Advanced Encryption Standard (AES)

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Advanced Encryption Standard (AES)

- Clear replacement for DES was needed
  - have theoretical attacks that can break it
  - have demonstrated exhaustive key search attacks
- Can use Triple-DES – but slow with small blocks
- US NIST issued call for ciphers in 1997
- 15 candidates accepted in Jun 98, 5 were shortlisted in Aug-99
- Rijndael was selected as the AES in Oct-2000
- Issued as FIPS PUB 197 standard in Nov-2001

AES Features

- Is a block cipher with a block length of 128 bits.
- allows 3 different key lengths: 128, 192, or 256 bits
- Stronger and faster than Triple-DES
- Active life of 20-30 years (+ archival use)
- Provided full specification and design details. NIST have released all submissions and unclassified analyses
- Both C and Java implementations
- Designed to be:
  - resistant against known attacks
  - speed and code compactness on many CPUs
  - design simplicity

AES Features

- Encryption consists of 10 rounds of processing for 128-bit keys, 12 rounds for 192-bit keys, and 14 rounds for 256-bit keys.
- Commonly used Key size is 128 bits
- Except for the last round in each case, all other rounds are identical
- Each round of processing includes
  - one single-byte based substitution step
  - a row-wise permutation step
  - a column-wise mixing step
  - Addition of the round key
- The order in which these four steps are executed is different for encryption and decryption

Plaintext Block and State Array

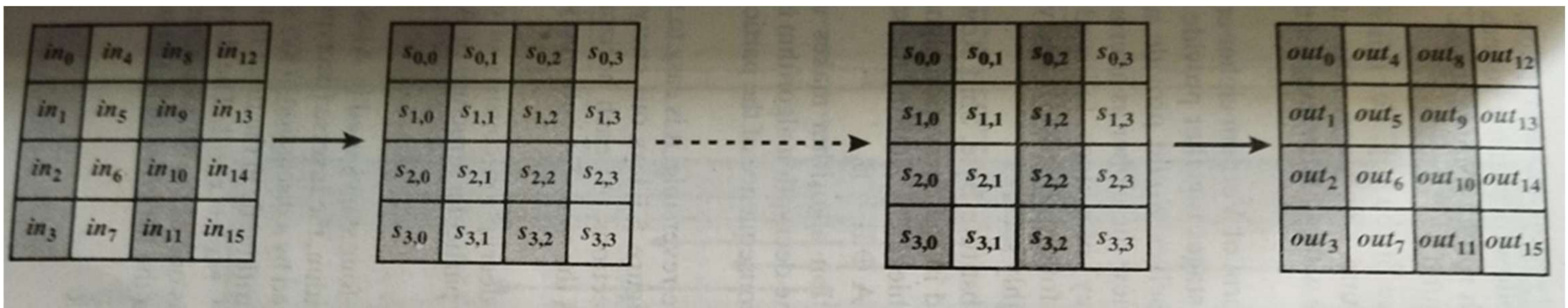
- AES is an **iterative** rather than **Feistel** cipher
- Operates an entire block in every round (Feistel operates of one half of the block at a time)
- Input plaintext is a block of 128 bits depicted as a 4x4 Input Matrix of bytes

$$\begin{bmatrix} byte_0 & byte_4 & byte_8 & byte_{12} \\ byte_1 & byte_5 & byte_9 & byte_{13} \\ byte_2 & byte_6 & byte_{10} & byte_{14} \\ byte_3 & byte_7 & byte_{11} & byte_{15} \end{bmatrix}$$

- First four bytes of a 128-bit input block occupy the first column in the Input Matrix. The next four bytes occupy the second column, and so on

Plaintext Block and State Array

- Input matrix is copied into a 4×4 array of bytes known as the State Array
- State array is modified at each stage of encryption and decryption
- After the final stage, the State array is copied to an Output Matrix.



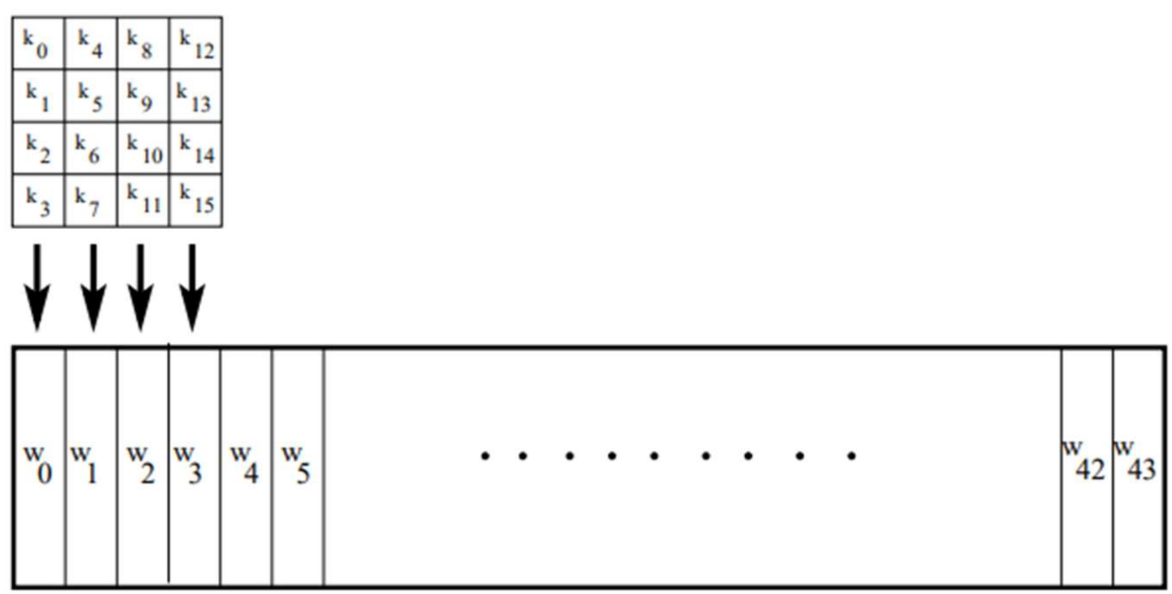
(a) Input, state array, and output

State Array and Word

- AES also has the notion of a word.
- A word consists of four bytes, that is 32 bits.
- Therefore, each column of the state array is a word, as is each row

Secret Key and Words

- Commonly used key size 128 bits
- 128 bit (16B) key is depicted as a 4x4 square matrix of bytes
- Then expanded into a 44 numbers size array of key schedule words.
- 4 distinct words (4x32 = 128 bits) serve as a round key for each round



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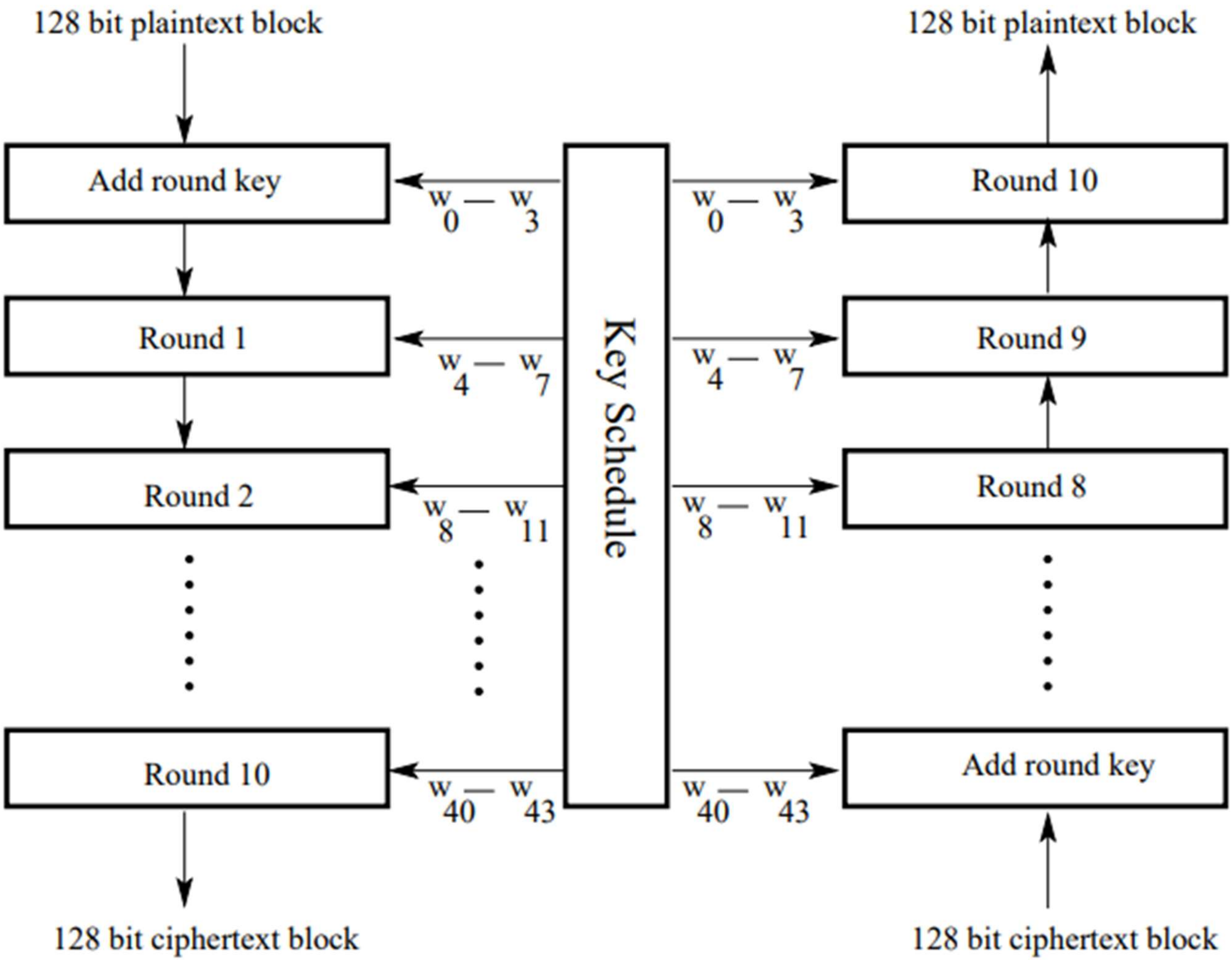
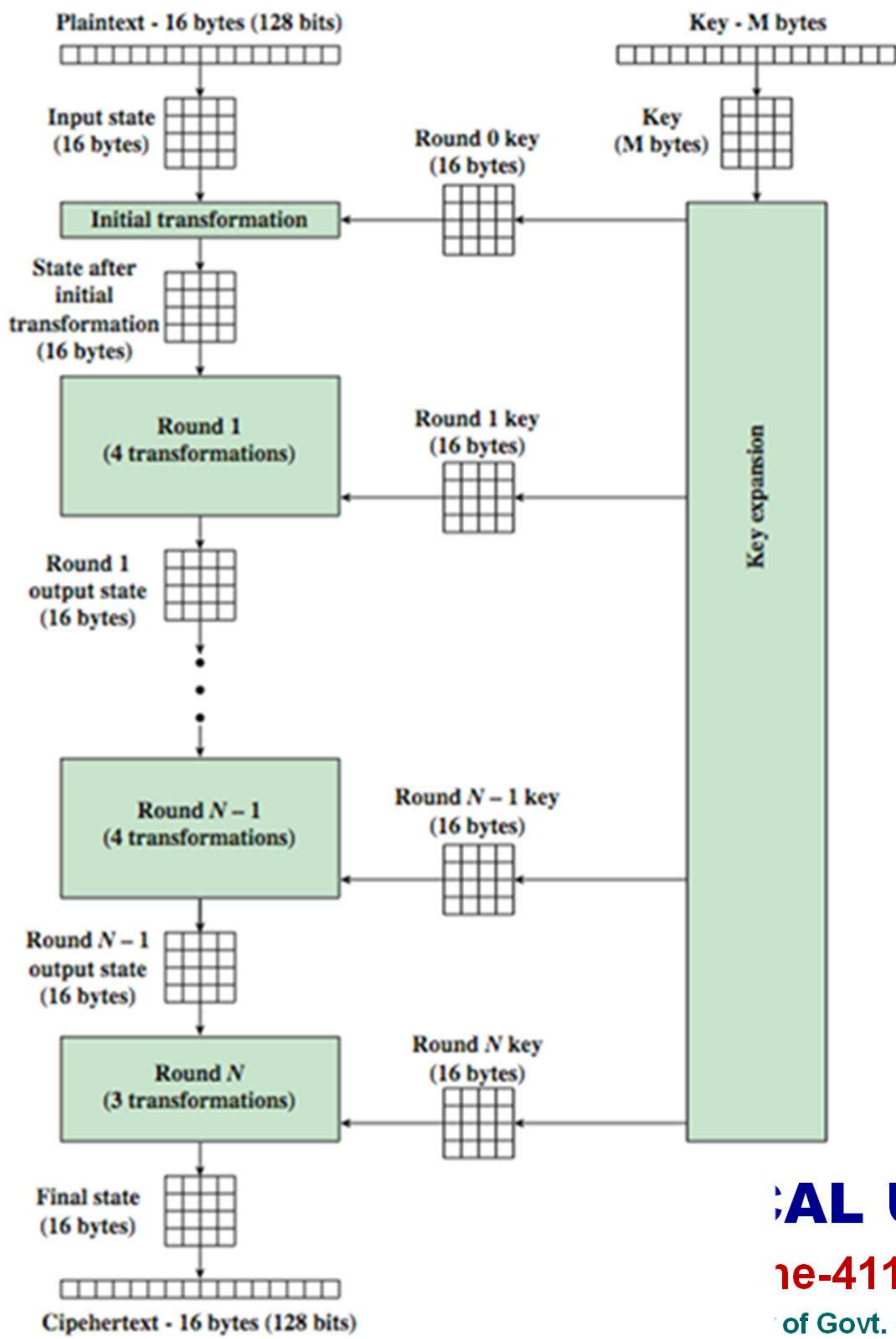
AES Encryption/Decryption



