MA-202 PROJECT PRESENTATION

Falling Dynamics of SARS - CoV - 2 as a Function of Human Height

Group Members

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PROBLEM STATEMENT

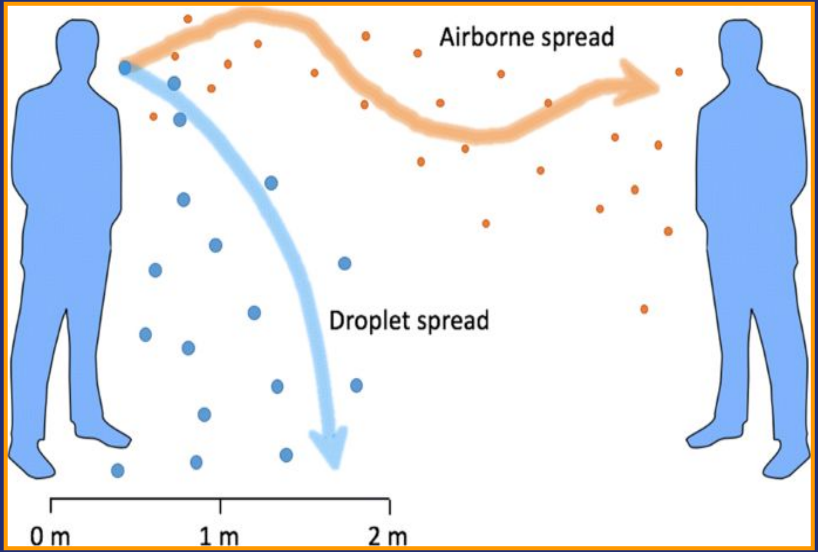
Presently, the Coronavirus disease has led to around 3.3 million deaths worldwide and has caused a global pandemic with serious impacts on the public and environmental health and human welfare. Therefore it is crucial to actually comprehend and control its spread system.

The objective of this project is to quantify the motion dynamics of the severe acute respiratory syndrome coronavirus 2 (Covid-19).

Observing the behavior of the SARs-CoV-2 particle is the first step in preventing the virus's transmission. We will figure out how far the particles travelled horizontally and how long it took them to settle down on ground. These measurements are taken when people with varying heights are talking, breathing, or sneezing. We'd measure them analytically (using Stokes Law and Newton's Laws of Motion) first, then numerically (using the Bisection Method, Euler's Method, Newton-Raphson Secant, and so on) and compare the results.

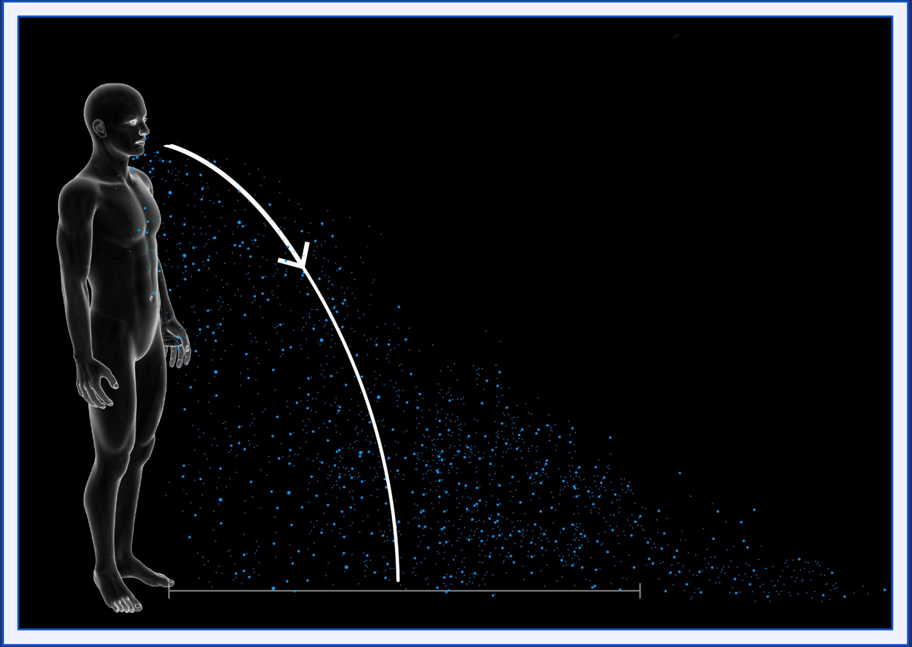
RELEVANCE/APPLICATION IN THE FIELD OF SCIENCE

With the help of this study, we would like to have an understanding of an estimated idea of the minimum distance between two or groups of individuals should maintain for the minimum risk of the transmission of the virus for the different height people. This will help the government and various health organisations to prepare their guidelines accordingly.



