Advances in Robotics and Control - Course Project

Rohan Reddy Bodgam   Deepak Sangeetham

International Institute of Information Technology, Hyderabad

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Adaptive Sliding Mode Control on Robotic Manipulator

Motivation for Choosing the Project

Robotic Manipulators are the most used systems in field of robotics. The type of work it does is same as that of a human hand. It is very simple to implement and gives a very good performance with almost any type of controller. So we wanted to see how it fares while doing trajectory tracking with Adaptive Sliding Mode Controller. There are also many trajectories to choose for tracking as any time of trajectory can be trackable as long as it is in joint space of the robot.

Dynamics of the System

The Dynamics of the 2-R system can generally be written as follows:

$$\tau = M(q)\ddot{q} + C(q, \dot{q}) + G(q)$$

Using Euler-Lagrangian equations, $\mathbf{M(q)} = \sum_{i=1}^{n}(m_i J_{v_i}^T J_{v_i} + J_{w_i}^T I_{C_i} J_{w_i})$

Writing in terms of Jacobian, $\mathbf{v_{p_{c_i}}} = \mathbf{J_{v_{p_{c_i}}}} \dot{\mathbf{q}}$, $i = 1, 2$ with

$\mathbf{q} = \theta = [\theta_1, \theta_2]^\mathbf{T}$, we get:

$$J_{v_{pc1}} = \begin{bmatrix} -\sin\theta_1 & 0 \\ \cos\theta_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{v_{pc2}} = \begin{bmatrix} -l_1 \sin\theta_1 - r_2 \sin(\theta_1 + \theta_2) & -r_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos\theta_1 - r_2 \cos(\theta_1 + \theta_2) & r_2 \cos(\theta_1 + \theta_2) \\ 0 & 0 \end{bmatrix}$$

Dynamics of the System

$$J_{v_{pc}} = \begin{bmatrix} \text{-l}_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2) & \text{-l}_2 \sin(\theta_1 + \theta_2) \\ \text{l}_1 \cos\theta_1 - l_2 \cos(\theta_1 + \theta_2) & \text{l}_2 \cos(\theta_1 + \theta_2) \\ 0 & 0 \end{bmatrix}$$

The angular velocities are given by:

$$\omega_1 = [0, 0, \dot{\theta}_1]^T;$$
$$\omega_2 = [0, 0, \dot{\theta}_1 + \dot{\theta}_2]^T$$

Therefore, we have:

$$J_{\omega_{c_1}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \quad J_{\omega_{c_1}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

such that $\omega_i = J_{\omega_{c_i}} \dot{\theta}, i = 1, 2.$

$m_{11} = (I_{c_1} + m_1 r_1^2) + (I_{c_2} + m_2(l_1^2 + r_2^2 + 2 l_1 r_2 \cos\theta_2)) + m_p(l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta_2),$
$m_{12} = I_{c_2} + m_2(r_2^2 + l_1 r_2 \cos\theta_2) + m_p(l_2^2 + l_1 l_2 \cos\theta_2)$
$m22 = (I_{c_2} + m_2 r_2^2) + m_p l_2^2.$

$$\mathbf{C}_{ij} = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{\partial \mathbf{M}_{ij}}{\partial \mathbf{q}_k} + \frac{\partial \mathbf{M}_{ik}}{\partial \mathbf{q}_j} + \frac{\partial \mathbf{M}_{kj}}{\partial \mathbf{q}_i}\right)\dot{\mathbf{q}}_k, \textit{where } \mathbf{i,j = 1,2,...,n}$$

$$\mathbf{C} = \text{-l}_1(m_2 r_2 + m_p l_2)\sin\theta_2 \begin{bmatrix} \dot{\theta}_2 & \dot{\theta}_1 + \dot{\theta}_2 \\ \dot{\theta}_1 & 0 \end{bmatrix}$$

$$\mathbf{G} = \text{g . M}$$

$$\textit{where}, M = \begin{bmatrix} (m_1 r_1 + (m_2 + m_p) l_1)\cos\theta_1 + (m_p + l_2 + m_2 r_2)\cos(\theta_1 + \theta_2) \\ (m_2 r_2 + m_p l_2)\cos(\theta_1 + \theta_2) \end{bmatrix}$$