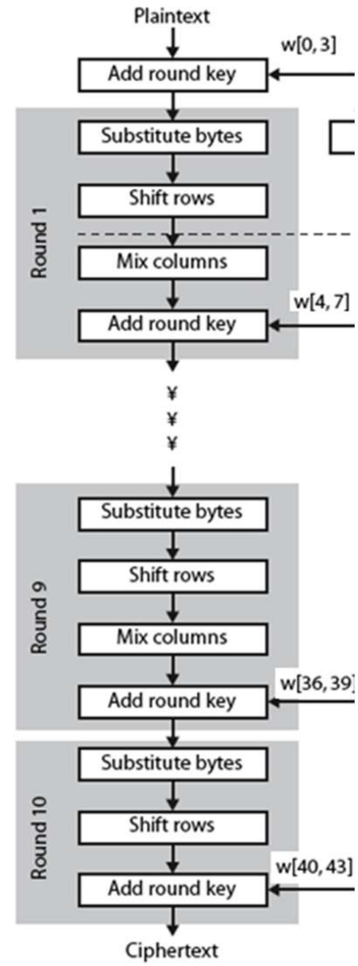
AES Encryption Process



No.of rounds	Key Length (bytes)
10	16
12	24
14	32

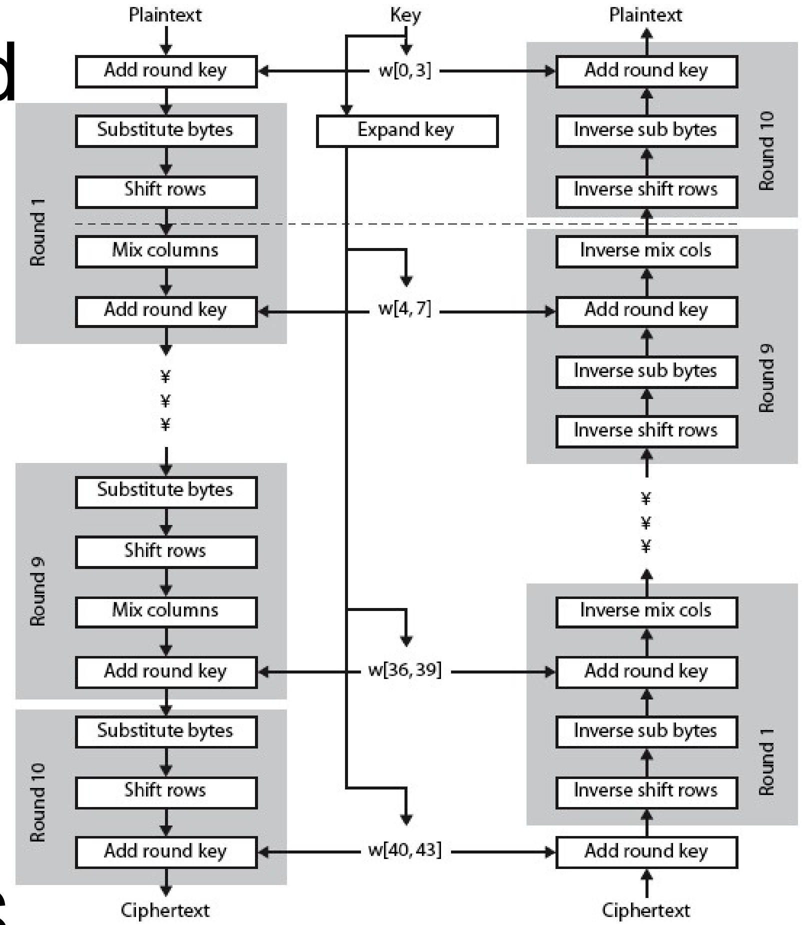
AES Encryption

- Commonly used key size 128 bits
- Initial Add Round key
- 9 rounds of 4 stages in which state undergoes:
  - Byte substitution (S-box lookup is done for every byte)
  - Shift rows (permute bytes between columns)
  - Mix columns (substitutions using matrix multiplication on finite fields)
  - Add round key (XOR state with key material)
- Final Round 10 with 3 stages
  - Byte substitution
  - Shift rows
  - Add round key
- All operations can be combined into XOR and table lookups - hence very fast and efficient



