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- Scales

① Nominal - Cannot be ordered. Example Male/Female
② Ordinal - Can be ordered but difference between there values cannot be interpreted in a meaningful way.
③ Continuous -
ⓐ Interval - Only differences, but not ratios [°C]
ⓑ Ratio - difference and ratio [Km/h]
ⓒ Absolute - same as ratio scale but values are measured in natural units. [1, 2, 3 ....]

- Probability.

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

B is occuring, A already occured.

$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}$

Bayes ⇒ $P(A|B) = \frac{P(B|A)\, P(A)}{P(B)}$ ⟵ $\therefore P(A \cap B) = P(A) \times P(B|A)$

$P(A \cap B) = P(A|B)\, P(B) = P(B|A)\, P(A)$

① 2 DVD stores
B: DVD without problems
A_1: Rented from store 1 A_2: Rented from store 2
$P(A_1) = 0.6$ $P(A_2) = 0.4$
$P(B|A_1) = 0.95$ $P(B|A_2) = 0.75$

→ i) $P(B) = ?$

$P(B|A_1) = \frac{P(B \cap A_1)}{P(A_1)}$ $P(B|A_2) = \frac{P(A_2 \cap B)}{P(A_2)}$

$P(A_1 \cap B) = P(A_1)\, P(B|A_1)$ $P(A_2 \cap B) = P(A_2)\, P(B|A_2)$

$P(B) = P(A_1)\, P(B|A_1) + P(A_2)\, P(B|A_2)$ ⟸
$= 0.6 \times 0.95 + 0.4 \times 0.75$
$= \underline{\underline{0.87}}$

b) From store S1 and working $P(A_1 \cap B)$

$\rightarrow \cancel{P(A_2|B) = \frac{P(A_2 \cap B)}{P(B)}}$ $\quad \cancel{P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)}}$

$\cancel{P(A_2 \cap B) = P(B) \times P(A_2|B)}$

$$P(B|A_1) = \frac{P(A_1 \cap B)}{P(A_1)}$$

$\therefore P(A_1 \cap B) = P(A_1) \times P(B|A_1) = 0.6 \times 0.95 = \underline{0.57}$

c) DVD working. What's prob that its from store 1?

$\rightarrow P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)}$ $\qquad P(B|A_1) = \frac{P(A_1 \cap B)}{P(A_1)}$

$$= \frac{P(A_1) P(B|A_1)}{P(B)}$$

$$= \frac{0.6 \times 0.95}{0.87} = \frac{0.57}{0.87} = \underline{0.65517}$$

d) DVD not works. $P(\bar{B}|A_1) = 0.05$
$P(\bar{B}|A_2) = 0.25$

Prob of DVD from store 1, given that not working

$\rightarrow P(A_1|\bar{B}) = \frac{P(A_1 \cap \bar{B})}{P(\bar{B})}$ $\quad \Big| \quad P(\bar{B}|A_1) = \frac{P(A_1 \cap \bar{B})}{P(A_1)}$

$$= \frac{P(A_1) P(\bar{B}|A_1)}{P(\bar{B})}$$

$$= \frac{P(A_1) P(\bar{B}|A_1)}{P(\bar{B}|A_1) P(A_1) + P(\bar{B}|A_2) P(A_2)}$$

$$= \frac{0.6 \times 0.05}{0.05 \times 0.6 + 0.25 \times 0.4}$$

$= \underline{\underline{0.2308}}$

- Independence
$P(A \cap B) = P(A) . P(B)$

(2) P(Neighbour takes care) $= P(A_2) = 2/3$ $\quad$ | P(B) = Dies
P(— " — not takes care) $= P(A_1) = 1/3$
P(Basil dies if care taken) $= P(B|A_2) = 1/2$
P(Basil dies if not taken) $= P(B|A_1) = 3/4$

a) P(Basil surviving)

$\rightarrow P(\bar{B}) = P(\bar{B}|A_1) P(A_1) + P(\bar{B}|A_2) P(A_2)$ or $\underline{\underline{1 - P(B)}}$

$P(B) = P(B|A_1) P(A_1) + P(B|A_2) P(A_2)$

$$= \frac{3}{4} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3} = \frac{1}{4} + \frac{1}{3} = \underline{\underline{\frac{7}{12}}}$$

$$P(\bar{B}) = 1 - P(B) = 1 - \frac{7}{12} = \underline{\frac{5}{12}}$$

b) P(Plant died, care not taken) $\Rightarrow P(A_1|B)$

$$P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{P(B|A_1) P(A_1)}{P(B)}$$

$$= \left(\frac{3}{4}\right)\left(\frac{1}{3}\right) \Big/ \left(\frac{7}{12}\right) = \underline{\underline{\frac{3}{7}}}$$

③ $P(A)$ = Fail in Practical Exam $= 0.25$
$P(B) =$ — " — Theory $= 0.15$
$P(A \cap B) = 0.1$
$P(A) + P(B) - P(A \cap B) = P(A \cup B)$
$P(A \cup B) = 0.25 + 0.15 - 0.1 = \underline{0.3}$

a) Atleast fail 1 examination
b) only fails Prac, not theory
→ $P(\text{Fails only Prac}) = 0.25 - 1 = P(A) - P(A \cap B) = \underline{0.15}$
c) successful passes both tests?
→ $P(\ ) = 1 - P(A \cup B) = 1 - 0.3 = \underline{0.7}$
d) Fails any two exams
→

④ 12 sided die
a) even no? → 0.5
b) no. greater than 9 → $3/12 = 0.25$
c) even greater than 9 → $2/12 = 0.166$
d) even no. or greater than 9 → $7/12$

⑤ Family of 6. 12 gifts and 2 for each member. Draw 2.
a) get his/her gift   Total $= {}^{12}C_2$
→ ~~all~~

⑥ $P(\text{salty}) = 0.2$ $P(A) = 0.2$ $P(B) =$ In love $= 0.3$
$P(A|B) = 0.6$
$P(\bar{A}) = 0.8$ $P(\bar{B}) = 0.7$

$P(A \cap B) = P(A|B) / P(B) = 0.6 \times 0.3 = 0.18$
~~$P(\ ) = P(\ )/P(\ )$~~

	B	$\bar{B}$	Total
A	0.18	0.02	0.2
$\bar{A}$	0.12	0.68	0.8
	0.3	0.7	1

$P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.2 - 0.18 = 0.02$
$P(\bar{A} \cap B) = P(B) - P(A \cap B) = 0.3 - 0.18 = 0.12$
$P(\bar{A} \cap \bar{B}) = P(\bar{A}) - P(\bar{A} \cap B) = 0.8 - 0.12 = 0.68$

⑦ 5% not able to pay
20% paid too late.
→ A → Bill never Paid $P(A) = 0.05$ $P(\bar{A}) = 0.95$
B → Paid late $P(B) = \cancel{0.20}?$ $B = M$
$P(B|\bar{A}) = 0.2$ ~~$P(A|B)$~~ $P(B|A) = 1$
a) P(not paying in a particular month) =
$P(M) = P(M|A)\,P(A) + P(M|\bar{A})\,P(\bar{A})$
$= 1 \times 0.05 + 0.2 \times 0.95$
$= 0.05 + 0.19 = 0.24$
b) Not paid bill in past month. P(never pays back) = ?
$P(A|B) = \dfrac{P(A)\,P(B|A)}{P(B)} = \dfrac{0.05 \times 1.0}{0.24} = \underline{0.208}$

c)
→ If bill not paid, prob. = 20.8% it will never be paid.
79.2% will pay even if late.
Not to block.

⑧ $P(A)=0.4$ $P(B)=0.5$

a) Atleast one succeeds.

→ $P(A\cap B)=P(A).P(B)$ ---- Independent

$=0.4\times0.5=\underline{0.20}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0.4+0.5-0.2=\underline{0.7}$

b) Exactly one hits.

$P(\text{Exactly one hits})=0.4-0.2+0.5-0.2$

$=\underline{0.5}$

c) Only B scores.

→ $P(\quad)=P(B)-0.2=0.5-0.2=\underline{0.3}$

⑨ $P(E\cup F)=0.7$ $P(E)=0.5$ $P(F)=0.3$ $P(E\cap F^c)=?$

→ $P(E\cup F)=P(E)+P(F)-P(E\cap B)$ ⇒ $P(E\cap B)=\underline{0.1}$

$P(E\cap F^c)=P(E)-P(E\cap F)=0.5-0.1=\underline{0.4}$

⑩ avg. of 6 = 23.5 ; 3 major avg = 42

difference in age of minor is same.

Mean of ages of minor?

→ $(a+b+c)/3=42$ ∴ $a+b+c=126$

Avg of 6 = 23.5 ∴ Total age = $6\times23.5=141$.

Ages sum of minor = $141-126=15$

Mean = $15/3=\underline{5}$

⑪ $P(BI)=6/10$ $P(WI)=4/10$ $P(BII)=3/7$ $P(WII)=4/7$

→ $P(I|B)=?$ $P(B)=(6/10)(3/7)=9/35$; $P(I)=0.5$

$P(B|II)=3/7$ $P(B|I)=3/5$ | $P(A|B)=[P(A\cap B)]/P(B)$

$$P(I|B)=\frac{P(I|B)\,P(I)}{P(I)P(B|B)+P(II)P(B|II)}=\frac{7}{12}$$

~~RANDOM VARIABLE~~

⑫ 1.8% infected $P(A)=0.018$ $P(\bar{A})=0.982$

A → Infected. B → detected

→ Given → $P(B|A)=0.95$, $P(B|\bar{A})=0.03$.

$P(A|B)=?$

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(B|A)P(A)}{P(B)}$$

$$=\frac{P(B|A)\,P(A)}{P(A)P(B|A)+P(\bar{A})P(B|\bar{A})}$$

$$=\frac{0.95\times0.018}{0.018\times0.95+0.982\times0.03}$$

$=\underline{0.3672}\approx37\%$

⑬ $P=\{a|a \text{ odd} <7\}$ $P=\{1,5,3\}$

$Q=\{0,1,2,3,4\}$

$P\cup B=\{0,1,2,3,4,5\}$

$n(P\cup B)=6$

No of proper sets formed by union $=2^6-1=63$

RANDOM VARIABLE

- Let Ω represent the sample space of random experiment and let R be the set of real nos. A random variable is a function of X which assigns to each element $\omega \in \Omega$ one and only one no. $X(\omega) = x, x \in R$ i.e.

$$x : \Omega \to R$$

- Cummulative Distribution Function [CDF]
CDF of random variable X is defined as

$$F(x) = P(X \le x)$$

- CDF for continuous

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

- $f(x)$ = Probability density function.

$$\frac{d}{dx} F(x) = f(x)$$

- For a func. $f(x)$ to be a probability density function of X, it needs to satisfy foll. conditions
① $f(x) \geq 0$ for all $x \in R$
② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

- If $x_1 < x_2$, both are known constants.

$$P(x_1 \le X \le x_2) = F(x_2) - F(x_1) = \int_{x_1}^{x_2} f(x)\,dx$$

$P(X = x_0) = 0$.

