

COMPILER DESIGN

Topic: Bottom-Up parsing

Introduction

- Constructs parse tree for an input string beginning at the leaves (the bottom) and working towards the root (the top)
- Example: $\text{id} * \text{id}$

$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid \mathbf{id}$

$\text{id} * \text{id}$

$F * \text{id}$
 \mid
 id

$T * \text{id}$
 \mid
 F
 \mid
 id

$T * F$
 $\mid \mid$
 $F \text{ id}$
 \mid
 id

F
 $\swarrow \downarrow \searrow$
 $T * F$
 $\mid \mid$
 $F \text{ id}$
 \mid
 id

E
 \mid
 F
 $\swarrow \downarrow \searrow$
 $T * F$
 $\mid \mid$
 $F \text{ id}$
 \mid
 id

Shift-reduce parser

- The general idea is to shift some symbols of input to the stack until a reduction can be applied
- At each reduction step, a specific substring matching the body of a production is replaced by the nonterminal at the head of the production
- The key decisions during bottom-up parsing are about when to reduce and about what production to apply
- A reduction is a reverse of a step in a derivation
- The goal of a bottom-up parser is to construct a derivation in reverse:
 - $E \Rightarrow T \Rightarrow T * F \Rightarrow T * id \Rightarrow F * id \Rightarrow id * id$

Handle pruning

- A Handle is a substring that matches the body of a production and whose reduction represents one step along the reverse of a rightmost derivation

Right sentential form	Handle	Reducing production
id*id	id	$F \rightarrow id$
F*id	F	$T \rightarrow F$
T*id	id	$F \rightarrow id$
T*F	T*F	$E \rightarrow T*F$

Shift reduce parsing

- A stack is used to hold grammar symbols
- Handle always appear on top of the stack
- Initial configuration:

Stack	Input
\$	w\$

- Acceptance configuration

Stack	Input
\$S	\$

Shift reduce parsing (cont.)

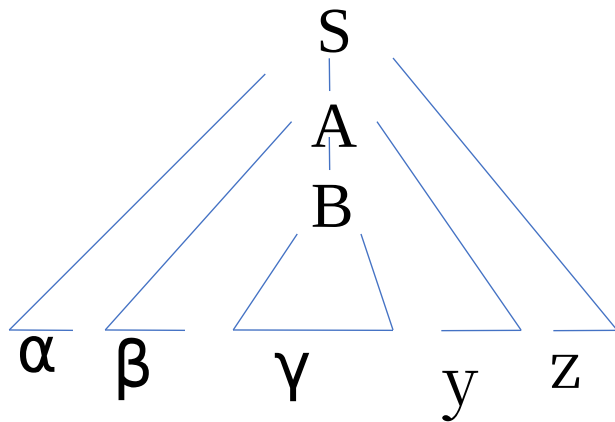
- Basic operations:

- Shift
- Reduce
- Accept
- Error

- Example: $id * id$

Stack	Input	Action
\$	$id * id \$$	shift
$\$id$	$* id \$$	reduce by $F \rightarrow id$
$\$F$	$* id \$$	reduce by $T \rightarrow F$
$\$T$	$* id \$$	shift
$\$T*$	$id \$$	shift
$\$T * id$	$\$$	reduce by $F \rightarrow id$
$\$T * F$	$\$$	reduce by $T \rightarrow T * F$
$\$T$	$\$$	reduce by $E \rightarrow T$
$\$E$	$\$$	accept

