

NLP CS5803 IITH Notes

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1 Introduction

The starting question is how to make computers understand human language. We need to find smart ways of representing the language.

2 Input Representation

Consider text modality, where the input is a document, it consists of words(also called tokens). The document is represented by the set of words/tokens it contains.

Lets see some methods for text representation

2.1 TF-IDF scheme

2.1.1 Formulation

The TF-IDF (Term Frequency-Inverse Document Frequency) scheme is a popular technique used to represent the importance of words in a document corpus.

It combines two factors: term frequency (TF) and inverse document frequency (IDF).

TF measures the frequency of a term in a document. It is calculated by counting the number of occurrences of a term in a document as a raw or by taking a log of it.

$$\text{tf}(t, d) = \begin{cases} 0 & \text{if } c(t, d) = 0 \\ 1 + \log(c(t, d)) & \text{if } c(t, d) \neq 0 \end{cases}$$

where $c(t, d)$ is the count of term t in document d .

IDF de-emphasizes the frequent words across the corpus (all documents combined is usually called corpus) and emphasizes the on words differentiating the documents.

$$\text{idf}(t) = \log_{10} \left(\frac{N}{\text{df}_t} \right)$$

where N is the total number of documents in the corpus and df_t is the number of documents containing the term t .

The TF-IDF score for a term in a document is obtained by multiplying its TF value with its IDF value. Mathematically, it can be represented as:

$$\text{tf-idf}(t, d) = \text{tf}(t, d) \times \text{idf}(t)$$

Now we get a table with TF-IDF values.

This way, each document is represented by a vector from the column of the table and each word is presented by a vector from the row of the table.

2.1.2 Limitation

- Words are considered at their lexical appearance
- Synonymy is not considered
- Polysemy(word with multiple meanings) is not considered

- long sparse vectors
- context is not considered

2.2 SVD - text representation

2.2.1 Formulation

Consider the term-document matrix and apply SVD to it to do dimensionality reduction, so that we can get dense representations.

Let A be the term-document matrix (matrix with freq. of words in docs), where each row represents a term and each column represents a document, by SVD we have

$$A = U\Sigma V^T$$

U , V are orthonormal matrices and Σ is a diagonal matrix of singular values of A in decreasing order.

By keep only the first k singular values, we have

$$A_k = U_k \Sigma_k V_k^T$$

The k here is much smaller than the original dimension of A . Terms can be represented by the rows of U_k and documents can be represented by the columns of V_k .

2.3 LDA - text representation(incomplete)

2.3.1 Formulation

LDA (Latent Dirichlet Allocation) is a text representation based on **topics**. It assumes that each document is a mixture of topics which are latent or unknown. Words in a document depend on topics of the document. LDA is a mechanism to identify the topics and connect words with topics. The generative process is as follows:

- For each document, draw a distribution over topics
- For each word in the document, draw a topic from the distribution over topics and then draw a word from the distribution over words for that topic.

The parameters of the model are the topic distributions for each document, the word distributions for each topic, and the topic distribution over the entire corpus.

The model is trained by maximizing the likelihood of the observed documents. The topic distributions for each document and the word distributions for each topic are learned from the data.

The learned topic distributions for each document can be used to represent the documents and the learned word distributions for each topic can be used to represent the topics.

2.3.2 Mathematical Formulation

The mathematical formulation of LDA is as follows:

For each document d in the corpus D :

1. Choose $N \sim \text{Poisson}(\xi)$.
2. Choose $\theta \sim \text{Dir}(\alpha)$.

3. For each of the N words w :
 - a. Choose a topic $z \sim \text{Multinomial}(\theta)$.
 - b. Choose a word w from $p(w|z, \beta)$, a multinomial probability conditioned on the topic z .

Here, ξ is the parameter of the Poisson distribution used to choose the number of words in a document, α is the parameter of the Dirichlet distribution used to generate the per-document topic distributions, and β is the parameter of the multinomial distribution used to generate the per-topic word distribution.

2.4 Word2Vec

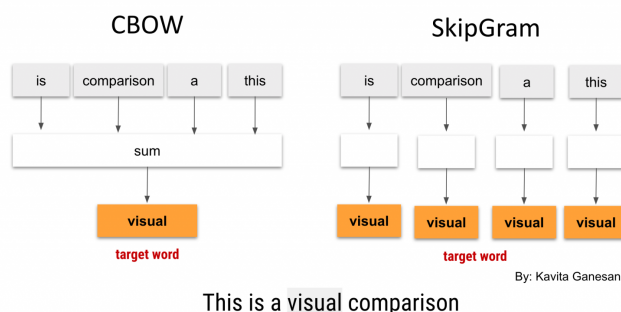
Context of words: Context of a word is the set of words that surround it. For example, in the sentence "I love to eat pizza", the context of the word "love" is {"I", "to", "eat"}.

