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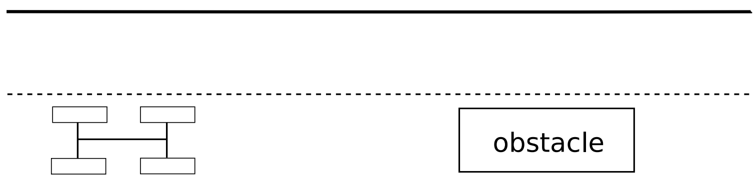
Relative Dynamics for Bicycle Model

1 Problem Formulation

Consider an autonomous vehicle driving at a constant highway speed on a straight road, attempting to avoid a static obstacle while staying within the road boundaries. This maneuver will require high vehicle lateral acceleration.

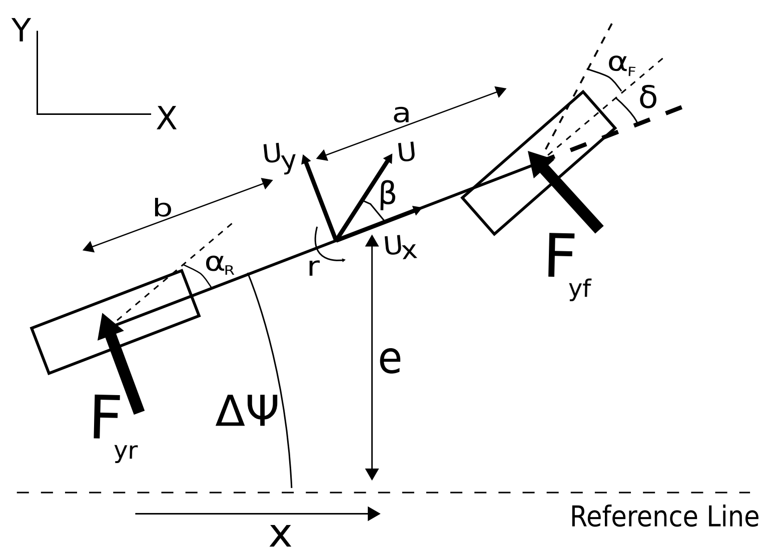


2 Tracker Dynamics

The dynamics of the vehicle in this situation can be described with the following four states:

$$s = [e \ \Delta\Psi \ r \ \ \beta]^T \tag{1}$$

where e is the lateral deviation from the desired path, $\Delta\Psi$ is the heading deviation from the desired path, r is the vehicle yaw rate, β is the vehicle sideslip (see below). Note that x is distance along the path and is needed to determine distance from the obstacle, but is not relevant for the lateral path tracking.



For a vehicle attempting a highly dynamic maneuver, the force generated by the tires begins to saturate, creating nonlinear dynamics. The dynamics are defined by the following ODEs:

$$\dot{\beta} = \frac{F_{\text{yf}} \cos\delta + F_{\text{yr}}}{mU_x} - r \tag{2a}$$

$$\dot{r} = \frac{aF_{\text{yf}} \cos\delta - bF_{\text{yr}}}{I_z} \tag{2b}$$

$$\dot{e} = U_x \sin\Delta\Psi + U_y \cos\Delta\Psi \tag{2c}$$

$$\Delta\dot{\Psi} = r \tag{2d}$$

$$\tag{2e}$$

The front and rear tire forces F_{yf} and F_{yr} are functions of the tire slip angles, given by:

$$\alpha_F = \frac{U_y + ar}{U_x} - \delta \tag{3}$$

$$\alpha_R = \frac{U_y - br}{U_x} \tag{4}$$

Note that each tire has separate cornering stiffnesses C_f and C_r and normal forces F_{zf} and F_{zr}

$$F_{\text{y}} = \begin{cases} -C_\alpha \tan\alpha + \frac{C_\alpha^2}{3\mu F_{\text{z}}} |\tan\alpha| \tan\alpha - \frac{C_\alpha^3}{27\mu^2 F_{\text{z}}^2} \tan^3\alpha, & |\alpha| < \arctan\left(\frac{3\mu F_{\text{z}}}{C_\alpha}\right) \\ -\mu F_{\text{z}} \text{sgn}\ \alpha, & \text{otherwise} \end{cases} \tag{5}$$

The control input is the steer angle, δ, which indirectly appears in the ODEs through the tire slip angle, then the tire force. Assume the following parameter set:

Table 1: Vehicle Parameters

Parameter	Symbol	Value	Units
Vehicle mass	m	1500	kg
Yaw moment of inertia	I_z	2250	$\text{kg} \cdot \text{m}^2$
Front axle to CG	a	1.04	m
Rear axle to CG	b	1.42	m
Gravity acceleration	9.81	m/s^2	
Front cornering stiffness	$\mathrm{C_F}$	8500	N
Rear cornering stiffness	$\mathrm{C_R}$	6300	N
Tire road friction (both tires)	μ	1.0	
Front normal force	F_{zf}	8560	N
Rear normal force	F_{zr}	6275	N
Vehicle Velocity	U_x	30	m/s

3 Planner Dynamics

While it is possible to run model-based online control with a fully nonlinear vehicle model, this is still an area of active research focus. A simpler planning model is to assume no saturation of tire force - i.e.:

$$F_y = -C\alpha \tag{6}$$

In this case, the vehicle dynamics can now be written as a linear dynamical system of the form:

$$\dot{s} = As + B\delta \tag{7}$$

where A and B are given by:

$$\dot{x} = Ax + B\kappa \tag{8a}$$

$$A = \begin{bmatrix} 0 & U_\mathrm{x} & 0 & U_\mathrm{x} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{a^2 C_\mathrm{F} + b^2 C_\mathrm{R}}{U_\mathrm{x} I_\mathrm{z}} & \frac{b C_\mathrm{R} - a C_\mathrm{F}}{I_\mathrm{z}} \\ 0 & 0 & \frac{b C_\mathrm{R} - a C_\mathrm{F}}{m U_\mathrm{x}^2} - 1 & -\frac{C_\mathrm{F} + C_\mathrm{R}}{m U_\mathrm{x}} \end{bmatrix} \tag{8b}$$

$$B = [0 \;\; 0 \;\; \frac{aC_f}{I_z} \;\; \frac{C_f}{mU_x}]^T \tag{8c}$$