Date / /

Tutorial - 4

Name - Deepak Kohli
Section - F
Roll no. - 53

Q-1 $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n^2)$

$a \geq 1, \quad b > 1$

One comparing

$a = 3, \quad b = 2, \quad f(n) = n^2$

Now, $c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.584} < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

Q-2 $T(n) = 4T(n/2) + n^2$

$a \geq 1, \quad b \geq 1$

$a = 4, \quad b = 2, \quad f(n)^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log n)$

Q-3 $T(n) = T(n/2) + 2^n$

$a = 1$

$b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

Date / /

$f(n) > n^c$

$T(n) = \Theta(2^n)$

Q-4 $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$

$b = 2, \quad f(n) = n^2$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c \rightarrow n^n$

$f(n) = n^c$

$T(n) = \Theta(n^2 \log_2 n)$

Q-5 $T(n) = 16T(n/4) + n$

$a = 16, \quad b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2 \log_4 4$

$= 2$

$n^c \Rightarrow n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

Q-6 $T(n) = 2T(n/2) + n \log n$

$a = 2, \quad b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n \log n > n$

$f(n) > n^c$

$T(n) = \Theta(n \log n)$

Date / /

Q-7 $T(n) = 2T(n/2) + n/\log n$

$a = 2, \quad b = 2, \quad f(n) = n/\log n$

$c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n$

$\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) \neq \Theta(n)$

Q-8 $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, \quad b = 4, \quad f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

$\because n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

Q-9 $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5, \quad b = 2$

$a \geq 1$ but here a is 0.5

So we cannot apply Master's Theorem.

Date / /

Q-10 $T(n) = 16T(n/4) + n!$

$a = 16, \quad b = 4, \quad f(n) = n!$

$\therefore \quad c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$T(n) = \Theta(n!)$

Q-11 $4T(n/2) + \log n$

$a = 4, \quad b = 2, \quad f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\because \log n < n^2$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$= \Theta(n^2)$

Q-12 $T(n) = \text{sqrt}(n)\, T(n/2) + \log n$

$a = \sqrt{n}, \quad b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$

$\therefore \frac{1}{2} \log_2 n < \log(n)$

$\therefore f(n) > n^c$

$T(n) = \Theta(f(n))$

$= \Theta(\log(n))$

Date / /

Q-13 $T(n) = 3T(n/2) + n$

$a = 3$; $b = 2$; $f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5849}$

$n < n^{1.5849}$

$\Rightarrow f(n) < n^c$

$T(n) = \Theta(n^{1.58494})$

Q-14 $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3, \quad b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\text{sqrt}(n) \leq n$

$f(n) < n^c$

$T(n) = \Theta(n)$

Q-15 $T(n) = 4T(n/2) + n$

$a = 4, \quad b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n < n^2$ (for any constant)

$f(n) < n^c$

$f(n) = \Theta(n^2)$

Date / /

Q-16 $T(n) = 3T(n/4) + n\log n$

$a = 3, \quad b = 4, \quad f(n) = n\log n$

$C = \log_b a = \log_4 3 = 0.792$

$n^C = n^{0.792}$

$O^{0.792} < n\log n$

$T(n) = \Theta(n\log n)$

Q-17 $T(n) = 3T(n/3) + n/2$

$a = 3, \quad b = 3$

$C = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^C = n^1 = n$

As $n/2 < n$

$f(n) < n^C$

$T(n) = \Theta(n)$

Q-18 $T(n) = 6T(n/3) + n^2\log n$

$a = 6, \quad b = 3$

$C = \log_b a = \log_3 6 = 1.6309$

$n^C = n^{1.6309}$

$n^{1.6309} < n^2\log n$

$\therefore T(n) = \Theta(n^2\log n)$

Date / /

Q-19 $T(n) = 4T(n/2) + n/\log n$

$a = 4, \quad b = 2, \quad f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \Theta(n^2)$

Q-20 $T(n) = 64T(n/8) - n^2 \log n$

$a = 64 \quad b = 8$

$c = \log_b a = \log_8 64 = \log_8 (8)^2$

$c = 2$

$n^c = n^2$

$\therefore \quad n^2 \log n > n^2$

$T(n) = \Omega(n^2 \log n)$

Q-21 $T(n) = 7T(n/3) + n^2$

$a = 7; \quad b = 3; \quad f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = \Theta(n^2)$

Date / /

Q-22 $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \quad b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$T(n) = \Theta(n(2 - \cos n))$