# A Reduced-order Homogenized Plate Model of Sandwich Structure Using Variational Asymptotic Method

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## 1. INTRODUCTION & OBJECTIVE

Sandwich structures offer high stiffness-to-weight ratio making them suitable for aerospace, automotive, and naval industries. These structures can be optimized for different scenarios and loading conditions by changing their core and facesheet geometry, and properties. However, modeling these structure in full $3D$ at the length-scale of components can be extremely costly, therefore, requiring the need to develop reduced-order homogenized plate models. The objective of the present work is to develop a mathematically rigorous and asymptotically correct homogenized reduced-order model for sandwich structures using the following two-step approach.

1. Variational Asymptotic Method (VAM) to reduce the $3D$ structure into an inhomogeneous $2D$ plate.
2. Homogenization of the fast periodic micro-response to obtain a homogeneous $2D$ plate model at the component scale.

## 2. METHODS OF ANALYSIS

Sandwich structures are $3D$ with one dimension (thickness) significantly smaller than the other two dimensions (length and width). VAM is applied to obtain an asymptotically correct $2D$ plate model by taking advantage of the small quantities inherent to the problem, like strains and thickness-to-length ratio [1,2]. The dimensional reduction is done by taking the through-the-thickness structure into account [3]. The resulting $2D$ plate model, comprising in-plane unit cells, is inhomogeneous. To homogenize it, this work uses two coordinate systems [4]:

- Global coordinates $(x_1, x_2)$ for the whole plate.
- Local coordinates $(y_1, y_2)$ for the unit cell.

Similar to the First-Order Shear Deformation Plate Theory (FSDT) [5, 6], the $2D$ plate model expresses the displacement field for the plate in terms of five in-plane primary variables – three displacements ($u_i,\ i = 1, 2, 3$) and two rotations ($\phi_\alpha,\ \alpha = 1, 2$). These variables are split into two parts:

$$\begin{aligned} u_i(x_1, x_2; y_1, y_2) &= v_i(x_1, x_2) + w_i(x_1, x_2; y_1, y_2), \\ \phi_\alpha(x_1, x_2; y_1, y_2) &= \psi_\alpha(x_1, x_2) + \theta_\alpha(x_1, x_2; y_1, y_2), \end{aligned} \tag{1}$$

where $v_i$ and $\psi_\alpha$ are the average displacements and rotations, respectively. $w_i$ and $\theta_\alpha$ are the fluctuations over the averaged values.

By applying the principle of virtual work, the governing equations for the fluctuating fields and the averaged variables are obtained. The equations for the fluctuating components and the averaged variables are solved analytically and numerically, respectively, considering the continuity of the displacements across the boundaries of the unit cells.

# 3. RESULTS AND HIGHLIGHTS OF IMPORTANT POINTS

One typical example of such structures is shown in Fig. 1, where different colors represent different materials. For such structures, analytical solutions for the fluctuating components in terms of the averaged displacements are obtained.
These solutions are given as follows:

$$w_i = y_\alpha \frac{\partial v_i}{\partial x_\alpha}, \qquad \theta_\alpha = y_\beta \frac{\partial \psi_\alpha}{\partial x_\beta} \tag{2}$$

The key highlights of the present work are as follows:

- The reduced-order model obtained using VAM captures the asymptotically correct deformation, avoiding *ad hoc* and *a priori* assumptions.
- The homogenized model includes the effect of in-plane periodic unit cells.
- The method reduces computational complexity while maintaining high accuracy.

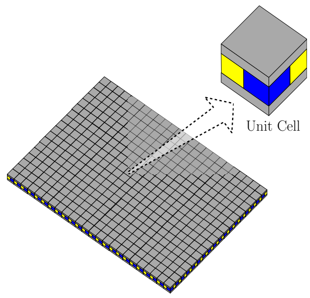


Figure 1: Inhomogeneous anisotropic plate with repetitive microstructure,

# 4. CONCLUSIONS

A simple and efficient method for analyzing inhomogeneous plates with repetitive microstructure is presented. The combination of VAM and two-scale homogenization gives a homogenized reduced-order plate model. This model can be used for advanced plate-like composite structures engineered for different scenarios. Future work will extend it to thermal and nonlinear problems.

# 5. REFERENCES

## References

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