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Chapter: Capacitor

A **capacitor** is an electrical device used to store electric charge or electric potential energy.

It consists of two conducting plates between which a dielectric (insulator) is used to increase its ability to store charge. While charging a capacitor, one of the plates is connected to positive terminal while the other to negative terminal of a battery.

Capacitance:

The ability of a capacitor to store charge or electric potential energy is called its capacitance.

The ratio of the charge stored (q) to the potential difference (V) between the plates of a capacitor is a constant quantity. This constant quantity is called the capacitance(C) of that capacitor.

i.e. Capacitance, $C = \frac{q}{V}$. Unit of capacitance is Coulomb/ volt or Farad.

Capacitance of an isolated charged sphere:

We know that, potential at the surface of a uniformly charged sphere with charge ‘q’ is, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

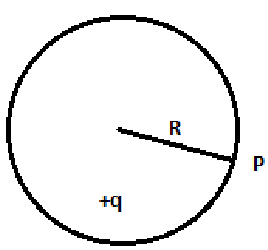

Fig. Isolated charged sphere

Capacitance of this charged sphere, $C = \frac{q}{V} = \frac{q}{\frac{1}{4\pi\varepsilon_0}\frac{q}{R}} \Rightarrow C = 4\pi\varepsilon_0 R$

Using this formula, we can find the capacitance of the Earth, assuming R(radius) as 6400 km.

Gauss’s law: It states that the total flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the charge enclosed by the surface. i.e. Total Flux, $\phi = \oint \vec{E}.\overrightarrow{dA} = \frac{1}{\varepsilon_0}q$.

The closed surface is the imaginary surface designed through the point of interest.

Types of Capacitor:

Three types of capacitors, basically, are studied here.

1. **Parallel plate capacitor:**
   Two parallel conducting plates, each having area ‘A’ placed small distance‘d’ apart, forms a parallel plate capacitor.

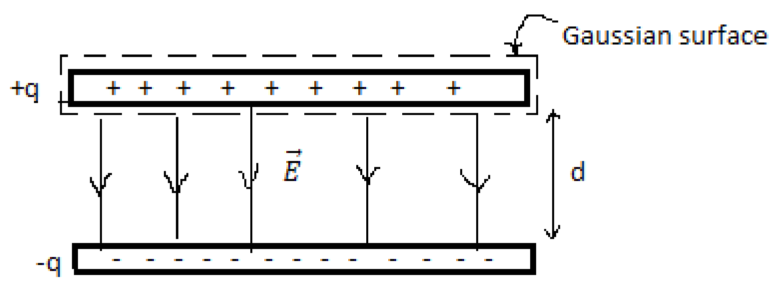

Fig. Parallel plate capacitor with Gaussian surface.

To find its capacitance, we draw a Gaussian surface enclosing charge ‘+q’ on positive plate.

Using Gauss law, $\oint \vec{E}.\overrightarrow{dA} = \frac{q}{\varepsilon_0}$

⇨ EA = $\frac{q}{\varepsilon_0}$

i.e. E = $\frac{q}{\varepsilon_0 A}$

Potential, V = $\int_0^d \vec{E}.\overrightarrow{dr} = E\int_0^d dr = Ed = \frac{qd}{\varepsilon_0 A}$

Therefore, Capacitance, C = q/ V = ε_0 EA /(Ed) = ε_0A/d

Here we see that capacitance depends upon geometry of the capacitor.

2. **Cylindrical capacitor**: Two cylinders of radii ‘a’ and ‘b’, placed co- axially, each of length ‘l’ forms a cylindrical capacitor. Let, ’+q’ charge be on the surface of inner cylinder,
   and ‘– q ’ be that on surface of outer one.

Charges are uniformly distributed throughout the respective surfaces.

