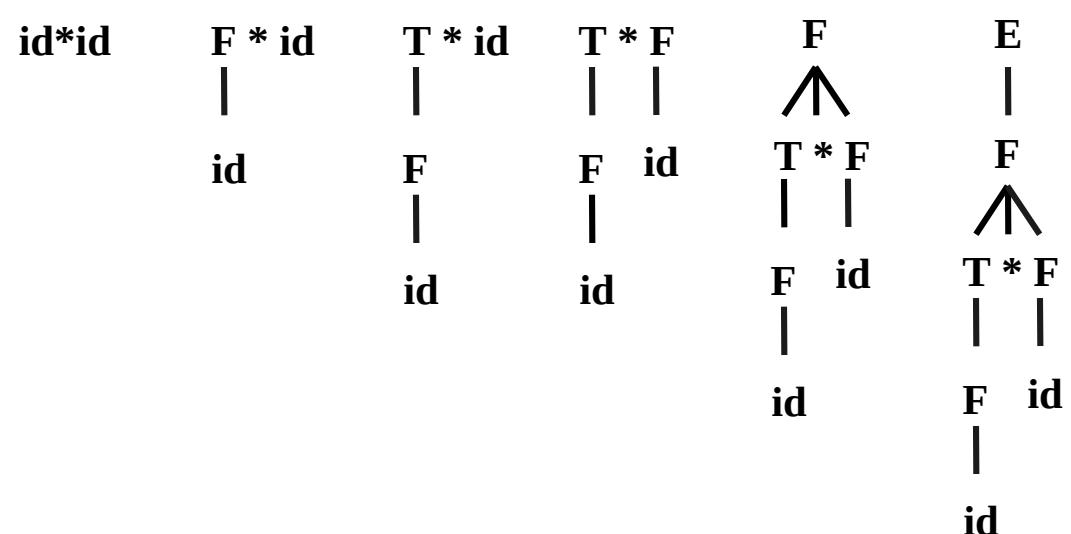


COMPILER DESIGN

Topic: Bottom-Up parsing

Introduction

- Constructs parse tree for an input string beginning at the leaves (the bottom) and working towards the root (the top)
- Example: id^*id

$$\begin{array}{l} E \rightarrow E + T \mid T \\ T \rightarrow T * F \mid F \\ F \rightarrow (E) \mid \text{id} \end{array}$$


Shift-reduce parser

- The general idea is to shift some symbols of input to the stack until a reduction can be applied
- At each reduction step, a specific substring matching the body of a production is replaced by the nonterminal at the head of the production
- The key decisions during bottom-up parsing are about when to reduce and about what production to apply
- A reduction is a reverse of a step in a derivation
- The goal of a bottom-up parser is to construct a derivation in reverse:
 - $E \Rightarrow T \Rightarrow T^*F \Rightarrow T^*id \Rightarrow F^*id \Rightarrow id^*id$

Handle pruning

- A Handle is a substring that matches the body of a production and whose reduction represents one step along the reverse of a rightmost derivation

Right sentential form	Handle	Reducing production
$\text{id}^* \text{id}$	id	$F \rightarrow \text{id}$
$F^* \text{id}$	F	$T \rightarrow F$
$T^* \text{id}$	id	$F \rightarrow \text{id}$
$T^* F$	$T^* F$	$E \rightarrow T^* F$

Shift reduce parsing

- A stack is used to hold grammar symbols
- Handle always appear on top of the stack
- Initial configuration:

Stack	Input
\$	w\$

- Acceptance configuration

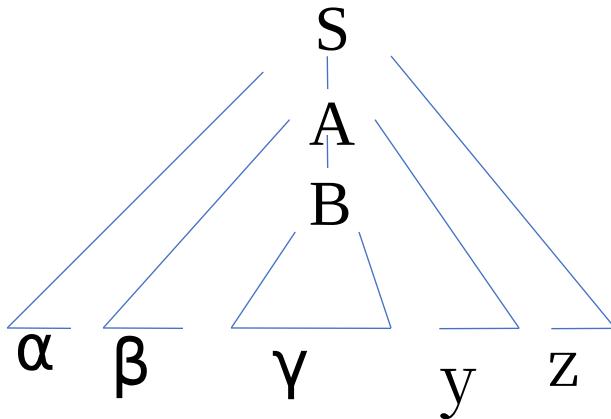
Stack	Input
\$S	\$

Shift reduce parsing (cont.)