Handle will appear on top of the stack



Stack

$\$ \alpha \beta \gamma$

$\$ \alpha \beta B$

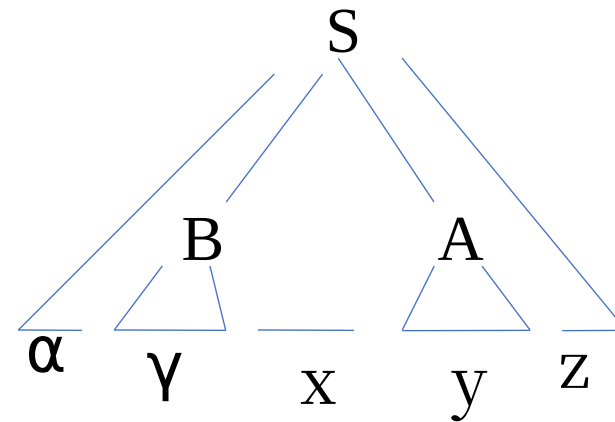
$\$ \alpha \beta B y$

Input

$yz\$$

$yz\$$

$z\$$



Stack

$\$ \alpha \gamma$

$\$ \alpha B x y$

Input

$xyz\$$

$z\$$

Conflicts during shift reduce parsing

- Two kind of conflicts
 - Shift/reduce conflict
 - Reduce/reduce conflict
- Example:

```
stmt → If expr then stmt
      | If expr then stmt else stmt
      | other
```

Stack

... if expr then stmt

Input

else ...\$

Reduce/reduce conflict

stmt -> id(parameter_list)

stmt -> expr:=expr

parameter_list->parameter_list, parameter

parameter_list->parameter

parameter->id

expr->id(expr_list)

expr->id

expr_list->expr_list, expr

expr_list->expr

Stack
... id(id

Input
,id) ...\$

LR Parsing

- The most prevalent type of bottom-up parsers
- LR(k), mostly interested on parsers with $k \leq 1$
- Why LR parsers?
 - Table driven
 - Can be constructed to recognize all programming language constructs
 - Most general non-backtracking shift-reduce parsing method