Note: Word2Vec models rely on "distributional symmetry" hypothesis, i.e. similar words have similar contexts.

Word2Vec is a popular technique used to represent words in a continuous vector space.

It is based on the idea that words that appear in similar contexts are semantically similar.

Word2Vec has two models: SkipGram and Continuous Bag of Words (CBOW).



2.4.1 Continuous Bag of Words (CBOW)

This model predicts the middle word based on the context words.

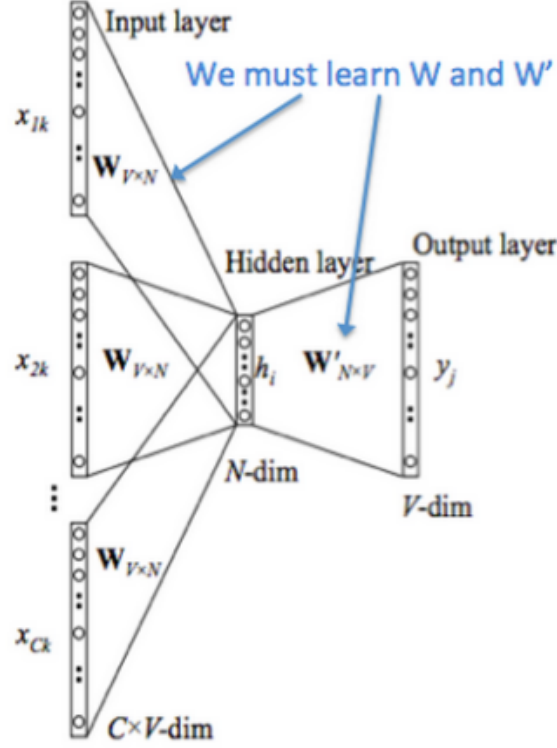
We create two matrices, $V \in \mathbf{R}^{n \times |V|}$ and $U \in \mathbf{R}^{|V| \times n}$. Where n is an arbitrary size which defines the size of our embedding space.

w_i : Word i from vocabulary V
 $V \in \mathbf{R}^{n \times |V|}$: Input word matrix
 v_i : i_{th} column of V , the input vector representation of word w_i
 $U \in \mathbf{R}^{n \times |V|}$: Output word matrix
 u_i : i_{th} row of U , the output vector representation of word w_i

Note that we do in fact learn two vectors for every word w_i (i.e. input word vector v_i and output word vector u_i). also, the below steps are done using a single hidden layer neural network.

We breakdown the way this model works in these steps:

1. We generate the one hot word vectors $(x_{c-m}, \dots, x_{c-1}, x_{c+1}, \dots, x_{c+m})$ for the input context of size m .



2. We get our embedded word vectors for the context
 $(v_{c-m} = Vx_{c-m}, v_{c-m+1} = Vx_{c-m+1}, \dots, v_{c+m} = Vx_{c+m})$.
3. Average these vectors to get $\hat{v} = \frac{v_{c-m} + v_{c-m+1} + \dots + v_{c+m}}{2m}$.
4. Generate a score vector $z = U\hat{v}$.
5. Turn the scores into probabilities $\hat{y} = \text{softmax}(z)$.
6. We desire our probabilities generated, \hat{y} , to match the true probabilities, y , which also happens to be the one hot vector of the actual word.

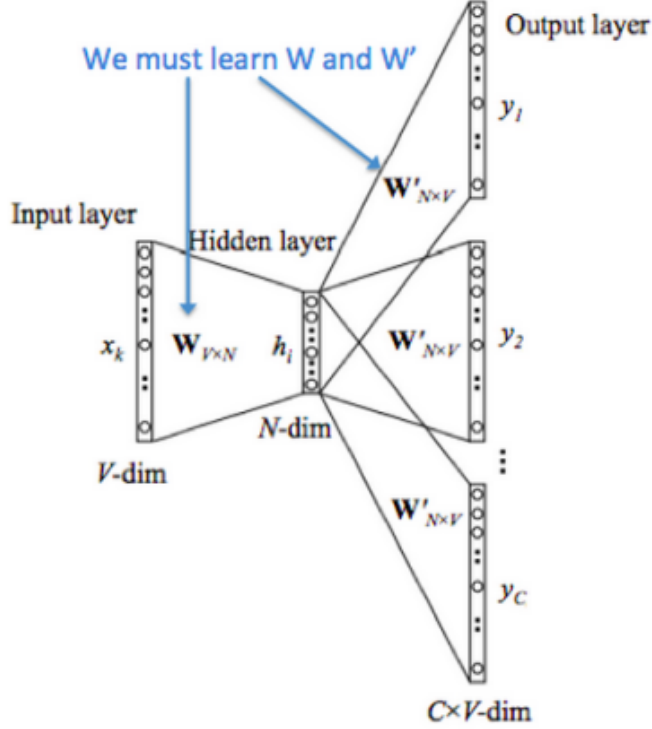
Since, we have a neural network we need objective function. we use a popular choice of distance/loss measure, cross entropy $H(\hat{y}, y)$.

