A Lyapunov Guidance Vector Field with Variable Guidance Gain for Elliptical Path Following

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1. INTRODUCTION & OBJECTIVE

There is a strong need for efficient real-time trajectory tracking algorithms in Unmanned Aerial Vehicles (UAVs) that can execute critical maneuvers and follow preplanned trajectories while adhering to the operational constraints of the vehicle. Several proposed algorithms for generating continuous curvature paths based on the Virtual Target Point (VTP) method include the carrot-chasing algorithm, pure pursuit, Line-of-Sight (LOS), the Nonlinear Guidance Law, and the Trajectory Shaping guidance law. Vector field (VF) guidance has emerged as a widely used path-following technique for autonomous systems. Nelson et al. [1] proposed a VF guidance law accounting for constant wind disturbances, which Zhou et al. [2] extended, combining adaptive control to mitigate the impact of varying wind speeds. A classical framework of VF is the Lyapunov Guidance Vector Field (LGVF), which incorporates course-rate and curvature constraints by defining converging and circulation terms. In this direction, Liang et al. [3] constructed a combined VF for arbitrary two-dimensional (2D) and three-dimensional (3D) differentiable paths using Helmholtz's theorem. Harinarayana et al. [4] incorporated curvature constraints to generate a near time-optimal and smooth path. To track the trajectories generated by such guidance laws, a variety of controllers, such as PID Control, Sliding Mode Control (SMC), Linear Quadratic Regulators (LQR), Model Predictive Control (MPC), etc, are used.

This work addresses the elliptical path following problem for UAVs in a two-dimensional (2D) environment through a LGVF Guidance Law, in which the convergence and circulation terms are constructed using a cross-track error-dependent guidance gain. The Lyapunov stability of the guidance law is established. A second-order course-hold autopilot is used for trajectory tracking. Convergence to the desired ellipse is shown for various sets of gain values.

2. METHODS OF ANALYSIS

An LGVF is proposed with convergence and circulation terms using normalised odd and even functions derived from the decomposition of an exponential function. These terms can also be written in terms of hyperbolic tangent and secant functions, respectively, given as,

$$f_1 = V_a \tanh\big(cg(x,y)\big)$$
$$f_2 = V_a \operatorname{sech}(cg(x,y))$$

where V_a is the airspeed, *g(x, y)* is the cross-track error, and *c* is the guidance gain proposed as,

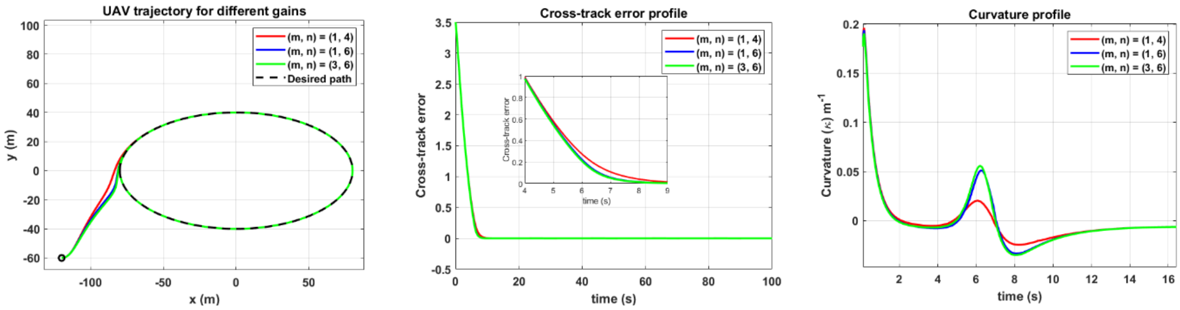
$$c = \frac{m(g(x,y))^2 + n}{(g(x,y))^2 + 1}$$

where *m* and *n* are positive constants.

Thus, the guidance gain *c*, used in the LGVF, is a function of the instantaneous cross-track error *g(x, y)*. It is observed that *c* converges to *m* at large distances $(g(x,y) \rightarrow \infty)$, and to *n* as the UAV is near the desired ellipse $(g(x,y) \rightarrow 0)$. Thus, it is possible to have better control over the convergence characteristics by suitably tuning the gains *(m, n)*. The Lyapunov stability of the proposed guidance law is examined using a Lyapunov candidate function. A second-order course-hold autopilot tracks the desired course angle generated by the guidance law. The changes in UAV trajectories by changing *m* and *n*, once at a time, are analysed. The time of asymptotic convergence of the cross-track error is compared across various gain settings. The variation of convergence and circulation terms is studied in the transition region (where the dominance of convergence and circulation terms switches), as well as in the area close to the desired ellipse.

3. RESULTS AND/OR HIGHLIGHTS OF IMPORTANT POINTS

The numerical simulations are conducted for a standard elliptical trajectory with $a = 80\ m$ and $b = 40\ m$, given the initial conditions as: position $(x_o, y_o) = (-120, -60)\ m$, course angle $(\chi_{in}) = 0\ rad$. A constant airspeed of $10\ m/s$ is considered. Three different sets of gain values are chosen as $(m, n) = (1, 4), (1, 6)$, and $(3, 6)$. The plots for UAV trajectory, cross-track error, and curvature profile are shown in Fig. (1). The response indicates that increasing the gain values increases the curvature, decreases the convergence time, as well as the path length.



a) UAV trajectory b) Cross-track error c) Curvature

Fig. (1) Numerical simulation results

4. CONCLUSIONS

We propose a varying gain LGVF guidance law to follow an elliptical path in a 2D environment. The UAV successfully converges to the desired elliptical path with different gain values, with some tradeoffs in curvature and convergence speed. This varying gain LGVF guidance law helps to have more controlled convergence characteristics than an LGVF guidance law with a constant guidance gain. Additions to this research include parameter tuning using optimisation of the curvature, and extending the algorithm to three-dimensional (3D) space.

5. REFERENCES

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