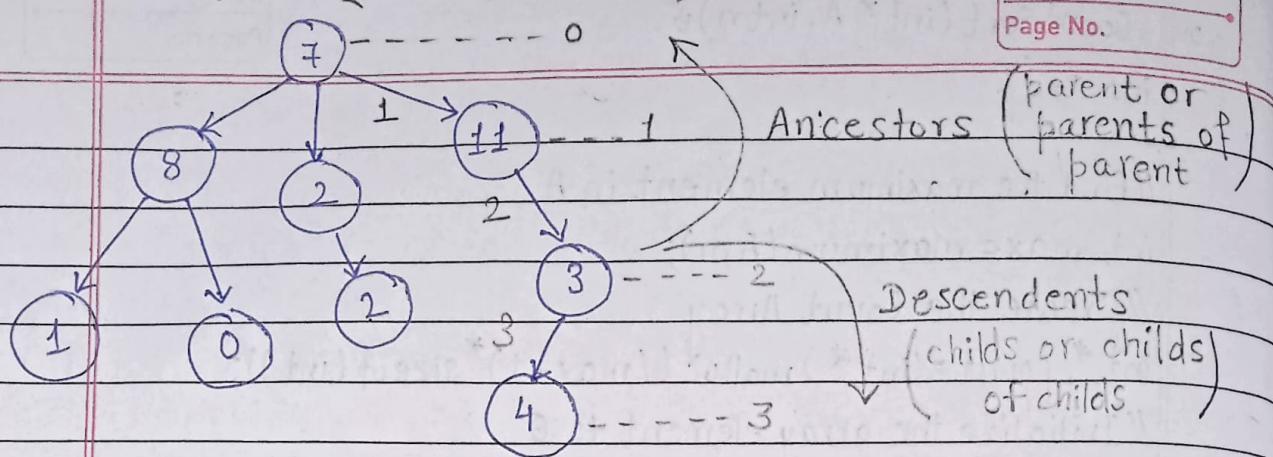


Introduction to Trees:

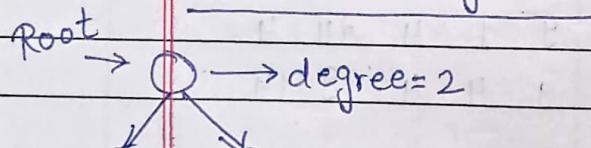
Trees!

Root of a Tree

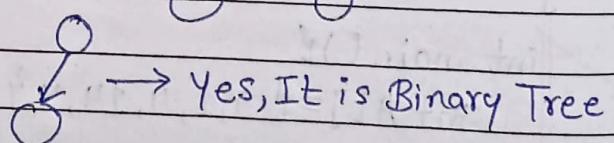
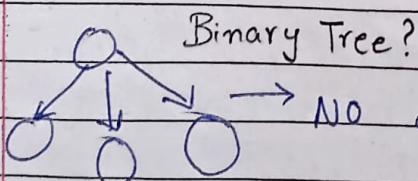
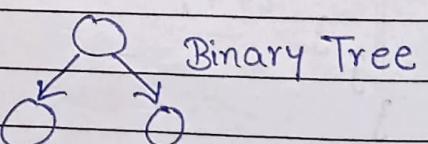
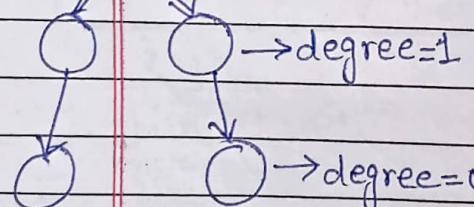
Common Terminologies:

- ① Root \rightarrow Topmost Node.
- ② Parent \rightarrow Node which connects to the child.
- ③ child \rightarrow Node which is connected by another Node is its child.
- ④ Leaf / External Node: Nodes with no children.
- ⑤ Internal Node: \rightarrow Node with atleast one child.
- ⑥ Depth \rightarrow No. of edges from root to the Node.
- ⑦ Height \rightarrow No. of edges from node to the deepest leaf!
- ⑧ Sibling \rightarrow Nodes belonging to the same parent!
- ⑨ Ancestors / Descendants

What is a Binary Tree?



\rightarrow Binary Tree is a tree, which has atmost 2. children for all the Nodes.



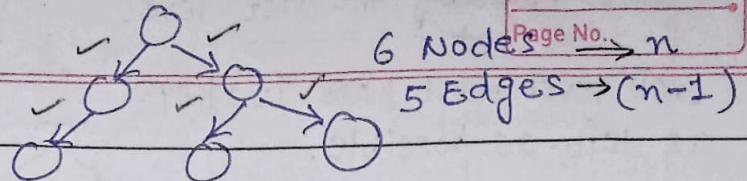
Quick Revision:

- ① Tree is made up of nodes & Edges!
- ② n nodes $\rightarrow n-1$ edges

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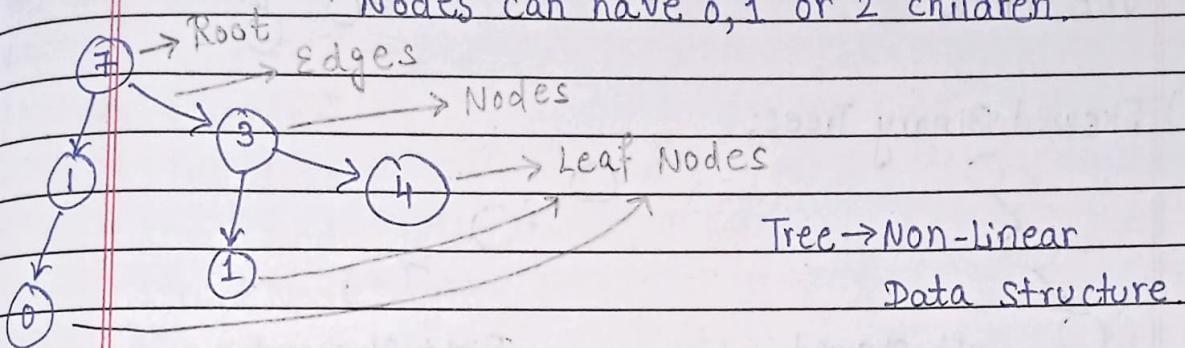
③ Degree?

Degree \rightarrow No. of Direct children. (for a Node)

④ Degree of a Tree is the highest degree of a Node among all the nodes present in the Tree.

⑤ Binary Tree \rightarrow Tree of degree $2 \leq 2$

Nodes can have 0, 1 or 2 children.



Lec-63 Types of Binary Trees:

Amb Tree \rightarrow Non Linear \rightarrow Ideal for representing hierarchical Data

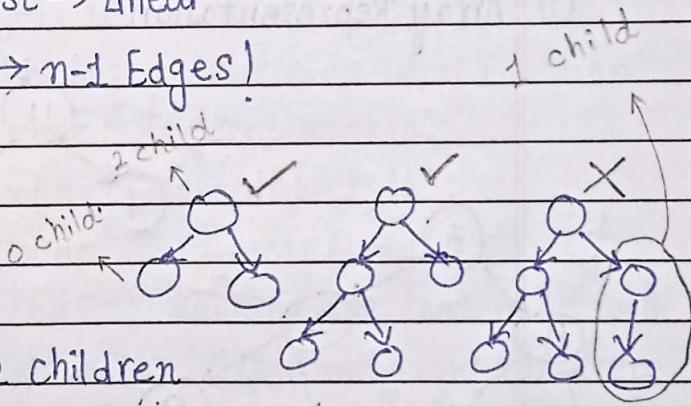
Array, Stack, Queue, LinkedList \rightarrow Linear

In a Tree with n Nodes $\rightarrow n-1$ Edges!

Types of Binary Trees:

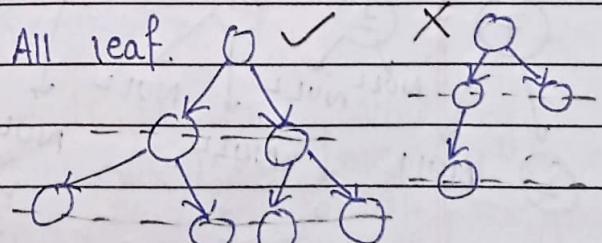
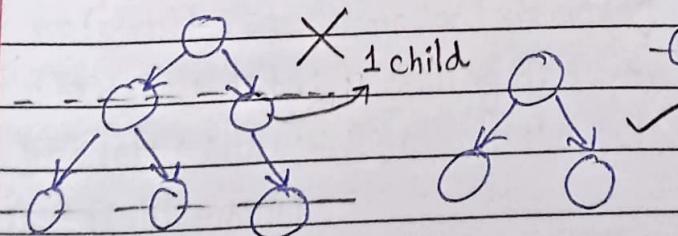
① Full or Strict Binary Tree

\rightarrow All Nodes have either 0 or 2 children



② Perfect Binary Tree.