Control Model Used : ASMC

Let us consider the tracking problem for desired trajectories satisfying $q, \dot{\mathbf{q}}, \ddot{\mathbf{q}} \in L^{\infty}$. Let $e(t) \triangleq \mathbf{q}(t) - \mathbf{q}_d(t)$ be the tracking error. We define a sliding variable s as

$$s(t) \triangleq \dot{e}(t) + \lambda e(t), \quad (A)$$

where $\lambda \in \mathbb{R}^{n \times n}$ is positive definite. In the following, let us omit variable dependency for compactness. Multiplying the derivative of (A) by M and using yields

$$\mathbf{M}\dot{\mathbf{s}} = M(\ddot{\mathbf{q}} - \ddot{\mathbf{q}}_d + \lambda \dot{e}) = \tau - Cs + \phi, \quad (B)$$

where $\phi \triangleq -(\mathbf{C}\dot{\mathbf{q}} + \mathbf{G} + \mathbf{M}\ddot{\mathbf{q}}_d - \mathbf{M}\ \lambda\dot{\mathbf{e}} - Cs)$ represents the overall uncertainty. Using (B) and Properties 1 and 2 we have

Control Model Used: ASMC

$$\|\phi\| \leq \overline{c}\|\dot{q}\|^2 + \overline{g} + \overline{m}(\|\ddot{q}_d\| + \|\lambda\|\|\dot{e}\|) + \overline{c}\|\dot{q}\|(\|\dot{e}\| + \|\lambda\|\|q\|).$$

Further, let us define $\xi = [\mathbf{e}^\top \ \dot{\mathbf{e}}^\top]^\top$. Then, using inequalities $\|\xi\| \geq \|e\|$, $\|\xi\| \geq \|\dot{e}\|$, boundedness of the desired trajectories, and substituting $\dot{q} = \dot{e} + \dot{q}_d$ into above equation yields,

$$\|\phi\| \leq K_0^* + K_1^*\|\xi\| + K_2^*\|\xi\|^2,$$

where $K_0^* \triangleq \{\overline{c}\|\dot{q}_d\|^2 + \overline{g} + \overline{m}\|\ddot{q}_d\|\}$, $K_1^* \triangleq \{\overline{c}\|\dot{q}_d\|(3 + \|\lambda\|) + +\overline{m}\|\lambda\|\}$, and $K_2^* \triangleq \overline{c}\|\dot{q}_d\|(2 + \|\lambda\|)$ are unknown finite scalars. Hence, a state-dependent upper bound structure naturally occurs for EL systems.

Controller Design

Problem: We don't know two cases here:
(i) no knowledge of the system dynamics
(ii)no a priori constant upper bound on the states

To solve this case, an adaptive sliding mode controller is designed with the following control law,

$$\tau(t) = -\Lambda s(t) - \rho(t)\,\mathrm{sgn}(s(t)), \tag{1}$$
$$\rho(t) = \hat{K}_0(t) + \hat{K}_1(t)\|\xi(t)\| + \hat{K}_2(t)\|\xi(t)\|^2, \tag{2}$$

where Λ is a positive definite user-defined matrix. The gain $\hat{K}_i$ are adapted via,

$$\dot{\hat{K}}_i(t) = \|s(t)\|\|\xi(t)\| - \alpha_i \hat{K}_i(t), \tag{3}$$
$$\hat{K}_i(0) > 0, \quad \alpha_i > 0, \tag{4}$$

where $\alpha_i \in \mathbb{R}^+$, $i = 0, 1, 2$ are design scalars.

Proof

Closed-loop stability is analysed using the Lyapunov function candidate:

$$V = \frac{1}{2}s^\top Ms + \sum_{i=0}^{2}(K_i - \hat{K}_i)^2. \tag{5}$$

using (A) and (1), the time derivative of above equation yields,

$$\begin{aligned}\dot{V} &= s^\top M\dot{s} + \tfrac{1}{2}s^\top M\dot{s} + \textstyle\sum_{i=0}^{2}(\dot{\hat{K}}_i - \dot{K}_i^*)\dot{\hat{K}}_i \\ &= s^\top(\tau - Cs + \phi) + \tfrac{1}{2}s^\top M\dot{s} + \textstyle\sum_{i=0}^{2}(\dot{\hat{K}}_i - \dot{K}_i^*)\dot{\hat{K}}_i \\ &= s^\top(-\Lambda s - \rho\,\mathrm{sgn}(s) + \phi) + \tfrac{1}{2}s^\top M\dot{s} + \textstyle\sum_{i=0}^{2}(\dot{\hat{K}}_i - \dot{K}_i^*)\dot{\hat{K}}_i.\end{aligned}$$

Using the property $\mathbf{s}^\top(\mathbf{M}\dot{\mathbf{s}} - 2\mathbf{C}\mathbf{s})\mathbf{s} = 0$.. Then, utilizing the upper bound structure and the fact that $\rho \geq 0$ from above equation, $\dot{V}$ gets simplified to,

