Reliability analysis of uncertain vibration response in bidirectional sandwich structures using a direct probability integral approach

Narayan Sharma[a], Pawan Kumar[b*], Prateek Chandrakar[c], and Dipak Kumar Maiti[d]

[a] Assistant Professor, Amity Institute of Defence Technology, Amity University, Noida, 201303, Uttar Pradesh, India
[b] Associate Professor, Department of Mechatronics, Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal, 576104, Karnataka, India
[c] Research Scholar, Department of Aerospace Engineering, IIT Kharagpur, Kharagpur, 721302, West Bengal, India
[d] Professor, Department of Aerospace Engineering, IIT Kharagpur, Kharagpur, 721302, West Bengal, India

1. INTRODUCTION & OBJECTIVE

Research on uncertainty quantification and propagation in complex structural systems has gained significant attention, particularly in systems where uncertainties have a critical influence on performance and reliability. The source of uncertainty could be the manufacturing defects, environmental conditions, and various other factors, and this randomness could significantly influence the system response. Thus, assessment of such systems under uncertain vibration is essential to avoid premature failure of the structures, or overly conservative designs. Since traditional deterministic based reliability evaluation assumes constant parameters and fails to capture real-world variability, hence, stochastic based reliability computation provides a realistic framework to develop robust and high-performance structures. There are several models available to solve the reliability of composite structures, however, these models fail to provide efficient solutions. Monte Carlo is one of the robust and versatile models to perform reliability analysis, however, the implementation of this model to complex structures is difficult due to high computation cost and time requirement to perform the simulations [1]. To mitigate the issue direct probability integral (DPI) based reliability model is developed to compute reliability parameters at various levels of uncertainty in the systems. DPI complies with the probability conservation principle and offers an efficient framework for solving complex structural dynamic problems with fast convergence and required accuracy [2]. Chen et al. [3] integrates Voronoi model and Generalized F-Discrepancy technique to generate better distributed samples, where each sample points assigned discrete probability value. The developed model was utilized for structural optimization and random vibration analysis and performed well for all the studied cases.

The present work introduces a novel DPI-based reliability model to evaluate the reliability parameters of bi-directional graded sandwich structures. Literature shows that while Monte Carlo simulation (MCS) delivers high accuracy in reliability estimation, its application becomes impractical for complex structures due to the significant computational cost. To address this, the proposed DPI-based model integrates Voronoi space and the Generalized F-Discrepancy (GFD) model, enabling accurate predictions with substantially fewer samples. The proposed meta-model, combined with a higher order layerwise FE model, offers an efficient solution for determining the reliability parameters of complex structural systems

2. METHODS OF ANALYSIS

DPI based reliability model is developed to estimate reliability parameters. For stochastic dynamic analysis, several selective points $\left(x_q\right)$ are generated through Voronoi space method and assigned some probabilistic weight to ensure non-overlapping domain. The total probability can be estimated through the following equation:

$$P_q = \int_{V_q} P_x\left(x_q\right) dx_q \qquad (1)$$

Where $q = 1, 2, 3, .., N$ is the selected points, uncertain parameters are $x_q = (x_{1,q}, x_{2,q}, x_{3,q}, ..., x_{n,q})$, and n representing the number of uncertain parameters. For each representative point the stochastic PDF curve is obtained using equation:

$$\wp_f\left(f\right) = \sum_{q=1}^{N} \frac{1}{\sqrt{2\pi\mathbb{S}}} \exp\left\{\frac{-(f - g\left(x_q\right))}{2\mathbb{S}^2}\right\}.P_q \qquad (2)$$

