Date . .

* One Sample Test :-

Data is assumed to arise as one sample from a defined Population.

* Two Sample Test :-

Data originates in the form of two samples from two different Population.

→ Two independent Sample Problem
→ Two Dependent Sample Problem.

* Hypothesis :-

A claim that we want to test.

(i) Null Hypothesis :-

Currently accepted value for a Parameter.

It is opposite of Alternative Hypothesis.
denoted by H_0.

(ii) Alternative Hypothesis :-

Involves the claim to be tested.
denoted by H_1.
It is formulated as deviation from the target value.

Date . .

Ex.1 A company has stated that their straw machine makes a straws that are 4mm diameter. A worker believes the machine no longer makes straws of this size and samples 100 ~~sample~~ straws to perform a Hypothesis test with 99% confidence.

→

H_0:

H_1:

Ex.2 Doctors believe that the average teen sleeps on average than 10 hrs. per day. A researcher believes that teens on average sleep longer. Write H_0 & H_1.

* ONE- and TWO SIDED TESTS:-

For an unknown population Parameter (θ) and a fixed value, θ_0, we've:-

Case	H_0	H_1	
a)	$\theta = \theta_0$	$\theta \neq \theta_0$	Two sided Test
b)	$\theta \geq \theta_0$	$\theta < \theta_0$	One Sided Test
c)	$\theta \leq \theta_0$	$\theta > \theta_0$	One Sided Test

Date . . .

Ex.3 A Sample of 100 tires is taken from a lot. The mean life of tires in the sample was found to be 39350 kms with the population Standard deviation of 3260 km. Test the Hypothesis, at 1% level of significance that the mean life of tire is 40,000 kms.

→

* Type I and II ~~Test~~ Error:-

Decision / Fact →	H_0 is True	H_0 is not True
H_0 is not Rejected	Correct Decision	Type II Error,
H_0 is Rejected	Type I Error	Correct Decision

(i) Type I Error:-

The Hypothesis H_0 is True, but is Rejected, ie. H_1 is accepted,

Date . .

(ii) Type II Error:-
Hypothesis H_0 is not rejected although it is not true.

(iii) Significance level:-
Probability of type I error, ie

$$P(H_1|H_0) = \alpha$$

ie.
Probability of accepting H_1 if H_0 is true.

(iv) Prob. of Type II error:- (level of Confidence)
denoted by β.
given by
$$P(H_0|H_1) = \beta$$

* We try to fix α and then minimize β.
* Power of Test:-
$$1-\beta = P(H_1|H_1)$$
ie. Prob. of making a decision in favour of the research hypothesis H_1, if it is true.
ie
the Prob. of detecting a correct research Hypothesis.

Date . .

* Steps to conduct a Statistical Test :-

Step 1: (i) Define the ~~Para~~ Assumptions for random variables of interest, and specify them in terms of population Parameters. (ie θ or μ or σ).

(ii) Formulate Null Hypothesis and Alternative Hypothesis.

(iii) fix a Significance value/level ie α.

Step 2:- Consider a statistic, $T(X) = T(X_1, X_2, \ldots X_n)$. The distribution has to be known under Null Hypothesis.

Step 3:- Consider a critical region, K for the statistic T ie a region where - if T falls in this region - H_0 is rejected, such that

$$P_{H_0}(T(X) \in K) \leq \alpha.$$

Step 4:- find $t(x) = T(x_1, x_2, \ldots x_n)$ based on sample values $X_1 = x_1, X_2 = x_2 \ldots X_n = x_n$

Date . . .

Step 5:- Decision Rule:

If $t(x)$ falls into critical region K, the null Hypothesis H_0 is rejected.

ie.

$t(x) \in K$: H_0 is rejected.

$\Rightarrow H_1$ is Significant.

If $t(x)$ falls outside K, null hypothesis is not rejected.

ie.

$t(x) \notin K$; H_0 is not rejected.

$\Rightarrow H_0$ accepted and is Significant.

* TESTS :-

used when $n \geq 30$

(Z-Test)

I. ONE-SAMPLE GAUSS TEST :-

* This test is used to test whether unknown mean differs from a specific value of mean of sample.

* We assume $\sigma^2 = \sigma_0^2$ is given.

Date . .

Steps:-

① (i) The random variable, X follows a $N(\mu, \sigma_0^2)$- distribution with known variance σ_0^2.

(ii) formulating H_0 and H_1.

$H_0 : \mu = \mu_0$; $H_1 : \mu \neq \mu_0$ (Two-sided test)
$H_0 : \mu \leq \mu_0$; $H_1 : \mu > \mu_0$ (One-sided test)
$H_0 : \mu \geq \mu_0$; $H_1 : \mu < \mu_0$, (One-sided test).

(iii) $\alpha = 0.05$ (used often, if not given in ques.)

② Constructing a Test Statistic:-
We know that if X_i's are i.i.d., then Sample Mean is Normally Distributed.

