculated from past values with complete confidence. On the other hand, a random signal has a lot of uncertainty about its behavior. The future values of a random signal cannot be accurately predicted and can be usually only be guessed based on the averages of sets of signals.
7. **Energy Vs. Power Signal**
   In electrical systems, a signal may represent a voltage or current. Consider a voltage v(t) developed across a resistor R, producing a current i(t). The *instantaneous power* dissipated in this resistor is defined by
   $$p(t) = \frac{v^2(t)}{R}$$
   $$or\ equivalently,\ \ p(t) = Ri^2(t)$$

In both cases, the instantaneous power p(t) is proportional to the squared amplitude of the signal. Furthermore, for a resistance R of 1Ω, we see that above equations take on the same mathematical form. Accordingly, in signal analysis it is customary to define power in terms of a 1Ω resistor, so that, regardless of whether a given signal x(t) represents a voltage or a current, we may express the instantaneous power of the signal as,

$$p(t) = x^2(t)$$

Based on this convention, we define the *total energy* of the continuous-time signal x(t) as

$$E = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt = \int_{-\infty}^{\infty} x^2(t)dt$$

And its *average power* as

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt$$

In the case of a discrete-time signal x[n], the integrals are replaced by corresponding sums. Thus the total energy of x[n] is defined by

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

And its average power is defined by

$$\textit{If the signal is aperiodic,} \qquad P = \lim_{\mathrm{N}\to\infty} \frac{1}{2\mathrm{N}+1} \sum_{\mathrm{n}=-\mathrm{N}}^{\mathrm{N}} |\mathrm{x[n]}|^2$$

$$\text{If the signal is periodic,} \qquad \mathrm{P} = \frac{1}{\mathrm{N}} \sum_{\mathrm{n}=0}^{\mathrm{N}-1} |\mathrm{x[n]}|^2$$

The second expression is the average power in a periodic signal x[n] with fundamental period N. If x[n] is real we can take $|x[n]|^2 = x^2[n]$.

A signal is referred to as an Energy signal, if and only if the total energy of the signal satisfies the condition $0 < E < \infty$.

On the other hand, it is referred to as a Power signal, if and only if the average power of the signal satisfies the condition, $0 < P < \infty$.

The energy and power classifications of signals are mutually exclusive. In particular, an energy signal has zero average power, whereas a power signal has infinite energy. It is also interest to note that periodic signals and random signals are usually viewed as power signals, whereas signals that are both deterministic and non-periodic are energy signals.

#Find energy or power of the following signals:

$$\#\ \boldsymbol{x[n] = \left(\frac{1}{2}\right)^n u[n]}$$

The given signal is aperiodic signal and we will find energy as,

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^n u[n]\right|^2 = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n} = \frac{1}{1-\left(\frac{1}{2}\right)^2} = \frac{4}{3}$$

$$\#\ \boldsymbol{x[n] = \cos\left(\frac{\pi n}{4}\right)}$$

The given signal is periodic signal with period N = 8 and we will find power as,

$$P = \frac{1}{N}\sum_{\mathrm{n}=0}^{\mathrm{N}-1} \mathrm{x}^2[\mathrm{n}] = \frac{1}{8}\sum_{\mathrm{n}=0}^{7} \cos^2\left(\frac{\pi n}{4}\right)$$

$$= \frac{1}{8}\sum_{\mathrm{n}=0}^{7}\left[\frac{1+\cos\left(\frac{\pi \mathrm{n}}{2}\right)}{2}\right] = \frac{1}{8}\sum_{\mathrm{n}=0}^{7}\frac{1}{2} + \frac{1}{8}\times\frac{1}{2}\sum_{\mathrm{n}=0}^{7}\cos\left(\frac{\pi \mathrm{n}}{2}\right) = \frac{1}{8}\times\frac{8}{2} + \frac{1}{16}\times 0 = \frac{1}{2}$$

Continuous-Time Sinusoidal Signals

A simple harmonic oscillation is mathematically described by the following continuous-time sinusoidal signal:

$$x_a(t) = A\cos(\Omega t + \theta), \quad -\infty < t < \infty$$

This signal is completely characterized by three parameters: *amplitude* A of the sinusoid, *frequency* Ω in radians per second, and *phase* θ in radians. Instead of Ω, we often use the frequency F in cycles per second or hertz (Hz), where $\Omega = 2\pi F$.

In terms of F the sinusoid becomes: $x_a(t) = A\cos(2\pi Ft + \theta), -\infty < t < \infty$
The analog sinusoidal signal is characterized by the following properties:

1. For every fixed value of the frequency F, $x_a(t)$ is periodic: $x_a(t + T_p) = x_a(t)$, where $T_p = 1/F$ is the fundamental period of the sinusoidal signal.
2. Continuous-time sinusoidal signals with distinct frequencies are themselves distinct.
3. Increasing the frequency F results in an increase in the rate of oscillation of the signal, in the sense that more periods are included in a given time interval.

