NLP CS5803 IITH Notes

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1 Introduction

The starting question is how to make computers understand human language. We need to find smart ways of representing the language.

2 Input Representation

Consider text modality, where the input is a document, it consists of words(also called tokens). The document is represented by the set of words/tokens it contains.

Lets see some methods for text representation

2.1 TF-IDF scheme

2.1.1 Formulation

The TF-IDF (Term Frequency-Inverse Document Frequency) scheme is a popular technique used to represent the importance of words in a document corpus.
It combines two factors: term frequency (TF) and inverse document frequency (IDF).
TF measures the frequency of a term in a document. It is calculated by counting the number of occurrences of a term in a document as a raw or by taking a log of it.

$$\text{tf}(t,d) = \begin{cases} 0 & \text{if } c(t,d) = 0 \\ 1 + \log(c(t,d)) & \text{if } c(t,d) \neq 0 \end{cases}$$

where $c(t,d)$ is the count of term t in document d.
IDF de-emphasizes the frequent words across the corpus (all documents combined is usually called corpus) and emphasizes the on words differentiating the documents.

$$\text{idf}(t) = \log_{10}\left(\frac{N}{\text{df}_t}\right)$$

where N is the total number of documents in the corpus and df_t is the number of documents containing the term t.
The TF-IDF score for a term in a document is obtained by multiplying its TF value with its IDF value. Mathematically, it can be represented as:

$$\text{tf-idf}(t,d) = \text{tf}(t,d) \times \text{idf}(t)$$

Now we get a table with TF-IDF values.

This way, each document is represented by a vector from the column of the table and each word is presented by a vector from the row of the table.

2.1.2 Limitation

- Words are considered at their lexical appearance
- Synonymy is not considered
- Polysemy(word with multiple meanings) is not considered

- long sparse vectors
- context is not considered

2.2 SVD - text representation

2.2.1 Formulation

Consider the term-document matrix and apply SVD to it to do dimensionality reduction, so that we can get dense representations.
Let A be the term-document matrix(matrix with freq. of words in docs), where each row represents a term and each column represents a document, by SVD we have

$$A = U\Sigma V^T$$

U, V are orthonormal matrices and Σ is a diagonal matrix of singular values of A in decreasing order.
By keep only the first k singular values, we have

$$A_k = U_k \Sigma_k V_k^T$$

The k here is much smaller than the original dimension of A. Terms can be represented by the rows of U_k and documents can be represented by the columns of V_k.

2.3 LDA - text representation

2.3.1 Formulation

LDA (Latent Dirichlet Allocation) is a text representation based on **topics**. It assumes that each document is a mixture of topics which are latent or unknown. Words in a document depend on topics of the document. LDA is a mechanism to identify the topics and connect words with topics The generative process is as follows:

- For each document, draw a distribution over topics
- For each word in the document, draw a topic from the distribution over topics and then draw a word from the distribution over words for that topic.

The parameters of the model are the topic distributions for each document, the word distributions for each topic, and the topic distribution over the entire corpus.
The model is trained by maximizing the likelihood of the observed documents. The topic distributions for each document and the word distributions for each topic are learned from the data.
The learned topic distributions for each document can be used to represent the documents and the learned word distributions for each topic can be used to represent the topics.

2.3.2 Mathematical Formulation

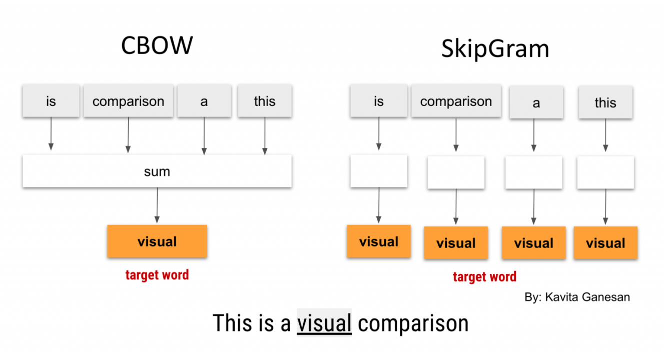
The mathematical formulation of LDA is as follows:
For each document d in the corpus D:
1. Choose $N \sim Poisson(\xi)$.
2. Choose $\theta \sim Dir(\alpha)$.

