Mohiem Agrawal
ICS Sheet #9

9.1)

| A | B | $C_{in}$ | S | $C_{out}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

~~a)~~ ~~Sum of~~ DNF

$\cancel{S \Rightarrow (\bar{A} \wedge \bar{B} \wedge C_{in}) \vee (\bar{A} \wedge B}$

$S =$

a) $(\neg A \wedge \neg B \wedge C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in})$
$\vee (A \wedge \neg B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$

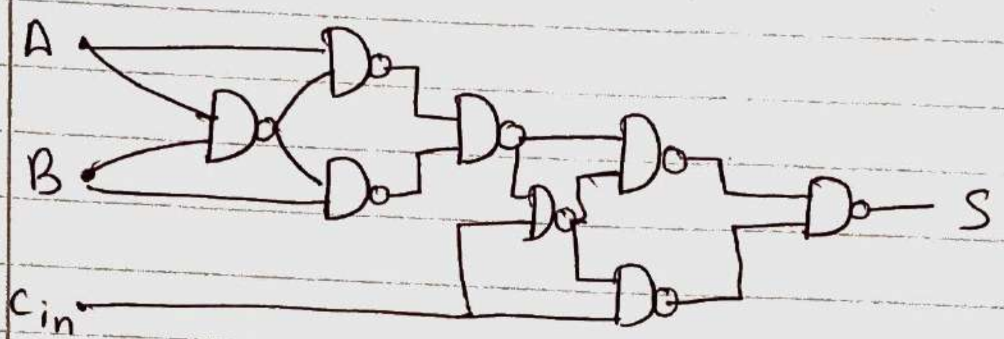
$C_{out} =$
$(\neg A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge C_{in}) \vee$
$(A \wedge B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$