The relationships we have described for sinusoidal signals carry over to the class of complex exponential signals

$$x_a(t) = Ae^{j(\Omega t + \theta)}$$

By definition, frequency is an inherently positive physical quantity. This is obvious if we interpret frequency as the number of cycles per unit time in a periodic signal. However, in many cases, only for mathematical convenience, we need to introduce negative frequencies.

Hence the frequency range for analog sinusoids is $-\infty < F < \infty$.

Discrete-Time Sinusoidal Signals

A discrete-time sinusoidal signal may be expressed as:

$$x[n] = A\cos(\omega n + \theta), \quad -\infty < n < \infty$$

Where n is an integer variable, called the sample number, A is the *amplitude* of the sinusoid, ω is the *frequency* in radians per second, and θ is *phase* in radians. Instead of ω, we often use the frequency variable f defined by $\omega = 2\pi f$.

In terms of f the sinusoid becomes: $x[n] = A\cos(2\pi f + \theta), \quad -\infty < n < \infty$
The frequency f has dimensions of cycles per sample.
In contrast to continuous-time sinusoids, the discrete-time sinusoids are characterized by the following properties:

1. ***A discrete-time sinusoid is periodic only if its frequency f is a rational number:***

By definition, a discrete-time signal x[n] is periodic with period N (>0) iff

$$x[n + N] = x[n] \qquad \text{for all } n$$

The smallest value of N for which the above equation is true is called the *fundamental period.*
Proof: For a sinusoid with frequency f_0 to be periodic, we should have

$$Cos[2\pi f_0(n + N) + \theta] = Cos[2\pi f_0 n + \theta]$$

This relation is true if and only if there exists an integer k such that

$$2\pi f_0 N = 2k\pi \Rightarrow f_0 = k/N$$

Hence, a discrete-time sinusoidal signal is periodic only if its frequency f_0 can be expressed as the ratio of two integers (i.e. f_0 is rational). To determine the fundamental period N of a periodic sinusoid, we express its frequency f_0 as the ratio of two integers and cancel common factors so that k and N are relatively prime. Then the fundamental period of the sinusoid is equal to N.

2. ***Discrete-time sinusoids whose frequencies are separated by an integer multiple of 2π are identical.***

Proof: Let us consider the sinusoid $\cos[\omega_0 n + \theta]$. It easily follows that

$$\cos[(\omega_0 + 2\pi)n + \theta] = \cos[\omega_0 n + 2\pi n + \theta] = \cos[\omega_0 n + \theta]$$

As a result, all sinusoidal sequences
$x_k[n] = A\cos[\omega_k n + \theta]$ $k = 0, 1, 2, 3, \ldots\ldots$
where $\omega_k = \omega_0 + 2\pi k$ $-\pi \le \omega_0 \le \pi$
are *indistinguishable (i.e. identical).* On the other hand, the sequences of any two sinusoids with frequencies with frequencies in the range $-\pi \le \omega \le \pi$ or $½ \le f \le ½$ are distinct. Consequently, discrete-time sinusoidal signals with frequencies $|\omega| \le \pi$ or $|f| \le ½$ are unique. Any sequence resulting from a sinusoid with a frequency $|\omega| > \pi$, or $|f| > ½$, is identical to a sequence obtained from a sinusoidal signal with frequency $|\omega| < \pi$. Thus we regard frequencies in the range $-\pi \le \omega \le \pi$ or $½ \le f \le ½$ as unique and all frequencies $|\omega| > \pi$, or $|f| > ½$, as aliases[1].

x[n] = cos(0.6πn) => f=0.3=3/10, rational number hence periodic.

x[n] = cos(0.8n) => f = 0.4/π, irrational number hence aperiodic.

[1]In signal processing and related disciplines, aliasing refers to an effect that causes different signals to become indistinguishable (or aliases of one another) when sampled. It also refers to the distortion or artifact that results when the signal reconstructed from samples is different from the original continuous signal.

3. ***The highest rate of oscillation in a discrete-time sinusoid is attained when $\omega = \pi$ (or $\omega = -\pi$) or, equivalently, $f = ½$ (or $f = -½$).***

To illustrate this property, let us investigate the characteristics of the sinusoidal signal sequence

$$x[n] = \cos\omega_0 n$$

When the frequency varies from 0 to π. To simplify the argument, we take values of $\omega_0 = 0, \pi/8, \pi/4, \pi/2, \pi$ corresponding to $f = 0, {}^1/_{16}, {}^1/_8, ¼, ½$; which results in periodic sequences having periods $N = \infty$, 16, 8, 4, 2. We note that the period of the sinusoid decreases (or rate of oscillation increases) as the frequency increases.