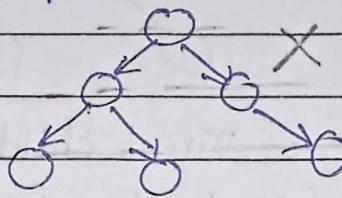
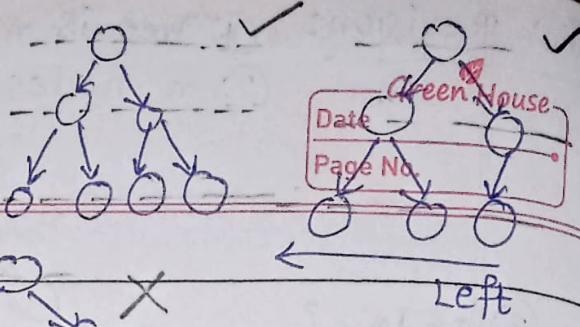
\rightarrow Internal nodes have 2 children + All leaf nodes are on same level.



③ Complete Binary Tree

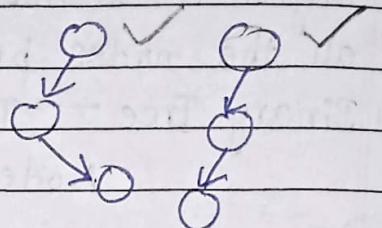
→ All levels are completely filled except possibly the last level

+ Last level must have its keys as left as possible.

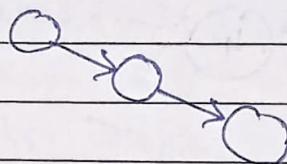
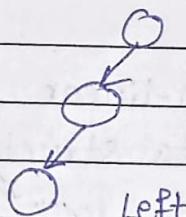


④ Degenerate Tree:

→ Every Parent Node has exactly one child.



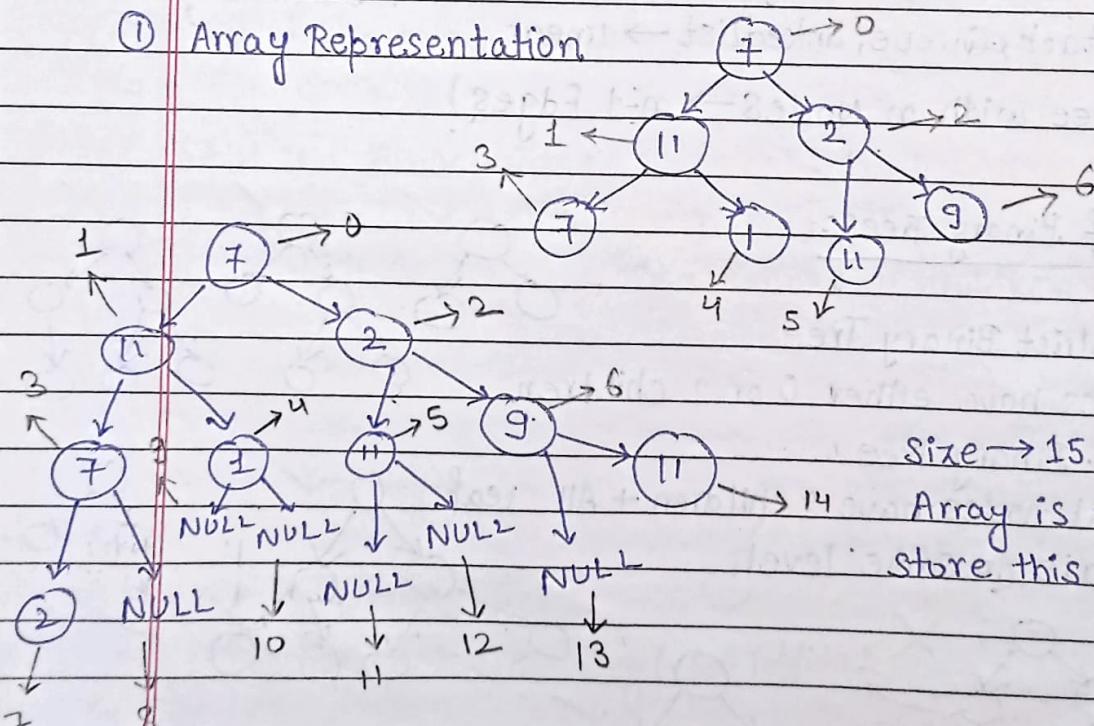
⑤ Skewed Binary Trees:



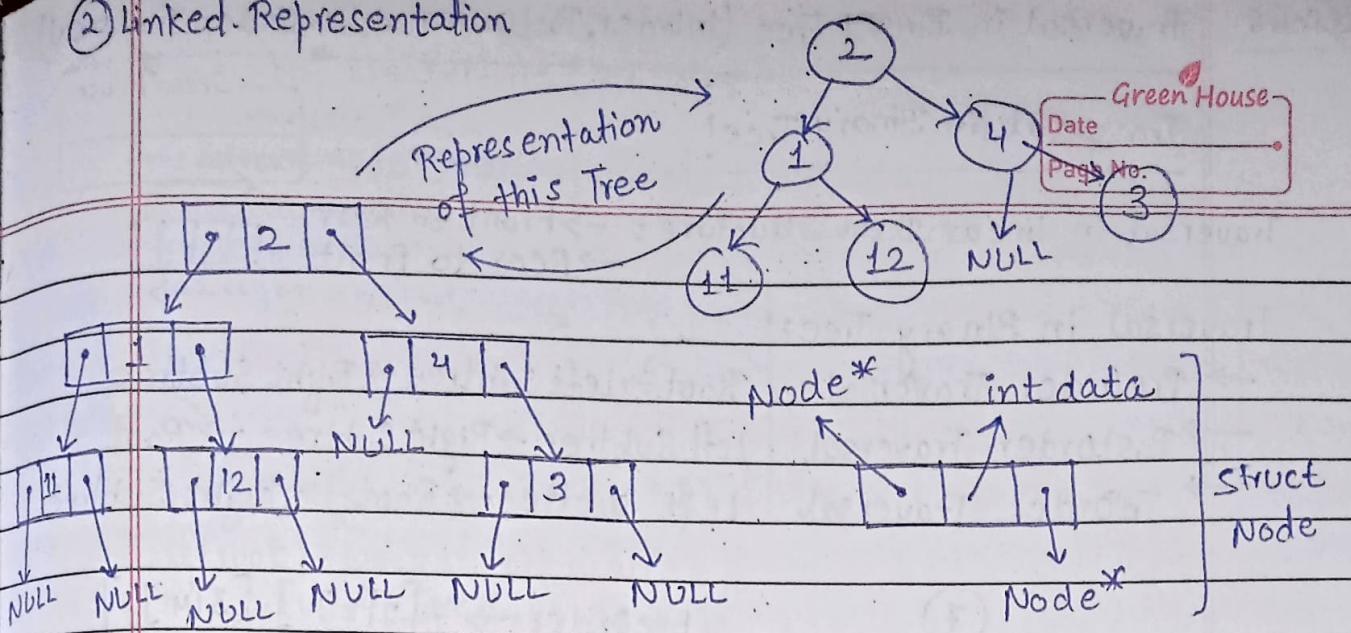
Dec-64

Representation of a Binary Tree:

① Array Representation



② Linked Representation



struct Node {

```
    int data;
    struct Node* left;
    struct Node* right;
};
```

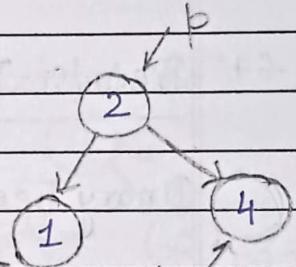
Lec-65 Linked Representation of Binary Tree in C:

```
#include <stdio.h>
#include <stdlib.h>

Struct node{
    int data;
    struct node* left;
    struct node* right;
};

struct node* createNode(int data){
    struct node*n; // Creating a node pointer
    n=(struct node*)malloc(sizeof(struct node)); // Allocating memory in heap
    n->data=data; // Setting the data
    n->left=NULL; // Setting the left and right children to NULL
    n->right=NULL;
    return n; // Finally returning the created node
}

int main(){
    // Constructing the root node - using Function
    struct node *P=createNode(2);
    struct node *P1=createNode(1);
    struct node *P2=createNode(4);
    P->left=P1; // Linking the root node with left child
    P->right=P2; // Left and right children
    return 0;
}
```



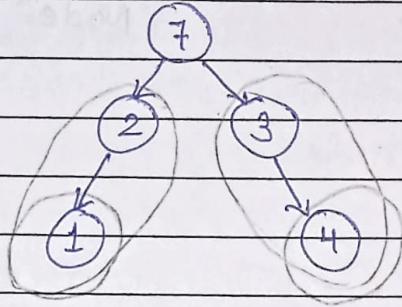
Traversal in Binary Tree!

