1. If $t1(n) \in O(g1(n))$ and $t2(n) \in O(g2(n))$, prove that $t1(n) + t2(n) \in O(\max\{g1(n), g2(n)\})$

For any four arbitrary real numbers, a_1, b_1, a_2, b_2
such that $a_1 \le b_1$ and $a_2 \le b_2$
we have $a_1 + a_2 \le 2\max\{b_1, b_2\}$

Since $t_1(n) \in O(g_1(n))$, then there exists some constant c_1 and non-negative integer n_1 such that

$$t_1(n) \le c_1 g_1(n) \text{ for all } n \ge n_1$$

Since $t_2(n) \in O(g_2(n))$, then there exists some constant c_2 and non-negative integer n_2 such that

$$t_2(n) \le c_2 g_2(n) \text{ for all } n \ge n_2$$

Let $c_3 = \max\{c_1, c_2\}$ and $n_0 = \max\{n_1, n_2\}$

$$\begin{aligned} t_1(n) + t_2(n) &\le c_1 g_1(n) + c_2 g_2(n) \\ &\le c_3 g_1(n) + c_3 g_2(n) \\ &= c_3\{g_1(n) + g_2(n)\} \\ &\le 2c_3 \max\{g_1(n), g_2(n)\} \end{aligned}$$

Hence, $t_1(n) + t_2(n) \in O(\max\{g_1(n), g_2(n)\}$, with constants c and n_0 required by the O definition being $2c_3 = 2\max\{c_1, c_2\}$ and $\max\{n_1, n_2\}$ respectively.

2. Find the time complexity of the below recurrence equation:

3)

$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + 1 & \text{if } n > 1 \\ 1 & \text{otherwise} \end{cases}$$

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$ Masters Theorem

$a = 2$
$b = 2$

$$\log_b a = \log_2 2 = 1$$

$k = 0$

$$1 > 0$$
$$\log_b a > k$$

case i)

$$\theta(n \cdot \log_b a)$$
$$\theta(n \cdot 1)$$
$$\theta(n)$$

4) $T(n) = \begin{cases} 2T(n-1) & \text{if } n>0 \\ 1 & \text{otherwise.} \end{cases}$

Backward substitution:-

$$T(n) = 2T(n-1) \rightarrow ①$$ Initial $T(0) = 0$

$n = n-1$

$$T(n-1) = 2T((n-1)-1)$$
$$T(n-1) = 2T(n-2) \rightarrow ②$$

sub ② in ①

$$T(n) = 2[2T(n-2)]$$
$$T(n) = 2^2 T(n-2) \rightarrow ③$$

$n = n-2$

$$T(n-2) = 2T((n-2)-1)$$
$$T(n-2) = 2T(n-3) \rightarrow ④$$

sub ④ in ③

$$T(n) = 2^2[2T(n-3)]$$
$$T(n) = 2^3 T(n-3) \rightarrow ⑤$$

$n = n-3$

$$T(n-3) = 2T((n-3)-1)$$
$$T(n-3) = 2T(n-4) \rightarrow ⑥$$

sub ⑥ in ⑤

$$T(n) = 2^3[2T(n-4)]$$
$$= 2^4 T(n-4) \rightarrow ⑦$$
$$T(n) = 2^k T(n-k)$$
$$n-k = 0 \Rightarrow n = k$$

if $T(0) = 1$

$$T(n) = 2^k \cdot T(0)$$

$$T(n) = 2^k \cdot 1$$
$$T(n) = 2^k$$
$$\therefore \quad n = k$$
$$T(n) = O(2^n)$$

5) Big O notation: show that $f(n) = n^2+3n+5$ is $O(n^2)$

To prove that $f(n) = n^2+3n+5$ is $O(n^2)$ we need to find constants c and n_0 such that $f(n) \leq c \cdot n^2$ for all $n \geq n_0$

$$f(n) = n^2+3n+5$$

For $n \geq 1$, $n^2 \geq n \ldots$ so on

$$f(n) = n^2+3n+5 \leq n^2+3n^2+5n^2$$
$$f(n) = n^2+3n+5 \leq 9n^2 \quad \text{for } n \geq 1$$

So, for $c=9$ and $n_0 = 1$.

$$f(n) \leq c \cdot n^2 \text{ for all } n \geq n_0$$

That proves $f(n)$ is $O(n^2)$

6. Big omega notation: prove that $g(n) = n^3+2n^2+4n$ is $\Omega(n^3)$

To prove that $g(n) = n^3+2n^2+4n$ is $\Omega(n^3)$ we need to find constants c and n_0 such that

$$g(n) \geq c \cdot n^3 \text{ for all } n \geq n_0$$

$$g(n) = n^3+2n^2+4n$$

For $n \geq 1$,

$$g(n) = n^3+2n^2+4n \geq n^3$$

Since $2n^2$ and $4n$ are both less than n^3 when $n \geq 1$

So, for $c=1$ and $n_0 = 1$

$$g(n) \geq c \cdot n^3 \text{ for all } n \geq n_0$$

That proves $g(n)$ is $\Omega(n^3)$

7. Big Theta Notation: Determine whether $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not

1. $h(n) = 4n^2 + 3n$ is $O(n^2)$:

   For $n \geq 1$, $h(n) \leq 4n^2 + 3n^2$
   (Since $3n$ is less than n^2 when $n \geq 1$)

   For This simplifies to $h(n) \leq 7n^2$
   for $n \geq 1$

   Therefore, $h(n)$ is $O(n^2)$

2. $h(n) = 4n^2 + 3n$ is $\Omega(n^2)$:

   For $n \geq 1$, $h(n) \geq 4n^2$
   (Since $3n$ is positive)
   Therefore $h(n)$ is $\Omega(n^2)$

Since $h(n)$ is both $O(n^2)$ and $\Omega(n^2)$, it is $\Theta(n^2)$

8. Let $f(n) = n^3 - 2n^2 + n$ and $g(n) = -n^2$ show whether $f(n) = \Omega(g(n))$ is true or false and justify your answer.

$n = 1$

$$f(1) = 1^3 - 2(1)^2 + 1 = 1 - 2 + 1 = 0 \qquad g(1) = -(n)^2 = (-1)^2 = 1$$

$n = 2$

$$f(2) = 2^3 - 2(2)^2 + 2 = 8 - 8 + 1 = 2 \qquad g(2) = (-2)^2 = 4$$

$n = 3$

$$f(3) = 3^3 - 2(3)^2 + 3 = 27 - 9 + 3 = 21 \qquad g(3) = (-3)^2 = 9$$

$n=4$

$$f(4) = 4^3 - 2(4)^2 + 4 \qquad g(n) = (-4)^2$$
$$= 64 - 32 + 4 \qquad = 16$$
$$= 32 + 4$$
$$= 36$$

$n=5$

$$f(5) = 5^3 - 2(5)^2 + 5 \qquad g(n) = (-5)^2$$
$$= 15 - 50 + 5 \qquad = 25$$
$$= 35 + 5$$
$$= 40$$

$$f(n) \geq g(n)$$

So it is best case according to asymptotic notation.

$$f(n) = \Omega(g(n))$$

9. Determine whether $h(n) = n\log n + n$ is in $\Theta(n \log n)$ prove a rigorous proof for your conclusion.

1. Upper Bound (O notation):

we need to find c_1 and n_0 such that
$h(n) \leq c_1 \cdot n\log n$ for all $n \geq n_0$

$$h(n) = n\log n + n$$
$$\leq n\log n + n\log n \text{ (since } \log n \text{ is increasing)}$$
$$= 2n\log n$$

Now, let $c_1 = 2$, then $h(n) \leq 2n\log n$ for all $n \geq 1$
so, $h(n)$ is $O(n\log n)$

2. Lower Bound (Ω notation):

we need to find c_2 and n_0 such that
$h(n) \geq c_2 \cdot n\log n$ for all $n \geq n_0$

$$h(n) = n\log n + n$$
$$\geq \frac{1}{2} \cdot n\log n \text{ (for } n \geq 2)$$

Now, let $c_2 = \frac{1}{2}$, then $h(n) \geq \frac{1}{2} \cdot n \log n$
for all $n \geq 2$. so, $h(n)$ is $\Omega(n \log n)$

3. **Combining Bounds:**

since $h(n)$ is both $O(n \log n)$ and $\Omega(n \log n)$,
it is also $\Theta(n \log n)$

Thus, $h(n) = n \log n + n$ is in $\Theta(n \log n)$

10. **solve the following recurrence relations and find the order of growth for solutions.**

$$T(n) = 4T(n/2) + n^2, \quad T(1) = 1$$

$$T(n) = aT(n/b) + f(n)$$

$a = 4$
$b = 2$

$$\log_b a = \log_2 4 = 2$$

$$k = 2$$

$$2 = 2$$

$$\log_b a = k$$

case ii)

