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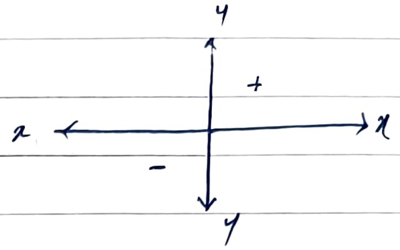
UNIT 2 Eigen Values
Eigen Vector

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$$



For Matrix A Sq. Matrix of order 'n' $n \neq 0$

$$AX = \lambda X$$

X is called eigen vector of A.
λ is called corresponding eigen value.

$$AX - \lambda X = 0$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \lambda \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y \\ 3x + 4y \end{bmatrix} - \begin{bmatrix} \lambda x \\ \lambda y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y - \lambda x \\ 3x + 4y - \lambda y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (1-\lambda) + 2y \\ 3x + (4-\lambda) y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

1. Consider.

$$|A - \lambda I| = 0$$

It is an eqn in λ of deg n.

2. Soln = of this eqn are eigen values.
$\lambda_1 \lambda_2 \dots \lambda_n$.

3. For such eigen value λ_i,
Solve $(A - \lambda_i I)x = 0$
Soln of the system is eigen vector x_1 corresponding to λ_i

- Find eigen value of vector A.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

1.) $$\begin{bmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(-1-\lambda) = 0$$

$1 - \lambda = 0$ or $-1 - \lambda = 0$

$\lambda = 1$ or $\lambda = -1$

$\boxed{\lambda = 1}$ & $\boxed{\lambda = -1}$ are two eigen values.

1.] Consider $\lambda = 1$
Consider $(A - \lambda I)x = 0$

$$\begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\sim \left[\begin{array}{cc|c} 0 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$-2y = 0$$

$$y = 0$$

$$x = r$$

$$x_1 = \begin{bmatrix} r \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} r$$

ii. Consider $\lambda = -1$

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x = 0$$

$$x = 0$$

$$y = r$$

$$x_2 = \begin{bmatrix} 0 \\ r \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

Find the eigen value and eigen vector.

$$A = \begin{bmatrix} 5 & -2 \\ 9 & -6 \end{bmatrix}$$

1. $\begin{bmatrix} 5-\lambda & -2 \\ 9 & -6-\lambda \end{bmatrix} = 0$

$= (5-\lambda)(-6-\lambda) + 18 = 0$

$= -30 - 5\lambda + 6\lambda + \lambda^2 + 18 = 0$

$= \lambda^2 + \lambda - 12 = 0$

$\boxed{\lambda_1 = 4}$ $\boxed{\lambda_2 = 3}$

1] Consider $\lambda_1 = -4$

$$\begin{bmatrix} 9 & -2 \\ 9 & 2 \end{bmatrix} \begin{bmatrix} x \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 9 & -2 & 0 \\ 9 & 2 & 0 \end{array}\right]$$

$R_2 - R_1 \sim \left[\begin{array}{cc|c} 9 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$9x - 2y = 0$	$X_1 = \begin{bmatrix} \frac{2}{9}r \\ r \end{bmatrix}$
$y = r$	$= \begin{bmatrix} 2/9 \\ 1 \end{bmatrix} r$
$9x - 2r$	
$x = \frac{2}{9} r$	$= \begin{bmatrix} 2 \\ 9 \end{bmatrix} r$

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Consider $\lambda_2 = 3$.

$$\begin{bmatrix} 2 & -2 \\ 9 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2 & -2 & 0 \\ 9 & -9 & 0 \end{array}\right] = R_2 - \frac{9}{2}R_1 \quad \left[\begin{array}{cc|c} 2 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$2x - 2y = 0$

$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} r$ $\qquad y = r.$

$2x - 2r = 0.$

$x = 1r$

(2.) $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

1. $\begin{bmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{bmatrix} = 0.$

$(1-\lambda)(3-\lambda) = 0$

$3 - \lambda - 3\lambda + \lambda^2 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$\boxed{\lambda_1 = 1}$ $\boxed{\lambda_2 = 3}$

1] Consider $\lambda_1 = 1$

$$\begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \qquad \left[\begin{array}{cc|c} 0 & 2 & 0 \\ 0 & 2 & 0 \end{array}\right]$$

$R_2 - R_1$ $\left[\begin{array}{cc|c} 0 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$2y = 0$ $\qquad x_1 \begin{bmatrix} r \\ 0 \end{bmatrix} = r \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$y = 0$

$x = r$

Consider $\lambda_2 : 3$

$$\left[\begin{array}{cc|c} -2 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$-2x + 2y = 0$

$y = r$

$-2x + 2r = 0$

$-2x = -2r$

$x = r$

$$x_2 = \begin{bmatrix} +r \\ r \end{bmatrix} = r\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

③. $\begin{bmatrix} 4 & 0 \\ 0 & -6 \end{bmatrix}$

④ $\begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

⑤ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

⑥ $\begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 4 & 0 \\ 0 & -6 \end{bmatrix}$

i. $\begin{bmatrix} 4-\lambda & 0 \\ 0 & -6-\lambda \end{bmatrix} = 0$

$(4-\lambda)(-6-\lambda) = 0$

$24 - 4\lambda + 6\lambda + \lambda^2 = 0$

$24 + 2\lambda + \lambda^2 = 0$ i.e $\lambda^2 + 2\lambda + 24 = 0$

$\lambda_1 = 4 \quad \lambda_2 = -6.$

consider $\lambda_1 = 4$.