① Interval [1, 10]

$$f(x) = \begin{cases} k & 1 \le x \le 10 \\ 0 & \text{otherwise.} \end{cases}$$

$$\to f(x) = \begin{cases} 1/9 & 1 \le x \le 10 \\ 0 & \text{otherwi} \end{cases}$$

$$1 = \int_{1}^{10} f(x)\,dx = \int_{1}^{10} k\,dx = 10k - 1k = 9k$$

$\therefore k = 1/9$

$$P(X \le 4) = \int_{1}^{4} f(x)\,dx = \int_{1}^{4} \frac{1}{9}\,dx$$

$$= \frac{1x}{9}\Big|_{1}^{4} = \frac{4}{9} - \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$$

$$P(2 < X \le 7) = \int_{2}^{7} f(x)\,dx = \int_{2}^{7} \frac{1}{9}\,dx = \frac{7}{9} - \frac{2}{9} = \frac{5}{9}$$

$P(X=5) = 0$

k if $P(k \le X < 9) = 0.5$

$P(9) - P(k) = 0.5$

$\frac{9-1}{9} - 0.5 = P(k)$

$\frac{8}{9} - \frac{1}{2} = \frac{k-1}{9}$

$\therefore k = 4.5$

$\to P(k \le X < 9) = 0.5$

$$0.5 = \int_{k}^{9} \frac{1}{9}\,dx$$

$$= \frac{1}{9}x\Big|_{k}^{9} = \frac{9}{9} - \frac{k}{9}$$

$1 - \frac{k}{9} = 0.5 \quad \therefore 0.5 = \frac{k}{9}$

$\therefore k = 4.5$

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- Expectation. $\Rightarrow$ Mean $= \mu$

① $E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$

$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$ — $f(x) = \begin{cases} 1/20 & \text{for } 0 \le x \le 20 \\ 0 & \text{otherwise} \end{cases}$

$= \int_{-\infty}^{0} x f(x)\, dx + \int_{0}^{20} x f(x)\, dx + \int_{20}^{\infty} x f(x)\, dx$

$= 0 + \int_{0}^{20} \frac{1}{20} x\, dx + 0$

$= \frac{1}{40} x^2 \Big|_0^{20} = \frac{1}{40}(20^2) = \frac{400}{40} = \underline{\underline{10}}$

② $x_i,\ i = 1, 2, \ldots 6$ are $P(X = x_i) = 1/6$

$E(X) = \sum_{i=1}^{6} x_i p_i = (1+2+3+4+5+6)\frac{1}{6} = \frac{21}{6} = \underline{\underline{3.5}}$

- Varience $\Rightarrow$ Standard Deviation.

$Var(X) = E[X - E(X)]^2$

$Var(X)$ for continuous:-

$Var(X) = \int_{-\infty}^{\infty} [X - E(X)]^2 f(x)\, dx$

$Var(X)$ for discrete

$Var(X) = \sum_{i=1} (x_i - E(X))^2 p_i$

- Exception is linear
Variance is not linear

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- Calculation rules of $E(X)$ and $Var(X)$
let $a, b \in \mathbb{R}$ and X, Y be r.v.

① $E(a) = a$

$\to$ Discrete

$E(X) = \sum_{i=1}^{k} x_i f(x_i) \Rightarrow E(a) = \sum_{i=1}^{k} x_i f(x_i)$

$\Rightarrow E(a) = \sum a f(a) = a \sum f(a)$

$= a$ — $f(a) = 1$, sum of probabilites is 1 for discrete

Continuous

$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_{-\infty}^{\infty} a f(x)\, dx$

$\underline{\underline{= a}}$ --- $\int_{-\infty}^{\infty} f(x)\, dx = \underline{\underline{1}}$

② $E(aX) = aE(X)$

$\to$ Discrete.

$E(X) = \sum_{i=1}^{k} x_i f(x_i) \Rightarrow E(aX) = \sum x_i a f(x_i) = a \sum x_i f(x_i)$

$\underline{\underline{= a E(X)}}$

Continuous

$E(X) = \int_{-\infty}^{\infty} x_i f(x)\, dx \Rightarrow E(aX) = \int_{-\infty}^{\infty} a x f(x)\, dx$

$E(aX) = a \int_{-\infty}^{\infty} x f(x)\, dx = aE(X).$

③ $E(a + bX) = a + bE(X)$

$\to$ Discrete

$E(X) = \sum_{i=1}^{k} x_i f(x_i) dx \Rightarrow E(a + bX) = E(a) + E(bX)$

$E(a) = \sum a f(a) = a \times 1 = \underline{a}$ — ①

$E(bX) = \sum b \sum x_i f(x_i) = b \sum x_i f(x_i) = \underline{bE}(X).$ — ②

$\therefore E(a+bX) = a + bE(X)$

Continuous

$E(a) = \int_{-\infty}^{\infty} a f(x)\, dx = a \int_{-\infty}^{\infty} f(x) = a.$

$E(bX) = \int_{-\infty}^{\infty} b x f(x)\, dx = b \int_{-\infty}^{\infty} x f(x)\, dx = bE(X)$

$\Big\} \to E(a+bX) = a + bE(X).$

μ = Mean
σ = variance

$\frac{x-\mu}{\sigma}$

• Exponential.

$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

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$F(x) = P(X \leq x) = 1 - e^{-\lambda x}$

$P(X > x) = e^{-\lambda x}$

• Discrete Distribution.

• Bernoulli Distribution.

$E(X) = p \qquad Var(X) = pq \qquad q = 1-p$

$f(x_i) = P(X = x_i) = \begin{cases} p & \text{if } X = 1 \\ 1-p & \text{if } X = 0 \end{cases}$

• Binomial Distribution.

$\boxed{{}^nC_r \, p^r \, q^{n-r}}$

$E(X) = np \qquad Var(X) = npq$

• Poisson

$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} \qquad E(X) = Var(X) = \lambda$

• $\mu = np \qquad \sigma = \sqrt{npq}$

① Patient recovers prob = 0.4 $\quad n = 15 \quad q = 0.6$

P(atleast 2 patient will survive) $= P(X \geq 2)$

$= 1 - P(X < 2)$

$= 1 - P(X \leq 1)$

$= 1 - 0.0052$

$= \underline{0.9948}$

$P(3 \leq X \leq 9) = P(X \leq 9) - P(X \leq 2)$

~~$= 0.9662 - 0.0905 \quad 0.8757$~~

$= 0.9662 - 0.0271 = \underline{0.9391}$

$P(X = 5) = {}^{15}C_5 (0.4)^5 (0.6)^{15-5}$

$= 3003 \times 0.01024 \times 6.04 \times 10^{-3}$

$= \underline{0.1859}$

TUTORIAL-2

① Random variable is a function of X which assigns to each element $\omega \in \Omega$ one and only one number
$X(\omega) = x \quad x \in R$

Discrete r.v. → When range of r.v is finite & countable
Continuous r.v. → Infinite or uncountable

② i) $f(x) = \frac{2(1+x)}{27}$ $\quad \int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow PDF$

$F(x) = \int_{-\infty}^{x} P(x)\,dx$

∴ Not PDF as interval is not given

ii) $f(x) = \begin{cases} x & , 0 < x < 1 \\ 2-x & , 1 \leq x < 2 \\ 0 & , \text{elsewhere} \end{cases}$

→ $\int_{-\infty}^{\infty} P(x)\,dx = 1$ — property

$\int_0^1 x\,dx + \int_1^2 (2-x)\,dx = \frac{x^2}{2}\Big|_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$

$= \frac{1}{2} + \left[\left(4 - \frac{4}{2}\right) - \left(2 - \frac{1}{2}\right)\right] = \frac{1}{2} + 4 - \frac{4}{2} - 2 + \frac{1}{2}$

$= \frac{1}{2} + 2 - 2 + \frac{1}{2} = 1$

$\int_{-\infty}^{\infty} P(x)\,dx = 1$

iii)

x	0	1	2	3	4	5	6
f(x)	0.41	0.35	0.15	0.10	0.04	0.01	0

→ $\sum_{x=1}^{n} P(x_i) = 1$

$f(0) + f(1) + \cdots + f(6)$
$= 0.41 + 0.35 + \cdots 0 > 1$
∴ Not PDF

iv)

x	1	2	3	4	5	6
f(x)	0.35	0.33	0.18	0.10	0.03	0.01

→ $\sum_{i=1}^{x} f(x_i) = 1$

$f(0) + \cdots + f(6) = 1.$ $\quad$ ∴ PDF

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(5)

$$F(x) = \begin{cases} 0 & \text{if } x<2 \\ \frac{-x^2}{4}+2x-3 & \text{if } -2 \le x \le 4 \\ 1 & \text{if } x>4 \end{cases}$$

(5) i) PDF of x

$\frac{d}{dx} F(x) = f(x)$

$\frac{d}{dx}\left(\frac{-x^2}{4}+2x-3\right) = \frac{-2x}{4}+2 = \frac{-x}{2}+2$

$f(x) = \left(\frac{-4}{2}+2\right) - \left(\frac{+2}{2}+2\right) = -2+2-1+2 = 1$

ii) $P(X<3)$ and $P(X=4)$

$\rightarrow P(X<3) = \frac{-x^2}{4}+2x-3 = \frac{-3^2}{4}+2\times3-3 = 0.75$

$P(X=4) = 0$

iii) $E(X)$, $Var(X)$

$\rightarrow E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{\infty} x\left[\frac{-x}{2}+2\right] dx$

$= \int_{-\infty}^{\infty} \left[2x - \frac{x^2}{2}\right] dx = \left[\frac{2x^2}{2} - \frac{x^3}{6}\right]_{-2}^{4}$

$= \left[\frac{2\times16}{2} - \frac{64}{6}\right] - \left[4+\frac{8}{6}\right]$

$E(X) = 3 \Rightarrow \mu$

$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

$= \int_{-2}^{4} [x-3]^2 \left[2-\frac{x}{2}\right]$

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(6)

$$F(x) = \begin{cases} 0 & \text{if } x<0 \\ 3x^2-2x^3 & 0 \le x \le 1 \\ 1 & \text{if } x>1 \end{cases}$$

