PARAMETER ESTIMATION OF NONLINEAR SYSTEMS

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ABSTRACT

In the current state of knowledge, the parameter estimation of linear dynamical systems with noisy measurements is comparatively easy because of the availability of precise estimation techniques. But the parameter estimation for nonlinear systems is challenging because of the non-availability of optimal performance and convergence of the nonlinear filter techniques. Consequently, a provable optimal closed form or iterative parameter estimation solution applicable to a wide class of nonlinear systems is not available and recourse is taken to approximate solutions. One of the earliest parameter estimation techniques is named the extended Kalman filter (EKF).

Keywords: Parameter estimation, BLDC motor, Gauss-Newton method, LM method

Nomenclature:

V	:	Voltage
R	:	Resistance
E	:	Back EMF
K_e	:	Back EMF constant
T	:	Torque
ω_m	:	Angular Velocity

1. Introduction

The parameter estimation is a research area of system identification, a scientific discipline used to obtain a description for a system. The system identification is mainly concerned with the determination of mathematical model structures of a dynamic system, which is unknown and not unique. The parameter estimation problem can be defined quite simply in general terms. The system under investigation is assumed to be modeled by a set of dynamic equations containing unknown parameters. To determine the values of the unknown parameters, the system is excited by a suitable input, and the input and actual system response are measured. The values of the unknown parameters are then inferred based on the requirement that the model response to the given input matches the actual system response. When formulated in this manner, the unknown parameters can be identified easily by many methods [7].

Parameter estimation is very useful in system modeling as well as in conventional, adaptive, and optimal control problems. Parameter estimation is used to determine the system parameters of a yet to be-modeled system and also the systems where the parameter changes with time or with different environmental conditions.

In the current state of knowledge, the parameter estimation of linear dynamical systems with noisy measurements is comparatively easy because of the availability of precise estimation techniques. But the parameter estimation for nonlinear systems is challenging because of the non-availability of optimal performance and convergence of the nonlinear filter techniques. Consequently, a provable optimal closed form or iterative parameter estimation solution applicable to a wide class of nonlinear systems is not available and recourse is taken to approximate solutions. One of the earliest parameter estimation techniques is named the extended Kalman filter (EKF).

Till now, it has not been reported that any nonlinear estimation technique can guarantee the optimality and iterative convergence of the estimation. So, the scope of relative improvement of estimation accuracy of existing estimation techniques or the scope for developing an alternative algorithm for nonlinear signal models still remains open. Knowledge in the domain of nonlinear estimation is still evolving and new algorithms continue to be contributed in the field of nonlinear parameter estimation.

To apply a nonlinear estimator to nonlinear systems, BLDC Motor model is selected. Brushless direct current (BLDC) motors are mostly preferred for dynamic applications in aerospace, automotive, pumping, and many other industries. In the future, BLDC motors will become mainstream power transmission in industries replacing traditional induction motors. Though the BLDC motors are gaining interest in industrial and commercial applications, the future of the BLDC motors faces indispensable concerns and open research challenges.

Recent advancements in aviation systems are paving the way for the deployment of Power Optimized Aircraft (POA) via Fly-by-Wire (FBW) and Powered-by-Wire (PBW) technologies. The goal is to replace existing actuators that are powered by the central hydraulic system with intelligent electromechanical (EMA) and electro-hydraulic (EHA) actuators [6]. The EMA uses mechanical gearing and ball screws to amplify the motor torque and provide linear actuation. This approach may make it difficult to meet the safety requirements of a primary flight control surface with multiple actuators to ensure continued operation in the event of a unit failure. When an EMA fails due to mechanical gearing, the result can be a jammed actuator, which may also prevent the backup units from operating [5]. The EHA is a local, electrically powered hydraulic actuation system that performs similar functions to a traditional hydraulic system. When multiple EHA actuators are used in parallel on a flight control surface, a simple hydraulic bypass valve can be used to accommodate the failure of one unit. This allows the failed unit' s hydraulic ram to move freely while the flight control surface moves under the control of the remaining healthy units [5]. A fundamental element that drives the EHA/EMA actuator is the DC motor. Nowadays, DC motors are being replaced BLDC motors [5].