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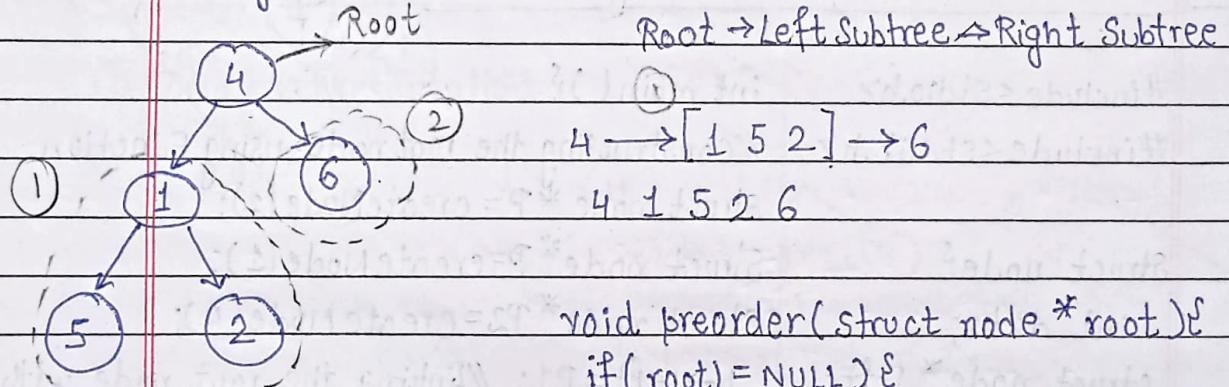
Traversal in Linear Data Structure: → Front to Rear
→ Rear to front

Ans: Traversal in Binary Tree:

- PreOrder Traversal Root → Left Subtree → Right Subtree
- PostOrder Traversal Left Subtree → Right Subtree → Root
- InOrder Traversal Left Subtree → Root → Right Subtree



preOrder → 7 [2[1]] [3[4]]
→ 7 2 1 3 4

Lec-67 PreOrder Traversal in a Binary Tree (with C Code):Binary Tree - PreOrder Traversal

```

void preorder(struct node *root) {
    if(root == NULL) {
        return;
    }
    printf("%d", root->data);
    preorder(root->left);
    preorder(root->right);
}
  
```

C-Code:

```

#include <stdio.h>
#include <stdlib.h>

struct node {
    int data;
    struct node *left;
    struct node *right;
}
  
```

3;

```
struct node* createNode (int data){  
    struct node *n;  
    n=(struct node*)malloc(sizeof(struct node));  
    n->data = data;  
    n->left = NULL;  
    n->right = NULL;  
    return n;  
}
```

4 1 5 2 6

```
void preOrder(struct node* root){  
    if(root!=NULL){
```

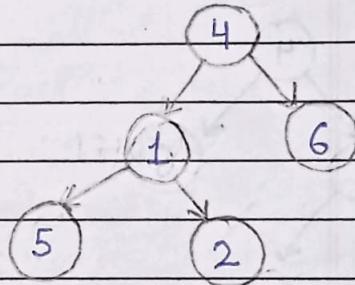
```
        printf("%d ",root->data);  
        preOrder(root->left);  
        preOrder(root->right);
```

}

```
int main(){
```

//Constructing the root node - using Function (Recommended)

```
struct node*p = createNode(4);  
struct node*p1 = createNode(1);  
struct node*p2 = createNode(6);  
struct node*p3 = createNode(5);  
struct node*p4 = createNode(2);
```



//Linking the root node with left and right children

p->left = p1;

p->right = p2;

p1->left = p3;

p1->right = p4;

```
PreOrder(p);
```

```
return 0;
```

}

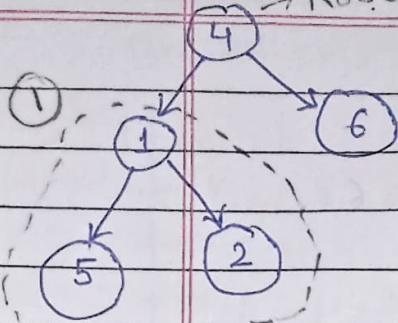
Lec-68

Postorder Traversal in a Binary Tree (with C Code):

Binary Tree - Postorder Traversal:

→ Root

Left Subtree → Right Subtree → Root



① (5 2 1) 6 4

→ 5 2 1 6 4

✓ void PostOrder(struct node* root){

if(root!=NULL){

postorder(root→left);

postorder(root→right);

printf("%d", root→data);

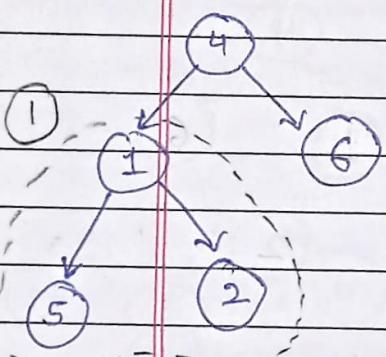
}

Lec-69

InOrder Traversal in a Binary Tree (with C Code):

Binary Tree - Inorder Traversal:

Left Subtree → Root → Right Subtree



① [5 1 2 7 4 6]

→ 5 1 2 4 6

✓ void InOrder(struct node* root){

if(root!=NULL){

Inorder(root→left);

printf("%d", root→data);

Inorder(root→right);

}

In(5)

→ print(5)

In(1)

→ print(1)

In(4)

→ Root→Right

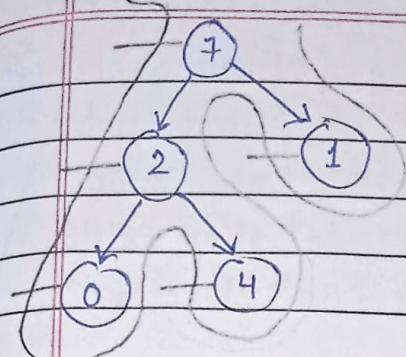
2 4 6

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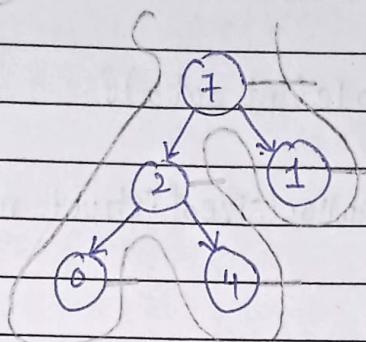
Lec-70 *Best* Trick to Find PreOrder, Inorder & PostOrder Traversals:

→ Trick to find Inorder, PreOrder, & PostOrder Traversal:

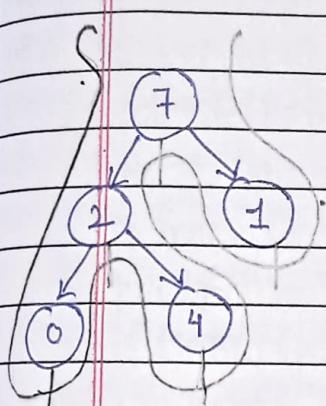
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7 2 0 4 1 → PreOrder Traversal



0 4 2 1 7
→ PostOrder Traversal



0 2 4 7 1 → InOrder Traversal

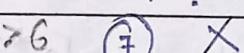
Lec-71 Binary Search Trees: Introduction and properties:

Binary Search Tree:

11 > 6

7 > 6

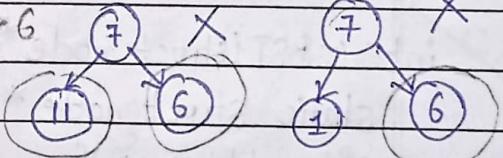
No!



