Assignments

1) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1 \; ; \; R_3 \to R_3 - 3R_1$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 \to R_4 - 4R_1 \; , \; R_4 \to R_4 - R_3$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ No. of non zero rows

Rank = 3

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2) $T: M \to P_2$

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$$

Find the rank and nullity of T.

$\rightarrow$ $$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

$$T(A) = (a-b) + (b-c)x + (c-a)x^2$$
$$= a - bx + c(x^2 - x + 1)$$

$\therefore$ The image of T is the set of all polynomials of degree at most 2, denoted as P_2.

Rank of T :-

The rank of T is the dimension of it's image. Since P_2 has a dimension of 3 (co-efficients for x^0, x^1 and x^2) the rank of T is 3.

The Null Space of Symmetric Matrix

$$T(A) = 0$$

this leads to System of equations

$a-b=0$, $b-c=0$, $c-a=0$

$\therefore a=b=c$

4) $3x-0.1y+0.2z=7.85$

$0.1x+7y-0.3z=-19.3$

$0.3x+0.2y+10z=71.4$

$$\Rightarrow x = \frac{7.85-0.2z+0.1y}{3}$$

$$\Rightarrow y = \frac{-19.3+0.3z-0.1x}{7}$$

$$\Rightarrow z = \frac{71.4+0.2y-0.3x}{10}$$

$x(0)=0$, $y(0)=0$, $z(0)=0$

<u>Ilteration-1</u>

$$x(1) = \frac{7.85}{3} = 2.616$$

$$y(1) = \frac{-19.3-0.1(2.616)}{7} = \frac{-19.5616}{7} = -2.794$$

$$z(1) = \frac{71.4+0.2(-2.794)-0.3(2.616)}{10}$$

$$= \frac{70.056}{10} = 7.0056$$

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Ilteration - 2

$$x(2) = \frac{7.85 - 0.2(7.00) + 0.1(2.79)}{3} = \frac{6.175}{3} = 2.05$$

$$y(2) = \frac{-19.3 + 0.3(7.00) - 0.1(2.05)}{7} = -2.487$$

$$z(2) = \frac{71.4 + 0.2(-2.487) - 0.3(2.27)}{10} = 7.023$$

Ilteration - 3

$$x(3) = \frac{7.85 - 0.2(7.023) + 0.1(-2.457)}{3} = 2.064$$

$$y(3) = \frac{-19.3 + 0.3(7.023) - 0.3(2.064)}{7} = -2.544$$

$$z(3) = \frac{71.4 + 0.2(-2.54) - 0.3(2.064)}{10} = 7.027$$

5) Define consistent and inconsistent system of equations. Hence. Solve the following system of equations if consistent

$x + 3y + 2z = 0$, $2x - y + 3z = 0$; $3x - 5y + 4z = 0$, $x + 17y + 4z = 0$

Sol • Consistent :- A system of equations is consistent if it has atleast one solution meaning the equation have a common solution.

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• Inconsistent :- A system of equations is inconsistent if it has no solution, meaning the equations do not intersect at any point are contradictory.

$$x+3y+2z=0$$
$$2x-y+3z=0$$
$$3x-5y+4z=0$$
$$x+17y+4z=0$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [A : B]$$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -17 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$ $\quad R_4 \to R_4 - R_1$ $\quad R_4 \to R_4 + R_3$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$\text{Rank}(C) = \text{Rank}(A) \neq$ no. of unknowns

infinite solutions.

$$AX = B$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x+3y+2z=0$$
$$-7y-z=0$$
$$-7y=z$$
$$y=\frac{-z}{7}$$

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Let $\boxed{z = k}$

$$x + 3\left(\frac{-k}{7}\right) + 2(k) = 0$$

$$7(x) + (-3k) + 14k = 0$$

$$x = \frac{-11k}{7}$$

6) $T: R_2 \to P_3$ is linear transformation

$$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$$

$\rightarrow T(a + bx + c) = (a+1) + (b+1)x + (c+1)x^2$

is a linear transformation, we need to check two properties

1) Additivity $T(u+v) = T(u) + T(v)$

2) Homogenity of degree 1:-

$T(ku) = kT(u)$ for all u in the domain of T and all scalars k

1) $T(u+v) = T((a_1 + b_1x + c_1) + (a_2 + b_2x + c_2))$

$$= T((a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2))$$

$$= (a_1 + a_2 + 1) + (b_1 + b_2 + 1)x + (c_1 + c_2 + 1)x^2$$

$= (a_1+1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1) + (b_2+1)x + (c_2+1)x^2$

