Homework: GATE CSE PYQs

Question

Evaluate the limit

$$\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{2n}.$$

https://www.hanbommoon.net/wp-content/uploads/2013/08/Midex_3_03003_sol.pdf

Solution

(1) (5 pts) Evaluate the limit

$$\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{2n}.$$

$$\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{2n}=\lim_{n\to\infty}\left(\left(1+\frac{3}{n}\right)^{n}\right)^{2}=\left(\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{n}\right)^{2}=(e^3)^2=e^6$$

- Using $\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{n}=e$: 2 pts.
- Getting correct answer e^6: 5 pts.

Question

Evaluate the limit

$$\lim_{n\to\infty} \left(n^2 - 1\right)^{\frac{1}{n}}.$$

Solution

(2) (8 pts) Evaluate the limit

$$\lim_{n\to\infty} (n^2-1)^{\frac{1}{n}}.$$

$$1^{\frac{1}{n}} \leq (n^2-1)^{\frac{1}{n}} \leq (n^2)^{\frac{1}{n}}$$

$$\lim_{n\to\infty} 1^{\frac{1}{n}} = \lim_{n\to\infty} 1 = 1$$

$$\lim_{n\to\infty} (n^2)^{\frac{1}{n}} = \lim_{n\to\infty} \left(n^{\frac{1}{n}}\right)^2 = 1^2 = 1$$

$$\Rightarrow \lim_{n\to\infty} (n^2-1)^{\frac{1}{n}} = 1$$

- Making two appropriate sequences $1^{\frac{1}{n}}$ and $(n^2)^{\frac{1}{n}}$ which give an upper and lower bound of given sequence: 3 pts.
- Evaluating the limit of one side $\lim_{n\to\infty} 1^{\frac{1}{n}} = 1$: +2 pts.
- Evaluating the limit of the other side $\lim_{n\to\infty} (n^2)^{\frac{1}{n}} = 1$: + 2 pts.
- By using the sandwich theorem, obtaining the conclusion $\lim_{n\to\infty} (n^2-1)^{\frac{1}{n}} = 1$: 8 pts.

Question

Evaluate the limit

$$\lim_{n \to \infty} \left(e^n + n\right)^{\frac{1}{n}}.$$

Solution

(2) (8 pts) Evaluate the limit

$$\lim_{n\to\infty} (e^n + n)^{\frac{1}{n}}\ .$$

$$(e^n)^{\frac{1}{n}} \leq (e^n + n)^{\frac{1}{n}} \leq (2e^n)^{\frac{1}{n}}$$

$$\lim_{n\to\infty} (e^n)^{\frac{1}{n}} = \lim_{n\to\infty} e = e$$

$$\lim_{n\to\infty} (2e^n)^{\frac{1}{n}} = \lim_{n\to\infty} 2^{\frac{1}{n}} (e^n)^{\frac{1}{n}} = \lim_{n\to\infty} 2^{\frac{1}{n}} e = e$$

$$\Rightarrow \lim_{n\to\infty} (e^n + n)^{\frac{1}{n}} = e$$

- Making two appropriate sequences $(e^n)^{\frac{1}{n}}$ and $(2e^n)^{\frac{1}{n}}$ which give an upper and lower bound of given sequence: 3 pts.
- Evaluating the limit of one side $\lim_{n\to\infty} (e^n)^{\frac{1}{n}} = e$: +2 pts.
- Evaluating the limit of the other side $\lim_{n\to\infty} (2e^n)^{\frac{1}{n}} = e$: + 2 pts.
- By using the sandwich theorem, obtaining the conclusion $\lim_{n\to\infty} (e^n + n)^{\frac{1}{n}} = e$: 8 pts.

