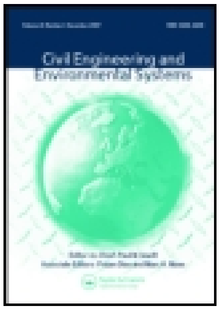

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A technical note on the inclusion of pressure dependent demand and leakage terms in water supply network models

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A technical note on the inclusion of pressure dependent demand and leakage terms in water supply network models

George Germanopoulos

This note suggests a technique for including pressure dependent demand and leakage terms in simulation models for water distribution systems. Empirical functions are used to relate consumer outflows and leakage losses to the network pressures and the inclusion of these functions in the mathematical formulation of the network analysis problem is described. An application to an existing distribution system is presented where it is shown that the extended network model proposed leads to more realistic simulation results when the network pressures are too low to provide specified consumer demands, or high enough to cause significant leakage losses. It is also found that the computational requirements of network analysis and simulation are not significantly affected by the inclusion of the additional terms.

Keywords: mathematical models, network analysis, simulation, water distribution

A water distribution system can be regarded as consisting of a number of sources supplying water to various points of time varying consumer demand through a network of pipes, valves, pumps and elevated storage reservoirs. The mathematical model of such a network comprises links interconnected at nodes. A consumption (i.e. the rate at which water is withdrawn from the network or supplied to it) and a hydraulic head are associated with each node. Each link is characterized by a nonlinear head-flow relationship relating the flow through the link to the head difference across it. The network is considered solved when the heads at all nodes and the flows in all the interconnecting hydraulic elements are known. A static network analysis gives an instantaneous solution of the network for given operating conditions. An extended period simulation consists of interconnected static solutions and gives the profile of the network heads and flows over a given period (usually 24 to 48 hours) under changing operating conditions.

Network simulation models developed to date assume that nodal consumptions are fixed and are provided irrespective of network pressures. Furthermore, they assume that no losses due to leakage occur in the network modelled, except for losses implicitly included in the fixed nodal consumptions. The motivation for this study has been the realization that the above assumptions are not valid for certain applications of these models.

It is clear that the amount of water flowing into consumption depends on the corresponding network pressures. If these fall below a minimum required level, (e.g. due to a network breakdown) the flow provided to consumers will be significantly reduced. On the other hand, if the network pressures keep rising the consumer outflows will not necessarily follow that increase as there is a limit to the total amount of water the consumers require at any given time. The assumption that nodal consumptions are fixed irrespective of the network pressures is therefore valid when the distribution system is simulated under normal conditions and the pressures can be expected to be adequate. If, however, the operation of the system is simulated under failure conditions, the relationship between pressure and supplied demand will have to be taken into account if the simulation results are to be realistic. Furthermore, such a model would permit direct analysis and assessment of pressure zoning and leakage control measures.

Water losses from a network due to leakage from mains and connections (as well as service reservoirs) may vary from 5% to 55% of the total supply[1] and can therefore have an important effect on the operation of the system. In a given network, leakage losses generally increase for higher pressures. A network simulation model taking into account the relationship between pressure and leakage flow can therefore be expected to provide more realistic results than if no leakage is assumed.

This note suggests a technique for incorporating pressure dependent demand terms and pressure dependent leakage terms in network analysis and simulation models. The application of the extended models to an existing water distribution system under varying operating conditions is also presented and the effect of the additional terms is assessed.

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Solution of a hydraulic network

The describing equations of the network can be formulated according to the following physical laws, each of which fully describes the network's operation:

(i) The algebraic sum of all flows at a node is zero, that is mass balance is preserved at each node
(ii) The algebraic sum of all hydraulic head drops around a closed loop in the network is zero

Irrespective of whether a node or loop formulation is used, the problem is one of solving the resulting system of simultaneous nonlinear equations for the unknown network flows and nodal heads. A static network solution is thus obtained. Different methods have been proposed for static network solutions (Hardy Cross, Newton–Raphson, linear theory) and various computer programs have been developed[2-12].

A brief description of the formulation of the problem in terms of the network's nodal equations and its solution using the Newton–Raphson procedure is now given. The formulation given at this stage assumes that consumer demands are fixed and that no leakage is taking place from the network pipes.

