CHAPTER-2

CREDIT HOUR → 12 Hrs

SIGNAL MEASUREMENT

Verdhman
PAGE NO.
DATE / /20

⇒ UNITS
→ fundamental
→ derived.

In measuring a quantity, we always compare it with some established reference standard. Such standard is called a unit of a quantity. Measurement implies a comparison with standard value. To say that a rope is 30 meter long is to say that is 30 times as long as on object whose length has been defined to be 1 meter.

The result of measurement of physical quantities must be defined both in kind and the magnitude.

FUNDAMENTAL UNITS:

Basic units of mass(M), Length(L), time(T) are known as fundamental units. These are also known as primary fundamental unit.

DERIVED UNITS:

All other units which can be expressed in terms of fundamental units with the help of physical equations are called derived units. For example; there is no unit for speed among the base unit, however a suitable unit can be derived from the equation.

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{m}{s}$$

2013/09/18 13:40

This suggest that the unit of speed is m/s.

similarly units of Volume is m^3, Area m^2 etc.

⇒ STANDARD:

1. International Standard (International bureau of measurement)
2. Primary Standard (National standard)
3. Secondary Standard (Laboratories Standard)
4. Working Standard (General Standard)

A standard is a physical representation of a unit of measurement. The term standard is applied to up piece of equipment having a known measure of physical quantity. They are used for obtaining the value of physical properties of other equipment by comparison measure.

1. International Standard:

They are defined on the basis of international agreement. They are check and evaluated regularly against absolute measurement in terms of fundamental units. They are maintained at the international bureau of weight and measurement. They are not available to the ordinary user of measuring instrument for the purpose of comparison or calibration.

2. Primary Standard:

The standard are maintained by national laboratories in different part of the world. They are not available for the use outside the national Laboratories. One of the main function of

2013/09/18 13:40

primary standard is the verification and calibration of secondary standard.

3. Secondary Standard:

They are basis reference standard used in industrial measurement, laboratories. These standards are checked locally against the reference standard. These are normally sent periodically to the national standard laboratories for calibration and comparison against primary standard. They are sent back to the industry by national laboratory. With a certification as regard their measured values in terms of primary standard.

4. Working Standard:

They are used to verify and calibrate general laboratories instruments for their accuracy and performance. These are used for general purpose measurement. They are calibrated and compare by secondary standard.

⇒ MEASUREMENT INSTRUMENT

→ Mechanical Instrument
→ Elecrrical Instrument
→ Electronics Instrument.

Mechanical Instrument:

They are reliab 2013/09/18 13:40 able conditions. They have moving parts that are rigid, heavy and bulky, consequently has large mass. During dynamic conditions, they are unable respond rapidly.

Electrical Instrument :

Electrical system normally depends upon mechanical measurement, movement as indicating device. They are too slow for present day requirement of fast measurement.

Electronic Instrument :

These instrument requires use of semiconductor devices. In electronic devices or instrument only movement involved is that of electrons, the response time is extremely small. Very weak signals can be detected by using pre-amplifiers and amplifiers. Power amplification provided by the electronic amplifiers which result in higher sensitivity

During electronic system non-electrical variable are converted into electrical variable using transducer, power consumption is very Low.

Similarly instrument can be classified into major categories depending upon the way of representing result of measurement.

→ Deflection type
→ Null type

2013/09/18 13:41

Vardhman
PAGE NO.
DATE: / /20

Deflection Type.

The measured quantity produces some physical effect which deflects or produces of mechanical displacement of the moving system of instrument. And opposing effect is built in the instrument which tries to oppose the deflection of moving system. The opposing system is so designed that its magnitude increases with increase in deflection of moving system caused by the quantity of measurement. The deflection of instrument produces up basis of determining the quantity to be measured. Opposing force is equal to deflection effect produced by the measured quantity or measuring.

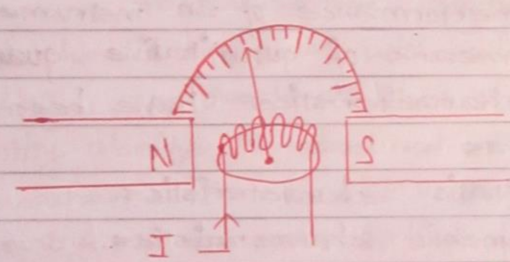


Fig:- Deflection type measuring instrument.

Null Type.

- A null type instrument attempts to maintain the deflection at zero by suitable application of an effect opposing that generated by the measured quantity. A detector that detects the null condition that is a device. Which indicates zero deflection, (balanced condition) when the effect produced by measured quantity is equal to the effect produced by opposing quantity. The accuracy is higher

2013/09/18 13:41

than deflection type and they are highly sensitive. For example; wheat stone bridge or meter bridge.

⇒ Characteristics of Instrument (Performance parameter)

Static Characteristics	Dynamic ~~Ch~~ Instrument.
→ Accuracy	→ Speed of response
→ Sensitivity	→ Measuring Lag
→ Reproducibility and Repeatability	→ Fidelity
→ Drift	→ Dynamic error.
→ Static error	
→ Dead zone.	

The performance of an instrument is described by means of quantitative quantity termed as characteristics have been ~~such~~ sub-divided as

- static characteristics
- dynamic characteristics.

(A) Static Characteristics

Static characteristics are set of criteria that gives of meaningful description of quality of measurement where the quantity to be measured are constant or change should with time.

2013/09/18 13:41

PAGE NO.
DATE: / /20

(B) Dynamic Characteristics

Dynamic characteristics are criteria based upon the dynamic relation many measurements are concerned with rapidly varying quantities and therefore for such a case we must test the dynamic relationship which exist between output and input. This is normally turned with the help of differential equation.

⇒ Static Characteristics

1. Accuracy

It is the degree of corectness with which a measuring means gives the true value with reference to accepted engineering standard. It is the closeness with which an instrument reading approaches the true value of the quantity being measured. The accuracy of measurement means confirmity to truth. Accuracy of measured signal depends upon following factors.

(a) Variation of the signal be measured.
(b) Accuracy of the observer
(c) Whether or not the quantity is being truely impressed upon the measurement

2013/09/18 13:42

2. Sensitivity

It is the ratio of magnitude of output signal to the magnitude of input signal or it is the ratio of response to the quantity be measured.