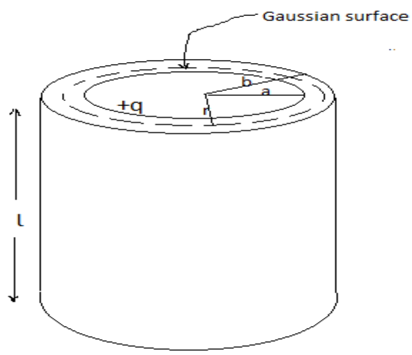

Fig. Cylindrical capacitor

A Cylindrical Gaussian surface of radius ‘r’ enclosing charge +q , in inner plate, is drawn.

Using Gauss law, $\int \vec{E}.\overrightarrow{dA} = \frac{q}{\varepsilon_0}$

⇨ EA = $\frac{q}{\varepsilon_0}$ ⇨ E = $\frac{q}{\varepsilon_0 A}$ = $\frac{q}{\varepsilon_0 2\pi r l}$

⇨ Charge enclosed by the Gaussian surface, q = ε_0 EA = ε_0.E.2πrl

Potential, Potential, V = $\int_a^b E.dr = \int_a^b \frac{q}{2\pi\varepsilon_0 r l} dr = \frac{q}{2\pi\varepsilon_0 l}\int_a^b \frac{1}{r} dr = \frac{q}{2\pi\varepsilon_0 l}\ln(\frac{b}{a})$

Capacitance, C = q/v

⇨ C = $\frac{2\pi\varepsilon_0 l}{\ln(\frac{b}{a})}$

3. **Spherical capacitor:** A spherical capacitor consists of two concentric spheres of radii ‘a’ and ‘b’ insulated with each other.
The inner sphere is uniformly charged with
‘+q’ charge and the outer one with ‘–q’ charge.

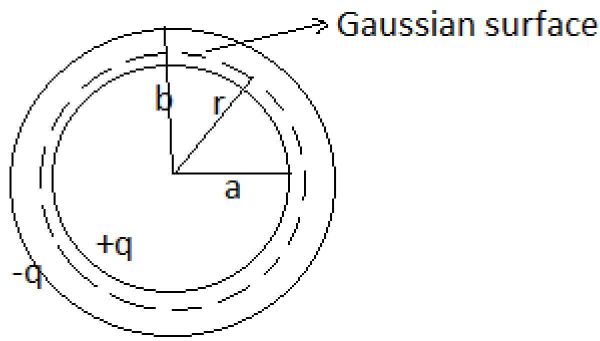

Fig. Spherical Capacitor

We proceed as below to find its
capacitance.
As a Gaussian surface, we draw a sphere of radius ‘r’ enclosing inner sphere, carrying +q charge.

From Gauss law, $\int \vec{E}.\overrightarrow{dA} = q/\varepsilon_0$ ⇨ E.4πr^2 = q/ε_0

⇨ $E = \frac{q}{4\pi\varepsilon_0 r^2}$ (as surface area of a sphere is $4\pi r^2$)

Then, Potential, $V = \int_a^b \vec{E}.\vec{dr}$

or, $V = \frac{q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2}$

Integrating, we get, $V = \frac{q}{4\pi\varepsilon_0}\frac{b-a}{ab}$

Then capacitance, $C = q/v = 4\pi\varepsilon_0(\frac{ab}{b-a})$

Energy stored in an electric field:

When charging a capacitor, some work must be done to store charges on the plates. This work done is stored in a capacitor in the form of electric potential energy.
Suppose at any instant, a small amount of charge 'dq' is transferred from one plate to another, by applying a potential difference of V.
Then, the small amount of work done, dW = V. dq
The total work done to store charge 'q' is,

$$W = \int_0^q V dq = \int_0^q \frac{q}{C} dq = \frac{q^2}{2C} \quad (\because c = q/v)$$

Using q = CV we get, $W = \frac{1}{2}CV^2$

Therefore, electric potential energy, $U = \frac{q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}qV$

Energy density:

In a parallel plate capacitor, the electric field has nearly the same value at all the points between the plates. Thus, energy density is uniform between the plates.