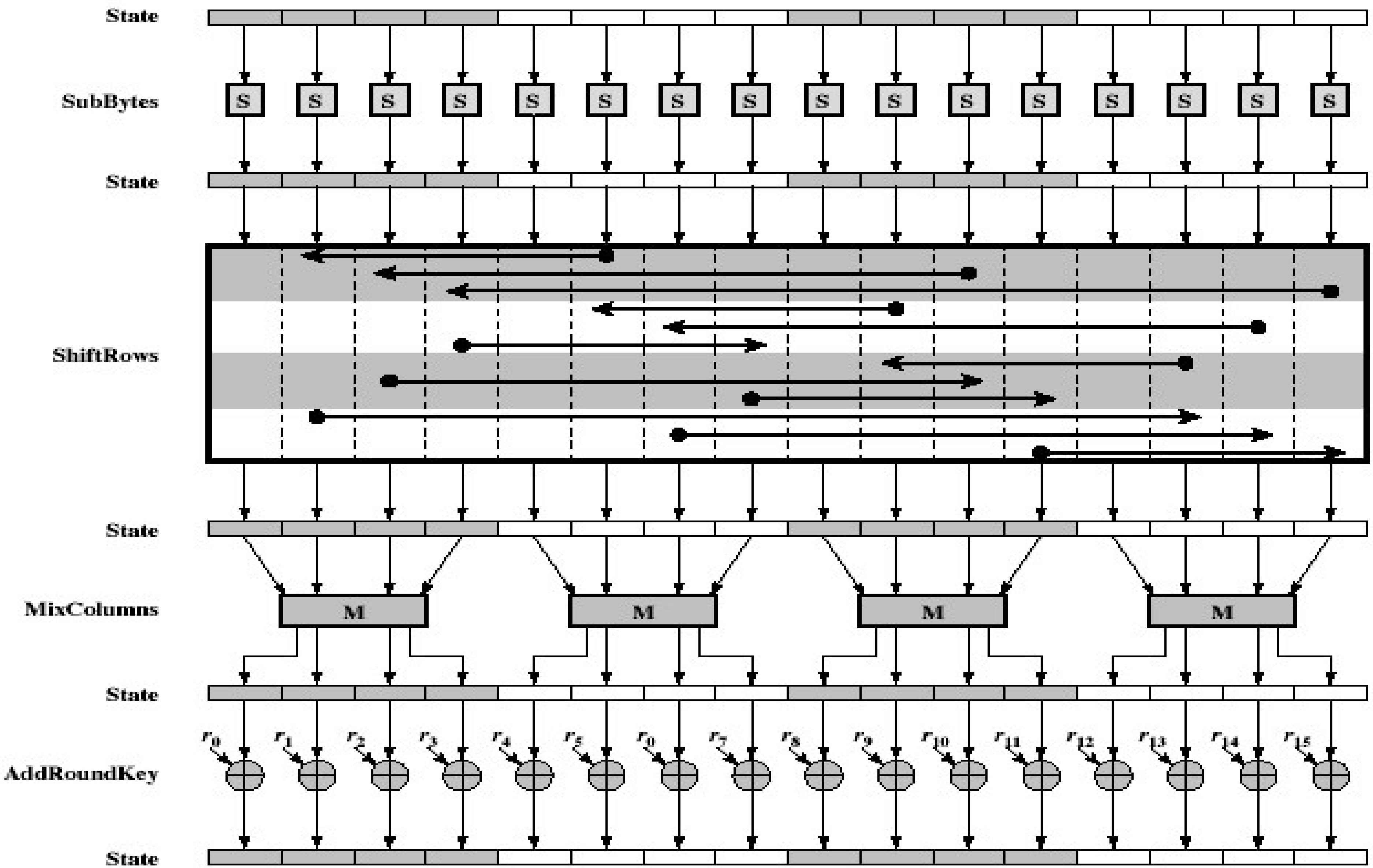
AES Encryption/Decryption

- Begins and ends with Add Round Key stage
- Each round includes Byte substitution, Shift rows and Mix columns to provide confusion, diffusion and non-linearity
  - Not using any key, and would add no security
- Each round of processing in AES involves byte-level substitutions followed by word-level permutations.



(a) Encryption (b) Decryption

A Round of AES

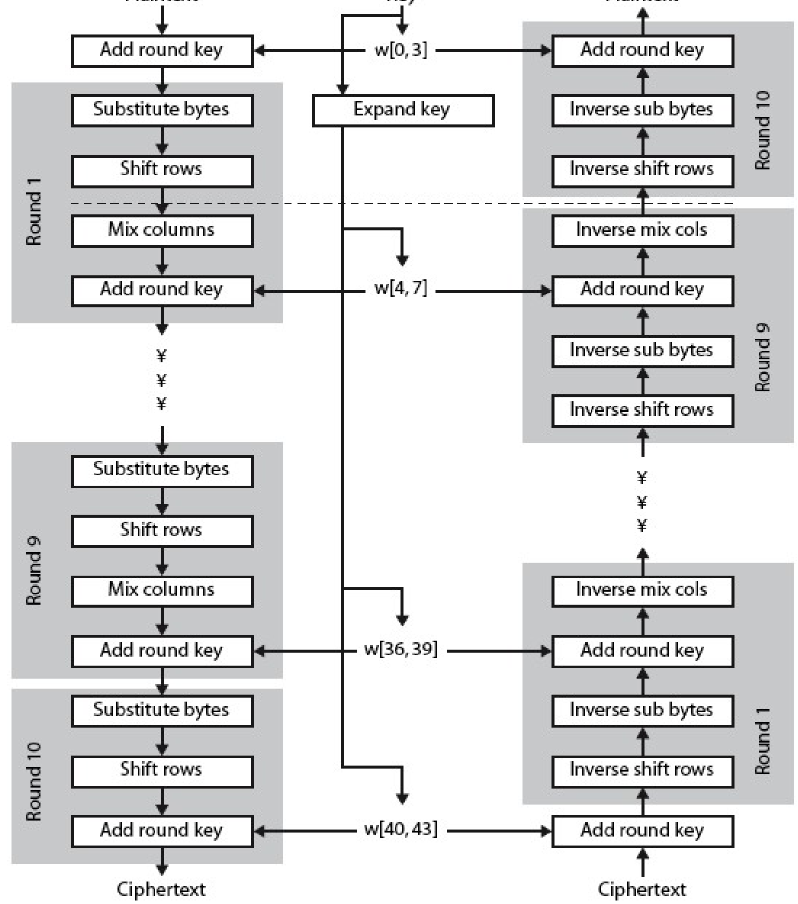


AES Encryption/Decryption

- Decryption uses the key words in reverse order
- Unlike DES, the decryption algorithm differs substantially from the encryption algorithm.
- For encryption, each round consists of 1) Substitute bytes, 2) Shift rows, 3) Mix columns 4) Add round key.
  - Add Round key consists of XORing the output of the previous three steps with four words from the key schedule
- For decryption, each round consists of 1) Inverse shift rows, 2) Inverse substitute bytes, 3) Add round key, and 4) Inverse mix columns
  - Add Round key consists of XORing the output of the previous two steps with four words from the key schedule.

AES Encryption/Decryption

- The last round for encryption does not involve the “Mix columns” step.
- The last round for decryption does not involve the “Inverse mix columns” step



(a) Encryption
(b) Decryption

Byte Substitution

- The goal of the substitution step is to reduce the correlation between the input bits and the output bits at the byte level
- The bit scrambling part of the substitution step ensures that the substitution cannot be described in the form of evaluating a simple mathematical function
- Is a table lookup into 16x16 matrix of S-box containing byte values of all 256 permutations of 8-bit values
- Forward S-Box

		Y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
X	0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
	1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
	2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
	3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
	4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
	5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
	6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
	7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
	8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
	9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
	a	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
	b	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
	c	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
	d	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
	e	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
	f	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

S-Box Construction (1)

1. Initialize S-box with byte values in ascending sequence row by row

	0	1	2	3	4	5	6	7	8	9	....
0	00	01	02	03	04	05	06	07	08	09	....
1	10	11	12	13	14	15	16	17	18	19	....
2	20	21	22	23	24	25	26	27	28	29	....

....

....

   – For example, for the cell located at row index 2 and column indexed 7, we place hex {27} in the cell

2. Replace the value in each cell by its multiplicative inverse in GF(2^8) based on the irreducible polynomial $x^8 + x^4 + x^3 + x + 1$

   – The hex value {00} is replaced by itself since this element has no multiplicative inverse.

Multi Inverse in GF(2^8) (2)

- On the irreducible polynomial $x^8 + x^4 + x^3 + x + 1$

X \ Y	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	00	01	8D	F6	CB	52	7B	D1	E8	4F	29	C0	B0	E1	E5	C7
1	74	B4	AA	4B	99	2B	60	5F	58	3F	FD	CC	FF	40	EE	B2
2	3A	6E	5A	F1	55	4D	A8	C9	C1	0A	98	15	30	44	A2	C2
3	2C	45	92	6C	F3	39	66	42	F2	35	20	6F	77	BB	59	19
4	1D	FE	37	67	2D	31	F5	69	A7	64	AB	13	54	25	E9	09
5	ED	5C	05	CA	4C	24	87	BF	18	3E	22	F0	51	EC	61	17
6	16	5E	AF	D3	49	A6	36	43	F4	47	91	DF	33	93	21	3B
7	79	B7	97	85	10	B5	BA	3C	B6	70	D0	06	A1	FA	81	82
8	83	7E	7F	80	96	73	BE	56	9B	9E	95	D9	F7	02	B9	A4
9	DE	6A	32	6D	D8	8A	84	72	2A	14	9F	88	F9	DC	89	9A
A	FB	7C	2E	C3	8F	B8	65	48	26	C8	12	4A	CE	E7	D2	62
B	0C	E0	1F	EF	11	75	78	71	A5	8E	76	3D	BD	BC	86	57
C	0B	28	2F	A3	DA	D4	E4	0F	A9	27	53	04	1B	FC	AC	E6
D	7A	07	AE	63	C5	DB	E2	EA	94	8B	C4	D5	9D	F8	90	6B
E	B1	0D	D6	EB	C6	0E	CF	AD	08	4E	D7	E3	5D	50	1E	B3
F	5B	23	38	34	68	46	03	8C	DD	9C	7D	A0	CD	1A	41	1C

27 x C9 mod $x^8 + x^4 + x^3 + x + 1$ (3)

- f(x) = 27 g(x) = C9 mod $x^8 + x^4 + x^3 + x + 1$
- f(x) = 27 (00100111) x g(x) = C9 (11001001) mod $x^8+x^4+x^3+x+1$ (00011011)
- b7 of f(x) =0, LS f(x).x = 01001110
- b7 of f(x).x =0, LS, f(x).x^2 = 10011100
- b7 of f(x).x^2 =1, LS, $\oplus$ f(x).x^3 = 00111000 $\oplus$ 00011011 = 00100011
- b7 of f(x).x^3 =0, LS, f(x).x^4 = 01000110
- b7 of f(x).x^4 =0, LS, f(x).x^5 = 10001100
- b7 of f(x).x^5 = 1, LS, $\oplus$ f(x).x^6 = 00011000 $\oplus$ 00011011 = 00000011
- b7 of f(x).x^6 = 0, LS, f(x).x^7 = 00000110
- 00100111 x 11001001 = f(x).1 $\oplus$ f(x).x^3 $\oplus$f(x).x^6 $\oplus$ f(x).x^7

= 00100111 $\oplus$ 00100011 $\oplus$ 00000011 $\oplus$ 00000110

= 00000001

S-Box Construction – Example 2 (4)

- A byte stored in a cell of the table by $b_7b_6b_5b_4b_3b_2b_1b_0$
- Byte stored in the cell (9, 5) of the S-Box is the multiplicative inverse of {95},
  - which is {8A},
  - After step 2, the bit pattern stored in the cell with row index 9 and column index 5 is 8A (1000 1010)
- 95 x 8A = (10010101) x (10001010)

$$= (x^7+x^4+x^2+1)\,(x^7+x^3+x) \bmod (x^8+x^4+x^3+x+1) = 1$$

X \ Y	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	00	01	8D	F6	CB	52	7B	D1	E8	4F	29	C0	B0	E1	E5	C7
1	74	B4	AA	4B	99	2B	60	5F	58	3F	FD	CC	FF	40	EE	B2
2	3A	6E	5A	F1	55	4D	A8	C9	C1	0A	98	15	30	44	A2	C2
3	2C	45	92	6C	F3	39	66	42	F2	35	20	6F	77	BB	59	19
4	1D	FE	37	67	2D	31	F5	69	A7	64	AB	13	54	25	E9	09
5	ED	5C	05	CA	4C	24	87	BF	18	3E	22	F0	51	EC	61	17
6	16	5E	AF	D3	49	A6	36	43	F4	47	91	DF	33	93	21	3B
7	79	B7	97	85	10	B5	BA	3C	B6	70	D0	06	A1	FA	81	82
8	83	7E	7F	80	96	73	BE	56	9B	9E	95	D9	F7	02	B9	A4
9	DE	6A	32	6D	D8	8A	84	72	2A	14	9F	88	F9	DC	89	9A
A	FB	7C	2E	C3	8F	B8	65	48	26	C8	12	4A	CE	E7	D2	62
B	0C	E0	1F	EF	11	75	78	71	A5	8E	76	3D	BD	BC	86	57
C	0B	28	2F	A3	DA	D4	E4	0F	A9	27	53	04	1B	FC	AC	E6
D	7A	07	AE	63	C5	DB	E2	EA	94	8B	C4	D5	9D	F8	90	6B
E	B1	0D	D6	EB	C6	0E	CF	AD	08	4E	D7	E3	5D	50	1E	B3
F	5B	23	38	34	68	46	03	8C	DD	9C	7D	A0	CD	1A	41	1C