$\begin{bmatrix} 4-\lambda & 0 \\ 0 & -6-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{cc|c} 4-4 & 0 & 0 \\ 0 & -6-4 & 0 \end{array}\right]$

$\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & -10 & 0 \end{array}\right]$

$0x + 0y = 0$

$y = \cancel{r}0$

$x = r$

$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} r \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} r$

Consider $\lambda = -6$

$\left[\begin{array}{cc|c} +10 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$10x + 0y = 0$

$x = 0$

$y = r$

$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 0 \\ r \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} r.$

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3. $\begin{bmatrix} 4 & 0 \\ 0 & -6 \end{bmatrix}$

i. $\begin{bmatrix} 4-\lambda & 0 \\ 0 & -6-\lambda \end{bmatrix} = 0$

$(4-\lambda)(-6-\lambda) = 0$

$24 - 4\lambda + 6\lambda + \lambda^2 = 0$

$24 + 2\lambda + \lambda^2 = 0$ i.e $\lambda^2 + 2\lambda + 24 = 0$

$\lambda_1 = 4 \qquad \lambda_2 = -6.$

consider $\lambda_1 = 4$.

$\begin{bmatrix} 4-\lambda & 0 \\ 0 & -6-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{cc|c} 4-4 & 0 & 0 \\ 0 & -6-4 & 0 \end{array}\right]$

$\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & -10 & 0 \end{array}\right]$

$0x + 0y = 0$

$y = 0$

$x = r$

$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} r \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} r$

Consider $\lambda = -6$

$\left[\begin{array}{cc|c} +10 & 0 & 0 \\ 0 & +0 & 0 \end{array}\right]$

$10x + 0y = 0.$

$x = 0$

$y = r.$

$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 0 \\ r \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} r.$

2) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

i) $\begin{bmatrix} 1-\lambda & 2 \\ 0 & 1-\lambda \end{bmatrix} = 0$

$(1-\lambda)(1-\lambda) = 0$

$1(1-\lambda) - \lambda(1-\lambda) = 0$

$1 - \lambda - \lambda + \lambda^2 = 0$

$\cancel{\lambda^2 + 1 = 0}$ $\quad \lambda^2 - 2\lambda + 1 = 0$

$\lambda_1 = 1 \quad \lambda_1 = 1$

consider $\lambda = 1$

$\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$.

$0x + 2y = 0$.

$x = r$.

$y = 0$.

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} r$.

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Ex $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

Step 1:]

$$\underbrace{(A}_{\text{Given matrix}} - \underbrace{\lambda}_{\text{const.}} \underbrace{I)}_{\text{ID}} \overbrace{X}^{\text{unknown}} = \underbrace{0}_{\text{Null matrix}}$$

Step 2.

$$\begin{bmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{bmatrix} = 0$$

Step 3:

$$\lambda^3 - \{\text{Sum of Diagonal elements}\}\lambda^2 + [\text{sum of diagonal minor}]\lambda - |A| = 0 \quad \text{— eqn.}$$

$$1\lambda^3 - \{6\lambda^2\} + 11\lambda - 6 = 0$$

Minor.

$$\begin{vmatrix} -3 & -2 \\ -4 & 1 \end{vmatrix} = -11$$

$$\begin{vmatrix} 8 & -2 \\ 3 & 1 \end{vmatrix} = 14$$

$$\begin{vmatrix} 8 & -8 \\ 4 & -3 \end{vmatrix} = 8$$

$\lambda_1 = 1 \qquad \lambda_2 = 2 \qquad \lambda_3 = 3$. — Eigen value.

~~Values~~.

Part 2. Eigen vector.

$\lambda_1 = 1$

$$\begin{bmatrix} 8-1 & -8 & -2 \\ 4 & -3-1 & -2 \\ 3 & -4 & 1-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix}$$

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Step 1: Crommer's Rule.

2 3
" 3.
' 2.

$$7x_1 - 8y - 2z = 0$$
$$4x - 4xy - 2z = 0.$$

$$\frac{x_1}{\begin{vmatrix}-8 & -2\\ -4 & -2\end{vmatrix}} = \frac{-y}{\begin{vmatrix}7 & -2\\ 4 & -2\end{vmatrix}} = \frac{z}{\begin{vmatrix}7 & -8\\ 4 & -4\end{vmatrix}}$$

$$\frac{x_1}{16-8} = \frac{-y}{-6} = \frac{z}{-28-(-32)}.$$

$$\frac{x}{8} = \frac{y}{6} = \frac{z}{4}.$$

$$X = \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}8\\ 6\\ 4\end{bmatrix} = \begin{bmatrix}4\\ 3\\ 2\end{bmatrix}$$

If a Matrix is a troingular matrix all

The NO. of repeated

The different between algebric Multiplicity and geometric Multiplicity is defect.

$\lambda^2 - \sigma_1\lambda + \sigma_2 = 0$.

$\sigma_1 \rightarrow$ Sum of diagonal elements.

$\sigma_2 \rightarrow |A|$

If A is a square Matrix of order 3 then.

eqn is given by. $\lambda^3 - \sigma_1\lambda^2 + \sigma_2\lambda - \sigma_3 = 0$

$\sigma_1 \rightarrow$ sum of diagonal element.

$\sigma_2 \rightarrow$ sum of det of order 2 containing 2 diagonal element of a time.

$\sigma_2 \rightarrow |A]$

$$A = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix}$$

Ch. eqn.

$$\lambda^3 - \sigma_1 \lambda^2 + \sigma_2 \lambda - \sigma_3 = 0$$

$\sigma_1 \rightarrow 4 + 1 + 2 = 7$

$\sigma_2 \rightarrow \begin{vmatrix} 4 & -2 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 6 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix}$

$= 0 + (-10) + 5 = -5$

$\sigma_3 |A| \rightarrow \begin{vmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{vmatrix}$

$= -75$

Ch eqn.

$$\lambda^3 - 7\lambda^2 - 5\lambda + 75 = 0$$

$\boxed{\lambda_1 = -3}$ $\boxed{\lambda_2 = 5}$

$-3 - 3 + 5 = 7$.