No!



1. It is a Type of Binary Tree!



Imp Properties:

→ All Nodes of the left Subtree are lesser.

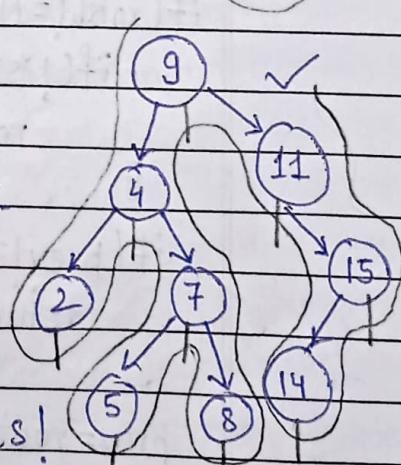
→ All nodes of the right Subtree are greater.

→ left & Right Subtrees are also BST.

→ There are no duplicate nodes.

Imp → InOrder Traversal of a BST gives an ascending Sorted Array!

yes!



2, 4, 5, 7, 8, 9, 11, 14, 15
Yes Ascending!

```
#include <stdio.h>
#include <stdlib.h>
struct node {
    int data;
    struct node *left;
    struct node *right;
};
```

```
struct node *createNode(int data) {
    struct node *n;
    n = (struct node *) malloc(sizeof(struct node));
    n->data = data;
    n->left = NULL;
    n->right = NULL;
    return n;
}
```

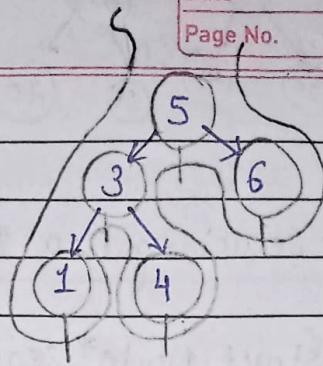
```
void inOrder(struct node *root) {
    if (root != NULL) {
        inOrder(root->left);
        printf("%d ", root->data);
        inOrder(root->right);
    }
}
```

```
int isBST(struct node *root) {
    static struct node *prev = NULL;
    if (root == NULL) {
        if (!isBST(root->left)) {
            return 0;
        }
        if (prev != NULL && root->data <= prev->data) {
            return 0;
        }
        prev = root;
    }
    return isBST(root->right);
}
```

```
else
    return 1;
}
```

```
int main()
```

```
struct node *p = createNode(5);
struct node *p1 = createNode(3);
struct node *p2 = createNode(6);
struct node *p3 = createNode(1);
struct node *p4 = createNode(4);
```



//Linking the root node with left and right children

```
p->left = p1;
```

```
p->right = p2;
```

```
p1->left = p3;
```

```
p1->right = p4;
```

```
inOrder(P);
```

```
printf("\n");
```

```
if (isBST(p)) {
```

```
    printf("This is a BST");
```

```
}
```

```
else {
```

```
    printf("This is not a BST");
```

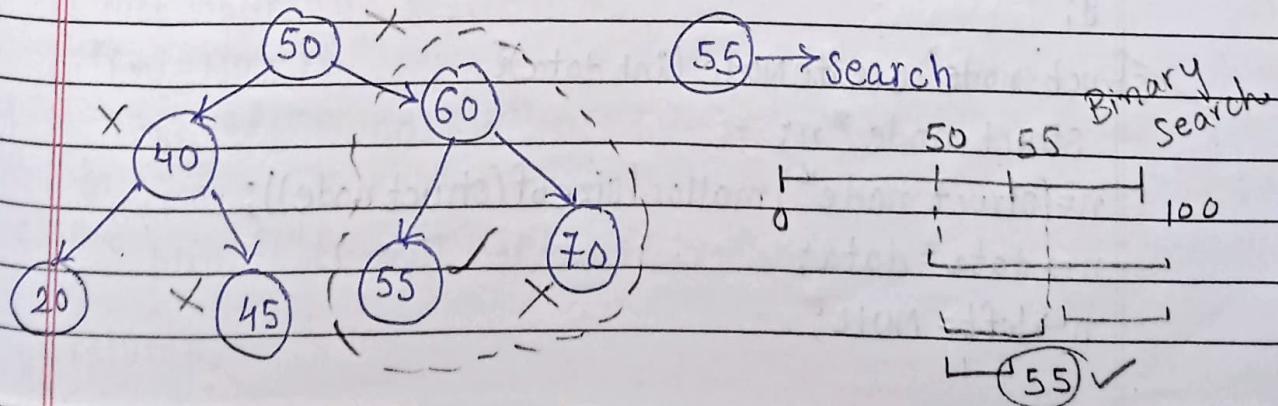
```
}
```

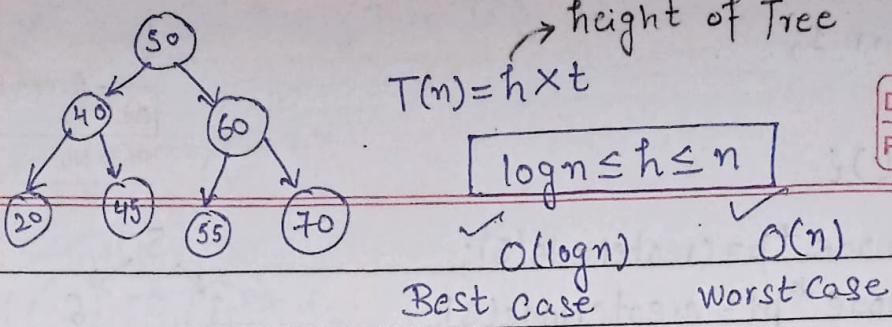
```
return 0;
```

```
}
```

c-73 Searching in a Binary Search Trees (Search Operation):

Searching in a Binary Search Tree



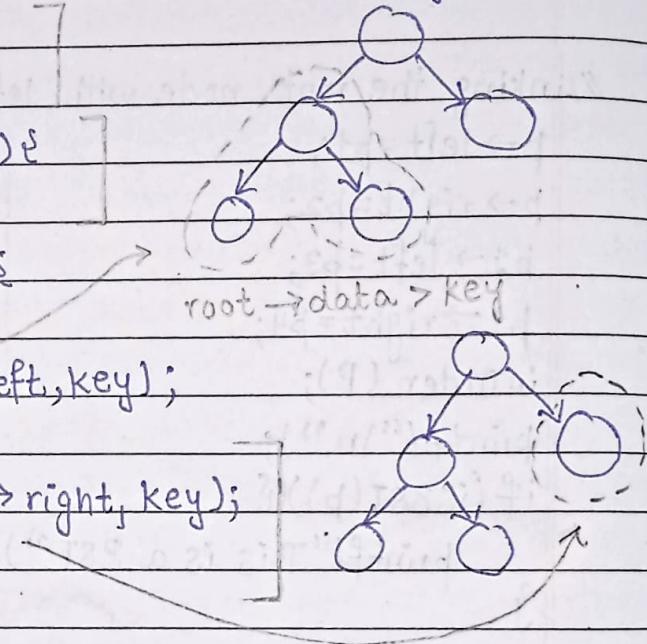


Code for Searching in a BST!

```

struct Node* search(Struct Node *root, int key) {
    if (root == NULL) {
        return NULL;
    }
    if (root->data == key) {
        return root;
    }
    else if (root->data > key) {
        return Search (root->left, key);
    }
    else {
        return Search (root->right, key);
    }
}

```



Lec-74 C. Code For Searching in a BST:

```

#include <stdio.h>
#include <stdlib.h>

Struct node {
    int data;
    Struct node *left;
    Struct node *right;
};

Struct node* createNode (int data) {
    Struct node *n;
    n=(Struct node*) malloc(sizeof(Struct node));
    n->data = data;
    n->left = NULL;
}

```

```
n->right = NULL;
return n;
```

```
struct node* search(struct node* root, int key) {
```

```
    if (root == NULL) {
        return NULL;
    }
```

```
    if (key == root->data) {
        return root;
    }
```

```
    else if (key < root->data) {
        return Search(root->left, key);
    }
```

```
    else {
        return Search(root->right, key);
    }
```

```
int main() {
```

