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Detailed Explanation of Maximum Volume of Cuboid

Introduction

Given a cuboid with:

- **Perimeter:** $P = 4(l + b + h)$
- **Area:** $A = 2(lb + bh + lh)$

We aim to maximize the **Volume** $V = l \cdot b \cdot h$.

Step-by-Step Approach

1. Express Volume in Terms of One Variable

The goal is to express the volume V of the cuboid in terms of a single variable, choosing the length l as the variable.

2. Use the Given Area Formula

$$A = 2(lb + bh + lh)$$

Rearrange to express the product of b and h:

$$bh = \frac{A}{2} - lb - lh$$

Substitute this into the volume formula:

$$V = lbh$$

$$V = l\left(\frac{A}{2} - lb - lh\right)$$

3. Use the Given Perimeter Formula

The given perimeter formula is:

$$P = 4(l + b + h)$$

Rearrange to express $b + h$:

$$b + h = \frac{P}{4} - l$$

Substitute this into the volume expression:

$$V = l\left(\frac{A}{2} - l(b + h)\right)$$

$$V = l\left(\frac{A}{2} - l\left(\frac{P}{4} - l\right)\right)$$

Expand and simplify:

$$V = \frac{lA}{2} - l^2 \cdot \frac{P}{4} + l^3$$

Thus, we have:

$$V = \frac{lA}{2} - \frac{l^2 P}{4} + l^3$$

4. Find the Critical Points by Differentiating

Differentiate V with respect to l:

$$\frac{dV}{dl} = \frac{A}{2} - \frac{lP}{2} + 3l^2$$

Set the derivative to zero to find the critical points:

$$\frac{A}{2} - \frac{lP}{2} + 3l^2 = 0$$

Rearrange to get a standard quadratic equation in l:

$$3l^2 - \frac{Pl}{2} + \frac{A}{2} = 0$$

5. Solve the Quadratic Equation

Solve the quadratic equation using the quadratic formula $l = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, where $a = 3$, $b = -\frac{P}{2}$, and $c = \frac{A}{2}$:

$$l = \frac{\frac{P}{2} \pm \sqrt{\left(\frac{P}{2}\right)^2 - 4 \cdot 3 \cdot \frac{A}{2}}}{2 \cdot 3}$$

Simplify:

$$l = \frac{P \pm \sqrt{P^2 - 24A}}{12}$$

6. Determine the Maximum Volume

Choose the positive root that maximizes the volume:

$$l = \frac{P - \sqrt{P^2 - 24A}}{12}$$

Substitute this l back into the volume equation V:

$$V = \frac{lA}{2} - \frac{l^2 P}{4} + l^3$$

After finding l, determine the maximum volume by evaluating this expression.

Algorithm

To maximize the volume of a cuboid given its perimeter and area, follow these steps:

1. **Compute the Length:**
   - Given the perimeter P and area A of the cuboid, calculate the optimal length l using:

   $$l = \frac{P - \sqrt{P^2 - 24A}}{12}$$

   - This formula is derived from solving the quadratic equation resulting from differentiating the volume function V.

2. **Calculate the Maximum Volume:**
   - With the computed length l, use the volume formula:

   $$V = \frac{l\left(P \cdot l - 8 \cdot l^2\right)}{4}$$

   - This formula calculates the volume using the length l, perimeter P, and area A.

CODE:

```
#include <cmath>

class Solution {
public:
    // Function to compute the optimal length l
    double length(double perimeter, double area) {
        // Calculate length using the derived
formula
        double l = (perimeter -
std::sqrt((perimeter * perimeter) - (24 * area))) /
12;
        return l;
    }

    // Function to compute the maximum volume V
    double maxVolume(double perimeter, double area)
{
        // Compute the optimal length l
        double l = length(perimeter, area);

        // Calculate the volume using the computed
length l
        double volume = ((perimeter * l * l) - (8 *
l * l * l)) / 4;
        return volume;
    }
};
```