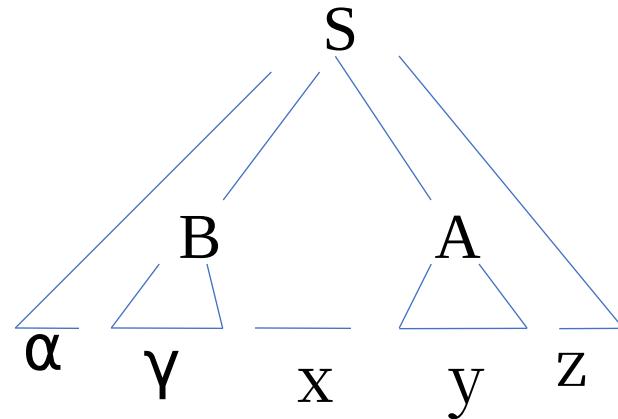
- Basic operations:
 - Shift
 - Reduce
 - Accept
 - Error
- Example: $\text{id}^* \text{id}$

Stack	Input	Action
\$	$\text{id}^* \text{id} \$$	shift
\$id	$* \text{id} \$$	reduce by $F \rightarrow \text{id}$
\$F	$* \text{id} \$$	reduce by $T \rightarrow F$
\$T	$* \text{id} \$$	shift
\$T*	$\text{id} \$$	shift
\$T*id	\$	reduce by $F \rightarrow \text{id}$
\$T*F	\$	reduce by $T \rightarrow T^*F$
\$T	\$	reduce by $E \rightarrow T$
\$E	\$	accept

Handle will appear on top of the stack



Stack	Input
$\$ \alpha \beta \gamma$	$yz \$$
$\$ \alpha \beta B$	$yz \$$
$\$ \alpha \beta B y$	$z \$$



Stack	Input
$\$ \alpha \gamma$	$xyz \$$
$\$ \alpha B x y$	$z \$$

Conflicts during shift/reduce parsing

- Two kinds of conflicts
 - Shift/reduce conflict
 - Reduce/reduce conflict
- Example:

stmt → If expr then stmt
| If expr then stmt else stmt
| other

Stack	Input
... if expr then stmt	else ...\$

Reduce/reduce conflict

stmt -> id(parameter_list)

stmt -> expr:=expr

parameter_list->parameter_list, parameter

parameter_list->parameter

parameter->id

expr->id(expr_list)

expr->id

expr_list->expr_list, expr

expr_list->expr

Stack
... id(id

Input
,id) ...\$

LR Parsing

- The most prevalent type of bottom-up parsers
- LR(k), mostly interested on parsers with $k \leq 1$
- Why LR parsers?
 - Table driven
 - Can be constructed to recognize all programming language constructs
 - Most general non-backtracking shift-reduce parsing method
 - Can detect a syntactic error as soon as it is possible to do so
 - Class of grammars for which we can construct LR parsers are superset of those which we can construct LL parsers

States of an LR parser

- States represent set of items
- An LR(0) item of G is a production of G with the dot at some position of the body:
 - For $A \rightarrow XYZ$ we have following items
 - $A \rightarrow .XYZ$
 - $A \rightarrow X.YZ$
 - $A \rightarrow XY.Z$
 - $A \rightarrow XYZ.$
 - In a state having $A \rightarrow .XYZ$ we hope to see a string derivable from XYZ next on the input.
 - What about $A \rightarrow X.YZ$?

Constructing canonical LR(0) item sets

- Augmented grammar:
 - G with addition of a production: $S' \rightarrow S$
- Closure of item sets:
 - If I is a set of items, closure(I) is a set of items constructed from I by the following rules:
 - Add every item in I to closure(I)
 - If $A \rightarrow \alpha.B\beta$ is in closure(I) and $B \rightarrow \gamma$ is a production then add the item $B \rightarrow .\gamma$ to closure(I).
- Example:

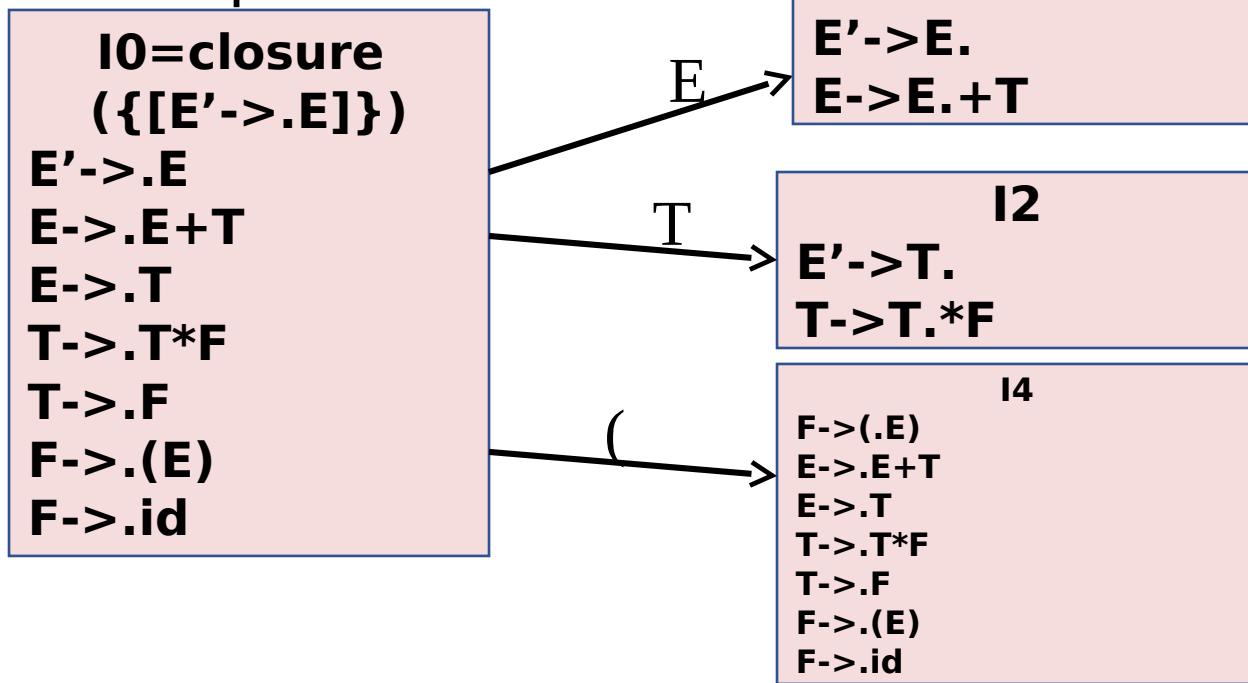
$E' \rightarrow E$
 $E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid id$

I₀=closure({[E'->.E]})

$E' \rightarrow .E$
 $E \rightarrow .E + T$
 $E \rightarrow .T$
 $T \rightarrow .T * F$
 $T \rightarrow .F$
 $F \rightarrow .(E)$
 $F \rightarrow .id$

Constructing canonical LR(0) item sets (cont.)

- Goto (I, X) where I is an item set and X is a grammar symbol is closure of set of all items $[A \rightarrow \alpha X . \beta]$ where $[A \rightarrow \alpha . X \beta]$ is in I
- Example



Closure algorithm

```
SetOfItems CLOSURE(I) {  
    J=I;  
    repeat  
        for (each item A-> α.Bβ in J)  
            for (each production B->γ of G)  
                if (B->.γ is not in J)  
                    add B->.γ to J;  
    until no more items are added to J on one round;  
    return J;
```

GOTO algorithm

```
SetOfItems GOTO(I,X) {  
    J=empty;  
    if (A-> α.X β is in I)  
        add CLOSURE(A-> αX. β ) to J;  
    return J;  
}
```