Continuity of flow at each network node can be written as:

$$\sum_{j=1}^{n} Q_{ij}+C_i=0 \qquad i=1,2,\ldots,n \tag{1}$$

in which C_i=consumption at node i; n=number of nodes; and Q_{ij}=flow from node i to node j. Any flow leaving a node in the above equation is taken to be positive. Using the Hazen–Williams equation for flow in a pipe[1], Eq (1) becomes:

$$\sum_{j=1}^{n} K_{ij}|H_i-H_j|^{-0.46}(H_i-H_j)+C_i=0$$
$$i=1,2,\ldots,n \tag{2}$$

in which H_i=piezometric head at node i; and K_{ij}= conveyance of the pipe connecting nodes i and j. Network elements other than pipes can be included in Eq (2) in a similar way[6,8,9]. Eq (2) can be written in vectorial form as:

$$\boldsymbol{f}(\boldsymbol{H},\boldsymbol{K},\boldsymbol{C})=\boldsymbol{0} \tag{3}$$

in which $\boldsymbol{H}$, $\boldsymbol{K}$, and $\boldsymbol{C}$ are the vectors of nodal heads, pipe conveyances and nodal consumptions respectively. Since there are n simultaneous equations one can solve for n unknowns, which may be heads at non-storage nodes, flows, or pipe conveyances, as well as valve and pump settings.

Let the vector of unknowns be X. The successive approximations in the Newton–Raphson procedure are given by:

$$X^{k+1}=X^k+f'(X^k)^{-1}f(X)^k \tag{4}$$

where $f'(X^k)$ is the Jacobian matrix of the set of equations in (3) evaluated at X^k. The criterion for convergence at each iteration is that the value of all elements of $f(X^k)$ must be less than a specified tolerance which represents the maximum unbalanced flow allowed at a node.

Modifications in the above procedure have been developed to improve its convergence when the assumed initial conditions are not close to the final solutions[5]. Furthermore, the system of linear equations in Eq (4) is sparse. This sparsity can be exploited to reduce the computer time and memory requirements for the solution[10].

Extended period simulation

An extended period simulation of a water distribution system gives the temporal variation of network pressure heads, pipe flows and reservoir levels for that period. The basic technique consists of a series of static solutions for pre-specified intervals of time using a schedule of pump and valve settings and consumer demand variations. Updated reservoir levels are obtained between each static solution by an integration scheme describing the dynamics of reservoir depletion. Typical simulation periods are 24–48 hours, with intervals of 1/2–2 hours between successive static solutions, depending on the degree of detail required. The problem of extended period simulation has not been documented as extensively as that of static analysis[13-15].

Inclusion of pressure dependent demand and leakage terms

In the network model developed, the pressure–consumption relationship for a given node is expressed as:

$$C_i=C_i^*(1-a_i\,\mathrm{e}^{-b_iP_i/P_i^*}) \tag{5}$$

where P_i=pressure at node i; C_i=the consumer outflow at node i; C_i^*=the nominal consumer demand; and a_i, b_i, P_i^*=constants for the particular node. C_i^* is the outflow normally provided to consumers assuming that the pressures in the system are adequate. P_i^* corresponds to the nodal pressure at which a given proportion of C_i^* is known to be provided. A graphical representation of the above relationship is given in Fig 1.

The exact form of the pressure–consumption relationship for each network node will depend on the hydraulic configuration between the node and the consumers downstream. For example, the nodal outflows may reach the consumers directly or via a service reservoir, and through pipelines of different possible lengths and diameters which may or may not include pressure reducing valves, flow control valves etc. Eq (5) does not take this explicitly into account. It displays the basic characteristics that are expected in a pressure–consumption relation, i.e. a fall in nodal outflow for pressures below a certain limit as well as a levelling out for higher pressures corresponding to the maximum flow that the consumers are likely to require. Field measurements

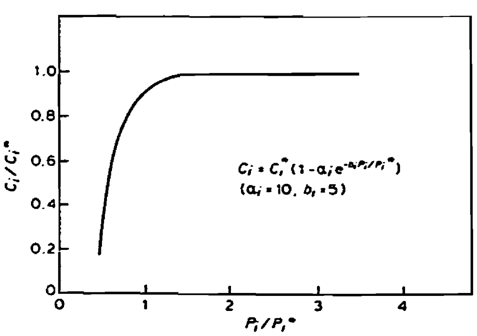


Fig 1 Nodal pressure-consumption relationship

and information from the distribution system operators can be used to determine the values of a_i, b_i, and P_i^* for each node.