S-Box Construction (5)

3. Apply bit scrambling to each bit b_i of the byte stored in a cell of the S-Box

$$b'_i = b_i \otimes b_{(i+4)\ mod\ 8} \otimes b_{(i+5)\ mod\ 8} \otimes b_{(i+6)\ mod\ 8} \otimes b_{(i+7)\ mod\ 8} \otimes c_i$$

- where c_i is the i^{th} bit of a specially designated byte c whose hex value is {63} (0110 0011)

Scrambling Matrix

$$\begin{bmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

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Role Played by the c Byte (6)

- In order for the byte substitution step to be invertible, the byte-to-byte the S-Box must give one-one mapping
- No input byte should map to itself, since a byte mapping to itself would weaken the cipher
  - multiplicative inverses in the construction of the table does give us unique entries in the table for each input byte — except for the input byte {00}
  - With the bit Scrambling, {00} input byte is mapped to {63}
- Bit-scrambling step also breaks the correlation between the bits before the substitution and the bits after the substitution

Scrambling Operation Example

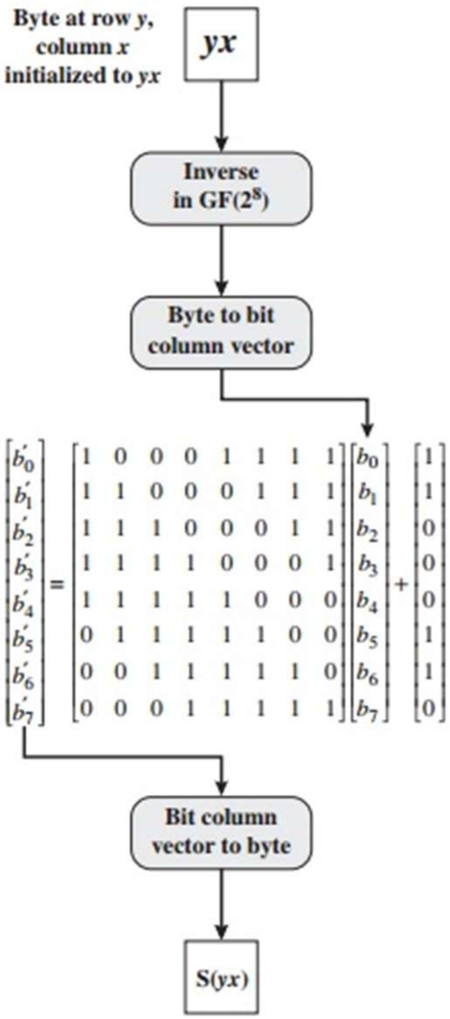
- Consider the byte {95}, the multiplicative Inverse is {8A} (10001010). Performing the scrambling operation to construct S-Box, you get {2A}

$$\begin{bmatrix}1&0&0&0&1&1&1&1\\1&1&0&0&0&1&1&1\\1&1&1&0&0&0&1&1\\1&1&1&1&0&0&0&1\\1&1&1&1&1&0&0&0\\0&1&1&1&1&1&0&0\\0&0&1&1&1&1&1&0\\0&0&0&1&1&1&1&1\end{bmatrix}\begin{bmatrix}0\\1\\0\\1\\0\\0\\0\\1\end{bmatrix}\oplus\begin{bmatrix}1\\1\\0\\0\\0\\1\\1\\0\end{bmatrix}=\begin{bmatrix}1\\0\\0\\1\\0\\0\\1\\0\end{bmatrix}\oplus\begin{bmatrix}1\\1\\0\\0\\0\\1\\1\\0\end{bmatrix}=\begin{bmatrix}0\\1\\0\\1\\0\\1\\0\\0\end{bmatrix}$$

- Row 9 and col 5 of the S-Box is with the value {2A}

		Y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
X	0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
	1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
	2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
	3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
	4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
	5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
	6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
	7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
	8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
	9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
	a	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
	b	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
	c	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
	d	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
	e	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
	f	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

Final S-Box for Encryption

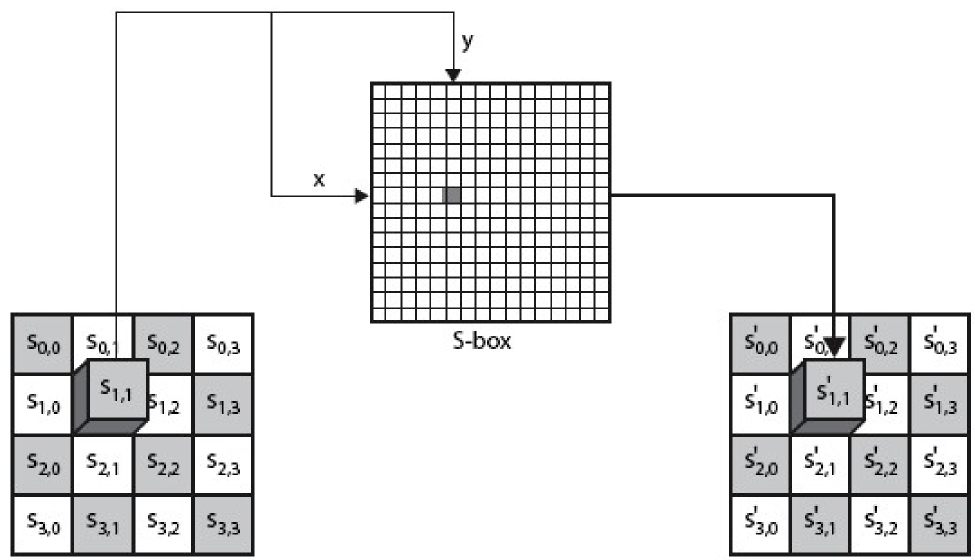


$$\begin{bmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

		Y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
X	0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
	1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
	2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
	3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
	4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
	5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
	6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
	7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
	8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
	9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
	a	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
	b	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
	c	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
	d	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
	e	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
	f	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

Byte Substitution through S-box

- Provides a local nonlinear transformation
- Each individual byte in a state is mapped to a new byte
- Leftmost 4 bits of the state as row and rightmost 4 bits as column are indexed into S-box to select an output byte



Inverse S-Box

- Inverse S-Box is not same as forward S-Box
- Forward S-Box

X \ Y	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
a	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
b	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
c	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
d	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
e	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
f	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

Inverse S-Box

X \ Y	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
1	7C	E3	39	82	9B	2F	FF	87	34	83	43	44	C4	DE	E9	CB
2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
a	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
b	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
c	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
d	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
e	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
f	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

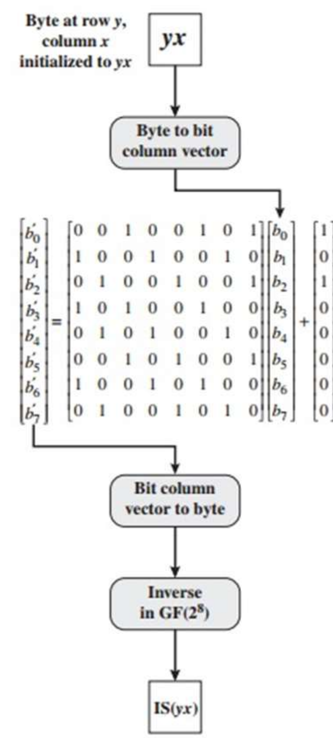
Inverse S-Box Contruction

- Decryption Substitute byte Transformation uses inverse S-box

1. Apply inverse bit scrambling with d = {05} (00000101)

$$b'_i = b_{(i+2)\ mod\ 8} \otimes b_{(i+5)\ mod\ 8} \otimes b_{(i+7)\ mod\ 8} \otimes d_i$$

$$\begin{bmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



2. Find the multiplicative Inverse in GF(2^8)

Inverse Substitution Bytes Proof

- Show that Inverse Substitution Byte is the inverse of Substitution Byte

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

- Let vector for c and d be C and D
- Let 8-bit vector be B
- $B' = XB \oplus C$
- We need to show that $Y(XB \oplus C) \oplus D = B$
- Expanding $YXB \oplus YC \oplus D = B$

Inverse Substitution Bytes Proof

- YXB⊕YC⊕D

$$\begin{bmatrix} 0&0&1&0&0&1&0&1\\ 1&0&0&1&0&0&1&0\\ 0&1&0&0&1&0&0&1\\ 1&0&1&0&0&1&0&0\\ 0&1&0&1&0&0&1&0\\ 0&0&1&0&1&0&0&1\\ 1&0&0&1&0&1&0&0\\ 0&1&0&0&1&0&1&0 \end{bmatrix} \begin{bmatrix} 1&0&0&0&1&1&1&1\\ 1&1&0&0&0&1&1&1\\ 1&1&1&0&0&0&1&1\\ 1&1&1&1&0&0&0&1\\ 1&1&1&1&1&0&0&0\\ 0&1&1&1&1&1&0&0\\ 0&0&1&1&1&1&1&0\\ 0&0&0&1&1&1&1&1 \end{bmatrix} \begin{bmatrix} b_0\\ b_1\\ b_2\\ b_3\\ b_4\\ b_5\\ b_6\\ b_7 \end{bmatrix} \oplus \begin{bmatrix} 0&0&1&0&0&1&0&1\\ 1&0&0&1&0&0&1&0\\ 0&1&0&0&1&0&0&1\\ 1&0&1&0&0&1&0&0\\ 0&1&0&1&0&0&1&0\\ 0&0&1&0&1&0&0&1\\ 1&0&0&1&0&1&0&0\\ 0&1&0&0&1&0&1&0 \end{bmatrix} \begin{bmatrix} 1\\ 0\\ 1\\ 0\\ 0\\ 1\\ 1\\ 0 \end{bmatrix} \oplus \begin{bmatrix} 1\\ 0\\ 1\\ 0\\ 0\\ 0\\ 0\\ 0 \end{bmatrix} =$$

