The assessment to study the free vibration of edge cracked bi-directional SFGM plate with modified inverse hyperbolic shear deformation theory (m-IHHSDT).

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ABSTRACT

Nowadays, the sectors like defence, construction, and nuclear energy are growing rapidly. In the applications of such fields the crack failure is needed to be avoided. The bidirectional variation of properties is chosen to compare similar studies with available literature with unidirectional approach. Hence, this work has addressed the free vibration behaviour of porous bi-directional sigmoid functionally graded (SFGM) edge cracked plate by newly modified inverse hyperbolic higher order shear deformation theory (m-IHHSDT). The gradation or variation of effective material properties is assumed x and z directions with the sigmoid law. Parametric analysis is employed to study the effect of volume fraction exponent, type of porosity, crack length, crack angle, and boundary conditions (BCs) on natural frequencies bi-directional functionally graded material. The value of porosity coefficient influences the natural frequency considerably. The natural frequency is strongly influenced by the volume fraction exponent (n) boundary conditions also. This work will help to design bidirectional functionally graded structures for the applications of combustion chambers, I.C. engine components, nuclear reactors, etc.

Keywords: Edge crack, Square plate, Free vibration, Bi-directional porous SFGM, Higher order shear deformation theory.

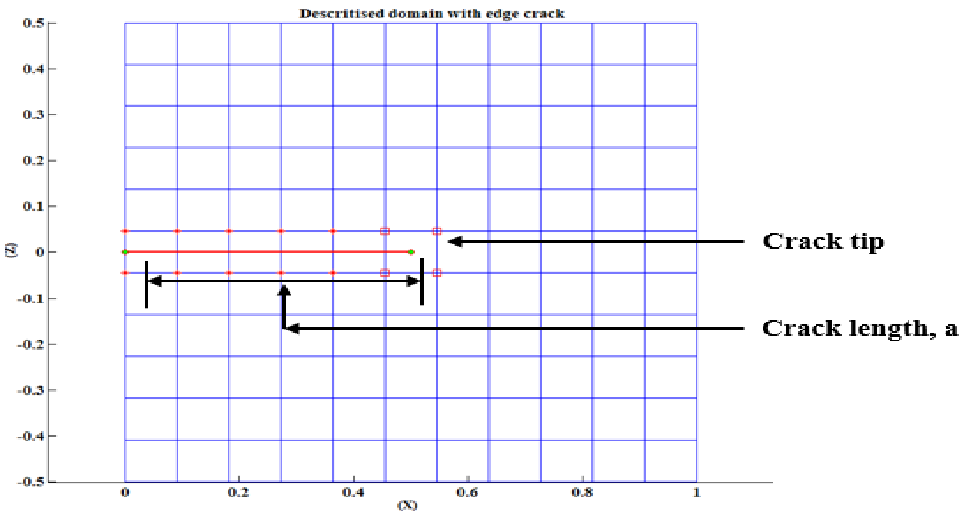

Figure 1. The edge crack propagation of bidirectional SFGM in free vibration.

Figure 1. Shows the square edge cracked FGM plate having length (l), and breadth (b), with gradation index in thickness direction (n_z), and longitudinal direction (n_x) are taken into consideration. X and Z are considered as cartesian coordinates respectively. The variation of material properties of bidirectional SFGM (Ramteke & Panda, 2021) with uniform porosity distribution can be expressed as:

$$\left.\begin{aligned} E(\mathrm{Z},\mathrm{X}) &= E_{\mathrm{m}} + 0.5 * (E_c - E_m)\left(\frac{2Z}{h}+1\right)^{n_z}\left(\frac{X}{a}\right)^{n_x} - 0.5 * Ve * (E_c + E_m) \;\; for\; -\frac{h}{2} \le Z \le 0 \text{ and } -\frac{a}{2} \le X \le 0 \\ E(\mathrm{Z},\mathrm{X}) &= E_{\mathrm{m}} + (E_c - E_m)\left\{1 - \frac{1}{2}\left(1 - \frac{2Z}{h}\right)^{n_z}\left(\frac{X}{a}\right)^{n_x}\right\} - 0.5 * Ve * (E_c + E_m) \;\; for\; 0 \le Z \le \frac{h}{2} \text{ and } 0 \le Z \le \frac{a}{2} \end{aligned}\right\} \quad (1)$$

The modified m-IHHSDT can be written as $f(Z) = Z\tan^{-1}h\left(\tanh\left(2\pi\left(\frac{PZ}{h}\right)^2\right)\right) - \left(\frac{3P\pi Z}{2}\right)$ (2)

Where, Ve = porosity fraction, Em and Ec are modulus of elasticities of aluminium and alumina respectively.