Experimental studies have confirmed that leakage losses from a distribution system increase with increasing pressures, with the effect being greater at higher pressures[16]. The network model incorporates leakage losses by using the empirical relationship:

$$V_{ij}=c_l(L_{ij}P_{ij}^{av})^{1.18} \tag{6}$$

where V_{ij} = leakage flow rate from the pipe connecting nodes i and j, c_l = a constant depending on the network, L_{ij} = pipe length, P_{ij}^{av} = average pressure along the pipe. In the proposed model, P_{ij}^{av} is approximated by the mean of the nodal pressures at the two ends of each pipe, i.e.:

$$P_{ij}^{av}=(P_i+P_j)/2 \tag{7}$$

where P_i = pressure at node i. Furthermore, the leakage flow rate V_{ij} given in Eq (6) is distributed equally in the form of additional nodal outflows at each end of the pipe. The total outflow for each network node will therefore be given, from (5) and (6), as:

$$Q_i^{out}=C_i^*(1-a_i\,e^{-b_iP_i/P_i^*})+\frac{1}{2}\sum_{j=1}^{n}c_l(L_{ij}P_{ij}^{av})^{1.18} \tag{8}$$

The set of nodal mass balance equations is therefore written as:

$$\sum_{j=1}^{n}Q_{ij}+Q_i^{out}=0 \qquad i=1,2,\ldots,n \tag{9}$$

or, analytically:

$$\sum_{j=1}^{n}K_{ij}|H_i-H_j|^{-0.46}(H_i-H_j)+C_i^*(1-a_i e^{-b_iP_i/P_i^*})$$

$$+\frac{1}{2}\sum_{j=1}^{n}C_l(L_{ij}P_{ij}^{av})^{1.18} \qquad i=1,2,\ldots,n \tag{10}$$

The pressure terms P_i and P_{ij}^{av} in Eq (10) can be expressed directly in terms of the network piezometric heads and the corresponding known ground levels. The terms a_i, b_i, P_i^*, c_l, are constants for the given network. It is therefore seen that the inclusion of additional pressure dependent demand and pressure dependent leakage terms in the simulation model does not introduce any new equations or unknowns in the formulation of the network analysis problem. The structure of the Jacobian matrix in Eq (4) will therefore be the same after the inclusion of the additional terms and the same sparse matrix programming techniques can be used to increase the efficiency of the Newton–Raphson solution.

Application

The effect of the inclusion of pressure dependent demand and leakage terms in the network simulation model was examined by using data on an existing water distribution system in the Avon Division of the Severn-Trent Water Authority. The simulation program developed was run with pressure dependent demand and leakage terms, with pressure dependent demand terms only, and without pressure dependent demand or leakage terms, and the simulation results were compared. The above simulations were carried out for normal operating conditions and for conditions arising from a network failure event.

The distribution system considered covers an area of approximately 270 km^2 and provides an average total daily demand of 100 megalitres per day (Mld). A schematic representation of the distribution system as modelled is given in Fig 2. Only trunk mains and major pipelines are included in the model. Details of the system's service reservoirs are given in Table 1. It is seen that the distribution system includes a system of trunk mains (Trunk Mains TM1, TM2, TM3, TM4) connected to a major aqueduct which delivers water pumped over a long distance from Water Treatment Works TW1 to Service Reservoir SR4; the aqueduct also supplies other areas en route. Service reservoir SR4 serves a large urban area. Water is fed into the trunk main system from the aqueduct via trunk mains TM1 and TM3 and from water treatment works TW2. The normal operating heads in the aqueduct are high and to reduce flow from the aqueduct into the trunk main system there is a pressure reducing valve at the end of TM1 just before the offtake to SR1. This gives priority to the local water treatment works TW2 for satisfying the demands of the area.

In the absence of detailed field data, the pressure–consumption relation of Eq (5) was chosen for all nodes to be:

$$C_i=C_i^*(1-10\,e^{-5P_i/P_i^*})$$

i.e. $a_i=10$, $b_i=5$. (See also Fig. 1.)
For this relationship P_i^* corresponds to the pressure that provides 93.2% of the nominal consumer demand C_i^*. The values of P_i^* were chosen for each node based on the corresponding ground levels as well as on the information available from the network operators for certain nodes on the minimum pressure needed to provide adequately for the nominal consumer demands.

