HPIPM reference guide

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Introduction

HPIPM - High-Performance Interior Point Method.

HPIPM is a library providing a collection of quadratic programs (QP) and routines to manage them. Aim of the library is to provide both stand-alone IPM solvers for the QPs and the building blocks for more complex optimization algorithms.

At the moment, three QPs types are provided: dense QPs, optimal control problem (OCP) QPs, and tree-structured OCP QPs. These QPs are defined using C structures. HPIPM provides routines to manage the QPs, and to convert between them.

HPIPM is written entirely in C, and it builds on top of BLASFEO [?], that provides high-performance implementations of basic linear algebra (LA) routines, optimized for matrices of moderate size (as common in embedded optimization).

Dense QP

OCP QP

The OCP QP is a QP in the form

$$\min_{x,u,s} \quad \sum_{n=0}^{N} \frac{1}{2} \begin{bmatrix} u_n \\ x_n \\ 1 \end{bmatrix}^T \begin{bmatrix} R_n & S_n & r_n \\ S_n^T & R_n & q_n \\ r_n^T & q_n^T & 0 \end{bmatrix} \begin{bmatrix} u_n \\ x_n \\ 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} s_n^l \\ s_n^u \\ 1 \end{bmatrix}^T \begin{bmatrix} Z_n^l & 0 & z_n^l \\ 0 & Z_n^u & z_n^u \\ (z_n^l)^T & (z_n^u)^T & 0 \end{bmatrix} \begin{bmatrix} s_n^l \\ s_n^u \\ 1 \end{bmatrix} \\
s.t. \quad x_{n+1} = A_n x_n + B_n u_n + b_n \\
\begin{bmatrix} u_n \\ \frac{x}{d}_n \end{bmatrix} \leq \begin{bmatrix} J_{u,n} & 0 \\ 0 & J_{x,n} \\ D_n & C_n \end{bmatrix} \begin{bmatrix} u_n \\ x_n \end{bmatrix} + \begin{bmatrix} J_{s^l,u,n} \\ J_{s^l,x,n} \\ J_{s^l,g,n} \end{bmatrix} s_n^l \\
\begin{bmatrix} J_{u,n} & 0 \\ 0 & J_{x,n} \\ D_n & C_n \end{bmatrix} \begin{bmatrix} u_n \\ J_{s^u,x,n} \\ J_{s^u,x,n} \\ J_{s^u,g,n} \end{bmatrix} s_n^u \leq \begin{bmatrix} \overline{u}_n \\ \overline{x}_n \\ \overline{d}_n \end{bmatrix} \\
n = 0, \dots, N \\
n = 0, \dots, N$$

where u_n are the control inputs, x_n are the states, s_n^l (s_n^u) are the slack variables of the soft lower (upper) constraints. The matrices $J_{\dots,n}$ are made of rows from identity matrices. Note that all quantities can vary stage-wise. Furthermore, note that the constraint matrix with respect to u and x is the same for the upper and the lower constraints.

```
int d_memsize_ocp_qp(int N, int *nx, int *nu, int *nb, int *ng, int *ns);
void d_create_ocp_qp(int N, int *nx, int *nu, int *nb, int *ng, int *ns,
    struct d_ocp_qp *qp, void *memory);

void d_cvt_colmaj_to_ocp_qp(double **A, double **B, double **b,
    double **Q, double **S, double **R, double **q, double **r,
    int **idxb, double **lb, double **ub,
    double **C, double **D, double **lg, double **ug,
    double **Z1, double **Zu, double **z1, double **zu, int **idxs,
    struct d_ocp_qp *qp);
```

Tree OCP QP