Energy density, μ = electric P.E./volume = $\frac{U}{A\times d} = \frac{\frac{1}{2}CV^2}{Ad}$ $(\because Volume\ between\ plates = Ad)$

Using capacitance of parallel plate capacitor, $C = \varepsilon_0 A/d$

We get, $\mu = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$

Since, V/d = E, the electric field

we get $\mu = \frac{1}{2}\varepsilon_0 E^2$

Dielectrics:

A dielectric is a non-conducting material, such as rubber, glass, plastic etc. It is used between the plates of a capacitor to provide mechanical support for the plates and to increase its capacitance. Two types of dielectrics are: Non – polar dielectric and polar dielectric.

1. **Non- polar dielectric:** The dielectric, in which positive and negative charges have same center of gravity, is called a non-polar dielectric. There is zero dipole moment due to

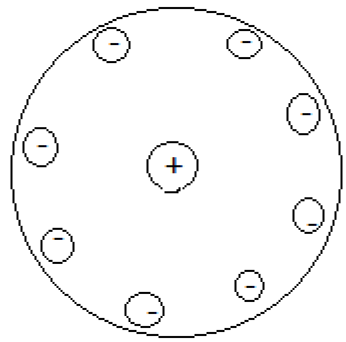

Fig. Non-polar dielectric

each molecule of the dielectric.
When external electric field is applied to the dielectric an induced dipole is formed that align with the external field. This phenomenon is called polarization. Examples of non-polar dielectric are H_2, N_2, CO_2 etc.

2. **Polar dielectric:** The dielectric in which positive and negative charges do not have same center of gravity is called polar dielectric. There is none - zero dielectric moment of each molecule, in the absence of external electric field. However, the dipoles are randomly

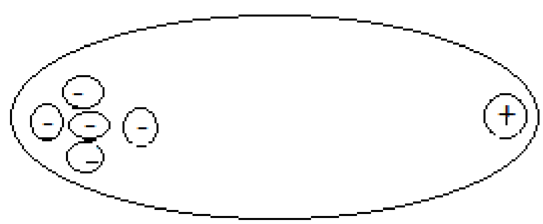

Fig. Polar dielectric

oriented.
When an external electric field is applied, the randomly oriented dipoles tend to align along the direction of applied field. Eg. N_2O, NH_3, HCl, H_2O etc.

Dielectric constant:

The dielectric constant (K) is defined as the ratio of capacitance of a capacitor with dielectric (C) to the capacitance without dielectric (C_0).

i.e. $K = \frac{C}{C_0}$ It is also called relative permittivity(ε_r).

Function of a dielectric in a capacitor:

1. We have, dielectric constant, $K = \frac{C}{C_0} \Rightarrow C = K\,C_0$ … (i)

2. If ‘q’ be the charge stored in a capacitor at any instant. ‘V_0’ be the potential, when there is air between the plates. When dielectric is introduced, let the potential be ‘V’.

Then, $C_0 = \frac{q}{V_0}$ and, $C = \frac{q}{V}$

Using these relations in equation (i) we get, $\frac{q}{V} = K\frac{q}{V_0}$ ⇨ $V = \frac{1}{K}V_0$ ....(ii)

3. If E_0 be the electric field, when there is no dielectric between the plates and 'E' be that in the presence of dielectric. Then, $E_0 = \frac{V_0}{d}$ ⇨$V_0 = E_0 d$

And, $E = \frac{V}{d}$ ⇨ $V = Ed$

Then, using these relations in (ii) we get, $E = \frac{1}{K}E_0$

4. Let us check for the electric potential energy.

For this, let U_0 and U be the electric P.E. before and after inserting the dielectric. Then,

$U_0 = \frac{1}{2}C_0{V_0}^2$ and $U = \frac{1}{2}CV^2$

Dividing these equations and using equations (i) and (ii), we get $U = \frac{1}{K}U_0$

Hence, electric field, potential and electric P.E. are reduced by factor $\frac{1}{K}$, however, capacitance is increased by K times, when a dielectric is introduced.