3. For each of the N words w:
a. Choose a topic $z \sim Multinomial(\theta)$.
b. Choose a word w from $p(w|z,\beta)$, a multinomial probability conditioned on the topic z.

Here, ξ is the parameter of the Poisson distribution used to choose the number of words in a document, α is the parameter of the Dirichlet distribution used to generate the per-document topic distributions, and β is the parameter of the multinomial distribution used to generate the per-topic word distribution.

2.4 Word2Vec

Context of words: Context of a word is the set of words that surround it. For example, in the sentence "I love to eat pizza", the context of the word "love" is {"I", "to", "eat"}.
Note: Word2Vec models rely on "distributional symmetry" hypothesis, i.e. similar words have similar contexts.
Word2Vec is a popular technique used to represent words in a continuous vector space.
It is based on the idea that words that appear in similar contexts are semantically similar.
Word2Vec has two models: SkipGram and Continuous Bag of Words (CBOW).



2.4.1 Continuous Bag of Words (CBOW)

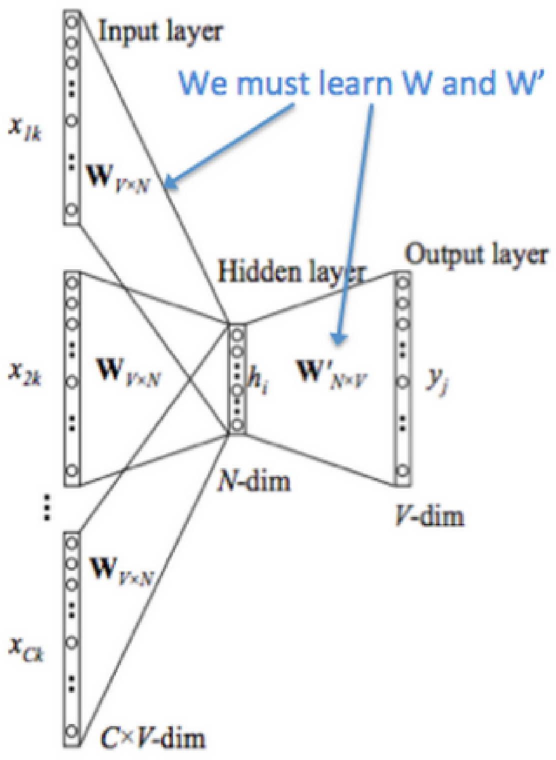
This model predicts the middle word based on the context words.

We create two matrices, $V \in \mathbf{R}^{n\times|V|}$ and $U \in \mathbf{R}^{|V|\times n}$. Where n is an arbitrary size which defines the size of our embedding space.

w_i: Word i from vocabulary V
$V \in \mathbf{R}^{n\times|V|}$: Input word matrix
v_i: i_{th} column of V, the input vector representation of word w_i
$U \in \mathbf{R}^{n\times|V|}$: Output word matrix
u_i: i_{th} row of U, the output vector representation of word w_i

Note that we do in fact learn two vectors for every word w_i (i.e. input word vector v_i and output word vector u_i). also, the below steps are done using a single hidden layer neural network.
We breakdown the way this model works in these steps:

1. We generate our one hot word vectors $(x_{c-m},\ldots,x_{c-1},x_{c+1},\ldots,x_{c+m})$ for the input context of size m.