The ratio of magnitude of measured quantity to the magnitude of response is defined as deflection factor or inverse sensitivity.

$$\text{Sensitivity} = \frac{\text{Magnitude of Output Signals}}{\text{Magnitude of input Signal}}$$

$$\text{or, Sensitivity} = \frac{\text{Response}}{\text{Quantity being measured}}$$

$$\text{Deflection Factor} = \frac{1}{\text{Sensitivity}}$$

The sensitivity of an element should be high.

Q. A wheat stone bridge required a change of 7Ω in unknown arm of the bridge to produce a change in deflection of 3 mm of Galvanometer, determine the sensitivity and deflection factor?

⇒ Solution,

Here, magnitude of input signal $= 7\Omega$

magnitude of output signal $= 3mm$

Then,

$$\text{Sensitivity} = \frac{\text{Magnitude of output signal}}{\text{Magnitude of input signal}}$$

$$= \frac{3mm}{7\Omega}$$

2013/09/18 13:42

$\therefore$ Sensitivity = 0.43 mm/Ω

$\therefore$ Deflection factor $= \frac{1}{\text{Sensitivity}}$

$= \frac{1}{0.43}$

$= 2.33\ \Omega/\text{mm}.$

3. Reproducibility and Repeatability

Reproducibility relates to the closeness of output reading for the same input when their changes in the method of measurement, observer, measuring instrument, location, conditions of use and time of measurement. It may be specified in terms of scale reading over a given period of time.

Repeatability relates to the closeness of output reading when the same input is applied repeatedly over a short period of time with the same measurement condition, same instrument, same location and same condition of use maintained throughout the observation by the same observer.

4. Drift

It is an undesired gradual departure of instrument output over a period of time i.e, unrelated to change in input operating condition or load. An instrument is said to have no drift if it reproduces sam[e reading at] different time for same variation in measured variable.

2013/09/18 13:42

The drift may be cause by following factors

- Mechanical vibrations
- ElectrOmagnetic field.
- Thermal emf.
- Temperature change etc.

5. Static Error.

Study error is defined as the difference between measured value and true value of the quantity.

i.e, $\delta_A = A_m - A_t$

where,

$\delta_A \rightarrow$ Static error

$A_m \rightarrow$ measure value.

$A_t \rightarrow$ true value.

δ_A is also called absolute study error of the quantity 'A' i.e, $\delta_A = \epsilon_0$

$\epsilon_0 \rightarrow$ absolute static error.

The relative static error (ϵ_r) is the ratio of absolute static error to the true value of quantity under measurement.

$$\epsilon_r = \frac{\epsilon_0}{A_t} = \frac{\delta_A}{A_t}$$

$$\Rightarrow \delta_A = A_t \epsilon_r$$

$$\because \delta_A = A_m - A_t$$

$$\therefore A_t \epsilon_r = A_m - A_t$$

or, $A_t \epsilon_r + A_t = A_m$

or, $A_t(1 + \epsilon_r) = A_m$

2013/09/18 13:42

$$\therefore A_t = \frac{A_m}{1+\epsilon_r}$$

Static Correctness $= -\delta_A$

$= A_t - A_m$

Q. A meter reads 127.50V and the true value of the voltage is 127.43V, determine static error and static correction of this instrument?

⇒ solution,

Here, measured value $(A_m) = 127.50V$

true value $(A_t) = 127.43V$

Then,

Static error $(\delta_A) = A_m - A_t$

$= 127.50 - 127.43$

$= 0.07$

∴ Static correction $= -\delta_A$

$= -0.07$.

6. Dead Zone.

It is defined as the largest change of the input quantity for which there is no change of the instrument. For example, the input to the instrument may not be sufficient to overcome the friction and will not move. It will only move when the input is such that it produces a driving force which can overcome friction force.

2013/09/18 13:42

→ Dynamic Characteristics.

1. Speed of Response.

It is defined as the rapidity with which a measurement system respond to change in the measurend quantity. The behaviour of the system when input vary with time (ie, input are dynamic in nature) and so does the output. Such a response in measurement system is found occur in industrial, aerospace, bilogical application.

2. Measuring Lag.

It is the retardation or delay in the response of measurement system to change in the measured quantity. The Lag is caused by conditions such as capacitance, inertia or resistance. They are of two types.

- Retardation type lag.
- Time delay type lag.

3. Fidelity:

It is defined as the degree to which a measurement system indicates change in measured quantity without any dynamic error. It refers to the ability of the system to reproduces the output in the same form as the input.

2013/09/18 13:43

4. Dynamic Error:

It is the difference between the true value of the quantity changing with the time and the value indicated by the measurement system. It is also called measurement error.

Errors are inherent in the act of measurement itself since the perfect accuracy is not attaindable, a description of each measurement should include an attempt to evaluate the magnitude and source of its error. The study of error is the first step in finding way to reduce them. Errors may arise from different source and are usually classified as below

Error
- Gross error
- Systematic error
  - Instrumental
  - Environmental
  - Observation
- Random error.

(A) Gross Error

This class of error mainly covers human mistakes in reading instrument, recording and calculating measurement result. This error is also known as human error. The responsibility of the mistake normally lies with the experiment.

For example; a person due to an over-sight read the voltage at 31.5 V while the actual reading was 21.5 V. But as long as human beings are involved some gross error will definitely be reduced. Complete elimination of gross error is probably impossible, some gross error are easily detected while other may be difficult to detect. Gross error may be of any amount and therefore, mathematical analysis of gross error is impossible.

Methods of elimination of reduction of gross error are

⇒ take careful attaination when making measurement and calculation.

⇒ Awareness of instrument limitations.

⇒ Use of two or more observers to take critical data.

⇒ Taking at least three readings to reduce possible accurance of gross error.

(B) Systematic Error

1. Instrumental Error

They are arised due to three main reasons

Due to inherent short coming in the instrument

These errors are inherent in instruments because of their mecha[illegible] 2013/09/18 13:43 [illegible]se errors may cause the instrument to read very low or very high data. For example;

if the spring of permanent magnet instrument has become weak, the instrument will always read high data.

Due to misuse of instrument.

The error cause in instrument are due to the fault of operator or observer than that of instrument. The zero adjustment is also an error of the observer.

Due to loading effect of instrument.

The error caused in the measurements are due to the loading effect. While calibrated voltmeter may give a mis-reading voltage when connected across the resistance due to loading effect.

