Unit- Three

Signals and Systems

- Signals and their classification: Periodic and non-periodic signals; Deterministic and
- Random signals; Energy and Power signals; Continuous and Discrete time signals
- Basic Elementary Signals - Unit Step Signal, Ramp Signal, Impulse Signal, Sinusoidal

Compiled by: Er. Ganesh Datt Joshi, Lecturer at NAST

Unit- Three

Signals and Systems

- Signal, Signum Signal
- System - Continuous and Discrete time system
- Basic system properties: Linearity, Causality, Stability, Static & Dynamic, and Time
- Invariance, Introduction to LTI System

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Signals and their classifications

Signals

It is defined as the physical parameters which changes their magnitude and direction with respect to time. Signals are electrical or electromagnetic representation of data. It can be also defined as the function of one or more independent variables which contains some information. For example: Radio-Tele-vision Signal, Voice and Video Signal etc.

Signals

A continuous time signal or analog signal may be defined as a signal which move continuously in time domain, the independent variable is time (t) and a continuous time signal is represented by x(t) as shown in the waveform.

A discrete time signal is defined only at certain time instant. For discrete time signal the independent variable is time (n) and is denoted by x(n).

Signals

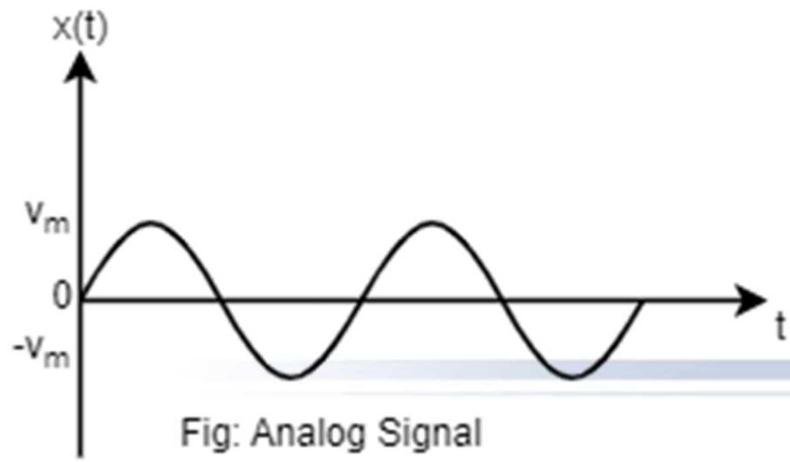


Fig: Analog Signal

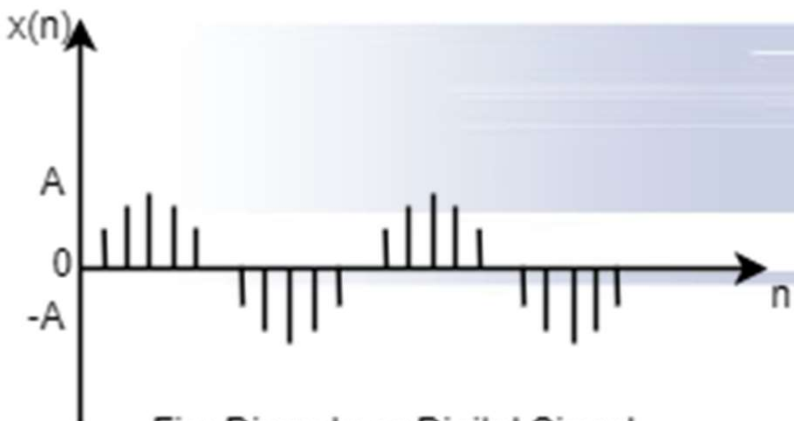


Fig: Discrete or Digital Signal

We may classify both continuous and discrete time signal as;

1. Periodic and Non-periodic

A periodic signal is that type of signal which has fixed patterns and repeats over and over within same interval of time period (t). A signal is said to be period if it satisfies the following conditions;

$x(t)=x(t+T)$ For all 'T'

