Prediction of Store Trajectory Using Domain-Decomposed Reduced-Order Modelling

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I. Introduction

A meticulous analysis is crucial to certify the safe-separation flight envelope of aircraft store (missile, payload etc.) duo, whenever an existing one undergoes any design modification or a new one is inducted. The trajectory followed by the store after its release from the aircraft is computed to predict the safe flight envelope of aircraft. There are mainly three conventional approaches to predict the store trajectory: flight testing, wind tunnel experiment, and computational fluid dynamics (CFD) investigation. All of these pose some constraints related to safety, time consumption, and economic viability. In the present work, we perform a store trajectory estimation based on empirical reduced-order modelling (ROM) integrated with domain-decomposition that we proposed recently [1, 2]. It is an efficient, albeit approximate, alternative to the steady RANS-based approach described above. The proposed ROM uses proper orthogonal decomposition (POD) to identify the underlying reduced-order topology of the flow which necessitates that all the empirical flow data supplied are for the same geometry (and mesh). the position of the store changes continuously throughout its trajectory. Consequently, the usual POD-ROM approach for single-body aerodynamics cannot be employed for the full-flow domain here. Considering this limitation, our proposed approach decomposes the full-domain into multiple subdomains such that POD-ROM can be employed for the maximal portion of the flow domain, and the remaining subdomain that involves changing geometry/mesh is solved using the full-order model (FOM). The overall approach is termed domain-decomposed ROM (DDROM). Our previous work [1–3] demonstrated the feasibility of the approach. Here, we extend the work for predicting the full trajectory of the store on more complicated and realistic three-dimensional problem.

II. Theory and approach

A. POD & Reduced-order model

Let us denote the flow vector field by $\boldsymbol{q}(\boldsymbol{x};\boldsymbol{\mu})$, where $\boldsymbol{x} := (x, y)$ is the 2D Cartesian coordinate and $\boldsymbol{\mu}$ is the parameter vector. In POD, we assume that the flow vector field can be approximated as $\boldsymbol{q}(\boldsymbol{x};\boldsymbol{\mu}) \approx \overline{\boldsymbol{q}}(\boldsymbol{x}) + \sum_{n=1}^{N_p} \eta^n(\boldsymbol{\mu})\widetilde{\boldsymbol{q}}^n(\boldsymbol{x})$. where, $\overline{\boldsymbol{q}}(\boldsymbol{x})$ is the mean flow vector field and the remaining 'fluctuations' are approximated as linear combinations of spatial basis functions $\{\widetilde{\boldsymbol{q}}^n(\boldsymbol{x})\}_{n=1}^{N_p}$ called POD modes, weighted by POD coefficients $\{\eta^n(\boldsymbol{\mu})\}_{n=1}^{N_p}$ whose determination follows the established 'snapshot' POD approach [4–6].

The ROM predicts the flow field for a new parameter vector $\boldsymbol{\mu}_0$ by invoking the governing equations. This is a more robust and accurate approach than the more straightforward interpolation in the parameter space. The POD-based ROM technique employed here was originally developed for steady single-body aerodynamics [7–11]. The particular details relevant to this work can be found in Ref. [6].

B. Domain decomposition

As discussed in section I, we can not implement POD-ROM for the full domain due to the continuous change in the geometry (mesh). To avoid this problem, We adopt the following domain decomposition strategy to deploy a POD-based reduced-order model (ROM) to solve the flow problem over the largest possible portion of the flow domain i.e., *Aircraftbox*, Ω_A and *Capsule*, Ω_C , along with a full-order model (FOM) solution restricted to the remaining small subdomain i.e., *Dropbox*, Ω_D [1, 2, 12], As shown in fig. 1a.

The capsule and aircraftbox are subdomains where the geometry (and even the mesh) can remain same across all possible configurations of the store. Thus, we will pursue POD-based ROM in these two subdomains. On the other hand, the dropbox is the only subdomain whose geometry (and hence the mesh) must change across various possible positions of the store. Hence, POD-ROM is inapplicable here, and we have to revert to FOM calculations instead. A more detailed discussion will be included in final paper.

III. Results

A. Geometry and mesh

The geometries of the aircraft and the store shown in fig. 1b are borrowed from a standard test case carried out by [13]. All geometry parameters, including those of the pylon, can be found in Heim1991.