In summary, $K = \frac{E_0}{E} = \frac{V_0}{V} = \frac{U_0}{U} = \frac{C}{C_0} = \frac{\varepsilon}{\varepsilon_0}$

Dielectrics and Gauss's law:

Q. Discuss how the Gauss's law is modified in dielectric medium.

Consider a parallel plate capacitor of plate area A and having magnitude of charge q on each plate. Let, E_0 be the magnitude of the field and ε_0 be the permittivity of air between the plates.

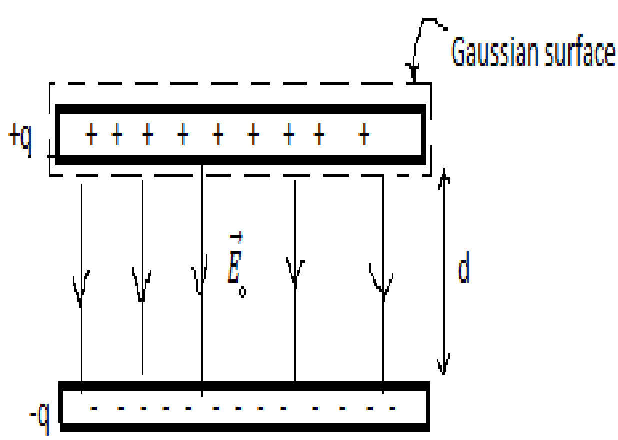

Fig. Parallel plate capacitor without dielectric

We draw a Gaussian surface in positive plate.

Using Gauss law,

$\oint \overrightarrow{E_0}.\overrightarrow{dA} = \frac{q}{\varepsilon_0}$ ⇨ $E_0A = \frac{q}{\varepsilon_0}$

$Therefore,$ $E_0 = \frac{q}{\varepsilon_0 A}$ ……(i)

Now, a dielectric having dielectric constant K is introduced between the plates of the same capacitor.

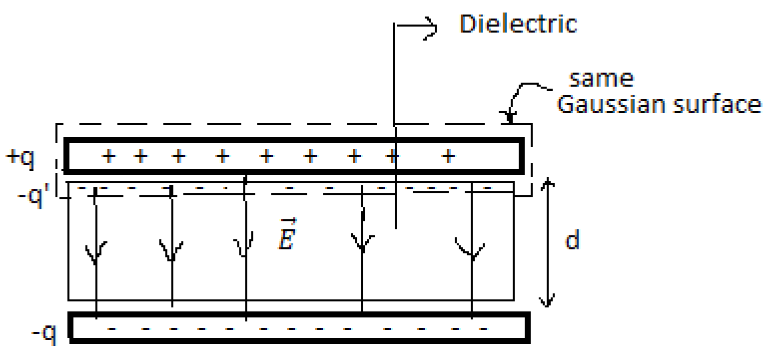

Fig. Parallel plate capacitor with dielectric

Then, charge - q' is induced on the top face of the dielectric, due to the charge q on the positive plate. Let, E be the electric field between the plates.

Taking the same Gaussian surface, net charge enclosed by the Gaussian surface is q – q'.

Using Gauss law, $\oint \vec{E}.\overrightarrow{dA} = \frac{q-q'}{\varepsilon_0}$

⇨ $EA = \frac{q-q'}{\varepsilon_0}$ ⇨ $E = \frac{q-q'}{\varepsilon_0 A}$ …..(ii)

Since, we have, $E = \frac{1}{K}E_0$ (i.e. electric field reduces by factor 1/K when dielectric is introduced)

Using (i) and (ii) we get

$\frac{q-q'}{\varepsilon_0 A} = \frac{q}{\varepsilon_0 A}\frac{1}{K}$

⇨$q - q' = \frac{q}{K}$ … (iii)

Now, the Gauss law can be written as, $\oint \vec{E}.\overrightarrow{dA} = \frac{q-q'}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{q}{K}$

⇨$\oint K\vec{E}.\overrightarrow{dA} = \frac{q}{\varepsilon_0}$ ⇨This is the expression for modified Gauss's law for a dielectric medium.