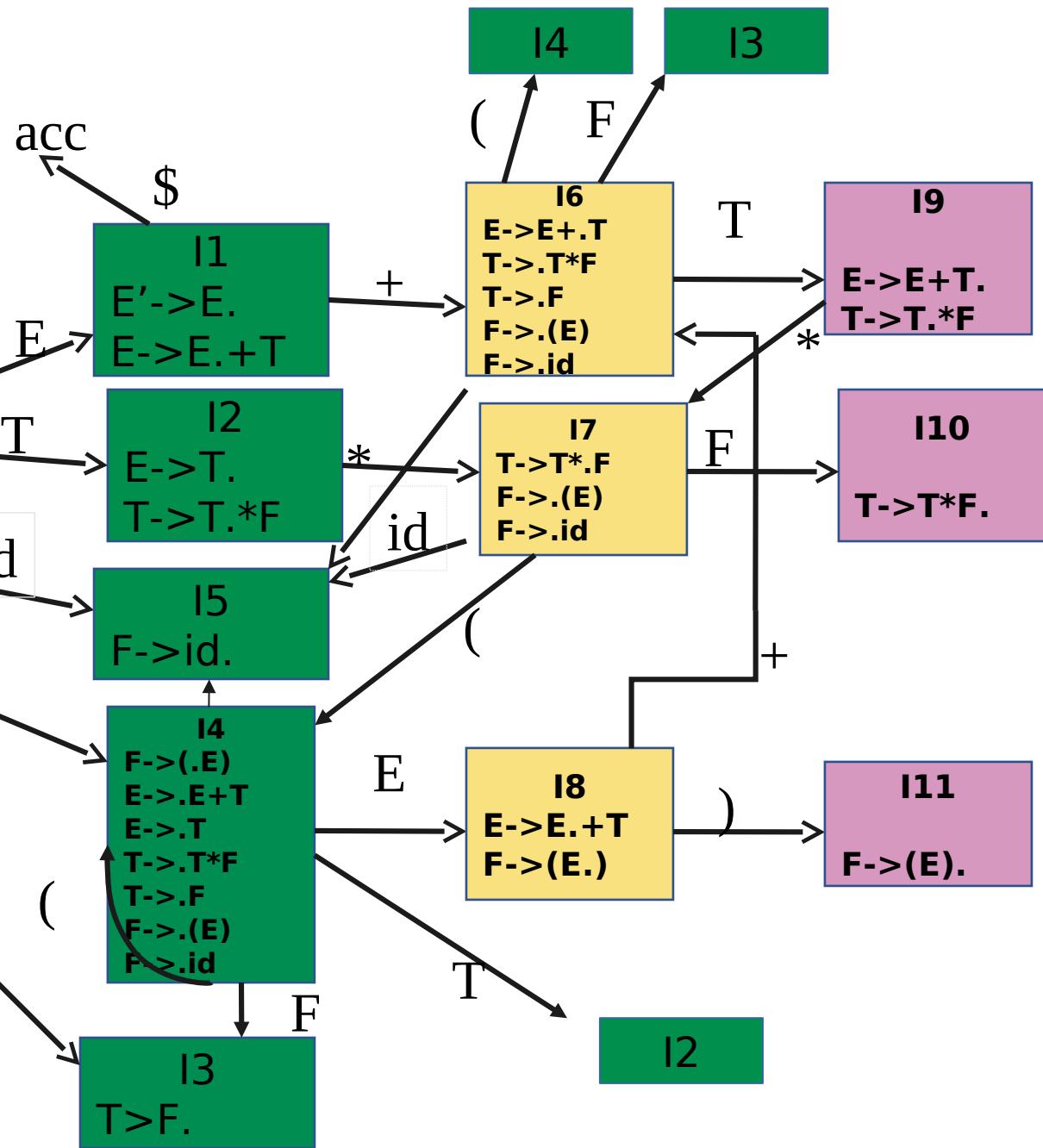
Canonical LR(0) items

```
Void items(G') {  
    C= CLOSURE({[S'->.S]});  
    repeat  
        for (each set of items I in C)  
            for (each grammar symbol X)  
                if (GOTO(I,X) is not empty and not in C)  
                    add GOTO(I,X) to C;  
    until no new set of items are added to C on a round;  
}
```

$E' \rightarrow E$ $E \rightarrow E + T \mid T$ $T \rightarrow T * F \mid F$ $F \rightarrow (E) \mid id$

I0=closure ($\{[E' \rightarrow .E]\}$)

$E' \rightarrow .E$
 $E \rightarrow .E + T$
 $E \rightarrow .T$
 $T \rightarrow .T * F$
 $T \rightarrow .F$
 $F \rightarrow .(E)$
 $F \rightarrow .id$