- YX = Identity Matrix and YC = D
- So YC⊕D = Null Vector
- YXB⊕YC⊕D = B

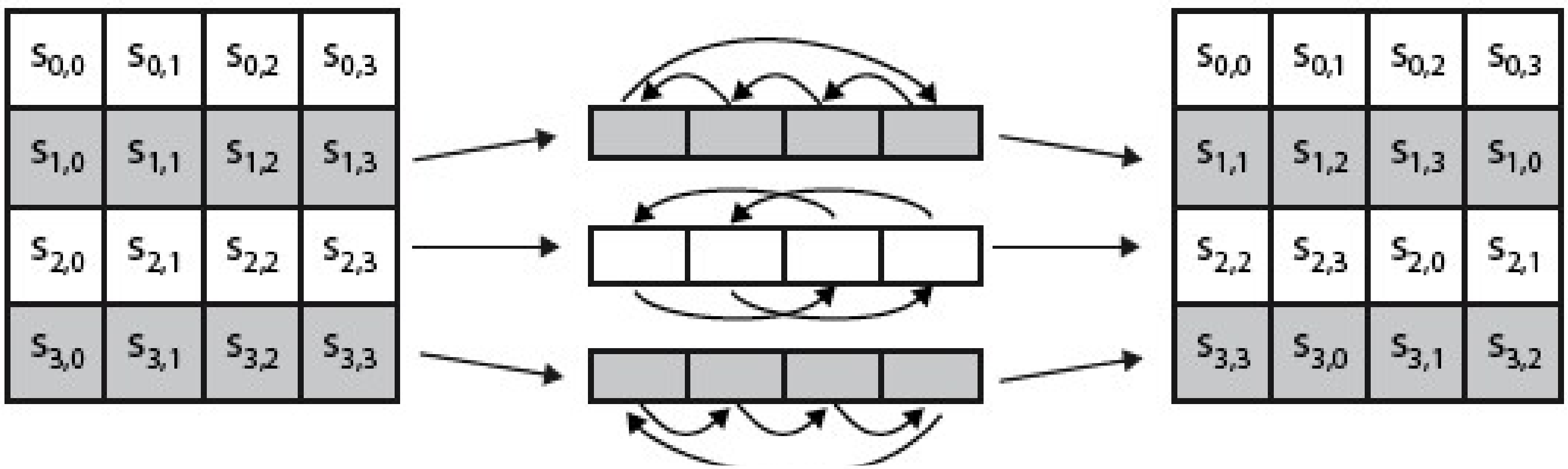
$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix} \begin{bmatrix} b_0\\ b_1\\ b_2\\ b_3\\ b_4\\ b_5\\ b_6\\ b_7 \end{bmatrix} \oplus \begin{bmatrix} 1\\ 0\\ 1\\ 0\\ 0\\ 0\\ 0\\ 0 \end{bmatrix} \oplus \begin{bmatrix} 1\\ 0\\ 1\\ 0\\ 0\\ 0\\ 0\\ 0 \end{bmatrix} = \begin{bmatrix} b_0\\ b_1\\ b_2\\ b_3\\ b_4\\ b_5\\ b_6\\ b_7 \end{bmatrix}$$

Shift Rows

- Scrambles (permutes) up the byte order of the input block
- a circular byte shift in each
  - 1st row is unchanged
  - 2nd row does 1 byte circular shift to left
  - 3rd row does 2 byte circular shift to left
  - 4th row does 3 byte circular shift to left

$$\begin{bmatrix} s_{0.0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1.0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2.0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3.0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} ===> \begin{bmatrix} s_{0.0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1.1} & s_{1,2} & s_{1,3} & s_{1,0} \\ s_{2.2} & s_{2,3} & s_{2,0} & s_{2,1} \\ s_{3.3} & s_{3,0} & s_{3,1} & s_{3,2} \end{bmatrix}$$

Shift Rows



87	F2	4D	97
EC	6E	4C	90
4A	C3	46	E7
8C	D8	95	A6

→

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

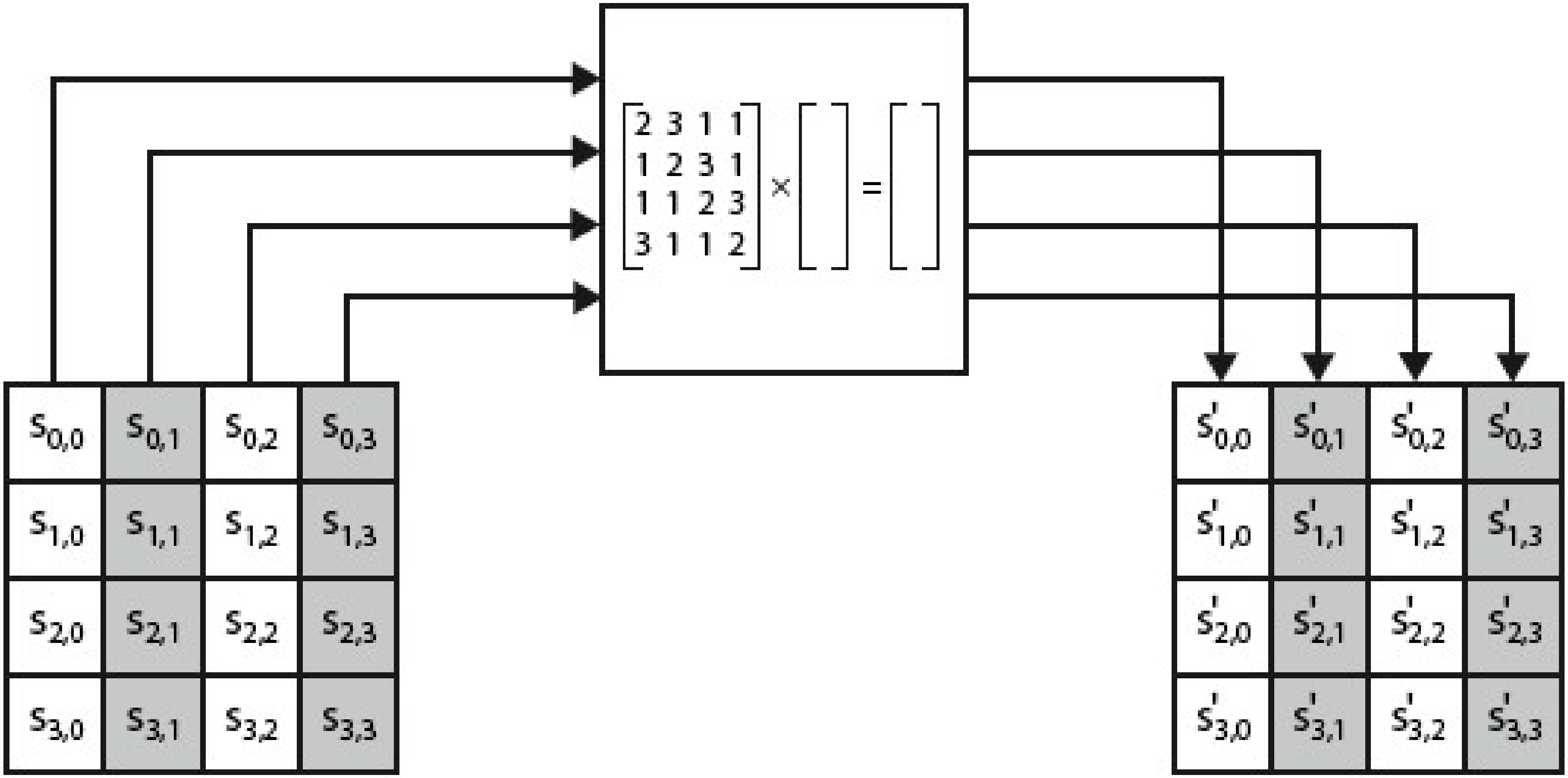
- For decryption, the corresponding step shifts the rows in exactly the opposite fashion

Mix Columns

- A linear mixing transformation that provides high diffusion.
- Each column is processed separately
- Each byte is replaced by a value dependent on all 4 bytes in the column
- Effectively a matrix addition and multiplication in GF(2^8) using prime polynomial m(x) =$x^8+x^4+x^3+x+1$

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

Mix Columns



$\begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} \times [\ ] = [\ ]$
$s_{0,0}$ $s_{0,1}$ $s_{0,2}$ $s_{0,3}$
$s_{1,0}$ $s_{1,1}$ $s_{1,2}$ $s_{1,3}$
$s_{2,0}$ $s_{2,1}$ $s_{2,2}$ $s_{2,3}$
$s_{3,0}$ $s_{3,1}$ $s_{3,2}$ $s_{3,3}$
$s'_{0,0}$ $s'_{0,1}$ $s'_{0,2}$ $s'_{0,3}$
$s'_{1,0}$ $s'_{1,1}$ $s'_{1,2}$ $s'_{1,3}$
$s'_{2,0}$ $s'_{2,1}$ $s'_{2,2}$ $s'_{2,3}$
$s'_{3,0}$ $s'_{3,1}$ $s'_{3,2}$ $s'_{3,3}$

Mix Columns Example

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \times \begin{bmatrix} 87 & F2 & 4D & 97 \\ 6E & 4C & 90 & EC \\ 46 & E7 & 4A & C3 \\ A6 & 8C & D8 & 95 \end{bmatrix} \rightarrow \begin{bmatrix} 47 & 40 & A3 & 4C \\ 37 & D4 & 70 & 9F \\ 94 & E4 & 3A & 42 \\ ED & A5 & A6 & BC \end{bmatrix}$$

$$(\{02\} \bullet \{87\}) \oplus (\{03\} \bullet \{6E\}) \oplus \{46\} \oplus \{A6\} = \{47\}$$

$$\{87\} \oplus (\{02\} \bullet \{6E\}) \oplus (\{03\} \bullet \{46\}) \oplus \{A6\} = \{37\}$$

$$\{87\} \oplus \{6E\} \oplus (\{02\} \bullet \{46\}) \oplus (\{03\} \bullet \{A6\}) = \{94\}$$

$$(\{03\} \bullet \{87\}) \oplus \{6E\} \oplus \{46\} \oplus (\{02\} \bullet \{A6\}) = \{ED\}$$