(6) $\rightarrow f(x) = \frac{d}{dx}[3x^2-2x^3] = 6x-6x^2$

$$f(x) = \begin{cases} 0 & x<0 \\ 6x-6x^2 & 0 \le x \le 1 \\ 1 & x>1 \end{cases}$$

Prob. of selling atleast 1/3 of wine but not more than 2/3

$P\left(\frac{1}{3} \le X \le \frac{2}{3}\right) = F\left(\frac{2}{3}\right) - F\left(\frac{1}{3}\right)$

$= \left[3\left(\frac{2}{3}\right)^2 - 2\left(\frac{2}{3}\right)^3\right] - \left[3\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right)^3\right]$

$= 1/9$

$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^1 x[6x-6x^2] = \frac{6}{12}$

$\therefore \mu = 1/2$

$Var(X) = \int_0^1 (x-\mu)^2 f(x)\,dx = \int_0^1 [x-(1/2)][6x-6x^2]\,dx$

(7) (9)

x	1	2	3	4	5	6
$f(x)$	0.35	0.33	0.18	0.10	0.03	0.01

10m fabric

imperfections in a 100m

$\rightarrow E(X) = 1\times0.35 + 2\times0.33 + \cdots + 6\times0.01$

$\mu = 2.16$

for 100m roll $= 10E(X) = 21.6$

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(8) $E(2X-1)=9$ and $E(2X-1)^2=89$, $\mu=?$, $\sigma=?$

(12) → $E(2X)-E(1)=9$

$E(2X)=10$ $\quad E(X)=\underline{5} = \mu$

let $2x-1=P$

$E(P)=9$ $\quad E(P)^2=89$

$Var(P) = E(X^2)-E(X)^2$

$=89-81$

$=8$

$V(2X-1)=8$

$V(2X)-V(1)=8$

$V(2X)=8$

$4V(X)=8$ $\quad \therefore Var(X)=\underline{2} = \sigma$

(9) (13) let $X_1, X_2, \cdots X_n$ be n r.v. with $E(X_i)=\mu$

$Var(X_i)=\sigma^2$, $i=1,2,\cdots n$.

Find $E(\bar{X})$ and $Var(\bar{X})$. $\bar{X}$= arithmetic mean

→ $E(\bar{X})=\mu$ $\quad var(\bar{X})=\frac{\sigma^2}{n}$

(10) (14) Coin tossed 10 times. P(Getting more heads than tail)

→ $P(X\geq 6) = 1-P(X\leq 5)$

$=1-0.6230 = \underline{0.377}$

(11) (24)

X	0	1	2	3	4	5	6
P(X=x)	k	3k	5k	7k	9k	11k	13k

i) k

→ $\sum k_i = 1$ $\quad \therefore 49k=1$ $\quad \therefore k=\underline{1/49}$

ii) $p(X\geq 2)= 5k+7k+9k+11k+13k$

$= \underline{45/49}$

iii) $p(0<X<5) = 3k+5k+7k+9k = \frac{24}{49}$

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iv) What is minimum value of c for which $p(X\leq c)>0.5$

→

(12) (25) $P(X=n)=2^{-n}$, $n=1,2,3,\cdots$ $\quad$ Is PDF=?

→ $\sum_{n=1}^{\infty} P(X=n) = \sum_{n=1}^{\infty} 2^{-n}$

This is a geometric series with the first term (a) equal to 2^{-1} and common ratio (r) also equal to 2^{-1}.
The sum of an infinite geometric series is with $|r|<1$ is given by $\frac{a}{1-r}$

$\therefore \sum_{n=1}^{\infty} 2^{-n} = \frac{2^{-1}}{1-2^{-1}} = \frac{1}{2}\left(\frac{1}{(1-\frac{1}{2})}\right) = \frac{1}{2}\left(\frac{1}{1/2}\right) = \underline{1}$ $\quad \therefore$ PDF

(13) (26) Box contains 8 items, 2 are defective. Draw 3 items.

→ $E(X=3) = 0P(X=0)+1\times P(X=1)+2P(X=2)+3P(X=3)$

$P(X=0) = {}^6C_3 / {}^8C_3 = 20/56 = 0.3571$

$P(X=1) = {}^6C_2\, {}^2C_1 / {}^8C_3 = 15\times 2/56 = 0.5371$

$P(X=2) = {}^6C_1\, {}^2C_2 / {}^8C_3 = 6\times 1/56 = 3/28 = 0.1071$

$P(X=3) = 0$

$E(X=3) = 0\times 0.3571 + 1\times 0.5371 + 2\times 0.1071$

$= 0.5371 + 0.2142$

$= \underline{0.7513}$

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(14) $\mu = 2$, $\sigma^2 = 0.5$

(27) → i) $E(2X-1)$

→ $E(2X-1) = E(2X) - E(1) = 2E(X) - 1$

$E(X) = \mu$ ∴ $E(2X-1) = 2 \times 2 - 1 = 3$

ii) $Var(X+2) \Rightarrow \sigma^2$

→ $Var(X+2) = Var(X) + Var(2) = ~~0.5~~ \; (1/2)^2 = (1/4)$

iii) $sd\left(\frac{3X-1}{-4}\right) = Var\left(\frac{1-3X}{4}\right) = Var\left(\frac{1}{4}\right) - Var\left(\frac{3X}{4}\right)$

$= \frac{9}{16} Var(X) = \frac{9}{16} \times \frac{1}{4} = \frac{9}{64} = \frac{3}{8}$

(15) 500 students. $\mu = 151$, $\sigma^2 = 15$

$z = (X-\mu)/\sigma$

(28)

$P(120 \le X \le 155) = P\left(Z \le \frac{155-151}{15}\right) - P\left(Z \le \frac{120-151}{15}\right)$

$= P(Z \le 0.266) - P(-2.066)$

$= 0.6026 - 0.0228$

$= 0.5798$

Students $= 500 \times 0.5798 = 290$

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(16) Fair pair of dice. X = Sum of points.

(29)

	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
E(X)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{6}{36}$								

(17) R.V. X $f(x) = \frac{c}{x^2+1}$, where $-\infty < x < \infty$

(30)

→ a) Value of c:

→ $1 = \int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{+\infty} \frac{c}{x^2+1} = 1.$

$c[\tan^{-1}(x)]_{-\infty}^{\infty} = 1.$

$c\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = 1$ ∴ $c = \frac{1}{\pi}$

b) X^2 lies betn $1/3$ and 1

$\frac{1}{3} \le X^2 \le 1$ ∴ $\frac{1}{\sqrt{3}} \le X \le 1$ or $\frac{-1}{\sqrt{3}} \le X \le -1$

$\int_{1/\sqrt{3}}^{1} f(x)\,dx = \int_{1/\sqrt{3}}^{1} \left(\frac{1/\pi}{x^2+1}\right) dx = \frac{1}{\pi}\left[\tan^{-1}(x)\right]_{1/\sqrt{3}}^{1}$

$= \frac{1}{\pi}[45 - 30) = \frac{1}{\pi} 15 = \frac{15}{\pi} = \frac{1}{12}$

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(18) Expectation of sum of points after tossing a fair dice
(31)
→ $E(X) = \sum_{i=1}^{k} x_i p_i$

$= \frac{1 \times 2}{36} + \frac{3 \times 2}{36} + \frac{4 \times 3}{36} \text{ ---- } + \frac{6 \times 5}{36} + \frac{7 \times 6}{36}$

$+ \frac{8 \times 5}{36} + \frac{7 \times 4}{36} \text{ ---- } + \frac{11 \times 2}{36} + \frac{12 \times 1}{36}$

$= \underline{\underline{7}}$

(19) 40 people visit shop per hour.
(16)
① Prob. that no one visits the store in 3 min break.
→ $\lambda = \frac{40}{60} = \frac{2}{3}$ For 3 min, $\lambda \times 3 = \underline{\underline{2}}$

$1 - (P(X=0) + P(X=1) + P(X=2) + P(X=3))$
$= 1 - P(X \leq 3)$
$= 1 - 0.8571$
$= \underline{\underline{0.1429}}$

② Prob. than 3 people visit during break.

→ $P(t=3, n=0) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-2}(2)^0}{0!} = \underline{\underline{0.1353}}$

Poisson = $\frac{e^{-\lambda}\lambda^x}{}$

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(20) Avg. no. of defects in 5 mtrs. is 12.
Find prob that fewer than 7 defects are found.
(17)
① In a given metre
→ $\lambda = \frac{12}{5} = \underline{\underline{2.4}}$ for 1 metre.

$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$

P(less than 7 defects) $= P(X \leq 7)$
$= \text{ppois}(7, 2.4)$
$\approx \underline{\underline{0.9958}}$

② on 7 of next 10 metre.
→ $P(X=7) = {}^{10}C_7 (0.9958)^7 (4.2 \times 10^{-3})^3$
$= 120 \times 0.97097 \times 7.4088 \times 10^{-8}$
$= 8.63247 \times 10^{-6}$

③ on 60 of next 75 metres.
→ $= {}^{75}C_{60} (0.9958)^{60} (4.2 \times 10^{-3})^{15}$

(21) Prove that binomial distribution tends to Poisson as
(15)
$n \to \infty$ and $p \to 0$
→ Binomial distribution $= {}^nC_r p^r q^{n-r}$
as $n \to \infty$ and $p \to 0$ then mean np remains constant
let $np = \lambda$ as constant.
$\therefore p = \lambda/n$

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$$\lim_{n\to\infty} {}^nC_r \left(\frac{\lambda}{n}\right)^r \left(1-\frac{\lambda}{n}\right)^{n-r} \quad \text{— (1)}$$

as $n \to \infty$

${}^nC_r = 1/r!$ $\qquad \dfrac{1}{(1-\frac{r}{n})!\,r!}$

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n = e^{-\lambda} \quad \text{(Formula)}$$

Continued (1)

$$\lim_{n\to\infty} \frac{1}{r!}\left(\frac{\lambda}{n}\right)^r e^{-\lambda} =$$

$$= \frac{e^{-\lambda}\lambda^r}{r!}$$

(22) mean = 100 $\qquad$ s.d. = 5

(20) (1) $P(95 < x < 110) = \phi\left(\frac{110-100}{5}\right) - \phi\left(\frac{95-100}{5}\right)$

$= \phi(2) - \phi(-1)$

$= 0.9772 - 0.1587$

$= \underline{0.8185}$

(2) $P(X<50) = \phi\left(\frac{50-100}{5}\right) = \phi(-10) = \underline{0}$

(3) k if $P(X>k) = 0.3192$

→ $1 - P(X \le k) = 0.3192$

$P(X \le k) = 1 - 0.3192$

$= 0.6808$

$\therefore k = \underline{0.47}$

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(4) X_1, X_2 if $P(X_1 < X < X_2) = 0.4176$ and $P(Z<z) = 0.7088$

→ $P(X_1 < X < X_2) = P(X < X_2) - P(X_1 < X)$

$= \phi\left(\frac{X_2-\mu}{\sigma}\right) - \phi\left(\frac{X_1-\mu}{\sigma}\right)$

$z = \frac{(X-\mu)}{\sigma}$ $\qquad$ $P(Z<z) = 0.7088$

$\therefore z = \underline{0.55}$

Doubtful

$\therefore 0.55 = \frac{X_2-\mu}{\sigma} = \frac{X_2-100}{5}$ $\quad \therefore X_2 = 102.75$

Same X_1.