The control moment gyro (CMG) is considered the primary actuator of the attitude control system of large spacecraft such as space stations and satellites. In comparison to traditional CMG supported by mechanical bearings, magnetically suspended CMG (MSCMG) has advantages such as low vibration, high precision, adjustable dumping, longevity, and so on [4]. Due to their unique design and absence of a commutator, high-speed brushless direct current (BLDC) motors have the advantages of high mechanical reliability, high power density and efficiency, greater longevity, and compact construction [4]. As a result, applications in space, where the operational environment is unique and harsh, are ideal for these motor

2. Methodology

2.1 BLDC Motor Model

The BLDC motors, which are called Permanent Magnet Direct Current (DC) Synchronous motors or electronically commuted motors, gaining popularity rapidly, mainly because of their better characteristics and performance than the other electrical motors [15]. In BLDC motor, to rotate the motor, commutation is established by energizing the stator windings. The commutation is performed electronically depending on the rotor position because there are no brushes on the rotor. The stator phase windings are inserted in the slots or wounded as one coil on the magnetic pole. There is no physical contact between the stator and the rotor. The polarity reversal is performed by semiconductors switches, which are to be switched in synchronization with the rotor position [15]. Hall effect sensors, which are systematically located on the rotor, are a vital part of the BLDC motor. The BLDC motor model is constructed with a permanent magnet rotor and three-phase star or delta-connected stator, and a drive unit [15]. **Error! Reference source not found.** shows the simple equivalent circuit of the BLDC motor.

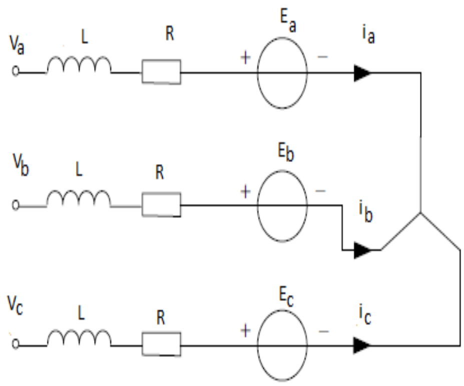

Figure 1:Equivalent circuit of BLDC motor

The drive unit regulates the switching of the DC supply voltage to the stator windings. The switching action of the drive unit is such that each armature phase conducts with either positive or negative polarity of 120 electrical degrees in a sequential manner to rotate the rotor of the motor [15]. This sequential operation is performed electronically at a certain rotor position. To energize stator windings in the next stage, the rotor position (pole signs of the windings for the previous condition) must be known. In every sequence, two phases are normally connected to the DC supply voltage and the remaining phase is disconnected.

The stator winding is wound as a coil on the magnetic pole. The magnetization of the permanent magnets and their displacement on the rotor are chosen in such a way that back EMF is trapezoidal [15]. In this respect, the conductors remain stationary, while permanent magnets rotate. Hall effect sensor sensed the rotor position. From the equivalent circuit of **Error! Reference source not found.**, equations of three phase BLDC motor are obtained [15]:

$$V_a = Ri_a + L\frac{di_a}{dt} + E_a \quad (1)$$

$$V_b = Ri_b + L\frac{di_b}{dt} + E_b \quad (1)$$

$$V_c = Ri_c + L\frac{di_c}{dt} + E_c \quad (2)$$

where, V_a, V_b and V_c are the stator voltages in Volts per phase; i_a, i_b, and i_c are the stator currents in Ampere per phase; L is the motor self-inductances in Henry in each phase; R is the stator resistance in each phase in ohms. E_a, E_b, and E_c are the respective back emf of the motor which are expressed as [12, 10]:

$$E_a = K_e\omega_m f_a(\theta) \quad (3)$$

$$E_b = K_e\omega_m f_b(\theta) \quad (7)$$

$$E_c = K_e\omega_m f_c(\theta) \quad (8)$$

where K_e is the back EMF constant in $(volt/rad.s^{-1})$; ω_m is the angular speed of the motor in rad/s; $f_a(\theta), f_b(\theta)$, and $f_c(\theta)$ are respective back EMF functions and tabulated in **Error! Reference source not found.**.

