Mahiem Agrawal
Sheet 7

7.1)

A	B	$\neg A$	$\neg B$	$(\neg A \to B)$	$(A \to \neg B)$	$\neg(A \to \neg B)$
0	0	1	0	0	1	0
0	1	1	1	1	1	0
1	0	0	0	1	1	0
1	1	0	1	1	0	1

Now another Table for booleon expressions.

A	B	$A \wedge B$	$A \vee B$	$A \leftrightarrow B$	$A \dot{\vee} B$
0	0	0	0	1	0
0	1	0	1	0	1
1	0	0	1	0	1
1	1	1	1	1	0

Therefore from this we see,
$A \wedge B$ has some expression as $\neg(A \to \neg B)$
therefore AND proved.

$A \vee B$ has some expression as $(\neg A \to B)$
therefore OR proved.

$A \leftrightarrow B$ we can say $(\neg A \to B) \leftrightarrow (\neg(A \to \neg B))$
has same ~~expression~~ values so equivelance
can also be proven.

Therefore after proving with AND OR operators we can say all boolean expressions can be expressed using just not and implication.

7.2)

a)

P	Q	R	S	φ
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	~~0~~ 1

Therefore condition is satisfied in only 2 ~~con~~ interpretation.

b) $(\neg P \land \neg Q \land \neg R \neg S) \lor (P \land Q \land R \land S)$

P.T.O

c) $(\neg P \vee Q) \wedge (\neg Q \vee R) \wedge (\neg R \vee S) \wedge (\neg S \vee P)$

Just doing $(\neg P \vee Q) \wedge (\neg Q \vee R)$

$(\neg P \wedge \neg Q) \vee (\neg P \wedge R) \vee (Q \wedge \neg Q) \vee (Q \wedge R)$

$\Rightarrow (\neg P \wedge \neg Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

Now by doing $\wedge (\neg R \vee S)$ as well

$(\neg P \wedge \neg Q) \vee (\neg P \wedge R) \vee (Q \wedge R) \wedge (\neg R \vee S)$

$= (\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \overbrace{R \wedge \neg R}^{\to 0}) \vee (Q \wedge \overbrace{R \wedge \neg R}^{\to 0})$

$(\neg P \wedge \neg Q \wedge S) \vee (\neg P \wedge R \wedge S) \vee (Q \wedge R \wedge S)$

$= (\neg P \wedge \neg Q \neg R) \vee (\neg P \wedge \neg Q \wedge S)$ ~~$R \wedge \neg R$~~

$\vee (\neg P \wedge R \wedge S) \vee (Q \wedge R \wedge S)$ as $(R \wedge \neg R) = 0$

Now by doing $\wedge (\neg S \vee P)$ as well.

$= (\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge S) \vee (\neg P \wedge R \wedge S)$

$\vee (Q \wedge R \wedge S) \wedge (\neg S \vee P)$

$=$ ~~$(\neg P \wedge \neg Q \neg R \neg S)$~~

Turns 0

$= [(\neg P \wedge \neg Q \wedge \neg R \wedge \neg S) \vee (\neg P \wedge \neg Q \wedge S \wedge \neg S) \vee$

$(\neg P \wedge R \cancel{\neg S} \neg S) \vee (Q \wedge R \wedge S \wedge \neg S)]$ ~~$\wedge P$~~

$\cancel{(\neg P \wedge \neg Q \wedge \neg R \wedge P) \vee (\neg P \wedge \neg Q \wedge S \wedge P)}$

$\to 0 \qquad \to 0$

$(\neg P \wedge \neg Q \wedge \neg R \wedge P) \vee (\neg P \wedge \cancel{\neg} Q \wedge S \wedge P] \vee$
$(\neg P \wedge R \wedge S \wedge P) \vee (Q \wedge R \wedge S \wedge P)$

Therefore just two remain.

$(\neg P \wedge \cancel{\neg Q \wedge \neg R \wedge \neg}$

$(\neg P \wedge \neg Q \wedge \neg R \wedge \neg S) \vee (P \wedge Q \wedge R \wedge S)$ ///