9.2.3

EE24BTECH11014 - Deepak Ahirwar

Question:
For the Differential Equation $\frac{dy}{dx} + \sin x = 0$, verify that $y = \cos x + C$ is a solution of the differential equation.
Solution: : We are given the first-order ordinary differential equation:

$$\frac{dy}{dx} + \sin x = 0 \tag{1}$$

This is a separable differential equation. We can rewrite it as:

$$\frac{dy}{dx} = -\sin x \tag{2}$$

Now, we integrate both sides with respect to x:

$$\int \frac{dy}{dx}\,dx = \int -\sin x\,dx \tag{3}$$

$$\int dy = -\int \sin x\,dx \tag{4}$$

$$\implies y = \cos x + C \tag{5}$$

where C is the constant of integration.
Therefore, the general solution to the given differential equation is:

$$y(x) = \cos x + C \tag{6}$$

Computational Solution:
Using classical defination of derivative we get,

$$f\prime(x) = \frac{f(x+h) - f(x)}{h} \tag{7}$$

$$\implies f(x+h) = f(x) + f\prime(x)\,h \tag{8}$$

For $y = f(x)$, we can get the points of the required graph by iterating the equation obtained in (8) where values of x increases in each iteration by h and obtaining the y-coordinate of it.
For,

$$x_0 = 0 \tag{9}$$

$$y_0 = 1 \tag{10}$$

$$h = 0.001 \tag{11}$$

Using Euler Method, we get difference equation,

$$y_{n+1} = y_n + h\frac{dy}{dx}\bigg|_{(x_n,y_n)} \tag{12}$$

$$y_{n+1} = y_n - h\sin x\bigg|_{(x_n,y_n)} \tag{13}$$

$$x_{n+1} = x_n + h \tag{14}$$

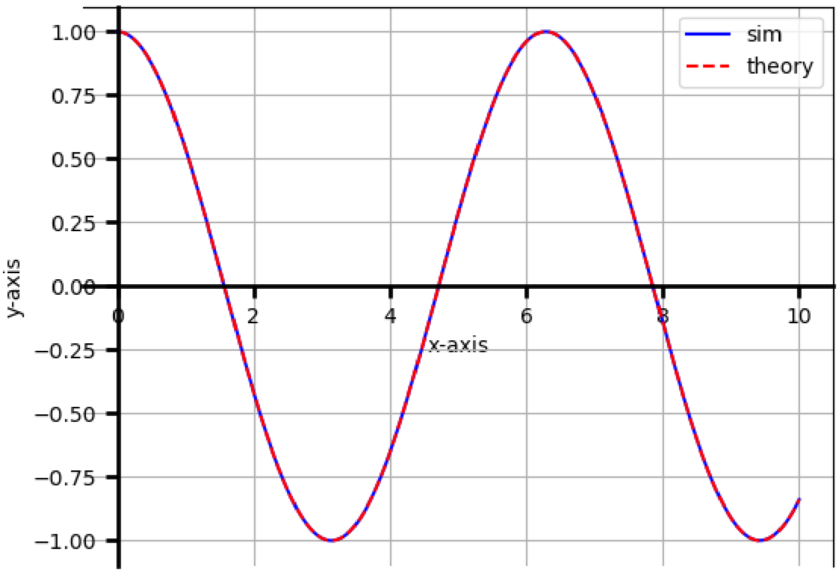

Fig. 0: Verification