Signals and their classifications

1. Periodic and Non-periodic

Fig: Continuous time period Signal

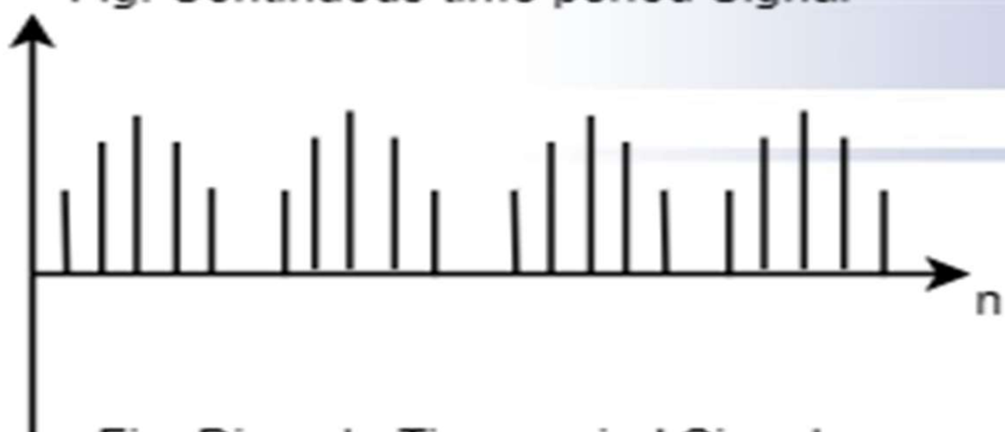


Fig: Discrete Time period Signal

Signals and their classifications

1. Periodic and Non-periodic

For discrete time signal;

$x(n)=x(n+N)$

A signal whose set of values or pattern does not retrieve after certain interval of time are known as aperiod or non-periodic signal.