$$\begin{aligned}
 H(\hat{y}, y) &= - \sum_{i=1}^{|V|} y_i \log(\hat{y}_i) \\
 &= - \log(\hat{y}_j) \\
 &= - \log \frac{\exp(z_c)}{\sum_{j=1}^{|V|} \exp(z_j)} \\
 &= - \log \frac{\exp(u_c^T \hat{v})}{\sum_{j=1}^{|V|} \exp(u_j^T \hat{v})} \\
 &= - \log P(u_c | \hat{v}) \\
 &= - \log P(w_c | w_{c-m}, \dots, w_{c-1}, w_{c+1}, \dots, w_{c+m})
 \end{aligned}$$

Hence, it perfectly makes sense to choose the cross entropy as our loss function.

2.4.2 SkipGram

Predicting surrounding context words given the middle word.



The setup is largely the same but we essentially swap our x and y i.e. x in the CBOW are now y and vice-versa. The input one hot vector (center word) we will represent with an x (since there is only one). And the output vectors as $y^{(j)}$. We define V and U the same as in CBOW.

We breakdown the way this model works in these 6 steps:

1. We generate the one hot input vector x .
2. We get our embedded word vectors for the input vector $v_c = Vx$.
3. Generate the logits of size V using $u = Uv_c$.
4. Turn the logits into probabilities, $\hat{y} = \text{softmax}(u)$.
5. We desire to minimize the negative log likelihood of probabilities.

As in CBOW, we need to generate an objective function for us to evaluate the model.

$$\begin{aligned}
J &= -\log P(w_{c-m}, \dots, w_{c-1}, w_{c+1}, \dots, w_{c+m} | w_c) \\
&= -\log \prod_{j=0, j \neq m}^{2m} P(w_{c-m+j} | w_c) \\
&= -\log \prod_{j=0, j \neq m}^{2m} P(u_{c-m+j} | v_c) \\
&= -\log \prod_{j=0, j \neq m}^{2m} \frac{\exp(u_{c-m+j}^T v_c)}{\sum_{k=1}^{|V|} \exp(u_k^T v_c)} \\
&= - \sum_{j=0, j \neq m}^{2m} u_{c-m+j}^T v_c + 2m \log \sum_{k=1}^{|V|} \exp(u_k^T v_c)
\end{aligned}$$

Similar to CBOW this can also be written as error between y^i and \hat{y}^i .

2.4.3 Negative Sampling

Note that the summation over $|V|$ is computationally huge! Any update we do or evaluation of the objective function would take $O(|V|)$ time which if we recall is in the millions. This is where negative sampling comes to the rescue. The idea is to sample a small number of negative examples (words that are not in the context of the center word).

While negative sampling is based on the Skip-Gram model, it is in fact optimizing a different objective.

Consider a pair (w, c) of word and context word. Let's denote by $P(D = 1 | w, c)$ the probability that (w, c) came from the corpus data. Correspondingly, $P(D = 0 | w, c)$ will be the probability that (w, c) did not come from the corpus data. First, let's model $P(D = 1 | w, c)$ with the sigmoid function:

$$P(D = 1 | w, c, \theta) = \frac{1}{1 + e^{-v_c^T v_w}}$$

Now, we build a new objective function that tries to maximize the probability of a word and context being in the corpus data if it indeed is, and maximize the probability of a word and context not being in the corpus data if it indeed is not. We take a simple maximum likelihood approach of these two probabilities. (Here we take θ to be the parameters of the model, and in our case it is V and U .)