PHYSICAL MODEL

The passage of respiratory droplets carrying viruses can be approximated as a half projectile motion from the human nose or mouth (highest point of the projectile) to the ground(lowest point of the projectile), as shown in the diagram. We are primarily interested in determining the time and distance travelled by the virus in this projectile motion before it settles on the ground in this report. In this analysis, we will consider three options: the first is to assume no air resistance, the second is to add air resistance according to Newton's Law, and the third is to apply Stokes Law for air resistance to determine the necessary values.



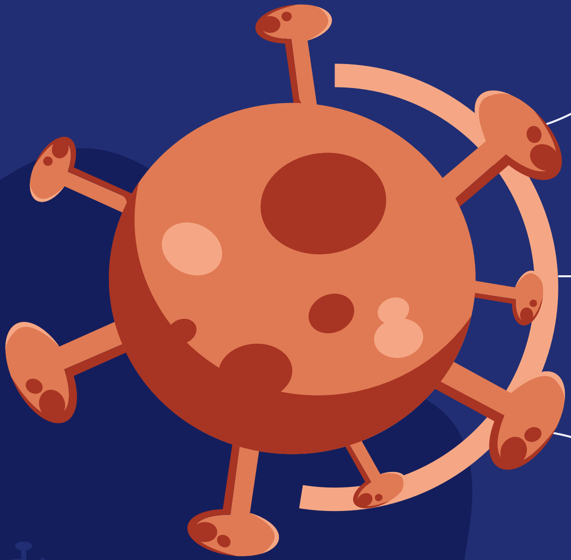
NOTATIONS

- ρp - particle density(kg m^{-3})
- ρ_w - density of water droplets(kg m^{-3})
- ρa - air density(1.2041 kg m^{-3} at 20°C and 1 atm)
- A - cross section area of object(m^2)
- a - particle acceleration(m s^{-2})
- v_y - downward velocity(m s^{-1})
- g - gravitational acceleration(9.81 m s^{-2})
- v_t - particle's terminal velocity according to newton's method(m s^{-1})
- V_0 - initial velocity of the particle(m s^{-1})
- m - mass of the falling particle(kg)
- F_g - gravitational force(kg m s^{-2})
- F_a - air resistance(kg m s^{-2})
- K - shape dependent coefficient(0.47 for spherical particle)
- h - falling distance(m)
- X - horizontal distance travelled(m)
- t - time taken by particle to fall from 'h' height(s)
- η - viscosity of fluid(kg m^{-1} s^{-1}) ($1.85\times10{-}5$ kg m^{-1} s^{-1})
- r - particle radius(m) ($5.0\times 10{-}8$ m)
- d - particle diameter (m)

ASSUMPTIONS

- The SARs-CoV-2 virus particle is assumed to have spherical shape with average diameter of 100 nm, radius of 5.0×10^{-8} m and a protein density of 1.35×10^{3} kg m^{-3}. Hence, we can use the sphere's volume formula for finding out the volume of the individual particle.
- Newton's law primarily refers to macro-scale objects flying with a high rate at a height of at least a few meters. Nonetheless, this law can adequately describe SARS-CoV-2's motion in calm air as its initial velocity in the case of sneezing/coughing was recorded to be 5.0 m s^{-1} and 1.5 m/s for breathing and talking.
- Terminal velocity is attained almost instantly.
- The interactions between SARS-CoV-2 and droplets were not taken into account in the calculations.
- The experiment had ignored the concept of evaporation. The droplets may evaporate and cause fluctuations in the result.
- 997 kg m^{-3} density can be assumed of a respiratory droplet(diameter less than 1 μm) that has SARs-CoV-2 in it.
- The cross-surface area for SARS-CoV-2 where a hemispherical surface area is equal to $2\pi r^2$ was assumed as 1.571×10^{-14} m^2 .

THREE CASES FOR THE FREE FALLING PARTICLE



01 **Without Air Resistance**

$t = \sqrt{(2h/g)}$

02 **With Air resistance, Newton's Method**

$t = h/V_t$

03 **With Air resistance, Stokes Law**

$t = h/V_t$

GOVERNING EQUATIONS

1. In the case of a free-falling particle with no air resistance, the particle would be affected only by gravitational force. The particle's vertical velocity would be

$$v_y = gt = \sqrt{(2gh)}$$

Rearranging to get h, we get

$$h = ½(gt^2)$$

Rearranging to get t, we get

$$t = \sqrt{(2h/g)}$$

Putting t value to get X, we get

$$X = V_o t = V_o \text{ x } \sqrt{(2h/g)}$$

Where V_o is the constant initial velocity of the particle in the direction he/she is facing.