Signals and their classifications

1. Periodic and Non-periodic

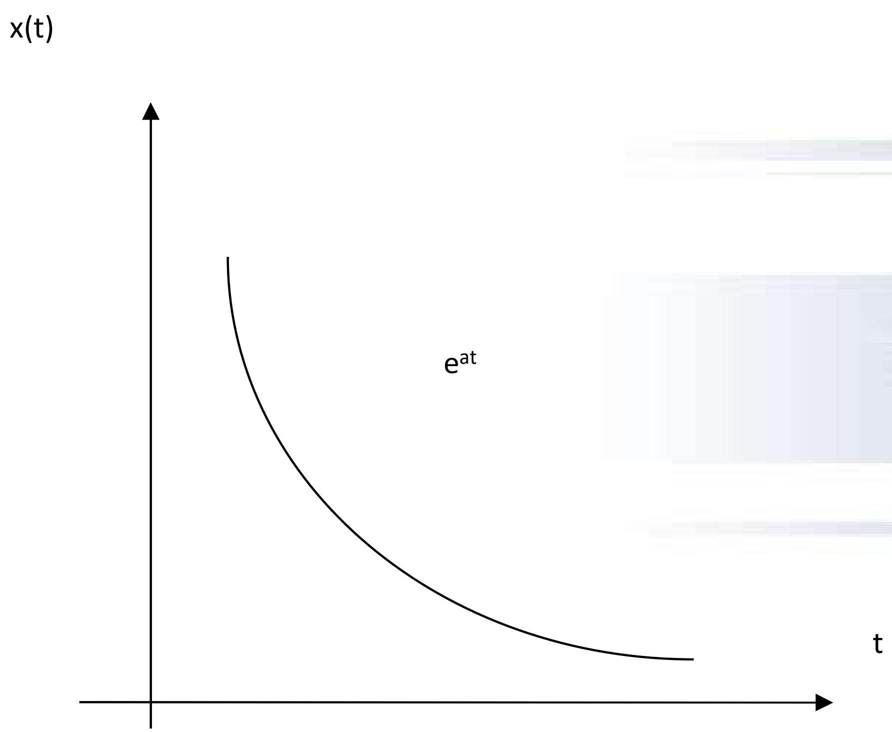


Fig: x(t)≠x(t+T) for all T

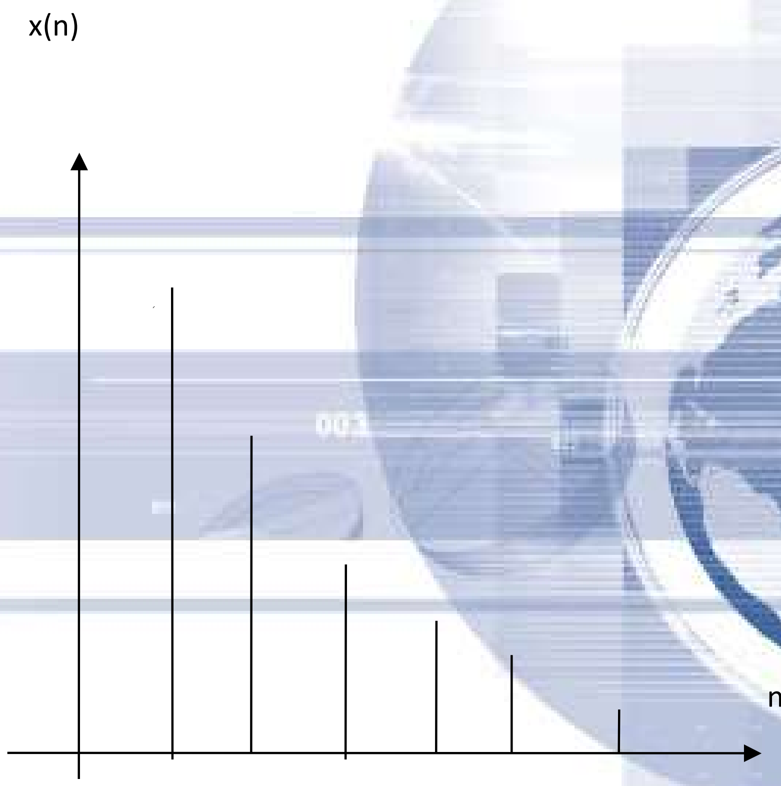


Fig: x(n)≠x(n+N) for all N

2. Deterministic and Random

The deterministic signal is one whose future value can be predicted from the knowledge of present and past value. A signal is said to be deterministic if it can be described without any uncertainty.

2. Deterministic and Random

A non- deterministic signal is one whose occurrence is always in random in nature, the pattern of such signal are quite irregular. Non-deterministic signals are also known as random signals. For example: Thunderstorm in nature, thermal noise generated in electrical circuit etc.

3. Energy and Power

The energy signal is one which has finite energy and average power. Hence, x(t) is an energy signal if;

$$0 < E < \infty \; \& \; P = 0$$

Mathematically, energy is calculated by;

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

A power signal is one which has finite power and infinite energy, a signal x(t) is a power signal if;

$$0 < P < \infty \;\&\; E = \infty$$

Mathematically,

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

3. Energy and Power

Non-periodic or periodic signals are energy signals and practical periodic signal are the power signals.

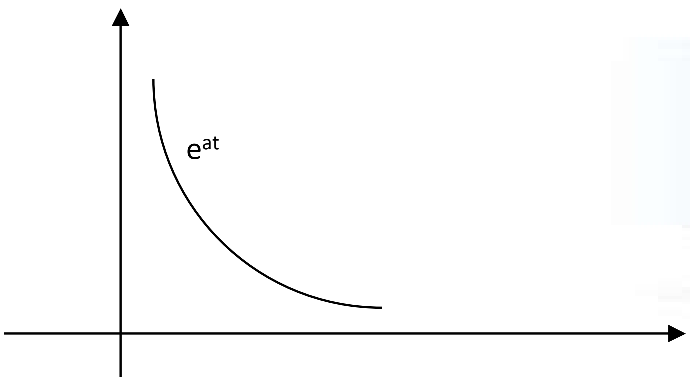


Fig: Energy Signal

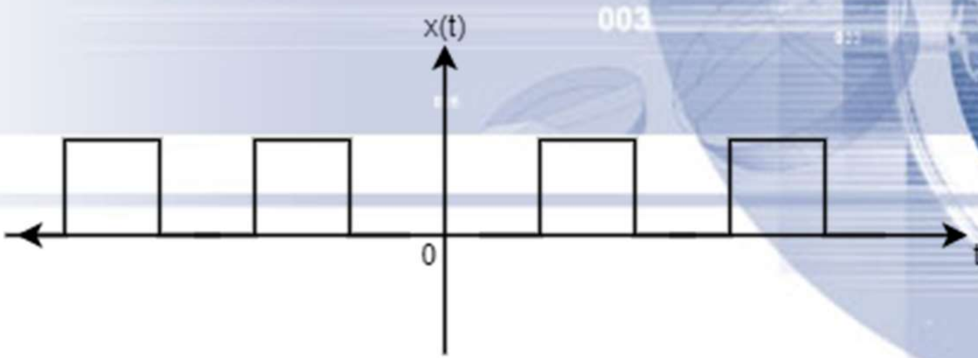


Fig: Power Signal

Example:

1. Check whether the signal $x(t) = A\cos\omega_0 t$ is a power or energy signal?

Solution:

Signals and their classifications

$$x(t) = A\cos\omega_0 t$$

To determine power of given signal,

$$p = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

$$p = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} (A^2\cos^2\omega_0 t) dt$$

$$p = \lim_{T\to\infty} \frac{A^2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\frac{1-\cos 2\omega_0 t}{2}\right) dt$$