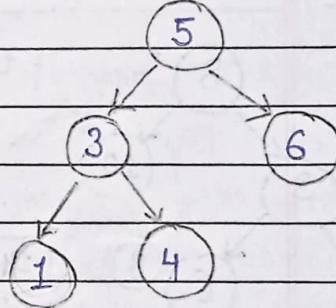
```
    struct node* p = CreateNode(5);
```

```
    struct node* p1 = CreateNode(3);
```

```
    struct node* p2 = CreateNode(6);
```

```
    struct node* p3 = CreateNode(1);
```

```
    struct node* p4 = CreateNode(4);
```



```
p->left = p1;
```

```
p->right = p2;
```

```
p1->left = p3;
```

```
p1->right = p4;
```

	Element not found
--	-------------------

```
struct node* n = search(p, 10);
```

```
if (n != NULL) {
```

```
    printf("Found: %d", n->data);
```

```
}
```

```
else {
```

```
}
```

```
    printf("Element not found");
```

```
}
```

```
return 0;
```



```

n = (struct node*) malloc(sizeof(struct node));
n->data = data;
n->left = NULL;
n->right = NULL;
return n;
}

```

```
void insert(struct node* root, int key) {
```

```
    struct node* prev = NULL;
```

```
    while (root != NULL) {
```

```
        prev = root;
```

```
        if (key == root->data) {
```

Inserted
node

```
            printf("Cannot Insert %d, already in BST", key);
```

```
            return n;
```

```
}
```

```
        else if (key < root->data) {
```

```
            root = root->left;
```

```
}
```

```
        else {
```

```
            root = root->right;
```

```
.
```

```
3
```

```
struct node* new = createNode(key);
```

```
if (key < prev->data) {
```

```
    prev->left = new;
```

```
else {
```

```
    prev->right = new;
```

```
}
```

```
3
```

```
int main() {
```

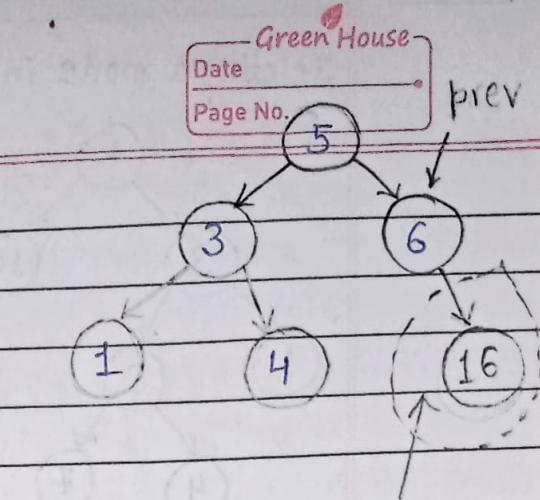
```
Struct node* p = createNode(5);
```

```
Struct node* p1 = createNode(3);
```

```
Struct node* p2 = createNode(6);
```

```
Struct node* p3 = createNode(1);
```

```
Struct node* p4 = createNode(4);
```



16

16

p->left = p1;

p->right = p2;

p1->left = p3;

p1->right = p4;

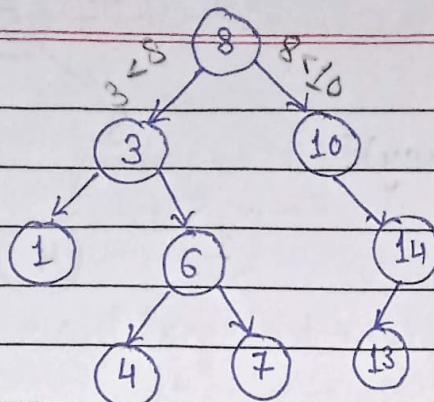
insert(p, 16);

printf("%d", p->right->
right->data);

return 0;

3

Deleting a node in a BST



Case1: The node is a leaf node.

Case2: The node is a non-leaf node

Case3: The node is the root node.

Case4: The node is not in Tree.

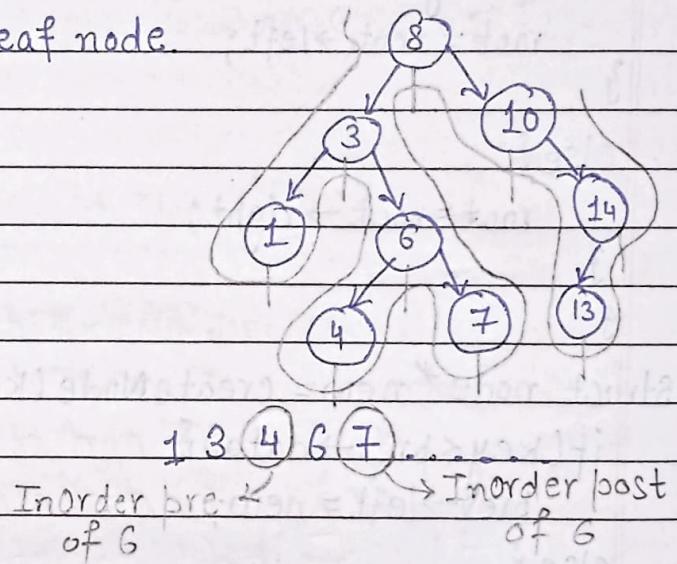
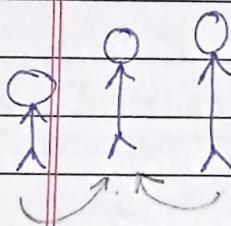
Case:1. The node is a leaf Node.

Step1 → Search the node

Step2 → Delete the node

 ↗ Inorder pre
 ↘ InOrder post

Case:2. The node is a non-leaf node.

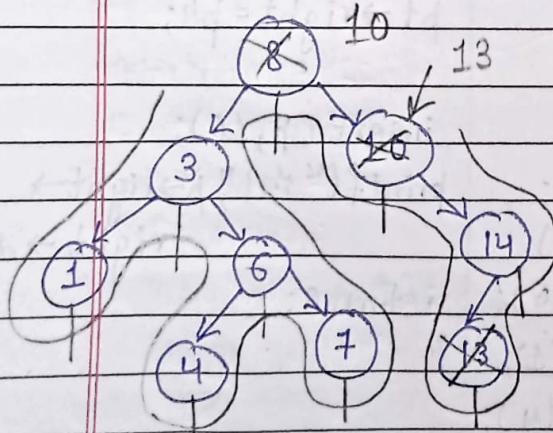


Case:3 The node is the root node / Node in Between BST

Step1 → Search for the node

Step2 → Search for InOrder pre & InOrder post.

Step3 → keep doing this until the tree has no Empty node.


 InOrder → 1 3 4 6 7 8 10 13 14
 ↑ ↑

C Code for Deletion in BST:

```
#include<stdio.h>
#include<stdlib.h>
Struct node{
```

```
    int data;
    Struct node *left;
    struct node *right;
```

};

```
Struct node* createNode(int data){
```

```
    Struct node* n;
    n=(Struct node*) malloc(sizeof(Struct node));
    n->data=data;
    n->left=NULL;
    n->right=NULL;
    return n;
```

};

```
Void inOrder(Struct node* root){
```

```
    if(root==NULL){
        inOrder(root->left);
        printf("%d ",root->data);
        inOrder(root->right);
```

};

```
Struct Node* inOrderPredecessor(Struct node* root){
```

```
    root=root->left;
    while(root->right!=NULL){
        root=root->right;
```

};

```
return root;
```

};

```
Struct node* deleteNode(Struct node* root, int value){
```

```
    Struct node* iPre;
    if(root==NULL){
        return NULL;
```

};

if($\text{root} \rightarrow \text{left} == \text{NULL}$ & & $\text{root} \rightarrow \text{right} == \text{NULL}$) {
 free(root);
 return NULL;
}

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// search for the node to be deleted

if ($\text{value} < \text{root} \rightarrow \text{data}$) {

 root \rightarrow left = deleteNode($\text{root} \rightarrow \text{left}$, value);

}

else if ($\text{value} > \text{root} \rightarrow \text{data}$) {

 root \rightarrow right = deleteNode($\text{root} \rightarrow \text{right}$, value);