The volume and mass of the virus can be estimated thus:

$$\text{Volume} = (4/3)*\pi r^3$$

$$\text{Mass} = \rho p\{(4/3)*\pi r^3\}$$

GOVERNING EQUATIONS

2. In fact, in addition to gravitational force, air resistance acts on particles passing through the atmosphere. The air resistance can be accounted for in two different ways -

<u>Newton's Law</u>: The first one is the application of Newton's Laws. Based on Newton's law, the following two forces can be introduced:

$$F_g = mg$$

$$F_a = (K \rho a A v_t^2)/2$$

The following equation can be used to integrate air resistance and gravitational forces -

$$ma = mg - (K \rho a A v_t^2)/2$$

Dividing by m, we get

$$a = g - (K \rho a A v_t^2)/2m$$

As a = 0 at terminal velocity,

$$g - (K \rho a A v_t^2)/2m = 0$$

$$V_t = \sqrt{(2mg/K \rho a A)}$$

The time (t) for the particles to fall down from a given height (h) in the calm air can be approximated thus:

$$t = h/V_t$$

GOVERNING EQUATIONS

Stoke's Law: As was stated above, a falling particle in a fluid (the air in this study) encounters a frictional resistance proportional to the product of its radius and velocity and to the viscosity of the fluid. The resisting force (Fa) due to friction was shown by Stokes in 1851 as follows:

$$F_a = 6\pi\eta r V_t$$

The downward force on a spherical droplet due to gravity can be estimated neglecting the air density thus (please note that $\rho_w \approx 830\rho_a$):

$$F_g \cong \rho_w(4/3\pi r^3)g$$

Given the conditions of terminal velocity, the following Eqs. can be written:

$$g - (6\pi\eta r V_t/m) = 0$$

$$V_t = mg/6\pi\eta r$$

Assuming that the terminal velocity is attained almost instantly, the falling time (t) from a height (h) can be expressed thus:

$$t = 18h\eta/\{d^2g(\rho p\text{-}\rho a)\} = h/V_t$$

PARAMETERS

1) The Stokes Law can be derived from the dimensional analysis as follows -

The following parameters are proportional to the viscous force acting on a sphere:

- ➢ the radius of the sphere(r)
- ➢ coefficient of viscosity(η)
- ➢ the velocity of the object(v)

This is represented mathematically as,

$$F \propto \eta^{a} r^{b} v^{c}$$

For finding values of a, b and c, substitution of the proportionality sign with an equality sign is done.

$$F = k(\eta^{a} r^{b} v^{c})$$

Where k is the proportionality constant here which is a numerical number of no dimensions.

PARAMETERS

Then the dimensional equality of left hand side and right hand side terms is done -

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^a \, [L]^b \, [LT^{-1}]^c$$

After simplification,

$$[MLT^{-2}] = M^a \cdot L^{-a+b+c} \cdot T^{-a-c}$$

Equating the powers with respect to their dimensions, we get the values of a, b and c as 1,1,1 respectively.

Hence, viscous force can be written as -

$$F = k\,(\eta\ r\ v)$$

For a spherical shaped body, the value of the proportionality constant is taken as 6π. Hence, the viscous force on a spherical body falling through a liquid according to Stokes law is -

$$F = 6\pi\eta rv$$

PARAMETERS

2) **K** - Shape dependent coefficient

Aerodynamic engineering uses the drag coefficient to model all of the complicated relationships between drag and shape, inclination, and certain flow conditions. Its formula is as follows -

$$K = \tfrac{1}{2} [D / (\rho\, v^2 a)]$$

Where:
'D' is drag, 'ρ' is density of object(sphere), 'v' is sphere velocity and 'a' is a cross sectional area.

3) Protein density of SARS-CoV-2

It is commonly assumed that the spatial average of protein density is 1.35×10^3 kg m^{-3}, regardless of the composition of the protein and, in fact, regardless of its molecular weight as well.

PARAMETERS

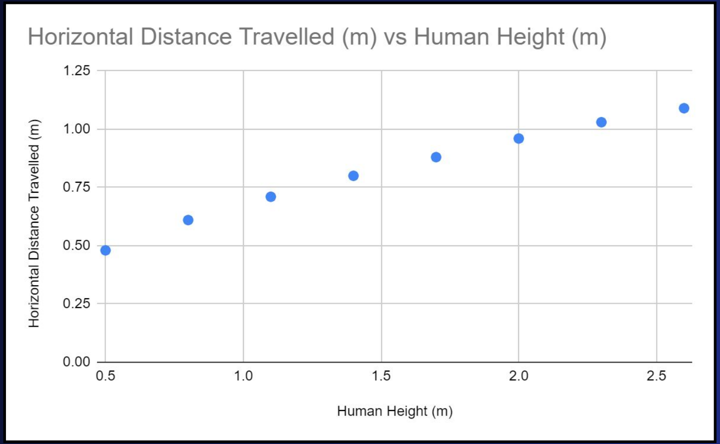
4) η - Coefficient of viscosity

A fluid's viscosity is a measure of its resistance to deformation at a given rate.