Signals and their classifications

$$p = \lim_{T\to\infty} \frac{A^2}{2T}\left\{\frac{T}{2} + \frac{T}{2}\right\}$$

$$p = \lim_{T\to\infty} \frac{A^2}{2T}.T$$

$$p = \frac{A^2}{2} \quad (0 < p < \infty)$$

Now, energy of given signal is

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E = \int_{-\infty}^{\infty} A^2 \cos^2 \ \omega_0 t \ dt$$

$$E = \frac{A^2}{2}\left[\int_{-\infty}^{\infty}(1 - \cos 2\,\omega_0 t)dt\right]$$

$$E = \infty$$

Since, power of given signal is finite and energy is infinite. So, the given is power signal.

2. $x(t) = \sin 15\pi t$. Determine whether the signal is power or energy.

Solution:

Here, $x(t) = \sin 15\pi t$

To determine power of given signal;

$$p = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

For periodic signal it must satisfy

$$x(t) = x(t+T)$$

Now,

$$x(t+T) = \sin[15\pi(t+T)]$$

$$= \sin(15\pi t + 15\pi T) \ldots \ldots . . (i)$$

Again at $T = \frac{2\pi}{\omega}$

$$2\pi = T\omega$$

Comparing the given equation with standard form

$$x(t) = \sin \omega t$$

We get, $\omega = 15\pi$

$$\therefore 2\pi = 15\pi T \ldots\ldots\ldots\ldots (ii)$$

Now, from equation (i) and (ii)

$$x(t+T) = \sin\{15\pi t + 2\pi\}$$

$$= \sin\{2\pi + 15\pi t\}$$

$$= \sin 15\pi t \quad [\because \sin(2\pi + \theta) = \sin\theta]$$

$x(t) = x(t+T)$ Is satisfied, hence, the given signal is periodic.

Basic Elementary Signals

Elementary discrete-time signals can be defined as those signals that can be used as building blocks to build more complex signals and are normally used for the study of signals and systems. Conversely, we can decompose complex signals into elementary signals and analyze the behavior of signals and systems. Following are the discrete-time elementary signals we study in this course:

1. Unit Step

A unit step signals is that type of elementary signal which exists only for positive values of time and it is zero for negative values $t<0$ for continuous time signal and $n<0$ for discrete time signal. Mathematically, it is represented by,