In equation (2), f representing the output vector, $\wp_f\left(f\right)$ gives PDF output vector. The input and output relation can be established using the mapping function $g\left(x\right)$. The accuracy of DPI model is predicted depending on the smoothing parameter $(\mathbb{S})$, and the selection of smoothing parameters is taken from refs. [4]. Implementing the GFD based point selection method, the reliability parameter of the stochastic system can be estimated as:

$$R_s = \sum_{q=1}^{N} \mathrm{H}\left[g\left(x_q\right)\right].P_q \tag{3}$$

where, $\mathrm{H}\left[g\left(x_q\right)\right] = \begin{cases} 0, \; g\left(x_q\right) \le 0 \\ 1, \; g\left(x_q\right) > 0 \end{cases}$

3. RESULTS AND DISCUSSIONS

To assess accuracy of DPI based reliability model, the PDF curve generated from ~1000 samples is compared with the Monte Carlo results obtained from 15,000 samples, as shown in Figure 1. The DPI-based PDF curve exhibits close agreement with the MCS results, and the converged sample size will be adopted for subsequent reliability analyses.

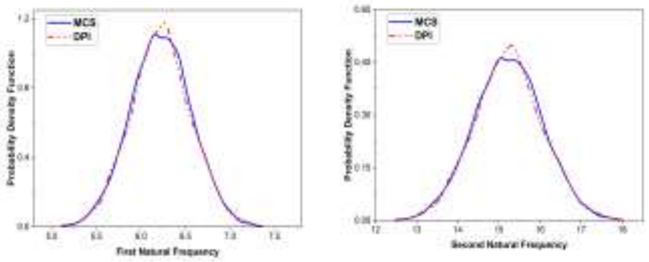
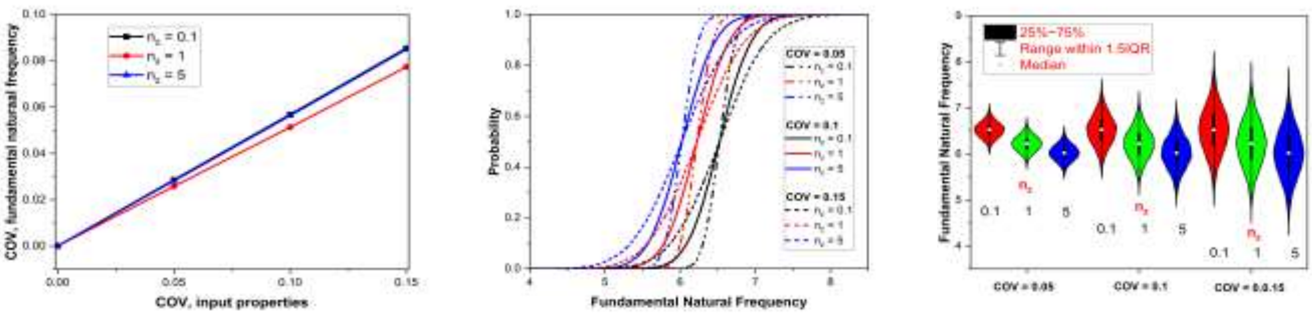


Figure 1: *Comparison of PDF curve from the DPI based stochastic model with Monte Carlo results for first and second modes ($\times 10^2$ Hz)*

The present DPI based reliability model evaluates the reliability curves for different levels of randomness in material properties, with violin plots included to capture the detailed scattering pattern of the natural frequencies under varying uncertainty levels. Figure 2 shows that as the randomness increases; the probability of structural failure significantly below the mean response of the structure. While deterministic approaches typically consider the mean value as the failure threshold, stochastic analysis highlights the importance of accounting for variability to prevent premature failures. It is also observed that increasing randomness does not shift the mean values but rather broadens the distribution, leading to greater output variability.



(a) (b) (c)

Figure 2: *COV plot (a), Reliability plot (b) and Violin plot (c) of the first natural frequency of bi-directional FG sandwich structures for various transverse gradation indices, (a) $n_x = 0$, (b) $n_x = 1$, (c) $n_x = 5$, for degrees of randomness in inputs*

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* Communicating Author: Pawan Kumar and pawan.k@manipal.edu