AES Multiplication

- Uses arithmetic in the finite field GF(2^8)
- with irreducible polynomial $m(x) = x^8+x^4+x^3+x+1$, which is `(10001|1011)` or `{11B}`
- Multiplication of a value by x ({02}) can be implemented as
  - if the b_7 bit is 1 (conditional XOR)
    - a 1- bit non-circular left shift followed by XOR with (0001 1011)
  - if the b_7 bit is 0
    - Only a 1- bit left shift
- Mix Column operation performs shift and XOR
-

Mix Colum Example (Cont)

- To find {02} • {87} ⊕ {03} • {6E} ⊕ {46} ⊕ {A6}
- To find {02} • {87} mod {11B}

  {87} = (1000 0111), here b_7 bit is 1

  1-bit SL →(10000 1110) → ⊕ (10001 1011) = (0001 0101)
- To find {03} • {6E}

  {03} • {6E} = {6E} ⊕ {02} • {6E}

  {6E} is (01101110), here b_7 bit is 0,

  so only 1-bit SL →(1101 1100)

  {6E} ⊕ {02} • {6E} = (0110 1110) ⊕ (1101 1100)

  =(1011 0010)

Mix Colum Example (Cont)

- To find $\{02\} \bullet \{87\} \oplus \{03\} \bullet \{6E\} \oplus \{46\} \oplus \{A6\}$

$\{02\} \bullet \{87\} = (0001\ 0101)$

$\{03\} \bullet \{6E\} = (1011\ 0010)$

$\{46\} = (0100\ 0110)$

$\{A6\} = (1010\ 0110)$

$\oplus \quad = (0100\ \ 0111) = \{47\}$

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

→

47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

Inv Mix Column

- Mix Column

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

- Inverse Mix Column with a different matrix

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

- Can be verified that product produces a unit matrix

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inv Mix Column

$$
\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix}
\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix}
$$

$$(\{0E\} \cdot \{02\}) \oplus \{0B\} \oplus \{0D\} \oplus (\{09\} \cdot \{03\}) = \{01\}$$

$$(\{09\} \cdot \{02\}) \oplus \{0E\} \oplus \{0B\} \oplus (\{0D\} \cdot \{03\}) = \{00\}$$

$$(\{0D\} \cdot \{02\}) \oplus \{09\} \oplus \{0E\} \oplus (\{0B\} \cdot \{03\}) = \{00\}$$

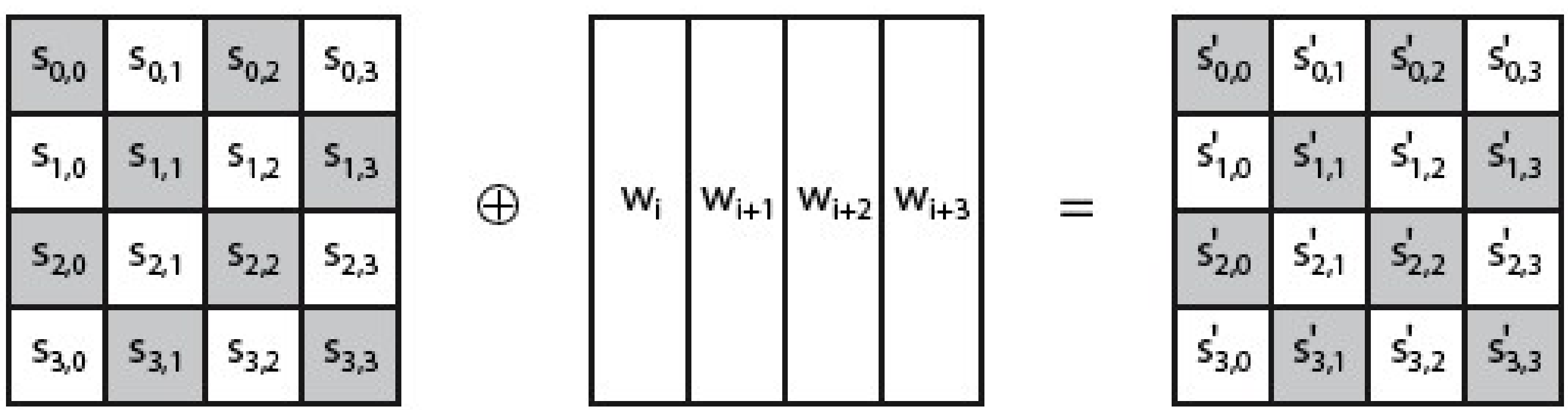
$$(\{0B\} \cdot \{02\}) \oplus \{0D\} \oplus \{09\} \oplus (\{0E\} \cdot \{03\}) = \{00\}$$

$$\{09\} \cdot \{03\} = \{09\} \oplus (\{09\} \cdot \{02\}) = 00001001 \oplus 00010010 = 00011011$$

$$
\begin{array}{lcl}
\{0E\} \cdot \{02\} & = & 00011100 \\
\{0B\} & = & 00001011 \\
\{0D\} & = & 00001101 \\
\{09\} \cdot \{03\} & = & 00011011 \\
 & & 00000001
\end{array}
$$

Add Round Key

- Vernam cipher: to XOR state with 128-bits of the round key
- Again process by column of state with a word of the round key
- Inverse for decryption is identical since XOR is own inverse, just with correct round key
- Designed to be as simple as possible



AES Key Expansion

- Takes 128-bit (16-byte) key and expands into an array of 44/52/60 32-bit words for (10/12/14 rounds)
- Designed to be simple to implement, but by using round constants, break symmetries, and make it much harder to deduce other key bits if just some are known
- Designed to resist known attacks

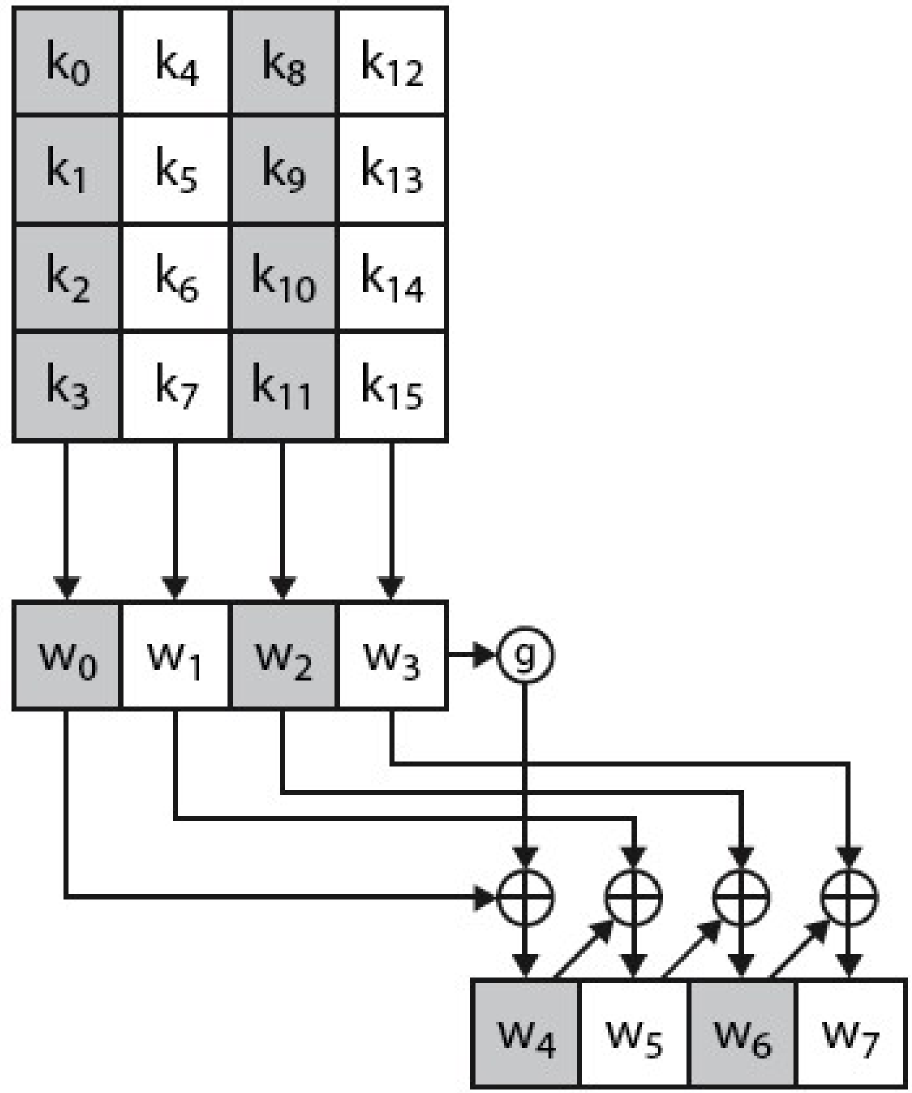
Key Expansion

- Each added word w[i] depends on the immediately preceding word, w[i-1], and the word four positions back, w[i- 4]
- In three out of four cases, a simple XOR is used

```
KeyExpansion (byte key[16], word w[44])
{
    word temp
    for (i = 0; i < 4; i++)   w[i] = (key[4*i], key[4*i+1],
                                      key[4*i+2],
                                      key[4*i+3]);
    for (i = 4; i < 44; i++)
    {
     temp = w[i - 1];
     if (i mod 4 = 0)   temp = SubWord (RotWord (temp))
                               ⊕ Rcon[i/4];
     w[i] = w[i-4] ⊕ temp
    }
}
```

AES Key Expansion

- For a word whose position in the w array is a multiple of 4, a more complex function g is used



