

Mahiem Agrawal

Problem 3.1

a)
$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{9n}{5n^3} = 0$$

$$\lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = \lim_{n \rightarrow \infty} \frac{5n^3}{9n} = \infty$$

Therefore it belongs to.

$$f \in o(g)$$

$$f \in O(g)$$

$$g \in \Omega(f)$$

$$g \in \omega(f)$$

b)
$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{9n^{0.8} + 2n^{0.3} + 14 \log n}{\sqrt{n}} = \infty$$

$$\lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{9n^{0.8} + 2n^{0.3} + 14 \log n} = 0.$$

Therefore it belongs to the notations.

$$f \in \Omega(g)$$

$$f \in \omega(g)$$

$$g \in O(f)$$

$$g \in o(f)$$

$$c) \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \frac{n^2}{\frac{\log n}{n \log n}} = \frac{n^3 \log n}{\log n} = \infty$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \frac{n \log n}{\frac{n^2}{\log n}} = \frac{\log n}{n^3 \log n} = 0$$

$$f \in \Omega(g)$$

$$f \in \omega(g)$$

$$g \in O(f)$$

$$g \in o(f).$$

$$a) \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \frac{(\log(3n))^3}{9 \log n} = \infty$$

$$\lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = \frac{9 \log n}{(\log(3n))^3} = 0.$$

$$f \in \Omega(g)$$

$$f \in \omega(g)$$

$$g \in O(f)$$

$$g \in o(f).$$

Problem 3.2

a) The selection sort has been implemented inside the file "SelectionSort.cpp".

b) To prove that the selection sort is correct we need to show the loop invariant passes the 3 conditions.

Initialization: Here we need to show loop invariant holds prior to the first iteration. So initially the sorted array has no element in it so therefore we can say it is sorted at that time.

Maintenance: Here we need to show that each iteration of the loop maintains the invariant. Here the left part of the sub array till the n element $A[0..n]$ is always sorted and the next element $A[n+1]$ gets added to the left sub-array so it maintains the loop invariant.

Termination: The loop invariant should ~~show~~ provide useful properties to show correctness when the loop terminates. At the end of the termination, the left sub array will reach the

Size of the loop and there will be no more elements to be sorted.

c) The random input sequence for Case A and B have been shown in the code with necessary conditions being shown.

d) In I & II have increased the value of n from 0 to 10,000 and found out the average case, best case, worst case for each condition. The values have been stored in "Input.txt" and later a GNU plot has been made with the help of them.

e) If we take a look at the code regardless of the best case or worst case the number of comparisons is always.

$$\sum_{i=0}^n (n-1) = \frac{n(n+1)}{2}$$

This shows the ~~time~~ complexity to be $\Theta(n^2)$ and the difference being generated in the graph is because of the other swap that need to take place.