COEP Technogical University

Department of Mathematics

(MA- 21001) Probability and Statistics for Engineers

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1 Tutorial: Week 10

1. Dishwasher powder is poured into the cartons in which it is sold by an automatic dispensing machine which is set to dispense 3 kg of powder into each carton. In order to check that the dispensing machine is working to an acceptable standard (i.e. does not need adjustment), a production engineer takes a random samples of 40 cartons and weighs them. It is found that the mean weight of the sample is 3.005 kg. It is known that the dispensing machine operates with a variance of $0.015^2 \ kg^2$ and that the manufacturer of the powder is willing to rely on a 5% level of significance. Does the sample provide the engineer with sufficient evidence that the true mean is not 3.00 kg and so the machine requires adjustment? (Ans: machine is not operating acceptably and needs adjustment)

2. A coin is tossed 10 times. We wish to test the hypothesis that the coin is fair. Let p be the probability that the coin shows a head. Which of the following represents the alternative hypothesis?

   a) $H1 : p > 0.5$
   b) $H1 : p < 0.5$
   c) $H1 : p = 0.5$
   d) $H1 : p \neq 0.5$

3. A large health study has found that 7% of the population suffers from a blood condition. A group of 15 people from an area near a mobile phone transmitter are tested for the condition and 2 people are found to have the condition. The locals believe that the transmitter increases the likelihood of having the condition. We wish to perform a test of significance on whether the mobile phone transmitter increases the incidence of the condition.

   Let p be the probability that an individual has the condition. Which of the following would be the appropriate null and alternative hypotheses?

   a) $H0 : p = 2/15$
      $H1 : p \neq 2/15$
   b) $H0 : p = 0.07$
      $H1 : p \neq 0.07$
   c) $H0 : p = 2/15$
      $H1 : p > 2/15$
   d) $H0 : p = 0.07$
      $H1 : p > 0.07$

4. A random sample of 30 households was selected as part of a study on electricity usage, and the number of kilowatt-hours (kWh) was recorded for each household in the sample for the March quarter of 2006. The average usage was found to be 375kWh. In a very large study in the March quarter of the previous year it was found that the standard deviation of the usage was 81kWh. Assuming the standard deviation is unchanged and that the usage is normally distributed provide an expression for calculating a 99% confidence interval for the mean usage in the March quarter of 2006.

   a) $375 \pm 2.756 \times \dfrac{81}{\sqrt{30}}$.

   b) $375 \pm 2.575 \times \dfrac{9}{\sqrt{30}}$.

   c) $375 \pm 2.33 \times \dfrac{81}{\sqrt{30}}$.

   d) $375 \pm 2.575 \times \dfrac{81}{\sqrt{30}}$.

5. Conduct an F-Test on the following samples:
   Sample-1 having variance = 109.63, sample size = 41.
   Sample-2 having Variance = 65.99, sample size = 21.

6. Test the null hypothesis that the following sample is from a population with median 100 against the alternative the median is greater than 100 i.e., Use normal approximation to Wilcoxon Signed Rank Test (without continuity correction) Assume that the distribution of differences is symmetric. 98.38, 115.33, 98.62, 114.38, 87.79, 84.06, 96.18, 98.74, 91, 107.82, 108.28, 112.62, 124.18, 101.99, 112.51, 75.65, 83.77, 84.91, 109.73, 109.41, 100.4, 95.37, 115.46, 111.78, 86.13, 82.14, 78.47, 98.18

   a) Null hypothesis H0 is rejected against the alternative H1 at 5% level of significance

   b) Null hypothesis H0 is rejected against the alternative H1 at 10% level of significance

   c) Null hypothesis H0 is not rejected against the alternative H1 at 5% level of significance

   d) Null hypothesis H0 is not rejected against the alternative H1 at 10% level of significance

7. Suppose that an allergist wishes to test the hypothesis that at least 30 percent of the public is allergic to some cheese products. Explain how the allergist could commit (a) a type I error; (b) a type II error.

8. A large manufacturing firm is being charged with discrimination in its hiring practices.
   (a) What hypothesis is being tested if a jury commits a type I error by finding the firm guilty?
   (b) What hypothesis is being tested if a jury commits a type II error by finding the firm guilty?

9. The proportion of adults living in a small town who are college graduates is estimated to be p=0.6. To test this hypothesis, a random sample of 15 adults is selected. If number of college graduates in our sample is anywhere from 6 to 12 , we shall not reject the null hypothesis that $p = 0.6$; otherwise we shall conclude that $p \neq 0.6$.
   (a) Evaluate α assuming that $p = 0.6$. Use binomial distribution.
   (b) Evaluate β for the alternatives $p = 0.5$ and $p = 0.7$.

(c) Is this a good test procedure?

10. Repeat example above for if 200 adults are selected and the fail to reject region is defined to be $110 \le x \le 130$, where x is the number of college graduates in our sample. Use the normal approximation. Ans : $\alpha = 0.1498$; $\beta = 0.0793\, for\, p = 0.5$; $\beta = 0.0618\, for\, p = 0.7$.

11. Lots of 40 components each are deemed unacceptable if they contain 3 or more defectives. The procedure for sampling a lot is to select 5 components at random and to reject the lot if a defective is found. What is the probability that exactly 1 defective is found in the sample if there are 3 defectives in the entire lot?

12. The PQR company claims that the lifetime of a type of battery that it manufactures is more than 250hrs. A consumer advocate wishing to determine whether the claim is justified measures the lifetimes of 24 of the company's batteries; the results are :
$271, 230, 198, 275, 282, 225, 284, 219, 253,$
$216, 262, 288, 236, 291, 253, 224, 264, 295, 211, 252, 294, 242, 272, 268$. Assuming the sample to be random, Using **Sign test**, determine whether the company's claim is justified at the 0.05 significance level.