2. Environmental Error

These are due to conditions external to the measuring devices including conditions in the area surrounding the instrument. These may be due to defect of temperature, pressure, dust, humidity, external magnetic and electric field etc. This error can be reduced by maintaining constant temperature, humidity by air conditioning.

2013/09/18 13:43

3. Observation Error

Fig1 Error due to parallex

Fig2: Correct Reading

Fig3 Error due to parallex

There are many sources of observation error. The parallex error arises due to pointer and the scale is not being in the same plane. We can eliminate this error by having the pointer and scale in the same plane or by taking the data perpendicularly with the pointer. There are human factors involved in the measurement, the sensing capability of individual observer affect the accuracy of measurement. Modern electric instruments have digital display of output which completely eliminates the error due to parallex.

(C) RANDOM ERROR.

The quantity being measured is affected by happening throughout the universe. We are aware about some of the factors influencing measurement but about the rest factors we are unawares. The disturbance which we are unaware is called random. Hence, the error caused by these happenings are called random error. For example; unknown events that cause small variation in measurement.

2013/09/18 13:43

Vardhmen
PAGE NO.
DATE: / /20

⇒ STATISTICAL ANALYSIS

1. Arithmetic Mean
2. Deviation From Mean
3. Average Deviation.
4. Standard Deviation.

1. Arithmetic Mean.

The based approximation is made when the number of reading of the same quantity is very large. Theoritically large number of reading would give the best result. The arithmetic mean is given by

$$\overline{X} = \frac{X_1 + X_2 + X_3 + \text{------} + X_n}{n}$$

where,

$\overline{X} \rightarrow$ Arithmetic mean

$n \rightarrow$ Toral no. of reading/observation.

$X_1, X_2, X_3, ---, X_n$ are the reading taken.

2. Deviation From Mean

Deviation is the departure of given reading from the arithmetic mean of the group of reading. If the deviation of first reading X_1 is called d_1, X_2 is called d_2 and so on. Then

$$d_1 = X_1 - \overline{X}$$
$$d_2 = X_2 - \overline{X}$$
$$\vdots$$
$$d_n = X_n - \overline{X}$$

2013/09/18 13:43

Where,

$\overline{X} \rightarrow$ Arithmetic mean

Note that the deviation from mean may have a positive or negative value and that algebraic sum of all deviation is zero.

3. Average Deviation.

The average deviation is an indication of precision of the instrument used in the measurement. Highly precise instrument will give a low average deviation. The average deviation is defined as the ratio of sum of absolute value of deviation to the total number of readings.

$$D = \frac{|d_1| + |d_2| + |d_3| + \text{------} + |d_n|}{N}$$

4. Standard Deviation:

Standard deviation of a finite number of data is the square root of sum of all the individual deviations square divided by (N-1)

$$S.D\ (\sigma) = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \text{------} + d_n^2}{N-1}}$$

Variance $= \sigma^2$

Q. A set of independent current measurement was taken by six observers and recorded as 12.8 mA, 12.2 mA, 12.5 mA, 13.1 mA, 12.9 mA and 12.4 mA. Calulate

i) arithmetic me 2013/09/18 13:44

ii) deviation from mean

iii) average deviation.

iv) standard deviation.

v) Varience

vi) probable error (0.674σ)

⇒ Soln.

Here,

Given data: 12.8, 12.2, 12.5, 13.1, 12.9, 12.4

$N = 6$

$$A.M(\bar{x}) = \frac{12.8+12.2+12.5+13.1+12.9+12.4}{6}$$

$$\therefore \bar{x} = 12.65$$

For standard deviation

$$d_1 = x_1 - \bar{x} = 12.8 - 12.65 = 0.15$$
$$d_2 = x_2 - \bar{x} = 12.2 - 12.65 = -0.45$$
$$d_3 = x_3 - \bar{x} = 12.5 - 12.65 = -0.15$$
$$d_4 = x_4 - \bar{x} = 13.1 - 12.65 = 0.45$$
$$d_5 = x_5 - \bar{x} = 12.9 - 12.65 = 0.25$$
$$d_6 = x_6 - \bar{x} = 12.4 - 12.65 = -0.25$$

$$\therefore \text{Average deviation } (D) = \frac{|d_1|+|d_2|+|d_3|+|d_4|+|d_5|+|d_6|}{N}$$

$$= \frac{0.15+0.45+0.15+0.45+0.25+0.25}{6}$$

$$= \frac{1.7}{6}$$

$$= 0.2833$$

$$\text{Standard deviation}(\sigma) = \sqrt{\frac{d_1^2+d_2^2+d_3^2+d_4^2+d_5^2+d_6^2}{N-1}}$$

$$= \sqrt{(0.15)^2+(0.45)^2+(0.15)^2+(0.45)^2+(0.25)^2+(0.25)^2}$$

2013/09/18 13:44

$$= \sqrt{\frac{0.575}{5}}$$

$$= 0.3391$$

$$\therefore \text{Variance} = \sigma^2$$
$$= (0.3391)^2$$
$$= 0.115$$

$$\text{probable error} = 0.674\sigma$$
$$= 0.674 \times 0.3391$$
$$= 0.2286$$

Probable Error.

The best value of normal distribution is obtained by taking arithmetic mean of the various value of the given reading. If 'σ' be the standard deviation then probable error is 0.670σ. The probable error has been used in experimental work in the past.

Limiting Error.

In most instrument the accuracy is guaranted to be within a certain percentage of full scale reading. The limit of thin deviation from the specified value are defined as limiting error. If A_a be the actual quantity, A_s be the nominal quantity. Then $\pm \delta A$ is

2013/09/18 13:44

Vardhman
PAGE NO.
DATE: / /20

the limiting error.

i.e,

$$A_a = A_s \pm \delta A$$

Relative limiting error; $\epsilon_r = \frac{A_a - A_s}{A_s} = \frac{\pm \delta A}{A_s} \Rightarrow \pm \delta A = \epsilon_r A_s$

Q. A 0-150V voltmeter has a guaranted accuracy of 1% of full deflection. The voltage measured by this instrument is 75V. Calculate the limiting error in percentage?