$$p > -1 \quad \Theta(n^k \log_n^{p+1})$$

$$\Theta(n^2 \cdot \log_n^{1+1})$$

$$\Theta(n^2 \cdot \log_n^2)$$

$$T(n) = \Theta(n^2 \cdot \log(n))$$

The order of growth for the solution is
$n^2 \cdot \log(n)$

12. Demonstrate the Binary search method to search key=23 from the array arr=[]={2,5,8,12,16,23,38,56,72,91}

0	1	2	3	4	5	6	7	8	9
2	5	8	12	16	23	38	56	72	91

low=0

high=9

$mid = \frac{low+high}{2}$

$= \frac{0+9}{2} = 4$

a[mid]==key

a[4]==23

16!=23

16<23

low=mid+1

5	6	7	8	9
23	38	56	72	91

low=5 high=9

$mid = \frac{5+9}{2} = \frac{14}{2} = 7$

a[mid]==key

a[7]==23

56!=23 56>23

high=mid-1

5	6	7
23	38	56

low=5 high=7

$mid = \frac{5+7}{2} = 6$

a[6]==23

38!=23

38>23

high=mid-1

5
23

low=5 high=5

$mid = \frac{5+5}{2} = 5$

a[5]==~~mid~~ key

23==23 ✓

Return the position of the key (i.e) 5

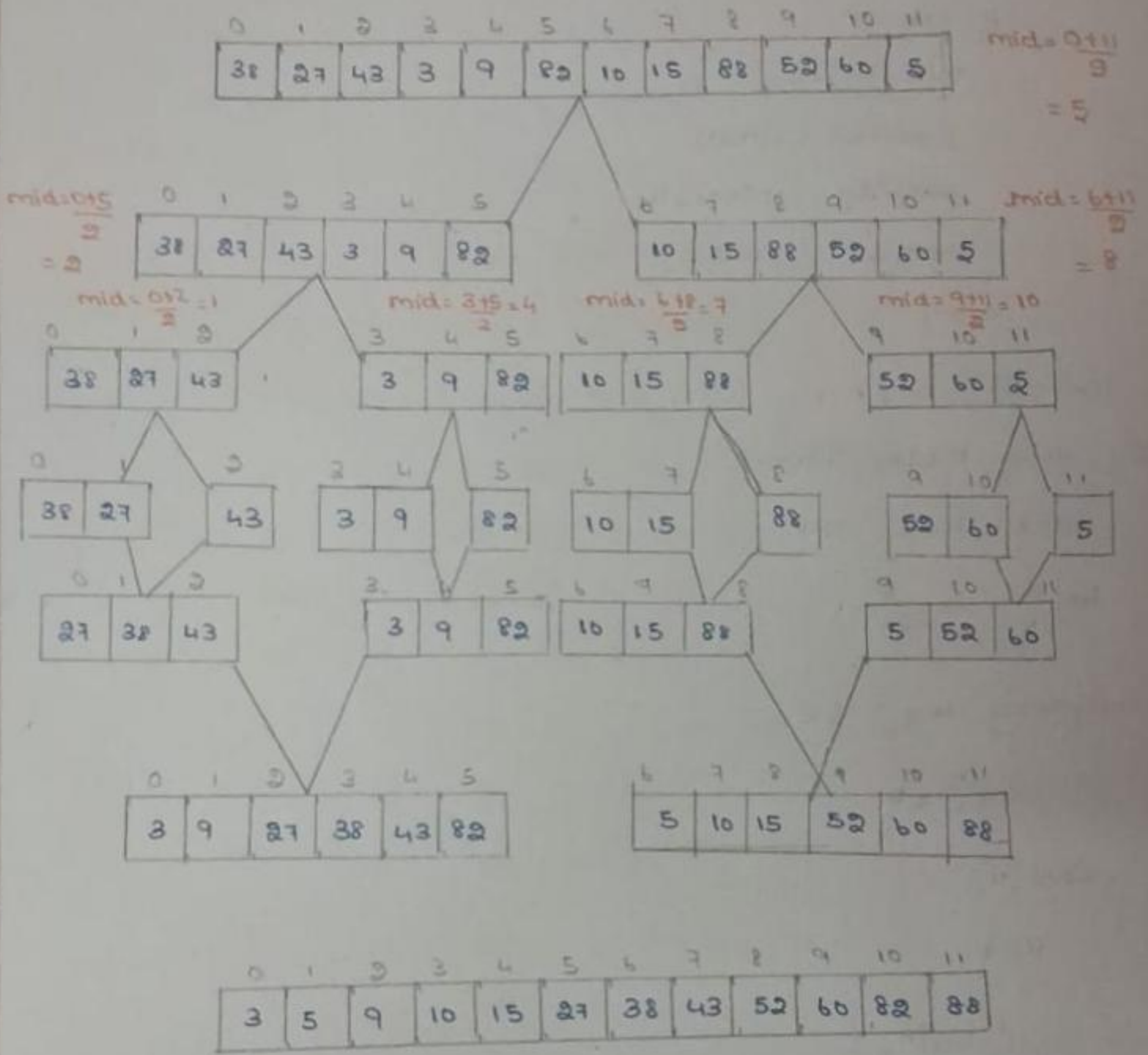
Pseudocode:

```
binary-search (a, n, key):
    low = 0
    high = n-1
    while (low <= high):
        mid = (high+ low)//2
        if a[mid] == key:
            return mid
        a[mid] > key
            high = mid-1
        a[mid] < key
            low = mid+1
    return -1 [If not found].
```

Time complexity:

O(nlogn).

16. Solve the elements using Merge sort divide & conquer strategy [38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5] & analyze the time complexity.



∴ The sorted list is: 3, 5, 9, 10, 15, 27, 38, 43, 52, 60, 82, 88.