}

// Deletion strategy when the node is found

else {

 iPre = inOrderPredecessor(root);

 root \rightarrow data = iPre \rightarrow data;

 root \rightarrow left = deleteNode($\text{root} \rightarrow \text{left}$, iPre \rightarrow data);

}

return root;

}

int main() {

 struct node * p = createNode(5);

 Struct node * p1 = CreateNode(3);

 Struct node * p2 = CreateNode(6);

 Struct node * p3 = CreateNode(1);

 Struct node * p4 = CreateNode(4);

// Linking the root node with left and right children

 p \rightarrow left = p1;

 p \rightarrow right = p2;

 p1 \rightarrow left = p3;

 p1 \rightarrow right = p4;

 inOrder(p);

 deleteNode(p, 5);

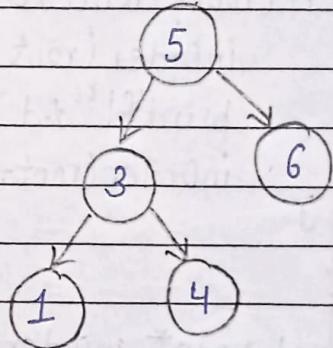
 printf("\n");

 printf("Data is %d\n", p \rightarrow data);

 inOrder(p);

 return 0;

}



1 3 4 5 6

Data is 4 | 1 3 4 6

AVL Trees - Introduction:AVL Trees :

Height Balanced Binary Tree.

Green House

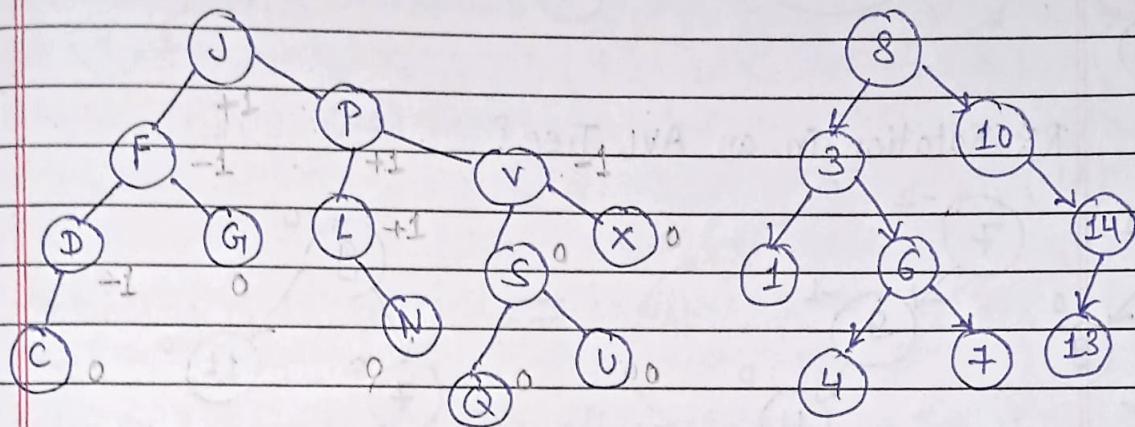
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why AVL Trees?

why do we need an AVL Tree?

- Almost all the operations in a binary search tree are of order $O(h)$ where h is the height of the tree.
- If we don't plan our tree properly, this height can get as high as n where n is the number of nodes in a BST (skewed tree).
- To guarantee an upper bound of $O(\log n)$ for all these operations, we use balanced trees.



what is an AVL Tree?

- Height balanced binary Search Trees.
- Height difference between heights of left and right subtrees for every node is less than or equal to ≤ 1 .
- Balanced factor = Height of right Subtree - Height of left Subtree
- Can be $-1, 0$ or 1 for a node to be balanced in a BST.
- Can be $-1, 0$ or 1 for all nodes of an AVL Tree.

BF → Balanced Factor

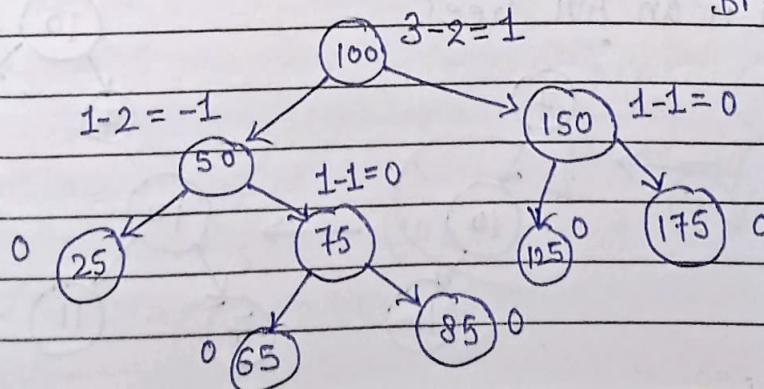
hl → Height of left Subtree

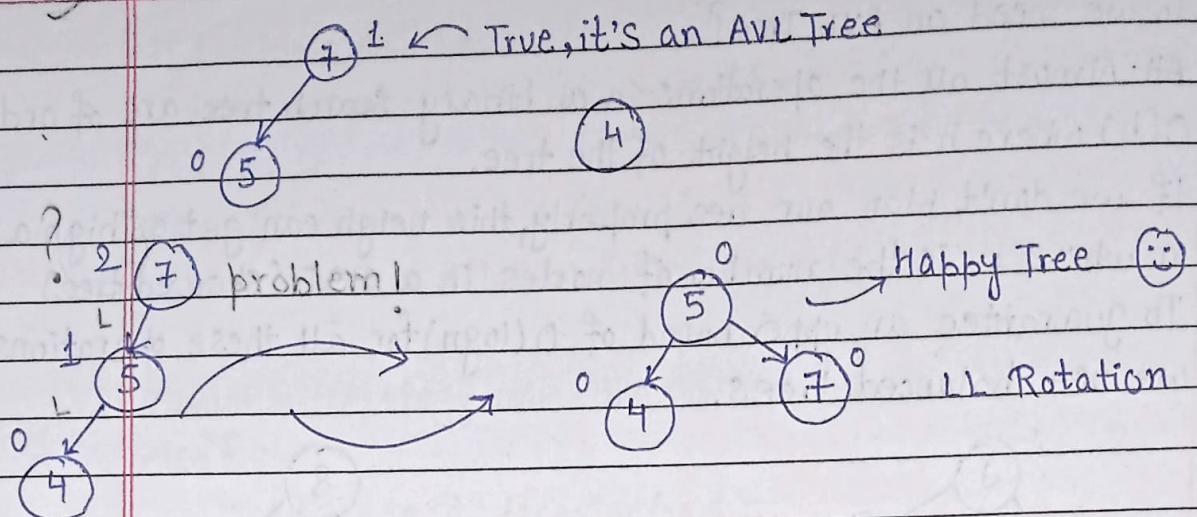
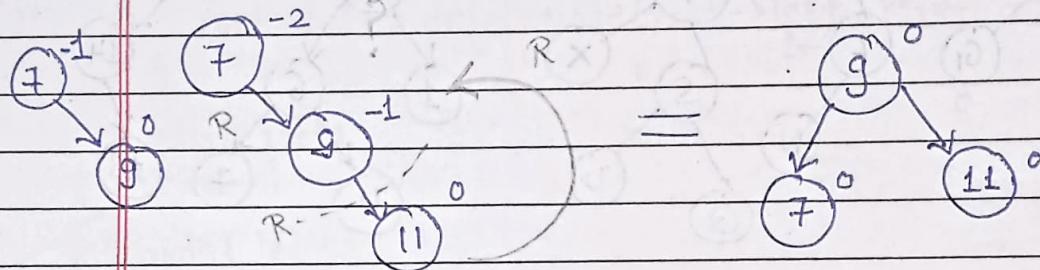
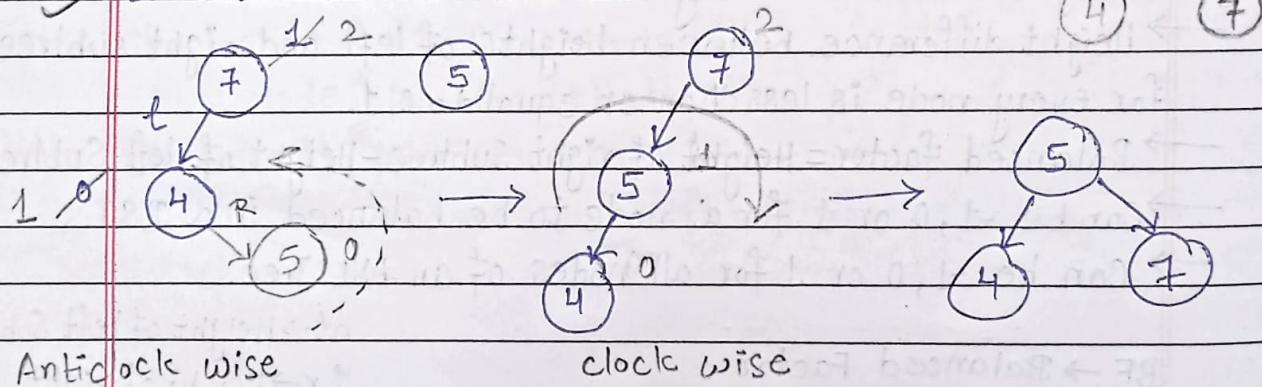
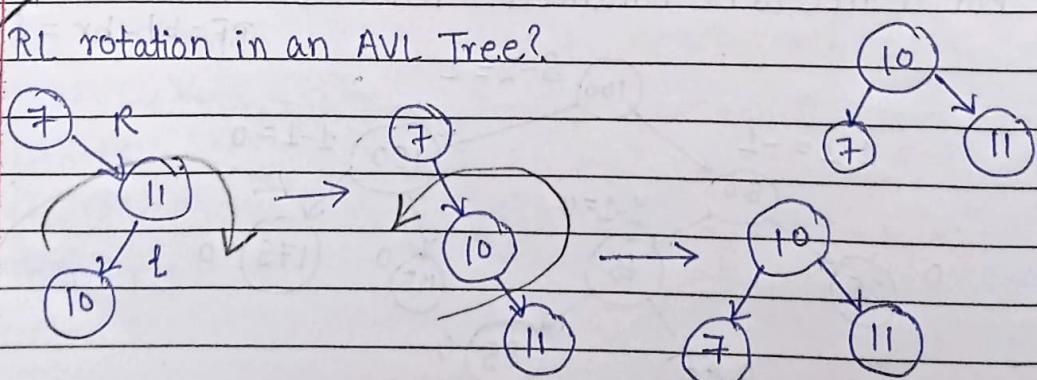
hr → Height of right

