Deformation Behavior of Soft Electroelastic Membranes Subjected to Large Electric Fields

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1. INTRODUCTION & OBJECTIVE

Dielectric elastomers are electroelastic materials that are capable of undergoing large strains under applied electric field. Modeling these materials requires integration of nonlinear electrodynamics with continuum mechanics for hyperelastic materials. When dielectric elastomers are placed between two deformable electrodes and electric field is applied, these materials undergo spontaneous polarization. The positive and negative charges produced due to potential difference compresses the elastomer due to force of attraction between them, which causes in-plane stretching due to incompressible nature of the elastomer. Due to the high strain ability of dielectric elastomers, these materials have wide applications as actuators and sensors in the field of soft robotics and biomedical engineering.
In the present work, dielectric elastomers are analyzed for the design of soft membrane actuators. The membrane actuators are modelled using an $O(h)$ non-linear membrane theory incorporating finite deformations. Specifically, deformations and limit point instability in circular and cylindrical membrane actuators and their dependence on material properties is analyzed for varying pressure and voltage inputs. The utility of this computational framework in design of actuators is demonstrated through these examples.

2. MATHEMATICAL FORMULATION

The electroelastic membranes are modelled using $O(h)$ membrane theory under applied internal pressure and electric field. The pointwise 2D electrostatic governing equations are given by [1]

$$\mathbf{Div}\,(\mathbf{1}\mathbf{D}_0) = 0, \qquad \mathbf{D}_0 \cdot \mathbf{N}_3 = -\sigma_F \tag{1}$$

where **Div** represent Lagrangian divergence operator, $\mathbf{1} = \mathbf{I} - \mathbf{N}_3 \otimes \mathbf{N}_3$, I is the identity tensor, $\mathbf{N}_3$ is the normal to the mid-surface of the membrane, $\mathbf{D}_0$ denotes the electric displacement tensor in reference configuration and σ_F is the applied surface charge density. It is important to note that the electric field outside the dielectric membrane is zero, which eliminates Maxwell-stress-induced traction on the exterior boundary. Similarly, the governing equations for the pressurized membrane are expressed as [1]:

$$\mathbf{Div}\,(\mathbf{1}\mathbf{P}_0) = -\{(p^+ - p^-)/h\,\}(J\,\mathbf{F}^{-T})_0\,\mathbf{N}_3, \qquad \mathbf{P}_0\,\mathbf{N}_3 = \mathbf{0}, \tag{2}$$

where $\mathbf{P}_0$ represents the Piola stress tensor on membrane mid-plane, p^+ and p^- are the applied pressure on the top and bottom surface of the membrane, respectively. J represents the determinant of the deformation tensor **F.** These equations are subsequently specialized for cylindrical and circular membranes subjected to axisymmetric loading and a through-thickness electric potential.

3. RESULTS AND DISCUSSION

The deformation, stretches and limit point instability in electroelastic membranes due to applied electromechanical load are analyzed. Our results are verified with literature for passive membranes (in the absence of applied electric field) for circular and cylindrical membranes and different cases are analyzed to study deformation behavior of electroelastic membranes.

For verification of our results, the limit point insatiability in passive hyperelastic circular membrane under nondimensionalized uniform pressure Δp^* is analyzed. The pressure gradient is evaluated for a specified nondimesional deformation z^* (R*=0) and load-deformation curves are presented in Fig. 1 for different values of nondimensional material parameter δ.

Our results are verified with Saxena et al. [2] for the case of Neo-Hookean material i.e. δ =1. It may be noted that the results of Neo-Hookean (δ =1) and Mooney-Rivlin (δ <1) hyperlelastic models are almost identical for smaller deformations $z^*<0.8$. However, the post limit point deformation is dependent on the choice of nondimensional material parameter δ. The pressure limit point is not observed for Mooney-Rivlin material $\delta > 0.96$.

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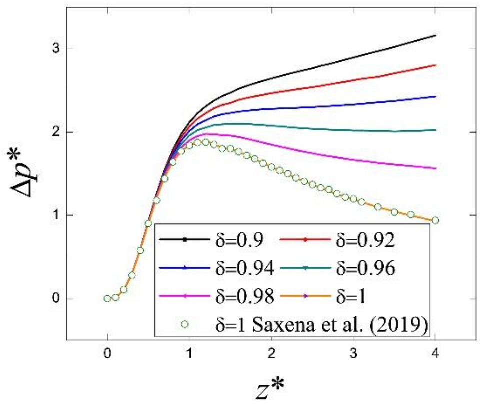

Fig. 1: Limit point instability plot for passive circular membrane for different nondimensional material parameter δ.

Further, the dielectric cylindrical membrane is analyzed under a constant pressure gradient $\Delta p^*=0.844$ and variable electric load $V^*=0.005\text{-}0.017$. The results are presented in Figure 2. It may be noted the bulge formation is observed at the center $z^*=0$ and the bulge evolves with the increase in the electric load.

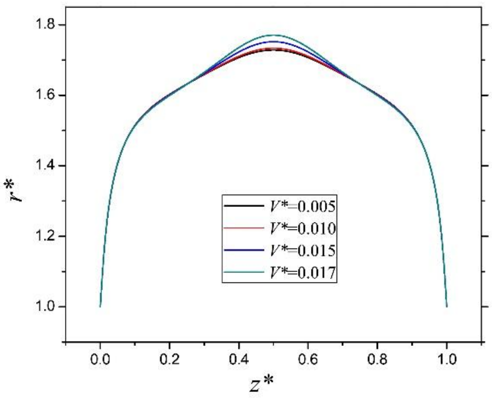

Fig. 2: Deformation plots for electroelastic cylindrical membrane underapplied pressure and electric load.

4. CONCLUSIONS

An $O(h)$ finite deformation membrane theory is presented for electroelastic membranes.The membrane theory is used to model circular and cylindrical membranes. The constitutive behavior of the dielectric elastomer membrane is modelled using Mooney-Rivlin type electroelastic model. The deformation, stretches, and limit point instability in electroelastic membranes under transverse pressure and an electric field is analyzed. Our limit point results are validated with existing literature for a passive hyperelastic membrane. The model is further used to study deformation and limit point behaviour due to applied electric field in electroelastic membranes. It is observed that applied electric field increases the radial deformation and axial stretches in the membrane actuators. The applied electric field tends to decrease the value of limit point pressure and also decrease the pressure required in post limit point deformation. Thus, the applicability of electroelastic membrane theory toward the analysis and computational design of dielectric elastomer membrane actuators is demonstrated.

5. REFERENCES

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