$-3 + 5 + 5 = 7$

We know that sum of eigenvalue = Trace

$\lambda = 5$ is repeated eigen value

Consider $\lambda_1 = -3$

$$\begin{bmatrix} 4-\lambda & -2 & 3 \\ -2 & 1-\lambda & 6 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} 7 & -2 & 3 \\ -2 & 4 & 6 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_{1 \to 3}$

$$\left[\begin{array}{ccc|c} 1 & +2 & \cancel{3}5 & 0 \\ -2 & 4 & 6 & 0 \\ 7 & -2 & 3 & 0 \end{array}\right]$$

$R_2 + 2R_1$ $\qquad$ $R_3 - 7R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 0 & 8 & 16 & 0 \\ 0 & -16 & -32 & 0 \end{array}\right]$$

$R_3 + 2R_2$.

$X_1 :$ $\begin{bmatrix} 1 \\ -2 \\ \; \end{bmatrix}$

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Find the eigen value and eigen vector.

$$A : \begin{bmatrix} 3 & 0 & 12 \\ -6 & 3 & 0 \\ 9 & 6 & 3 \end{bmatrix}$$

ch eqn.

$$\lambda^3 - \sigma_1\lambda^2 + \sigma_2\lambda - \sigma_3 = 0$$

$\sigma_1 = 9$

$$\sigma_2 : \begin{vmatrix} 3 & 0 \\ -6 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 0 \\ 6 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 12 \\ 9 & 3 \end{vmatrix}$$

$$= 9 + 9 \cancel{+} - 99 = -81.$$

$\sigma_3 = -729.$

$$\lambda^3 - 9\lambda^2 \cancel{+} - 81\lambda \cancel{+} 729 = 0$$

$\lambda_1 = -9 \qquad \lambda_2 = 9.$ — Vector value.

For eqn ... consider $\lambda = -9$

$$\begin{bmatrix} 3-\lambda & 0 & 12 \\ -6 & 3-\lambda & 0 \\ 9 & 6 & 3-\lambda \end{bmatrix} = \begin{bmatrix} 12 & 0 & 12 \\ -6 & 12 & 0 \\ 9 & 6 & 12 \end{bmatrix}$$

Complex conjugate of $a+ib$ is $a-ib$.

$$(a+ib)(a-ib) = a^2-(ib)^2$$
$$= a^2-(ib)^2.$$
$$= a^2-i^2b^2$$
$$= a^2+b^2.$$

division.

$$\frac{a+ib}{c+id} = \frac{(a+ib)(c-id)}{(c+id)(c-id)}$$

$$= \frac{1}{c^2+d^2}\left[(ac+bd)+i(bc-ad)\right]$$

[Multiply by conjugate is real no.]

$\boxed{\text{We donote complex by } z} = \boxed{a+ib}$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

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Find the eigen value & vector

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\lambda^2 \cancel{+} - 0\lambda + 1 = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda^2 = -1$$

$$\lambda = \pm i$$

$$\lambda_1 = i \qquad \lambda_2 = -i$$

Consider $\lambda_1 = i$

$$\begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -i & 1 & 0 \\ -1 & -i & 0 \end{array}\right]$$

$R_2 \to R_1$

$$\left[\begin{array}{cc|c} -1 & -i & 0 \\ -i & 1 & 0 \end{array}\right]$$

$R_2 - i R_1$

$$\left[\begin{array}{cc|c} -1 & -i & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$-x - iy = 0 \qquad -x = ir \qquad X_1 = \begin{bmatrix} -ir \\ r \end{bmatrix} = \begin{bmatrix} -i \\ 1 \end{bmatrix} r$$

$$y = r \qquad x = -ir$$

$$-x - ir = 0$$

eigen Vector

Consider $-i$

$x_2 = \begin{bmatrix} i \\ 1 \end{bmatrix}$

$$\left[\begin{array}{cc|c} i & 1 & 0 \\ -1 & i & 0 \end{array}\right]$$

$R_2 \to R_1$

$$\left[\begin{array}{cc|c} -1 & i & 0 \\ i & 1 & 0 \end{array}\right]$$

$R_2 + iR_1$

$$\left[\begin{array}{cc|c} -1 & i & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$-x + iy = 0$

$y = r$

$-x + ir = 0$

$-x = -ir$

$x = ir$

$x_1 = \begin{bmatrix} ir \\ r \end{bmatrix} = \begin{bmatrix} i \\ r \end{bmatrix} r$

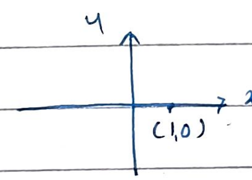
Rotation Matrix.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$

rotation matrix is this where θ is a angle through which the system is rotated anticlockwise.

let $\theta = 90^\circ$.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\boxed{\text{rotating in anticlockwise}}$

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1/2 & 0 \\ 1 & 0 & 4 \end{bmatrix}$$

ch eqn : $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 \rightarrow 6.5$

$S_2 \rightarrow \begin{vmatrix} 2 & 0 \\ 0 & 1/2 \end{vmatrix} + \begin{bmatrix} 1/2 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} = 10$

$S_3 \rightarrow 3.5$

$$\lambda^3 - 6.5\lambda^2 + 10\lambda - 3.5 = 0$$

$\lambda_1 = 0.5 \quad \lambda_2 = 1.5 \quad \lambda_3 = 4.4$ — eigen value.