$$\begin{aligned}
\theta &= \arg \max_{\theta} \prod_{(w,c) \in D} P(D=1|w, c, \theta) \prod_{(w,c) \in \tilde{D}} P(D=0|w, c, \theta) \\
&= \arg \max_{\theta} \prod_{(w,c) \in D} P(D=1|w, c, \theta) \prod_{(w,c) \in \tilde{D}} (1 - P(D=1|w, c, \theta)) \\
&= \arg \max_{\theta} \sum_{(w,c) \in D} \log P(D=1|w, c, \theta) + \sum_{(w,c) \in \tilde{D}} \log(1 - P(D=1|w, c, \theta)) \\
&= \arg \max_{\theta} \sum_{(w,c) \in D} \log \frac{1}{1 + \exp(-u_w^T v_c)} + \sum_{(w,c) \in \tilde{D}} \log \left(1 - \frac{1}{1 + \exp(-u_w^T v_c)} \right) \\
&= \arg \max_{\theta} \sum_{(w,c) \in D} \log \frac{1}{1 + \exp(-u_w^T v_c)} + \sum_{(w,c) \in \tilde{D}} \log \left(\frac{1}{1 + \exp(u_w^T v_c)} \right)
\end{aligned}$$

This changes as follows by considering k negative samples for each positive.

$$\log \sigma(u_{c-m+j}^T v_c) + \sum_{k=1}^K \log \sigma(-\tilde{u}_k^T v_c)$$

In the above formulation, $\{\tilde{u}_k | k = 1, \dots, K\}$ are sampled from $P_n(w)$. the n value of 0.75 seems work well in practice, since it increase the high probability words less, while low probability words more.

2.5 GloVe

2.5.1 Formulation

Emphasizes on co-occurrence with context/ Probe word

Co-occurrence Matrix:

- X : word-word co-occurrence matrix
- X_{ij} : number of times word j occur in the context of word i
- $X_i = \sum_k X_{ik}$: the number of times any word k appears in the context of word i
- $P_{ij} = P(w_j|w_i) = \frac{X_{ij}}{X_i}$: the probability of j appearing in the context of word i

Focus on ratio of co-occurrence probabilities Given words w_i , w_j , and w_k , the ratio of their co-occurrence probabilities is:

$$F(w_i, w_j, \tilde{w}_k) = \frac{P_{ik}}{P_{jk}}$$

Word embeddings are linear in structures so, the natural way of defining F is using subtraction and multiplication

$$F((w_i - w_j)^T \tilde{w}_k) = \frac{P_{ik}}{P_{jk}}$$

, the distinction between a word and a context word is arbitrary and that we are free to exchange the two roles. To do so consistently, we must not only exchange $w \leftrightarrow \tilde{w}$ but also $X \leftrightarrow X^T$. Our final model should be invariant under this relabeling.

However, the symmetry can be restored in two steps.

First, we require that F be a homomorphism between the groups $(\mathbf{R}, +)$ and $(\mathbf{R}_{>0}, \times)$, i.e.,

$$F((w_i - w_j)^T \tilde{w}_k) = \frac{F(w_i^T \tilde{w}_k)}{F(w_j^T \tilde{w}_k)}$$

which implies $F(w_i^T \tilde{w}_k) = P_{ik} = \frac{X_{ik}}{X_i}$.

then $F = \exp$, or, $w_i^T \tilde{w}_k = \log(P_{ik}) = \log(X_{ik}) - \log(X_i)$

We can observe the equation becomes symmetric if not for $\log(X_i)$. This term is independent of k so it can be absorbed into a bias b_i for w_i . Finally, adding an additional bias \tilde{b}_k for \tilde{w}_k restores the symmetry,

$$w_i^T \tilde{w}_k + b_i + \tilde{b}_k = \log(X_{ik}).$$

Need to weigh all co-occurrences differently, as some words that happen to occur rarely or never.

Proposed a new weighted least squares regression model that addresses these problems.

$$J = \sum_{i,j=1}^V f(X_{ij})(w_i^T \tilde{w}_j + b_i + \tilde{b}_j - \log X_{ij})^2$$

2.5.2 Embeddings reflect Social bias

Embeddings Reflect Societal Bias

Extreme <i>she</i>	Extreme <i>he</i>	Gender stereotype <i>she-he</i> analogies		
1. homemaker	1. maestro	sewing-carpentry	registered nurse-physician	housewife-shopkeeper
2. nurse	2. skipper	nurse-surgeon	interior designer-architect	softball-baseball
3. receptionist	3. protege	blond-burly	feminism-conservatism	cosmetics-pharmaceuticals
4. librarian	4. philosopher	giggle-chuckle	vocalist-guitarist	petite-lanky
5. socialite	5. captain	sassy-snappy	diva-superstar	charming-affable
6. hairdresser	6. architect	volleyball-football	cupcakes-pizzas	lovely-brilliant
7. nanny	7. financier	Gender appropriate <i>she-he</i> analogies		
8. bookkeeper	8. warrior	queen-king	sister-brother	mother-father
9. stylist	9. broadcaster	waitress-waiter	ovarian cancer-prostate cancer	convent-monastery
10. housekeeper	10. magician			

$$S_{(a,b)}(x, y) = \cos(\vec{a} - \vec{b}, \vec{x} - \vec{y}) \text{ if } \|\vec{x} - \vec{y}\| \leq \delta, \quad 0 \text{ else}$$