Table 1:Functions of $f_a(\theta), f_b(\theta)$ *and* $f_c(\theta)$

θ	$f_a(\theta)$	$f_b(\theta)$	$f_c(\theta)$
$0 \leq \theta < \frac{\pi}{6}$	$\frac{6}{\pi}\theta$	-1	1
$\frac{\pi}{6} \leq \theta < \frac{\pi}{2}$	1	-1	$-\frac{6}{\pi}\theta + 2$
$\frac{\pi}{2} \leq \theta < \frac{5\pi}{6}$	1	$\frac{6}{\pi}\theta - 4$	-1
$\frac{5\pi}{6} \leq \theta < \frac{7\pi}{6}$	$-\frac{6}{\pi}\theta + 6$	1	-1
$\frac{7\pi}{6} \leq \theta < \frac{9\pi}{6}$	-1	1	$\frac{6}{\pi}\theta - 8$
$\frac{9\pi}{6} \leq \theta < \frac{11\pi}{6}$	-1	$-\frac{6}{\pi}\theta + 10$	1
$\frac{11\pi}{6} \leq \theta < 2\pi$	$\frac{6}{\pi}\theta - 12$	-1	1

3. Result and Discussion

The parameter of the BLDC motor is estimated using Gauss-Newton and Levenberg-Marquardt methods. The parameters estimated are resistance (R), inductance (L), back-EMF constant (Ke), and moment of inertia (J). The step size α used in these iterative methods changes in every iteration. The initial value of α is chosen as 0.01 and it is reduced by 0.005 after each iteration. The initial values of the parameters are $R = 1.4$, $L = 6.15 \times 10^{-3}$, $K_e = 0.105$, and $J = 7.1 \times 10^{-3}$ for both estimation methods.

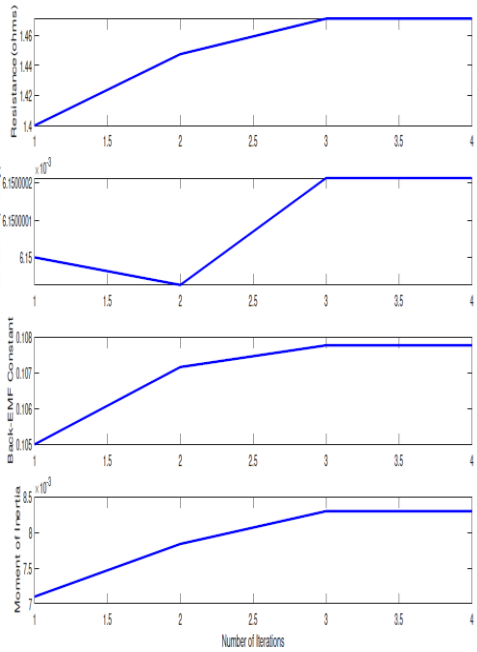

Figure 2: Parameter convergence graph

4. Conclusion

In this study of the parameter estimation of the BLDC motor, the Continuous BLDC motor model is discretized by using Euler forward discretization methods for simulation to generate the inputoutput data for the parameter estimation. All the state variables are taken as the outputs of the system, which are three phases current, angular speed, and angular position of the rotor and the three phases of voltage are taken as input to the system. The parameters resistance (R), inductance (L), back-EMF constant (K_e), and moment of inertia (J) are estimated using Gauss-Newton methods and Levenberg-Marquardt methods. It is evident from the estimated result that Gauss-Newton and Levenberg-Marquardt methods have a very good agreement with each other. These two estimation methods for the BLDC motor system are too much sensitive to the initial values of the parameters, with little changes in an initial value of a parameter change the estimated parameter value drastically, and it will diverge.

A comparison of the dynamic responses of the bldc motor, the angular speed, and the angular position is done between the estimated parameters and actual parameters. It is observed that the angular speed of the bldc motor with estimated parameters has good agreement with the angular speed of the bldc motor with actual parameters. In the case of angular position, till 1 second, they have a very good agreement but after 1 second as time goes, the bldc motor with actual parameters does an extra revolution compared to the bldc motor with estimated parameters in a certain interval of time.

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