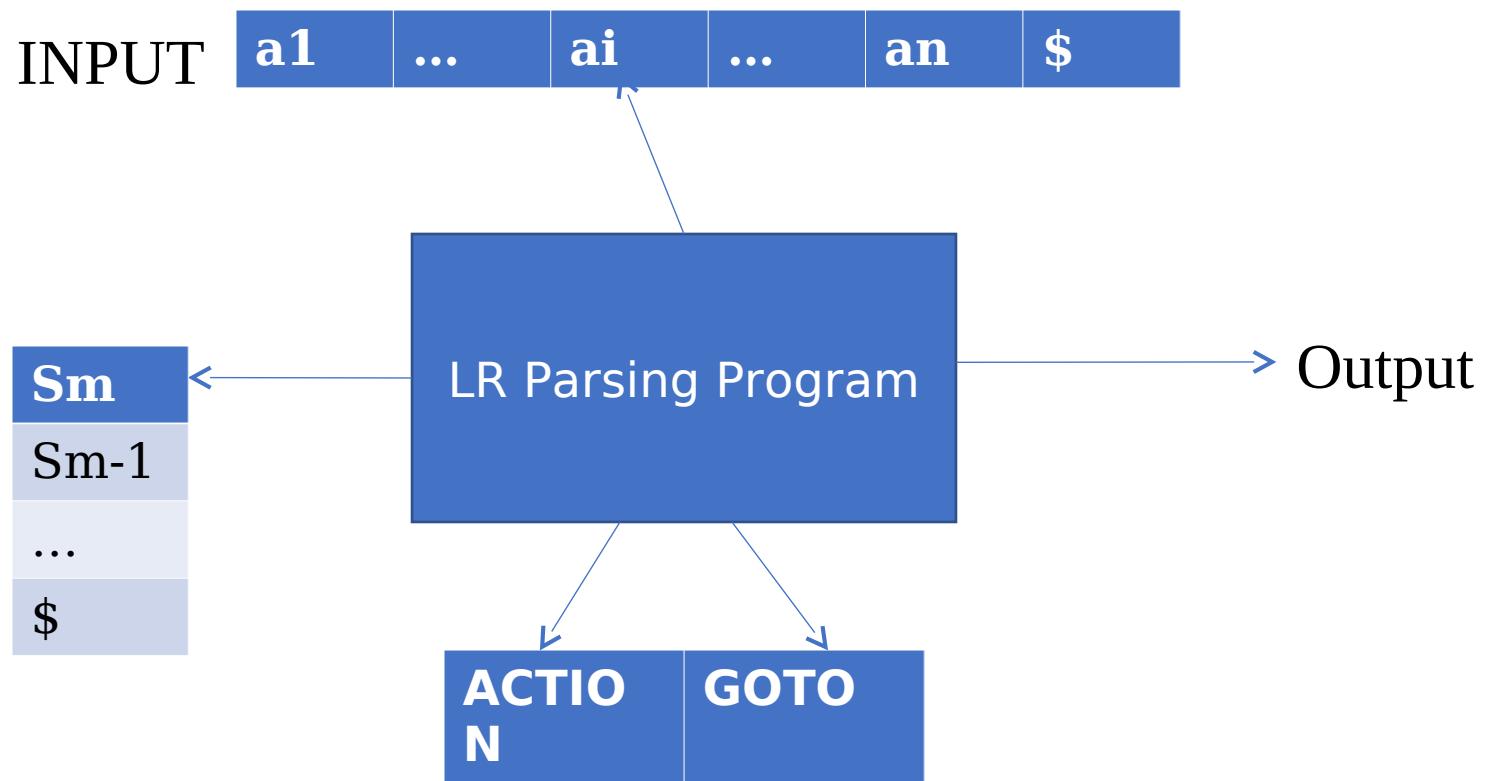
STATE	ACTION							GOTO		
	id	+	*	()	\$	E	T	F	
0	S5				S4			1	2	3
1		S6					Acc			
2		R2	S7			R2	R2			
3		R4	R4			R4	R4			
4	S5			S4				8	2	3
5		R6	R6			R6	R6			
6	S5			S4					9	3
7	S5			S4						10
8		S6				S11				
9		R1	S7			R1	R1			
10		R3	R3			R3	R3			
11		R5	R5			R5	R5			