 - Can detect a syntactic error as soon as it is possible to do so
 - Class of grammars for which we can construct LR parsers are superset of those which we can construct LL parsers

States of an LR parser

- States represent set of items
- An LR(0) item of G is a production of G with the dot at some position of the body:
 - For $A \rightarrow XYZ$ we have following items
 - $A \rightarrow .XYZ$
 - $A \rightarrow X.YZ$
 - $A \rightarrow XY.Z$
 - $A \rightarrow XYZ.$
 - In a state having $A \rightarrow .XYZ$ we hope to see a string derivable from XYZ next on the input.
 - What about $A \rightarrow X.YZ$?

Constructing canonical LR(0) item sets

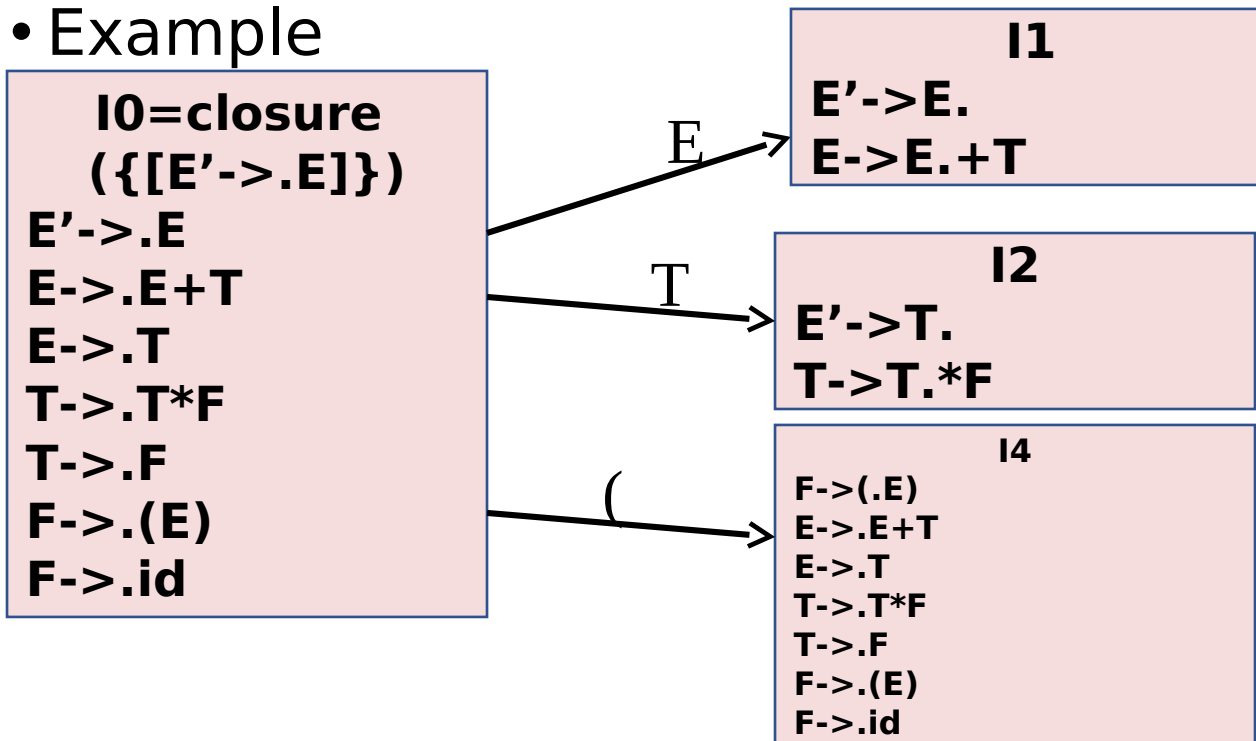
- Augmented grammar:
 - G with addition of a production: $S' \rightarrow S$
- Closure of item sets:
 - If I is a set of items, $\text{closure}(I)$ is a set of items constructed from I by the following rules:
 - Add every item in I to $\text{closure}(I)$
 - If $A \rightarrow \alpha.B\beta$ is in $\text{closure}(I)$ and $B \rightarrow \gamma$ is a production then add the item $B \rightarrow \gamma$ to $\text{closure}(I)$.
- Example:

$E' \rightarrow E$
 $E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid \text{id}$

$I_0 = \text{closure}(\{[E' \rightarrow \cdot E]\})$
 $E' \rightarrow \cdot E$
 $E \rightarrow \cdot E + T$
 $E \rightarrow \cdot T$
 $T \rightarrow \cdot T * F$
 $T \rightarrow \cdot F$
 $F \rightarrow \cdot (E)$
 $F \rightarrow \cdot \text{id}$

Constructing canonical LR(0) item sets (cont.)

- Goto (I, X) where I is an item set and X is a grammar symbol is closure of set of all items $[A \rightarrow \alpha X \beta]$ where $[A \rightarrow \alpha.X \beta]$ is in I
- Example



Closure algorithm

SetOfItems CLOSURE(I) {

 J=I;

 repeat

 for (each item $A \rightarrow \alpha.B\beta$ in J)

 for (each production $B \rightarrow \gamma$ of G)

 if ($B \rightarrow \gamma$ is not in J)

 add $B \rightarrow \gamma$ to J;

 until no more items are added to J on one round;

 return J;

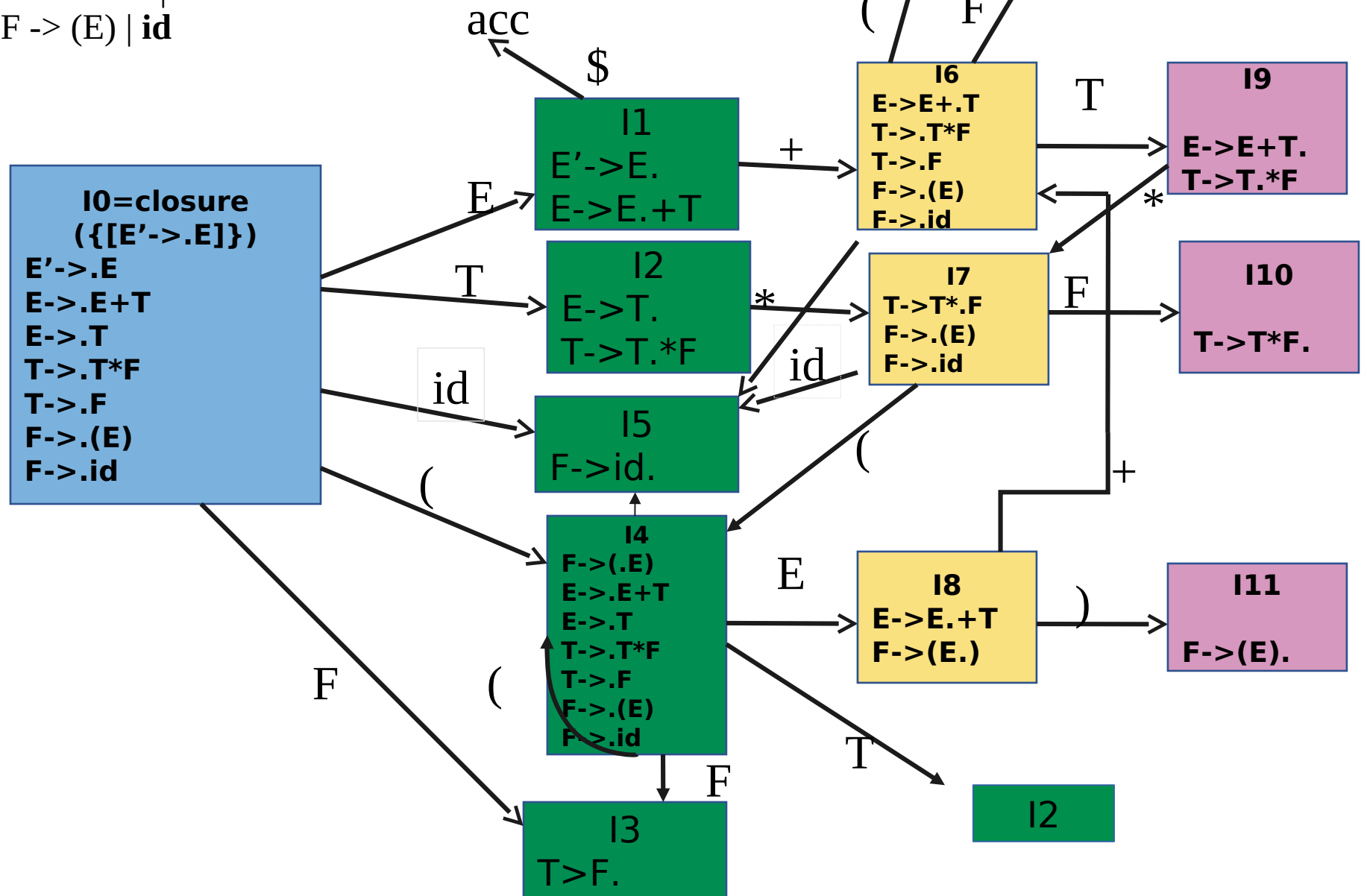
GOTO algorithm

```
SetOfItems GOTO(I,X) {  
    J=empty;  
    if ( $A \rightarrow \alpha.X \beta$  is in I)  
        add CLOSURE( $A \rightarrow \alpha.X \beta$ ) to J;  
    return J;  
}
```

Canonical LR(0) items

```
Void items( $G'$ ) {  
     $C = \text{CLOSURE}(\{[S' \rightarrow \cdot S]\})$ ;  
    repeat  
        for (each set of items  $I$  in  $C$ )  
            for (each grammar symbol  $X$ )  
                if ( $\text{GOTO}(I, X)$  is not empty and not in  $C$ )  
                    add  $\text{GOTO}(I, X)$  to  $C$ ;  
    until no new set of items are added to  $C$  on a round;  
}
```


$E' \rightarrow E$
 $E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid id$