To see what happens for $\pi \le \omega_0 \le 2\pi$, we consider the sinusoids with frequencies $\omega_1 = \omega_0$ and $\omega_2 = 2\pi - \omega_0$. Note that as ω_1 varies from π to 2π, ω_2 varies from π to 0. It can be easily seen that

$x_1[n] = A\cos\omega_1 n = A\cos\omega_0 n$
$x_2[n] = A\cos\omega_2 n = A\cos(2\pi - \omega_0)n = A\cos(-\omega_0 n) = x_1[n]$

Hence ω_2 is an alias to ω_1. If we had used a sine function instead of a cosine function, the result would basically be the same, except for a 180^0 phase difference between the sinusoids $x_1[n]$ and $x_2[n]$. In any case, as we increase the relative frequency ω_0 of a discrete-time sinusoid form π to 2π, its rate of oscillation decreases. For $\omega_0 = 2\pi$ the result is a constant signal, as in the case of $\omega_0 = 0$. Obviously, for $\omega_0 = \pi$ (or $f = ½$) we have the highest rate of oscillation.

Since discrete-time sinusoidal signals with frequencies that are separated by an integer multiple of 2π are identical, it follows that the frequencies in any interval $\omega_1 \le \omega \le \omega_1 + 2\pi$ constitute all the existing discrete-time sinusoids or complex exponentials. Hence the frequency range for discrete-time sinusoids is finite with duration 2π. Usually, we choose the range $0 \le \omega \le 2\pi$ or $-\pi \le \omega \le \pi$ ($0 \le f \le 1$, $-½ \le f \le ½$), which we call the *fundamental range.*

Harmonically Related Complex Exponentials

These are sets of periodic complex exponentials with fundamental frequencies that are multiple of a single positive frequency.

$$s_k(t) = e^{jk\Omega_0 t} = e^{jk2\pi F_0 t} k = 0, \pm 1, \pm 2, ---$$

$$FundamentalPeriod = \frac{1}{kF_0} = \frac{T_p}{k}$$

Similarly for discrete-time, $s_k[n] = e^{j2\pi k f_0 n}$

Analog to Digital Convertor (ADC):

1. **Sampling**: Continuous-time signal to discrete-time signal

   x(t) ===> x(nT) ≡ x[n], T is the sampling interval.

2. **Quantization**: Discrete-time continuous value signal to discrete-time discrete-value signal.

   x[n] ===> $x_q[n]$. Quantization error, $q_e = x[n] - x_q[n]$.

3. **Coding**: In the coding process, each discrete value $x_q[n]$ is represented by a b – bit binary sequence.

Sampling of Analog Signal:

We limit our discussion to periodic or uniform sampling.

$$x(t) = Acos(2\pi Ft + \theta)$$

$$x[n] \cong x(nT), \qquad -\infty < n < \infty$$

$$t = nT = \frac{n}{F_s}$$

Sampling periodically at a rate $F_s = 1/T$ samples/sec

$$x(nT) \cong x[n] = Acos(2\pi FnT + \theta) = Acos\left(2\pi\left(\frac{F}{F_s}\right)n + \theta\right)$$

If we compare with discrete-time sinusoid $x[n] = Acos(2\pi fn + \theta)$ we get,

$$f = \frac{F}{F_s}, \ \ or, \omega = \Omega T$$

Is called relative or normalized frequency. We can use f to determine the frequency F in hertz only if the sampling frequency F_s is know.

Recall,

$$-\infty < F < \infty \& -\frac{1}{2} \leq f \leq \frac{1}{2} cycles/samples$$

$$-\infty < \Omega < \infty \& -\pi \leq \omega \leq \pi radians/samples$$

We find that the frequency of the continuous-time sinusoid when sampled at a rate F_s= 1/T must fall in the range

$$-\frac{1}{2T} = -\frac{F_s}{2} \leq F \leq \frac{F_s}{2} = \frac{1}{2T}$$

Mapping of infinite range to finite frequency range of variable f. since highest frequency in a discrete-time signal is ω = π or f = ½ with sampling rate F_s, the corresponding highest value of F is, $F_{max} = \frac{F_s}{2} = \frac{1}{2T}$

Some terms related to sampling of analog signals:

Sampling Frequency, F_s: It is the number of samples per second while converting continuous-time signal to discrete-time signal.

Nyquist Criteria, $F_s \geq 2F_{max}$: It is the criteria that has to be fulfilled for the reconstruction of signal from discrete-time signal. F_s is the sampling frequency and F_{max} is the maximum frequency contained in continuous-time signal.