(23) Pair of dice rolled 420 times.

(21) Prob that 8 occurs at least 50 times

→ Getting 8 = {(6,2), (2,6), (5,3), (3,5), (4,4)}

$p = 5/36$ $\qquad q = 31/36$

$P(X \ge 50) = 1 - P(X \le 49)$

$= 1 - \sum_{x=0}^{49} b(x, 420, 5/36)$

Between 70-90 times

$P(70 \le X \le 90) = P(X \le 90) - P(X \le 69)$

$= \sum_{x=0}^{90} b(x, 420, 5/36) - \sum_{x=0}^{69} b(x, 420, 5/36)$

(24) $\beta = 2$ avg life $\qquad$ 1000 switches

(22) P(300 getting failed)? $\quad \Big| \; \dfrac{\lambda^x e^{-\lambda}}{x!}$

→ $P(X<1) = 1 - e^{-1/2} = 1 - e^{-1/2} = \underline{0.3935}$

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(25) ~~of the mean of sd of mean of size 36=2~~

(23) If sd. of mean for sampling distribution of a random sample of size 36 is 2. Change in sample size for reducing sd by 0.8. Means s.d = 1.2

→ $\boxed{\sigma_{\bar{x}} = \frac{6}{\sqrt{n_1}}}$ $2 = \frac{6}{\sqrt{36}}$ ∴ $6 = 12$

We want sd = 1.2

∴ $1.2 = \frac{6}{\sqrt{n_2}}$ ⇒ $1.2 = \frac{12}{\sqrt{n_2}}$ ∴ $n_2 = 100$

(26) P(bad reaction) = 0.001

(33) Determine probability that out of 2000 individuals

(a) exactly 3

→ $p = 0.001$ $q = 0.999$

$P(X=3) = {}^{2000}C_3 [0.001]^3 [0.999]^{1997}$

$= 0.18053$

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(b) more than 2

→ $P(X>2) = 1 - P(X \le 2)$

$= 1 - (P(X=0) + P(X=1) + P(X=2))$

$= 1 - \left[{}^{2000}C_0 [0.001]^0 [0.999]^{2000} + {}^{2000}C_1 [0.001]^1 [0.999]^{1999} + {}^{2000}C_2 [0.001]^2 [0.999]^{1998} \right]$

$= 1 - [0.135210 + 0.270670 + 0.2708]$

$= 1 - 0.67667$

$= 0.3233$

(27) Patient recovers a disease = 0.4 prob.

(37) 15 people

(a) at least 10 survive

→ $p = 0.4$ $q = 0.6$

$P(X \geq 10) = 1 - P(X \le 9)$

$= 1 - 0.9662 = 0.0338$

(b) from 3 to 8 survive

→ $P(3 \le X \le 8) = P(X \le 8) - P(X \le 2)$

$= 0.9050 - 0.0271$

$= 0.8779$

(c) $P(5 \le X \le 8) = P(X \le 8) - P(X \le 4)$

$= 0.9050 - 0.2173$

$= 0.6877$

(d) Exactly 5.

$P(X=5) = {}^{15}C_5 (0.4)^5 (0.6)^{10} = 0.1859$

$= P(X \le 5) - P(X \le 4)$

$= 0.1859$

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(28) Avg no. of tanks each day = 10
(38) Can handle maximum of 15.
P(Tanks will have to be sent away on a day) = ?
→ $\lambda = 10$.
$P(X \le 15) = 1 - \cancel{pbinom}\ ppois(15,10)$
$= 1 - 0.9513 = \underline{0.0487}$

(29) Batting avg. = 0.25
(40) P(He hits exactly 1 in next 4 hits) = ?
→ ~~dpois~~
Prob of hitting 1 = $P(x=1) = e^{-\lambda x}\lambda^{x} = e^{-0.25}(0.25)$
$= \underline{0.195}$
$P(X=1) = {}^4C_1 (0.195)^1 (0.805)^3$

(30) 2 error per page on avg. Prob on next page —
(a) 4 or more errors
→ $P(X \ge 4) = 1 - P(X \le 3)$
$= 1 - ppois(3,2) = 1 - 0.8571 = \underline{0.1429}$

(b) no errors
→ $P(X=0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{2^0 e^{-2}}{0!} = \underline{0.1353}$

(31) Standard normal distribution. Find area under the curve
(a) lies to right of z = 1.84
→ $1 - \phi(1.84) = 1 - 0.9671 = \underline{0.0329}$

(b) between z = -1.97 and z = 0.86
→ $\phi(0.86) - \phi(-1.97) = 0.8051 - 0.0244$
$= \underline{0.7807}$

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(32) live on avg. of 40 months sd = 6.3
(44) (a) $P(X>32) = 1 - \phi\left(\frac{32-40}{6.3}\right) = 1 - \phi(-1.26)$
$= 1 - 0.1038$
$= \underline{0.8962}$

(b) less than 28 months
→ $P(X<28) = \phi\left(\frac{28-\cancel{32}40}{6.3}\right) = \phi(-1.90) = \underline{0.0287}$

(c) $P(37 < X < 49) = P(X < 49) - P(X < 37)$
$= \phi\left(\frac{49-40}{6.3}\right) - \phi\left(\frac{37-40}{6.3}\right)$
$= \phi(1.428) - \phi(-0.47)$
$= 0.9222 - 0.3192$
$= \underline{0.603}$

(33) avg. for one way trip = 24 mins s.d. = 3.8 mins
(45) (a) trip takes at least 1/2 hr.
→ $z = \frac{x-\mu}{\sigma} = \frac{30-24}{3.8} = 1.58$.
$P(z \ge 30) = 1 - \phi(1.58)$
$= 1 - 0.9429 = \underline{0.0571}$

(b) Office opens at 9am, he leaves house at 8:45 am daily.
Percentage of time is he late for work?
→ $P(X > 15) = 1 - \phi\left(\frac{15-24}{3.8}\right) = 1 - \phi(-2.36) = 1 - 0.0091$
$= 99.09\%$

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- Z-test $\quad Z = \frac{\bar{X} - \mu_0}{(\sigma/\sqrt{n})}$

① $H_0 : \mu = \mu_0$ $\quad$ CR ⇒ $Z > Z_{1-\alpha/2}$ or $Z < Z_{\alpha/2}$
$H_1 : \mu \neq \mu_0$

② $H_0 : \mu \geq \mu_0$ $\quad$ CR ⇒ $Z < Z_\alpha$
$H_1 : \mu < \mu_0$

③ $H_0 : \mu \leq \mu_0$ $\quad$ CR ⇒ $Z > Z_{-\alpha}$
$H_1 : \mu > \mu_0$

- t-test $\quad t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$

① $H_0 : \mu = \mu_0$ $\quad$ CR ⇒ $t > t_{\alpha/2}$ or $t < t_{1-\alpha/2}$
$H_1 : \mu \neq \mu_0$

② $H_0 : \mu \geq \mu_0$ $\quad$ CR ⇒ $t < t_{1-\alpha}$
$H_1 : \mu < \mu_0$

③ $H_0 : \mu \leq \mu_0$ $\quad$ CR ⇒ $t > t_\alpha$
$H_1 : \mu > \mu_0$

- χ^2 test $\quad \frac{(n-1)S_x^2}{\sigma^2}$

① $H_0 : \sigma = \sigma_0$ $\quad \chi^2 > \chi^2_{\alpha/2}$ or $\chi^2 < \chi^2_{1-\alpha/2}$
$H_1 : \sigma \neq \sigma_0$

② $H_0 : \sigma \geq \sigma_0$ $\quad \chi^2 < \chi^2_{1-\alpha}$
$H_1 : \sigma < \sigma_0$

③ $H_0 : \sigma \leq \sigma_0$ $\quad \chi^2 > \chi^2_\alpha$
$H_1 : \sigma > \sigma_0$

- f test $\quad \frac{\chi_1^2}{\chi_2^2} \Rightarrow \frac{S_1/\sigma_1}{S_2/\sigma_2}$

① $H_0 : \sigma = \sigma_0$ $\quad f > f_{\alpha/2}$ or $f < f_{1-\alpha/2}$
$H_1 : \sigma \neq \sigma_0$

② $H_0 : \sigma \geq \sigma_0$ $\quad f > f_\alpha$
$H_1 : \sigma < \sigma_0$

③ $H_0 : \sigma \leq \sigma_0$ $\quad f < f_{1-\alpha}$
$H_1 : \sigma > \sigma_0$

SAMPLING DISTRIBUTION

← ans

- χ^2 stats

$$\frac{(n-1) \cdot S^2_x}{\sigma^2} = \chi^2$$

$$P(\chi^2 < \chi^2_\alpha) = 1 - P(\chi^2_\alpha)$$

- T-stats

dof $= n-1$ $\quad t = \frac{\bar{X} - \mu}{S/\sqrt{n}}$ $\quad t_{1-\alpha} = -t_\alpha$

- F stats

$$f_\alpha(v_1, v_2) = \frac{1}{f_{1-\alpha}(v_2, v_1)} \qquad F = \frac{\chi_1^2}{\chi_2^2} = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$$

$$Z = \frac{\bar{X} - \mu_0}{(\sigma/\sqrt{n})}$$

- Hypothesis Testing

① z test : σ is known and have to make inference on μ
② t test : σ is unknown and have to make inference on
③ χ^2 test : when inferences are on σ and for one sample pro

TUTORIAL-4

① Statistic

→ Statistic is any function on a random variable.

② Unbiased estimate → The estimator (T_S) is said to be unbiased of θ if $E(T_S) = \theta$ ← Actual Value

Biased ⇒ $E(T_S) \neq \theta$

$Bias(T_S) = E(T_S) - \theta$

③ Let $X_1, X_2, \ldots X_n$ be n random variables with $E(X_i) = \mu$ $Var(X_i) = \sigma^2$ $i = 1, 2, 3 \ldots n$

What is unbiased estimator of mean μ and that of σ^2?

