Used when $n < 30$, and σ is unknown

Date . .

II t- TEST:-

* This test is used when σ^2 is known.
* Sample Variance $S_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$.

Population Variance, $\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \mu)^2$.

Steps:-

① (i), (ii), (iii) } Same as Z-test

② Constructing a Test Statistic:-

$$T(X) = \frac{\bar{X} - \mu_0}{S_x} \sqrt{n}$$

with $n-1$ degrees of freedom.

③ Critical Regions:- (degree of freedom, $\nu = n-1$)

Case	H_0	H_1	K
(a)	$\mu = \mu_0$	$\mu \neq \mu_0$	$(-\infty, -t_{1-\frac{\alpha}{2}}) \cup (t_{1-\frac{\alpha}{2}}, \infty)$
(b)	$\mu \geq \mu_0$	$\mu < \mu_0$	$(-\infty, -t_{1-\alpha})$
(c)	$\mu \leq \mu_0$	$\mu > \mu_0$	$(t_{1-\alpha}, \infty)$

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(4) ~~Relia~~ Realization of test Statistic:—
for an observed sample $x_1, x_2, \ldots x_n$,

$$\bar{x} = \frac{\sum x_i}{n}.$$

and $$t(x) = \frac{(\bar{x} - \mu_0)\sqrt{n}}{S_x}$$

(5) Decision Rule:—
(Same as Z-test)

Ques: A manufacturer of a certain brand of energy bar claims that the average saturated fat content in the bar is 0.5 gms. Will you support his claim if the 8 bars that you examined for fat content were found to ~~be~~ contain 0.6, 0.7, 0.7, 0.3, 0.4, 0.5, 0.4 and 0.2 gms of Saturated fat? Take $\alpha = 0.05$.

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III. χ^2 - Test :-

* This test is used when $\sigma^2 = \sigma_0^2$ is given.
* We assume the Distribution of the Population sample is Normal.

Steps :-

① (i)

(ii)

(iii)

② Constructing a Test Statistics :-

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

③ Critical Regions :-

Case	H_0	H_1	K
(a)	$\theta = \theta_0$	$\theta \neq \theta_0$	$(0, \chi^2_{1-\frac{\alpha}{2}}) \cup (\chi^2_{\frac{\alpha}{2}}, \infty)$
(b)	$\theta \geq \theta_0$	$\theta < \theta_0$	$(0, \chi^2_{1-\alpha})$
(c)	$\theta \leq \theta_0$	$\theta > \theta_0$	$(\chi^2_{\alpha}, \infty)$

④ Realization of test Statistic :-

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

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(5) Decision Rule
(same as z-test)

Que. A manufacturer of car batteries claims that the life of a Company's batteries is approximately normally distributed with a standard deviation of 0.9 yrs. If a random sample of 10 of these batteries has a standard Deviation of 1.2 yrs, do you think $\sigma > 0.9$ yrs? Use $\alpha = 0.05$.

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IV. F Test :-

* We use this test for comparing variances. ie whether $\sigma_1^2 = \sigma_2^2$ or $\sigma_1^2 \neq \sigma_2^2$.

* We assume the Population to be Normally Distributed.

Steps :-

① (i)

(ii)

(iii)

② Constructing a Test Statistic :-

$$f = \frac{S_1^2}{S_2^2}$$

Also, we've $f_\alpha(\nu_1, \nu_2) = \dfrac{1}{f_{1-\alpha}(\nu_2, \nu_1)}$

③ Critical Regions :-

Case	H_0	H_1	K
(a)	$\sigma_1^2 = \sigma_2^2$	$\sigma_1^2 \neq \sigma_2^2$	$\left(0, f_{1-\frac{\alpha}{2}}^{(\nu_1,\nu_2)}\right) \cup \left(f_{\frac{\alpha}{2}}^{(\nu_1,\nu_2)}, \infty\right)$
(b)	$\sigma_1^2 \geq \sigma_2^2$	$\sigma_1^2 < \sigma_2^2$	$\left(0, f_{1-\alpha}^{(\nu_1,\nu_2)}\right)$
(c)	$\sigma_1^2 \leq \sigma_2^2$	$\sigma_1^2 > \sigma_2^2$	$\left(f_\alpha^{(\nu_1,\nu_2)}, \infty\right)$

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(4) Realization of Test Statistic:-

$$f = \frac{S_1^2}{S_2^2}$$

(5) Decision Rule.

Ques. A study is conducted to compare the lengths of time required by men and women to assemble a certain Product. Past experience indicates that the distribution of times for both men and women is approximately normal but variance of times for a women is less than that of a man. A random sample of times of times for 11 men and 14 women gives respective standard Deviations. 6.1 and 5.3. Test the Hypothesis $\sigma_1^2 = \sigma_2^2$ against $\sigma_1^2 > \sigma_2^2$.

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V χ^2 goodness of fit :-

* used when observed absolute frequencies are compared with the expected absolute frequencies under H_0.

Steps:-

(1) (i)

(ii)

(iii)

(2) Test Statistic,

$$T(X) = \chi^2 = \sum_{i=1}^{k} \frac{(N_i - np_i)^2}{np_i},$$

where N_i are the absolute frequencies of observations of the sample X in class i, N_i is a random variable with realization n_i in the observed sample.

(3) Critical Regions:-

$K = \chi^2_\alpha$ for $\nu = k-1$ degree of freedom.

(4) $\chi^2 = \sum_{i=1}^{k} \frac{(N_i - np_i)}{np_i}$.

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(5) If $\chi^2 > K$, then
H_0 is Rejected
and
vice versa.

Ques. Gregor Mendel conducted crossing experiments with pea plants of different shape and colour. let us look at the outcome of a pea crossing exp with the following results:-

Crossing Result	Round Yellow	Round Green	Edged Yellow	Edged Green
Observation	315	108	101	32.

Mendel has hypothesis that the four different types occur in proportions of 9:3:3:1 i.e.

$p_1 = \frac{9}{16}$, $p_2 = \frac{3}{16}$, $p_3 = \frac{9}{16}$, $p_4 = \frac{1}{16}$, $n = 556$

(1) The Hypothesis are

$H_0 : P(X=i) = p_i$

$H_1 : P(X=i) \neq p_i$

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(2)
$$\chi^2 = \sum_{i=1}^{k} \frac{(N_i - np_i)^2}{np_i}$$

$n = 556$

(3) we've

i	N_i	p_i	np_i
1	315	$\frac{9}{16}$	312.75
2	108	$\frac{3}{16}$	104.25
3	101	$\frac{3}{16}$	104.25
4	32	$\frac{1}{16}$	34.75

$K: \chi^2_{k-1,\alpha} = \chi^2_{0.05}$ for degree of freedom, $\nu = 4-1 = 3$.

$\therefore K = 7.815$

(4) $\chi^2 = 0.47$

(5) $\because 0.47 < 7.815$

$\therefore H_0$ is Accepted.