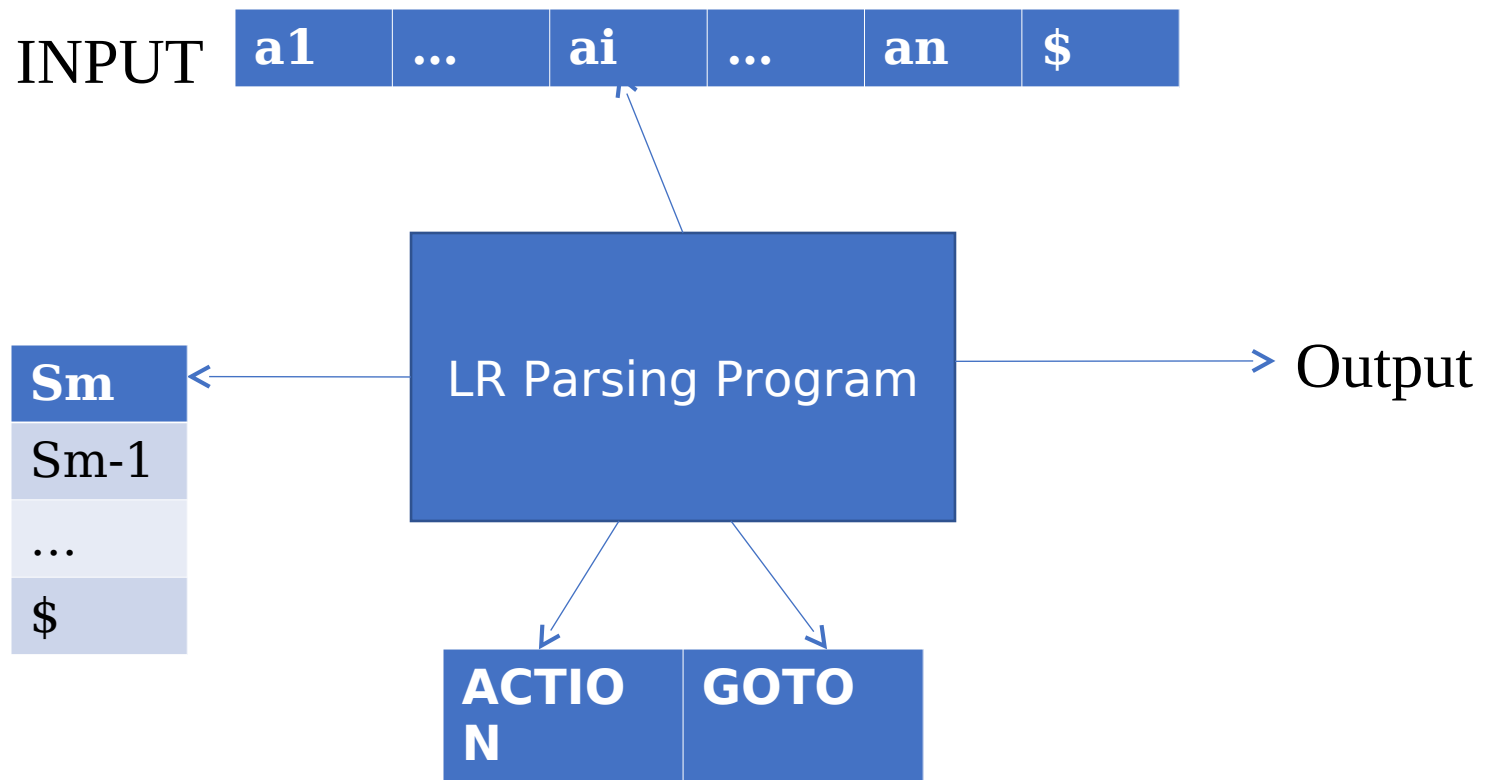
STATE	ACTON						GOTO		
	id	+	*	()	\$	E	T	F
0	S5			S4			1	2	3
1		S6				Acc			
2		R2	S7		R2	R2			
3		R4	R4		R4	R4			
4	S5			S4			8	2	3
5		R6	R6		R6	R6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S11				
9		R1	S7		R1	R1			
10		R3	R3		R3	R3			
11		R5	R5		R5	R5			

Use of LR(0) automaton

- Example: $\text{id} * \text{id}$

Line	Stack	Symbols	Input	Action
(1)	0	\$	id*id\$	Shift to 5
(2)	05	\$id	*id\$	Reduce by $F \rightarrow id$
(3)	03	\$F	*id\$	Reduce by $T \rightarrow F$
(4)	02	\$T	*id\$	Shift to 7
(5)	027	\$T*	id\$	Shift to 5
(6)	0275	\$T*id	\$	Reduce by $F \rightarrow id$
(7)	02710	\$T*F	\$	Reduce by $T \rightarrow T*F$
(8)	02	\$T	\$	Reduce by $E \rightarrow T$
(9)	01	\$E	\$	accept