→ Unbiased estimator of $\mu = \bar{x} = \frac{1}{n}\sum_{i=0}^{n} x_i$

—"— of $\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

④

⑥ Random sample of 25 obs. normal population with variance $\sigma^2 = 6$.

ν \ α	0.05 ← ans
24	36.4

ⓐ P(S^2 greater than 9.1)

→ $\chi^2 = \frac{(n-1)S_x^2}{\sigma^2} = \frac{24 \times 9.1}{6} = 36.4$

∴ $\alpha = \underline{0.05}$

ⓑ S^2 lying in betⁿ 3.462 and 10.745

→

$\chi_1^2 = \frac{(n-1)S_x^2}{\sigma^2} = \frac{24 \times 3.462}{6.0} = 13.848$ $\alpha_1 = 0.95$

$\chi_2^2 = \frac{(n-1)S_x^2}{\sigma^2} = \frac{24 \times 10.745}{6} = 42.98$. $\alpha_2 = 0.01$

∴ $\alpha_1 - \alpha_2 = 0.95 - 0.01 = \underline{0.94}$

⑭ i) $\chi^2_{0.025}$ dof=15

→ 27.488

ii) $\chi^2_{0.01}$ dof=18 ⇒ 34.805

iii) $\chi^2_{0.05}$ dof=28 ⇒ 37.652

iv) $t_{0.025}$ dof=15 ⇒ 2.131

v) $t_{0.995}$, dof=17 ⇒ $t_{0.995} = -t_{1-0.995}$

$t_{0.995} = -t_{0.005} = -2.898$

vi) $f_{0.05}$ dof$(v1, v2) = (7, 15)$ ⇒ 2.71

vii) $f_{0.99}$ dof(28,12) ⇒ $\frac{1}{f_{1-0.99}(12,28)} = \frac{1}{f_{0.01}(12,28)} = \frac{1}{2.90} = 0.3$

viii) $f_{0.01}$ dof(24, 19) ⇒ 2.92

✱ ix) χ^2_α if $P(\chi^2 < \chi_\alpha^2) = 0.95$ dof = 6

→ $P(\chi^2 > \chi_\alpha^2)$ 1−0.95 = 0.05

$\chi_\alpha^2 = 12.592$

ν \ α	0.05
6	12.59

x) χ^2_α if $P(\chi^2 > \chi^2_\alpha) = 0.05$ dof = 16

$\rightarrow$ $\chi_\alpha^2 = \underline{\underline{26.296}}$

xi) χ_α^2 if $P(\chi_\alpha^2 < \chi^2 < 23.209) = 0.015$, dof = 10.

$\rightarrow$ $P(\chi^2 > \chi_\alpha^2) - P(\chi^2 > 23.209) = 0.015$

$P(X > \chi_\alpha^2) - 0.01 = 0.015$

$P(X > \chi_\alpha^2) = 0.025$

$\chi^2_\alpha = \underline{\underline{20.483}}$

xii) $P(T < 2.365)$ dof = 7

$\rightarrow$ $P(T > 2.365) = 0.025$

$P(T < 2.365) = 1 - 0.025 = \underline{\underline{0.975}}$

xiii) $P(-t_{0.005} < T < t_{0.01})$ dof = 20

$\rightarrow$ $P(T > -t_{0.005}) - P(T > t_{0.01})$

$= P(T > t_{0.995}) - P(T > t_{0.01})$

$= 0.995 - 0.01 = \underline{0.985}$

xiv) $P(T > -2.567)$ dof = 17

$\rightarrow$ $P(T > 2.567) = 0.01$

$P(T > -2.567) = 1 - 0.01 = \underline{\underline{0.99}}$

xv) k ? $P(k < T < 2.807) = 0.095$ ss = 24 dof = 23

$\rightarrow$ $P(T > k) - P(T > 2.807) = 0.095$

$P(T > k) - 0.005 = 0.095$

$P(T > k) = 0.095 + 0.005$

$P(T > k) = 0.1$

$k = \underline{\underline{1.319}}$

$\sigma \Rightarrow \chi^2, F$
$\mu \Rightarrow z, t$

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Ex. 100 tests. Avg. lifespan $= 71.8$ yrs. $= \mu_0$
s.d $= 8.9$ yrs $\alpha = 5\%$
Does it seem to indicate that mean lifetime today is greater than 70 yrs?

→ $\bar{x} = 71.8$ $\sigma = 8.9$ $n = 100$ $\alpha = 0.05$

$H_1 = \mu > 70$ $\therefore H_0 = \mu \leq 70$

$$z = \frac{\bar{x} - \mu_0}{(\sigma/\sqrt{n})} = \frac{71.8 - 70}{(8.9/10)} = \frac{1.8}{0.89} = 2.022$$

H_1 is tested. $H_1 : \mu > 70$

$z_{1-\alpha} = z_{1-0.05} = z_{0.95} = 0.45$

CR $=$ $z > z_{1-\alpha}$ $z > z_{1-\alpha} = z > z_{1-0.05} = z_{0.95} = 0.45$

$z = 2.022 > 0.45$
$\therefore$ Accept H_0, reject H_1

(17) Fair coin. Accept if no. of heads of 100 tosses in betn 40-60
(a) $n = 100$ $p = 0.5$ $q = 0.5$ $\mu = 50$ $\sigma = 5$
Type I $\Rightarrow$ $\alpha = P(X < 40) \cup P(X > 60)$ $=$

$= P\left(z \leq \frac{39 - 50}{5}\right) + 1 - P(X \leq 60) = P(z \leq -2.2) + 1 - P(z < 2)$

$= 1 + 0.0139 - 0.9772 = 0.0367\%$

(b)

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(18) 64 tosses of coin
(a) Level of significance $= 0.05$
$p = 0.5$ $q = 0.5$ $\mu = np = 64 \times 0.5 = 32$ $\sigma = 4$
LOS $= 0.05$

(19) Normally distributed s.d $= 0.9$ yrs.
If random sample $= 10$ then s.d $= 1.2$ yrs, do you think that $\sigma > 0.9$ yrs. LOS $= 0.05$

→ $\boxed{\text{I}}$ To claim : $H_0 : \sigma \leq 0.9$
$H_1 : \sigma > 0.9$

$\boxed{\text{II}}$ $\alpha = 0.05$

$\boxed{\text{III}}$ test stats : χ^2 stats.

$$\chi^2 = \frac{(n-1)\, s^2}{\sigma^2} = \frac{9 \times 1.2^2}{0.9^2} = 16$$

$\boxed{\text{IV}}$ CR: $\chi^2 > \chi^2_{\alpha}$ $\chi^2 > \chi^2_{0.05}$
$\chi^2_{0.05} = 16.919$ $\chi^2 > 16.919$

$\boxed{\text{V}}$ $16 < 16.919$, we accept H_0 and reject H_1
σ is not greater than 0.9 yrs.

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Ex. In past the s.d of a certain 40 kg packages filled by a machine was $\overset{\sigma}{0.25}$ kg. Random sample of $\overset{n}{20}$ packages shows a s.d. of $\overset{s}{0.32}$ kg. Is apparent increase in variability significance of $\alpha = 0.05$. What is conclusion for $\alpha = \cancel{1}1\%$.

→ $n = 20$ s.d. of population $= 0.25$
s.d. of sample $= 0.32$

$\boxed{I}$ $H_0 : \sigma \leq 0.25$
$H_1 : \sigma > 0.25$

$\boxed{II}$ $\alpha = 0.05$

$\boxed{III}$ χ^2 test

$$\chi^2 = \frac{(n-1)\, S^2}{\sigma^2} = \frac{(20-1)\, 0.32^2}{0.25} = \underline{\underline{31.1296}}$$

$\boxed{IV}$ Critical region:-
$\chi^2 > \chi_\alpha^2$
$\chi^2 > \chi_{0.05}^2$ $\quad \chi^2 > 30.144$

$\chi^2 = 31.296 \;|\; \chi^2 > 30.144 \;|\; 31.296 > 30.144$

$\boxed{V}$ ∴ We accept H_1 and reject H_0

For $\cancel{1}1\%$,
$\boxed{IV}$ $\chi^2 > \chi_\alpha^2$ $\quad \chi^2 > \chi_{0.01}^2$
$\chi^2 > 36.191$ $\quad$ dof $= n-1 = 20-1 = \underline{19}$
$31.296 > 30.144$ — False

$\boxed{V}$ ∴ We will accept H_0

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Ex. In the past a machine produced washer mean thickness of $\overset{\mu_0}{0.05}$ inches. To determine weather machine is in proper working. The sample of $\overset{n}{10}$ bottle is selected at random and mean thickness is found to be $\overset{\bar{x}}{0.053}$ inches. $\overset{S}{\text{s.d}} = 0.003$. Test hypothesis for 5% and 1% LOS

→ $n = 10$ $\quad \bar{x} = 0.053$ $\quad \mu_0 = 0.05$

$\boxed{I}$ $H_0 : \mu \leq 0.05$
$H_1 : \mu_0 > 0.05$

$\boxed{II}$ $\alpha = 0.05 = 5\%$

$\boxed{III}$ ~~[illegible]~~ As σ is unknown. $\quad S = 0.003$
t test

$$t = \frac{\bar{x} - \mu_0}{(S/\sqrt{n})} = \frac{0.053 - 0.05}{0.003/3.162} = \underline{\underline{3.162}}$$

$\boxed{IV}$ Critical Region.
Right tailed
$\gamma = 10 - 1 = 9$
$t > t_{0.05}$ $\quad t > 1.833$

$\boxed{V}$ $3.162 > 1.833$
H_0 rejected and H_1 accepted.

$\boxed{IV}$ CR $\quad \gamma = 9 \quad \alpha = 0.01$
$t > t_{0.01}$ $\quad t > 02.821$
$3.162 > 2.821$
∴ H_1 is accepted, H_0 rejected.

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(20) $n_1 = 11$ $n_2 = 14$ { Test hypothesis that $\sigma_1^2 = \sigma_2^2$ against alternative that $\sigma_1^2 > \sigma_2^2$
$\sigma_1 = 6.1$ $\sigma_2 = 5.3$
$dof_1 = 10$ $dof_2 = 13$

$\boxed{I}$ $H_0 : \sigma_1^2 \leq \sigma_2^2$
$H_1 : \sigma_1^2 > \sigma_2^2$

$\boxed{II}$ $\alpha = 5\% = 0.05$

$\boxed{III}$ f stats.

$$f = \frac{S_1^2}{S_2^2} = \frac{6.1^2}{5.3^2} = 1.32$$

$\boxed{IV}$ Critical region :
$f > f_\alpha$
$f > f_{0.05}$ $f > 2.67$

$\boxed{V}$ $f = 1.32 < 2.67$
$\therefore H_1$ rejected, H_0 is accepted.