⇒ Solution,

Here, nominal value $= 150V$

Relative % error $(\epsilon_r) = 1\%$

$$\epsilon_r = 0.01$$

Since, $\epsilon_r = \frac{\delta A}{A_s}$

$$\therefore \delta A = \epsilon_r \times A_s = 0.01 \times 150 = 1.5\,V$$

Now,

$$\delta A = 1.5V$$
$$A_s = 75V$$
$$\epsilon_r = ?$$

Then,

$$\epsilon_r = \frac{\delta A}{A_s} = \frac{1.5V}{75V} = 0.02$$

$$\epsilon_r = 2\%$$

$\therefore$ Actual value $(A_a) = A_s \pm \delta A$

$$= 75 \pm 0.02$$

2013/09/18 13:44

DATE: / /20

Q. The resistance of an unknown resister is determine by wheat-stone bridge meter. The solution of unknown resistor is stated as $R_x = \frac{R_1 R_2}{R_3}$ where,

$R_1 = 500 \pm 1\%$
$R_2 = 615 \pm 1\%$
$R_3 = 100 \pm 0.5\%$

Calculate
i) Nominal value of R_x
ii) Limiting error in ohm(Ω) of R_x and percentage error.

⇒ Solution,

Here, we have given

$R_1 = 500 \pm 1\%$
$R_2 = 615 \pm 1\%$
$R_3 = 100 \pm 0.5\%$

Then,

$$\text{Nominal value of } R_x = \frac{R_1 R_2}{R_3} = \frac{500 \times 615}{100} = 3075\Omega$$

Now for limiting error,

$$\frac{\delta R_x}{R} = \epsilon_r = \pm\left(\frac{\delta R_1}{R_1} + \frac{\delta R_2}{R_2} + \frac{\delta R_3}{R_3}\right) \quad \text{(i)}$$

where,

$R_1 = 500 \pm 1\%$, $\delta R_1 = 1\%$ of 500
$= R_n \pm \delta R_1$ $\quad = 5\Omega$

$\delta R_2 = 1\%$ of $650 = 6.5\Omega$
$\delta R_3 = 0.5\%$ of $100 = 0.5\Omega$

Then from (i)

$$\epsilon_r = \pm\left(\frac{5}{500} + \frac{6.5}{650} + \frac{0.5}{100}\right)$$

2013/09/18 13:45

$\epsilon_r = 0.025$

$\epsilon_r = 2.5\%$

Again,

$R_x = R_x \pm \delta R_x$

$\because \frac{\delta R_x}{R_x} = \epsilon_r$

or, $\delta R_x = \epsilon_r \times R_x$

$= 0.025 \times 3075$

$= 76.875$

$\therefore R_x = (3075 \pm 76.875)$

⇒ WHEAT-STONE BRIDGE

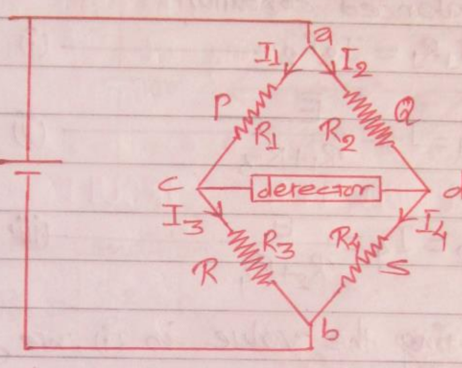


Fig: Wheat-Stone Bridge

A bridge is the name used to denote a special class of measuring circuit. They are most often used for making measurement of resistance, capacitance and inductance. Bridge are used for resistance measurement when the very accurant determination of a particular resistance is required. The most well known and widely used resistance bridge is wheat-stone bridge, it is used for accurately measuring resistance value from mΩ to MΩ range. The value of re[illegible] from

from the bridge are far more accurate then the value obtained from ohm meter.

The bridge has four arms together with a source of emf and a null detector usually as a galvanometer or other sensitive meter. The current through the galvanometer depends upon the potential difference between point C and d. The bridge is said to be balanced when the potential difference across the galvanometer is zero. So, that there is no current through the galvanometer.

At balanced condition

$$I_1 R_1 = I_2 R_2 \text{ ———— (i)}$$

and,

$$I_1 = I_3 = \frac{E}{R_1 + R_3} \text{ ———— (ii)}$$

$$I_2 = I_4 = \frac{E}{R_2 + R_4} \text{ ———— (iii)}$$

substituting the value in (i), we get

$$\frac{E \times R_1}{R_1 + R_3} = \frac{E \times R_2}{R_2 + R_4}$$

or, $R_1(R_2 + R_4) = R_2(R_1 + R_3)$

or, $R_1 R_2 + R_1 R_4 = R_2 R_1 + R_2 R_3$

or, $R_1 R_4 = R_2 R_3$

$$R_4 = \frac{R_2 R_3}{R_1}$$

$$\therefore R_4 = \frac{R_2}{R_1} \times R_3$$

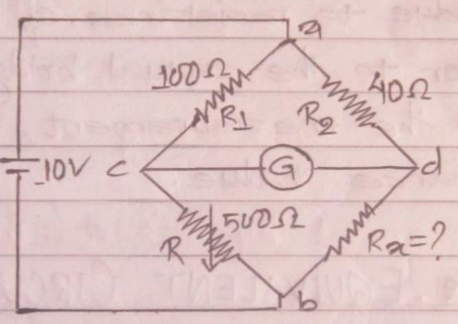
2013/09/18 13:45

PAGE NO.
DATE: / /20

In practical bridge, the ratio of R_2/R_1 is controlled by a variable switch, R_3 is continuously adjustable variable resistor. A number of detector including galvanometer or even headphone are available to determine a null or balanced condition.

Q. Given a wheat-stone bridge with $R_1 = 100\Omega$, $R_2 = 40\Omega$, $R_3 = 500\Omega$, $V_0 = E = 10V$, $R_G = 600\Omega$. Find the value of R_x when the bridge is in balanced condition?

⇒ soln.



At balanced condition

$$R_x = \frac{R_2}{R_1} \times R_3$$

$$= \frac{40}{100} \times 500$$

$$= 200\,\Omega$$

2013/09/18 13:45

⇒ Causes of error of wheat-stone bridge or (Measurement error)

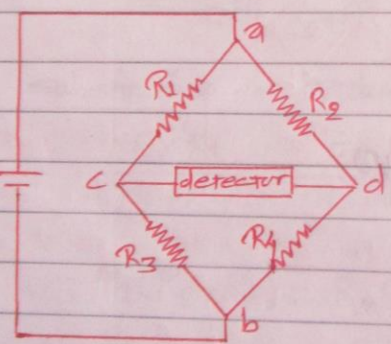
1. Insufficient sensitivity of null detector.