Q. Show that induced charges in dielectric is always less than free charges in conducting plates.

Equation (iii) gives, $q' = q\,(1-\frac{1}{K})$

When there is air between the plates of a capacitor, K = 1 ⇨ q' = 0 (no induced charges)

However, if there is dielectric between the plates, its dielectric constant K > 1 ⇨ q' < q i.e. induced charges in the dielectric is always less than free charges.

Define displacement vector and Polarization vector and find the relationship between them.

We have a relation, E = $\frac{q-q'}{\varepsilon_0 A}$ Where the symbols have their usual meanings.

or, E = $\frac{1}{\varepsilon_0}\left(\frac{q}{A} - \frac{q'}{A}\right)$

The quantity $\frac{q}{A}$ is called Electric displacement vector, $\vec{D}$. It is equivalent to free surface charge density.

The quantity $\frac{q'}{A}$ is called polarization vector$\vec{P}$. It represents the capacity of dipole formation due to applied field and is equivalent to induced surface charge density.

Therefore,

$\vec{E} = \frac{1}{\varepsilon_0}(\vec{D} - \vec{P})$ ⇨$(\vec{D} - \vec{P}) = \varepsilon_0 \vec{E}$

i.e. $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ ⇨ required expression

The expression $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ *can be easily converted to* $\vec{P} = \varepsilon_0(K-1)\vec{E}$ *by using the formula* $\vec{D} = \frac{q}{A}$ *and* $\vec{E} = \frac{1}{K}\vec{E}_0$ *with* $\vec{E}_0 = \frac{q}{\varepsilon_0 A}$

i.e. $\vec{P} = \varepsilon_0 \chi \vec{E}$ .Here, $\chi = (K-1) = (\varepsilon_r - 1)$ is called dielectric susceptibility.

Charging and discharging of a capacitor:

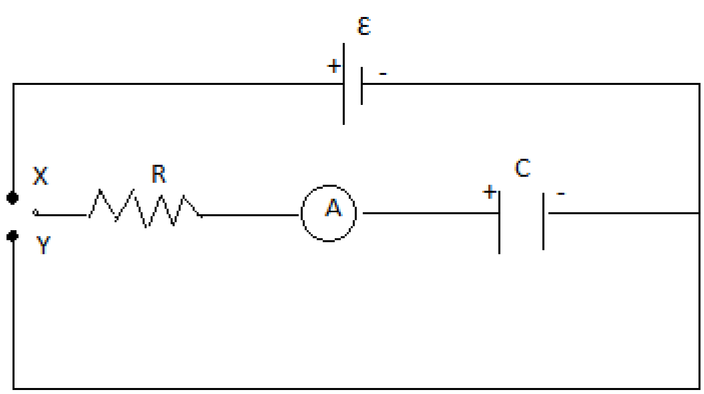

fig. circuit diagram for charging and discharging of a capacitor

Referring to the fig. above, a capacitor with capacitance C is connected in series with a resistor R, ammeter A and a battery with emf Ɛ.

Charging : When key X is switched on, charges start accumulating on the plates of the capacitor, and current goes on decreasing exponentially. Let, q and 'I' be the charge on capacitor and current on the circuit at any instant of time't'.

Applying Kirchhoff's loop rule to the upper circuit,

$$\varepsilon = V_C + V_R = q/c + IR$$
$$or, q/c + IR = \varepsilon$$

$$or, q/c + (dq/dt)R = \mathcal{E}$$
$$R\, dq/dt = \mathcal{E} - q/c = (\mathcal{E}C - q)/C = (q_0 - q)/C$$
$$Where, q_0 = \mathcal{E}C \text{ is the maximum charge stored in the capacitor.}$$
$$\Rightarrow dq/(q_0 - q) = dt/RC$$