The leakage constant c_l in Eq (6) was given the value of 1×10^{-7}. As will be seen below this leads to leakage losses in the order of 10% to 15% of the total supply. Since c_l is dimensionally dependent, it should be added that pipe lengths and piezometric heads are expressed in metres and flows in megalitres per day (Mld).

Normal operating conditions

Typical simulation results for normal operating conditions are presented in Figs 3a–3e. Under normal operating conditions the network pressures are sufficient to provide fully the specified nominal nodal consumptions, so the inclusion of pressure dependent demands is seen to have little effect.

The total leakage flow from the network is found to vary from 11% to 13.5% of the total flow supplied from water treatment works TW1 and TW2 during the 24-hour simulation period. In the aqueduct, the additional nodal outflows due to leakage result in lower heads and in higher flows being provided from water treatment works TW1 (Figs 3a and 3b). Similarly, the flow entering trunk main TM1 from the aqueduct is found to be higher than if leakage flows are not considered (Fig 3c). On the other hand, in the part of the system where pressures are relatively low, i.e. trunk mains TM2, TM3, and TM4, the losses due to leakage are found to be low and their effect

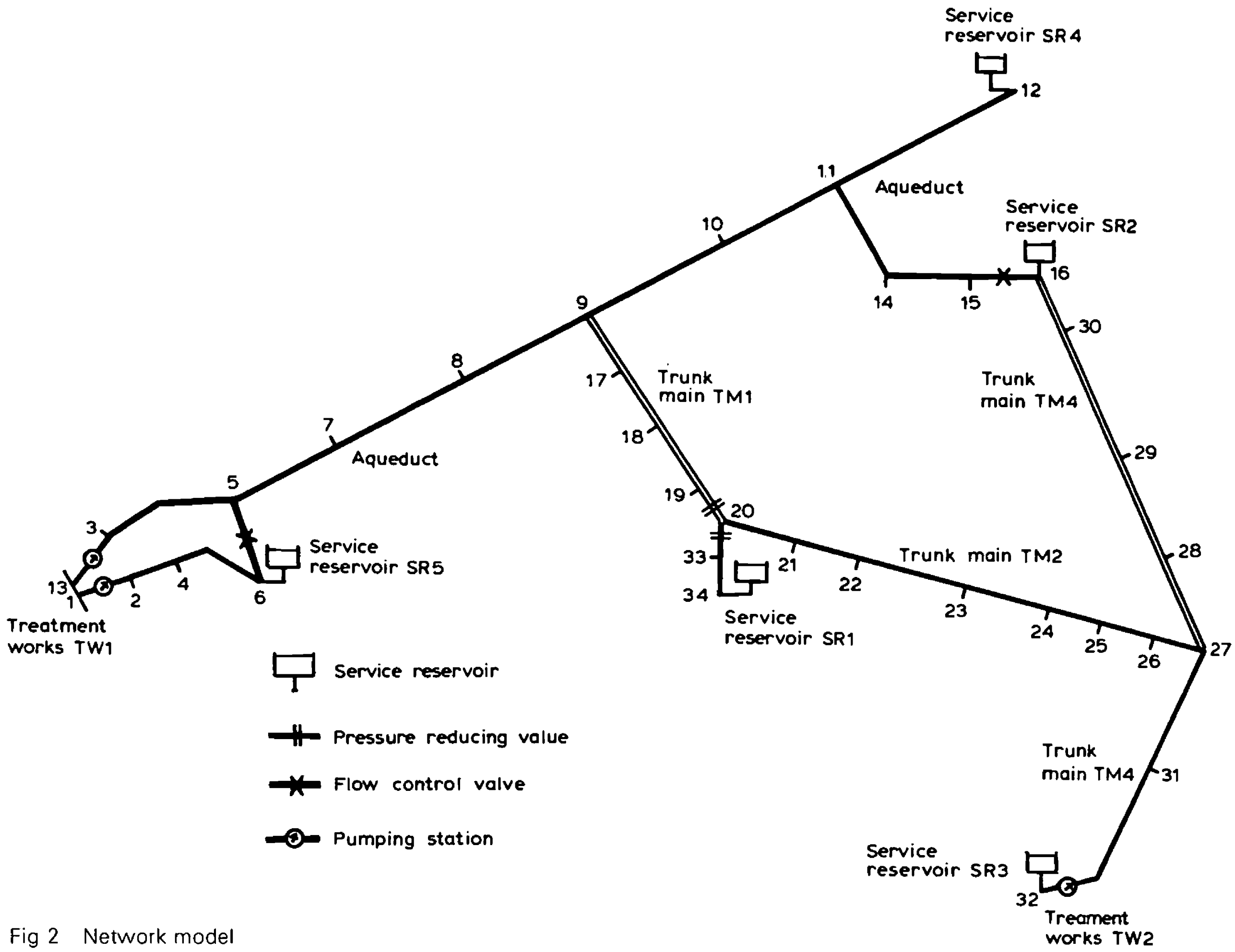


Fig 2 Network model

Table 1 Service reservoir data

Service reservoir	Bottom water level (m AOD)	Top water level (m AOD)	Capacity (M1)
SR1	81.4	84.1	1.8
SR2	113.8	120.4	4.5
SR3	93.0	98.1	14.9
SR4	150.4	157.9	177.0
SR5	205.9	213.4	6.35

on the network heads and flows is not significant (Figs 3d and 3e).

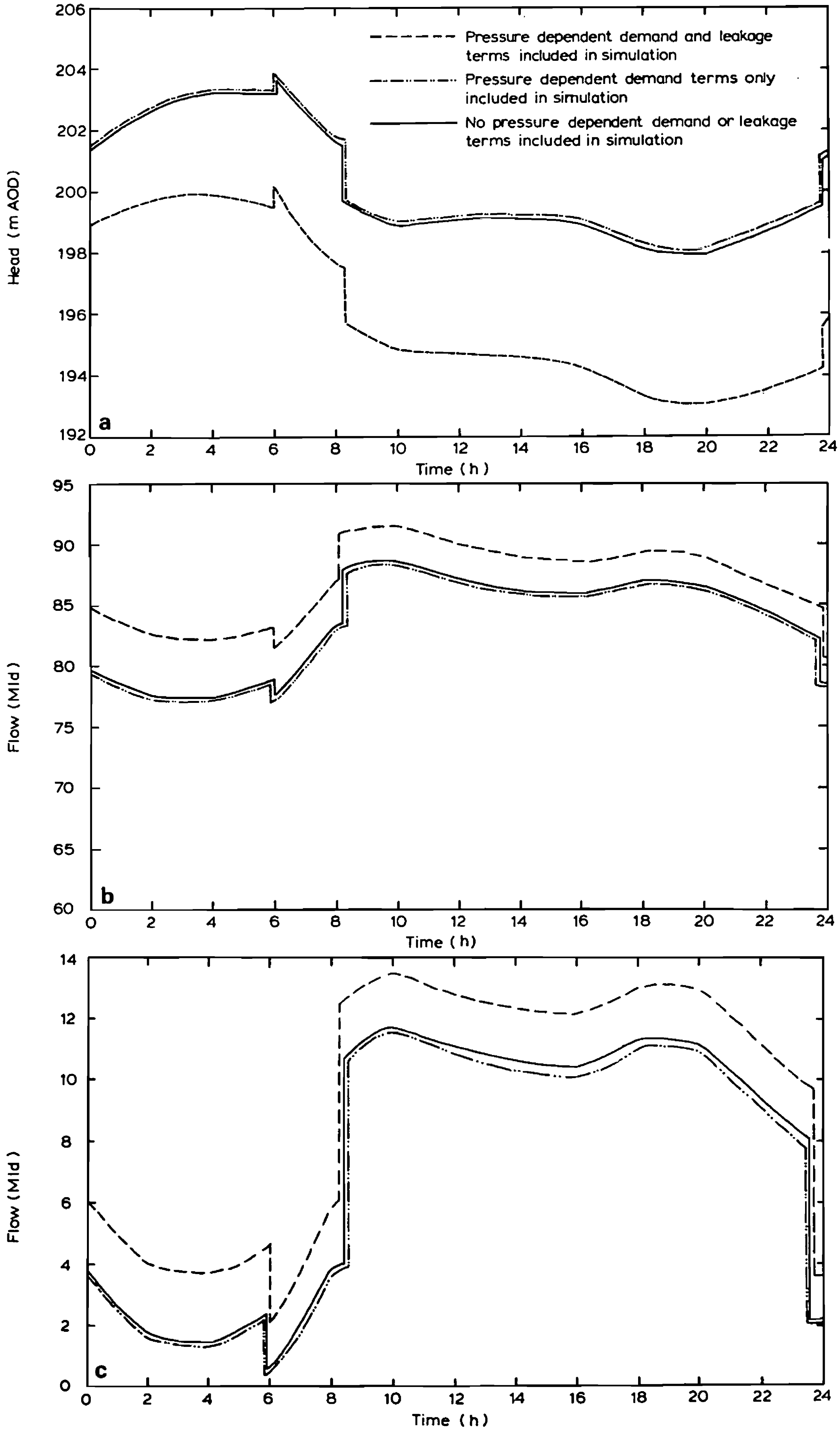
It must be noted at this point that the sudden changes observed in the simulated head and flow profiles correspond to changes in the operating conditions of the system, such as the start-up and interruption in the operation of the pumps, changes in valve settings and pipe bursts and their isolation. These are assumed to occur instantaneouslv in the simulation program.