14. Find the no. of times to perform swapping for selection sort. Also estimate the time complexity. S = [12, 7, 5, -2, 18, 6, 13, 4]

12	7	5	-2	18	6	13	4
Start			min				

-2	7	5	12	18	6	13	4
	Start						min

min

-2	4	5	12	18	6	13	7
		Start					

-2	4	5	12	18	6	13	7
			Start		min		

-2	4	5	6	18	12	13	7
				Start			min

-2	4	5	6	7	12	13	18
					min		

-2	4	5	6	7	12	13	18
						Start	

-2	4	5	6	7	12	13	18
							min

Sorted List: -2, 4, 5, 6, 7, 12, 13, 18

Usually, the number of swaps required will be n-1. But for this question there are only 4 swaps.

Time Complexity: $O(n^2)$. It is n^2 in all the three cases.

15. Find the index of the target value 10 using binary search from the following list of elements. [2, 4, 6, 8, 10, 12, 14, 16, 18, 20]

0	1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18	20

low = 0 high = 9

mid = $\frac{0+9}{2}$ = 4

a[mid] == key

a[4] == ~~27~~ 10

10 == 10 ✓

Return the position of the key (i.e) 4

Pseudocode:

```
binary-search (array, size of array, key)
        low = 0
        high = size -1
        while (low <= high)
              mid = (high + low) / 2
              if a[mid] == key
                    return mid
                a[mid] > key
                    high = mid-1
                a[mid] < key
                    low = mid+1
        return -1  [If not found]
```

17 Pseudocode:

~~def~~

```
partition (low, high)
    if l<h:
        mid = h+l/2
        partition (l, mid)
        partition (mid+1, h)
        merge (l, mid, h)
    end if
```

$T(n) = 2T(n/2) + n-1$

By using Master theorem.

$a=2 \quad b=2 \quad k=1$

$\log_b^a = \log_2^2 = 1$

Comparing $\log_a^b$ & k

$\log_a^b = k$

$\therefore$ case (ii)

$p<-1$

$\therefore O(n^k \log^{p+1} n)$

$= O(n \log n)$

$\therefore$ Time complexity: $O(n\log n)$

17. Sort the array 64, 34, 25, 12, 22, 11, 90 using Bubble sort. What is the time complexity of Selection Sort in Best, average & worst case)

$\boxed{64 \quad 34}$ 25 12 22 11 90

34 $\boxed{64 \quad 25}$ 12 22 11 90

34 25 $\boxed{64 \quad 12}$ 22 11 90 — Phase - I

34 25 12 $\boxed{64 \quad 22}$ 11 90

34 25 12 22 $\boxed{64 \quad 11}$ 90

34 25 12 22 11 $\boxed{64 \quad 90}$

34 25 12 22 11 64 $\boxed{90}$

$\boxed{34 \quad 25}$ 12 22 11 64 90

25 $\boxed{34 \quad 12}$ 22 11 64 90

25 12 $\boxed{34 \quad 22}$ 11 64 90 — Phase - II

25 12 22 $\boxed{34 \quad 11}$ 64 90

25 12 22 11 $\boxed{34 \quad 64}$ 90

25 12 22 11 34 $\boxed{64}$ $\boxed{90}$

$\boxed{25 \quad 12}$ 22 11 34 64 90

12 $\boxed{25 \quad 22}$ 11 34 64 90 — Phase - III

12 22 $\boxed{25 \quad 11}$ 34 64 90

12 22 11 $\boxed{25 \quad 34}$ 64 90

12 22 11 25 $\boxed{34}$ $\boxed{64}$ $\boxed{90}$

$\boxed{12 \quad 22}$ 11 25 34 64 90

12 $\boxed{22 \quad 11}$ 25 34 64 90 — Phase - IV

12 11 $\boxed{22 \quad 25}$ 34 64 90

12 11 22 $\boxed{25}$ $\boxed{34}$ $\boxed{64}$ $\boxed{90}$

$\boxed{12 \quad 11}$ 22 25 34 64 90

11 $\boxed{12 \quad 22}$ 25 34 64 90 - Phase $\overline{V}$

11 12 $\boxed{22}$ $\boxed{25}$ $\boxed{34}$ $\boxed{34}$ $\boxed{90}$

$\boxed{11 \quad 12}$ 22 25 34 64 90

$\boxed{11}$ $\boxed{12}$ $\boxed{22}$ $\boxed{25}$ $\boxed{34}$ $\boxed{64}$ $\boxed{90}$ - Phase $\overline{VI}$

∴ The sorted list: 11, 12, 22, 25, 34, 64, 90

Time Complexity: $O(n^2)$

Selection Sort: Best Case: $O(n^2)$

Worst Case: $O(n^2)$

Average case: $O(n^2)$

Sort the array 64, 25, 12, 22, 11 using selection sort. What is the time complexity.

64	25	12	22	11	90
start				min	

11	25	12	22	64	90
	start	min			

11	12	25	22	64	90
		start	min		

11	12	22	25	64	90
			Start.		

min

11	12	22	25	64	90

front

11	12	22	25	64	90

The sorted list is : 11, 12, 22, 25, 64, 90

Time Complexity:

The time complexity of the Selection sort Algorithm is always $O(n^2)$. Because, for each element in the array, the algorithm iterate throughs the remaining elements to find the minimom.

