

# 12.8.ex.13

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## Question:

Find the area bounded by the curve  $y = \cos x$  between  $x = 0$  and  $x = 2\pi$ .

**Solution:** :

## Theoretical logic:

### 1) Set up the integral:

The area under the curve can be calculated as:

$$\text{Area} = \int_{x_1}^{x_2} f(x) dx \quad (1)$$

Here:

$$f(x) = \cos(x), \quad x_1 = 0, \quad x_2 = 2\pi \quad (2)$$

Check whether the curve  $y = \cos(x)$  crosses the x-axis in the interval  $x \in [0, 2\pi]$ :

$$y = 0 = \cos(x) \implies x = \frac{\pi}{2}, \frac{3\pi}{2} \quad (3)$$

Thus, the integral becomes:

$$\text{Area} = \int_0^{\pi/2} \cos(x) dx - \int_{\pi/2}^{3\pi/2} \cos(x) dx + \int_{3\pi/2}^{2\pi} \cos(x) dx \quad (4)$$

### 2) Compute each integral:

$$\int_0^{\pi/2} \cos(x) dx = [\sin(x)]_0^{\pi/2} = \sin\left(\frac{\pi}{2}\right) - \sin(0) = 1 - 0 = 1 \quad (5)$$

$$\int_{\pi/2}^{3\pi/2} \cos(x) dx = [\sin(x)]_{\pi/2}^{3\pi/2} = \sin\left(\frac{3\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right) = -1 - 1 = -2 \quad (6)$$

$$\int_{3\pi/2}^{2\pi} \cos(x) dx = [\sin(x)]_{3\pi/2}^{2\pi} = \sin(2\pi) - \sin\left(\frac{3\pi}{2}\right) = 0 - (-1) = 1 \quad (7)$$

### 3) Add the areas:

$$\text{Area} = 1 + |-2| + 1 \quad (8)$$

$$= 1 + 2 + 1 = 4 \quad (9)$$

The total area bounded by the curve  $y = \cos(x)$  between  $x = 0$  and  $x = 2\pi$  is 4

**Computational Logic:** Using the trapezoidal rule to get the area. The trapezoidal rule is as follows.

$$\int_a^b f(x) dx \approx \sum_{k=1}^N \frac{f(x_{k+1}) + f(x_k)}{2} h \quad (10)$$

where

$$h = \frac{b-a}{N} \quad (11)$$

∴ The difference equation obtained is

$$A = \int_a^b f(x) dx \approx h \left( \frac{1}{2} f(a) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2} f(b) \right) \quad (12)$$

$$h = \frac{b-a}{n} \quad (13)$$

$$A = j_n, \text{ where, } j_{i+1} = j_i + h \frac{f(x_{i+1}) + f(x_i)}{2} \quad (14)$$

$$\rightarrow j_{i+1} = j_i + h(x_{i+1}^2 + x_i^2) \quad (15)$$

$$x_{i+1} = x_i + h \quad (16)$$

$$n = 1000000 \quad (17)$$