Integrating, we get, $\int_0^q \frac{dq}{q_0 - q} = \frac{1}{RC}\int_0^t dt$

$\Rightarrow$ - $\int_0^q \frac{-dq}{q_0 - q} = \frac{1}{RC}.t$

$\Rightarrow$ -$[\ln(q_0 - q)]_0^q = \frac{t}{RC}$ $\Rightarrow \ln(q_0 - q) - \ln(q_0) = -\frac{t}{RC}$

$\Rightarrow \ln\left(\frac{q_0 - q}{q_0}\right) = -\frac{t}{RC}$

$\Rightarrow \frac{q_0 - q}{q_0} = e^{-t/RC}$

$\Rightarrow q_0 - q = q_0\, e^{-t/RC}$ $\Rightarrow$ q = $q_0\left(1 - e^{-t/RC}\right)$... (i) Required charging equation.

Differentiating this equation w.r.t. 't' the current during charging is given by,

I = $\frac{dq}{dt} = -q_0\left(-\frac{1}{RC}\right)e^{-t/RC} = \frac{q_0}{RC}.e^{-t/RC} = I_0\, e^{-t/RC}$

Therefore, I = $I_0\, e^{-t/RC}$.....(ii) Where, $I_0 = \frac{q_0}{RC}$ is the maximum current

The term' RC' in equations (i) and (ii) is called capacitive time constant of the charging circuit.

When t= RC, equation (i) becomes, q = $q_0\,(1 - e^{-1}) = q_0(1 - 0.37) = 0.63\, q_0$

i.e. q = 63% of q_0

Hence, time constant of a charging circuit is defined as the time at which the capacitor charges by 63% of its maximum value.

The plots of current and charge with time are as shown below.

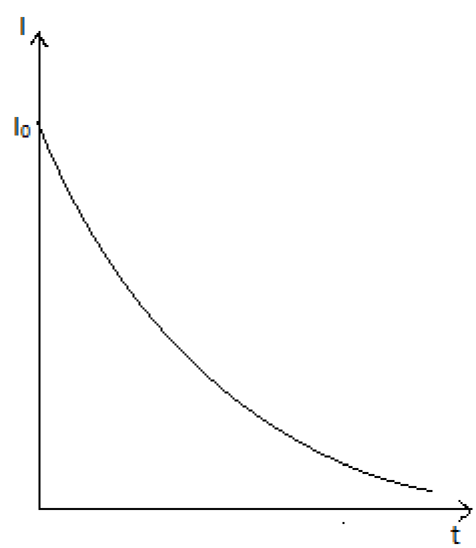

Fig. variation of current with time

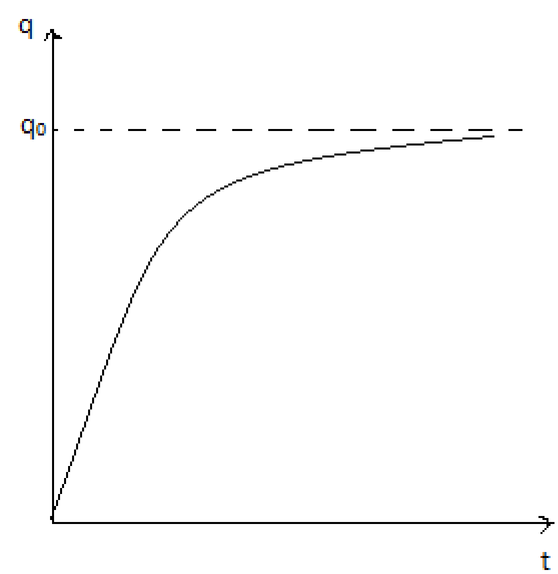

Fig. Variation of charge with time

Discharging:

When the capacitor is fully charged, switch X is switched off and Y is switched on. Then, discharging of the capacitor through resistor, takes place.