Validation and parametric study:

Table 1. Effect of thickness ratio on the vibration responses (nondimensional) of bidirectional SFGM porous square plate. (a/h = 10, SSSS, $\varpi = \omega\left(\frac{b^2}{h}\right)\sqrt{\frac{\rho_c}{E_c}}$)

Power law index, (n)		Porosity distribution (Ve)				Ref.
		Even		Uneven		
		0.1	0.2	0.1	0.2	
$n_z = 0$	$n_x = 0$	4.7593	4.7593	4.8152	4.876	(Ramteke & Panda, 2021)
	$n_x = 0.5$	4.2426	4.1683	4.3394	4.3818	
	$n_x = 1$	3.9285	3.7979	4.0545	4.0834	
$n_z = 0.5$	$n_x = 0$	4.5393	4.4787	4.6216	4.6635	
	$n_x = 0.5$	4.0862	3.9541	4.2063	4.2323	
	$n_x = 1$	3.8103	3.6263	3.9565	3.9713	
$n_z = 0$	$n_x = 0$	5.7488	5.8219	5.7672	5.8534	Present study
	$n_x = 0.5$	5.2499	5.4649	5.2194	5.3472	
	$n_x = 1$	1.1221	2.6048	2.0647	1.4880	
$n_z = 0.5$	$n_x = 0$	4.6206	4.5663	4.7057	4.7526	
	$n_x = 0.5$	4.3871	4.4249	4.4503	4.5090	
	$n_x = 1$	2.8836	1.8309	3.2716	3.0998	

In Table 1. (Ramteke & Panda, 2021) have utilised polynomial HSDT (Kar & Panda, 2016) and discussed the results of non-dimensional frequency (NDF) of Al/Al_2O_3 SFGM bi-directional plate in free vibration with simply supported boundary condition. The effects of type of porosity and varying power indices NDF's are observed in decreasing trend. Similarly at fixed value of power indices the NDF's resulted into decreasing pattern of values. The results obtained in this work by using modified version of IHHSDT (Kumar et al., 2019) are matching with the (Ramteke & Panda, 2021). On this note, it is concluded that the developed code can be employed to study the free vibration behavior of bidirectional SFGM square plate with edge crack. So, further analyses by varying values of temperature, volume fraction exponent, type of porosity, and boundary conditions (BCs) on natural frequencies of bi-directional functionally graded material.

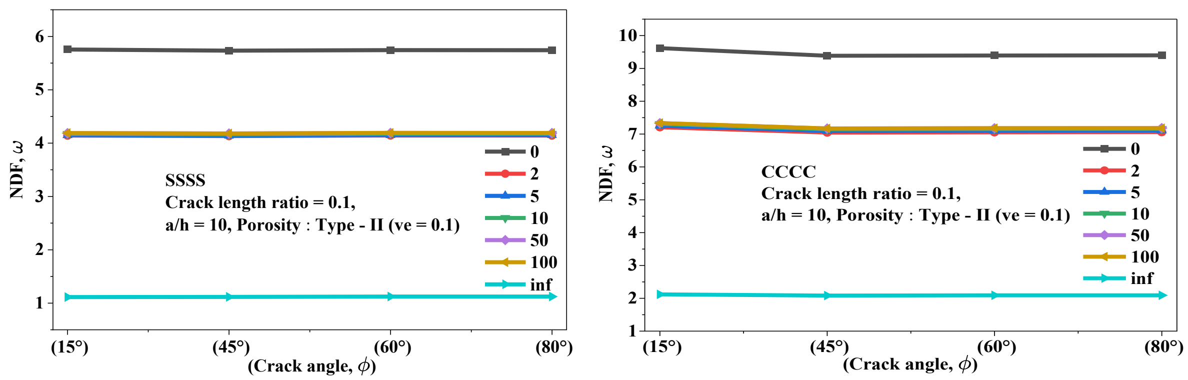

Figure 2. The effect of crack angle on nondimensional frequency of bidirectional SFGM

From Figure 2. values of NDF for CCCC (clamped on all sides) support condition are higher as compared to SSSS (simply supported on all sides). With the increase of power law index NDF decreases. But the frequency distribution pattern is kind of similar in nature.

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