Precision Flight Control of Quadrotor Drones Using LQR with Integral Action.

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I. Abstract

This paper presents an LQR-based control strategy with integral action for precise quadrotor trajectory tracking. We develop a hierarchical architecture combining an inner attitude control loop and outer position tracking loop, supported by efficient state estimation using Madgwick's filter and Kalman filtering. The controller demonstrates 0.05 m/s tracking accuracy with <0.01° yaw deviation in simulation and maintains sub-0.11 m position errors in real-world tests on a Parrot AR.Drone 2.0. The method offers significant advantages including computational efficiency, modularity, and robust performance in both simulated and physical environments, making it particularly suitable for embedded flight control applications requiring precise autonomous operation.

II. Nomenclature

Ω	=	$[p\ q\ r]^T$ is the angular velocity in body frame.
M	=	$\left[M_x\ M_y\ M_z\right]^T$ is control input including the aerodynamic moments and differential thrust moments.
F_T	=	$[0\ 0\ F_{des}]^T$ Summation of thrust forces from the twin rotors as control inputs
g	=	$[\ 0\ 0 - g]^T$ is acceleration due to gravity.
R	=	represents the rotational matrix that encodes the attitude (ϕ, θ, ψ) of the tailsitter.
(ϕ, θ, ψ)	=	Euler angles representing the orientation of the tailsitter.
J	=	Moment of Inertia parameters for tailsitter.
m	=	Mass of the tailsitter.

III. Motivation

The growing adoption of quadrotor drones in commercial and industrial applications demands robust control systems capable of precise trajectory tracking. Traditional PID controllers often struggle with dynamic operating conditions, while nonlinear control methods can be computationally intensive. This work addresses these challenges by developing an optimal control approach that balances performance and computational efficiency, enabling reliable autonomous operation in real-world environments.

IV. Methodology

The control system design follows a rigorous model-based approach, beginning with derivation of the quadrotor dynamics and proceeding through controller synthesis and implementation. The key components are:

Nonlinear Dynamics Modeling

The quadrotor dynamics are derived using Newton-Euler formalism:

Translational dynamics:

$$m\ddot{x} = (cos\varphi sin\theta cos\psi + sin\varphi sin\psi)U_1$$
$$m\ddot{y} = (cos\varphi sin\theta sin\psi - sin\varphi cos\psi)U_1$$
$$m\ddot{z} = (cos\varphi cos\theta)U_1 - mg$$

Rotational dynamics:

$$I_x\ddot{\varphi} = \theta\dot{\psi}(I_y - I_z) + U_2$$
$$I_y\ddot{\theta} = \dot{\varphi}\dot{\psi}(I_z - I_x) + U_3$$
$$I_z\ddot{\psi} = \dot{\varphi}\theta(I_x - I_y) + U_4$$

where $[U_1\ U_2\ U_3\ U_4]^T = \Gamma[T_1\ T_2\ T_3\ T_4]^T$ represents the control inputs mapped from individual rotor thrusts.

Equilibrium Linearization

The system is linearized around hover conditions ($\varphi=\theta=\psi=0$):

$$\dot{x} = Ax + Bu$$

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where for the height subsystem:

$$A_z = [0\ 1;\ 0\ 0], B_z = [0;\ 1/m]$$

and for attitude subsystems (φ,θ,ψ):

$$A_{att} = [0\ 1;\ 0\ 0], B_{att} = [0;\ 1/I_i] \quad (i = x, y, z)$$

LQR with Integral Action
The augmented state-space for tracking control:

$$\dot{x}_{aug} = [A\ 0;\ -C\ 0]x_{aug} + [B;\ 0]u + [0;\ I]r$$

The LQR cost function:

$$J = \int (x_{aug^{\top}} Q x_{aug} + u^{\top} R u) dt$$

with Q = diag(q_1,q_2,q_3), R diagonal, solved via Riccati equation:

$$A_{aug}{}^{\top} P + P A_{aug} - P B_{aug} R^{-1} B_{aug}{}^{\top} P + Q = 0$$

yielding optimal control:

$$u = -K x_{aug} = -[K_x\ K_i][x;\ \int e]$$

Hierarchical Control Implementation
Outer loop (position):

$$u_{xy} = K_p(x_{ref} - x) + K_i \int (x_{ref} - x) dt$$

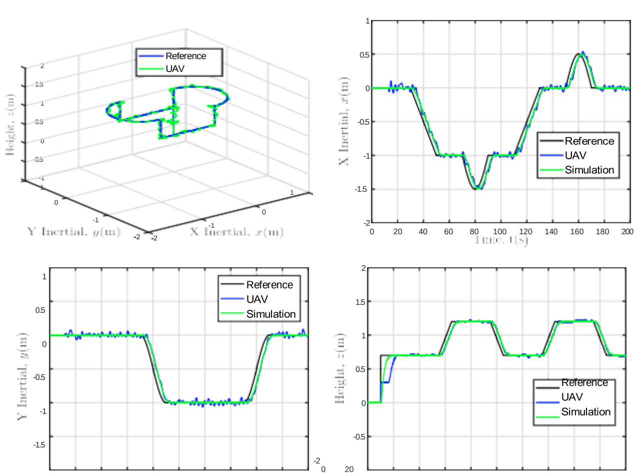
Inner loop (attitude):