Complex Operation g

```
SubWord (RotWord (temp)) ⊕ Rcon[i/4];
```

- RotWord performs a one-byte circular left shift on a word. This means that an input word [b_0, b_1, b_2, b_3] is transformed into [b_1, b_2, b_3, b_0]
- SubWord performs a byte substitution on each byte of its input word, using the S-box
- The result of steps 1 and 2 is XORed with a round constant, Rcon[j]

j	1	2	3	4	5	6	7	8	9	10
RC[j]	01	02	04	08	10	20	40	80	1B	36

Complex Operation g

- Inclusion of a round-dependent Rcon eliminates the symmetry between the ways in which round keys are generated in different rounds
- Rcon is a word in which the three rightmost bytes are always 0
- Effect of an XOR of a word with Rcon is to only perform an XOR on the leftmost byte of the word
- Rcon is different for each round and is defined as Rcon[j] = (RC[j], 0, 0, 0), with RC[1] = 1, RC[j] = 2 · RC[j - 1] and with multiplication defined over the field GF(2^8).

j	1	2	3	4	5	6	7	8	9	10
RC[j]	01	02	04	08	10	20	40	80	1B	36

Key Expansion Rationale

- Designed to resist known attacks
- Design criteria included
  - Knowing part key insufficient to find many more
  - Invertible transformation
  - Fast on wide range of CPU's
  - Use round constants to break symmetry
  - Diffuse key bits into round keys
  - Enough non-linearity to hinder analysis
  - Simplicity of description

Key Expansion Example

- for Key: 0f1571c947d9e8590cb7add6af7f6798

Key Words	Auxiliary Function
w0 = 0f 15 71 c9 w1 = 47 d9 e8 59 w2 = 0c b7 ad w3 = af 7f 67 98	RotWord(w3)= 7f 67 98 af = x1 SubWord(x1)= d2 85 46 79 = y1 Rcon(1)= 01 00 00 00 y1 ⊕ Rcon(1)= d3 85 46 79 = z1
w4 = w0 ⊕ z1 = dc 90 37 b0 w5 = w4 ⊕ w1 = 9b 49 df e9 w6 = w5 ⊕ w2 = 97 fe 72 3f w7 = w6 ⊕ w3 = 38 81 15 a7	RotWord(w7)= 81 15 a7 38 = x2 SubWord(x4)= 0c 59 5c 07 = y2 Rcon(2)= 02 00 00 00 y2 ⊕ Rcon(2)= 0e 59 5c 07 = z2
w8 = w4 ⊕ z2 = d2 c9 6b b7 w9 = w8 ⊕ w5 = 49 80 b4 5e w10 = w9 ⊕ w6 = de 7e c6 61 w11 = w10 ⊕ w7 = e6 ff d3 c6	RotWord(w11)= ff d3 c6 e6 = x3 SubWord(x2)= 16 66 b4 8e = y3 Rcon(3)= 04 00 00 00 y3 ⊕ Rcon(3)= 12 66 b4 8e = z3
w12 = w8 ⊕ z3 = c0 af df 39 w13 = w12 ⊕ w9 = 89 2f 6b 67 w14 = w13 ⊕ w10 = 57 51 ad 06 w15 = w14 ⊕ w11 = b1 ae 7e c0	RotWord(w15)= ae 7e c0 b1 = x4 SubWord(x3)= e4 f3 ba c8 = y4 Rcon(4)= 08 00 00 00 y4 ⊕ Rcon(4)= ec f3 ba c8 = 4
w16 = w12 ⊕ z4 = 2c 5c 65 f1 w17 = w16 ⊕ w13 = a5 73 0e 96 w18 = w17 ⊕ w14 = f2 22 a3 90 w19 = w18 ⊕ w15 = 43 8c dd 50	RotWord(w19)= 8c dd 50 43 = x5 SubWord(x4)= 64 c1 53 1a = y5 Rcon(5)= 10 00 00 00 y5 ⊕ Rcon(5)= 74 c1 53 1a = z5
w20 = w16 ⊕ z5 = 58 9d 36 eb w21 = w20 ⊕ w17 = fd ee 38 7d w22 = w21 ⊕ w18 = 0f cc 9b ed w23 = w22 ⊕ w19 = 4c 40 46 bd	RotWord(w23)= 40 46 bd 4c = x6 SubWord(x5)= 09 5a 7a 29 = y6 Rcon(6)= 20 00 00 00 y6 ⊕ Rcon(6)= 29 5a 7a 29 = z6
w24 = w20 ⊕ z6 = 71 c7 4c c2 w25 = w24 ⊕ w21 = 8c 29 74 bf w26 = w25 ⊕ w22 = 83 e5 ef 52 w27 = w26 ⊕ w23 = cf a5 a9 ef	RotWord(w27)= a5 a9 ef cf = x7 SubWord(x6)= 06 d3 df 8a = y7 Rcon(7)= 40 00 00 00 y7 ⊕ Rcon(7)= 46 d3 df 8a = z7
w28 = w24 ⊕ z7 = 37 14 93 48 w29 = w28 ⊕ w25 = bb 3d e7 f7 w30 = w29 ⊕ w26 = 38 d8 08 a5 w31 = w30 ⊕ w27 = f7 7d a1 4a	RotWord(w31)= 7d a1 4a f7 = x8 SubWord(x7)= ff 32 d6 68 = y8 Rcon(8)= 80 00 00 00 y8 ⊕ Rcon(8)= 7f 32 d6 68 = z8
w32 = w28 ⊕ z8 = 48 26 45 20 w33 = w32 ⊕ w29 = f3 1b a2 d7 w34 = w33 ⊕ w30 = cb c3 aa 72 w35 = w34 ⊕ w32 = 3c be 0b 38	RotWord(w35)= be 0b 38 3c = x9 SubWord(x8)= ae 2b 07 eb = y9 Rcon(9)= 1B 00 00 00 y9 ⊕ Rcon(9)= b5 2b 07 eb = z9
w36 = w32 ⊕ z9 = fd 0d 42 cb w37 = w36 ⊕ w33 = 0e 16 e0 1c w38 = w37 ⊕ w34 = c5 d5 4a 6e w39 = w38 ⊕ w35 = f9 6b 41 56	RotWord(w39)= 6b 41 56 f9 = x10 SubWord(x9)= 7f 83 b1 99 = y10 Rcon(10)= 36 00 00 00 y10 ⊕ Rcon(10)= 49 83 b1 99 = z10
w40 = w36 ⊕ z10 = b4 8e f3 52 w41 = w40 ⊕ w37 = ba 98 13 4e w42 = w41 ⊕ w38 = 7f 4d 59 20 w43 = w42 ⊕ w39 = 86 26 18 76	

AES Example Encryption

- Plaintext: 0123456789abcdeffedcba9876543210
- Key: 0f1571c947d9e8590cb7add6af7f6798
- Ciphertext: ff0b844a0853bf7c6934ab4364148fb9