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* An elastic membrane in the $x_1 x_2$ plane with boundary circle $x_1^2 + x_2^2 = 1$
is streatched so that a pt. $P(x_1, x_2)$.
goes over into the pt. $Q(y_1, y_2)$
given by

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Find the principle directions.
what shape does the boundry circle take P.

example :

$$A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$$

ch. eqn is given by. $\lambda^2 - \sigma_1 \lambda + \sigma_3 = 0$

$\sigma_1 = 10$

$\sigma_2 = \begin{vmatrix} 5 & 3 \\ 3 & 5 \end{vmatrix} = 16.$

$\lambda^2 - 10\lambda + 16 = 0$

$\lambda_1 = 8 \quad \lambda_2 = 2.$

consider $\lambda_1 = 8$.

$$\begin{bmatrix} 5-\lambda & 3 \\ 3 & 5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -3 & 3 & 0 \\ 3 & -3 & 0 \end{array}\right]$$

$R_2 + R_1$

$$\left[\begin{array}{cc|c} -3 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$-3x_1 + 3x_2 = 0$

$3x_1 = 3x_2.$

$x_1 = x_2.$

$x_2 = r \quad x_1 = r$

$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} r.$

Consider. $\lambda_2 = +2.$

$$\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 3 & 3 & 0 \\ 3 & 3 & 0 \end{array}\right]$$

$R_2 - R_1$

$$\left[\begin{array}{cc|c} 3 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$3x_1 + 3x_2 = 0$

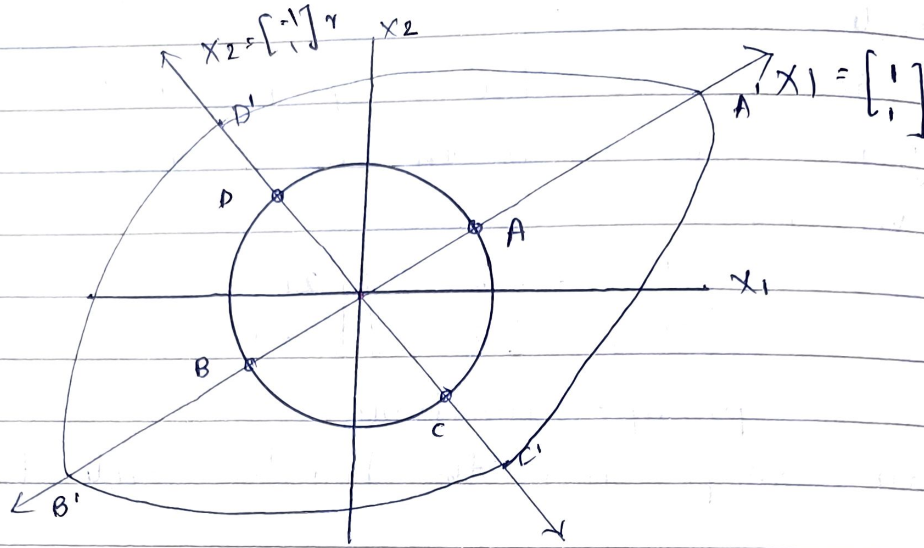
$3x_1 = -3x_2.$

$x_1 = -x_2$

$x = -r \quad x_2 = +r$

$X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$x_1^2 + x_2^2 = 1$



$A X_1 = 8 X_1$.

$A X_2 = 2 X_2$.

$x_1^2 + x_1^2 = 1 \quad \therefore 2x_1^2 = 1 \quad x_1^2 = \frac{1}{2} \quad x_1 = \frac{1}{\sqrt{2}}$.

$A = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$B = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

$C = \left(\frac{1}{\sqrt{2}} \quad -\frac{1}{\sqrt{2}}\right)$

$D = \left(\frac{-1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}\right)$

* Product of eigen value is det of matrix.

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Properties of Eigen value and Eigen vector.

If $\lambda_1 \lambda_2 \cdots \lambda_n$ are Eigen values of a matrix A of order n. counted according to their Multiplicity. then prove that

1. $\lambda_1 \lambda_2 \cdots \lambda_n = |A|$
2. $\lambda_1 + \lambda_2 \cdots + \lambda_n =$ Trace of A

characteristic equation $\rightarrow$ $\lambda^n - \sigma_1 \lambda^{n-1} + \sigma_2 \lambda^{n-2} \cdots (-1)^n \sigma_n$

$\sigma_1 =$ Trace $\downarrow$ (sum of diagonal) & $\sigma_n = |A|$

second property.

1. The Matrix A is invertible if an only if 0 is not an Eigen value of A $\Rightarrow \lambda \neq 0$

   A is invertible $\boxed{\forall i} \rightarrow$ (for all)

   $|A| \neq 0$

   but $|A| = \lambda_1 \lambda_2 \cdots \lambda_n$

   $\Leftrightarrow \lambda_1 \lambda_2 \cdots \lambda_n \neq 0$

3.)

1. The matrix A

4. If λ is a eigen value of $[A]$, then λ^{-1} is eigen value of $A^{-1} = 1/\lambda$.

$\boxed{Ax = \lambda x}$ → defination.

Proof : Let us assume that

A^{-1} exists.

$|A| \neq 0$

∴ All eigen values of A are nonzero.

∴ Let $\lambda \neq 0$ be an eigen value of A.

(There exists)

Then $\exists$ a nonzero vector X s.t.

$$AX = \lambda X.$$

$$\therefore \; A^{-1}(A\lambda) = A^{-1}(\lambda X)$$

$$X = \lambda A^{-1} X$$

$$\frac{1}{\lambda} X = A^{-1} X.$$

$$\boxed{A^{-1}X = \frac{1}{\lambda} X}$$

$\frac{1}{\lambda}$ is an eigen value of A^{-1}

(5) If λ is an eigen value of A λ^2 is a eigen value of A^2.

$$AX = \lambda X$$

$$A(AX) = A(\lambda X)$$

$$A^2 X = \lambda AX.$$

$$= \lambda(\lambda x)$$

$$A^2 X = \lambda^2 X$$

$$\therefore \boxed{A^2 X = \lambda^2 X}$$

6. If λ is a eigen value of A then $k\lambda$ is a eigen value of kA

$$AX = \lambda X.$$
$$kAX = k\lambda X.$$
$$(kA)X = (k\lambda)X.$$

7. The eigen vector corrosponding to two distint eigen value are linearly independent

Let λ_1 λ_2 be two distinct eigen value of A then their exist ($\exists$) X_1 & X_2 $\rightarrow$ (non zero) s.t.

$$AX_1 = \lambda_1 X_1 \quad AX_2 = \lambda_2 X_2.$$

Claim: X_1 & X_2 are L.I.

