Analytical Modelling of Electro-Mechanical Impedance (EMI) in Adhesively Bonded Beams for Multi-Damage Detection

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Abstract

The present study demonstrates an analytical approach for modelling multiple damages inside adhesively bonded aluminum beam structure using PZT-host structure interaction electro-mechanical impedance (EMI) technique. The EMI responses under the healthy and different possible damage states have been theoretically obtained using Euler-Bernoulli's beam theory. The 1D theoretical model which includes the effect of dynamic stiffness of the host beam, predicts the natural frequencies as well as the corresponding mode shapes which in turn gives rise to determination of EMI response of the PZT-host structure model. Multiple damage states such as single, double and triple disbonds at a time have been introduced inside the model by division of the structure into multiple parts/segments, with the application of exact boundary, continuity and compatibility conditions to achieve the closed-form solutions. The impedance response of the healthy as well as various unhealthy states of the structure have been obtained within a user specified frequency range of 20kHz-80kHz which consists of reasonable amount of resonance peaks for damage detection. The pristine state impedance response of the1D analytical model has been validated with the EMI-response obtained from FEM-based 3D ANSYS model. The conclusive sign of higher degree of damage progression has been demonstrated through comparative assessment of unhealthy and pristine state statistical damage indices such as root mean square deviation (RMSD) and correlation coefficient (CC).

Keywords: Damage detection; Analytical Model; Adhesively bonded beams; Structural Health Monitoring (SHM), Non-destructive Testing (NDT); Electro-Mechanical Impedance (EMI); Conductance (G); Disbond; Root Mean Square Deviation (RMSD); Correlation Coefficient (CC); Lead Zirconate Titanate (PZT).

1. Introduction

Weight efficient adhesively bonded joints are commonly found in various engineering applications due to their associated advantages such as smooth stress distribution, preservation of material integrity, improved fatigue and vibration resistance etc. However, these structures are susceptible to dsibonds/delaminations which poses a potential threat to long term structural integrity and reliability. The EMI method which incorporates surface-bonded PZT transducers has emerged as a promising NDT tool for damage localization and quantification due to its sensitivity to small minor defects that leads to modification of local stiffness and boundary conditions.

The present method proposes a low-computational cost 1D analytical EMI model for damage detection that preserves the principal structural dynamic properties such as mass, stiffness and damping etc. of a real-life 3D structure. The analytical framework includes the modelling of single, double and triple disbonds inside an adhesively bonded metallic beams.

2. Methodology

The test structure represents a cantilever beam with a surface bonded PZT 5H transducer and is formed by joining two individual aluminum beam by means of epoxy adhesive (refer Fig.1). The electro-mechanical coupling of the PZT-structure interaction has been captured in terms of electro-mechanical impedance (EMI) signatures. The EMI response which is a function of dynamic stiffness of the structure can be represented mathematically as follows:

$$Z(\omega) = \frac{1}{i\omega C}\left(1 - {k_{31}}^2 \frac{K_{str}(\omega)}{K_{PZT}+K_{str}(\omega)}\right)^{-1} \qquad (1)$$

where $K_{str}(\omega)$' is the dynamic stiffness of the structure; 'K_{PZT}' is the stiffness of the PZT transducer and the other terms represent the piezo-electric properties of the attached PZT.

As axial vibration modes are more prone to higher natural frequencies, the axial vibration is neglected in the present model. The structural stiffness (K_{str}) experienced by the PZT due to PZT-host structure interaction which is a function of the natural frequencies and mode-shapes of the structure is given as follows:

$$\boldsymbol{K_{str}} = \frac{\boldsymbol{F_{PZT}}}{\widehat{\boldsymbol{u_{PZT}}}} = \left\{ \frac{\left(\frac{\boldsymbol{d}}{\boldsymbol{2}}\right)^{\boldsymbol{2}}}{\boldsymbol{\rho A}} \sum_{\boldsymbol{n}} \frac{[\boldsymbol{W_n}'(\boldsymbol{Loc1}) - \boldsymbol{W_n}'(\boldsymbol{Loc2})]^{\boldsymbol{2}}}{\boldsymbol{\omega_n}^{\boldsymbol{2}} + \boldsymbol{2i\zeta\omega\omega_n} - \boldsymbol{\omega^2}} \right\}^{-\boldsymbol{1}} \qquad (2)$$

Where 'Loc_1' and 'Loc_2'are the location of two extreme ends of the PZT patch.

$W_n{}'$ is the mode shape of the structure.

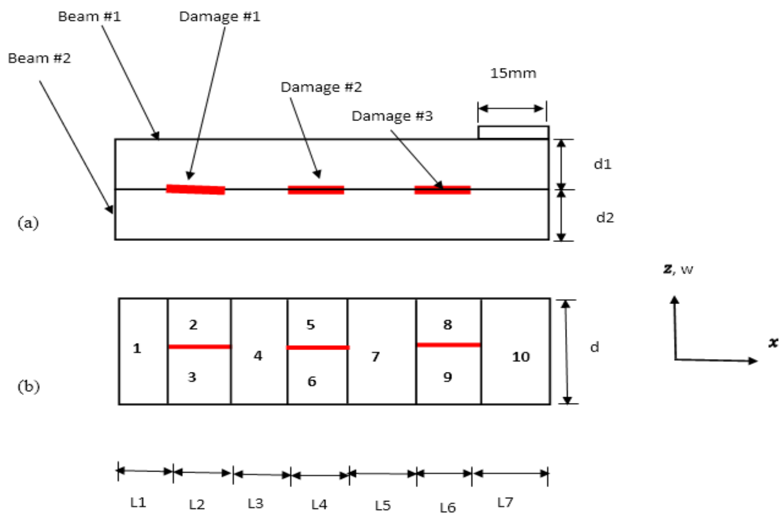

Figure 1 Two adhesively bonded Aluminum beam with multiple disbonds connected with a PZT transducer on top

