Section 1.5

1 Complements of Numbers

- Complements are used in digital computers to simplify subtraction operation and for logical manipulation
- There are two types of complment for base-r system
  1. Diminished radix complement-(r-1)'s complement
  2. Radix complement-r's Complement

1.1 Diminished Radix Complement

- Given a number N in base r having n digits, the (r-1)'s complement of N is its diminished radix complement , is defined as $(r^n - 1) - N$.
- For example in decimal system,

$$\text{The 9's complement of 546700 is } 999999 - 546700 = 453299. \quad (1)$$
$$\text{The 9's complement of 012398 is } 999999 - 012398 = 987601. \quad (2)$$

  From the above example it is clear that 9's complement can be obtained by subtracting each digit with 9
- In binary

$$\text{The 1's complement of 1011000 is 0100111.} \quad (3)$$
$$\text{The 1's complement of 0101101 is 1010010.} \quad (4)$$

  From the above example it is clear that the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.
- Generally, in any base-r system (r-1)'s complement is obtained by subtracting each digit in (r-1)

1.2 Radix Complement

- The r's complement of an n-digit number N in base r is defined as $r^n - N$ for N $\neq$ 0 and as 0 for N= 0.

- Notice, r's complement = (r-1)'s complement + 1. i,e

$$r^n - N = (r^n - 1) - N + 1 \tag{5}$$

- For example, in decimal system

$$\text{the 10's complement of 012398 is 987602} \tag{6}$$
$$\text{the 10's complement of 246700 is 753300} \tag{7}$$

- In binary

$$\text{the 2's complement of 1101100 is 0010100} \tag{8}$$
$$\text{the 2's complement of 0110111 is 1001001} \tag{9}$$

- The original number N contains a radix point, the point should be removed temporarily in order to form the r's or (r- 1)¿s complement. The radix point is then restored to the complemented number in the same relative position.

1.3 Subtraction with complement

- When we subtract borrow carry when the minuend is smaller than the subtrahend. This works when using pen and paper but very inefficient than using complements.

- The subtraction of two n-digit unsigned numbers M- N in base r can be done as follows:

  1. Add the minuend M to the r's complement of the subtrahend N. Mathematically,$M + (r^n - N) = M - N + r^n$
  2. If $M \geq N$, the sum the sum will produce an end carry r^n , which can be discarded; what is left is the result M- N.
  3. if $M < N$, the sum does not produce an end carry and is equal to r^n - (N- M), which is the r's complement of (N- M). To obtain the answer in a familiar form, take the r's complement of the sum and place a negative sign in front.