LR-Parsing model



LR parsing algorithm

```
let a be the first symbol of w$;
while(1) { /*repeat forever */
    let s be the state on top of the stack;
    if (ACTION[s,a] = shift t) {
        push t onto the stack;
        let a be the next input symbol;
    } else if (ACTION[s,a] = reduce A-> $\beta$ ) {
        pop  $|\beta|$  symbols of the stack;
        let state t now be on top of the stack;
        push GOTO[t,A] onto the stack;
        output the production A-> $\beta$ ;
    } else if (ACTION[s,a]=accept) break; /* parsing is done */
    else call error-recovery routine;
}
```

Example

STAT E	ACTON						GOTO		
	id	+	*	()	\$	E	T	F
0	S5			S4			1	2	3
1		S6				Acc			
2		R2	S7		R2	R2			
3		R4	R4		R4	R4			
4	S5			S4			8	2	3
5		R6	R6		R6	R6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S11				
9		R1	S7		R1	R1			
10		R3	R3		R3	R3			
11		R5	R5		R5	R5			

- (0) $E' \rightarrow E$
- (1) $E \rightarrow E + T$
- (2) $E \rightarrow T$
- (3) $T \rightarrow T * F$
- (4) $T \rightarrow F$
- (5) $F \rightarrow (E)$
- (6) $F \rightarrow id$

id*id+id?

Line	Stack	Symbols	Input	Action
(1)	0		id*id+id\$	Shift to 5
(2)	05	id	*id+id\$	Reduce by $F \rightarrow id$
(3)	03	F	*id+id\$	Reduce by $T \rightarrow F$
(4)	02	T	*id+id\$	Shift to 7
(5)	027	T*	id+id\$	Shift to 5
(6)	0275	T*id	+id\$	Reduce by $F \rightarrow id$
(7)	02710	T*F	+id\$	Reduce by $T \rightarrow T*F$
(8)	02	T	+id\$	Reduce by $E \rightarrow T$
(9)	01	E	+id\$	Shift
(10)	016	E+	id\$	Shift
(11)	0165	E+id	\$	Reduce by $F \rightarrow id$

Constructing SLR parsing table

- Method

- Construct $C = \{I_0, I_1, \dots, I_n\}$, the collection of LR(0) items for G'
- State i is constructed from state I_i :
 - If $[A \rightarrow \alpha.a\beta]$ is in I_i and $\text{Goto}(I_i, a) = I_j$, then set $\text{ACTION}[i, a]$ to “shift j ”
 - If $[A \rightarrow \alpha.]$ is in I_i , then set $\text{ACTION}[i, a]$ to “reduce $A \rightarrow \alpha$ ” for all a in $\text{follow}(A)$
 - If $\{S' \rightarrow .S\}$ is in I_i , then set $\text{ACTION}[i, \$]$ to “Accept”
- If any conflicts appears then we say that the grammar is not SLR(1).
- If $\text{GOTO}(I_i, A) = I_j$ then $\text{GOTO}[i, A] = j$
- All entries not defined by above rules are made “error”
- The initial state of the parser is the one constructed from the set of items containing $[S' \rightarrow .S]$

Example grammar which is not SLR(1)

$S \rightarrow L=R \mid R$

$L \rightarrow *R \mid \text{id}$

$R \rightarrow L$

I0
 $S' \rightarrow .S$
 $S \rightarrow .L=R$
 $S \rightarrow .R$
 $L \rightarrow .*R \mid$
 $L \rightarrow .\text{id}$
 $R \rightarrow .L$

I1
 $S' \rightarrow S.$

I2
 $S \rightarrow L.=R$
 $R \rightarrow L.$

I3
 $S \rightarrow R.$

I4
 $L \rightarrow *.R$
 $R \rightarrow .L$
 $L \rightarrow .*R$
 $L \rightarrow .\text{id}$

I5
 $L \rightarrow \text{id}.$

I6
 $S \rightarrow L=.R$
 $R \rightarrow .L$
 $L \rightarrow .*R$
 $L \rightarrow .\text{id}$

I7
 $L \rightarrow *R.$

I8
 $R \rightarrow L.$

I9
 $S \rightarrow L=R.$

Action

=

More powerful LR parsers

- Canonical-LR or just LR method
 - Use lookahead symbols for items: LR(1) items
 - Results in a large collection of items
- LALR: lookaheads are introduced in LR(0) items