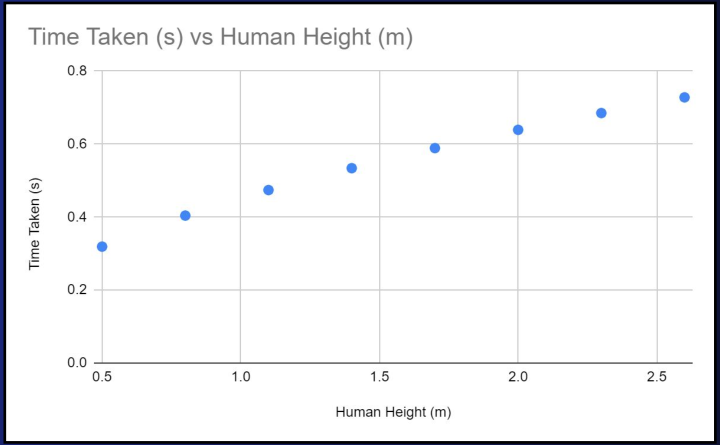
Other Parameters

- r = radius of SARS-CoV-2 taken as 5×10^{-8} m
- A = cross sectional area taken as 1.571×10^{-14} m^2
- K = for a sphere , shape coefficient is considered here as 0.47
- ρ_a = 1.2041 kg m^{-3} at 1 atm, 20^0C
- m = 7.07×10^{-19} kg
- Volume = $5.236\times10{-}22$ m^3
- Protein density = 1.35×10^3 kg m^{-3}
- $\rho_w = 830{*}\rho_a$
- $\eta = 1.85 \times 10^{-5}$ kg m^{-1} s^{-1}

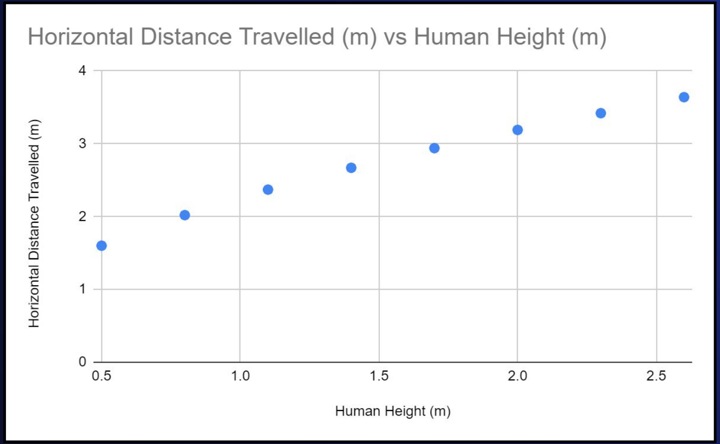
ANALYTICAL ANALYSIS



Horizontal distance travelled by SARS-CoV-2 without a respiratory droplet due to breathing and talking as a function of human height.

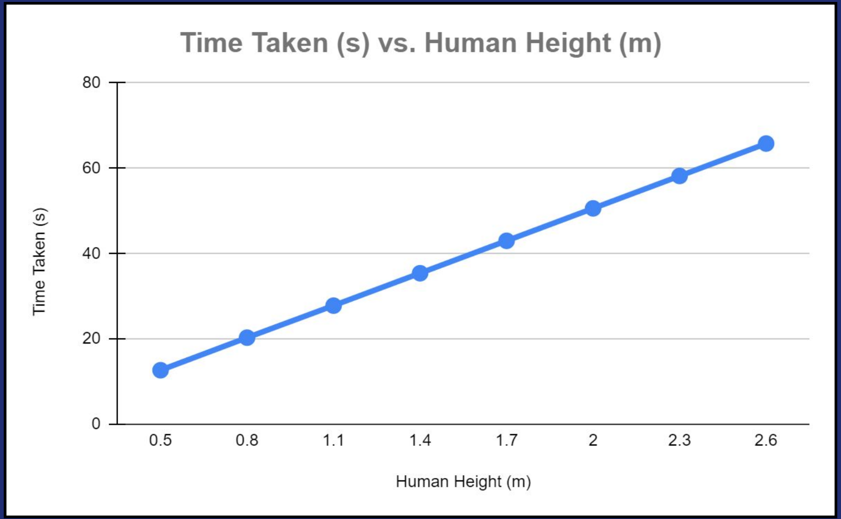


Free-fall time of SARS-CoV-2 without a respiratory droplet.