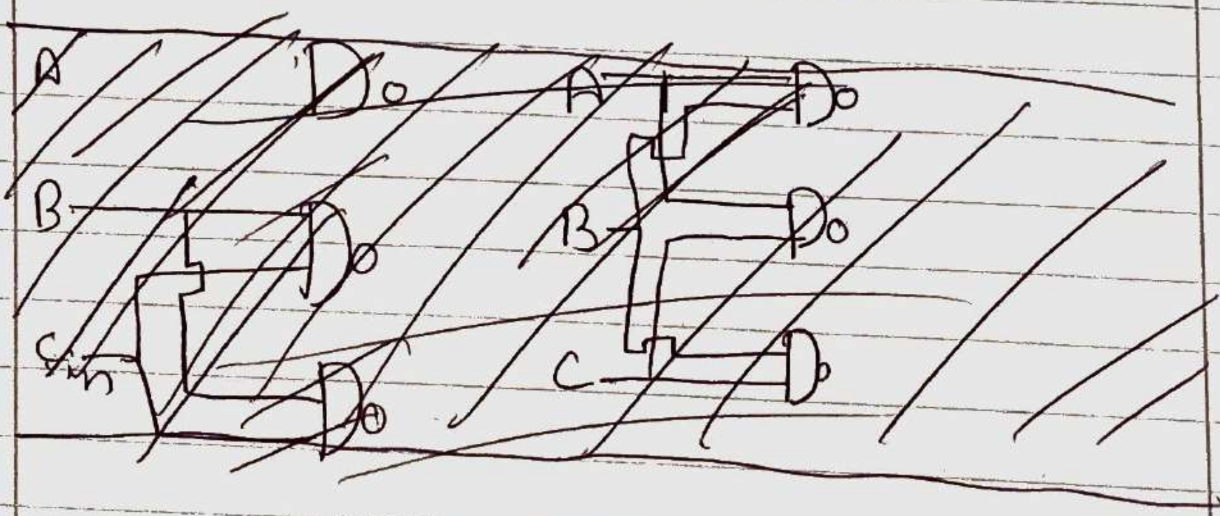
b) CNF

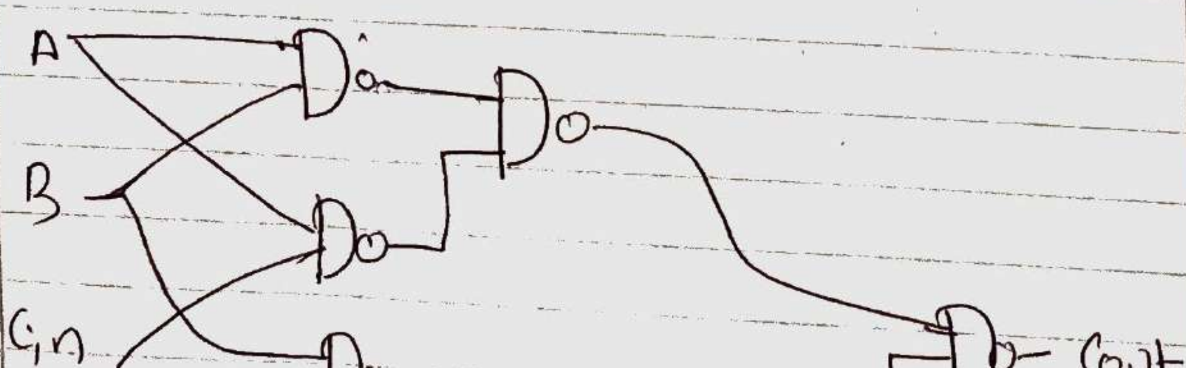
$S = (A \vee B \vee C_{in}) \wedge (A \vee \neg B \vee \neg C_{in}) \wedge$
$(\neg A \vee B \vee \neg C_{in}) \wedge (\neg A \vee \neg B \vee C_{in})$

$$C_{out} = (A \vee B \vee C_{in}) \wedge (A \vee B \vee \neg C_{in}) \wedge (A \vee \neg B \vee C_{in}) \wedge (\neg A \vee B \vee C_{in})$$

d)



same

c)

| $A$ | $B$ | $A \dot{\vee} B$ | $\bar{A}$ | $\bar{B}$ | $A \uparrow B$ | $\bar{A} \uparrow \bar{B}$ | $\overline{A \uparrow B \uparrow (\bar{A} \uparrow \bar{B})}$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | $\cancel{1}$ 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 | $\cancel{0}$ 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | $\cancel{0}$ 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | $\cancel{1}$ 0 |

As we can see
$A \dot{\vee} B$ can also be expressed as
$\overline{(A \uparrow B) \uparrow (\bar{A} \uparrow \bar{B})}$

$S = A \dot{\vee} B \dot{\vee} C_{in}$

$= \overline{(A \uparrow B) \uparrow (\bar{A} \uparrow \bar{B})} \dot{\vee} C_{in}$

$=($

$$= \overline{\left(\left(\overline{(A \uparrow B) \uparrow (\bar{A} \uparrow \bar{B})}\right) \uparrow C_{in}\right) \uparrow \left(\left(\overline{(A \uparrow B) \uparrow (\bar{A} \uparrow \bar{B})}\right) \uparrow \bar{C}_{in}\right)}$$

$$= \overline{\left(\left(\overline{(A \uparrow B) \uparrow (\bar{A} \uparrow \bar{B})}\right) \uparrow C_{in}\right) \uparrow \left(\left((A \uparrow B) \uparrow (\bar{A} \uparrow \bar{B})\right) \uparrow \bar{C}_{in}\right)}$$

//

Now $A \vee B$ is represented by $\bar{A} \uparrow \bar{B}$ and
and $A \wedge B$ is represented by $\overline{A \uparrow B}$ so.

~~$C = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$~~

$$C = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$

$$= (A \wedge B) \vee (C_{in} \wedge (\overline{(A \uparrow B) \uparrow (\bar{A} \uparrow \bar{B})}))$$

$$= (A \wedge B) \vee (\overline{C_{in} \uparrow ((A \uparrow B) \uparrow (\bar{A} \uparrow \bar{B}))})$$

$$= (\overline{A \uparrow B}) \vee (\overline{C_{in} \uparrow (\overline{(A \uparrow B) \uparrow (\bar{A} \uparrow \bar{B})})})$$

$$= (\overline{\overline{A \uparrow B}}) \uparrow (\overline{\overline{C_{in} \uparrow (\overline{(A \uparrow B) \uparrow (\bar{A} \uparrow \bar{B})})}})$$

$$= (A \uparrow B) \uparrow (C_{in} \uparrow (\overline{(A \uparrow B) \uparrow (\bar{A} \uparrow \bar{B})}))$$

Problem 9.2

a) To prove

foldr op e xs = foldl op e xs.