Canonical LR(1) items

- In LR(1) items each item is in the form: $[A \rightarrow \alpha.\beta, a]$
- An LR(1) item $[A \rightarrow \alpha.\beta, a]$ is valid for a viable prefix γ if there is a derivation $S \xRightarrow{*} \delta A w \xRightarrow{rm} \delta \alpha \beta w$, where
 - $\Gamma = \delta \alpha$
 - Either a is the first symbol of w , or w is ϵ and a is $\$$
- Example:
 - $S \rightarrow BB$
 - $B \rightarrow aB | b$

$$S \xRightarrow{*} aaBab \xRightarrow{rm} aaaBab$$

Item $[B \rightarrow a.B, a]$ is valid for $\gamma = aaa$
and $w = ab$

Constructing LR(1) sets of items

```
SetOfItems Closure(I) {  
    repeat  
        for (each item  $[A \rightarrow \alpha.B\beta, a]$  in I)  
            for (each production  $B \rightarrow \gamma$  in  $G'$ )  
                for (each terminal  $b$  in  $\text{First}(\beta a)$ )  
                    add  $[B \rightarrow \gamma, b]$  to set I;  
    until no more items are added to I;  
    return I;  
}
```

```
SetOfItems Goto(I, X) {  
    initialize J to be the empty set;  
    for (each item  $[A \rightarrow \alpha.X\beta, a]$  in I)  
        add item  $[A \rightarrow \alpha X.\beta, a]$  to set J;  
    return closure(J);  
}
```

```
void items( $G'$ ) {  
    initialize C to  $\text{Closure}(\{[S' \rightarrow \cdot S, \$]\})$ ;  
    repeat  
        for (each set of items I in C)  
            for (each grammar symbol X)  
                if ( $\text{Goto}(I, X)$  is not empty and not in C)  
                    add  $\text{Goto}(I, X)$  to C;  
    until no new sets of items are added to C;  
}
```

Example

$S' \rightarrow S$

$S \rightarrow CC$

$C \rightarrow cC$

$C \rightarrow d$

Canonical LR(1) parsing table

- Method

- Construct $C = \{I_0, I_1, \dots, I_n\}$, the collection of LR(1) items for G'
- State i is constructed from state li :
 - If $[A \rightarrow \alpha.a\beta, b]$ is in li and $\text{Goto}(li, a) = lj$, then set $\text{ACTION}[i, a]$ to “shift j ”
 - If $[A \rightarrow \alpha., a]$ is in li , then set $\text{ACTION}[i, a]$ to “reduce $A \rightarrow \alpha$ ”
 - If $\{S' \rightarrow .S, \$\}$ is in li , then set $\text{ACTION}[i, \$]$ to “Accept”
- If any conflicts appears then we say that the grammar is not LR(1).
- If $\text{GOTO}(li, A) = lj$ then $\text{GOTO}[i, A] = j$
- All entries not defined by above rules are made “error”
- The initial state of the parser is the one constructed from the set of items containing $[S' \rightarrow .S, \$]$

