CS60073: Advanced Machine Learning

Class Test I

Time: 1 hrs, Marks: 20 (4 X 5)

Solve the problem <u>neatly</u> on paper. Write your <u>Name and Roll number</u> clearly on top of the paper. Take photograph of the paper(s) and convert to a SINGLE pdf file. Upload the file in MS Teams.

---

1. Let vector $\mathbf{V}_t$ denote the values of a set stocks on the t-th day. The change of stock values is governed by the following model -

$\mathbf{V}_t = M\mathbf{V}_{t-1} + \boldsymbol{\eta}_t$ for $t > 1$

where M is a given matrix and $\boldsymbol{\eta}_t$ is a zero mean Gaussian noise vector with covariance $\sigma^2\mathbf{I}$.

Also, $p(\mathbf{V}_1) = \mathcal{N}(\mathbf{0}, \Sigma)$ is a Gaussian. Further, let $\mathbf{Y}_t$ be an economic index with the linear relation –

$\mathbf{Y}_t = N\mathbf{V}_t + \boldsymbol{\varepsilon}_t$

where N, is known, and $\boldsymbol{\varepsilon}_t$ is a zero mean Gaussian noise with covariance $\tau^2\mathbf{I}$. The $\boldsymbol{\eta}$ and $\boldsymbol{\varepsilon}$ noise vectors are uncorrelated.

i. Show that $\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_t$ is Gaussian distributed.

ii. Show that the covariance matrix of $\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_t$ has elements, $M^{t'-t}\Sigma$ if $t \neq t'$, and $M^t\Sigma(M^t)^\mathbf{T}$ if $t = t'$.

M^t is the M matrix raised to power t.

iii. Explain if $\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_t$ is a Gaussian Process.

iv. Show that the sequence $\mathbf{Y}_1, \mathbf{Y}_2, \ldots, \mathbf{Y}_t$ is a Gaussian Process.