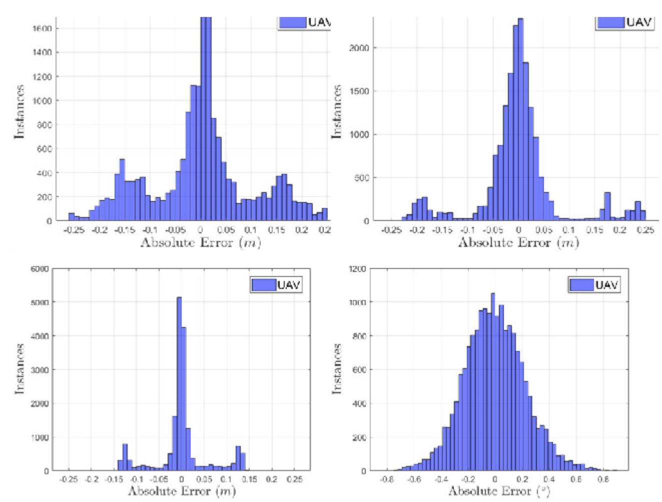
$$\tau_{\varphi}\theta\psi = K_{att}(\eta_{ref} - \eta) + K\_i_{att} \int (\eta_{ref} - \eta) dt$$

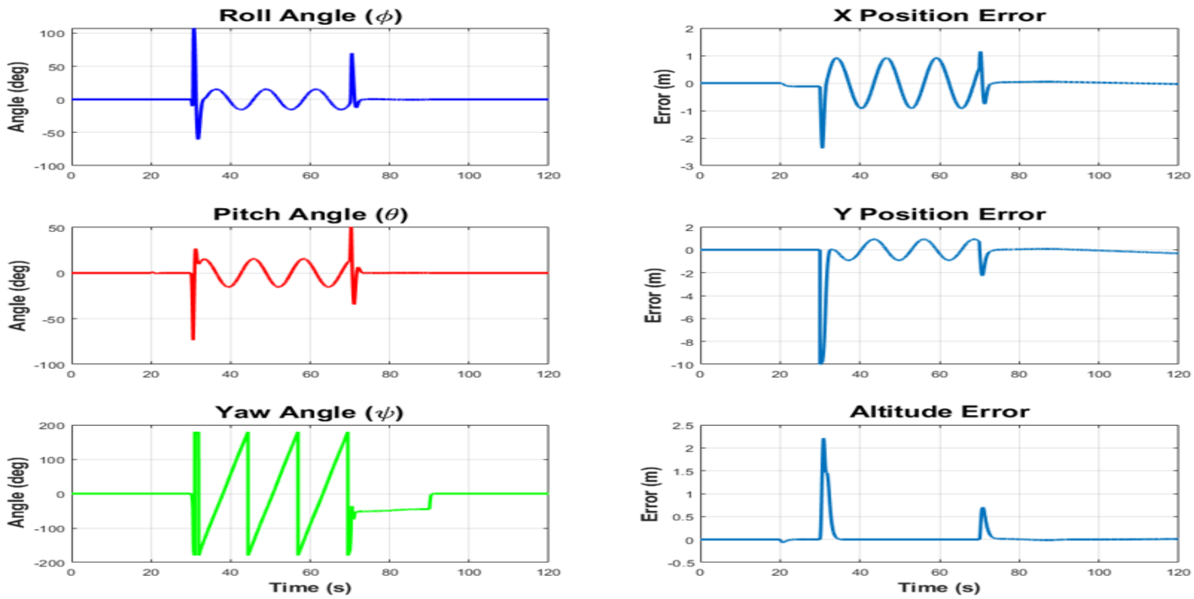
This methodology provides systematic approach from first principles to implementable control laws, with stability guarantees from LQR theory and disturbance rejection through integral action

V. Results

The dynamic model of the UAV considered proved to be accurate and sufficient, even though higher order effects were neglected. The inner-outer loop control structure, constituted by linear quadratic controllers whose design relying on the linearization of the referred model, provided good results in trajectory tracking, presenting robustness to perturbations not only in simulation but also in the real system:







VI. Conclusion

As per the early simulations the proposed controller has successfully track the orientation while performing smooth transitions to and fro. The performance of the proposed controller has been found effective while performing positional and trajectory tracking based on the simulation results.

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