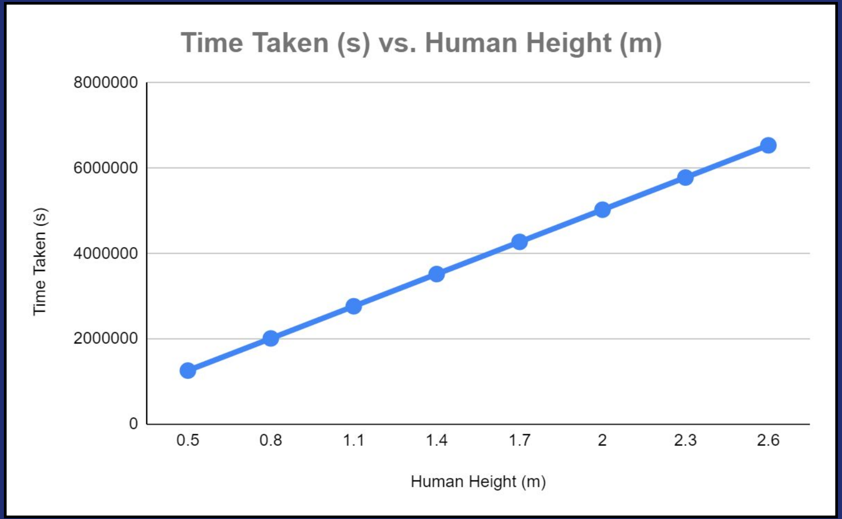


Horizontal distance travelled by SARS-CoV-2 without a respiratory droplet due to sneezing or coughing as a function of human height.

ANALYTICAL ANALYSIS



Falling dynamics of SARS-CoV-2 without a respiratory droplet under the gravity with air resistance according to Newton's laws as a function of human height.



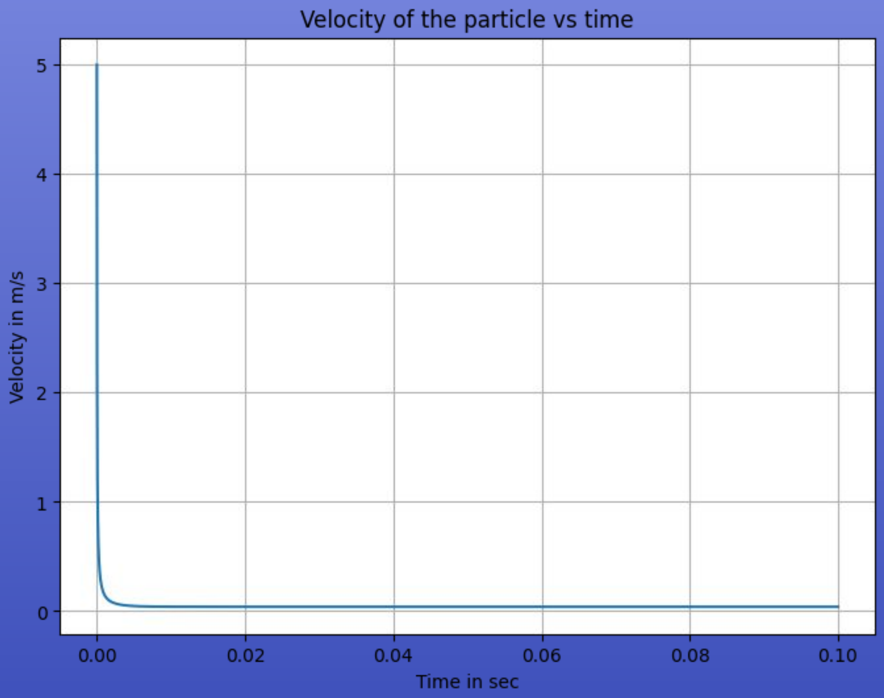
Falling dynamics of SARS-CoV-2 without a respiratory droplet under the gravity with air resistance according to Stoke's laws as a function of human height.