<u>Base case when xs has 1 element,</u>

foldr op e [x] = x op e
foldl op e [x] = e op x.

∴ As e is a neutral element this is true.

## Induction step,

foldr op e $[x_1 \ldots x_n]$ = foldl op e $[x_1 \ldots x_n]$

For n+1 element then,

foldr op e $[x_1, \ldots x_n, x_{n+1}]$

$= x_1 \text{ op } (x_2 \text{ op } \ldots (x_n \text{ op } (x_{n+1} \text{ op } e)))$

$= x_1 \text{ op } (x_2 \text{ op } \ldots (x_n \text{ op } (x_{n+1})))$ → As e is neutral,

$= (x_1 \text{ op } (x_2 \text{ op } \ldots (x_{n-1} \text{ op } (x_n)))) \text{ op } x_{n+1}$

$= \underbrace{(x_1 \text{ op } (x_2 \text{ op } (x_{n-1} \text{ op } (x_n \text{ op } e))))}$ op $x_{n+1}$

↓ This is equal to foldl op e $[x_1, \ldots x_n]$ (Induction hypothesis)

= foldl op e $[x_1, \ldots x_n]$ op $x_{n+1}$

= foldl op e $[x_1, \ldots x_n, x_{n+1}]$ // proved //

b) To prove.

foldr ~~op1~~ e xs = foldl op2 e xs.

Base case when xs has 1 element.

foldr op1 e [x] = x $op_1$ e
foldl op2 e [x] = e $op_2$ x.

As given x $op_1$ e = e $op_2$ x it is proved for base case.

Induction step.

Hypothesis for n

foldr $op_1$ e $[x_1, \ldots x_n]$ = foldl op2 e $[x_1 \ldots x_n]$

For n+1 element,

( foldr $op_1$ e $[x_1, \ldots x_n, x_{n+1}]$

$= x \; op_1 (x_2 \; op_1 \ldots (x_n \; op_1 \; \underbrace{(x_{n+1} \; op_1 \; e)}_{= e \; op2 \; x_{n+1}}))$

$= x \; op_1 (x_2 \; op_1 \ldots (x_n \; op_1 (e \; op_2 \; x_{n+1})$

$= \underbrace{x \; op_1 (x_2 \; op_1 (x_n \; op_1 \; e))}_{\text{Induction hypothesis,}} \; op_2 \; x_{n+1}$

$= \text{foldl op2 } e\ [x \ldots x_n] \text{ op2 } x_{n+1}$

$= \text{foldl op2 } e\ [x \ldots x_n, x_{n+1}]$ ///

proved.

c) To prove,

$\text{foldr op } a\ xs = \text{foldl op' } a\ (\text{reverse } xs)$

Base condition when xs has 1 element:

$\text{foldr op } a\ [x] = x \text{ op } a$

$\text{foldl op' } a\ [x] = a \text{ op' } x.$

$\therefore$ As $x \text{ op } a = a \text{ op' } x$ given from question it is proved.

Induction step.

$\text{foldr op } a\ \underbrace{\cancel{xs}}^{[x_1, \ldots x_n]} = \text{foldl op' } a\ \underbrace{(\cancel{\text{reverse } xs})}^{[x_n \ldots x_1]}$

For n+1 element.

$\text{foldr op } a\ [x_1, \ldots x_n, x_{n+1}]$

$$= (x_1 \ op \ (x_2 \ op \ (x_n \ (x_{n+1} \ op \ a))))$$

$$= x_1 \ op \ (x_2 \ op \ (x_n \ op \ a) \ \underbrace{op \ x_{n+1}}_{\text{associativity}}$$

$$= \underbrace{foldl \ op' a \ [x_n \cdots x_1]}_{} \ op \ x_{n+1}$$

Induction hypothesis

$$= \underbrace{x_{n+1} \ op'}_{} (foldl \ op' \ a \ [x_n \ldots x_1])$$

given in questio

$$= foldl \ op' a \ [x_{n+1}, x_n, \ldots x_1]$$ // proved.