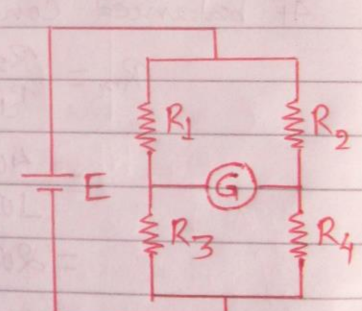
2. Change in resistance of bridge arm due to heating effect of current through the resistor.

3. Thermal emf in the bridge circuit or galvanometer circuit can also cause problem when low value resistor are being measured.

4. Error due to resistance of lead and contact exterior to the actual bridge circuit play a roll in the measurement of very low resistance value.

⇒ THEVENIN'S EQUIVALENT CIRCUIT



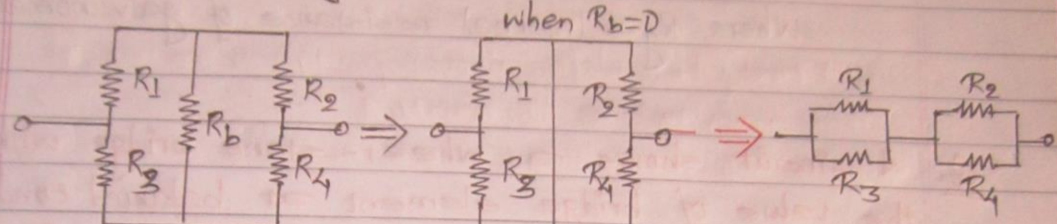


To determine whether or not the galvanometer has required sensitivity to detect unbalanced condition, it is necessary to calculate galvanometer current Ig. Different galvanometer not only may require different current sensitivity but they also have

2013/09/18 13:45

different internal resistance. since, we are interested in the current through the galvanometer, the thevenin's equivalent circuit is determined by looking into the galvanometer terminal.

For detecting R_{th},

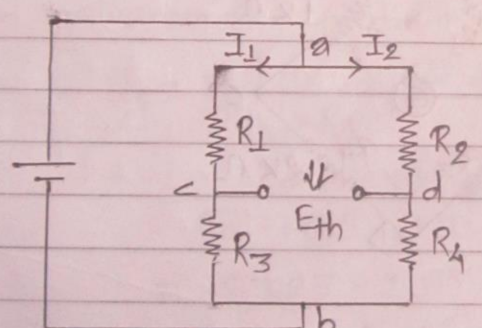


$R_b \rightarrow$ internal resistance

$\therefore$ Equivalent Resistance.

$$R_{th} = (R_1 // R_3) + (R_2 // R_4)$$

$$R_{th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$



$$\therefore E_{th} = E_{ac} - E_{ad}$$

$$= I_1 R_1 - I_2 R_2$$

$$= \left(\frac{E}{R_1 + R_2}\right) R_1 - \left(\frac{E}{R_2 + R_4}\right) R_2$$

$$= E\left(\frac{R_1}{R_1+R_3} - \frac{R_2}{R_2+R_4}\right)$$

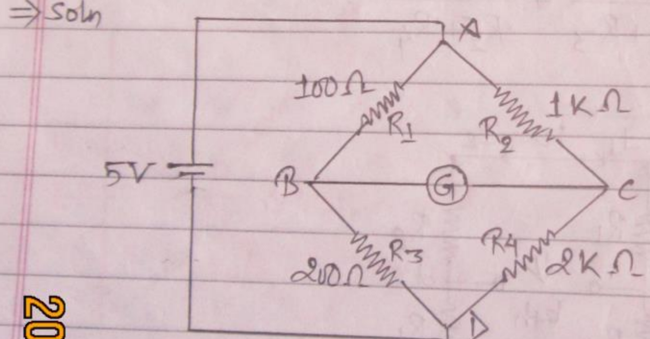
Now, galvanometer current is given by

$$\boxed{I_g = \frac{E_{th}}{R_{th}+R_g}}$$

Where, $R_g \rightarrow$ Internal resistance of galvanometer.

Q. A circuit shows a wheat-stone bridge with the value of bridge element at balanced condition. The battery is 5V, internal resistance is negligible. The galvanometer has a current sensitivity of 10 mm/μA and internal resistance of 100 Ω. Calculate the deflection of galvanometer caused by +5 Ω unbalanced in the arm 'CD'.

⇒ Soln



Here,

Given,

$R_1 = 100\Omega$, $R_2 = 1K\Omega$, $R_3 = 200\Omega$ and $R_4 = 2000+5$
$= 2005\,\Omega$

Sensitivity = 10 mm/μA

Equivalent resistance

$$R_{th} = (R_1 // R_3) + (R_2 + R_4)$$

$$= \frac{R_1 R_2}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$= \frac{100 \times 200}{100 + 200} + \frac{1000 \times 2005}{1000 + 2005}$$

$$= \frac{20000}{300} + \frac{2005000}{3005}$$

$$R_{th} = 733.89\,\Omega$$

For E_{th},

$$E_{th} = E\left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4}\right)$$

$$= 5\left(\frac{100}{100 + 200} - \frac{1000}{1000 + 2005}\right)$$

$$= 5\left(\frac{100}{300} - \frac{1000}{3005}\right)$$

$$= 2.77$$

Then galvanometer current

$$I_g = \frac{E_{th}}{R_{th} + R_g}$$

$$= \frac{2.77 \times 10^{-3}}{733.89 + 100}$$

$$= 3.325 \times 10^{-6} A$$

$$= 3.32\,\mu A$$

Deflection $= I_g \times$ sensitivity

$$= 3.32\,\mu A \times 10\,mm/\mu A$$

$$= 33.2\,mm$$

2013/09/18 13:46

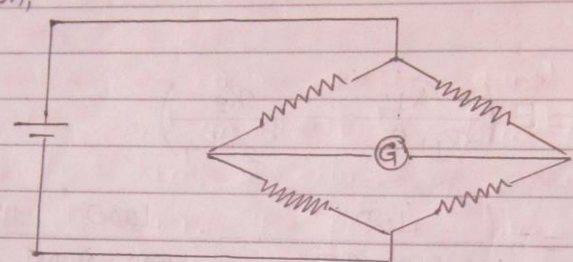
Q. Given a wheat-stone bridge with $R_1 = 100\Omega$, $R_2 = 40\Omega$, $R_3 = 50\Omega$, $V_0 = 10V$, $R_m = R_g = 600\Omega$

(i) Find the value of R_x at balanced condition.