Use of LR(0) automaton

- Example: $\text{id}^* \text{id}$

Line	Stack	Symbols	Input	Action
(1)	0	\$	$\text{id}^* \text{id} \$$	Shift to 5
(2)	05	\$id	*id\$	Reduce by F->id
(3)	03	\$F	*id\$	Reduce by T->F
(4)	02	\$T	*id\$	Shift to 7
(5)	027	\$T*	id\$	Shift to 5
(6)	0275	\$T*id	\$	Reduce by F->id
(7)	02710	\$T*F	\$	Reduce by T->T*F
(8)	02	\$T	\$	Reduce by E->T
(9)	01	\$E	\$	accept

LR-Parsing model



LR parsing algorithm

```
let a be the first symbol of w$;  
while(1) { /*repeat forever */  
    let s be the state on top of the stack;  
    if (ACTION[s,a] = shift t) {  
        push t onto the stack;  
        let a be the next input symbol;  
    } else if (ACTION[s,a] = reduce A->β) {  
        pop |β| symbols of the stack;  
        let state t now be on top of the stack;  
        push GOTO[t,A] onto the stack;  
        output the production A->β;  
    } else if (ACTION[s,a]=accept) break; /* parsing is done */  
    else call error-recovery routine;  
}
```

Example

STAT E	ACTON						GOTO		
	id	+	*	()	\$	E	T	F
0	S5			S4			1	2	3
1		S6				Ac c			
2		R2	S7		R2	R2			
3		R4	R4		R4	R4			
4	S5			S4			8	2	3
5		R6	R6		R6	R6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S1 1				
9		R1	S7		R1	R1			
10		R3	R3		R3	R3			
11		R5	R5		R5	R5			

- (0) $E' \rightarrow E$
- (1) $E \rightarrow E + T$
- (2) $E \rightarrow T$
- (3) $T \rightarrow T * F$
- (4) $T \rightarrow F$
- (5) $F \rightarrow (E)$
- (6) $F \rightarrow id$

$id * id + id ?$

Lin e	Stac k	Symbo ls	Input	Action
(1)	0		$id * id + id$ \$	Shift to 5
(2)	05	id	$* id + id $$	Reduce by $F \rightarrow id$
(3)	03	F	$* id + id $$	Reduce by $T \rightarrow F$
(4)	02	T	$* id + id $$	Shift to 7
(5)	027	T^*	$id + id $$	Shift to 5
(6)	027 5	$T^* id$	$+ id $$	Reduce by $F \rightarrow id$
(7)	027 10	$T^* F$	$+ id $$	Reduce by $T \rightarrow T^* F$
(8)	02	T	$+ id $$	Reduce by $E \rightarrow T$
(9)	01	E	$+ id $$	Shift
(10)	016	$E +$	$id $$	Shift
(11)	016 5	$E + id$	\$	Reduce by $F \rightarrow id$

Constructing SLR parsing table

- Method
 - Construct $C = \{I_0, I_1, \dots, I_n\}$, the collection of LR(0) items for G'
 - State i is constructed from state I_i :
 - If $[A \rightarrow \alpha.a\beta]$ is in I_i and $\text{Goto}(I_i, a) = I_j$, then set $\text{ACTION}[i, a]$ to “shift j ”
 - If $[A \rightarrow \alpha.]$ is in I_i , then set $\text{ACTION}[i, a]$ to “reduce $A \rightarrow \alpha$ ” for all a in $\text{follow}(A)$
 - If $[S' \rightarrow .S]$ is in I_i , then set $\text{ACTION}[i, \$]$ to “Accept”
 - If any conflicts appears then we say that the grammar is not SLR(1).
 - If $\text{GOTO}(I_i, A) = I_j$ then $\text{GOTO}[i, A] = j$
 - All entries not defined by above rules are made “error”
 - The initial state of the parser is the one constructed from the set of items containing $[S' \rightarrow .S]$