The outer loop runs $n+1$ times and the inner loops runs $n-1$, $n-2$ till 1 time.

Best case: $O(n^2)$

Wort case: $O(n^2)$

Average case: $O(n^2)$

16 19. Solve the following using Insertion sort using Brute-force approach [38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5) & analyze the complexity

$\boxed{38 \quad 27}$ 43 3 9 82 10 15 88 52 60 5

27 $\boxed{38 \quad 43}$ 3 9 82 10 15 88 52 60 5

27 38 $\boxed{43 \quad 3}$ 9 82 10 15 88 52 60 5

27 $\boxed{38 \quad 3}$ 43 9 82 10 15 88 52 60 5

$\boxed{27 \quad 3}$ 38 43 9 82 10 15 88 52 60 5

3 27 38 $\boxed{43 \quad 9}$ 82 10 15 88 52 60 5

3 27 $\boxed{38 \quad 9}$ 43 82 10 15 88 52 60 5

3 $\boxed{27 \quad 9}$ 38 43 82 10 15 88 52 60 5

$\boxed{3 \quad 9}$ 27 38 43 82 10 15 88 52 60 5

3 9 27 38 $\boxed{43 \quad 82}$ 10 15 88 52 60 5

3 9 27 38 43 $\boxed{82 \quad 10}$ 15 88 52 60 5

3 9 27 38 $\boxed{43 \quad 10}$ 82 15 88 52 60 5

3 9 27 $\boxed{38 \quad 10}$ 43 82 15 88 52 60 5

3 9 $\boxed{27 \quad 10}$ 38 43 82 15 88 52 60 5

3 $\boxed{9 \quad 10}$ 27 38 43 82 15 88 52 60 5

3 9 10 27 38 43 $\boxed{82 \quad 15}$ 88 52 60 5

3 9 10 27 38 $\boxed{43 \quad 15}$ 82 88 52 60 5

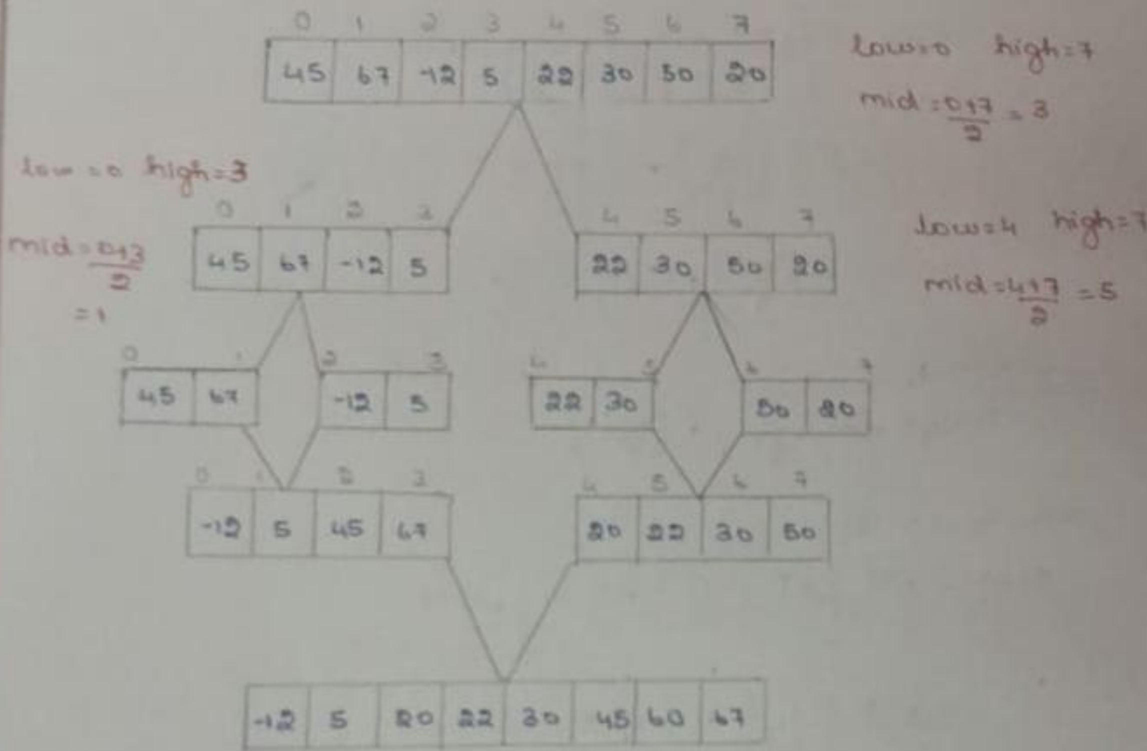
3 9 10 27 $\boxed{38 \quad 15}$ 43 82 88 52 60 5

3 9 10 $\boxed{27 \quad 15}$ 38 43 82 88 52 60 5

3 9 $\boxed{10 \quad 15}$ 27 38 43 82 88 52 60 5

3 9 10 15 27 38 43 $\boxed{82 \quad 88}$ 52 60 5

13. Apply merge sort and order the list of 8 elements $d = [45, 67, -12, 5, 22, 30, 50, 20]$. Set up a recurrence relation for the number of key comparisons made by merge sort.