2. We get our embedded word vectors for the context $(v_{c-m} = Vx_{c-m}, v_{c-m+1} = Vx_{c-m+1}, \ldots, v_{c+m} = Vx_{c+m})$.
3. Average these vectors to get $\hat{v} = \frac{v_{c-m}+v_{c-m+1}+\ldots+v_{c+m}}{2m}$.
4. Generate a score vector $z = U\hat{v}$.
5. Turn the scores into probabilities $\hat{y} = \text{softmax}(z)$.
6. We desire our probabilities generated, $\hat{y}$, to match the true probabilities, y, which also happens to be the one hot vector of the actual word.

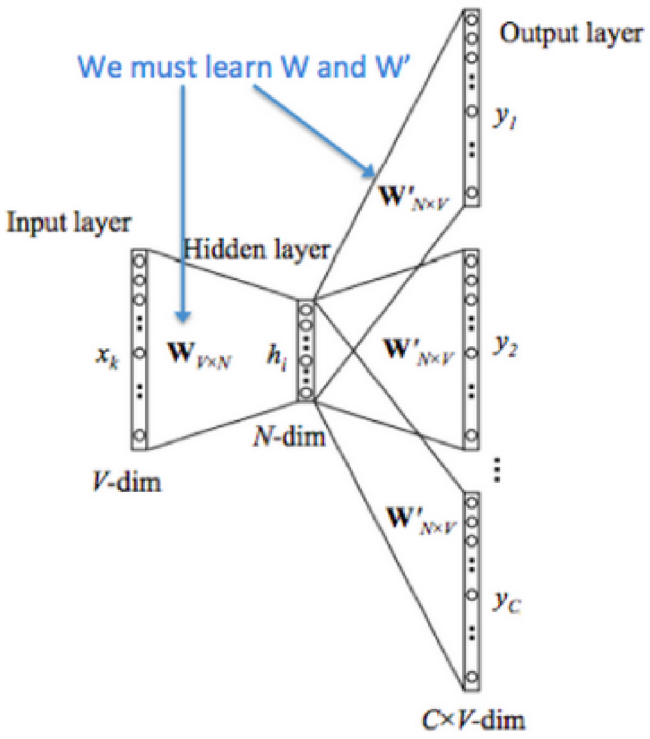
Since, we have a neural network we need objective function. we use a popular choice of distance/loss measure, cross entropy $H(\hat{y}, y)$.

$$\begin{aligned}
H(\hat{y}, y) &= -\sum_{i=1}^{|V|} y_i \log(\hat{y}_i) \\
&= -\log(\hat{y}_j) \\
&= -\log \frac{\exp(z_c)}{\sum_{j=1}^{|V|} \exp(z_j)} \\
&= -\log \frac{\exp(u_c^T \hat{v})}{\sum_{j=1}^{|V|} \exp(u_j^T \hat{v})} \\
&= -\log P(u_c|\hat{v}) \\
&= -\log P(w_c|w_{c-m}, \ldots, w_{c-1}, w_{c+1}, \ldots, w_{c+m})
\end{aligned}$$

Hence, it perfectly makes sense to choose the cross entropy as our loss function.

2.4.2 SkipGram

Predicting surrounding context words given the middle word.



The setup is largely the same but we essentially swap our x and y i.e. x in the CBOW are now y and vice-versa. The input one hot vector (center word) we will represent with an x (since there is only one). And the output vectors as $y^{(j)}$. We define V and U the same as in CBOW.

We breakdown the way this model works in these 6 steps:

1. We generate our one hot input vector x.
2. We get our embedded word vectors for the context $v_c = Vx$.
3. Since there is no averaging, just set $\hat{v} = v_c$.
4. Generate $2m$ score vectors, $u_{c-m}, \dots, u_{c-1}, u_{c+1}, \dots, u_{c+m}$ using $u = Uv_c$.
5. Turn each of the scores into probabilities, $\hat{y} = \text{softmax}(u)$.
6. We desire our probability vector generated to match the true probabilities which is $y_{c-m}, \dots, y_{c-1}, y_{c+1}, \dots, y_{c+m}$, the one hot vectors of the actual output.