Example grammar which is not SLR(1)

$$S \rightarrow L=R \mid R$$
$$L \rightarrow *R \mid id$$
$$R \rightarrow L$$

I0

$S' \rightarrow S.$

$S \rightarrow .L=R$

$S \rightarrow .R$

$L \rightarrow .*R \mid$

$L \rightarrow .id$

$R \rightarrow .L$

I1

$S' \rightarrow S.$

I2

$S \rightarrow L.=R$

$R \rightarrow L.$

I3

$S \rightarrow R.$

I4

$L \rightarrow *.R$

$R \rightarrow .L$

$L \rightarrow .*R$

$L \rightarrow .id$

I5

$L \rightarrow id.$

I6

$S \rightarrow L.=R$

$R \rightarrow .L$

$L \rightarrow .*R$

$L \rightarrow .id$

I7

$L \rightarrow *R.$

I8

$R \rightarrow L.$

I9

$S \rightarrow L=R.$

2

Action

=

Shift 6
Reduce $R \rightarrow L$

More powerful LR parsers

- Canonical-LR or just LR method
 - Use lookahead symbols for items: LR(1) items
 - Results in a large collection of items
- LALR: lookahead symbols are introduced in LR(0) items

Canonical LR(1) items

- In LR(1) items each item is in the form: $[A \rightarrow \alpha.\beta, a]$
- An LR(1) item $[A \rightarrow \alpha.\beta, a]$ is valid for a viable prefix γ if there is a derivation $S \Rightarrow^* \delta A w \Rightarrow^* \delta \alpha \beta w$, where
 - $\Gamma = \delta \alpha$
 - Either a is the first symbol of w , or w is ϵ and a is $\$$
- Example:
 - $S \rightarrow BB$
 - $B \rightarrow aB | b$

$$S \Rightarrow^* aaBab \xrightarrow{rm} aaaBab$$

Item $[B \rightarrow a.B, a]$ is valid for $\gamma = aaa$ and $w = ab$

Constructing LR(1) sets of items

```
SetOfItems Closure(I) {
    repeat
        for (each item [A-> $\alpha$ .B $\beta$ ,a] in I)
            for (each production B-> $\gamma$  in G')
                for (each terminal b in First( $\beta$ a))
                    add [B->. $\gamma$ , b] to set I;
```

until no more items are added to I;

return I;

}

```
SetOfItems Goto(I,X) {
    initialize J to be the empty set;
    for (each item [A-> $\alpha$ .X $\beta$ ,a] in I)
        add item [A-> $\alpha$ X. $\beta$ ,a] to set J;
    return closure(J);
}
```

```
void items(G') {
    initialize C to Closure({[S'->. $S$ , $]});
    repeat
        for (each set of items I in C)
            for (each grammar symbol X)
                if (Goto(I,X) is not empty and not in C)
                    add Goto(I,X) to C;
    until no new sets of items are added to C;
}
```

Example

$S' \rightarrow S$

$S \rightarrow CC$

$C \rightarrow cC$

$C \rightarrow d$

Canonical LR(1) parsing table

- Method
 - Construct $C = \{I_0, I_1, \dots, I_n\}$, the collection of LR(1) items for G'
 - State i is constructed from state I_i :
 - If $[A \rightarrow \alpha.a\beta, b]$ is in I_i and $\text{Goto}(I_i, a) = I_j$, then set $\text{ACTION}[i, a]$ to “shift j ”
 - If $[A \rightarrow \alpha., a]$ is in I_i , then set $\text{ACTION}[i, a]$ to “reduce $A \rightarrow \alpha$ ”
 - If $[S' \rightarrow .S, \$]$ is in I_i , then set $\text{ACTION}[i, \$]$ to “Accept”
 - If any conflicts appears then we say that the grammar is not LR(1).
 - If $\text{GOTO}(I_i, A) = I_j$ then $\text{GOTO}[i, A] = j$
 - All entries not defined by above rules are made “error”
 - The initial state of the parser is the one constructed from the set of items containing $[S' \rightarrow .S, \$]$