SOLUTION METHODOLOGY

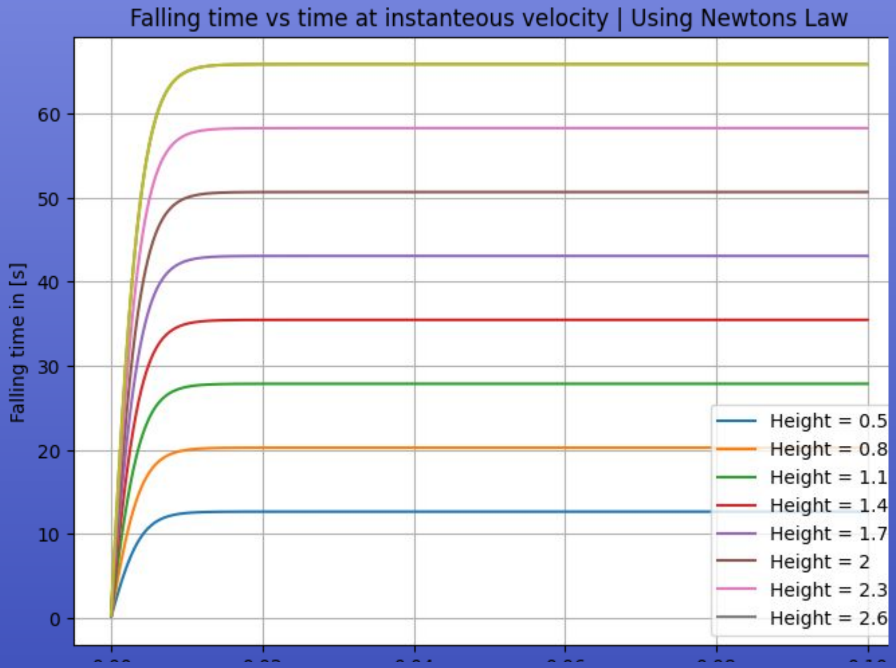
To begin solving the governing equations with the help of their parameters, we employ the following methodology to compare between the Stokes and Newton's Law:

- Considering the Newton's Law of motion (with air resistance)
  - We consider the Governing Equation (6a), **ma = mg - (K ρaA(vt)^2)/2**
  - we numerically analyse the Terminal Velocity of the falling air droplet.
  - The equation is solved with the help of the Euler's Method.
  - With the help of the above approach we generate the Magnitude of the Terminal Velocity
  - Next, to approximate the equation (8), **t = h/Vt** we numerically plotted a curve to compare the duration of falling time vs the time at which it is attained
  - After that, we studied the variation of falling time with different height, using Euler's Implicit method for solving the differential equation (8, 6b), **dv/dt** = **ma = mg - (K ρaA(vt)^2)/2**
  - At last we used Bisection method to find the horizontal distance covered by the particle for different heights and initial velocity(sneezing, breathing/ talking) with the help of Equation (3)
- Now we did the same steps by considering Stoke's equation for the terminal velocity.
- Next, we also compared all the numerical obtained solutions with their analytical results.
- Following are the step by step method followed for the Solution

NUMERICAL ANALYSIS (NEWTON'S LAW)

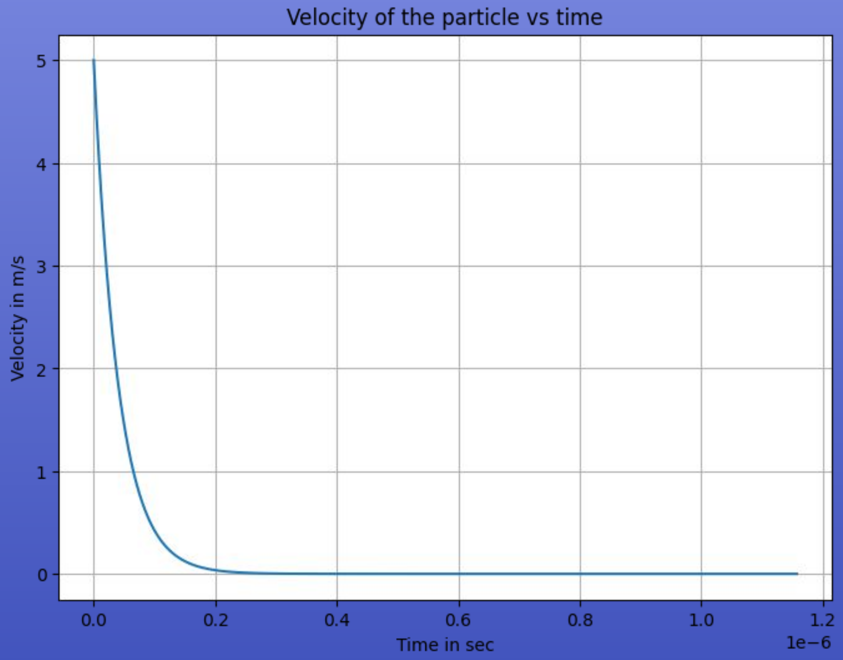


Velocity of particle VS Time
The terminal velocity (in m/s) is :
0.03947790388948962