(ii) If the R_x is changed by +2 from its value at balanced, find I_g that will flow through galvanometer.

(iii) Find the deflection of galvanometer, if sensitivity is 10 mm/μA.

⇒ Solution,



We have given,

Resistance of arm AB (R_1) = 100Ω

Resistance of arm AC (R_2) = 40Ω

Resistance of arm BD (R_3) = 50Ω

Resistance of arm CD (R_4) = R_x = ?

Supplied voltage (V_0) = 10V

Sensitivity of galvanometer = 10mm/μA

Internal resistance (R_g) = 600Ω

i) Value of R_x/R_4

Now, when bridge is in balanced condition then

$$R_4 = R_x = \frac{R_3}{R_1} \times R_2$$

$$= \frac{40\Omega \times 50\Omega}{100\Omega}$$

$$R_x = R_4 = 20\Omega$$

DATE: / /20

ii) since, R_x is change by $+2\Omega$ from its value at balance i.e, $R_x = 20\Omega$.

2013/09/18 13:47

$R_x = R'_x = 20\Omega + 2\Omega = 22\Omega$

Then,

equivalent resistance $(R_{th}) = (R_1 // R_3) + (R_2 // R'_x)$

$= (100\Omega // 50\Omega) + (40\Omega // 22\Omega)$

$= \frac{100\Omega \times 50\Omega}{100\Omega + 50\Omega} + \frac{40\Omega \times 22\Omega}{40\Omega + 22\Omega}$

$R_{th} = 47.526\,\Omega$

Also,

Thevenin voltage $(V_{Th}) = E_{th} = E_{AB} - E_{AC}$

$= I_1 R_1 - I_2 R_2$

$= \left(\frac{E}{R_1 + R_3}\right) R_1 - \left(\frac{E}{R_2 + R'_x}\right) \times R_2$

$= E\left[\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R'_x}\right]$

$= 10V\left[\frac{100\Omega}{100\Omega + 50\Omega} - \frac{40\Omega}{40\Omega + 22\Omega}\right]$

$= 10V \times 0.02184$

$= 0.2184\,V$

So, galvanometer current $(I_g) = \frac{E_{th}}{R_{th} + R_g}$

$= \frac{0.2184V}{47.526\Omega + 600\Omega}$

$I_g = 337.3\,\mu A$

iii) Deflection of galvanometer is

$= I_g \times$ sensitivity of galvanometer

$= 337.3\mu A \times 10\text{ mm}/\mu A$

$= 3373$ mm.

⇒ AC BRIDGE

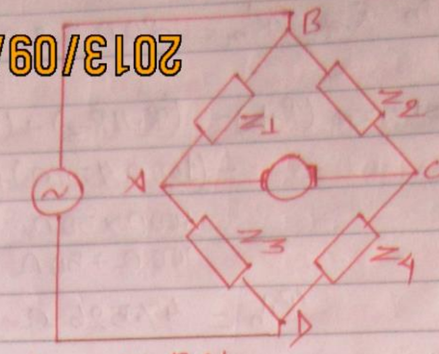


Fig: AC Bridge.

The ac bridge is a natural outgrowth of dc bridge and in its basic form it consist of four bridge arm, a source of excitation and a null detector. The power source supply and ac voltage to the bridge at the desired frequency. For measurement at low frequency the power line may serve as source of excitation at higher frequencies and oscillator generally supply the excitation voltage. The null deflection must response to ac unbalanced current and in its form consist of a pair of headphone or any means.

The four bridge arm Z_1, Z_2, Z_3 and Z_4 are indicated as shown in figure. The balanced condition in this ac bridge is reached when the detector response is zero. The condition for bridge balance requires that the potential difference from 'A' to 'C' is zero. This will be the case when the voltage drop from

'B' to 'A' is equal to the voltage drop from 'B' to 'C' in both magnitude and phase.

In balanced condition

$$E_{BA} = E_{BC} \quad \text{and} \quad I_1 = I_2$$

$$\Rightarrow I_1 Z_1 = I_2 Z_2$$

$$\text{or, } \frac{E}{Z_1+Z_3} \times Z_1 = \frac{E}{Z_2+Z_4} \times Z_2$$

$$\text{or, } Z_1(Z_2+Z_4) = Z_2(Z_1+Z_3)$$

$$\text{or, } Z_1Z_2 + Z_1Z_4 = Z_2Z_1 + Z_2Z_3$$

$$\text{or, } \boxed{Z_1Z_4 = Z_2Z_3} \rightarrow \text{Impedance form.}$$

$$\boxed{Z_4 = \frac{Z_2Z_3}{Z_4}}$$

$$\boxed{Y_1Y_4 = Y_2Y_3} \rightarrow \text{Admittance form.}$$

From above equation, it states that the product of impedance of one pair of opposite arm must be equal to the product of impedance of the other pair of opposite arm with the impedance expressed in complex notation.

If impedances are expressed in terms of magnitude and phase angle then balanced equation can be written as

$$Z_1Z_4 = Z_2Z_3$$

$$\text{or, } Z_1\angle\theta_1 \, Z_4\angle\theta_4 = Z_2\angle\theta_2 \, Z_3\angle\theta_3$$

$$\text{or, } Z_1Z_4\angle(\theta_1+\theta_4) = Z_2Z_3\angle(\theta_2+\theta_3)$$

$$\Rightarrow \quad Z_1 Z_4 = Z_2 Z_3 \quad \text{——— (A)}$$

$$\text{and } \measuredangle(\theta_1+\theta_4) = \measuredangle(\theta_2+\theta_3) \quad \text{——— (B)}$$

The product of magnitude of opposite arm must be equal from equation (A) and the sum of phase angle of the opposite arm must be equal from equation (B).