1. Unit Step

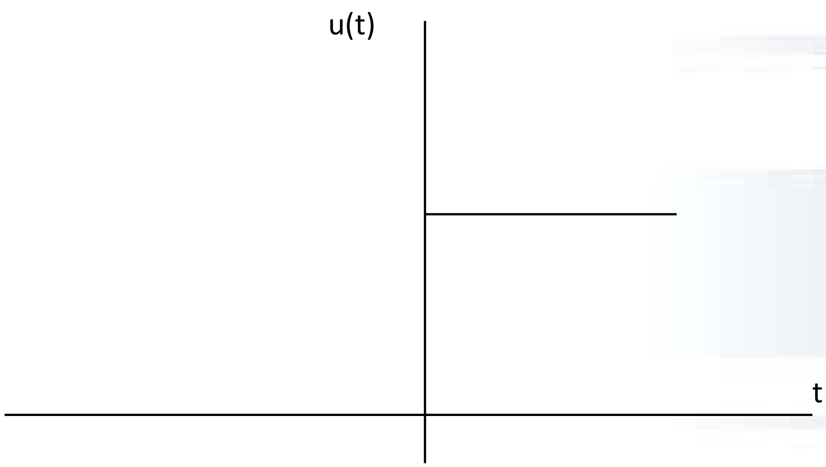
For continuous time,

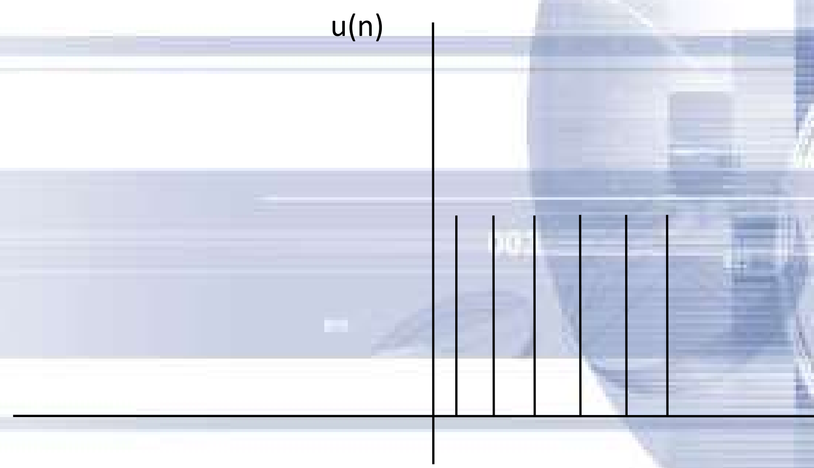
For discrete time

$$u(t) = \begin{cases} 1 \; for \; t \geq 0 \\ 0 \; for \; t < 0 \end{cases}$$

$$u(n) = \begin{cases} 1 \; for \; n \geq 0 \\ 0 \; for \; n < 0 \end{cases}$$

1. Unit Step



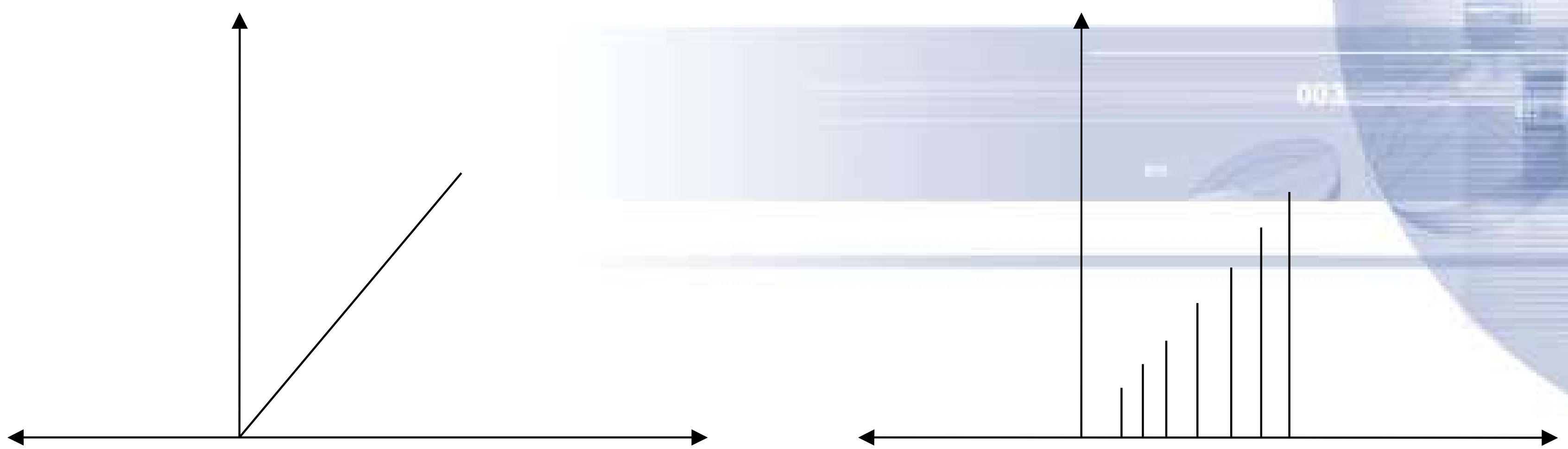


2. Ramp Signal

A continuous time ramp signal is that type of signals whose value start at time t=0 and increases linearly with the increase in time towards positive axis. It is represented by r(t) or $u_r(t)$ and mathematically,

2. Ramp Signal

$$u_r t = \begin{cases} t \; for \; t \geq 0 \\ 0 \; for \; t < 0 \end{cases}$$



3. Impulse Signal (Unit Sample Signal)

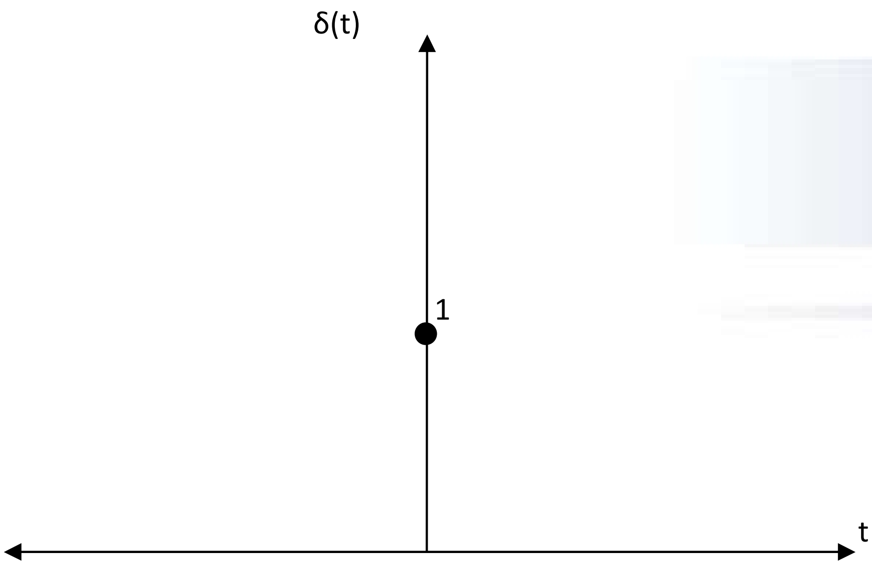
It is defined as δ[n] and mathematically expressed,