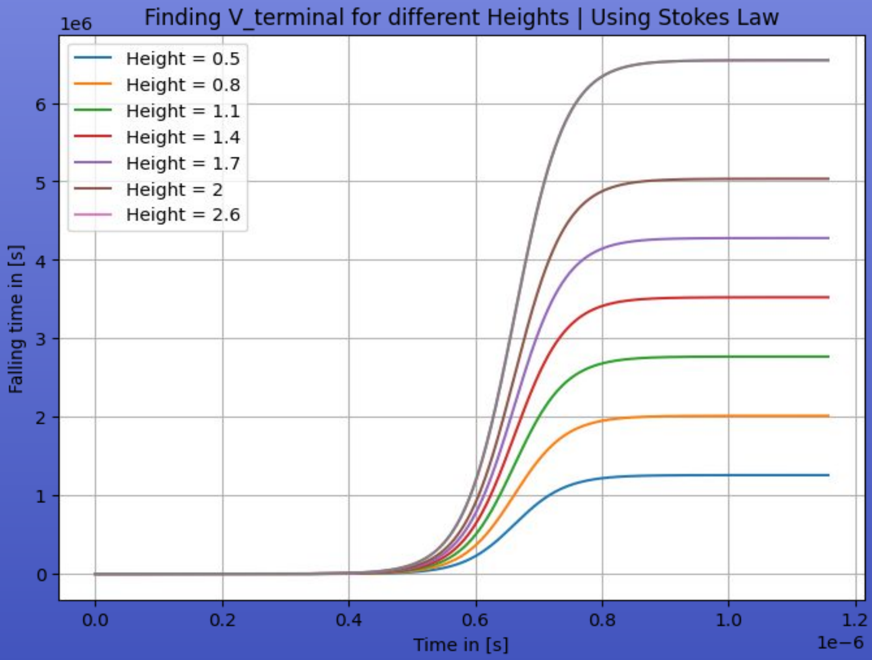


Falling Time VS Instantaneous Time

NUMERICAL ANALYSIS (STOKE'S LAW)