Nyquist Rate, $F_N = 2F_{max}$: It is the minimum sampling frequency required for the proper reconstruction of the signal.

Folding Frequency (or Nyquist Frequency), $F_s/2$: The highest frequency that can be reconstructed or measured using discretely sampled data. It is the half of sampling frequency.

 # Consider the two analog sinusoidal signals

$$x_1(t) = \cos(2\pi(10)t) \& x_2(t) = \cos(2\pi(50)t)$$

Sample the two signals at a rate F_s = 40 Hz and find the discrete-time signals obtained.

[Hint: The two sinusoidal signals are identical & consequently indistinguishable. We say that the frequency F_2 = 50 Hz is an alias of the frequency F_1 = 10 Hz at the sampling rate of 40 Hz]

 # Consider the analog signal

$$x(t) = 3cos100\pi t$$

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a. Determine the minimum sampling rate required to avoid aliasing.
b. Suppose that the signal is sampled at F_s = 200 Hz, What is the discrete-time signal obtained after sampling?
c. Suppose that the signal is sampled at the rate Fs = 75 Hz, what is the discrete-time signal obtained after sampling?
d. What is the frequency 0 < F < Fs/2 of a sinusoid that yields samples identical to those obtained in part(c) ?

 # Consider the analog signal

$$x(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$$

a. What is the Nyquist rate for this signal?
b. Assume now that we sample this signal using a sampling rate Fs = 5000 samples/s. What is the discrete-time signal obtained after sampling?

What is the analog signal y(t) we can reconstruct from the samples if we use ideal interpolation?

 # A digital communication link carries binary-coded words representing samples of an input signal,

$$x(t) = 3cos600\pi t + 2cos1800\pi t$$

The link is operated at 10,000 bits/s and each input sample is quantized into 1024 different voltage levels.

i. What is the sampling frequency & folding frequency?
ii. What is the Nyquist rate for the signal x(t) ?
iii. What are the frequencies in the resulting discrete time signal x[n] ?
iv. What is the resolution Δ ?

Solution: ➔

As the link is operated at 10,000 bits/s & each input sample is quantized into 1024 different voltage levels the each sampled value is represented by $\log_2 1024$ = 10 bits/sample

i. Then maximum sampling frequency, $F_s = \frac{10{,}000\ bits/sec}{10\ bits/sample} = 1000\ samples/sec$

Folding frequency (is the maximum frequency that can be represented uniquely by sampled signal), $\frac{F_s}{2} = 500\ samples/sec.$

ii. $x(t) = 3cos600\pi t + 2cos1800\pi t$

Here, F_1 = 300 Hz and F_2 = 900 Hz. Thus F_{max} = 900 Hz.

The Nyquist rate, F_N = 2Fmax = 1800 Hz.

iii. For F_s = 1000 Hz,

$$x[n] \cong x(nT) = x\left(\frac{n}{F_s}\right) = 3cos2\pi\left(\frac{300}{1000}\right)n + 2cos2\pi\left(\frac{900}{1000}\right)n$$

$$= 3cos2\pi\left(\frac{3}{10}\right)n + 2cos2\pi\left(\frac{9}{10}\right)n$$

$$= 3cos2\pi\left(\frac{3}{10}\right)n + 2cos2\pi\left(1-\frac{1}{10}\right)n$$

$$= 3cos2\pi\left(\frac{3}{10}\right)n + 2cos2\pi\left(\frac{1}{10}\right)n$$

$$\therefore f_1 = \frac{3}{10} \& f_2 = \frac{1}{10}$$

$$Here\ both\ frequencies\ f_1 and\ f_2\ lies\ in\ the\ interval -\frac{1}{2} \le f \le \frac{1}{2}$$

iv. ADC resolution = 10 bits

Voltage resolution, $\Delta = \frac{x_{max}-x_{min}}{L-1} = \frac{5-(-5)}{1024-1} = 9.76\ mV$.

Some Trigonometric Identities:

$$\sin(2\pi+\theta) = sin\theta;\ \sin(2\pi-\theta) = -sin\theta$$

$$\sin(\pi \pm \theta) = \mp sin\theta$$

$$\cos(\pi \pm \theta) = -cos\theta$$

$$cos(2\pi \pm \theta) = cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin(A \pm B) = sinAcosB \pm cosAsinB$$

$$\cos(A \pm B) = cosAcosB \mp sinAsinB$$

References:

1. J. G. Proakis, D. G. Manolakis, "Digital Signal Processing, Principles, Algorithms and Applications", 3rd Edition, Prentice-hall, 2000. Chapter 1.
2. S. Sharma, "Digital Signal Processing", Third Revised Edition, S.K. Kataria & Sons, 2007.
3. Analog Devices, "Mixed Signal and DSP Design Techniques", Prentice-hall 2000.