Proof

$$\dot{V} = s^{\top}(-\Lambda s - \rho\,\mathrm{sgn}(s) + \phi) + \textstyle\sum_{i=0}^{2}(\dot{\hat{K}}_i - \dot{K}_i^*)\dot{\hat{K}}_i$$
$$\leq -s^{\top}\Lambda s - \textstyle\sum_{i=0}^{2}\{(\hat{K}_i - K_i^*)(\|\xi\|_i\|s\| - \dot{\hat{K}}_i)\}(3)$$

From above equations, we have

$\dot{\hat{K}}_i - \dot{K}_i^* = \|s\|(\hat{K}_i - K_i^*)\|\xi\|_i + \alpha_i\hat{K}_iK_i^* - \alpha_i\hat{K}_i^2.$

substituting above equation in (3)

$$\dot{V} \leq -\lambda_{\min}(\Lambda)\|s\|_n^2 + \textstyle\sum_{i=0}^{2}(\alpha_i\hat{K}_iK_i^* - \alpha_i\hat{K}_i^2)$$
$$\leq -\lambda_{\min}(\Lambda)\|s\|_n^2 - \textstyle\sum_{i=0}^{2}\left(\frac{\alpha_i(\hat{K}_i - K_i^*)^2}{2} - \frac{\alpha_i(K_i^*)^2}{2}\right).(6)$$

The above inequality shows that is a Negative-definite Matrix. The upper bound of Lyapunov function v can be given as,

$$V \leq \frac{\overline{m}}{2}\|s\|^2 + \textstyle\sum_{i=0}^{2}\frac{1}{2}(\hat{K}_i - K_i^*)^2.(7)$$

Proof

Using above condition , the above equality can be further simplified as

$$\dot{V} \leq -\rho V + \frac{1}{2}\sum_{i=0}^{2}\alpha_i K_i^2.$$

where $\varrho \triangleq \frac{\min\left\{\frac{\lambda_{\min}(\Lambda)}{n}, \frac{\alpha_i}{2}\right\}}{\max\left\{\frac{m}{2}, \frac{1}{2}\right\}} > 0$ by design. Defining a scalar $0 < \kappa < \varrho$ simplifies to,

$$\dot{V} \leq -\kappa V - (\varrho - \kappa)V + \frac{1}{2}\sum_{i=0}^{2}\alpha_i K_i^2.$$

Defining a scalar B as

$$B \triangleq \frac{1}{2(\varrho - \kappa)}\sum_{i=0}^{2}\alpha_i K_i^2,$$

$$V \leq \max\{V(0), B\}, \quad \forall t \geq 0,$$

This Lyapunov function enters in finite time inside the ball defined by B. This yields $V \geq (\frac{m}{2n})\|s\|^2$

Results

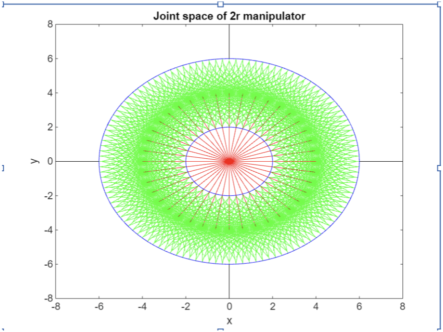

Figure: Joint space

Link for Working Video: Click here

Results(Robust Sliding Mode)

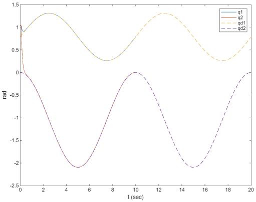

Figure: Individual Trajectories

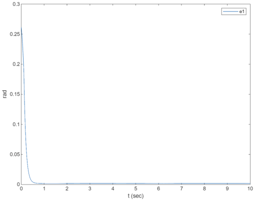

Figure: Error in x

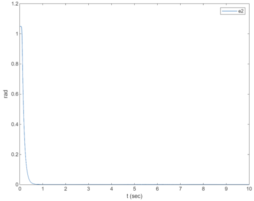

Figure: Error in y

Conclusion

In this project, we have tried to do trajectory tracking using a specific controller from dynamics of a robotic system. The controller used was adaptive sliding mode controller and robotics system used was a robot 2 revolute joint manipulator. The theory on how it is done along with required proofs are properly mentioned and satisfied. Robust sliding mode for the same robot was implemented to compare with the implemented controller.

THANK YOU