"2007-MA-(52-68)"

EE24BTECH11014 - Deepak

1) Consider the quadrature formula
$$\int_{-1}^{1} |x| f(x) dx = \frac{1}{2}[f(x_0) + f(x_1)],$$
where x_0 and x_1 are quadrature points. Then the highest degree of the polynomial, for which the above formula is exact, equals

a) 1 b) 2 c) 3 d) 4

2) Let $A, B$and C be three events such that
$P(A) = 0.4, P(B) = 0.5, P(A \cup B) = 0.6, P(C) = 0.6$ and $P(A \cap B \cap C) = 0.1$.
Then $P(A \cup B|C)$=

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{1}{5}$

3) Consider two identical boxes B_1 and B_2, where the box $B_i (i = 1, 2)$ contains $i + 1$ red and $5 - i - 1$ white balls. A fair die is cast. Let the number of dots shown on th e top face of the die be N. If N is even or 5, two balls are drawn with replacement from the box B_1, otherwise, two balls are drawn with replacement from the box B_2. The probability that the two drawn balls are of different colours is

a) $\frac{7}{25}$ b) $\frac{9}{25}$ c) $\frac{12}{25}$ d) $\frac{16}{25}$

4) Let $X_1, X_2, \cdots$ be a sequence of independent and identically distributed random varables with
$$P(X_1 = -1) = P(X_1 = 1) = \frac{1}{2}.$$
Suppose for the standard normal random variable Z, $P(-0.1 < Z \leq 0.1) = 0.08$. If $S_n = \sum_{i=1}^{n^2} X_i$, then $\lim_{n\to\infty} P(S_n > \frac{n}{10}) =$

a) 0.42 b) 0.46 c) 0.50 d) 0.54

5) Let $X_1, X_2, \cdots, X_5$ be a random sample of size 5 from a population having standard normal distribution. Let
$$\overline{X} = \frac{1}{5}\sum_{i=1}^{5} X_i \text{ and } T = \sum_{i=1}^{5} (X_i - \overline{X})^2$$
Then $E(T^2\overline{X}^2) =$

a) 3 b) 3.6 c) 4.8 d) 5.2

6) Let $x_1 = 3.5, x_2 = 7.5$ and $x_3 = 5.2$ be the observed values of random sample of size three from a population having uniform distribution over the interval $(\theta, \theta + 5)$, where $\theta \in (0, \infty)$ is unknown and is to be estimated. Then which of the following is NOT a maximum likelihood estimate of θ?

a) 2.4 b) 2.7 c) 3.0 d) 3.3

7) The value of

$$\int_0^\infty \int_{\frac{1}{y}}^\infty x^4 e^{-x^3 y}\, dx\, dy$$

equals

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) 1

8) $\lim_{n\to\infty} \left[(n+1)\int_0^1 x^n \ln(1+x)\, dx\right] =$

a) 0 b) $ln2$ c) $ln3$ d) ∞

9) Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} x^4, \text{if x is rational} \\ 2x^2 - 1, \text{if x is irrational} \end{cases}$$

Let S be the set of points where f is continuous. Then

a) $S = \{1\}$ b) $S = \{-1\}$ c) $S = \{-1, 1\}$ d) $S = \phi$

10) For a positive real number p, let $\{f_n : n = 1, 2, \cdots\}$ be a sequence of functions defined on $[0, 1]$ by

$$f_n(x) = \begin{cases} n^{p+1}x, & \text{if } 0 \le x \le \frac{1}{n} \\ \frac{1}{x^p}, & \text{if } \frac{1}{n} < x \le 1 \end{cases}$$

Let $f(x) = \lim_{n\to\infty} f_n(x), x \in [0, 1]$. Then, on $[0, 1]$,

a) f is Riemann integrable
b) the improper integral $\int_0^1 f(x)dx$ converges for $p \ge 1$
c) the improper integral $\int_0^1 f(x)dx$ converges for $p < 1$
d) f_n converges uniformly

11) Which of the following inequality is NOT true for $x \in [\frac{1}{4}, \frac{3}{4}]$:

a) $e^{-x} > \sum_{j=0}^{2} \frac{(-x)^j}{j!}$
b) $e^{-x} < \sum_{j=0}^{3} \frac{(-x)^j}{j!}$
c) $e^{-x} > \sum_{j=0}^{4} \frac{(-x)^j}{j!}$
d) $e^{-x} > \sum_{j=0}^{5} \frac{(-x)^j}{j!}$

12) Let $u(x, y)$ be the solution to the Cauchy problem

$$xu_x + u_y = 1, u(x, 0) = 2ln(x), x > 1.$$

Then $u(e, 1) =$

a) -1 b) 0 c) 1 d) e

13) Suppose

$$y(x) = \lambda \int_0^{2\pi} y(t) \sin(x+t)\, dt, \quad x \in [0, 2\pi]$$

has eigenvalues $\lambda = \frac{1}{\pi}$ and $\lambda = \frac{-1}{\pi}$ with corresponding eigenfunctions $y_1(x) = \sin(x) + \cos(x)$ and $y_2(x) = \sin(x) - \cos(x)$, respectively . Then the integral equation

$$y(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} y(t) \sin(x+t)\, dt, \quad x \in [0, 2\pi]$$

has a solution when $f(x) =$

a) 1
b) $\cos(x)$
c) $\sin(x)$
d) $1 + \sin(x) + \cos(x)$

14) Consider the Neumann problem

$$u_{xx} + u_{yy} = 0, \quad 0 < x < \pi, \quad -1 < y < 1$$
$$u_x(0, y) = u_x(\pi, y) = 0$$
$$u_y(x, -1) = 0, \quad u_y(x, 1) = \alpha + \beta \sin(x)$$

The problem admits solution for

a) $\alpha = 0, \beta = 1$
b) $\alpha = -1, \beta = \frac{\pi}{2}$
c) $\alpha = 1, \beta = \frac{\pi}{2}$
d) $\alpha = 1, \beta = -\pi$

15) The functional

$$\int_0^1 (1 + x)(y')^2 \, dx, \quad y(0) = 0, \quad y(1) = 1,$$

possesses

a) strong maxima
b) strong minima
c) weak maxima but NOT a strong maxima
d) weak minima but NOT a strong minima

16) The value of α for which the integral equation

$$u(x) = \alpha \int_0^1 e^{x-t} u(t) \, dt$$

has a non-trivial solution is

a) -2
b) -1
c) 1
d) 2

17) Let $P_n(x)$ be the Legendre polynomial of degree n and let

$$P_{m+1}(0) = -\frac{m}{m+1} P_{m-1}(0), \quad m = 1, 2, \cdots$$

If $P_n(0) = -\frac{5}{16}$,then $\quad \int_{-1}^1 P_n^2(x) dx =$

a) $\frac{2}{13}$
b) $\frac{2}{9}$
c) $\frac{5}{16}$
d) $\frac{2}{5}$