

9.2.3

EE24BTECH11014 - Deepak Ahirwar

Question:

For the Differential Equation $\frac{dy}{dx} + \sin x = 0$, verify that $y = \cos x + C$ is a solution of the differential equation.

Solution: : We are given the first-order ordinary differential equation:

$$\frac{dy}{dx} + \sin x = 0 \quad (1)$$

This is a separable differential equation. We can rewrite it as:

$$\frac{dy}{dx} = -\sin x \quad (2)$$

Now, we integrate both sides with respect to x :

$$\int \frac{dy}{dx} dx = \int -\sin x dx \quad (3)$$

$$\int dy = - \int \sin x dx \quad (4)$$

$$\implies y = \cos x + C \quad (5)$$

where C is the constant of integration.

Therefore, the general solution to the given differential equation is:

$$y(x) = \cos x + C \quad (6)$$

Computational Solution:

Using classical definition of derivative we get,

$$f'(x) = \frac{f(x+h) - f(x)}{h} \quad (7)$$

$$\implies f(x+h) = f(x) + f'(x)h \quad (8)$$

For $y = f(x)$, we can get the points of the required graph by iterating the equation obtained in (8) where values of x increases in each iteration by h and obtaining the y -coordinate of it.

For,

$$x_0 = 0 \quad (9)$$

$$y_0 = 1 \quad (10)$$

$$h = 0.001 \quad (11)$$

Using Euler Method, we get difference equation,

$$y_{n+1} = y_n + h \frac{dy}{dx} \Big|_{(x_n, y_n)} \quad (12)$$

$$y_{n+1} = y_n - h \sin x_n \quad (13)$$

$$x_{n+1} = x_n + h \quad (14)$$

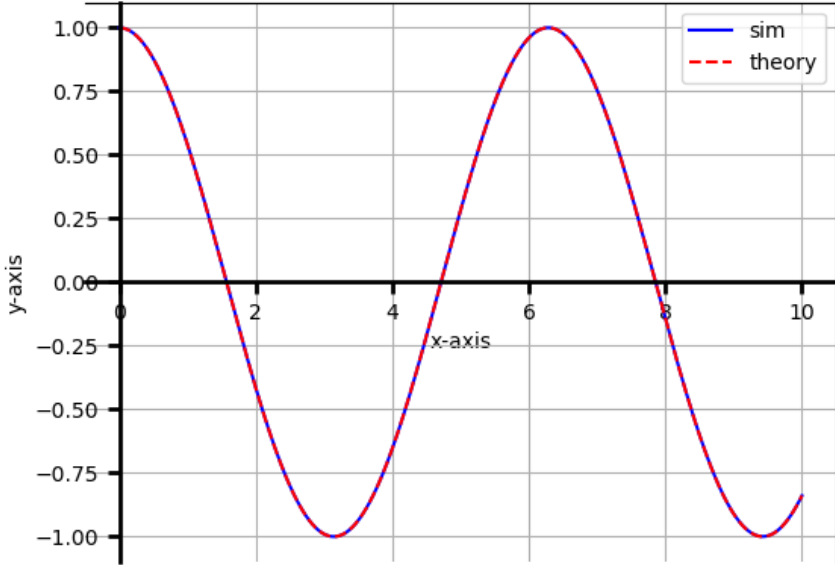


Fig. 0: Verification