(21) Avg. saturated fat = 0.5 gms.
8 bars examined have
0.6, 0.7, 0.7, 0.3, 0.4, 0.5, 0.4, 0.2 } will we support this claim
$\alpha = 0.05$

→ $\bar{x} = 0.475$ ← Sample avg.

$$S = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{8}}$$

$= 0.17$

$\boxed{I}$ $H_0 : \mu \leq 0.5$ $H_1 : \mu > 0.5$ dof = 7

$\boxed{II}$ $\alpha = 0.05$

$\boxed{III}$ t-test

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{0.475 - 0.5}{0.17/\sqrt{8}} = -0.416$$

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$\boxed{IV}$ Critical region
~~test~~ $t > t_\alpha$
$t > 1.895$

$\boxed{V}$ $-0.416 < 1.895$
$\therefore$ We reject H_1 and accept H_0

(22) Sample
100 tyres. Mean life = 39350 sd = 3260
Test Hypo. at 1% LOS. Mean life of tyre = 40000

→ $\mu_0 = 40000$ $\bar{x} = 39350$ $\sigma = 3260$
$n = 100$

$\boxed{I}$ $H_0 : \mu = 40000$ $H_1 : \mu \neq 40000$

$\boxed{II}$ $\alpha = 0.01$

$\boxed{III}$ σ is known and inference of μ $\therefore$ z test

$$z = \frac{\bar{x} - \mu_0}{(\sigma/\sqrt{n})} = \frac{39350 - 40000}{(3260/10)} = -1.99$$

$\boxed{IV}$ CR $z > z_{1-\alpha/2}$ or $z < z_{\alpha/2}$
$z > z_{0.995}$ or $z < z_{0.005}$
$z > 2.57$ or $z < -2.57$

$\boxed{V}$ $z = -1.99 < 2.57$ $-1.99 > -2.57$
H_0 accepted

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(23) 10 lambs 38, 40, 45, 53, 47, 43, 55, 48, 52, 49. $\alpha = 0.05$

Can we say variance of population = 20.

→ $\bar{x} = \frac{\Sigma \text{ sample}}{10} = \frac{470}{10} = 47$

$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{10}} = \sqrt{26} = 5.09$

8.1
4.9
0.4
2.5
0
1.6
6.4
0.1
1.6
0.4

$\bar{x} = 47$ $s = 1.61$ $n = 10$ $\alpha = 0.05$

variance $= \sigma^2 = 20$ $\therefore \sigma = \sqrt{20} = 4.47$

$H_0: \sigma^2 \leq 20$ $H_1: \sigma^2 > 20$

$$\chi^2 = \frac{(n-1)s_x^2}{\sigma^2} = \frac{(10-1)(5.09)^2}{20} = 11.6586$$

CR: $\chi^2 > \chi_\alpha^2$ dof $= 10 - 1 = 9$

$\chi^2 > \chi_{0.05}^2$

$\chi^2 > 16.919$

$\chi^2 = 11.658 < 16.919$

$\therefore$ Accept H_0

(24) A: 66 67 75 76 82 84 88 90 92

B: 64 66 74 78 82 85 87 92 93 95 97

Both same variance at 10% LOS.

→ $\bar{x}_A = 80$ $\bar{x}_B = 83$

$n_A = 9$ $n_B = 11$

$dof_A = 8$ $dof_B = 10$

$S_A = 9.03$ $S_B = 10.35$

$\sigma_A = 81.5409$ $\sigma_B = 107.1225$

$H_0: \sigma_A^2 = \sigma_B^2$ $H_1: \sigma_A^2 \neq \sigma_B^2$

$\alpha = 10\% = 0.1$

f stats $\Rightarrow f = \frac{S_A^2}{S_B^2} = \frac{81.54}{107.12} = 0.76$

CR:- $f > f_{\alpha/2}$ or $f < f_{1-\alpha/2}$

$f > f_{0.05}(8,10)$ $f < f_{0.95}(8,10)$

$f > f_{0.05}(8,10)$ $f < 1/f_{0.05}(10,8)$

$f > 3.07$ $f < 1/3.35$

$f > 3.07$ $f < 0.2985$

$0.76 < 3.07$ $0.76 > 0.2985$

↓

Accept H_0

(25) 18 students

9, 12, 18, 14, 12, 14, 12, 10, 16, 14, 13, 15, 13, 11, 13, 11, 9, 11

Claims median = 12. 2% LOS

(I) $H_0: \tilde{\mu}_0 = 12$

$H_1: \tilde{\mu}_0 \neq 12$

(II) $\alpha = 2\% = 0.02$

(III) Test statistics:- sign Test

− + + + − + + + + + − + − − − (Not include hypo val) i.e. 12 here

$n = 15$ $x = 9$ $n/2 = 15/2 = 7.85 \approx 8$

$p = 2P[X \leq 9 \text{ when } p = 0.5]$

$= 2 \times b(9, 15, 1/2) = 0.8491$

$P = 2p[X \geq 9 \text{ when } p = 0.5]$

$= 2 \times [1 - P(X \leq 8 \text{ when } p = 0.5)]$

$= 2 \times [1 - 0.6964] = 2 \times 0.3036 = 0.6072$

$p > \alpha$, we accept H_0

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(26) 11 players
7, 16, 121, 51, 101, 81, 1, 16, 9, 11, 16
→ Mean = $\frac{7+16+121+51+101+81+1+16+9+11+16}{11}$

$= 430/11 = \underline{39.10}$

Mode = 16
1, 7, 9, 11, 16, 16, 16, 51, 81, 101
Median = 16

(27) 92, 95, 85, 80, 75, 50
→ $\bar{x} = \frac{92+95+85+80+75+50}{6} = \frac{477}{6} = \underline{79.5}$

$x_i - \bar{x} \Rightarrow$ 12.5, 15.5, 0.5, 5.5, −4.5, −29.5
$\sigma = 14.81$ $\sigma^2 = 219.58$

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(28) Mean = 527 SD = 112
→ $P(X \geq 500) = 1 - P(X \leq 500)$
$= 1 - P\left(Z \leq \frac{500-527}{112}\right)$
$= 1 - P(Z \leq -0.24)$
$= 1 - 0.4052$
$= \underline{0.5948}$

How ~~many~~ high must an individual score on to get highest 5%?
→ $P(X > k) = 0.05$
$1 - P(X \leq k) = 0.05$
$P(X \leq k) = 0.95$
$0.95 = P\left(Z \leq \frac{k - \mu}{\sigma}\right)$
$0.95 = P\left(Z \leq \frac{k-527}{112}\right)$
$1.645 = \frac{k-527}{112}$ $\therefore k = 1.645 \times 112 + 527$
$= \underline{711.24}$

(29)

Population	Population s.d.	Sample s.d.
Men	30 σ	35 S
Women	50	45

→ $F_{stats} = \frac{\chi_m^2}{\chi_w^2} = \frac{S_m^2/\sigma_m^2}{S_w^2/\sigma_w^2} = \frac{30^2/35^2}{50^2/45^2}$
$= \frac{0.734}{1.2345} = 0.594$

$\frac{35^2/30^2}{45^2/50^2} = \frac{1.3611}{0.81} = \underline{1.68}$

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(30) ~~$\mu=300$~~ chains average $=300$ μ
select 15 random, avg $=290$ sd $=50$
n $\bar{x}$ S

t stats $\Rightarrow \dfrac{\bar{X}-\mu_0}{S/\sqrt{n}} = \dfrac{290-300}{50/\sqrt{15}} = -0.7746$

$P(X \leq 290) = P(Z \leq -0.7746) \approx 1-0.7746$ as z is right handed
$=$ ~~0.2236~~ 0.2254

If true avg. life is 300 days, then 22.56% chance that 15 randomly selected bulbs will have avg less than equal to 290 days.

(31) Random $=60$, 59 passed.
avg. % of passing $= 90\%$
$S = 10$
$\rightarrow X \sim Binomial(60, p)$
$H_0 : p = 0.90$ $\quad H_1 : p > 0.90$

$\mu = np = 60 \times 0.90 = 54$
$\sigma = \sqrt{np(1-p)} = 2.32$

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each carton was set to 3kg.
(32) $n = 40$ cartons mean $= \bar{X} = 3.005$ $\quad \sigma = 0.015$
5% LOS ~~the~~ $\mu_0 = 3.00$
$\rightarrow H_0 : \mu = \mu_0$
$H_1 : \mu \neq \mu_0$

$\alpha = 0.05\%$

z statistics $= \dfrac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = \dfrac{3.005-3}{0.015/\sqrt{40}} = 2.108$

CR $\Rightarrow$ $z > z_{1-\alpha/2}$ $\quad$ $z < z_{\alpha/2}$
$z > z_{1-0.025}$ and $z < z_{0.025}$
$z > z_{0.975}$ $\quad$ $z < z_{0.025}$
$z > 1.96$ $\quad$ $z < -1.96$

$2.108 > 1.96$
H_0 reject, H_1 accepted. Machine requires adjustment.