Operation under failure conditions

The failure event simulated is a burst on trunk main TM1 at node 19, just before the inlet of service reservoir SR1. The consequences of a burst are an immediate loss of pressure and loss of water from the network, and a partial restoration of flows and pressures following isolation of the burst. The level of service to consumers following the burst isolation depends on the capacity of the reconfigured system. In this case the trunk main burst was taken to occur at 07.00 hrs and it was assumed that the burst was isolated and repair work started after one hour. Operation under failure conditions therefore concerns the period after 08.00 hrs, as, until the time of the burst occurrence and isolation, the system operates normally. Typical simulation results for this event over a 24-hour period are shown in Figs 4a to 4d.

The heads along the aqueduct and trunk main TM1 are still found to be sufficient to provide fully for the prespecified nodal demands, so the inclusion of pressure dependent demand terms if found to have little effect for that part of the system. The inclusion of leakage losses was found to have a significant effect on the aqueduct and trunk main TM1, which is similar to that observed under normal operation.

Low pressures along TM2, in particular following the isolation of the burst, have resulted in the specified nominal demands not being fully satisfied for certain nodes. The inclusion of pressure dependent demand terms in the simulation model indicated that for node 21 about 70% of the specified nominal demand is provided, for node 22 almost no consumer flow at all is supplied, and for node 23 about 80% of the specified demand is provided. As a



Pressure dependent demand and leakage terms included in simulation
Pressure dependent demand terms only included in simulation
No pressure dependent demand or leakage terms included in simulation
Head (m AOD)
Time (h)
a
Flow (Mld)
Time (h)
b
Flow (Mld)
Time (h)
c

