Mahiem Agrowal
Computer Science '22.
Sheet #4.

Problem 4.1

~~a)~~
a) For a relation to be partial order($\preceq$)
we need to prove that is. ~~reflive~~ reflexive, antisymmetric and transitive.

Reflexive
If p is just a subset of ~~the~~ ~~word~~ ω then
$p \subseteq \beta$

$p \preceq p \Rightarrow \exists \omega \; \forall \; p = p$. for all values.
Example.
ω = computer.
p = computer also

$\therefore$ $p = \omega$
or $p = p$ $\therefore$ proved,, reflexive

set ~~can~~ also be included which in our case is q so,

Anti-Symmetric.

$(p, w) \in \Sigma^*$, $\exists$ $pq = w \wedge w = pq$.

This can be proven easily as we have said p is a subset of w where $p = w$ and there exists an empty set which in our case is q so

$\therefore p = w$ and $w = p$.

Example.

$w =$ "Name" $p =$ "Name" and $q =$ "_"

$\therefore pq = w$ and $w = pq$

or $p = w$ or $w = p$. $\therefore$ proved.

or Name = Name   Name = Name.

Transitive.

$p \preceq q \wedge q \preceq r \in \Sigma^*$, $\exists$ $p \preceq r$ so

$\exists$ Example,

p = Comp q = Computer r = ComputerSystem.

Comp is a prefix of Computer
or p is prefix of q.

Again

"Computor" is prefix of Computor Network.

$\therefore$ q is a prefix of r.

$\therefore$ p is also a prefix of r.

$\therefore$ proved Transitive

$\therefore$ ~~It~~.

$\therefore$ It is a partial order

b) For this question we say p is a proper prefix of ω such that $p \neq \omega$.

$p \preceq p \Rightarrow E\omega \wedge p \neq p$ for all values

Example

$p =$ Comput

$\omega =$ computer

$p \neq \omega$

or $p \neq p$

$\therefore$ irreflixive. proved.

c) we know $(p, \omega) \in \preceq r$ as well as $(p, \omega) \prec r$ so any combination of p and ω will either be strict partial order or partial order

$(p, \omega) \in \prec r \vee (p, \omega) \in \preceq$ $\therefore$ It is total.

A-symmetric

$(p, w) \in \Sigma^*$; $\nexists$ ~~$p \neq w$~~ $pq \neq w \wedge w \neq pq$

we have defined P as a proper subset so p cannot be equal to w ~~and~~ even when we include empty set to q.

Example.

$w =$ Hello

$p =$ Hell. and $q = \emptyset$

$pq \neq w$

or Hell $\neq$ Hello

$\therefore$ It is ~~anti~~ A-Symmetric.

Transitive.

$p \preceq q \wedge q \preceq r \in E^*$, $\exists\, p \preceq r$ so.

$p =$ "N" and $q =$ "Na" and $r =$ "Name".

"N" is a subset of "Na" so

$p \subset q$

"Na" is a subset of "Name"

$q \subset R$

$\therefore$ $p \subset R$ and $p \preceq r$ $\therefore$ prove a.

Therefore as it is A-symmetric, ~~irreflive~~ irreflexive and Transitive it is strict partial order.

Problem 4.2

$f: A \to B \wedge g: B \to C$,

a) $g \circ f$ is bijective then f should be injective and g should be surjective.

let $a \in A$, $b \in B$ and $c \in C$

~~There exists $g(f(a)) = c$~~

There exists $a \in A$ with $g(f(a)) = c$

Then if we take $b \in f(a) \in B$, $g(b) = c$

As ~~B~~ $c \in C$ is an image of g. c must be mapped by atleast one of the element of B

g is surjective

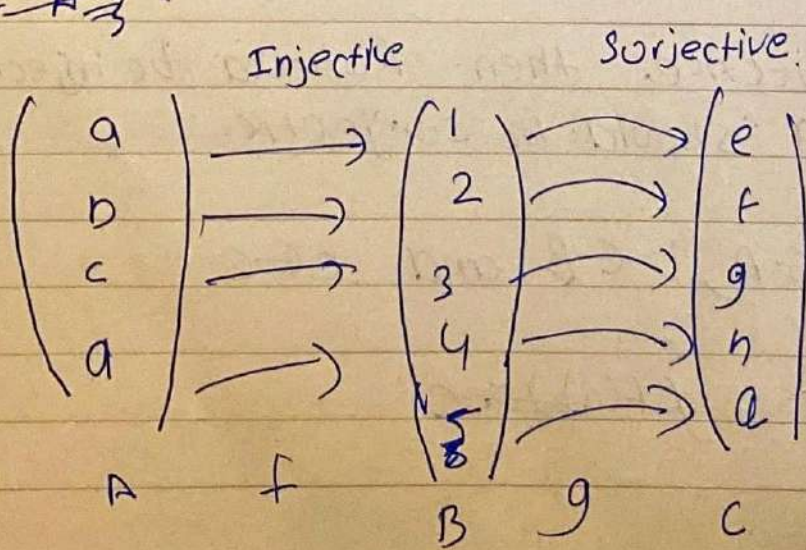
Consider

$\cancel{g}$ ~~f(A)~~ $f(a) = A$
$f(b) = B$. and as we know

~~gf~~ $g \circ f$ is bijective all domain of a should match to A. for it to be bijective.

$\therefore$ All domain of a match to A therefore it is ~~bijective~~ injective

b) ~~f: A{~~ .



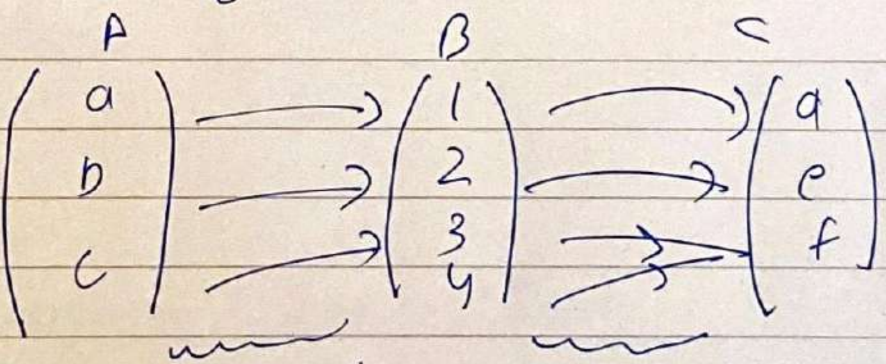
Here.

~~⟺~~ $f: A \rightarrow B$ is injective as every element of x a valle of y comes.

~~f~~ ~~g~~ $f: B \rightarrow C$ is surjective as every element in codomain ~~C~~ is mapped to one element of B.

However it is not bijective as every codomain Y is not matched by one domain X. look at the alphabet a

c)



not soijetive

not injective

r

$F: A \rightarrow B$ is not sorjective as evely codomain B is not mapped by one element in A

$f: B \rightarrow C$ is not injective as every element ~~X do~~ B does not have a differenty.

However $f: A \rightarrow C$ is bijective as every element of A is matched exactly by one element in C.