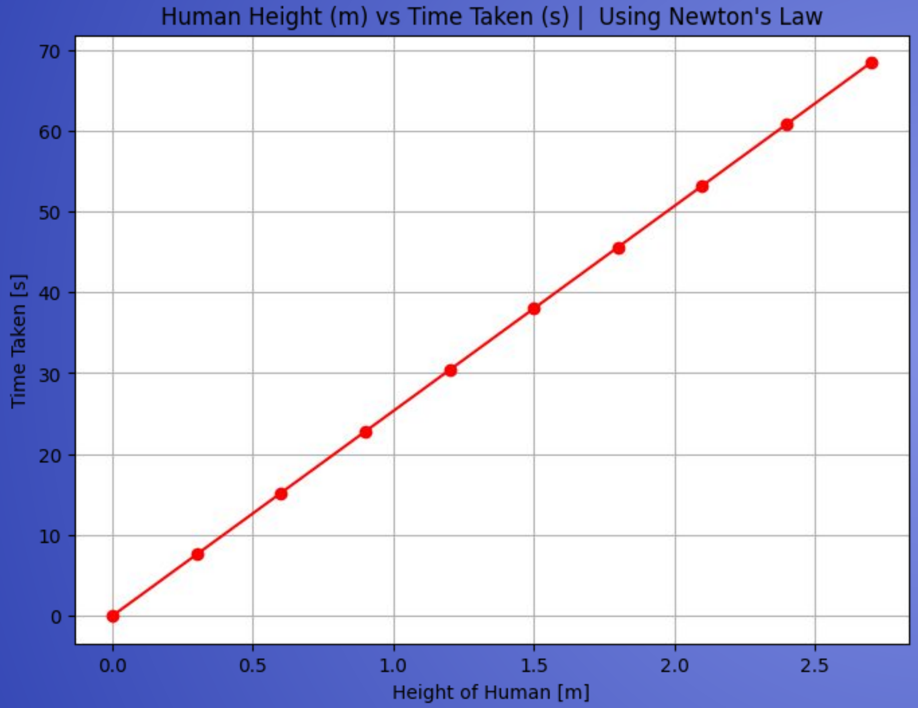
Velocity of particle VS Time



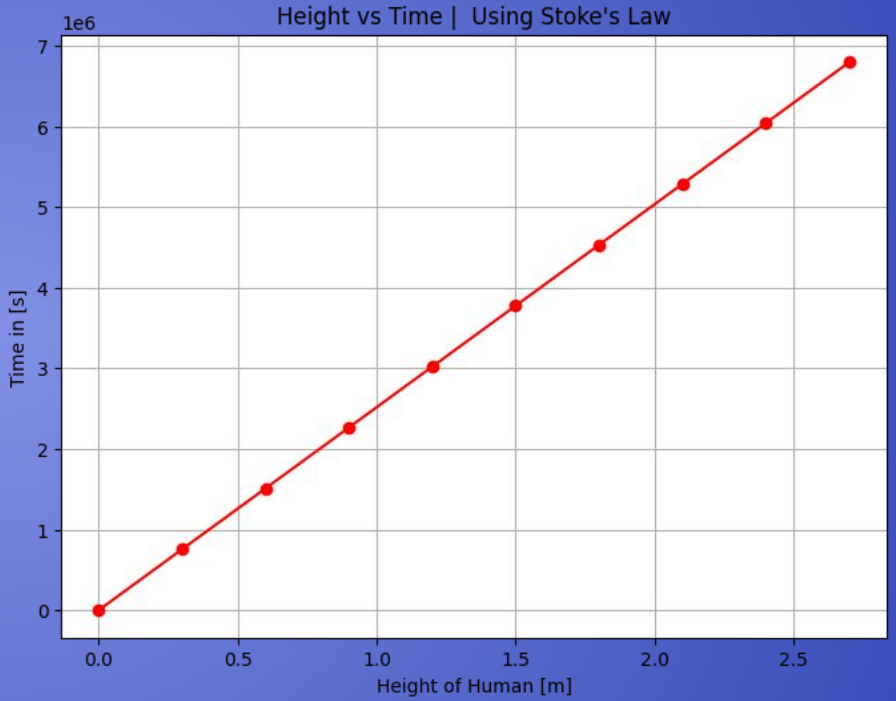
Falling Time vs Instantaneous Time

The terminal velocity (in m/s) is :
3.972992187935091e-07

NUMERICAL ANALYSIS



Height vs Time Taken



Height vs Time Taken

ALGORITHMS AND METHODS USED

The Equations were modelled using the Euler's and Bi-Section Methods, we wrote the code in python. The basic algorithmic flow is :

A. Euler's Method:

1. Define the constants required for each equation
2. Set the number of iterations to balance between computation time and accuracy. The step size is also set accordingly for each case. As the step size reduces, the accuracy improves and the computation time increases.
3. Input the boundary conditions required for each equation. This includes initial velocities (5 m/s for coughing/sneezing, 1.5 m/s for breathing/talking) and time count starts from t = 0.
4. Defining the functions for approximating derivatives using Euler's explicit function:
   a. dV / dt at $V = v_i = (v_{i+1} - v_i) / \Delta t$
5. Using these equations, we numerically compute the magnitude of velocity at each time stamp by iterating using a for loop
6. Finally, we plot various curves using the data and inferences obtained by analysis of the above data.

ALGORITHMS AND METHODS USED

B. Bisection Method:

1. We used bisection method to find the horizontal distance (h) covered by the droplet
2. We first plot the function F(h) = 0 and identify limits x_l and x_r such that $F(x_l)*F(x_r) < 0$ to ensure a unique solution in the domain.
3. Now find $x_{mid} = (x_l + x_r) / 2$ and find $F(x_{mid})$
4. If $F(x_{mid})*F(x_r) > 1$, reduce domain by setting $x_r = x_{mid}$
5. If $F(x_{mid})*F(x_l) > 1$, reduce domain by setting $x_l = x_{mid}$
6. Iterate until you get error = 0 or less than the tolerance specified

In Euler's Explicit Method, the error obtained is of the order O(▵h), so its consistent, and the tolerance taken in bisection method = 10^{-6}

RESULTS AND DISCUSSION

- The results that we are getting from the analytical analysis and the numerical analysis are exactly the same.
- The predictions of the two physical models show a large variation in the falling time of the SARS-CoV-2 particle under gravity with air resistance.
- **For an average human height of 1.7 m, the terminal velocity of the falling particle according to Newton's and Stoke's laws are 3.95×10^{-2} m s^{-1} and 3.98×10^{-7} m s^{-1} (negligible**) respectively.
- For an average human height of 1.7 m, the falling time of the covid particle according to Newton's and Stoke's laws are 43 s and 4274578.828 s (about 50 days) respectively.
- When air resistance is neglected during the fall of covid particles, the time it takes to reach the ground for an average human height of 1.7 m is 0.589 s.
- **In the case when air resistance is neglected, assuming the velocity of SARS-CoV-2 of 1.5 m s−1 for breathing and talking and of 5.0 m s−1 for sneezing and coughing, the horizontal distance travelled by SARS-CoV-2 for a height of 1.7 m was 0.88 m and 2.94 m, respectively.**

CONCLUSION

Thus, according to our study, it is necessary to maintain a social distance of about 1 m during breathing and talking and about 3 m during sneezing or coughing to protect ourselves from getting infected from the coronavirus.

THANK YOU