subtree

→ For a tree to be balanced $|BF| \leq 1$

$$BF = hl - hr = \{-1, 0, 1\}$$



Insertion and Rotation in AVL Tree:Rotation in AVL TreesLL Rotation in an AVL Tree?RR Rotation in an AVL Tree?LR rotation in an AVL Tree?RL rotation in an AVL Tree?

AVL Trees - LL, LR, RL and RR rotations:

Rotation in AVL Trees with multiple Nodes

Green House

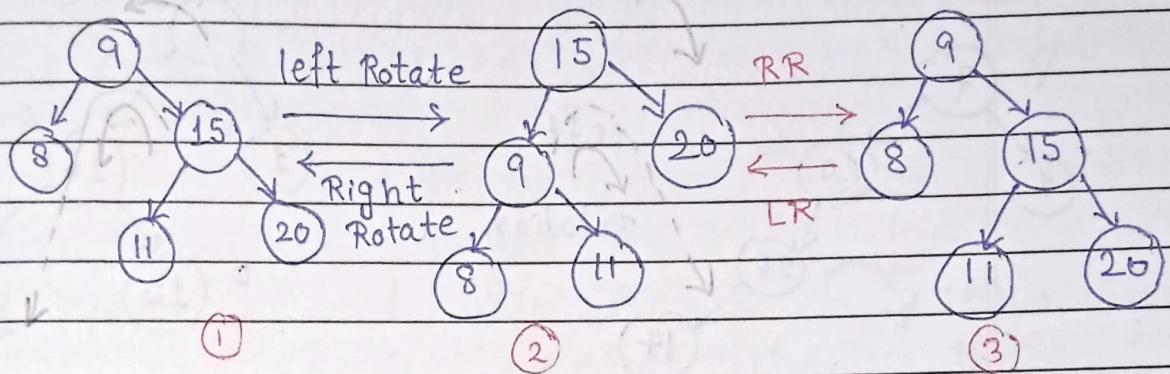
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Rotate Operations:

We can perform rotate operations to balance a binary search tree such that the newly formed tree satisfies all the properties of a BST. Following are two basic rotate operations:

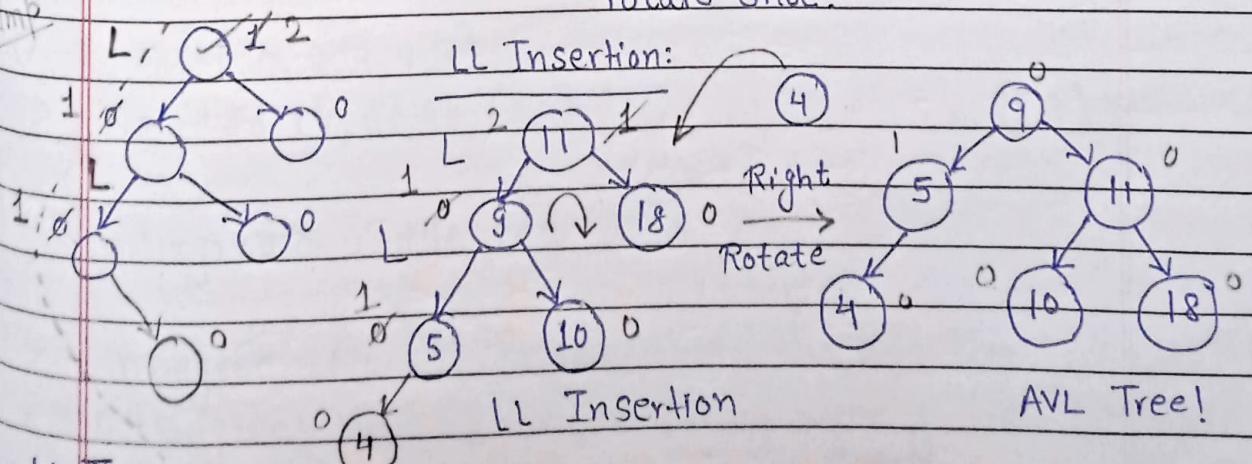
1. Left Rotate w.r.t a node → Node is moved towards the left.
2. Right Rotate w.r.t a node → Node is moved towards the right.