2.5.3 Identifying and quantifying bias in embeddings

Aspect of bias is known: Say "Gender"

- Direct bias - this refers to the alignment of a word vector with the "gender" dimension explicitly extracted from gender-specific word pairs. If a gender-neutral word like "doctor" leans closer to "he" than "she" in the embedding space, it exhibits direct gender bias.

- We create a matrix with each row representing a word vector from the gendered pairs.
- We apply PCA to identify the most significant components capturing gender.
- We use the first principal component (or a combination of the first few) as the "gender" direction vector g .
- We calculate the cosine similarity between w_vec and g :

$$\text{direct_bias}(w_vec) = |\cos(w_vec, g)|$$

- Compute the gender component in elements from N gender neutral words:

$$\text{DirectBias} = \frac{1}{|N|} \sum_{w \in N} |\cos(w, g)|$$

- Indirect Bias and Perpendicular Component: Indirect bias arises from subtle associations in the training data. These associations might not directly align with the "gender" direction, but they can still influence the position of a word vector in the embedding space.

- Need to find the component to the perpendicular of the "gender" dimension
- Component of vector a along vector b :
 - * Scalar Component: $\text{comp}_b(a) = \frac{a \cdot b}{|b|}$
 - * Vector component: $\text{comp}_b(a) \cdot b$
- $w_g = (w \cdot g)g$, $w_{\perp} = w - w_g$
- IndirectBias $B(w, v) = \frac{w \cdot v - \frac{w_{\perp} \cdot v_{\perp}}{|w_{\perp}| \cdot |v_{\perp}|}}{w \cdot v}$

Modified objective of GloVe to minimize the bias

$$J = \sum_{i,j=1}^V f(X_{ij})(w_i^T \tilde{w}_j + b_i + \tilde{b}_j - \log X_{ij})^2 + \lambda \cos(w_i, g) + \gamma \cos(\tilde{w}_j, g),$$

2.6 FastText

Represent words by sum of its character n-grams

2.6.1 Formulation

As in skip-gram, model probability of a context word given a word Given word w and context word w_c ,

$$P(w_c|w) = P(u_c|v) = \frac{\exp(u_c^T v)}{\sum_{j=1}^{|V|} \exp(u_j^T v)}$$

Given a word w , let us denote by $G_w \subset \{1, \dots, G\}$ the set of n-grams appearing in w and itself. We associate a vector representation z_g to each n-gram g . In FastText the key is to compute feature of a word using its n-grams.

$$v = \sum_{g \in G} z_g$$

The objective function is the same as in Skip-gram.

2.7 Summary - GloVe, Word2Vec, FastText

- **Word2vec:**

- *Method:* skip-gram - predicts surrounding words and CBOW - predicts center word based on surrounding words.
- *Advantages:* Simple and efficient, good at capturing semantic relationships between words.
- *Disadvantages:* Doesn't handle rare words well, can be computationally expensive for large datasets.

- **GloVe:**

- *Method:* Analyzes word co-occurrence statistics from a corpus to build word embeddings.
- *Advantages:* Often performs well on analogy tasks, considers global word co-occurrence information.
- *Disadvantages:* May not capture fine-grained semantic relationships as effectively as word2vec, can be slower to train compared to word2vec.

- **FastText:**

- *Method:* Combines word n-grams (subword information) with word embeddings, allowing it to handle rare words and morphologically related words.
- *Advantages:* Handles out-of-vocabulary (OOV) words better than word2vec and GloVe, effective for languages with rich morphology.
- *Disadvantages:* Can be more complex to train compared to word2vec, might require more data for optimal performance.

Feature	Word2vec	GloVe	FastText
Captures Semantics	✓	✓	✓
Analogy Tasks	✓(skip-gram)	✓	✓
Handles OOV Words	×	×	✓
Morphology	×	×	✓
Efficiency	✓	×	✓
Training Speed	✓	×	×
Context Sensitivity	×	×	×
Syntax Awareness	×	×	×
Fairness	×	×	×
Reasoning	×	×	×

3 language Modelling and Smoothing