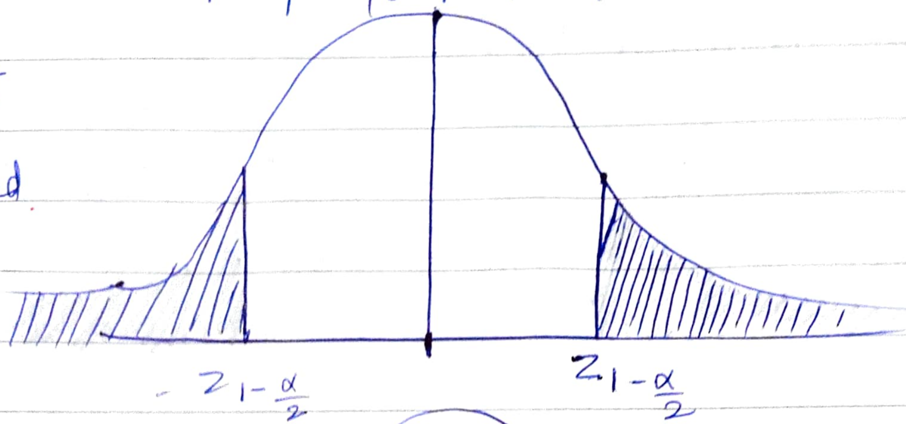
$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \sim N\left(\mu_0, \frac{\sigma_0^2}{n}\right)$$

$$\therefore \quad T(X) = \frac{\bar{X} - \mu_0}{\sigma_0} \sqrt{n} \sim N(0,1)$$

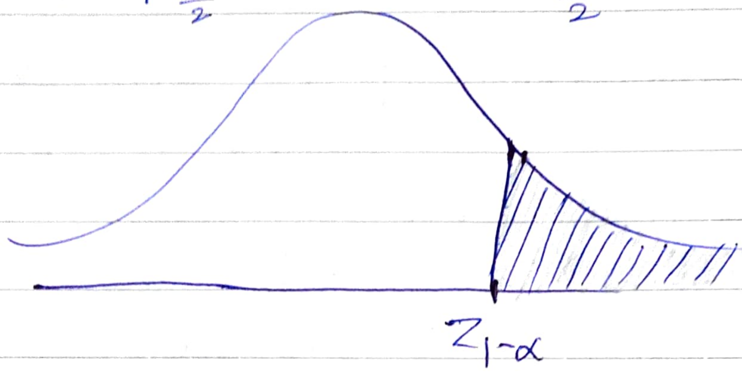
③ Critical Region:-
Consider the Table

Case	H_0	H_1	Critical Region
Two sided (a)	$\mu = \mu_0$	$\mu \neq \mu_0$	$K = (-\infty, -Z_{1-\frac{\alpha}{2}}) \cup (Z_{1-\frac{\alpha}{2}}, \infty)$
one sided (b)	$\mu \leq \mu_0$	$\mu > \mu_0$	$K = (Z_{1-\alpha}, \infty)$
one sided (c)	$\mu \geq \mu_0$	$\mu < \mu_0$	$K = (-\infty, Z_\alpha = -Z_{1-\alpha})$

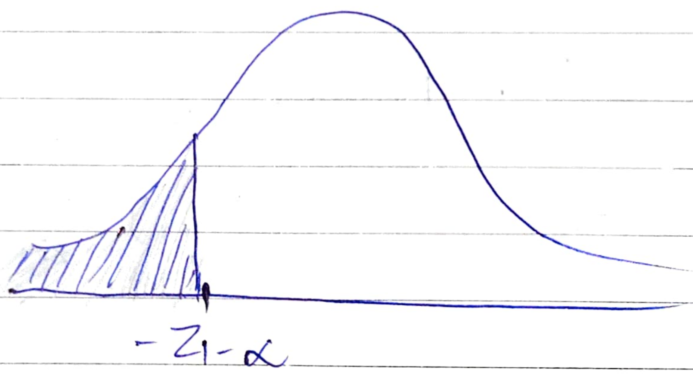
(a) Two sided



(b)



(c)



4) Realization of Test Statistic:
for an observed $x_1, x_2, \ldots, x_n$,

Date . .

the arithmetic mean

$\bar{x} = \frac{\sum x_i}{n}$ is used to

calculate realized test statistic, $t(x)$ ie

$t(x) = T(x_1, x_2, \dots x_n)$ as

$$t(x) = \frac{\bar{x} - \mu_0}{\sigma_0} \sqrt{n}.$$

5. Decision Rule :-

If $t(x) \in K \Rightarrow H_0$ is Rejected.

$\Rightarrow H_1$ is Accepted.

If $t(x) \notin K \Rightarrow H_0$ is Accepted

$\Rightarrow H_1$ is Rejected.

Ques. A Random Sample of 100 recorded deaths in the US during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 yrs. does this seem to indicate that mean life span today is greater than 70 yrs. ? Use a 0.05 level of significance.