(33) A coin tossed 10 times. Which is correct alternative?
(a) $H1 : p > 0.5$
(b) $H1 : p < 0.5$
(c) $H1 : p = 0.5$
(d) $H1 : p \neq 0.5$ ✓

(35) 30 households $n=30$

$\mu = 375$ $\sigma = 81$

Cofidence level = 99% $\therefore$ LOS = 0.01%

$Z > Z_{1-\alpha/2}$ or $Z < Z_{\alpha/2}$

$Z > Z_{1-0.005}$ or $Z < Z_{0.005}$

$Z > Z_{0.995}$ or $Z < Z_{0.005}$

$Z > 2.575$ or $Z < -02.575$

$$P(-2.575 \le Z \le 2.575) = \left(-2.575 \le \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \le +2.575\right)$$

$$= -2.575 \times \frac{\sigma}{\sqrt{n}} \le \bar{x} - \mu \le 2.575 \times \frac{\sigma}{\sqrt{n}}$$

$$= \bar{x} - 2.575 \times \frac{\sigma}{\sqrt{n}} \le \mu \le 2.575 \times \frac{\sigma}{\sqrt{n}} + \bar{x}$$

$$= -2.575 \frac{\sigma}{\sqrt{n}} + \bar{x} \le \mu \le \frac{\sigma}{\sqrt{n}} 2.575 + \bar{x}$$

$$= 375 \pm 2.575 \times \frac{81}{\sqrt{30}}$$

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(56) Sample-1 ⇒ variance = 109.63, sample size = 41
Sample-2 ⇒ variance = 65.99, sample size = 21

$$f = \frac{\chi_1^2}{\chi_2^2} = \frac{S_1}{S_2} = \frac{\text{variance}_1}{\text{variance}_2} = \frac{109.63}{65.99} = 1.6613$$

$n_1 = 41$, $n_2 = 21$
$dof_1 = 40$ $dof_2 = 20$

Choose alpha as not given : 0.1
Halved for two tailed test : 0.05

$F(40, 20)$ at $\alpha = 0.05 = 1.99$

CR ⇒ $f > f_{\alpha/2}$ or $f < f_{1-\alpha/2}$
if $f < 1.99$ $F < f_{0.95}$

$f < \frac{1}{f_{1-\alpha}(v_2, v_1)}$ $\therefore f < \frac{1}{f_{0.05}(20,40)}$

$f < 4.84$

$1.6613 < 1.99$ $1.6613 < 4.84$

So cannot rejet null hypothesis

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$\mu = 1.8$ claim.

data	d_i (data − μ_0)	abs	rank
1.5	−0.3	0.3	6 = 5.5
2.2	0.4	0.4	7
0.9	−0.9	0.9	10
1.3	−0.5	0.5	8
2.0	0.2	0.2	4 = 3.33 = 3
1.6	−0.2	0.2	3 = 3.33 = 3
~~1.8~~	~~0.0~~		
1.5	−0.3	0.3	5 = 5.5
2.0	0.2	0.2	2 = 3.33 = 3
1.2	−0.6	0.6	9
1.7	−0.1	0.1	1

n = 10 not taken the μ value from data.

$W_+ = 13$ $W_- = 42$
$\therefore W = 13 \rightarrow$ min. value
$\alpha = 0.05$

$H_0 : \bar{\mu} = 100$

$H_1 : \bar{\mu} > 100$

$\mu = 100$

(37)

	data	di	abs	Rank
1	98.38	-1.62	1.62	4
2	115.33	15.33	15.33	21
3	98.62	-1.38	1.38	3
4	114.38	14.38	14.38	19
5	87.79	-12.21	12.21	15
6	84.06	-15.94	15.94	23
7	96.18	-3.82	3.82	7
8	98.74	-1.26	1.26	2
9	91	-9	9	11
10	107.82	7.82	7.82	9
11	108.28	8.28	8.28	10
12	112.62	12.62	12.62	17
13	124.18	24.18	24.18	27
14	101.99	1.99	1.99	6
15	112.51	12.51	12.51	16
16	75.65	-24.35	24.35	28
17	83.77	-16.23	16.23	24
18	84.91	-15.09	15.09	20
19	109.73	9.73	9.73	13
20	109.41	9.41	9.41	12
21	100.4	0.4	0.4	1
22	95.37	-4.63	4.63	8
23	115.46	15.46	15.46	22
24	111.78	11.78	11.78	14
25	86.13	-13.87	13.87	18
26	82.14	-17.86	17.86	25
27	78.47	-21.53	21.53	26
28	98.18	-1.82	1.82	5

$W_+ = 187$

$W_- = 237$

$W = 187$

$W_+ < W_-$

accept H_1 and reject H_0

(38) Test hypothesis that 30% public is allergic.

(a) Type 1 error [reject H_0, even H_0 is true]

→ $H_1 : X > 30\%$ $\quad H_0 : X \le 30\%$

30% people have allergy but we got that more than 30% are allergic.

(b) Type 2 error [accept H_0, even H_0 is false]

→ $H_1 : X > 30\%$ $\quad H_0 : X \le 30\%$

More than 30% have allergy still we got that less than 30% people have allergy

(39)

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(40) estimated $p = 0.6$
$n = 15$, not reject if 6-12 are graduated.
→ $H_0: p = 0.6$ $H_1: p \neq 0.6$
(a) Evaluate α assuming $p = 0.6$. Use binomial dist.
→ Type I → Reject even if H_0 is true.
x = no. of college graduates.
$R = (X < 6) \cup (X \geq 13)$
$R = (X \leq 6) \cup (X \geq 13)$

$\alpha = \sum_{x=0}^{5} b(x, 15, 0.6) + \sum_{x=13}^{15} b(x, 15, 0.6)$

$= 0.0338 + \sum_{x=0}^{15} b(x, 15, 0.6) - \sum_{x=0}^{x=12} b(x, 15, 0.6)$

$= 0.0338 + (1 - 0.9729)$
$= 0.0338 + 0.0271$
$= \underline{0.0609}$

(b) Evaluate β for alternatives $p = 0.5$ and $p = 0.7$
→ Type II error, accept H_0 even its false.
We reject if 0-5 and 13-15 college graduates.
$H_0: p = 0.6$ $H_1: p \neq 0.6$
$p = 0.5$
$\beta = P(X \geq 6) \cap P(X \leq 12)$
$= P(6 \leq X \leq 12)$
$= P(X \leq 12) - P(X \leq 5)$
$= 0.9963 - 0.1509$
$= \underline{0.8454}$

$p = 0.7$
$\beta = P(X \geq 6) \cap P(X \leq 12)$
$= P(X \leq 12) - P(X \leq 5)$
$= 0.8732 - 0.0037 = \underline{0.8695}$

$p = 0.5$ $q = 0.5$
$\mu = np = 0.5 \times 200 = 100$
$\sigma = \sqrt{npq} = 7.07$

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(41) 200 adults
fail to reject region $110 \leq x \leq 130$.
→ (a) $\alpha = P(X \leq 109) \cup P(X \geq 131)$
$= P(X \leq 109) + (1 - P(X \leq 130))$
$= P\left(Z \leq \frac{109 - 100}{7.07}\right) + \left(1 - P\left(Z \leq \frac{130 - 100}{7.07}\right)\right)$
$= P(Z \leq 1.272) + (1 - P(Z \leq 4.24))$
$= 0.8980 + 1 - 1$
$\alpha = \underline{0.8980}$

(b) $\beta = P(110 \leq x \leq 130)$
$= P(x \leq 130) - P(x \leq 109)$
$= P\left(Z \leq \frac{130 - 100}{7.07}\right) - P\left(Z \leq \frac{109 - 100}{7.07}\right)$
$= P(Z \leq 4.24) - P(Z \leq 1.272)$
$= 1 - 0.8980$
$= \underline{0.102}$

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(42) 40 components
unacceptable if contain 3 or more defectives.
Select 5 at random.

→ $N=40$, $n=5$, $k=3$

$$P(X=1) = \frac{{}^{3}C_1 \times {}^{40-3}C_{5-1}}{{}^{40}C_5} = \frac{{}^{3}C_1 \times {}^{37}C_4}{{}^{40}C_5} = \underline{\underline{0.3011}}$$

(43) H_0: lifetime $= 250$
H_1: lifetime > 250 $\quad \alpha = 0.05$

Sign test
+ − − + + − + − + − + + − + + − + + − + + − + +

$+ = 15 \quad - = 9 \quad n = 24 \quad x = 15$

$p = 0.5 \quad q = 0.5$
$\mu = 0.5 \times 24 = 12$
$\sigma = \sqrt{0.5 \times 0.5 \times 24} = 2.45$

$P = P(X \geq x \text{ when } p = 1/2)$
$= P(X \geq 15)$
$= 1 - b(14, 24, 0.5)$
$= 1 - P\left(Z \leq \frac{14-12}{2.45}\right) = 1 - P(Z \leq 0.82) = 1 - 0.7939$
$= \underline{\underline{0.2061}}$

$p > \alpha$
$p > 0.05$ ∴ Accept H_0

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(21) $\mu = 0.5 \quad n = 8$

Data = 0.6, 0.7, 0.7, 0.3, 0.4, 0.5, 0.4, 0.2

0.025	0.225	0.225	0.175	0.075	0.025	0.075	0.275

$$\bar{x} = \frac{\sum_{i=0}^{n=6} x_i}{n} = \frac{3.8}{8} = \underline{\underline{0.475}}$$

$$S = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}} = \sqrt{\frac{0.235}{8}} = \sqrt{0.029375} = \underline{\underline{0.17139}}$$

$\boxed{\text{I}}$ $H_0: \mu = 0.5$
$H_1: \mu \neq 0.5$

$\boxed{\text{II}}$ $\alpha = \underline{\underline{0.05}}$

$\boxed{\text{III}}$ t-test

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{0.475 - 0.5}{0.1713/\sqrt{8}} = \frac{-0.025}{0.060} = \underline{\underline{-0.416}}$$

$\boxed{\text{IV}}$ CR ⇒

$t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$ $\quad$ dof $= 8-1 = \underline{\underline{7}}$
$t > t_{0.025}$ $\quad t < -t_{0.025}$
$t > 2.365$ $\quad t < -2.365$

$\boxed{\text{V}}$ $t = -0.416$
$-0.416 < 2.365$ $\quad -0.416 > -2.365$

H_0 accepted, H_1 rejected

TUT-5

- Linear Model

$Y = \alpha + \beta x + e$ $\qquad y_i = \alpha + \beta x_i + e_i$

Error which represents deviation from regression line x_i, y_i

- e_i have mean 0 $\qquad$ variance $= \sigma^2$

$E(e_i) = 0$ $\qquad var(e_i) = \sigma^2$

$\min \sum_{i=1}^{n} e_i^2 = \min \sum_{i=1}^{n} (y_i - \alpha - \beta x_i)^2$

$$\boxed{\begin{aligned}\sum y_i &= \sum \alpha + \beta \sum x_i \\ \sum x_i y_i &= \alpha \sum x_i + \beta \sum x_i^2\end{aligned}}$$ Normal equations

$y = \alpha + \beta x$ Regression line

Ex-1

x_i	0	1	2	3
y_i	1	4	7	10

→ $n = 4$

$\sum y_i = 22 \qquad \sum x_i = 6 \qquad \sum x_i y_i = 4 + 14 + 30 = 48$

$\sum x_i^2 = 14$

$\sum y_i = \sum \alpha + \beta \sum x_i$

$22 = \sum \alpha + 6\beta$

$22 = 4\alpha + 6\beta$ — (1)

$\sum x_i y_i = \alpha \sum x_i + \beta \sum x_i^2$

$48 = 6\alpha + 14\beta$ — (2)

$\alpha = 1, \ \beta = 3$

Hence, regression line is $Y = 1 + 3x$

- $\hat{\alpha}$ and $\hat{\beta}$ be estimates of α and β

$\hat{\alpha} = \frac{\sum y_i - \beta \sum x_i}{n}$ $\qquad \hat{\beta} = \frac{\sum x_i y_i + \alpha \sum x_i}{\sum x_i^2}$

$$\boxed{y = \hat{\alpha} + \hat{\beta} x}$$

Every sample point satisfies the regression line eqn

$y_i = \hat{\alpha} + \hat{\beta} x_i$

$\Rightarrow \sum y_i = \sum \hat{\alpha} + \hat{\beta} \sum x_i$

divide by n

$\frac{\sum y_i}{n} = \frac{\sum \hat{\alpha} + \hat{\beta} \sum x_i}{n}$ $\qquad \left(\frac{\sum}{n} = \text{avg}\right)$

$$\boxed{\bar{y} = \hat{\alpha} + \hat{\beta} \bar{x}}$$

- $\hat{\beta} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$ $\qquad \boxed{\hat{\beta} = \frac{S_{xy}}{S_{xx}}}$

$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y})$

$S_{xx} = \sum (x_i - \bar{x})^2$

$S_{yy} = \sum (y_i - \bar{y})^2$

- Residual :-

let, (x_i, y_i) be observed data point

$(x_i, \hat{y}_i)$ predicted by regression line

$(y_i - \hat{y}_i)$ = residual.