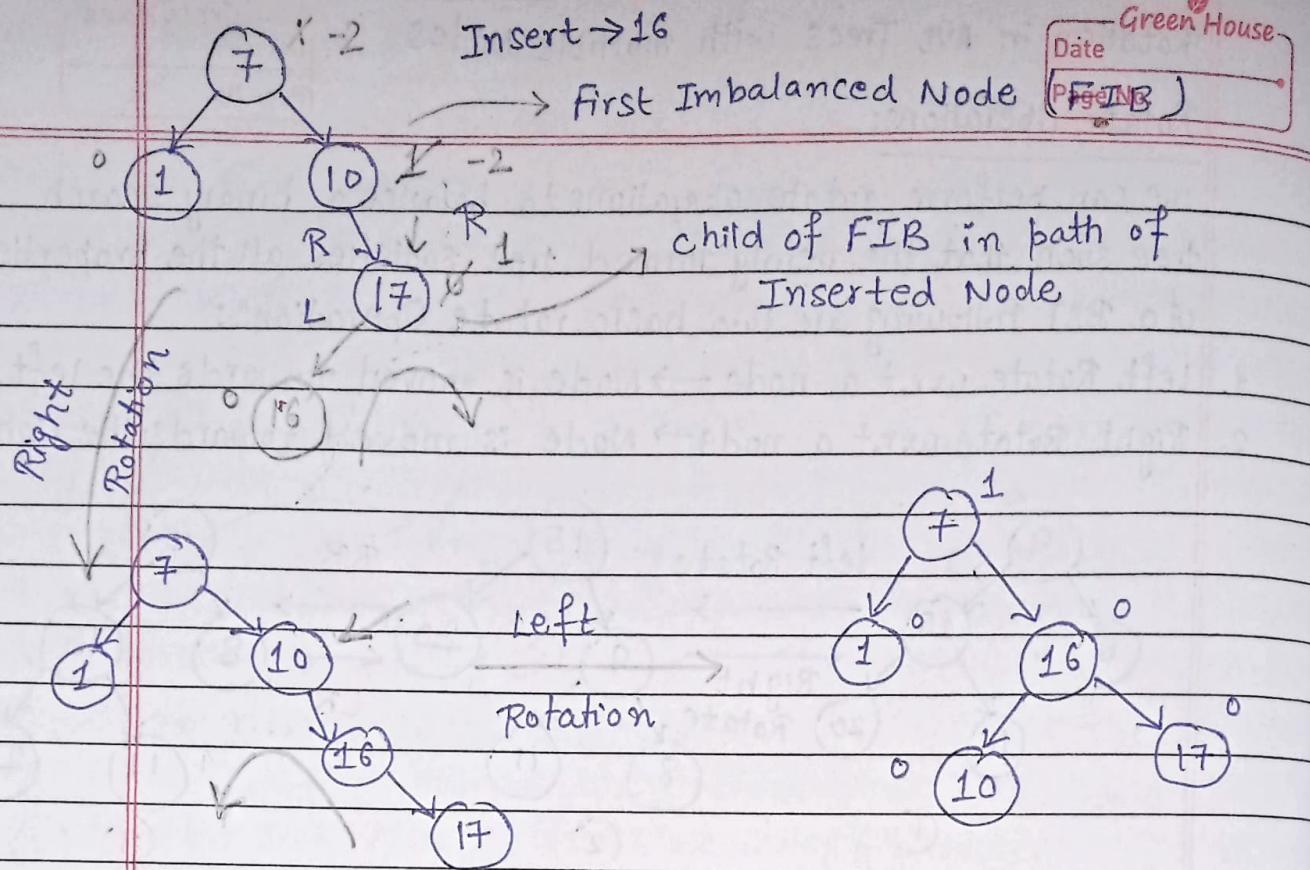
Balancing a AVL tree after Insertion:

In order to balance an AVL tree after insertion, we can follow the following rules:

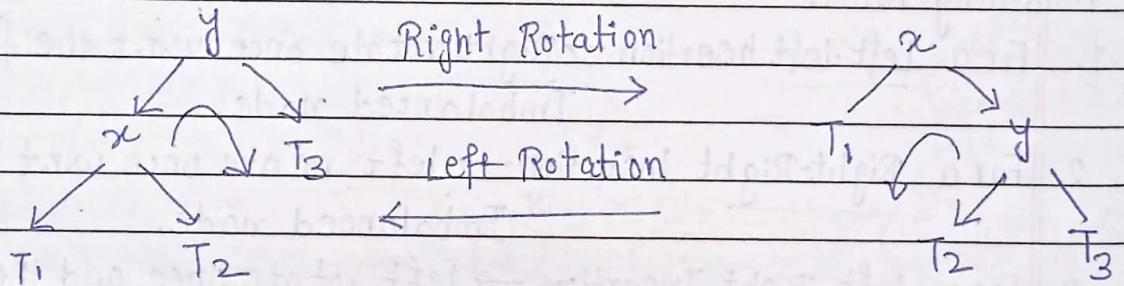
1. For a Left-Left Insertion → Right rotate once w.r.t the first Imbalanced node.
2. For a Right-Right Insertion → Left rotate once w.r.t the first Imbalanced node.
3. For a Left-Right Insertion → Left rotate once and then Right rotate Once.
4. For a Right-Left Insertion → Right rotate once and then left rotate once.



Right-Left Insertion



Lec-82. C Code for AVL Tree Insertion & Rotation (LL, RR, LR and RL Rotation) :-



Introduction to Graphs (Graph Data Structure)

what is a Graph?

- Array / Linked-lists & Stacks → Linear Data Structure.
- BST & AVL Trees → Non-linear Data Structure.

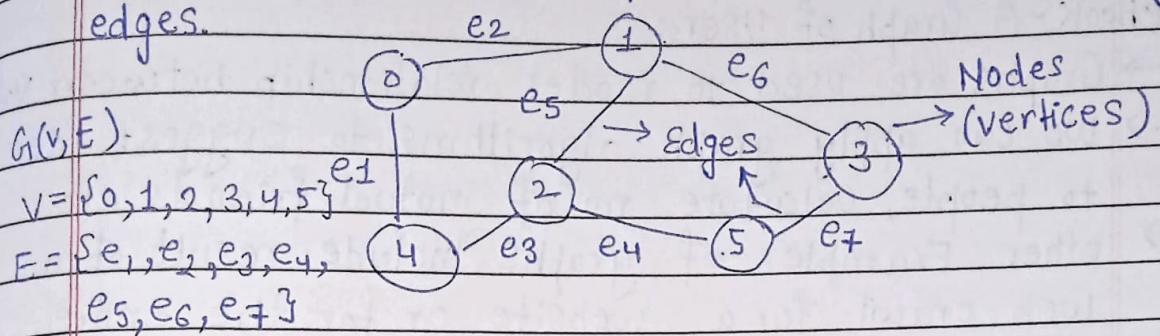
Green House

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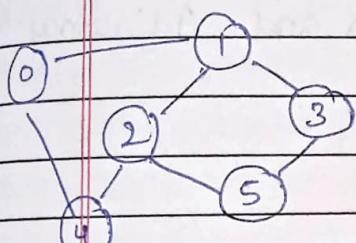
Graph is an Example of Non-linear DS.

A Graph is a Collection of Nodes Connected through edges.



Formal Definition of a Graph:

→ A Graph $G = (V, E)$ is a collection of vertices and edges connecting these vertices.

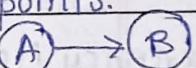


→ Used to model Paths in a city, Social Networks, website backlinks, Internal employee network etc.

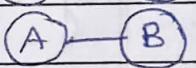
→ A vertex or a Node is one of fundamental unit/entity of which graphs are formed.

→ A Edge is uniquely defined by it's 2 endpoints.

→ Directed edges - one way Connection



→ Undirected edges - Two way Connection



→ Directed Graph - All directed Edges.

→ Undirected Graph - All Undirected Edges.

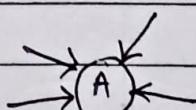
Here,

$$V = \{1, 0, 2, 3, 4, 5\}$$

$$E = \{\{0, 1\}, \{1, 4\}, \{0, 4\}, \{2, 4\}, \{1, 2\}, \{1, 3\}, \{2, 5\}, \{5, 3\}\}$$

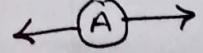
InDegree & Outdegree of a Node:

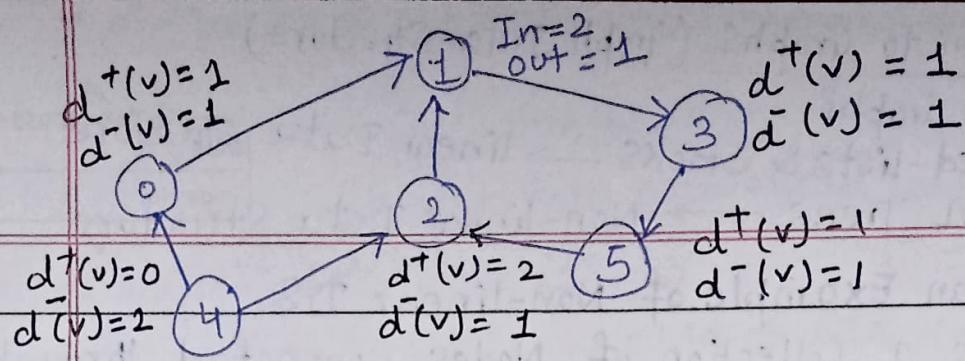
entering



→ InDegree = No. of Edges going out of the Node.

→ OutDegree = No. of Edges going coming into the Node.





Facebook - A Graph of Users:

- Graphs are used to model relationship between Nodes
- We can apply graph algorithms to suggest friends to people, calculate no of mutual friends etc.
- other Example of graphs include result of a web crawl for a website or for the entire world wide web, city routes etc.

Lec-84

Representation of Graph - Adjacency Link and Adjacency Matrix and other Representations:-

- Graphs = Nodes & Edges
- Used to model real world problems like managing a Social network, website links etc.
- Helps to Solve problems like is there a path between Delhi and California by road. If yes which one is shortest?

Adjacency Matrix:-