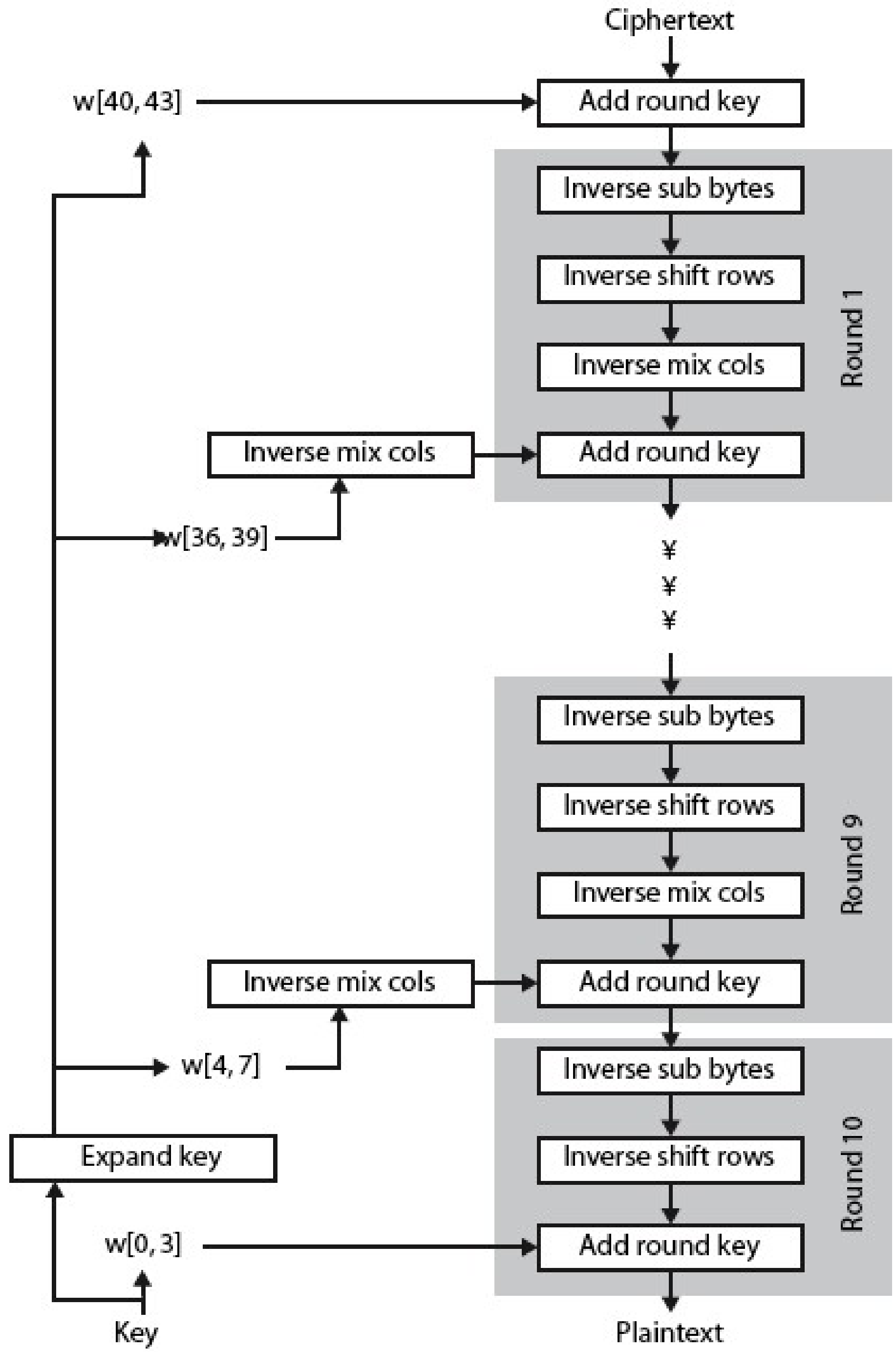
Start of round	After SubBytes	After ShiftRows	After MixColumns	Round Key
01 89 fe 76 23 ab dc 54 45 cd ba 32 67 ef 98 10				0f 47 0c af 15 d9 b7 7f 71 e8 ad 67 c9 59 d6 98
0e ce f2 d9 36 72 6b 2b 34 25 17 55 ae b6 4e 88	ab 8b 89 35 05 40 7f f1 18 3f f0 fc e4 4e 2f c4	ab 8b 89 35 40 7f f1 05 f0 fc 18 3f c4 e4 4e 2f	b9 94 57 75 e4 8e 16 51 47 20 9a 3f c5 d6 f5 3b	dc 9b 97 38 90 49 fe 81 37 df 72 15 b0 e9 3f a7
65 0f c0 4d 74 c7 e8 d0 70 ff e8 2a 75 3f ca 9c	4d 76 ba e3 92 c6 9b 70 51 16 9b e5 9d 75 74 de	4d 76 ba e3 c6 9b 70 92 9b e5 51 16 de 9d 75 74	8e 22 db 12 b2 f2 dc 92 df 80 f7 c1 2d c5 1e 52	d2 49 de e6 c9 80 7e ff 6b b4 c6 d3 b7 5e 61 c6
5c 6b 05 f4 7b 72 a2 6d b4 34 31 12 9a 9b 7f 94	4a 7f 6b bf 21 40 3a 3c 8d 18 c7 c9 b8 14 d2 22	4a 7f 6b bf 40 3a 3c 21 c7 c9 8d 18 22 b8 14 d2	b1 c1 0b cc ba f3 8b 07 f9 1f 6a c3 1d 19 24 5c	c0 89 57 b1 af 2f 51 ae df 6b ad 7e 39 67 06 c0
71 48 5c 7d 15 dc da a9 26 74 c7 bd 24 7e 22 9c	a3 52 4a ff 59 86 57 d3 f7 92 c6 7a 36 f3 93 de	a3 52 4a ff 86 57 d3 59 c6 7a f7 92 de 36 f3 93	d4 11 fe 0f 3b 44 06 73 cb ab 62 37 19 b7 07 ec	2c a5 f2 43 5c 73 22 8c 65 0e a3 dd f1 96 90 50
f8 b4 0c 4c 67 37 24 ff ae a5 c1 ea e8 21 97 bc	41 8d fe 29 85 9a 36 16 e4 06 78 87 9b fd 88 65	41 8d fe 29 9a 36 16 85 78 87 e4 06 65 9b fd 88	2a 47 c4 48 83 e8 18 ba 84 18 27 23 eb 10 0a f3	58 fd 0f 4c 9d ee cc 40 36 38 9b 46 eb 7d ed bd
72 ba cb 04 1e 06 d4 fa b2 20 bc 65 00 6d e7 4e	40 f4 1f f2 72 6f 48 2d 37 b7 65 4d 63 3c 94 2f	40 f4 1f f2 6f 48 2d 72 65 4d 37 b7 2f 63 3c 94	7b 05 42 4a 1e d0 20 40 94 83 18 52 94 c4 43 fb	71 8c 83 cf c7 29 e5 a5 4c 74 ef a9 c2 bf 52 ef
0a 89 c1 85 d9 f9 c5 e5 d8 f7 f7 fb 56 7b 11 14	67 a7 78 97 35 99 a6 d9 61 68 68 0f b1 21 82 fa	67 a7 78 97 99 a6 d9 35 68 0f 61 68 fa b1 21 82	ec 1a c0 80 0c 50 53 c7 3b d7 00 ef b7 22 72 e0	37 bb 38 f7 14 3d d8 7d 93 e7 08 a1 48 f7 a5 4a
db a1 f8 77 18 6d 8b ba a8 30 08 4e ff d5 d7 aa	b9 32 41 f5 ad 3c 3d f4 c2 04 30 2f 16 03 0e ac	b9 32 41 f5 3c 3d f4 ad 30 2f c2 04 ac 16 03 0e	b1 1a 44 17 3d 2f ec b6 0a 6b 2f 42 9f 68 f3 b1	48 f3 cb 3c 26 1b c3 be 45 a2 aa 0b 20 d7 72 38
f9 e9 8f 2b 1b 34 2f 08 4f c9 85 49 bf bf 81 89	99 1e 73 f1 af 18 15 30 84 dd 97 3b 08 08 0c a7	99 1e 73 f1 18 15 30 af 97 3b 84 dd a7 08 08 0c	31 30 3a c2 ac 71 8c c4 46 65 48 eb 6a 1c 31 62	fd 0e c5 f9 0d 16 d5 6b 42 e0 4a 41 cb 1c 6e 56
cc 3e ff 3b a1 67 59 af 04 85 02 aa a1 00 5f 34	4b b2 16 e2 32 85 cb 79 f2 97 77 ac 32 63 cf 18	4b b2 16 e2 85 cb 79 32 77 ac f2 97 18 32 63 cf	4b 86 8a 36 b1 cb 27 5a fb f2 f2 af cc 5a 5b cf	b4 8e f3 52 ba 98 13 4e 7f 4d 59 20 86 26 18 76
ff 08 69 64 0b 53 34 14 84 bf ab 8f				

AES Example Avalanche

Round		Number of bits that differ
	0123456789abcdeffedcba9876543210 0023456789abcdeffedcba9876543210	1
0	0e3634aece7225b6f26b174ed92b5588 0f3634aece7225b6f26b174ed92b5588	1
1	657470750fc7ff3fc0e8e8ca4dd02a9c c4a9ad090fc7ff3fc0e8e8ca4dd02a9c	20
2	5c7bb49a6b72349b05a2317ff46d1294 fe2ae569f7ee8bb8c1f5a2bb37ef53d5	58
3	7115262448dc747e5cdac7227da9bd9c ec093dfb7c45343d689017507d485e62	59
4	f867aee8b437a5210c24c1974cffeabc 43efdb697244df808e8d9364ee0ae6f5	61
5	721eb200ba06206dcbd4bce704fa654e 7b28a5d5ed643287e006c099bb375302	68
6	0ad9d85689f9f77bc1c5f71185e5fb14 3bc2d8b6798d8ac4fe36a1d891ac181a	64
7	db18a8ffa16d30d5f88b08d777ba4eaa 9fb8b5452023c70280e5c4bb9e555a4b	67
8	f91b4fbfe934c9bf8f2f85812b084989 20264e1126b219aef7feb3f9b2d6de40	65
9	cca104a13e678500ff59025f3bafaa34 b56a0341b2290ba7dfdfbddcd8578205	61
10	ff0b844a0853bf7c6934ab4364148fb9 612b89398d0600cde116227ce72433f0	58

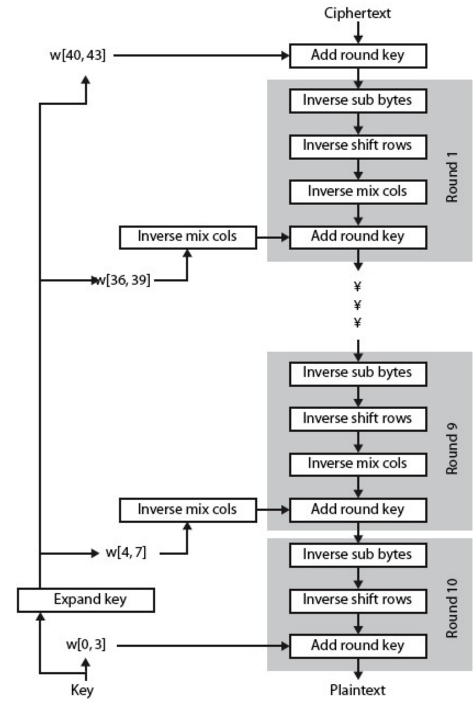
AES Decryption

- AES decryption is not identical to encryption since steps done in reverse
- But can define an equivalent inverse cipher with steps as for encryption
  - but using inverses of each step
  - with a different key schedule



AES Decryption

- works since the result is unchanged when
  - swap byte substitution and shift rows
    - For a given State S_i ,
    - InvShiftRows [InvSubBytes (S_i)] = InvSubBytes [InvShiftRows (S_i)]
  - swap mix columns and add (tweaked) round key



Interchanging AddRoundKey and InvMixColumns

- Transformations AddRoundKey and InvMixColumns do not alter the sequence of bytes in State
- AddRoundKey and InvMixColumns are linear with respect to the column input and both operate on State one column at a time
- For a given State S_i and a given round key w_j

$$\text{InvMixColumns}(S_i \oplus w_j) = [\text{InvMixColumns}(S_i)] \oplus [\text{InvMixColumns}(w_j)]$$

Implementation on 8-bit CPU

- Can efficiently implement on 8-bit CPU
  - Byte substitution works on bytes using a table of 256 entries
  - Shift rows is simple byte shifting
  - Add round key works on byte XORs
  - Mix columns requires matrix multiply in $GF(2^8)$ which works on byte values, can be simplified to use a table lookup to avoid timing attacks

Implementation on 32-bit CPU

- Can efficiently implement on 32-bit CPU
  - Redefine steps to use 32-bit words
  - Can precompute 4 tables of 256-words
  - Then each column in each round can be computed using 4 table lookups + 4 XORs
  - At a cost of 16Kb to store tables
- Designers believe this very efficient implementation was a key factor in its selection as the AES cipher

Average Time Required for Exhaustive Key Search

Key size (bits)	Cipher	Number of Alternative Keys	Time Required at 10^9 decryptions/s	Time Required at 10^{13} decryptions/s
56	DES	$2^{56} \approx 7.2 \times 10^{16}$	2^{55} ns = 1.125 years	1 hour
128	AES	$2^{128} \approx 3.4 \times 10^{38}$	2^{127} ns = 5.3×10^{21} years	5.3×10^{17} years
168	Triple DES	$2^{168} \approx 3.7 \times 10^{50}$	2^{167} ns = 5.8×10^{33} years	5.8×10^{29} years
192	AES	$2^{192} \approx 6.3 \times 10^{57}$	2^{191} ns = 9.8×10^{40} years	9.8×10^{36} years
256	AES	$2^{256} \approx 1.2 \times 10^{77}$	2^{255} ns = 1.8×10^{60} years	1.8×10^{56} years
26 characters (permutation)	Monoalphabetic	$26! = 4 \times 10^{26}$	2×10^{26} ns = 6.3×10^{9} years	6.3×10^{6} years