Section 1.3: Number-Base Conversion

Key Concepts

- Two number representations are **equivalent** if they have the same decimal value (e.g., $(0011)_8$ and $(1001)_2$ both represent 9).

- Conversion from base r to decimal involves expanding the number into a power series and summing the terms.

- Conversion from decimal to base r requires separating the number into its integer and fractional parts, as each part is converted differently.

Decimal to Base-r Conversion

- **Integer Part**: Divide the number by r repeatedly, accumulating remainders. The remainders (in reverse order) form the base-r representation.

- **Fractional Part**: Multiply the fraction by r repeatedly, accumulating integers. The integers form the base-r representation.

Examples

- **Decimal to Binary**:

    - Convert $(41)_{10}$ to binary:

    $$41 \div 2 = 20 \quad \text{remainder } 1$$
    $$20 \div 2 = 10 \quad \text{remainder } 0$$
    $$10 \div 2 = 5 \quad \text{remainder } 0$$
    $$5 \div 2 = 2 \quad \text{remainder } 1$$
    $$2 \div 2 = 1 \quad \text{remainder } 0$$
    $$1 \div 2 = 0 \quad \text{remainder } 1$$

    Result: $(41)_{10} = (101001)_2$.

- **Decimal to Octal**:

– Convert $(153)_{10}$ to octal:

$$153 \div 8 = 19 \quad \text{remainder } 1$$
$$19 \div 8 = 2 \quad \text{remainder } 3$$
$$2 \div 8 = 0 \quad \text{remainder } 2$$

Result: $(153)_{10} = (231)_8$.

- **Decimal Fraction to Binary**:

  – Convert $(0.6875)_{10}$ to binary:

$$0.6875 \times 2 = 1.375 \quad \text{integer } 1$$
$$0.375 \times 2 = 0.75 \quad \text{integer } 0$$
$$0.75 \times 2 = 1.5 \quad \text{integer } 1$$
$$0.5 \times 2 = 1.0 \quad \text{integer } 1$$

Result: $(0.6875)_{10} = (0.1011)_2$.

- **Decimal Fraction to Octal**:

  – Convert $(0.513)_{10}$ to octal:

$$0.513 \times 8 = 4.104 \quad \text{integer } 4$$
$$0.104 \times 8 = 0.832 \quad \text{integer } 0$$
$$0.832 \times 8 = 6.656 \quad \text{integer } 6$$
$$0.656 \times 8 = 5.248 \quad \text{integer } 5$$
$$0.248 \times 8 = 1.984 \quad \text{integer } 1$$
$$0.984 \times 8 = 7.872 \quad \text{integer } 7$$

Result: $(0.513)_{10} = (0.406517)_8$.

Combining Integer and Fractional Parts

- For numbers with both integer and fractional parts, convert each part separately and combine the results.

- Example: $(41.6875)_{10} = (101001.1011)_2$.

- Example: $(153.513)_{10} = (231.406517)_8$.

Table of Powers of Two

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024 (1K)	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096 (4K)	20	1,048,576 (1M)
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

Table 1: Powers of Two