Question

Evaluate the limit $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{2x}$

https://sites.pitt.edu/~evt3/a/0220/2017_Fall/Exams/e2(0220)_2017_fall_sln.pdf

Solution

bonus problem Evaluate the limit $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x}$

Solution: By the definition of the number e $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=e.$

Then $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x}=\lim\limits_{x\to\infty}\left(\left(1+\frac{1}{x}\right)^{x}\right)^{2}=\left(\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right)^{2}=e^{2}.$

Question

Evaluate the limit

$$\lim_{n\to\infty} \frac{(\ln n)^2}{n}.$$

Solution

(1) (5 pts) Evaluate the limit

$$\lim_{n\to\infty} \frac{(\ln n)^2}{n}.$$

$$\lim_{n\to\infty} \frac{(\ln n)^2}{n} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{n\to\infty} \frac{2(\ln n)\frac{1}{n}}{1} = \lim_{n\to\infty} \frac{2\ln n}{n} = 2\lim_{n\to\infty} \frac{\ln n}{n} = 0$$

- Applying L'Hôpital's rule and getting $\lim_{n\to\infty} \frac{2(\ln n)\frac{1}{n}}{1}$: 3 pts.
- Obtaining correct answer 0: 5 pts.

Question

Evaluate the limit

$$\lim_{n\to\infty} \frac{3n^3+n}{e^n}$$

Solution

(1) (5 pts) Evaluate the limit

$$\lim_{n\to\infty} \frac{3n^3+n}{e^n}$$

By L'Hôpital's rule,

$$\lim_{n\to\infty} \frac{3n^3+n}{e^n} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{n\to\infty} \frac{9n^2+1}{e^n} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{n\to\infty} \frac{18n}{e^n} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{n\to\infty} \frac{18}{e^n} = 0 \text{ (5 pts)}$$

Question

Evaluate the limit

$$\lim_{n\to\infty} \frac{n^2 - 5n}{e^n}.$$

Solution

(1) (5 pts) Evaluate the limit

$$\lim_{n\to\infty} \frac{n^2 - 5n}{e^n}.$$

$$\lim_{n\to\infty} \frac{n^2 - 5n}{e^n} = \lim_{n\to\infty} \frac{2n - 5}{e^n} \text{ (2 pts)} = \lim_{n\to\infty} \frac{2}{e^n} = 0 \text{ (5 pts)}$$

GATE CSE 1993 | Question: 02.1

asked in Calculus Sep 13, 2014 • recategorized Apr 22, 2021 by **Lakshman Bhaiya** 2,493 views

12

$$\lim_{x\to 0} \frac{x(e^x-1)+2(\cos x-1)}{x(1-\cos x)} \text{ is } \_\_\_\_\_\_\_\_$$

gate1993 limits calculus normal fill-in-the-blanks

https://gateoverflow.in/605/gate-cse-1993-question-02-1

17

Best answer

Use LH rule:

First Derivative: $$\frac{[x(e^x) + (e^x - 1) - 2(\sin x)]}{[x \sin x + (1 - \cos x)]}$$

Second Derivative: $$\frac{[xe^x + e^x + e^x - 2\cos x]}{[x\cos x + \sin x + \sin x]}$$

Third Derivative: $$\frac{[xe^x + e^x + e^x + e^x + 2\sin x]}{[-x\sin x + \cos x + \cos x + \cos x]}$$

Put $x = 0 : \dfrac{[0+1+1+1+0]}{[0+1+1+1]} = \dfrac{3}{3} = 1.$

GATE CSE 2008 | Question: 1

24

$$\lim_{x \to \infty} \frac{x - \sin x}{x + \cos x}$$ equals

A. 1

B. -1

C. ∞

D. $-\infty$

https://gateoverflow.in/399/gate-cse-2008-question-1

55

Best answer

$$\lim_{x\to\infty}\frac{x-\sin x}{x+\cos x}$$

$$=\lim_{x\to\infty}\frac{x(1-\frac{\sin x}{x})}{x(1+\frac{\cos x}{x})}$$

$$=\lim_{x\to\infty}\frac{1-\frac{\sin x}{x}}{1+\frac{\cos x}{x}}$$

now to calculate values of $\frac{\sin x}{x}$ and $\frac{\cos x}{x}$ we use Squeezing Theorem.

$$-1\leq\sin x\leq+1\frac{-1}{x}\leq\frac{\sin x}{x}\leq\frac{+1}{x}$$

$$-1\leq\cos x\leq+1\frac{-1}{x}\leq\frac{\cos x}{x}\leq\frac{+1}{x}$$

now as $x\to\infty$ we get $\frac{1}{x}\to 0$, this implies that:

$$0\leq\frac{\sin x}{x}\leq 00\leq\frac{\cos x}{x}\leq 0$$

Hence,

$$\lim_{x\to\infty}\frac{x-\sin x}{x+\cos x}$$

$$=\lim_{x\to\infty}\frac{1-\frac{\sin x}{x}}{1+\frac{\cos x}{x}}$$

$$=\lim_{x\to\infty}\frac{1-0}{1+0}=1$$

answer = **option A**

GATE CSE 2010 | Question: 5

asked in **Calculus** Sep 21, 2014 • edited Jun 17, 2021 by **Lakshman Bhaiya**

29

What is the value of $\lim_{n \to \infty} \left(1 - \frac{1}{n}\right)^{2n}$?

