10.4.ex.11

EE24BTECH11014 - Deepak Ahirwar

Question: Find two consecutive odd positive integers, sum of whose squares is 290.

Solution: Let the smaller of the two consecutive odd positive integers be x. Then, the second integer will be $x + 2$.
According to the question:

$$x^2 + (x + 2)^2 = 290 \tag{1}$$

Simplify:

$$x^2 + x^2 + 4x + 4 = 290 \tag{2}$$
$$2x^2 + 4x + 4 - 290 = 0 \tag{3}$$
$$2x^2 + 4x - 286 = 0 \tag{4}$$

Divide the equation by 2:

$$x^2 + 2x - 143 = 0 \tag{5}$$

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{6}$$

Here, $a = 1$, $b = 2$, and $c = -143$:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-143)}}{2(1)} \tag{7}$$
$$x = \frac{-2 \pm \sqrt{4 + 572}}{2} \tag{8}$$
$$x = \frac{-2 \pm \sqrt{576}}{2} \tag{9}$$
$$x = \frac{-2 + 24}{2} \quad \text{or} \quad x = \frac{-2 - 24}{2} \tag{10}$$
$$x = 11 \quad \text{or} \quad x = -13 \tag{11}$$

Since x is given to be an odd positive integer:

$$x = 11 \tag{12}$$

Thus, the two consecutive odd integers are 11 and 13.

CODING LOGIC:-
Eigen value method

1) Characteristics polynomial is given by

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \tag{13}$$

where $a_n \neq 0$

2) Divide Characteristics equation by a_n

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \tag{14}$$

$$p(x) = x^n + \frac{a_{n-1}}{a_n} x^{n-1} + \cdots + \frac{a_1}{a_n} x + \frac{a_0}{a_n} \tag{15}$$

3) Companion Matrix of characteristic polynomial is given by:
Let

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & \cdots & -\frac{a_{n-1}}{a_n} \end{bmatrix} \tag{16}$$

4) QR decomposition

$$A = QR \tag{17}$$

a) Q is an $m \times n$ orthogonal matrix
b) R is an $n \times n$ upper triangular matrix.

Given a matrix $A = [a_1, a_2, \ldots, an]$, where each a_i is a column vector of size $m \times 1$.

5) Normalize the first column of A:

$$q_1 = \frac{a_1}{\|a_1\|} \tag{18}$$

6) For each subsequent column a_i, subtract the projections of the previously obtained orthonormal vectors from a_i :

$$a_i' = a_i - \sum_{k=1}^{i-1} \langle a_i, q_k \rangle q_k \tag{19}$$

Normalize the result to obtain the next column of Q:

$$q_i = \frac{a_i'}{\|a_i'\|} \tag{20}$$

Repeat this process for all columns of A.

7) Finding R:-
After constructing the ortho-normal columns $q_1, q_2, \ldots, q_n$ of Q, we can compute the elements of R by taking the dot product of the original columns of A with the columns of Q:

$$r_{ij} = \langle a_j, q_i \rangle \text{ , for } i \leq j \tag{21}$$

8) **QR-Algorithm**

a) Initialization
Let $A_0 = A$, where A is the given matrix.

b) QR Decomposition
For each iteration $k = 0, 1, 2, \ldots$:

i) Compute the QR decomposition of A_k, such that:

$$A_k = Q_k R_k \tag{22}$$

where:

A) Q_k is an orthogonal matrix ($Q_k^\top Q_k = I$).

B) R_k is an upper triangular matrix.

The decomposition ensures $A_k = Q_k R_k$.

ii) Form the next matrix A_{k+1} as:

$$A_{k+1} = R_k Q_k \tag{23}$$

c) Convergence
Repeat Step 2 until A_k converges to an upper triangular matrix T. The diagonal entries of T are the eigenvalues of A.

d) The eigenvalues of matrix will be the roots of the equation.