COEP Technogical University

Department of Mathematics
(MA- 21001) Probability and Statistics for Engineers
T.Y. B. Tech. Semester V
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1 Tutorial: Week 4

1. Define random variable, discrete and continuous random variables giving five examples of each.

2. For each of the following exercises, determine the range (possible values) of the random variable.

   i. A software program has 5000 lines of code. The random variable is the number of lines with a fatal error.
   ii. A batch of 500 machined parts contains 10 that do not conform to customer requirements. The random variable is the number of parts in a sample of 5 parts that do not conform to customer requirements.
   iii. There is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted.

3. Determine, with reasons, which of the following functions are probability distribution functions and if it is then say what type of distribution it represents:

   (i) $f(x) = 2(1 + x)/27$ Hint/Ans: not a pdf since no interval is specified and the area under the curve is not one.

   (ii) $f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \leq x < 2, \\ 0, & \text{elsewhere} \end{cases}$

   (iii)

x	0	1	2	3	4	5	6
$f(x)$	0.41	0.35	0.15	0.10	0.04	0.01	0

   (iv)

x	1	2	3	4	5	6
$f(x)$	0.35	0.33	0.18	0.10	0.03	0.01

   Further, If X represents the number of imperfections per 10 m of a fabric in example 2 (iv) above, then how many imperfections do you expect in a 100m roll of fabric?

   (v) $f(x) = \dfrac{-x^2}{2024}$ on $(-1, 1)$ and zero otherwise.

   By making suitable changes in the required functions above, make them into probability distribution functions. Is it the only way? Hint/Ans:(i) can be made into a pdf in infinitely many ways since we get only one eqn involving two unknown limits. For example 2 < x < 5 is one interval which makes it into a prob distn of a cts r.v.

Hint/Ans:(iii) if we have to change only one of the probabilities then then there is a unique way but then we can not do this by using the last three probabilities.

4. Find the cumulative distribution functions for the above distribution functions.

5. Consider the following distribution function of a random variable X :

$$F(x) = \begin{cases} 0 & \text{if } x < 2 \\ \dfrac{-x^2}{4} + 2x - 3 & \text{if } -2 \leq x \leq 4 \\ 1 & \text{if } x > 4. \end{cases}$$

   (i)What is the PDF of X?

   (ii) Calculate $P(X < 3)$ and $P(X = 4)$.

   (iii) Determine $E(X)$ and $\mathrm{Var}(X)$.

6. An innovative winemaker experiments with new grapes and adds a new wine to his stock. The percentage sold by the end of the season depends upon weather and various other factors. It can be modelled using the random variable X with the CDF as

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 3x^2 - 2x^3 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1. \end{cases}$$

   Plot the cumulative distribution function with R. Determine $f(x)$. What is the probability of selling at least one-third of his wine, but not more than two-thirds? Define CDF in R and calculate the probability of above question again. What is the variance of X?

7. A continuous random variable X defined in the interval $[1, 10]$ has a constant density function. Find it. Hence find

   (i) $P(X \leq 4)$

   (ii) $P(2 < X \leq 7)$

   (iii) $P(X = 5)$

   (iv) k if $P(k \leq X < 9) = 0.5$

8. Let the random variable X denote the number of semiconductor wafers that need to be analyzed in order to detect a large particle of contamination. Assume that the probability that a wafer contains a large particle is 0.01 and that the wafers are independent. Determine the probability distribution of X.

   Hint: Let p denote a wafer in which a large particle is present, and let a denote a wafer in which it is absent. The sample space of the experiment is infinite, and it can be represented as all possible sequences that start with a string of a's and end with p. Write sample space and probability formula for the general element in the sample space.

   Ans: $P(X = x) = (0.99)^{x-1}(0.01)$

9. A random variable X has the following p.m.f.

X	0	1	2	3	4	5	6
p(X=x)	k	3k	5k	7k	9k	11k	13k

Find (i) k (ii) $p(X \geq 2)$ (iii) $p(0 < X < 5)$ (iv) What is the minimum value of C for which $p(X \leq c) > 0.5$?
(Ans: (i) $k = \frac{1}{49}$ (ii) $\frac{45}{49}$ (iii) $\frac{15}{49}$ (iv) Minimum value of C is 4.)

10. Verify whether the assignment $P(X = n) = 2^{-n},\ n = 1, 2, 3, \ldots$ is a probability mass function for random variable X. (Ans: It is a p.m.f.)

2 Some general questions

11. Suppose measurements of an item with a metric micrometer A yield a mean of 4.20 mm and a standard deviation of 0.015 mm. Measurements of another item with an English micrometer B yield a mean of 1.1 inches and a standard deviation of 0.005 inches. Which micrometer is relatively "more" precise?
Ans: Micrometer A is more precise. Why?

12. Statistics and Probability is the title of a book. If each letter was carved into a block and dropped into a bag, what are the chances a person would draw either the letter A or I from the bag? (Ans: 7/24)

13. A manufacturing company is set up in two different locations. If the number of employees in one location are 663, and the average monthly salary for their employees is Rs.13454, and the number of employees in the other location are 504, and the average monthly salary for their employees is Rs.17591. Find the combined arithmetic mean of the monthly salary? Ans: Rs. 15240.67

14. Given 2 samples, Sample 1 = [13.3, 2.4, 10, 13.3, 11] and Sample 2 = [8.5, 7.1, 12.6, 11.5, 10.3]. Find the sample which has a relatively greater spread of values from the mean? Ans: Sample 1

15. Ben is the customer relation manager at a hotel. Recently, Ben has been receiving customer feedback saying that the customers had to wait too long to be served by a customer service representative. Ben decides to note down the customer's waiting time in minutes. What kind of graph would be appropriate to check the frequency distributions of customers' waiting time? Ans: Histogram

16. 3 natural numbers are chosen at random. What is the probability that their product yields an odd number? (Ans: 1/8)

17. The mean of the first n natural numbers is Ans: (n+1)/2

18. 128 players are participating in a knockout tournament. How many games are required to decide the winner? Note: In a knockout tournament, whenever two people play, the loser is eliminated and the winner advances to the next round.

    (Ans: 127)

19. A fair coin is tossed 10 times. What is the probability of getting more heads than tails?

20. A fair coin is tossed 10 times. What is the probability of getting less heads than tails?