A. 0
B. e^{-2}
C. $e^{-1/2}$
D. 1

56

Best answer

I will solve by two methods

Method 1:

$$y = \lim_{n \to \infty} \left(1 - \frac{1}{n}\right)^{2n}$$

Taking log

$$\log y = \lim_{n \to \infty} 2n \log\left(1 - \frac{1}{n}\right)$$

$$= \lim_{n \to \infty} \frac{\log\left(1 - \frac{1}{n}\right)}{\left(\frac{1}{2n}\right)}$$ (converted this so as to have form $\left(\frac{0}{0}\right)$)

Apply L' hospital rule

$$\log y = \lim_{n \to \infty} \frac{\left(\frac{1}{1 - \frac{1}{n}}\right) \cdot \frac{1}{n^2}}{\left(\frac{-1}{2n^2}\right)}$$

$$\log y = -2$$

$$y = e^{-2}.$$

Method 2:

It takes 1 to power infinity form

$$\lim_{x \to \infty} f(x)^{g(x)}$$

$$= e^{\lim_{x \to \infty} (f(x) - 1) g(x)}$$

where, $(f(x) - 1) * g(x) = \frac{-1}{n} * 2n = -2.$

i.e., -2 constant.

so we get final ans is $= e^{-2}$.

You can refer this link for second method

http://www.vitutor.com/calculus/limits/one_infinity.html

Correct Answer: B

GATE CSE 2015 Set 1 | Question: 4

asked in **Calculus** Feb 11, 2015 • edited Jun 21, 2021 by **Lakshman Bhaiya**

23

$\lim_{x \to \infty} x^{\frac{1}{x}}$ is

A. ∞
B. 0
C. 1
D. Not defined

https://gateoverflow.in/8021/gate-cse-2015-set-1-question-4

35

Best answer

Apply an exponential of a logarithm to the expression.

$$\lim_{x\to\infty} x^{\frac{1}{x}} = \lim_{x\to\infty} \exp\left(\log\left(x^{\frac{1}{x}}\right)\right) = \lim_{x\to\infty} \exp\left(\frac{\log(x)}{x}\right)$$

Since the exponential function is continuous, we may factor it out of the limit.

$$\lim_{x\to\infty} \exp\left(\frac{\log(x)}{x}\right) = \exp\left(\lim_{x\to\infty} \frac{\log(x)}{x}\right)$$

Logarithmic functions grow asymptotically slower than polynomials.

Since $\log(x)$ grows asymptotically slower than the polynomial x as x approaches ∞,

$$\lim_{x\to\infty} \frac{\log(x)}{x} = 0:$$

$$e^0 = 1$$

Correct Answer: C

GATE CSE 2015 Set 3 | Question: 9

asked in Calculus Feb 14, 2015 • retagged Aug 2, 2015 by Arjun

31

The value of $\lim\limits_{x \to \infty} (1 + x^2)^{e^{-x}}$ is

A. 0
B. $\frac{1}{2}$
C. 1
D. ∞

https://gateoverflow.in/8403/gate-cse-2015-set-3-question-9

44

Best answer

Apply an exponential of a logarithm to the expression.

$$\lim_{x\to\infty}(x^2+1)^{e^{-x}} = \lim_{x\to\infty}\exp\left(\log((x^2+1)^{e^{-x}})\right)$$

$$= \lim_{x\to\infty}\exp\left(\frac{\log(x^2+1)}{e^x}\right)$$

Since the exponential function is continuous, we may factor it out of the limit.

$$\lim_{x\to\infty}\exp\left(\frac{\log(x^2+1)}{e^x}\right)$$

$$= \exp\left(\lim_{x\to\infty}\frac{\log(x^2+1)}{e^x}\right)$$

The numerator of $e^{-x}\log(x^2+1)$ grows asymptotically slower than its denominator as x approaches ∞.

