12.8.ex.13

EE24BTECH11014 - Deepak Ahirwar

Question:

Find the area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$.

Solution: :

Theoretical logic:

1) **Set up the integral:**
   The area under the curve can be calculated as:

$$\text{Area} = \int_{x_1}^{x_2} f(x)\,dx \tag{1}$$

   Here:

$$f(x) = \cos(x), \quad x_1 = 0, \quad x_2 = 2\pi \tag{2}$$

   Check whether the curve $y = \cos(x)$ crosses the x-axis in the interval $x \in [0, 2\pi]$:

$$y = 0 = \cos(x) \implies x = \frac{\pi}{2}, \frac{3\pi}{2} \tag{3}$$

   Thus, the integral becomes:

$$\text{Area} = \int_0^{\pi/2} \cos(x)\,dx - \int_{\pi/2}^{3\pi/2} \cos(x)\,dx + \int_{3\pi/2}^{2\pi} \cos(x)\,dx \tag{4}$$

2) **Compute each integral:**

$$\int_0^{\pi/2} \cos(x)\,dx = [\sin(x)]_0^{\pi/2} = \sin\left(\frac{\pi}{2}\right) - \sin(0) = 1 - 0 = 1 \tag{5}$$

$$\int_{\pi/2}^{3\pi/2} \cos(x)\,dx = [\sin(x)]_{\pi/2}^{3\pi/2} = \sin\left(\frac{3\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right) = -1 - 1 = -2 \tag{6}$$

$$\int_{3\pi/2}^{2\pi} \cos(x)\,dx = [\sin(x)]_{3\pi/2}^{2\pi} = \sin(2\pi) - \sin\left(\frac{3\pi}{2}\right) = 0 - (-1) = 1 \tag{7}$$

3) **Add the areas:**

$$\text{Area} = 1 + |-2| + 1 \tag{8}$$

$$= 1 + 2 + 1 = 4 \tag{9}$$

The total area bounded by the curve $y = \cos(x)$ between $x = 0$ and $x = 2\pi$ is 4

Computational Logic: Using the trapezoidal rule to get the area. The trapezoidal rule is as follows.

$$\int_a^b f(x)\,dx \approx \sum_{k=1}^{N} \frac{f(x_{k+1}) + f(x_k)}{2} h \tag{10}$$

where

$$h = \frac{b-a}{N} \tag{11}$$

$\therefore$The difference equation obtained is

$$A = \int_a^b f(x)\; dx \approx h\left(\frac{1}{2} f(a) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2} f(b)\right) \tag{12}$$

$$h = \frac{b-a}{n} \tag{13}$$

$$A = j_n, \;\text{ where, }\; j_{i+1} = j_i + h\frac{f(x_{i+1}) + f(x_i)}{2} \tag{14}$$

$$\rightarrow j_{i+1} = j_i + h\left(x_{i+1}^2 + x_i^2\right) \tag{15}$$

$$x_{i+1} = x_i + h \tag{16}$$

$$n = 100000 \tag{17}$$

Using the code answer obtained is 4.0000000000