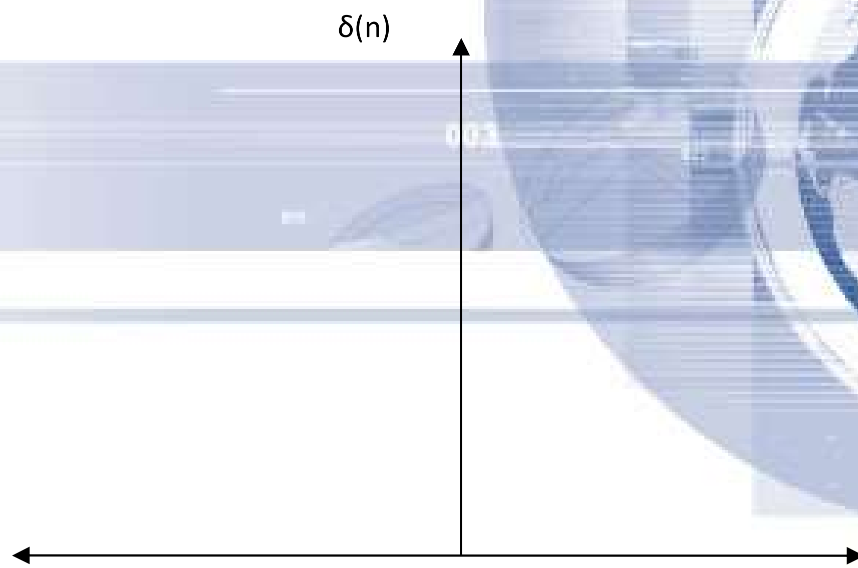
$$\delta[n] = \begin{cases} 1, for\ n = 0 \\ 0, for\ n \neq 0 \end{cases}$$

3. Impulse Signal (Unit Sample Signal)

The unit sample signal exists only at origin or the magnitude of unit sample signal is unity at n=0 for discrete time and t=0 for continuous time signals. At other points, the magnitude of this signal is zero.

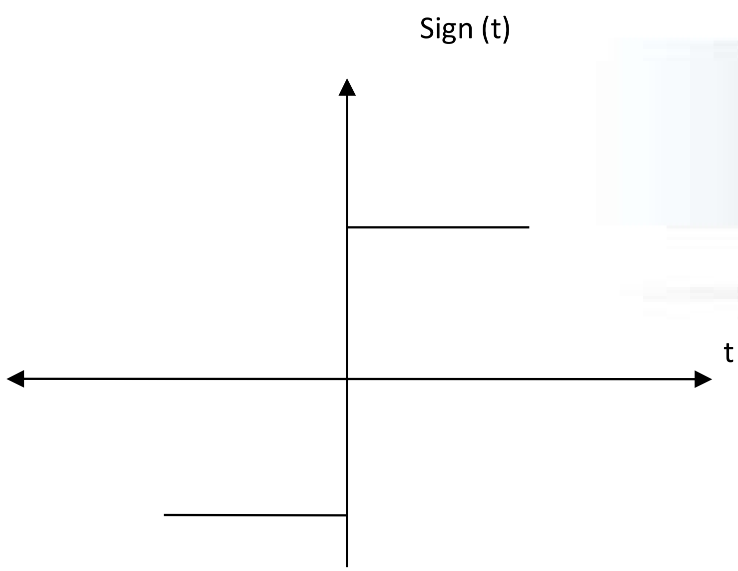
3. Impulse Signal (Unit Sample Signal)