Since $\log(x^2+1)$ grows asymptotically slower than e^x as x approaches ∞, $\lim_{x\to\infty} e^{-x}\log(x^2+1) = 0 : e^0$.

Evaluate e^0.

$e^0 = 1$:

Answer: 1.

Correct Answer: C

GATE CSE 2016 Set 1 | Question: 3

asked in Calculus Feb 12, 2016 • edited Mar 27, 2021 by soujanyareddy13

5,266 views

24

$$\lim_{x\to 4} \frac{\sin(x-4)}{x-4} = \_\_\_\_\_\_\_\_\_$$

gatecse-2016-set1 calculus limits easy numerical-answers

https://gateoverflow.in/39630/gate-cse-2016-set-1-question-3

30

Substitute $h = x - 4$, it becomes $\lim\limits_{h \to 0} \frac{\sin h}{h}$.

This is a standard limit and answer is 1.

Best answer

answered Feb 12, 2016 • selected Feb 15, 2016 by **Pooja Palod**

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abhilashpanicker29

1 comment

Vishal_kumar98 commented Nov 11, 2021

Very nicely explained.

1

GATE CSE 2017 Set 1 | Question: 28

asked in **Calculus** Feb 14, 2017 • edited Mar 27, 2021 by **soujanyareddy13**

21

The value of $\lim\limits_{x \to 1} \dfrac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

A. is 0
B. is -1
C. is 1
D. does not exist

gatecse-2017-set1 calculus limits normal

https://gateoverflow.in/118309/gate-cse-2017-set-1-question-28

27

Since substituting $x = 1$ we get $\frac{0}{0}$ which is indeterminate.

After applying L'Hospital rule, we get $\frac{(7x^6 - 10x^4)}{(3x^2 - 6x)}$

Now substituting $x = 1$ we get $\left(\frac{-3}{-3}\right) = 1$.

Best answer

Hence, answer is 1.

Correct Answer: C

GATE CSE 2019 | Question: 13

asked in **Calculus** Feb 7, 2019 • retagged Dec 1, 2022 by **Lakshman Bhaiya**

13

Compute $\lim_{x \to 3} \dfrac{x^4 - 81}{2x^2 - 5x - 3}$

A. 1
B. $53/12$
C. $108/7$
D. Limit does not exist

gatecse-2019 engineering-mathematics calculus limits 1-mark

https://gateoverflow.in/302835/gate-cse-2019-question-13

21

Let $y = \lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

When we put 3 in the equation we get $\frac{0}{0}$ form, so we can apply L Hospital′s rule.

Best answer

Differentiate the numerator and denominator separately

$$y = \lim_{x \to 3} \frac{4x^3 - 0}{4x - 5 - 0}$$

$$y = \lim_{x \to 3} \frac{4x^3}{4x - 5}$$

Put the limit and get the value

$$y = \frac{4 \times (3)^3}{4 \times (3) - 5}$$

$$y = \frac{4 \times 27}{7}$$

$$y = \frac{108}{7}$$

Correct Answer is C.

answered Feb 7, 2019 • edited Mar 29, 2021 by **soujanyareddy13**

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Lakshman Bhaiya

GATE CSE 2021 Set 1 | Question: 20

asked in **Calculus** Feb 18, 2021 • retagged Nov 30, 2022 by **Lakshman Bhaiya**

3,905 views

3

Consider the following expression.

$$\lim_{x \to -3} \frac{\sqrt{2x+22}-4}{x+3}$$

The value of the above expression (rounded to 2 decimal places) is __________.

gatecse-2021-set1 | calculus | limits | numerical-answers | 1-mark

https://gateoverflow.in/357431/gate-cse-2021-set-1-question-20

8

$$\lim_{x\to-3}\frac{\sqrt{2x+22}-4}{x+3}\ \left(\frac{0}{0}\ form\right)$$

Using L'Hôpital's rule

$$\lim_{x\to-3}\frac{\frac{1}{2\sqrt{2x+22}}(2)-0}{1+0}=\lim_{x\to-3}\frac{1}{\sqrt{2x+22}}=\frac{1}{\sqrt{2(-3)+22}}=\frac{1}{4}=0.25$$

Best answer

answered Feb 20, 2021 • selected Apr 25, 2021 by **gatecse**