∴ The sorted list is :

-12, 5, 20, 22, 30, 45, 60, 67.

Time Complexity: $O(n \log n)$

Recurrence relation: ~~O(nlogn)~~ $T(n) = 2T(n/2) + (n-1)$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad 82 \quad \boxed{88 \quad 52} \quad 60 \quad 5$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad \boxed{82 \quad 52} \quad 88 \quad 60 \quad 5$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad \boxed{43 \quad 52} \quad 82 \quad 88 \quad 60 \quad 5$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad 52 \quad 82 \quad \boxed{88 \quad 60} \quad 5$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad 52 \quad \boxed{82 \quad 60} \quad 88 \quad 5$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad \boxed{52 \quad 60} \quad 82 \quad 88 \quad 5$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad 52 \quad 60 \quad 82 \quad \boxed{88 \quad 5}$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad 52 \quad 60 \quad \boxed{82 \quad 5} \quad 88$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad 52 \quad \boxed{60 \quad 5} \quad 82 \quad 88$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad \boxed{52 \quad 5} \quad 60 \quad 82 \quad 88$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad \boxed{43 \quad 5} \quad 52 \quad 60 \quad 82 \quad 88$

$3 \quad 9 \quad 10 \quad 15 \quad 27 \quad \boxed{38 \quad 5} \quad 43 \quad 52 \quad 60 \quad 82 \quad 88$

$3 \quad 9 \quad 10 \quad 15 \quad \boxed{27 \quad 5} \quad 38 \quad 43 \quad 52 \quad 60 \quad 82 \quad 88$

$3 \quad 9 \quad 10 \quad \boxed{15 \quad 5} \quad 27 \quad 38 \quad 43 \quad 52 \quad 60 \quad 82 \quad 88$

$3 \quad 9 \quad \boxed{10 \quad 5} \quad 15 \quad 27 \quad 38 \quad 43 \quad 52 \quad 60 \quad 82 \quad 88$

$3 \quad \boxed{9 \quad 5} \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad 52 \quad 60 \quad 82 \quad 88$

$\boxed{3 \quad 5} \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad 52 \quad 60 \quad 82 \quad 88$

$3 \quad 5 \quad 9 \quad 10 \quad 15 \quad 27 \quad 38 \quad 43 \quad 52 \quad 60 \quad 82 \quad 88$

$\therefore$ The sorted list is: 3, 5, 9, 10, 15, 27, 38, 43, 52, 60, 82, 88.

Pseudocode:

```
Insertion_sort (a, size of a):
    for i in range (2, size): — (n-2)
        key = a[i] — (1)
        j = i-1 — (1)
        while j>0 and a[j]> key   n*(n-2)
            a[ j+1] = a[j] — (1)
            j-=1 — (1)
        a[i+1] = key — (1)
    return a — (1)
```

Time complexity:

$$T(n) = n^{*}(n-2) + n-2+1+1+1+1+1+1$$

$$= n^2 - 2n + n - 2 + 6$$

$$= n^2 - n + 4$$

$\therefore$ $O(n) = O(n^2)$

Time complexity is $O(n^2)$

2. Given an array of [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9] integers. Sort the following elements using insertion sort using Brute Force approach strategy analysis complexity of the algorithm

4	-2	5	3	10	-5	2	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-2	4	5	3	10	-5	2	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-2	4	5	3	10	-5	2	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-2	4	3	5	10	-5	2	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-2	3	4	5	10	-5	2	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-2	3	4	5	10	-5	2	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-2	3	4	5	-5	10	2	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-2	3	4	-5	5	10	2	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-2	3	-5	4	5	10	2	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-2	-5	3	4	5	10	2	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-5	-2	3	4	5	10	2	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-5	-2	3	4	5	2	10	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-5	-2	3	4	2	5	10	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-5	-2	3	2	4	5	10	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-5	-2	2	3	4	5	10	8	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-5	-2	2	3	4	5	8	10	-3	6	7	-4	1	9	-1	0	-6	-8	11	-9
-5	-2	2	3	4	5	8	-3	10	6	7	-4	1	9	-1	0	-6	-8	11	-9
-5	-2	2	3	4	5	-3	8	10	6	7	-4	1	9	-1	0	-6	-8	11	-9
-5	-2	2	3	4	-3	5	8	10	6	7	-4	1	9	-1	0	-6	-8	11	-9
-5	-2	2	3	-3	4	5	8	10	6	7	-4	1	9	-1	0	-6	-8	11	-9
-5	-2	2	-3	3	4	5	8	10	6	7	-4	1	9	-1	0	-6	-8	11	-9
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-5	-3	-2	2	-4	3	4	5	6	7	8	10	1	9	-1	0	-6	-8	11	-9
-5	-3	-2	-4	2	3	4	5	6	7	8	10	1	9	-1	0	-6	-8	11	-9