Using Kirchoff's loop rule for lower circuit,

$0 = V_c + V_R = q/C + IR = q/C + R.dq/dt$

⇨ $R.dq/dt = -q/C$

⇨ $dq/q = -\frac{-1}{RC}dt$ Integrating,

⇨ $\int_{q_0}^{q} \frac{dq}{q} = \frac{-1}{RC}\int_0^t dt$

⇨ $[\ln q]_{q_0}^{q} = \frac{-t}{RC}$ ⇨ $\ln q - \ln q_0 = \frac{-t}{RC}$ ⇨ $\ln\left(\frac{q}{q_0}\right) = \frac{-t}{RC}$

⇨ $\frac{q}{q_0} = e^{\frac{-t}{RC}}$ ⇨ $q = q_0\, e^{\frac{-t}{RC}}$ …. (iii)

$Differentiating(iii)$ w.r.t. 't' we get, current,

$$I = -I_0\, e^{-\frac{t}{RC}} \ldots..(iv)$$

Equations (iii)and (iv)are called discharging equations.

The factor 'RC' in equations (iii) and (iv) is called capacitive time constant of discharging circuit.

Using, t = RC in (iii) we get q = 37% of q_0

Hence, the time constant of the discharging circuit is the time at which the charge stored in capacitor falls to 37% of its initial value.

The plots of current and charge with respect to time are as shown below.

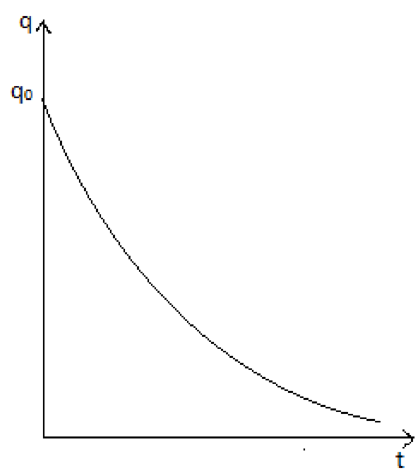

Fig. variation of charge with time

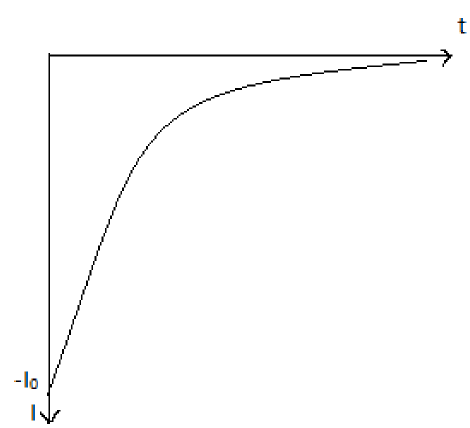

Fig. Variation of current with time

Physical meaning of polarization of dielectric:

When a dielectric is placed in an external electric field, the charged particles (atoms, ions, molecules) arrange in such an order that dielectric acquires a certain electric moment; such a process is called polarization.

The electric moment P (induced moment) acquired by the particle is directly proportional to the local electric field or internal field inside the dielectric, $E_{int.}$

i.e. $P \propto E_{int.}$

$P = \alpha E_{int.}$ The proportionality constant α is known as polarizability of given dipole.

For 'N' dipoles per unit volume of the matter, we write

$P = N\alpha E_{int.}$

Here, P is the average dipole moment of N dipoles

Unit of α is Farad.meter^2

Types of Polarization:

1. **Electronic polarization:** Center of gravity of the electrons and protons may change resulting in induced dipole moment. i.e. $P_e = \alpha_e E_{int.}$ Here α_e is called electronic polarizability.
2. **Molecular polarization:** Molecules having permanent dipole moment i.e. polar molecules are affected in two ways:
   a) **Atomic polarization:** The field may cause the atoms to be displaced from the initial position and hence changing the dipole moment. Such a polarizability is called atomic polarizability (α_a).
   b) **Orientational or dipolar polarization:** The molecules may rotate about its axis so that dipoles align with the field. This is called orientational polarizability (α_d).
3. **Interfacial polarization:** It arises due to the large number of defects in the structure of crystals, such as lattice vacancies, impurity center, dislocation etc. The polarizability is called interfacial polarizability (α_i).
   Hence, total polarizability, $\alpha = \alpha_e + \alpha_a + \alpha_d + \alpha_i$

Lorentz local field:

The internal intensity, $E_{int.}$ on a molecule is the geometrical sum of the external applied intensity (E) and the component of the intensity caused by the action of other polarized molecules of the matter on that molecule.

i.e. $\vec{E}_{int.} = \vec{E} + \vec{E_1}$

We can derive that $\vec{E_1} = \frac{\vec{P}}{3\varepsilon_0}$. This electric filed $\vec{E_1}$ is called Lorentz field.