$= T(a_1+b_1x+c_1) + T(a_2+b_2x+c_2)$

So function is additive

$\therefore$ Homogenity of Degree 1

$T(kv) = T(k(a+bx+c))$

$= T(ka+kbx+kc) = (k(a+1) + (kb+1)x + (kc+1))$

$= k(a+1) + k(b+1)x + k(c+1)x^2$

$= kT(a+bx+c)$

So, the function is homogenous of degree 1

$\therefore$ It indeed linear transformul

3) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{4-1}\begin{bmatrix} 2 & +1 \\ +1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ 1/3 & 2/3 \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{3}-\lambda & 1/3 \\ 1/3 & 2/3-\lambda \end{bmatrix} = \left(\frac{2}{3}-\lambda\right)^2 - \left(\frac{1}{3}\right)^2 = 0$

$\boxed{a^2-b^2 = (a+b)(a-b)}$

$\left(\frac{1}{3}-\lambda\right)(1-\lambda) = 0$

Eigen values $\lambda = 1, 1/3$

$\lambda = 1$

$$\begin{bmatrix} -1/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$-x\left(\frac{1}{3}\right) + y\left(\frac{1}{3}\right) = 0$$

$$\boxed{\text{Eigen vector} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$$x = y = k$$

$\lambda = \frac{1}{3}$

$$\begin{bmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x = -y \quad \boxed{\text{Eigen vector} = \begin{bmatrix} k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$Z = A + 4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

$$Z - \lambda I = \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}$$

$$[a^2 - b^2 = (a-b)(a+b)]$$

$$(6-\lambda)^2 - 1 = (6-\lambda-1)(6-\lambda+1)$$

$$= (5-\lambda)(7-\lambda) = 0$$

$$\boxed{\text{Eigenvalues } \lambda = 5, 7} \quad \boxed{\text{Eigenvector} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$\lambda = 5$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad x - y = 0 \quad k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x = y$$

$x = 7$

$\boxed{\text{Eigenvector} = k\begin{bmatrix}1\\-1\end{bmatrix}}$

$$\begin{bmatrix}-1 & -1\\-1 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = 0 \quad x = -y \quad k\begin{bmatrix}1\\-1\end{bmatrix}$$

7) $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In cases S is not a basis determinennt Sub Space - Spanned by S.

→ $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$

can be arranged as a matrix

$$A = \begin{bmatrix}1 & 3 & -2\\2 & 1 & 1\\3 & 0 & 3\end{bmatrix}$$

Now, lets perform row reduction to obtained the echelon form:

$$\begin{bmatrix}1 & 3 & -2\\2 & 1 & 1\\3 & 0 & 3\end{bmatrix} \quad R_2 \leftarrow R_2 - 2R_1, \text{ and } R_3 \leftarrow R_3 - 3R_1$$

$$\begin{bmatrix}1 & 3 & -2\\0 & -5 & 5\\0 & -9 & 9\end{bmatrix} \quad R_3 \to R_3 + \frac{9}{5}R_2 \quad \begin{bmatrix}1 & 3 & -2\\0 & -5 & 5\\0 & 0 & 0\end{bmatrix}$$

∴ Third row of zeros indicates that the vectors is S are linearly dependent for basis of the subspace spanned by S.

$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \end{bmatrix}$ (1,3,2) and (0,-5,5) these vectors form a basis for supspace spanned by S.

∴ Dimension of subspace spanned by S = 2

∴ Set 'S' is not a basis of R^3 because the row reduced form has a row of zero's.

∴ The basis for the subspace spanned by S is {(1,3,-2), (0,-5,5)}

∴ The dimension of subspase is 2.