Q. The impedance of the AC bridge are given as follow $Z_1 = 100\Omega \measuredangle 80$, $Z_2 = 250\Omega \measuredangle 0$, $Z_3 = 400\Omega \measuredangle 30$ and $Z_4 = ?$, $\theta_4 = ?$

$\Rightarrow$ Solution,

We have given magnitude and phase angle of arms of AC-bridge as follows

$Z_1 = 100\Omega \qquad \phi_1 = 80^\circ$

$Z_2 = 250\Omega \qquad \phi_2 = 0^\circ$

$Z_3 = 400\Omega \qquad \phi_3 = 30^\circ$

$Z_4 = \text{unknown} = ? \qquad \phi_4 = ?$

Then,

The first condition for bridge balance gives that $Z_1 Z_4 = Z_2 Z_3$ and on solving we get Z_4

i.e, $$Z_4 = \frac{Z_2 Z_3}{Z_1}$$

$$= \frac{400\Omega \times 250\Omega}{100\Omega}$$

$$Z_4 = 1000\Omega$$

$$\therefore Z_4 = 1K$$

Again,

To find phase angle ϕ_4, second condition for bridge balance gives that

$$\measuredangle\phi_1 + \measuredangle\phi_4 = \measuredangle\phi_2 + \measuredangle\phi_3$$

or, $80^\circ + \phi_4 = 0^\circ + 30^\circ$

$\measuredangle \phi_4 = 30^\circ - 80^\circ = -50^\circ$

Hence, the unknown impedance Z_4 can be written in polar form as,

$Z_4 = 1k \measuredangle -50^\circ$

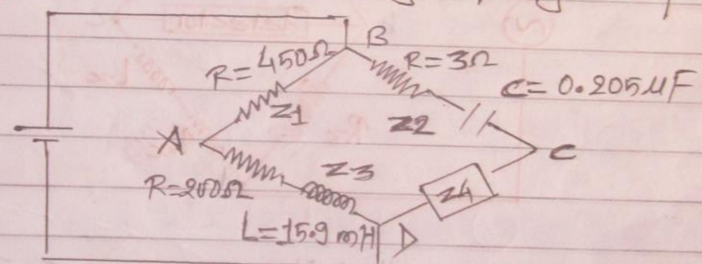
indicating that we are dealing with capacitive element, possibly in series.

Q. The Ac-bridge is in balanced with the following constant arm AB, $R = 450\,\Omega$, BC = R, $3\,\Omega$ in series with $C = 0.205\,\mu F$, CD = ? DA, $R = 200\,\Omega$ series with $L = 15.9\,mH$. The oscillator frequency is 1 kHz. Find the constant of CD?

⇒ solution..

We have given, the parameter of constant arm AB, BC, DA and need to find constant of CD.

First we draw a Ac-bridge for given parameter.



From figure, we get.

$Z_1 = R = 450\,\Omega$

$Z_2 = R - jX_C = 3\,\Omega - j/\omega C$ —— (i).

So, frequency of oscillator (F) = 1 KHz = 1000 Ω

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000 \times 0.205\,\mu F}$$

$= 776.76\,\Omega$

and $X_L = 2\pi f L = 2\pi \times 1000 \times 15.9\,mH$

$= 99.852\,\Omega$

Then from equation ii.

$Z_2 = 3\Omega - j\,776.76\,\Omega$

 $= R + j\omega L = 200\Omega + j\,99.852\,\Omega$

$Z_4 = ?$

Now, the general equation for bridge balance state is

$$Z_1 Z_4 = Z_2 Z_3.$$

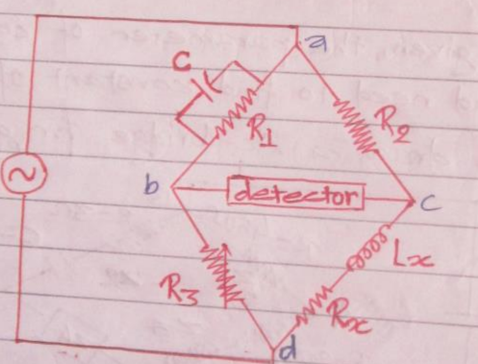
or, $$450\Omega \times Z_4 = (3\Omega - j776.76\Omega)(200\Omega + j99.852\Omega)$$

$$Z_4 = \frac{(600 + j299.556 - j155352 - j^2 77561.04)}{450}$$

$$Z_4 = (173.6912 - j\,344.56)\,\Omega$$

$$Z_4 = 385.86 \angle -63.25^\circ$$

⇒ MEASUREMENT OF INDUCTANCE USING MAXWELL BRIDGE



Maxwell bridge measures as an unknown inductance in terms of a known capacitance. One of the ratio arm has a resistance and a capacitance in parallel and the opposite arm has a combination of resistance and inductance in series. Z_1, Z_2, Z_3 and Z_4/Z_x are the corresponding impedances of arm ab, ac, bd and cd respectively.

At balanced condition

$$Z_1 Z_x = Z_2 Z_3$$

$$\text{or, } Z_x = \frac{Z_2 Z_3}{Z_1}$$

$$\text{or, } Z_x = Z_2 Z_3 Y_1 \text{ ——— (i)}$$

Now,

$$Z_1 = R_1 // (-jX_{C_1})$$

$$\Rightarrow Z_1 = \frac{-jR_1 X_{C_1}}{R_1 - jX_{C_1}}$$

$$\Rightarrow Y_1 = \frac{1}{Z_1} = \frac{R_1 - jX_{C_1}}{-jR_1 X_{C_1}}$$

$$= \frac{R_1 - j\frac{1}{\omega C_1}}{-jR_1 \frac{1}{\omega C_1}}$$

$$= \frac{(R_1 \omega C_1 - j)/\omega C_1}{-jR_1 \frac{1}{\omega C_1}}$$

$$= \frac{(R_1 \omega C_1 - j)}{-jR_1}$$

$$= \frac{R_1 \omega C_1}{-jR_1} - \frac{j}{-jR_1}$$

$$\Rightarrow Y_1 = \frac{1}{R_1} - \frac{\omega C_1}{j}$$

$$\Rightarrow \boxed{Y_1 = \frac{1}{R_1} + j\omega C_1}$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$\therefore Z_x = R_x + jX_{L_x}$$

$$= R_x + j\omega L_x$$

Substituting Z_1, Z_2, Z_3, Z_x in eq^n (i)