The governing differential equation applying Euler-Bernoulli's beam theory is given as

$$EI\,\frac{d^4W(x,t)}{dx^4} + \rho A\,\frac{d^2W(x,t)}{dt^2} = 0 \qquad (3)$$

The free vibration of the model can be given as:

$$W(x,t) = \sum_{n=1}^{\infty} w(x)e^{i\omega t} \qquad (4)$$

Where '$w(x)$' are the corresponding transverse modes of the structure.

Hence, the mode shapes can be expressed as follows:

$$w_{in}(x_{in}) = A_{in1}\sin(\lambda_{in}x_{in}) + A_{in2}\cos(\lambda_{in}x_{in}) + A_{in3}\sinh(\lambda_{in}x_{in}) + A_{in4}\cosh(\lambda_{in}x_{in}) \quad (5)$$

Where A_{in1}-A_{in4} (i=1,2,3 ... 10) are arbitrary constants and $\lambda_{in} = \left(\frac{\rho A\omega_{in}{}^2}{EI_i}\right)^{\frac{1}{4}}$

Application of appropriate boundary condition gives rise to determination of natural frequencies and corresponding mode shapes which can be utilized to determine the EMI response of the structure. Each damage scenarios involve solving roots of system of transcendental equations to find the natural frequencies and hence the mode shapes, with the continuity and compatibility conditions at region interfaces.

3. Numerical Example

Consider two aluminum beams of dimension $150mm * 25mm * 1mm$ are attached by means of epoxy adhesive. A PZT patch of dimension $15mm * 15mm * 0.4mm$ is fitted on the surface of beam as shown in Figure 1. The frequency range 20kHz-80kHz has been investigated for the pristine state structure which consists of 11 numbers of natural modes. The analytical model has been validated with ANSYS simulation. A comparison of impedance spectrum between analytical and ANSYS model demonstrates an excellent agreement in terms of number of resonant peaks, peak locations and overall pattern of the spectrum (refer Fig. 2). The impedance responses of the structure are now obtained for (i) Single damage state, (ii) Two damage state and (iii) Three damage state from the analytical model respectively. Figure 3 describes a comparison of impedance response between pristine state and multiple damage states of the structure obtained from respective analytical models.

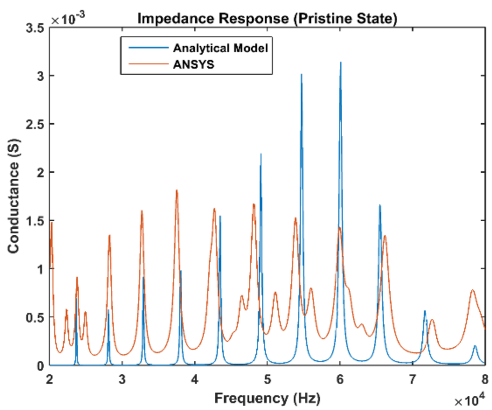

Figure 2 . Comparison of Impedance Response between the Analytical Model and ANSYS

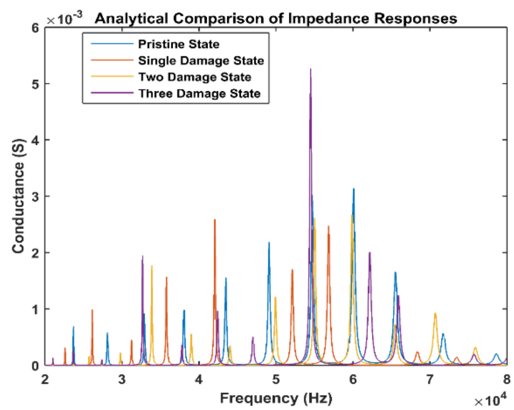

Figure 3 Comparison of Impedance Responses for Different Damage State Obtained from Respective Analytical Model

4. Results and discussions

From Figure 3 it can be seen that presence of any damage significantly modifies the EMI response of the structure as compared to the healthy state. Resonance frequency shifting can be spotted due to the multiple damages present in the structure. Presence of damage reduces the stiffness of the structure which in turn decreases the natural frequencies of the system. The conclusive sign of damage present inside the structure can be seen through the left shift of resonant frequencies which in turn indicates the reduction in structural stiffness and hence decrement of natural frequencies.

5. Conclusive remarks

The present method proposes a computationally inexpensive 1D alternative model to a full-scale 3D-simulation, especially for preliminary diagnostics. The proposed analytical model is robust enough to preserve the basic structural dynamics properties when compared with a 3D simulation model, which significantly reduces the computational complexity of EMI data generation. The model offers an accurate prediction of resonant frequencies and hence subsequent impedance-spectrum, making it a valuable tool for early damage prediction.