Example

$S' \rightarrow S$

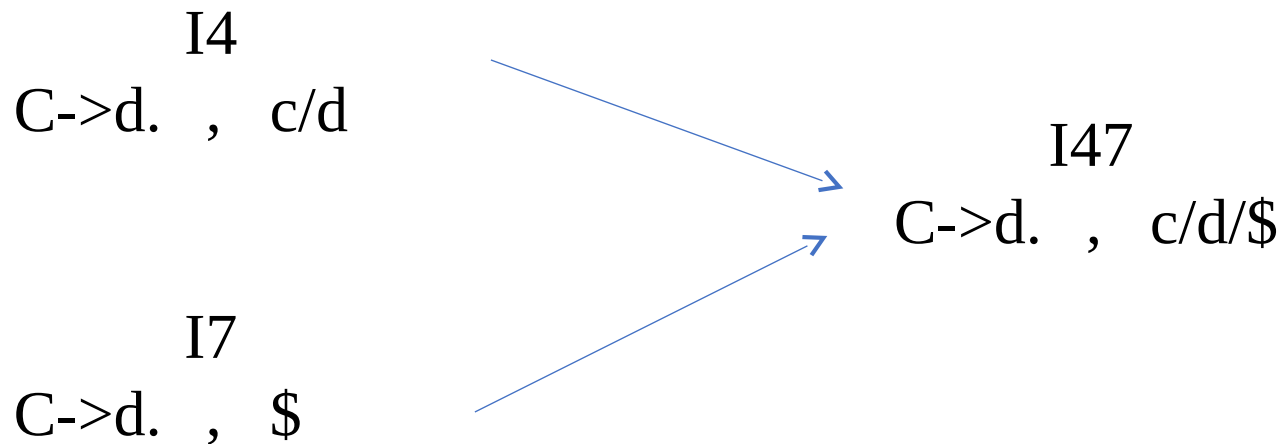
$S \rightarrow CC$

$C \rightarrow cC$

$C \rightarrow d$

LALR Parsing Table

- For the previous example we had:



- State merges can't produce Shift-Reduce conflicts. Why?
- But it may produce reduce-reduce conflict

Example of RR conflict in state merging

$S' \rightarrow S$

$S \rightarrow aAd \mid bBd \mid aBe \mid bAe$

$A \rightarrow c$

$B \rightarrow c$

An easy but space-consuming LALR table construction

- Method:

1. Construct $C = \{I_0, I_1, \dots, I_n\}$ the collection of LR(1) items.
2. For each core among the set of LR(1) items, find all sets having that core, and replace these sets by their union.
3. Let $C' = \{J_0, J_1, \dots, J_m\}$ be the resulting sets. The parsing actions for state i , is constructed from J_i as before. If there is a conflict grammar is not LALR(1).
4. If J is the union of one or more sets of LR(1) items, that is $J = I_1 \cup I_2 \dots \cup I_k$ then the cores of $\text{Goto}(I_1, X)$, ..., $\text{Goto}(I_k, X)$ are the same and is a state like K , then we set $\text{Goto}(J, X) = K$.

- This method is not efficient, a more efficient one is discussed in the book

Compaction of LR parsing table

- Many rows of action tables are identical
 - Store those rows separately and have pointers to them from different states
 - Make lists of (terminal-symbol, action) for each state
 - Implement Goto table by having a link list for each nonterminal in the form (current state, next state)

Using ambiguous grammars

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$E \rightarrow id$

I0: $E' \rightarrow .E$

$E \rightarrow .E + E$

$E \rightarrow .E * E$

$E \rightarrow .(E)$

$E \rightarrow .id$

I1: $E' \rightarrow E.$

$E \rightarrow E. + E$

$E \rightarrow E. * E$

I4: $E \rightarrow E + .E$

$E \rightarrow E. + E$

$E \rightarrow E. * E$

$E \rightarrow .(E)$

$E \rightarrow .id$

I2: $E \rightarrow (.E)$

$E \rightarrow .E + E$

$E \rightarrow .E * E$

$E \rightarrow .(E)$

$E \rightarrow .id$

I5: $E \rightarrow E * .E$

$E \rightarrow (.E)$

$E \rightarrow .E + E$

$E \rightarrow .E * E$

$E \rightarrow .(E)$

$E \rightarrow .id$

STATE	ACTION						GO TO
	id	+	*	()	\$	
0	S3			S2			1
1		S4	S5			Acc	
2	S3		S2				6
3		R4	R4		R4	R4	
4	S3			S2			7
5	S3			S2			8
6		S4	S5				
7		R1	S5		R1	R1	
8		R2	R2		R2	R2	
9		R3	R3		R3	R3	

I3: $E \rightarrow .id$

I6: $E \rightarrow (E.)$

$E \rightarrow E. + E$

$E \rightarrow E. * E$

I8: $E \rightarrow E * E.$

$E \rightarrow E. + E$

$E \rightarrow E. * E$

I7: $E \rightarrow E + E.$

$E \rightarrow E. + E$

$E \rightarrow E. * E$

I9: $E \rightarrow (E).$

Thank You