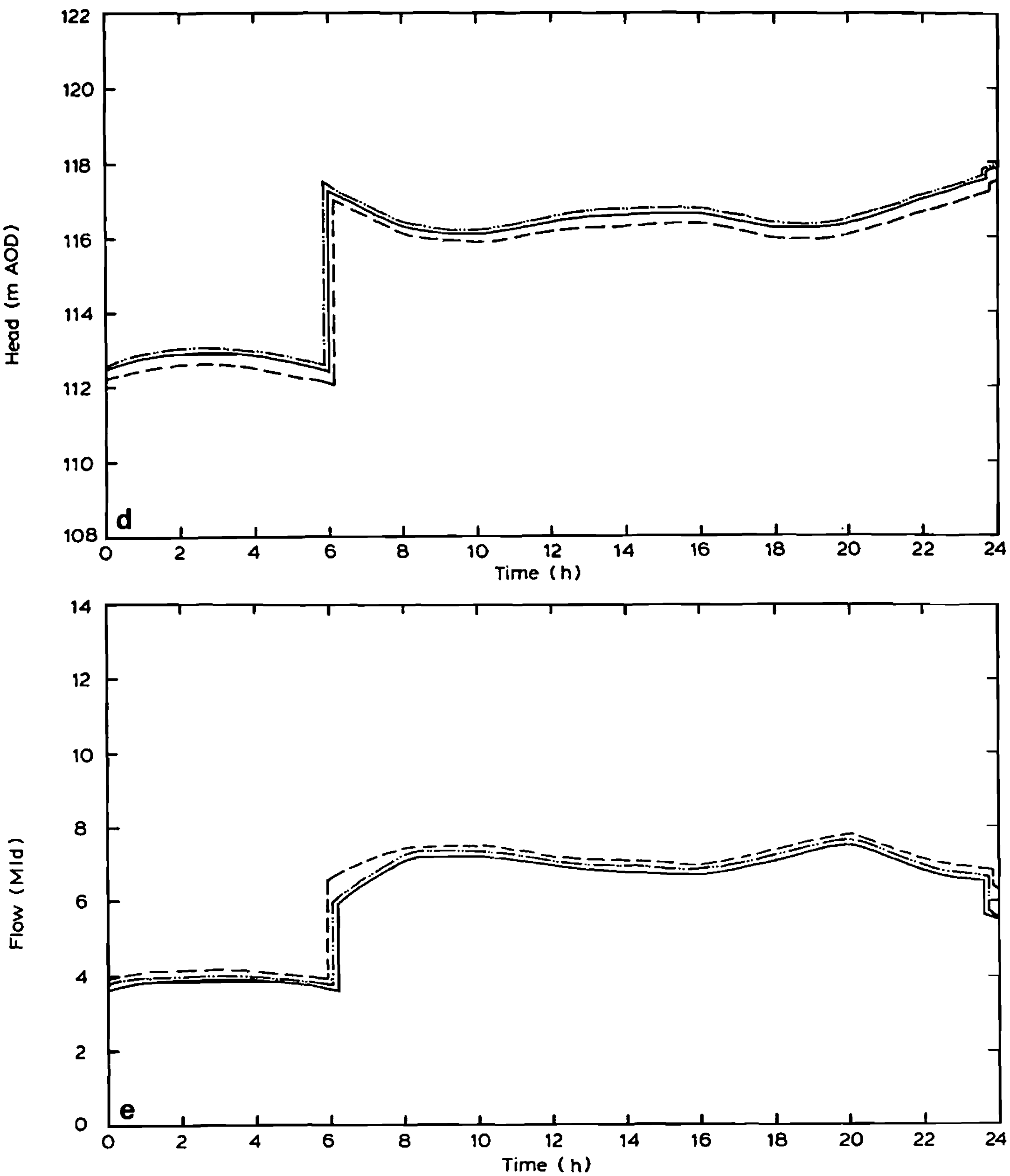


Fig 3 Simulated network heads and flows for normal operation. (a), head at node 7. (b), flow from node 5 to 7. (c), flow from node 9 to 17. (d), head at node 27. (e), flow from node 27 to 26

result, the inclusion of pressure dependent demands in the simulation model leads to significantly different heads and flows in TM2 than the assumption of fixed demands. (Figs 4a–c)

As the inclusion of pressure dependent demand terms results in only partial provision of certain demands along TM2, the flow reaching service reservoir SR1 from TM2 is much higher than if nodal demands are assumed to be fully provided independent of the pressures (Fig 4c). This has an important effect on the level variation of SR1, particularly as following the isolation of trunk main TM1, SR1 is supplied via TM2 only. Fig 4d shows that if fixed nodal demands are assumed, SR1 will empty just before 14.00 hrs. On the other hand, if pressure dependent demands are assumed, it is found that the reservoir exhaustion problem is significantly deferred. Clearly therefore, the inclusion of pressure dependent demand terms has an important effect in the assessment of the level of service in the area served by service reservoir SR1 for the failure event considered.

Finally, the above application has shown that computation times and convergence characteristics of the network solution are not largely affected by the introduction of the additional pressure dependent terms. Computation times were found to be higher by about 10% if pressure dependent demand and leakage terms are included in the network model. The number of iterations required to reach a solution by the Newton–Raphson procedure is generally not affected.

Discussion and conclusions

The results presented above demonstrate that the assumption of fixed nodal consumptions is not valid when