$\sum y_i = nA + B \sum x_i + C \sum x_i^2$

$\sum x_i y_i = A \sum x_i + B \sum x_i^2 + C \sum x_i^3$

$\sum x_i^2 y_i = A \sum x_i^2 + B \sum x_i^3 + C \sum x_i^4$

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Ex-2

	1	2	3	4	5
x	24	35	64	20	33
y	90	65	30	60	60

x_i	y_i	$(x_i-\bar{x})$	$(y_i-\bar{y})$	$(x_i-\bar{x})(y_i-\bar{y})$	$(x_i-x)^2$
24	90	-11.2	29	-324.8	125.44
35	65	-0.2	4	-0.8	0.04
64	30	28.8	-31	-892.8	795.24
20	60	-15.2	-1	15.2	231.04
33	60	-2.2	-1	2.2	4.84
Σ 176	305			-1021	1156.6

$\bar{x} = \frac{176}{5} = 35.2$ $\quad \bar{y} = \frac{305}{5} = 61$

$\hat{\beta} = \frac{S_{xy}}{S_{xx}} = \frac{-1021}{1156.6} = -1.038$

$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$

$= 61 - (-1.038)(35.2)$

$= 97.537$

$\therefore y = \hat{\alpha} + \hat{\beta}x = 97.537 + (-1.038)x$

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- Multilinear regression

Straight line eqⁿ in 3-D

$Y = \alpha + \beta_1 x_1 + \beta_2 x_2$

Sum of square of errors $= \sum_{i=0}^{m} e_i^2 = \Sigma(y_i - \hat{y})^2$

$SSE = \sum_{i=1}^{n} (y_i - \alpha - \beta_1 x_{1i} - \beta_2 x_{2i})^2$

$\Sigma x_{2i} y_i = \alpha \Sigma x_{2i} + \beta_1 \Sigma(x_{1i} \cdot x_{2i}) + \beta_2 \Sigma x_{2i}^2$

① $\Sigma y_i = n\alpha + \beta_1 \Sigma x_{1i} + \beta_2 \Sigma x_{2i}$

② $\Sigma x_{1i} y_i = \alpha \Sigma x_{1i} + \beta_1 \Sigma x_{1i}^2 + \beta_2 \Sigma x_{1i} \cdot x_{2i}$

③ $\Sigma x_{2i} y_i = \alpha \Sigma x_{2i} + \beta_1 \Sigma x_{1i} \cdot x_{2i} + \beta_2 \Sigma x_{2i}^2$

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Ex-3

y	85	74	76	90	85	87
x_1	65	50	55	65	55	70
x_2	01	7	5	2	6	3

→ $n=6$

$\Sigma y_i = 497$

$\Sigma x_{1i} = 360$

$\Sigma x_{2i} = 24$

$\Sigma x_{1i} y_i = 30020$

$\Sigma x_{1i}^2 = 21900$ $\quad \Sigma x_{1i} x_{2i} = 1360$

$\Sigma x_{2i} y_i = 1934$

$\Sigma x_{2i}^2 = 124$

① $\Sigma y_i = n\alpha + \beta_1 \Sigma x_{1i} + \beta_2 \Sigma x_{2i}$

$497 = 6\alpha + \beta_1 360 + 24\beta_2$

② $\Sigma x_{1i} y_i = \alpha \Sigma x_{1i} + \beta_1 \Sigma x_{1i}^2 + \beta_2 \Sigma x_{1i} \cdot x_{2i}$

$30020 = 360\alpha + 21900\beta_1 + 1360\beta_2$

③ $\Sigma x_{2i} y_i = \alpha \Sigma x_{2i} + \beta_1 \Sigma x_{1i} x_{2i} + \beta_2 \Sigma x_{2i}^2$

$1934 = 24\alpha + 1360\beta_1 + 124\beta_2$

$\alpha = 44.83$ $\quad \beta_1 = 0.64$ $\quad \beta_2 = -0.1$

$y = 44.83 + 0.64 x_1 + (-0.1) x_2$

(4) $\{X(t) \mid ; -\infty < t < \infty\}$ $\{-1, 1\}$

$P[X(t) = -1] = 1/2 = P[X(t) = 1]$, $-\infty < t < \infty$

⇒ ⓐ $\mu(t) = E(X(t)) = -1 \times 1/2 + 1 \times 1/2 = \underline{0}$ ∴ Independent of t.

ⓑ Auto correction function $R(0) = E(X^2(t))$

$R(0) = E(X^2(t)) = 1 \times 1/2 + 1 \times 1/2 = \underline{\underline{1}}$

ⓒ Yes, it is stationary in wide sense.

(i) $\mu(t) = E(X(t))$ is independent of t.

(ii) $R(t_1, t_2) = R(0, t_2 - t_1) = R(\tau)$ $\quad t_2 \geq t_1 \geq \square$

(iii) $R(0) = E(X^2(t)) < \infty$

$$\chi^2 = \sum_{i=1}^{k} \frac{(N_i - nP_i)^2}{nP_i}$$ observed, expected

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P-Test

① Ratio 5:2:2:1

Can containing 500; 269 A, 112 B, 74 C and 45 D.

At 5% LOS test the ratio.

→ $\boxed{I}$ H_0 : Ratio is 5:2:2:1

H_1 : Not 5:2:2:1

$\boxed{II}$ $\alpha = 0.05$

$\boxed{III}$

$$\chi^2 = \sum_{i=1}^{k} \frac{(N_i - nP_i)^2}{nP_i}$$

$k = 4$, $n = 500$

k	N_i	nP_i	$\frac{(N_i - nP_i)^2}{nP_i}$
1	269	$500 \times \frac{5}{10} = 250$	$\left(\frac{(269-250)^2}{250}\right)$
2	112	$500 \times \frac{2}{10} = 100$	
3	74	100	
4	45	$500 \times \frac{1}{10} = 50$	

$$\chi^2 = \frac{(269-250)^2}{250} + \frac{(112-100)^2}{250} + \frac{(74-100)^2}{100} + \frac{(45-50)^2}{50}$$

$\chi^2 = \underline{\underline{10.144}}$

$\boxed{IV}$ Two tailed test

$\chi^2 > \chi^2_{\alpha/2}$ $\quad \chi^2 < \chi^2_{1-\alpha/2}$

$\chi^2 > \chi^2_{0.025}$ $\quad \chi^2 < \chi^2_{0.975}$

$\chi^2 > 9.348$ $\quad \chi^2 < 0.216$ $\quad$ dof $= k-1 = \underline{\underline{3}}$

$\boxed{V}$ $\chi^2 = 10.144$

$\chi^2 > 9.348$ $\quad \chi^2 < 0.216$ ✗

Reject H_0.

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FORMULAS

- Probability

$P(A|B) = \frac{P(A\cap B)}{P(B)}$ $P(B|A) = \frac{P(A\cap B)}{P(A)}$

$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$

Independent

$P(A\cap B) = P(A) \cdot P(B)$

$P(A) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B})$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(A\cap B^c) = P(A) - P(A\cap B)$

- $F(x) = P(X \le x)$ → CDF

$\frac{d}{dx}F(x) = f(x)$ → PDF

- $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$

$Var(X) = \int_{-\infty}^{\infty} E[X-E(X)]^2$ $Var(X) = \int_{-\infty}^{\infty} [X-E(X)]^2 f(x)\,dx$

$E(a) = a$ $E(a+bx) = E(a) + bE(X) = a + bE(X)$

$Var(a) = 0$ $Var(a+bx) = Var(a) + b^2 Var(X) = b^2 Var(X)$

μ = Expectation σ = Variance

$P(X \le x) = P\left(Z \le \frac{x-\mu}{\sigma}\right)$

$\mu = np$ $\sigma = \sqrt{npq}$ variance $= \sigma^2 = npq$

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- Bernoulli Distribution

$q = 1-p$

$E(X) = p$ $Var(X) = pq$

$f(x_i) = P(X=x_i) = \begin{cases} p & \text{if } X=1 \\ 1-p & \text{if } X=0 \end{cases}$

Binomial Distribution

$E(X) = np$ $Var(X) = npq$ ${}^nC_r\, p^r q^{n-r}$

Poisson Distribution

$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $E(X) = Var(X) = \lambda$

Exponential Distribution

$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$ $F(x) = P(X \le x) = 1 - e^{-\lambda x}$

$P(X > x) = e^{-\lambda x}$

- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

- $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$ $f = \frac{\chi_1^2}{\chi_2^2} = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2}$

$$\Sigma y_i = \Sigma \alpha + \beta \Sigma x_i$$
$$\Sigma x_i y_i = \alpha \Sigma x_i + \beta \Sigma x_i^2$$

$$Y = \alpha + \beta x$$

$$\hat{\beta} = \frac{S_{xy}}{S_x} = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - x)^2}$$

$$\Sigma y_i = nA + B\Sigma x_i + C\Sigma x_i^2$$
$$\Sigma x_i y_i = A\Sigma x_i + B\Sigma x_i^2 + C\Sigma x_i^3$$
$$\Sigma x_i^2 y_i = A\Sigma x_i^2 + B\Sigma x_i^3 + C\Sigma x_i^4$$

$$\chi^2 = \frac{(N_i - nP_i)^2}{nP_i}$$

$$LCL = \frac{\mu - 3\sigma}{\sqrt{n}}$$
$$UCL = \mu + 3\sigma/\sqrt{n}$$