

Primality test in $O(\sqrt{N})$:-

→ To determine if the given int is prime or not.

Br →

```
bool isPrime (int n)
{
```

```
    if (n == 1) return false;
```

```
    for (i < n, i = 2)
```

```
        if (n % i == 0) return false;
```

```
    return true;
```

```
}
```

→ Better Approach:

- All ~~numbers~~ divisors of a number M occur in Pairs (a, b) , $a \times b = M$.

- 12: 1, 2, 3, 4, 6, 12

pairs: (1,12) (4,3) (2,6)

Uaim: for a divisor pair (a, b) one of them lies below \sqrt{N} and other lies above \sqrt{N}

Proof:

There would be three cases

Case 1: Both a & b are below \sqrt{N}

Case 2: Both a & b are above \sqrt{N}

Case 3: One is below \sqrt{N} and the other above.

Case 1: let's assume that this statement is true, hence

$$a < \sqrt{N} \quad \text{and} \quad b < \sqrt{N}$$

But then,

$$a \times b < N$$

which contradicts the fact that $a \times b = N$

hence, Case 1 is not true.

Case 2: let's assume it is true, then,

$$a > \sqrt{N} \quad \& \quad b > \sqrt{N}$$

But then,

$$a \times b > N$$

Hence, Case 2 is utter bullshit.

Case 3: One is - - .

$$a = \text{sqrt}(M) \times \text{sqrt}(M) / b \quad \text{--- (1)}$$

Subcase 1: $b < \text{sqrt}(M)$ gives $1 < \text{sqrt}(M) / b$
 $a = \text{sqrt}(M) \times (1 + \alpha)$

Hence, $a > \text{sqrt}(M)$

Subcase 2: $b > \text{sqrt}(M)$ gives $1 > \text{sqrt}(M) / b$
 $a = \text{sqrt}(M) \times (1 - \alpha)$

Hence, $a < \text{sqrt}(M)$

Therefore, Case - 3 is correct:

⇒ Considering the above stuff we can conclude that we only need to check till sqrt(M).