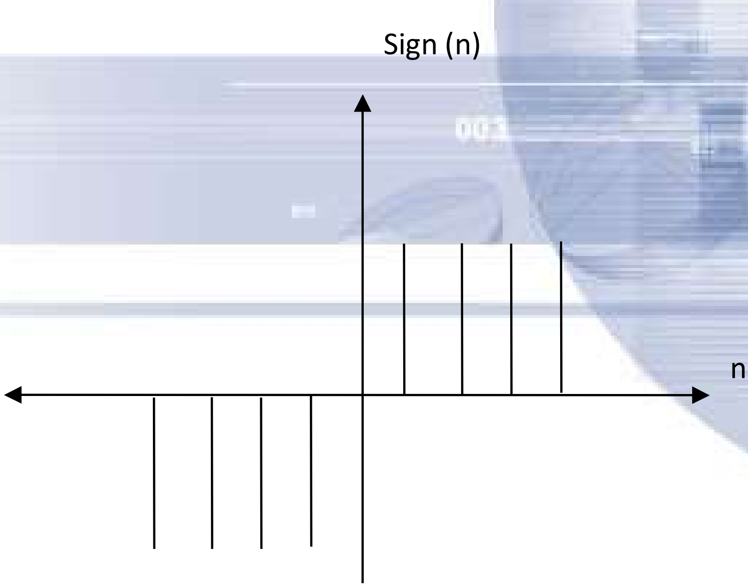




4. Signum Signal

Signum signal is defined for both positive and negative values of discrete and continuous time signals.





System: Continuous and Discrete Time System

A system may be defined as a set of elements or functional blocks which are together connected and produces an output which response to an input signal. The response of the output of the system depends upon the transfer function of the system or device used. Mathematically;

$$y(t) = F[x(t)]$$

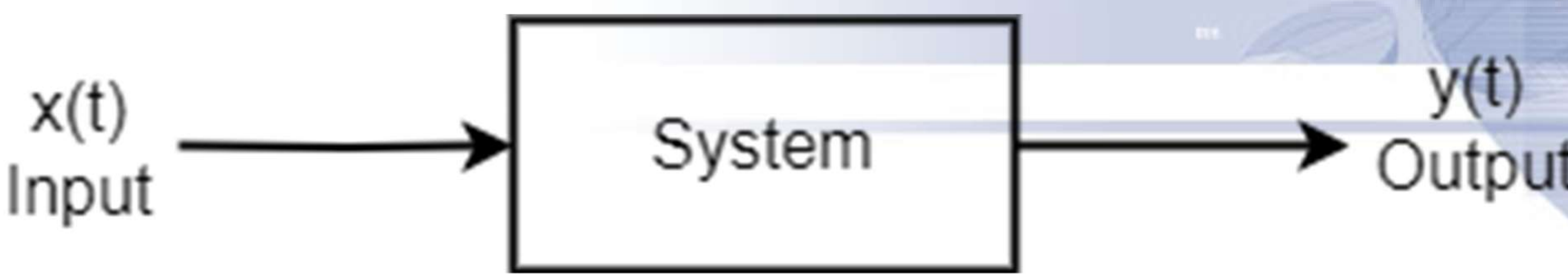
System: Continuous and Discrete Time System

Where,

$y(t)$: output

$x(t)$: input

F: Function or device used



System: Continuous and Discrete Time System

1. Continuous Time System

It is defined as the system in which the associated signals are continuous in nature, it means if the continuous signal x(t) given as input to the system and the output is also continuous with time.

For e.g.: Audio-video amplifier, power suppliers.

2. Discrete Time System

It may be defined as a system in which the associated signals are also discrete time signal. This means that the discrete time system both input and output signals discrete in nature.

For e.g.: Microprocessor, S/C Memory, Shift Registers etc.

Basic System Properties

Linearity

A system is said to be linear I superposition principle applies to that system i.e. a linear system may be defined as a system whose response to the sum of the weighted input is same as the sum of weighted output. For e.g.: Filters communication channels etc.

Linearity

Let us consider two systems:

$$y_1(t) = F[x_1(t)]$$

$$y_2(t) = F[x_2(t)]$$

Where, $x_1(t)$ and $x_2(t)$ are input and $y_1(t)$ and $y_2(t)$ are output response. Now, for linear system,

$$F[a_1x_1(t) + a_2x_2(t)] = a_1y_1(t) + a_2y_2(t)$$

Where, a_1 and a_2 are constants.

Causality

A system is said to be causal if the output at any time depends only on the values of the input at the present and past time. For e.g.:

$$y(t) = x^2(t) + x(t-2)$$

A system is said to be non-causal if its output at any time does not depends on the value of the input and the present and past time but also on the value of future time. For e.g.:

$$y(t) = x^2(t) + x(t-2)$$

Causality

A system is said to be non-casual if its output at any time does not depend on the value of the input at the present and past time but also on the value of future time.