As in CBOW, we need to generate an objective function for us to evaluate the model. A key difference here is that we invoke a Naive Bayes assumption to break out the probabilities

$$\begin{aligned} J &= -\log P(w_{c-m}, \dots, w_{c-1}, w_{c+1}, \dots, w_{c+m} | w_c) \\ &= -\log \prod_{j=0, j\neq m}^{2m} P(w_{c-m+j} | w_c) \\ &= -\log \prod_{j=0, j\neq m}^{2m} P(u_{c-m+j} | v_c) \\ &= -\log \prod_{j=0, j\neq m}^{2m} \frac{\exp(u_{c-m+j}^T v_c)}{\sum_{k=1}^{|V|} \exp(u_k^T v_c)} \\ &= -\sum_{j=0, j\neq m}^{2m} u_{c-m+j}^T v_c + 2m \log \sum_{k=1}^{|V|} \exp(u_k^T v_c) \end{aligned}$$

Similar to CBOW this can also be written as error between y^i and $\hat{y^i}$.

2.4.3 Negative Sampling

Note that the summation over |V| is computationally huge! Any update we do or evaluation of the objective function would take O(|V|) time which if we recall is in the millions. This is where negative sampling comes to the rescue. The idea is to sample a small number of negative examples (words that are not in the context of the center word).

While negative sampling is based on the Skip-Gram model, it is in fact optimizing a different objective.

Consider a pair (w, c) of word and context word. Let's denote by $P(D = 1|w, c)$ the probability that (w, c) came from the corpus data. Correspondingly, $P(D = 0|w, c)$ will be the probability that (w, c) did not come from the corpus data. First, let's model $P(D = 1|w, c)$ with the sigmoid function:

$$P(D = 1|w, c, \theta) = \frac{1}{1 + e^{-v_c^T v_w}}$$

Now, we build a new objective function that tries to maximize the probability of a word and context being in the corpus data if it indeed is, and maximize the probability of a word and context not being in the corpus data if it indeed is not. We take a simple maximum likelihood approach of these two probabilities. (Here we take θ to be the parameters of the model, and in our case it is V and U.)

$$
\begin{aligned}
\theta &= \arg\max_{\theta} \prod_{(w,c)\in D} P(D=1|w,c,\theta) \prod_{(w,c)\in \tilde{D}} P(D=0|w,c,\theta) \\
&= \arg\max_{\theta} \prod_{(w,c)\in D} P(D=1|w,c,\theta) \prod_{(w,c)\in \tilde{D}} (1-P(D=1|w,c,\theta)) \\
&= \arg\max_{\theta} \sum_{(w,c)\in D} \log P(D=1|w,c,\theta) + \sum_{(w,c)\in \tilde{D}} \log(1-P(D=1|w,c,\theta)) \\
&= \arg\max_{\theta} \sum_{(w,c)\in D} \log \frac{1}{1+\exp(-u_w^T v_c)} + \sum_{(w,c)\in \tilde{D}} \log\left(1-\frac{1}{1+\exp(-u_w^T v_c)}\right) \\
&= \arg\max_{\theta} \sum_{(w,c)\in D} \log \frac{1}{1+\exp(-u_w^T v_c)} + \sum_{(w,c)\in \tilde{D}} \log\left(\frac{1}{1+\exp(u_w^T v_c)}\right)
\end{aligned}
$$

This changes as follows by considering k negative samples for each postive.

$$
\log\sigma(u_{c-m+j}^T v_c) + \sum_{k=1}^{K} \log\sigma(-\tilde{u}_k^T v_c)
$$

In the above formulation, $\{\tilde{u}_k | k=1,\ldots,K\}$ are sampled from $P_n(w)$. the n value of 0.75 seems work well in practice, since it increase the high probability words less, while low probability words more.