Let $X_1 = kX_2$.

$$\therefore AX_1 = AkX_2.$$
$$\lambda_1 X_1 = k(AX_2)$$
$$\lambda_1 X_1 = k\lambda_2 X_2$$
$$\lambda_1 (kX_2) = k\lambda_2 X_2$$
$$k(\lambda_1 - \lambda_2) X_2 = 0$$

Since $X_2 \neq 0$ $\because$ (eigen vector).
$\lambda_1 - \lambda_2 \neq 0$ (distinct)

Problem.

Suppose that in the year 2004 land used in the city is as follow C: Commercial → 25%.
I : Industrial - 20%. R: Resid - ~~7~~55%.
Find the state of distribution 2009, 2014 Assuming that the transition probability for 5 year intervals are given by the Matrix A.

$$
\begin{array}{cccc}
\text{From } C & \text{From } I & \text{From } R & \\
\end{array}
$$

$$
A = \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix} \begin{array}{l} \text{To } C \\ \text{To } I \\ \text{To } R, \end{array}
$$

Find the limiting state or Saturation state.

In 2009.

$$
\text{State} = \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix} \begin{bmatrix} 25 \\ 20 \\ 55 \end{bmatrix}
$$

$$
= \begin{bmatrix} 19.5 \\ 34 \\ 46.5 \end{bmatrix}
$$

State after 5 years =

$$A^T = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.1 & 0.9 & 0 \\ 0 & 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore$ A eigen value = A^T eigen vector. [1 is an eigen value of A]

Consider $\lambda = 1$.

vector.

$$A: \begin{bmatrix} 0.7-1 & 0.2 & 0.1 \\ 0.1 & 0.9-1 & 0 \\ 0 & 0.2 & 0.8-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -0.3 & 0.1 & 0 & 0 \\ 0.2 & -0.1 & 0.2 & 0 \\ 0.1 & 0 & -0.2 & 0 \end{array}\right]$$

$R_1 \to R_3$.

$$\left[\begin{array}{ccc|c} 0.1 & 0 & -0.2 & 0 \\ 0.2 & -0.1 & 0.2 & 0 \\ -0.3 & 0.1 & 0 & 0 \end{array}\right]$$

$R_2 - 2R_1$ $\quad$ $R_3 + 3R_1$

$$\left[\begin{array}{ccc|c} 0.1 & 0 & -0.2 & 0 \\ 0 & -0.1 & +0.2 & 0 \\ 0 & 0.1 & -0.6 & 0 \end{array}\right]$$

$I : \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

constant.

* Cayley Hamilton.

Theorem: Any square matrix follow its ch eqn.

Verify this thm for:

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

use it to find the value of $A^5 + 5A^4 - 6A^3 + 2A^2 - 4A + 7I$.

Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

If ch eqn is $\lambda^2 - \sigma_1 \cancel{\lambda} + \sigma_2 = 0$.

where $\sigma_1 = 1 + 3 = 4$ $\quad \sigma_2 = \begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} = 3 - 8 = 5$.

ch eqn $\lambda^2 - 4\lambda - 5 = 0$.

Claim: A satisfies this ch eqn eqn i.e $A^2 - 4A - 5I = 0$.

$$A^2 = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1\times1 + 4\times2 & 1\times4 + 4\times3 \\ 2\times1 + 3\times2 & 2\times4 + 3\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix}$$

LHS : $A^2 - 4A \cdot 5I$

$$= \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - 4\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 16 \\ 8 & 12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

C.H Thm is verified.

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$A^5 + 5A^4 - 6A^3 + 2A^2 - 4A + 7I$

$$
\begin{array}{r l}
 & A^3 + 9A^2 + 35A \\
A^2 - 4A - 5I \,\big) & \cancel{A^5} + 5A^4 - 6A^3 + 2A^2 - 4A + 7I \\
 & -\cancel{A^5} \pm (-4A^4) \mp 5A^3 \\
\hline
 & 9A^4 - 1A^3 + 2A^2 - 4A + 7I \\
 & -\cancel{9A^4} - 36A^3 - 45A^2 \\
\hline
 & 35A^3 - 47A^2 - 4A + 7I \\
 & -\cancel{35A^3} - 140A^2 - 175A \\
\hline
 & 187A^2 + 171A + 7I
\end{array}
$$

* Diagonalization.
    * Similar matrices have same eigen values..

A

* Let A & B be similar Matrices
$\exists C$ - nonsingular Matrix s.t.
$B = C^{-1}AC$

Let λ be an eigen value of B & X be the corresponding eigen vector.

$$BX = \lambda X.$$
$$(C^{-1}AC)X = \lambda X.$$
$$C(C^{-1}AC)X = C\lambda X.$$
$$(AC)X = \lambda CX.$$
$$AC(X) = \lambda(CX).$$

Since C is non singular matrix and X
$\therefore$ λ is an eigen value of A

Problem of Diagonalization.

$A = \begin{bmatrix} 8 & -1 \\ 5 & 2 \end{bmatrix}$ $\quad \lambda^2 - 6\lambda + 6x = 0$

$\lambda^2 - 10\lambda + 21 = 0$

$(\lambda - 3)(\lambda - 7)$

$\lambda = 3 \quad \lambda = 7$ — eigen value.

