Control Allocation Matrix (Mixer) for PX4 Iris X-Configuration Quadrotor

Contents

1 Introduction

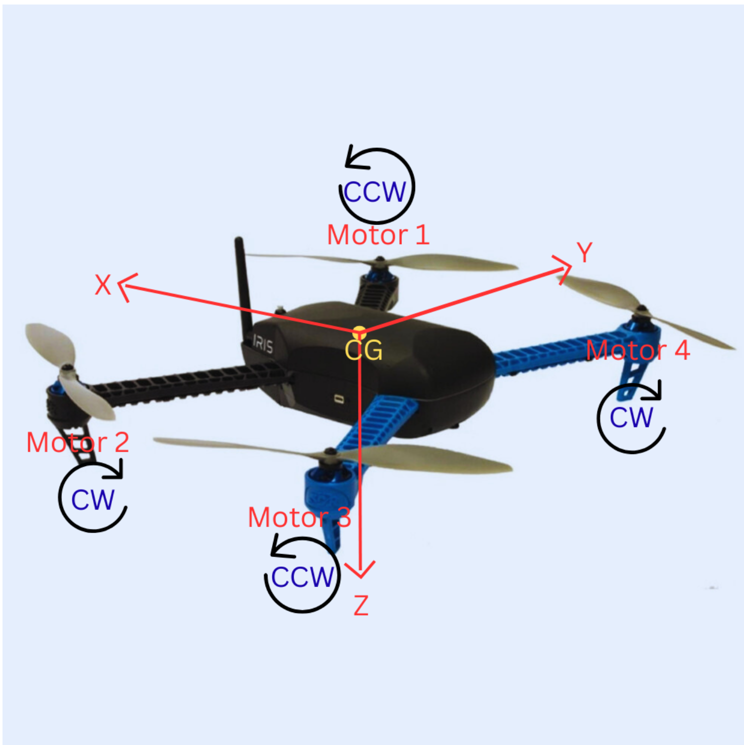
Multirotor flight control systems generate high-level control commands in terms of total thrust and body moments. The **mixer (control allocation block)** converts these commands into individual motor speed-squared commands.

This document derives the control allocation matrix for the **PX4 Iris quadrotor**, which uses an **X-configuration** geometry.

2 Vehicle Configuration

2.1 PX4 Iris Geometry

The quadrotor is arranged in an X-configuration, with motors located at 45° to the body axes.



2.2 Motor Layout

Motor	Position	Arm Angle	Spin Direction
1	Front-Right	$+45°$	CCW
2	Front-Left	$-45°$	CW
3	Rear-Left	$-135°$	CCW
4	Rear-Right	$+135°$	CW

All motors are at distance l from the center of mass.

3 Coordinate Frames and Sign Conventions

3.1 Body Frame Definition

- X_B: Forward
- Y_B: Right
- Z_B: Downward

3.2 Moment Sign Convention (Right-Hand Rule)

- Roll moment M_x: rotation about X_B
- Pitch moment M_y: rotation about Y_B
- Yaw moment M_z: rotation about Z_B

3.3 Thrust Direction

Thrust acts **upward**, opposite to the $+Z_B$ axis.

4 Motor Position Vectors

Because the motors lie in the x–y plane and are equally spaced at 45°, the position vectors are:

$$\mathbf{r}_i = \begin{bmatrix} \frac{l}{\sqrt{2}} & \frac{l}{\sqrt{2}} & 0 \\ \frac{l}{\sqrt{2}} & -\frac{l}{\sqrt{2}} & 0 \\ -\frac{l}{\sqrt{2}} & -\frac{l}{\sqrt{2}} & 0 \\ -\frac{l}{\sqrt{2}} & \frac{l}{\sqrt{2}} & 0 \end{bmatrix}$$

Each row corresponds to motors 1 through 4 respectively.

5 Thrust Force Model

The thrust force generated by motor i is

$$\mathbf{F}_i = \begin{bmatrix} 0 \\ 0 \\ -T_i \end{bmatrix}$$

where $T_i = k_t \omega_i^2$.

6 Moment from Thrust

Moments arise from the cross product:

$$\boldsymbol{\tau}_i = \mathbf{r}_i \times \mathbf{F}_i$$

Let

$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ 0 \end{bmatrix} \quad \mathbf{F}_i = \begin{bmatrix} 0 \\ 0 \\ -T_i \end{bmatrix}$$

6.1 Cross Product Expansion

$$\boldsymbol{\tau}_i = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_i & y_i & 0 \\ 0 & 0 & -T_i \end{vmatrix}$$

6.1.1 Roll Moment (M_x)

$$\tau_{x,i} = y_i T_i$$

6.1.2 Pitch Moment (M_y)

$$\tau_{y,i} = -x_i T_i$$

6.1.3 Yaw Moment

$$\tau_{z,i} = 0$$

6.2 Final Moment Vector

$$\boxed{\boldsymbol{\tau}_i = \begin{bmatrix} y_i T_i \\ -x_i T_i \\ 0 \end{bmatrix}}$$

7 Total Moments

7.1 Roll Moment

$$M_x = \sum y_i T_i$$

$$M_x = \frac{l}{\sqrt{2}}(T_1 - T_2 - T_3 + T_4)$$

7.2 Pitch Moment

$$M_y = -\sum x_i T_i$$

For positive nose-up pitch:

$$M_y = \frac{l}{\sqrt{2}}(T_1 + T_2 - T_3 - T_4)$$

7.3 Yaw Moment

Yaw is generated by reaction torque:

$$Q_i = k_m \omega_i^2$$

Motor	Spin	Yaw Contribution
1	CCW	$+Q_1$
2	CW	$-Q_2$
3	CCW	$+Q_3$
4	CW	$-Q_4$

$$M_z = Q_1 - Q_2 + Q_3 - Q_4$$

7.4 Why Thrust Does Not Produce Yaw

- Thrust vectors are parallel
- No moment arm about Z_B
- Yaw arises purely from motor reaction torque

8 Total Thrust

$$T = T_1 + T_2 + T_3 + T_4$$

9 Control Allocation Matrix

9.1 Force–Moment Mapping

$$\begin{bmatrix} T \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} k_t & k_t & k_t & k_t \\ \frac{k_t l}{\sqrt{2}} & -\frac{k_t l}{\sqrt{2}} & -\frac{k_t l}{\sqrt{2}} & \frac{k_t l}{\sqrt{2}} \\ \frac{k_t l}{\sqrt{2}} & \frac{k_t l}{\sqrt{2}} & -\frac{k_t l}{\sqrt{2}} & -\frac{k_t l}{\sqrt{2}} \\ k_m & -k_m & k_m & -k_m \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}$$

9.2 Mixer Equation

$$\boxed{\begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} T \\ M_x \\ M_y \\ M_z \end{bmatrix}}$$

where **A** is the control allocation matrix above.

10 Conclusion

This document derives the mixer matrix for an X-configuration quadrotor from first principles using rigid-body mechanics. The formulation is directly applicable to PX4, Simulink-based controllers, and real-time embedded implementations.