Example:

$$y(t) = x(t) + 2x(t-2) + 4x(t+4)$$

Where, $[x(t) + 2x(t-2)]$ part is casual

$[4x(t+4)]$ part is non-casual and overall is non-casual.

Basic System Properties

Causality

A system is said to be non-casual if its output at any time does not depend on the value of the input at the present and past time but also on the value of future time.

Example:

$$y(t) = x(t) + 2x(t-2) + 4x(t+4)$$

Where, $[x(t) + 2x(t-2)]$ part is casual

$[4x(t+4)]$ part is non-casual and overall is non-casual.

Basic System Properties

Causality

A register is a casual system i.e. voltage across R depends upon the present time current.

Basic System Properties

Causality

Following equation describe casual system:

$i)\ y(t) = 0.2x(t) - x(t-1)$

$ii) y(t) = 0.8x(t-1)$

$iii) y(n) = x(n-1)$

Following are non-casual system:

$i)\ y(t) = x(t+1)$

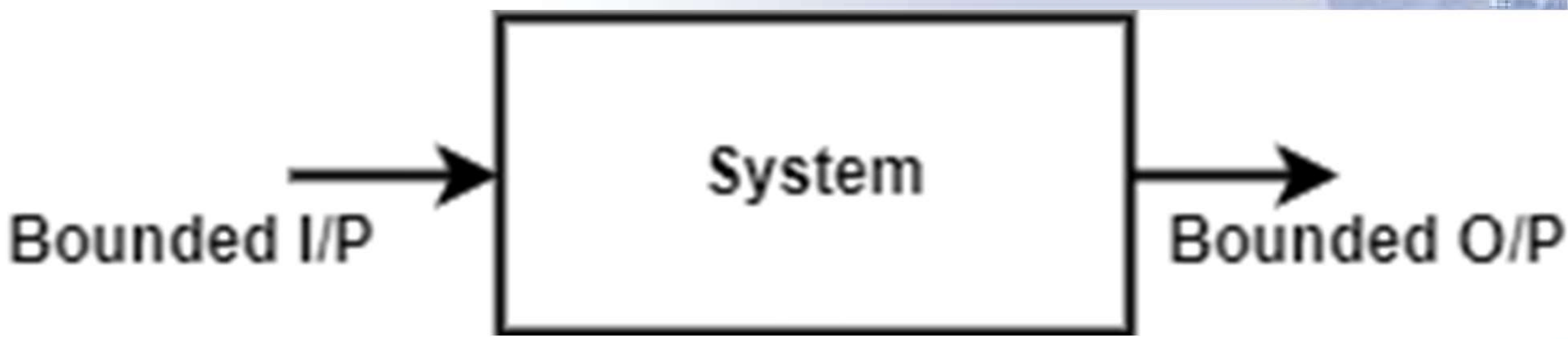
$ii) y(n-2) = x(n)$

$iii) y(n) = x(n) - x(n+1)$

Basic System Properties

Stability

A system having finite magnitude value called as bounded signal like $|g(t)| \leq M$. Where, M is positive real time finite number.



Basic System Properties

Stability

If $|x(n)| \leq M_x < \infty$, then

$$y(n) \leq M_y < \infty$$

A system is called stable if different bounded input results in bounded output (BIBO), then the output of such system does not divert or does not grow unnecessary large.

Basic System Properties

Static/Dynamic

The system is memory-less whereas dynamic system is a memory system.

Example: $y(t) = 2x(t)$ For present value $t = 0$, the system output is $y(0) = 2x(0)$. Here, the output only is dependent upon present input. Hence the system is memory less or static.

$y(t) = 2x(t) + 3x(t-3)$ for present value $t = 0$, the system output is $y(0) = 2x(0) + 3x(-3)$. Here $x(-3)$ is past value for the present input for which the system requires memory to get this output. Hence, the system is a dynamic system.

Basic System Properties

Time Invariance:

A system is time invariant if the input-output relationship doesn't vary with time. This means that a system is time invariant if its behavior and input characteristics does not change with time.

For a time-invariant system, the time shift in the input signal results in corresponding time shift in the output.

Basic System Properties

Time Invariance:

Mathematically,

$$y(t) = F(x(t))$$

If time $'t'$ is changed by t_1, then

$$y(t - t_{1\_} = F[x(t - t_1]$$

So, for time variance

$$y(t) = y(t - t_1)$$

Introduction to LTI System

If a system is both linear and time invariant then that system is called linear time invariant (LTI) system.