$$R_x + j\omega L_x = \left(\frac{1}{R_1} + j\omega C_1\right) R_2 R_3$$

$$\text{or, } R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3$$

Now, comparing real and imaginary part, we get

$$\boxed{R_x = \frac{R_2 R_3}{R_1}}$$

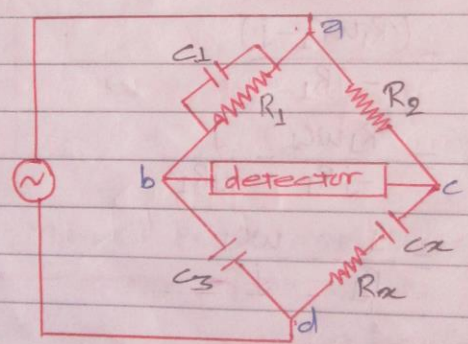
and,

$$\omega L_x = \omega C_1 R_2 R_3$$

$$\boxed{L_x = C_1 R_2 R_3}$$

Where resistance is expressed in Ohm (Ω) and capacitance in Farad (F) and inductance in Henry (H).

⇒ MEASUREMENT OF CAPACITANCE OF SCHERING BRIDGE



Schering bridge is used for the measurement of capacitance. R_1 contains parallel combination of resistor and capacitor and the corresponding opposite arm contains series combination of resistor and capacitor, the standard arm 'bd' contains only a capacitor.

At balanced condition

$Z_x Z_1 = Z_2 Z_3$

$Z_x = Z_2 Z_3 Y_1$ ——— (i)

Now,

$Y_1 = \frac{1}{R_1} + j\omega C_1$

$Z_2 = R_2$

$Z_3 = -jX_{C_3} = -j\frac{1}{\omega C_3} = \frac{-j}{\omega C_3}$

$Z_x = R_x - jX_{C_x}$

$= R_x - j\frac{1}{\omega C_x}$

Substituting Y_1, Z_2, Z_3, Z_x in eq^n (i), We get

$R_x - j\frac{1}{\omega C_x} = R_2\left(\frac{-j}{\omega C_3}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$

$= -j\frac{R_2}{R_1 \omega C_3} - j^2\frac{\omega C_1 R_2}{\omega C_3}$

$R_x - j\frac{1}{\omega C_x} = \frac{R_2 C_1}{C_3} - j\frac{R_2}{R_1 \omega C_3}$

On comparing real part and imaginary part, We get

$\boxed{R_x = \frac{C_1}{C_3} \times R_2}$ ——— (A)

and, $\frac{1}{\omega C_x} = \frac{R_2}{R_1 \omega C_3}$

or $\boxed{C_x = \frac{R_1}{R_3} \times C_3}$ ——— (B)

This is the required expression for measurement of capacitance using Schering bridge.

→ WEIN BRIDGE FOR the MEASUREMENT OF FREQUENCY

2013/09/18 13:49

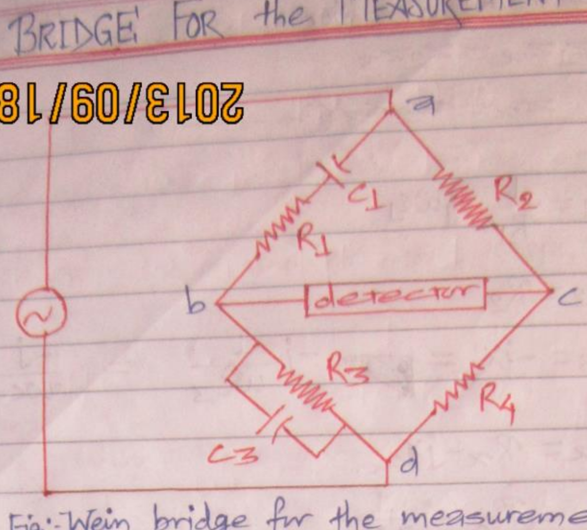


Fig:- Wein bridge for the measurement of frequency.

The Wein bridge is used to measure frequency and also for various other useful circuit. The Wein bridge also finds application in audio and high frequency oscillator as the frequency determining elements. The wein bridge has a series combination of resistor and capacitor in arm 'ab' and parallel combination of resistor and capacitor in the adjusting arm 'bd'.

At balanced condition

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_2 = Z_1 Z_4 Y_3 \quad \text{—— (i)}$$

Now,

$$Z_2 = R_2 + j0$$

$$Z_3 = R_3 // -jX_3$$

$$= \frac{-jR_3 X_3}{R_3 - jX_3}$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{R_3} + j\omega C_3$$

and, $Z_4 = R_4$

Also, $Z_1 = R_1 - jX_{C_1}$

$$= R_1 - \frac{j}{\omega C_1}$$

substituting in equation (i),

$$Z_2 = Z_1 Z_4 Y_3$$

$$R_2 = \left(R_1 - \frac{j}{\omega C_1}\right) R_4 \left(\frac{1}{R_3} + j\omega C_3\right)$$

$$= \left(R_1 R_4 - \frac{jR_4}{\omega C_1}\right)\left(\frac{1}{R_3} + j\omega C_3\right)$$

$$= \frac{R_1 R_4}{R_3} + j\omega C_3 R_1 R_4 - \frac{jR_4}{\omega C_1 R_3} + \frac{R_4 \omega C_3}{\omega C_1}$$

$$= \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1} + j\left(\omega C_3 R_1 R_4 - \frac{R_4}{\omega C_1 R_3}\right)$$

Equating imaginary part to zero

Here,
Real part is

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}$$

and imaginary part is (gives)

$$\omega C_3 R_1 R_4 - \frac{R_4}{\omega C_1 R_3} = 0$$

or, $\omega^2 C_1 C_3 R_1 R_3 R_4 - R_4 = 0$

or, $R_4(\omega^2 C_1 C_3 R_1 R_3 - 1) = 0$

or, $\omega^2 C_1 C_3 R_1 R_3 - 1 = 0$

or, $\omega^2 C_1 C_3 R_1 R_3 = 1$

or, $$\omega^2 = \frac{1}{C_1 C_3 R_1 R_3}$$

or, $$\omega = \frac{1}{\sqrt{C_1 C_3 R_1 R_3}}$$

or, $$2\pi f = \frac{1}{\sqrt{C_1 C_3 R_1 R_3}}$$

$$\boxed{f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}}}$$