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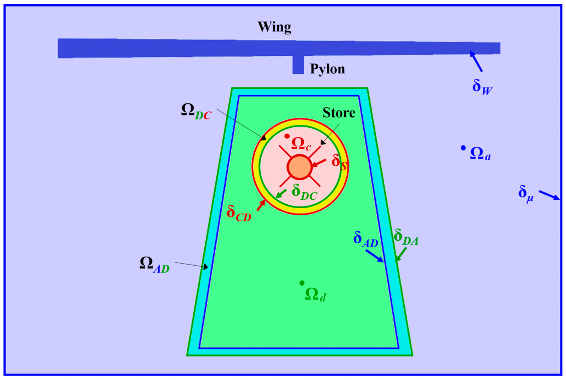
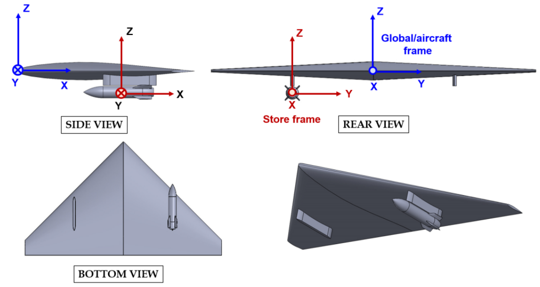


(a) Three domain-decomposed ROM approach for the two-body problem.



(b) Model and coordinate setup for the 3D store separation problem. The store frame is not used in this work.

The mesh is generated using the commercial software Pointwise (v18.2). The total cell count of the grid is 3.54 million. To achieve the y^+ constraint of unity at any wall for the $k-\omega$ (SST) turbulence model, prism layers have been generated on the store with the first cell layer height of 1 micron, and stretching factor of 1.2. We do not aim to capture the boundary layer on the wing, because we need accurate force and moment predictions only on the store surface. Therefore, a few prism layers were generated on the wing with the sole aim of capturing the flow gradient around it. Considering the page limitation final paper will discuss about geometry and mesh in more detail.

B. Performance of DDROM

In this preliminary work, we present the verification of the DDROM technique to predict force and moment coeffs. on the store for a new parameter set. Performance is evaluated in the sets of parameters of the 'test' and the results are presented in table 1. All coefficients are multiplied by 100 to obtain numbers that are easier to present in the table. The 'truth' values of the coefficients come from a FOM trajectory calculation for the test case. The absolute errors of the DDROM predictions are not insignificant as a fraction of the corresponding truth values; indeed, there is 180% relative error in roll moment prediction for the case of $M = 0.6$ and $\alpha = 7.5°$. However, this is somewhat misleading as the truth value of the same happens to be very close to zero. A more representative metric is the maximum absolute error as a percentage of the range of the respective coefficient observed in the learning database, as presented in the final row. Even in this perspective, the DDROM performance may only be deemed as adequate, but it leaves much ground for improvement. The prediction of roll is always difficult for such slender bodies, and we too encounter this challenge here.

Table 1 Performance of DDROM in terms of absolute prediction error of force and moment coefficients at the initial points of the 4 test trajectories. The last row gives the maximum error encountered in tests as a percentage of the respective truth ranges.

M	α [deg]	Quantity	C_x	C_y	C_z	$C_{m,x}$	$C_{m,y}$	$C_{m,z}$
0.6	2.5	Error	1.02	1.92	0.36	0.05	9.23	2.43
0.6	7.5	Error	0.56	3.84	4.78	0.18	0.24	1.89
0.7	2.5	Error	1.59	4.97	0.79	0.38	1.57	0.49
0.7	7.5	Error	2.34	5.88	0.30	0.08	7.71	2.20
Max error as % of range of truth values			7.66	6.89	7.28	35.23	9.03	6.50

IV. Conclusions

We extend our previous work on efficient albeit approximate computation of store-separation trajectories from aircraft. The adopted strategy has at heart an empirical reduced-order model (ROM) based on proper orthogonal decomposition (POD) that is applied to the majority of the flow domain. To overcome the issue of the changing grid and geometry inherent in the store-separation problem, a domain decomposition strategy has been adopted wherein the remainder of the domain is solved with the full-order model, and the subdomains' solutions are iteratively matched at the interfaces. Encouraging preliminary results are presented in table 1 but approach needs to be improved substantially for predicting full trajectory. We foresee that these will be overcome in final paper, thus delivering a viable efficient alternative to CFD-intensive store separation trajectory calculation.

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