Therefore, $\vec{E}_{int.} = \vec{E} + \frac{\vec{P}}{3\varepsilon_0}$

Clausius Mosotti Equation:

We have, internal intensity or local field, $\vec{E}_{\text{int.}} = \vec{E} + \vec{E_1}$ Where, $\vec{E}$ is applied electric field and $\vec{E_1} = \frac{\vec{P}}{3\varepsilon_0}$ is Lorentz field.

i.e. $\vec{E}_{\text{int.}} = \vec{E} + \frac{\vec{P}}{3\varepsilon_0}$

Also, $\vec{P} = \varepsilon_0(\mathrm{K}-1)\,\vec{E}$ $\quad (\because q' = q\left(1-\frac{1}{K}\right) \Rightarrow \frac{q'}{A} = \frac{q}{A}\left(1-\frac{1}{K}\right) = \vec{D}(1-\frac{1}{K})$ *and* $\vec{D} = K\varepsilon_0\vec{E}\,)$

So, $\vec{E}_{\text{int.}} = \vec{E} + \frac{\varepsilon_0(\mathrm{K}-1)\vec{E}}{3\varepsilon_0} \quad = \left[1 + \frac{\varepsilon_0(\mathrm{K}-1)}{3\varepsilon_0}\right]\vec{E} \quad = \left[\frac{K+2}{3}\right]\vec{E}$

$\Rightarrow \vec{E}_{\text{int}} = \left[\frac{K+2}{3}\right]\vec{E}$ ...... (i) This is known as Mosotti equation.

[In vacuum, K = 1. So, $\vec{E}_{\text{int}} = \vec{E}$]

We have, $\vec{P} = \mathrm{N}\alpha\,\vec{E}_{\text{int}}$

Using (i), we get

$\vec{P} = \mathrm{N}\alpha\left[\frac{K+2}{3}\right]\vec{E}$ .... (ii)

Comparing equation (ii) with $\vec{P} = \varepsilon_0(\mathrm{K}-1)\,\vec{E}$ We get,

$\varepsilon_0(\mathrm{K}-1) = \mathrm{N}\alpha\left[\frac{K+2}{3}\right]$

$\Rightarrow \frac{\mathbf{K-1}}{\boldsymbol{K+2}} = \frac{\mathbf{N\alpha}}{\mathbf{3\varepsilon_0}}$ ... (iii).

Equation (iii) is known as **Clausius Mosotti** equation.

Using $\mathrm{K} = \frac{\varepsilon}{\varepsilon_0}$, Clausius Mosotti equation becomes, $\frac{\varepsilon-\varepsilon_0}{\varepsilon+2\varepsilon_0} = \frac{\mathrm{N}\alpha}{3\varepsilon_0}$

The magnitude $\frac{\mathrm{K}-1}{K+2} = \frac{\varepsilon-\varepsilon_0}{\varepsilon+2\varepsilon_0}$ is known as specific polarization of a dielectric. This is a dimensionless quantity. For air or free space, specific polarization vanishes as K = 1.

The dielectric susceptibility χ is defined as, $\chi = (K-1)$

So,

$$\boxed{\frac{\mathrm{K}-1}{K+2} = \frac{\varepsilon-\varepsilon_0}{\varepsilon+2\varepsilon_0} = \frac{\chi}{\chi+3} = \frac{\mathrm{N}\alpha}{3\varepsilon_0}}$$

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