Consider $\lambda = 3$:

$\left[\begin{array}{cc|c} 5 & -1 & 0 \\ 5 & -1 & 0 \end{array}\right]$

$R_2 - R_1$

$\left[\begin{array}{cc|c} 5 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$5x - y = 0$

$y = r$

$5x - r = 0$

$x = \frac{r}{5}$

$X_1 \begin{bmatrix} 1/5 \\ r \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix} r$

Consider $\lambda = 7$

$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 5 & 5 & 0 \end{array}\right]$

$R_2 - 5R_1$

$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$x - y = 0$

$X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} r.$

Since they are two Independent linear vector they are diagonalizgation.

Let $C = \begin{bmatrix} 1 & 1 \\ 5 & 1 \end{bmatrix}$

Let $C = \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}$

$C^{-1} = \frac{1}{|A|}$ cofactor.

$C^{-1} = \frac{1}{|4|} \begin{bmatrix} 5 & -1 \\ -1 & 1 \end{bmatrix}$ cofactor.

$= C^{-1} A C$.

$= \frac{1}{4} \begin{bmatrix} 5 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 8 & -1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}$

$= \frac{1}{4} \begin{bmatrix} 35 & -7 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}$

$= \frac{1}{4} \begin{bmatrix} 28 & 0 \\ 0 & 12 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 3 \end{bmatrix}$

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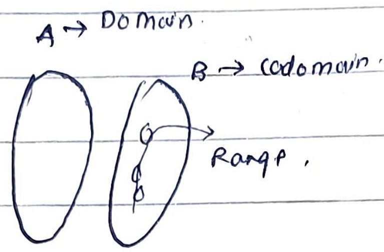
CALCULAS.

univarite calculus.

Function



$F(x) = \sin x$.

$y(x) = \ln x$.

$y = e^x$

$y = \tan^{-1} x$

$y = \bar{x} + 3x - 4$.

$y = \cosh x$.

$[-1 \ 1] \to$ close.

$(-1 \ 1)$ open.

$(-1 \ 1]$ one open/ one close.

$y = \sqrt{x}$ $\quad [0 \ \infty)$

$F(x) = \sqrt{9 - x^2}$

$x^2 \geq 9$

$(-\infty \ -3] \cup [3, \infty)$

$$F(x) = \frac{1}{\sqrt{9 - x^2}}$$

$(-3, 3)$

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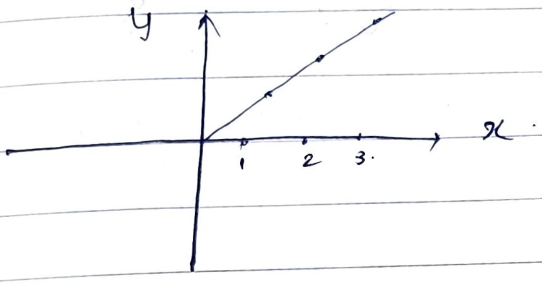
$$y = f(x) = \frac{x^2 - 9}{x - 3} \cdot \quad x \neq 3.$$

$$\frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = (x+3)$$



$$\text{Limit } x \to 3 \quad \frac{x^2-9}{x-3}.$$

$$\text{limit } x \to 3 \quad \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$$

$$= -6.$$

$$\text{limit } x = 3 \to \quad 9x^2 + 2 = 9(3)^2 + 2 = 83.$$

DERIVATIVES : the rate of chang of function with respect to an indewended variabl.

$y = f(x)$

- Differentiability
- Derivative.

Division rule of derivative. = $d\left(\frac{u}{v}\right) = \frac{v\,d(u) - u\,d(v)}{v^2}$.

Change rule $y = \sin(x^2)$

$= \cos(x^2)\, 2x$.

formula. $\frac{dy}{dx}\, x^2 = 2x$.

let $t = x^2$.

$y = \sin t$ $\qquad t = x^2$.

$\sin x = \cos x$.

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \cos t \times 2x$

$= \cos(x^2) \times 2x$.

* $y = e^{3x^2+2x} \cdot \sin (5x+9)^2]$.

$y' = e^{3x^2+2x} \left[\cos \left[(5x+9)^2 \right] \right] \times 2(5x+9) \times 5]$

$e^{3x^2+2x}. \;(6x+2) \times \sin (5x+9)^2$

Geometry definition of curve. → slope of a tangent to the curve.

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FORMULAS.

$\frac{dy}{dx} x^2 = 2x$.

$\frac{dy}{dx} (\sin x) = \cos x$.

$\frac{dy}{dx} (\cos x) = -\sin x$.

$\frac{dy}{dx} (\tan x) = \sec^2(x)$.

$\frac{dy}{dx} (\cot x) = -\operatorname{cosec}^2(x)$

$\frac{dy}{dx} (\operatorname{cosec} x) = -(\operatorname{cosec} x)(\cot x)$

$\frac{dy}{dx} (\sec x) = (\sec x)(\tan x)$

$\frac{dy}{dx} \ln(x) = \frac{1}{x}$.

$\frac{dy}{dx} a^x = a^x \log a$.

$\frac{dy}{dx} x^x = x^x + \ln x$

$\frac{dy}{dx} e^x = e^x$.

$\frac{dy}{dx} (k) = 0$; k is any constant.