-5 -4 -3 -2 -1 0 $\boxed{1 \quad -6}$ 2 3 4 5 6 7 8 9 10 -8 11 -9

-5 -4 -3 -2 -1 $\boxed{0 \quad -6}$ 1 2 3 4 5 6 7 8 9 10 -8 11 -9

-5 -4 -3 -2 $\boxed{-1 \quad -6}$ 0 1 2 3 4 5 6 7 8 9 10 -8 11 -9

-5 -4 -3 $\boxed{-2 \quad -6}$ -1 0 1 2 3 4 5 6 7 8 9 10 -8 11 -9

-5 -4 $\boxed{-3 \quad -6}$ -2 -1 0 1 2 3 4 5 6 7 8 9 10 -8 11 -9

-5 $\boxed{-4 \quad -6}$ -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 -8 11 -9

$\boxed{-5 \quad -6}$ -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 -8 11 -9

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 $\boxed{10 \quad -8}$ 11 -9

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 $\boxed{9 \quad -8}$ 10 11 -9

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 $\boxed{8 \quad -8}$ 9 10 11 -9

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 $\boxed{7 \quad -8}$ 8 9 10 11 -9

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 $\boxed{6 \quad -8}$ 7 8 9 10 11 -9

-6 -5 -4 -3 -2 -1 0 1 2 3 4 $\boxed{5 \quad -8}$ 6 7 8 9 10 11 -9

-6 -5 -4 -3 -2 -1 0 1 2 3 $\boxed{4 \quad -8}$ 5 6 7 8 9 10 11 -9

-6 -5 -4 -3 -2 -1 0 1 2 $\boxed{3 \quad -8}$ 4 5 6 7 8 9 10 11 -9

-6 -5 -4 -3 -2 -1 0 1 $\boxed{2 \quad -8}$ 3 4 5 6 7 8 9 10 11 -9

-6 -5 -4 -3 -2 -1 0 $\boxed{1 \quad -8}$ 2 3 4 5 6 7 8 9 10 11 -9

-6 -5 -4 -3 -2 -1 $\boxed{0 \quad -8}$ 1 2 3 4 5 6 7 8 9 10 11 -9

-6 -5 -4 -3 -2 $\boxed{-1 \quad -8}$ 0 1 2 3 4 5 6 7 8 9 10 11 -9

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-6 -5 -4 $\boxed{-3 \quad -8}$ -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 -9

-6 -5 $\boxed{-4 \quad -8}$ -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 -9

-6 $\boxed{-5 \quad -8}$ -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 -9

$\boxed{-6 \quad -8}$ -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 -9

-8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 $\boxed{10 \quad 11}$ -9

-8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 $\boxed{11 \quad -9}$

-8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 $\boxed{10 \quad -9}$ 11

-8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 $\boxed{9 \quad -9}$ 10 11

-8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 $\boxed{8 \quad -9}$ 9 10 11

-8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 $\boxed{7 \quad -9}$ 8 9 10 11

-8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 $\boxed{6 \quad -9}$ 7 8 9 10 11

-8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 $\boxed{5 \quad -9}$ 6 7 8 9 10 11

-8 -6 -5 -4 -3 -2 -1 0 1 2 3 $\boxed{4 \quad -9}$ 5 6 7 8 9 10 11

-8 -6 -5 -4 -3 -2 -1 0 1 2 $\boxed{3 \quad -9}$ 4 5 6 7 8 9 10 11

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-8 -6 -5 -4 -3 -2 -1 $\boxed{0 \quad -9}$ 1 2 3 4 5 6 7 8 9 10 11

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-8 -6 -5 -4 -3 $\boxed{-2 \quad -9}$ -1 0 1 2 3 4 5 6 7 8 9 10 11
-8 -6 -5 -4 $\boxed{-3 \quad -9}$ -2 -1 0 1 2 3 4 5 6 7 8 9 10 11
-8 -6 -5 $\boxed{-4 \quad -9}$ -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11
-8 -6 $\boxed{-5 \quad -9}$ -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11
-8 $\boxed{-6 \quad -9}$ -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11
$\boxed{-8 \quad -9}$ -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11
-9 -8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11

pseudo code:

```
Insertion_ sort (a, size of a):
  for i in range (2, size):
    key = a[i]
    j = j -1
    while j >0 and a[j] >key
      a[j+1] = a[j]
        j -= 1
      a[i+1] = key
      return a
```

Time complexity:

$T(n) = O(n^2)$

-5 $\boxed{-3\ \ -4}$ -2 2 3 4 5 6 7 8 10 1 9 -1 0 -6 -8 11 -9

-5 -4 -3 -2 2 3 4 5 6 7 8 $\boxed{10\ \ 1}$ 9 -1 0 -6 -8 11 -9

-5 -4 -3 -2 2 3 4 5 6 7 $\boxed{8\ \ 1}$ 10 9 -1 0 -6 -8 11 -9

-5 -4 -3 -2 2 3 4 5 6 $\boxed{7\ \ 1}$ 8 10 9 -1 0 -6 -8 11 -9

-5 -4 -3 -2 2 3 4 5 $\boxed{6\ \ 1}$ 7 8 10 9 -1 0 -6 -8 11 -9

-5 -4 -3 -2 2 3 4 $\boxed{5\ \ 1}$ 6 7 8 10 9 -1 0 -6 -8 11 -9

-5 -4 -3 -2 2 3 $\boxed{4\ \ 1}$ 5 6 7 8 10 9 -1 0 -6 -8 11 -9

-5 -4 -3 -2 2 $\boxed{3\ \ 1}$ 4 5 6 7 8 10 9 -1 0 -6 -8 11 -9

-5 -4 -3 -2 $\boxed{2\ \ 1}$ 3 4 5 6 7 8 10 9 -1 0 -6 -8 11 -9

-5 -4 -3 -2 1 2 3 4 5 6 7 8 $\boxed{10\ \ 9}$ -1 0 -6 -8 11 -9

-5 -4 -3 -2 1 2 3 4 5 6 7 8 9 $\boxed{10\ \ -1}$ 0 -6 -8 11 -9

-5 -4 -3 -2 1 2 3 4 5 6 7 8 $\boxed{9\ \ -1}$ 10 0 -6 -8 11 -9

-5 -4 -3 -2 1 2 3 4 5 6 7 $\boxed{8\ \ -1}$ 9 10 0 -6 -8 11 -9

-5 -4 -3 -2 1 2 3 4 5 6 $\boxed{7\ \ -1}$ 8 9 10 0 -6 -8 11 -9

-5 -4 -3 -2 1 2 3 4 5 $\boxed{6\ \ -1}$ 7 8 9 10 0 -6 -8 11 -9

-5 -4 -3 -2 1 2 3 4 $\boxed{5\ \ -1}$ 6 7 8 9 10 0 -6 -8 11 -9

-5 -4 -3 -2 1 2 3 $\boxed{4\ \ -1}$ 5 6 7 8 9 10 0 -6 -8 11 -9

-5 -4 -3 -2 1 2 $\boxed{3\ \ -1}$ 4 5 6 7 8 9 10 0 -6 -8 11 -9

-5 -4 -3 -2 1 $\boxed{2\ \ -1}$ 3 4 5 6 7 8 9 10 0 -6 -8 11 -9

-5 -4 -3 -2 $\boxed{1\ \ -1}$ 2 3 4 5 6 7 8 9 10 0 -6 -8 11 -9

-5 -4 -3 -2 -1 1 2 3 4 5 6 7 8 9 $\boxed{10\ \ 0}$ -6 -8 11 -9

-5 -4 -3 -2 -1 1 2 3 4 5 6 7 8 $\boxed{9\ \ 0}$ 10 -6 -8 11 -9

-5 -4 -3 -2 -1 1 2 3 4 5 6 7 $\boxed{8\ \ 0}$ 9 10 -6 -8 11 -9

-5 -4 -3 -2 -1 1 2 3 4 5 6 $\boxed{7\ \ 0}$ 8 9 10 -6 -8 11 -9

-5 -4 -3 -2 -1 1 2 3 4 5 $\boxed{6\ \ 0}$ 7 8 9 10 -6 -8 11 -9

-5 -4 -3 -2 -1 1 2 3 4 $\boxed{5\ \ 0}$ 6 7 8 9 10 -6 -8 11 -9

-5 -4 -3 -2 -1 1 2 3 $\boxed{4\ \ 0}$ 5 6 7 8 9 10 -6 -8 11 -9

-5 -4 -3 -2 -1 1 2 $\boxed{3\ \ 0}$ 4 5 6 7 8 9 10 -6 -8 11 -9

-5 -4 -3 -2 -1 1 $\boxed{2\ \ 0}$ 3 4 5 6 7 8 9 10 -6 -8 11 -9

-5 -4 -3 -2 -1 $\boxed{1\ \ 0}$ 2 3 4 5 6 7 8 9 10 -6 -8 11 -9

-5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 $\boxed{10\ \ -6}$ -8 11 -9

-5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 $\boxed{9\ \ -6}$ 10 -8 11 -9

-5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 $\boxed{8\ \ -6}$ 9 10 -8 11 -9

-5 -4 -3 -2 -1 0 1 2 3 4 5 6 $\boxed{7\ \ -6}$ 8 9 10 -8 11 -9

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-5 -4 -3 -2 -1 0 1 2 3 4 $\boxed{5\ \ -6}$ 6 7 8 9 10 -8 11 -9

-5 -4 -3 -2 -1 0 1 2 3 $\boxed{4\ \ -6}$ 5 6 7 8 9 10 -8 11 -9

-5 -4 -3 -2 -1 0 1 2 $\boxed{3\ \ -6}$ 4 5 6 7 8 9 10 -8 11 -9

-5 -4 -3 -2 -1 0 1 $\boxed{2\ \ -6}$ 3 4 5 6 7 8 9 10 -8 11 -9

-5 -4 -3 -2 -1 0 1