9-9.4-2

EE24BTECH11014 -DEEPAK

Question:
Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.
Solution: The parameters of the conics are

Variable	Description
V_1, u_1, f_1	Parameters of Parabola
V_2, u_2, f_2	Parameters of circle
P_1, P_2	Points of intersection
A	Area between the conics

TABLE 0: Variables Used

$$V_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ u_1 = \begin{pmatrix} 0 \\ -2 \end{pmatrix}, \ f_1 = 0 \tag{0.1}$$

$$V_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ u_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ f_2 = -\frac{9}{4} \tag{0.2}$$

The intersection of two conics with parameters $V_i, u_i, f_i, \ i = 1, 2$ is defined as

$$x^T \left(V_1 + \mu V_2\right) x + 2\left(u_1 + \mu u_2\right)^T x + \left(f_1 + \mu f_2\right) = 0 \tag{0.3}$$

Solving this the points of intersection are

$$\begin{pmatrix} \sqrt{2} \\ \frac{1}{2} \end{pmatrix}, \begin{pmatrix} -\sqrt{2} \\ \frac{1}{2} \end{pmatrix} \tag{0.4}$$

Area between the curves is,

$$2\int_0^{\frac{1}{2}} \left(\sqrt{\frac{9}{4} - y^2} - \sqrt{4y} \right) dy \tag{0.5}$$

By solving the integration, we get area is equal to 3.005 sq.units

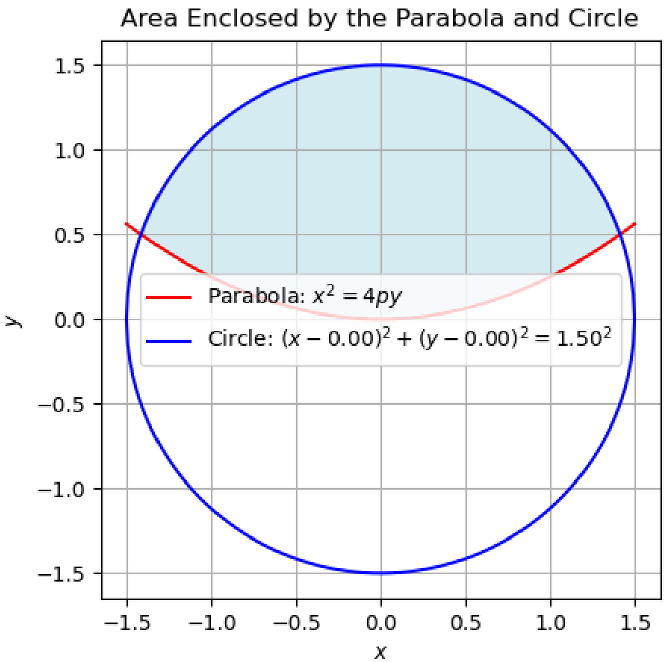
Area Enclosed by the Parabola and Circle
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
y
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
x
Parabola: $x^2 = 4py$
Circle: $(x - 0.00)^2 + (y - 0.00)^2 = 1.50^2$

Fig. 0.1