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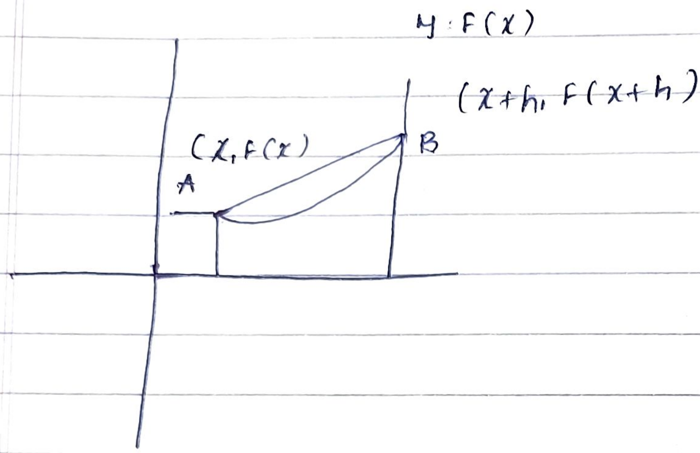
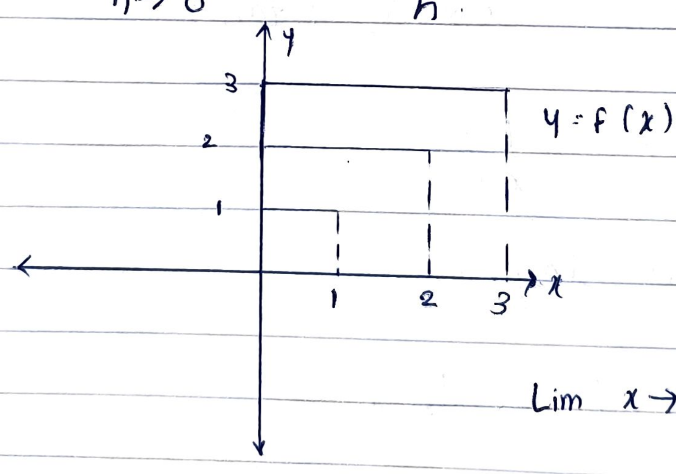
$$\boxed{\text{slope} : \frac{y_2 - y_1}{x_2 - x_1}}$$ of line joining ~~y~~ (x_1, y_1) & (x_2, y_2)

* Derivative.



Slope of tgt at A to the curve $y = f(x)$ $= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

$$\frac{dy}{dx} = y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$



$\lim_{x \to 1^-} \neq \lim_{x \to 1^+}$

Mode → if x is 1 it will be 1 if x is -1 it will be 1 only. Sign changes

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Calculate the derivative $y = x^2$ using first principle of derivative.
$\rightarrow f(x) = x^2$.

Soln : $y' = \lim_{h \to 0} \frac{(x+h)^2 - f(x)}{h}$

$= \frac{(x+h)^2 - x^2}{h}$

$= \lim_{h \to 0} \frac{(\cancel{x^2} + 2hx + h^2 - \cancel{x^2})}{h}$ → by quadratic.

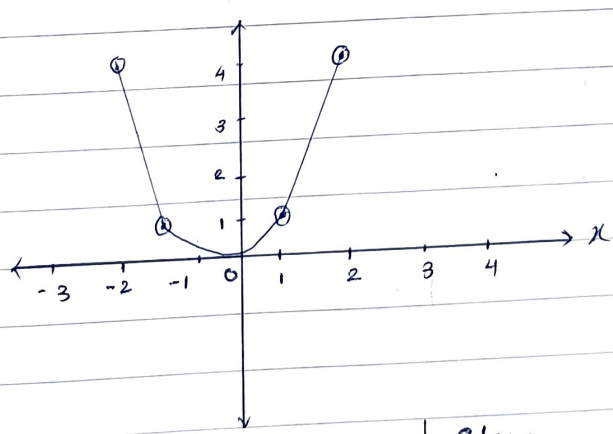
$= \frac{2hx + h^2}{h}$

$= \frac{\cancel{h}(2x + h)}{\cancel{h}}$

limit $h = 0$: $2x + h$

$= \underline{\underline{2x}}$.

PARABOLA. $\longrightarrow y = x^2$.



At origin tgt is x axis
slope = 0

$y'|_{x=0}$

Slope of tgt at $x = 1$

$y'|_{x=1}$
$= 2$

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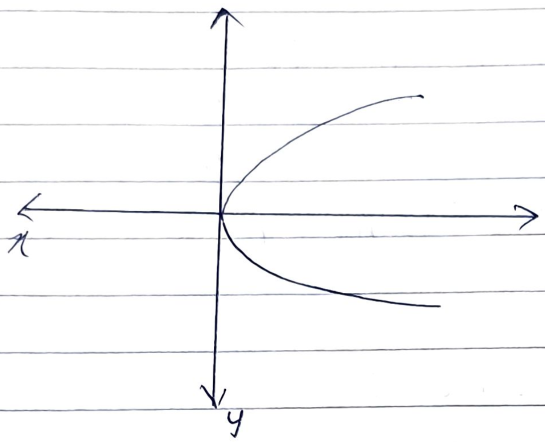
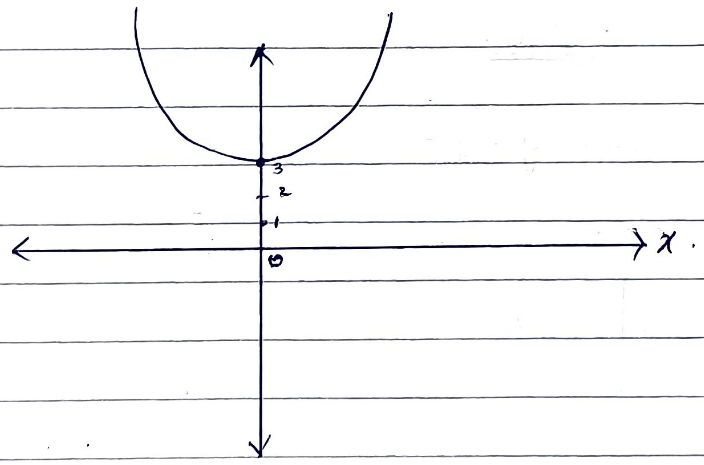
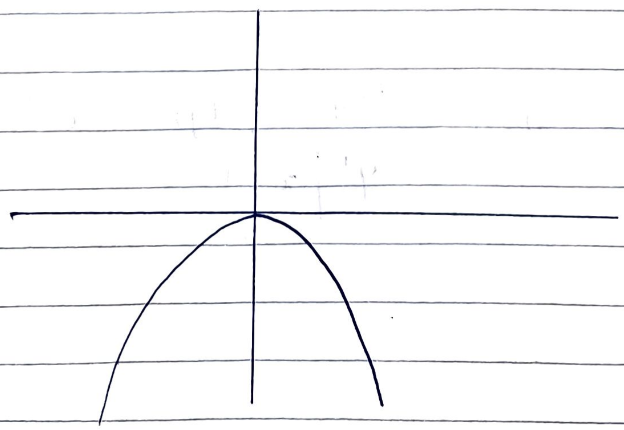
$x = y^2.$