Example

$S' \rightarrow S$

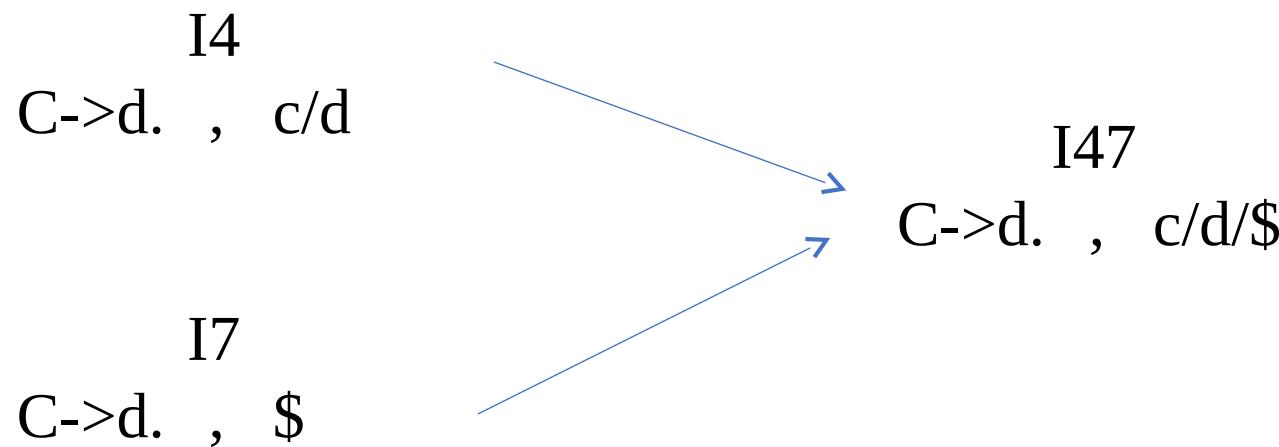
$S \rightarrow CC$

$C \rightarrow cC$

$C \rightarrow d$

LALR Parsing Table

- For the previous example we had:



- State merges can't produce Shift-Reduce conflicts. Why?
- But it may produce reduce-reduce conflict

Example of RR conflict in state merging

S'->S

S -> aAd | bBd | aBe | bAe

A -> c

B -> c

An easy but space-consuming LALR table construction

- Method:
 1. Construct $C = \{I_0, I_1, \dots, I_n\}$ the collection of LR(1) items.
 2. For each core among the set of LR(1) items, find all sets having that core, and replace these sets by their union.
 3. Let $C' = \{J_0, J_1, \dots, J_m\}$ be the resulting sets. The parsing actions for state i , is constructed from J_i as before. If there is a conflict grammar is not LALR(1).
 4. If J is the union of one or more sets of LR(1) items, that is $J = I_1 \cup I_2 \dots \cup I_k$ then the cores of $\text{Goto}(I_1, X), \dots, \text{Goto}(I_k, X)$ are the same and is a state like K , then we set $\text{Goto}(J, X) = k$.
- This method is not efficient, a more efficient one is discussed in the book

Compaction of LR parsing table

- Many rows of action tables are identical
 - Store those rows separately and have pointers to them from different states
 - Make lists of (terminal-symbol, action) for each state
 - Implement Goto table by having a link list for each nonterminal in the form (current state, next state)

Using ambiguous grammars

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$E \rightarrow id$

I0: $E' \rightarrow .E$

$E \rightarrow .E + E$

$E \rightarrow .E * E$

$E \rightarrow .(E)$

$E \rightarrow .id$

I1: $E' \rightarrow E.$

$E \rightarrow E.+E$

$E \rightarrow E.*E$

$E \rightarrow .(E)$

$E \rightarrow .id$

I3: $E \rightarrow .id$

I4: $E \rightarrow E+.E$

$E \rightarrow .E + E$

$E \rightarrow .E * E$

$E \rightarrow .(E)$

$E \rightarrow .id$

I5: $E \rightarrow E*.E$

$E \rightarrow .(E)$

$E \rightarrow .E + E$

$E \rightarrow .E * E$

$E \rightarrow .(E)$

$E \rightarrow .id$

STATE	ACTION							GO TO
	id	+	*	()	\$		
0	S3			S2				1
1		S4	S5				Acc	
2	S3		S2					6
3		R4	R4			R4	R4	
4	S3			S2				7
5	S3			S2				8
6		S4	S5					
7		R1	S5			R1	R1	
8		R2	R2			R2	R2	
9		R3	R3			R3	R3	

I6: $E \rightarrow (E.)$

$E \rightarrow E.+E$

$E \rightarrow E.*E$

$E \rightarrow .(E)$

$E \rightarrow .id$

$E \rightarrow E.*E$

I7: $E \rightarrow E+E.$

$E \rightarrow E.E + E$

$E \rightarrow E.E * E$

$E \rightarrow .(E)$

$E \rightarrow E.+E$

$E \rightarrow E.*E$

I8: $E \rightarrow E*E.$

$E \rightarrow E.E + E$

$E \rightarrow E.*E$

I9: $E \rightarrow (E).$

Thank You