8) using Jocobi's method (perform 3 iterations) Solve $3x-6y+2z=23$; $-4x+y-z=-15$, $x-3y+7z=16$, with initial values $x_0=1$, $y_0=1$, $z_0=1$

→ $3x-6y+2z=23$
$-4x+y-z=-15$
$x-3y+7z=16$

With initial velocities $x=1, y=1, z=1$

Iteration-1:-

$$x(1) = \frac{23+6y(0)-2(z(0))}{3} = 9.0$$

$$y(1) = \frac{-15+4(x(1))+z(0)}{1} = -9.0$$

$$z(1) = \frac{16 - x_1(y_1) - 3(y(1))}{7} = 2.0$$

Iteration-2

$$x(2) = \frac{23 + 6(y)_1 - 2z(1)}{3} = 5.0$$

$$y(2) = \frac{-15 + 4(x)(1) + z(1)}{7} = -5.0$$

$$z(2) = \frac{16 - x(1) + 3(y)(1)}{7} = \approx 3.0$$

Iteration-3

$$x(3) = \frac{23 + 6y(2) - 2z(2)}{3} = 6.0$$

$$y(3) = \frac{-15 + 4x(2) + z(2)}{1} \approx -6.0$$

$$z(3) = \frac{16 - (x)(2) + 3y(2)}{7} \approx 2.0$$

9) Explain one application of matrix operations in processing with example

Sol. • Matrix Magic in Image processing: convoluting takes centre stage.

Digital images are essentially grids of tiny squares called pixels, each holding an intensity value (gray scale) or colour information (RGB). To manipulate these images effectively, image processing relies heavily on the power of matrices.

1) Image as a Matrix

we represent the image as a matrix, where each pixels value becomes an element in the grid

2) Kernel : The tiny Power house :-

we define a small matrix called a kernel or filter. Different kernel designs produce different results.

3) The Convolution slide :-

We slide the kernel across the image matrix, one element at a time.

4) Summing up for a New Value pixel

The multiplied values from the element-wise multiplication are then summed up.

5) Iterating Across : The Image :-

we repeat this process (sliding, multiplication, summarating for every position of the kernel with in the image matrix

Here's an example of how convolution works using a simple image filtering task.

→ Let's consider small 3x3 grey scale image matrix :-

$$\begin{bmatrix} 100 & 150 & 200 \\ 120 & 180 & 220 \\ 110 & 160 & 190 \end{bmatrix}.$$

→ Now we want to apply a simple 3x3 filter / kernal to this image. lets take a blur filter kernal :-

$$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

→ To apply this filter, we perform element-wise multiplycation of the filter kernel with the corresponing pixels in the image matrix and then sum up the results.

This sum becomes the new value for the filter center pixel in the output

$(100 \times \frac{1}{9}) + (150 \times \frac{1}{9}) + (200 \times \frac{1}{9}) + (120 \times \frac{1}{9}) +$

$\left(180 \times \frac{1}{9}\right) + \left(220 \times \frac{1}{9}\right) + \left(110 \times \frac{1}{9}\right) + \left(160 \times \frac{1}{9}\right) +$

$+ \left(190 \times \frac{1}{9}\right) = 159$

→ Each pixel value is weighted average of it's neighbour pixels effectively creating a blurred image of the original image.

→ Convolution allows for the application of various filters to image; enabling tasks such as blurring, sharpening, edge detection and more.

10) Give a brief description of linear transformation for computer vision for rotating 2D image.

Sol) Linear transformations are a powerful tool in computer vision for tasks like image rotation. There's a breakdown.

1) Image as a Vector

We can represent a 2D image as a collection of points where each point corresponding to a pixel's location (x, y) co-ordinates

2) Rotation Matrix :-

To rotate the image, we use a specific rotation matrix. This matrix encodes the rotation angle and how it affects the x and y coordinates of each point (pixel) in the image

3) Transformation Magic :-

By multiplying the image vector (representing all pixel locations) with the rotation matrix, we perform a linear transformation on all the points simultaneously.

Affine transformation include operations such as rotation, translation, scaling and shearing. when rotating a 2D image the affine transformation, involves by applying a rotation matrix to co-ordinate of each pixel in the image.

For rotating a 2D image clock-wise by an angle θ, the rotation matrix R is typically used

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Each pixel co-ordinate (x, y) in the original image is multplied by this rotation matrix to obtain the new co-ordinates (x', y') in the rotated image. The new pixel value at (x', y') is then determined by interpolating the values of neighbouring in the original image.