$y = x^2 + 3$



$y = \cancel{3} - x^2.$ inverted parabola.



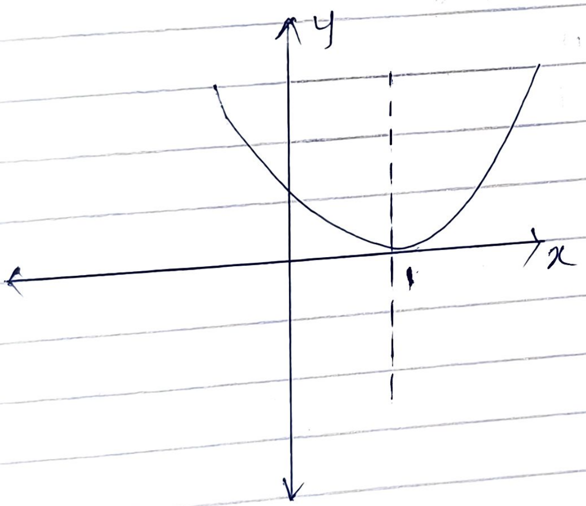
$x^2+y^2=a^2$ — circle.

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ — ellipse.

$x^2-y^2=a^2$ — hyperbola.

$y=x^2$ — parabola.

* $y=(x-1)^2$.



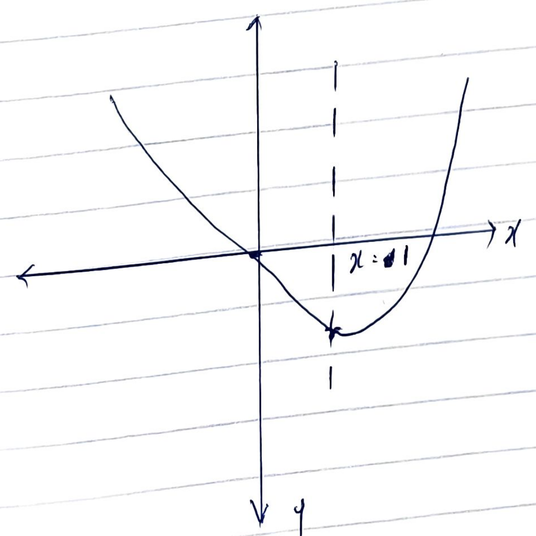
* $y=x^2-2x$

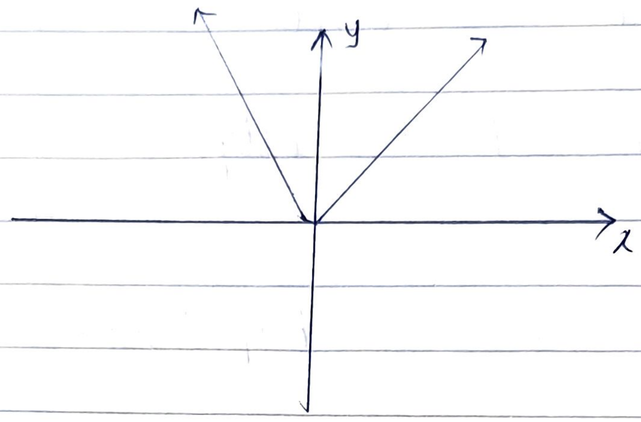
$= x^2-2x+1-1$

$= (x-1)^2-1$

axis $\rightarrow$ $x-1=0$

$x=1$





Mod x.

$y = |x|$

$\lim_{h \to 0^-} \; \dfrac{F(0+h) - F(0)}{h}$

$\lim_{h \to 0^-} = \dfrac{-h}{h} = -1$

(Actual syllabus)

* Extreme Values

Absolute extreme values.

$F(0) = 1/2$
$F(x) = 0$ $\quad\quad 0x = 1$
$F(1) = 1/2$.

first step → find the derivative.

Q. Determine absolute extreme values for the function in the given domain

$F(x) = \sqrt{4-x^2} \quad -2 \le x = 1$

f is continuous on. $[-2, 1]$
Both the absolute extreme value exists.

1.] $y' = F'(x) - \frac{1}{\cancel{2}\sqrt{4-x^2}} \times (-\cancel{2}x) = \frac{-x}{\sqrt{4-x^2}}$.

(i) at $x = 0$
$F'(x) = 0$.

(ii) y' does not exist.
if $\sqrt{4-x^2} = 0$
$x^2 = 4$.
$x = \pm 2$

-2 is an end point. It is not a critical point.

$x = 2 \notin [-2, 1]$

pt	$f(x)$
$x=0$ critical.	2
$x=-2$ end	0
$x=1$	$\sqrt{3}$.

Absolut max is 2 and it occur $x=0$.
Absolute min is 0 and it occur at $x=-2$.

2. $F(x) = \dfrac{-1}{x+3}$ $\quad -2 \leq x \leq 3$.

$F'(x) = \dfrac{1}{(x+3)^2}$.

No critical point.