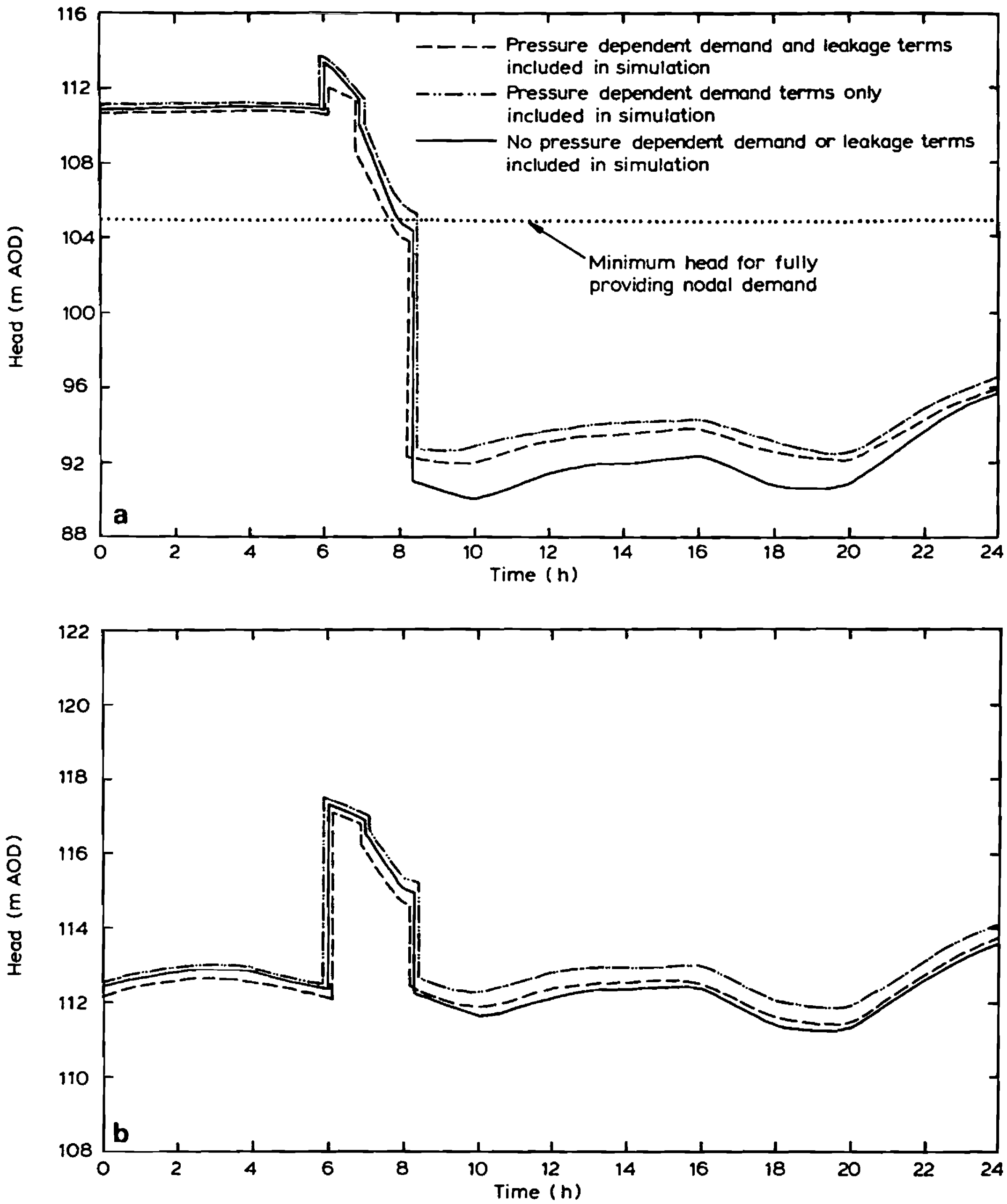


Fig 4 Simulated network heads and flows for failure conditions (a), head at node 23. (b), head at node 27

the distribution system is operating under failure conditions and pressures are lower than normal. Indeed, when the simulation model is used to evaluate the level of service offered to consumers following possible network breakdowns, the assumption of fixed known nodal consumptions independent of the network pressures contradicts the objective of the simulation. It is more realistic to work in terms of the interaction between consumer outflows and pressures based on the hydraulic characteristics of the system, rather than in terms of a demand driven model. The difficulties with using demand driven models in such cases have been highlighted elsewhere[17].

The application presented has also shown that losses due to leakage can be expected to have an important effect on the operation of a water distribution system when the pressures are high. Incorporating leakage losses explicitly in a simulation program in the form of a pressure–leakage relation is therefore expected to enable the problem of leakage evaluation and reduction in a distribution system to be approached more realistically than by a post-simulation examination of the network pressures. It must be made clear at this point that leakage losses at the consumers' end of the distribution system are taken into account implicitly in the nodal consumer outflows. The evaluation of leakage losses concerns only the elements included in the network model.

The main drawback in the extension of network models proposed in this study is the additional data required for calibrating the pressure–demand and

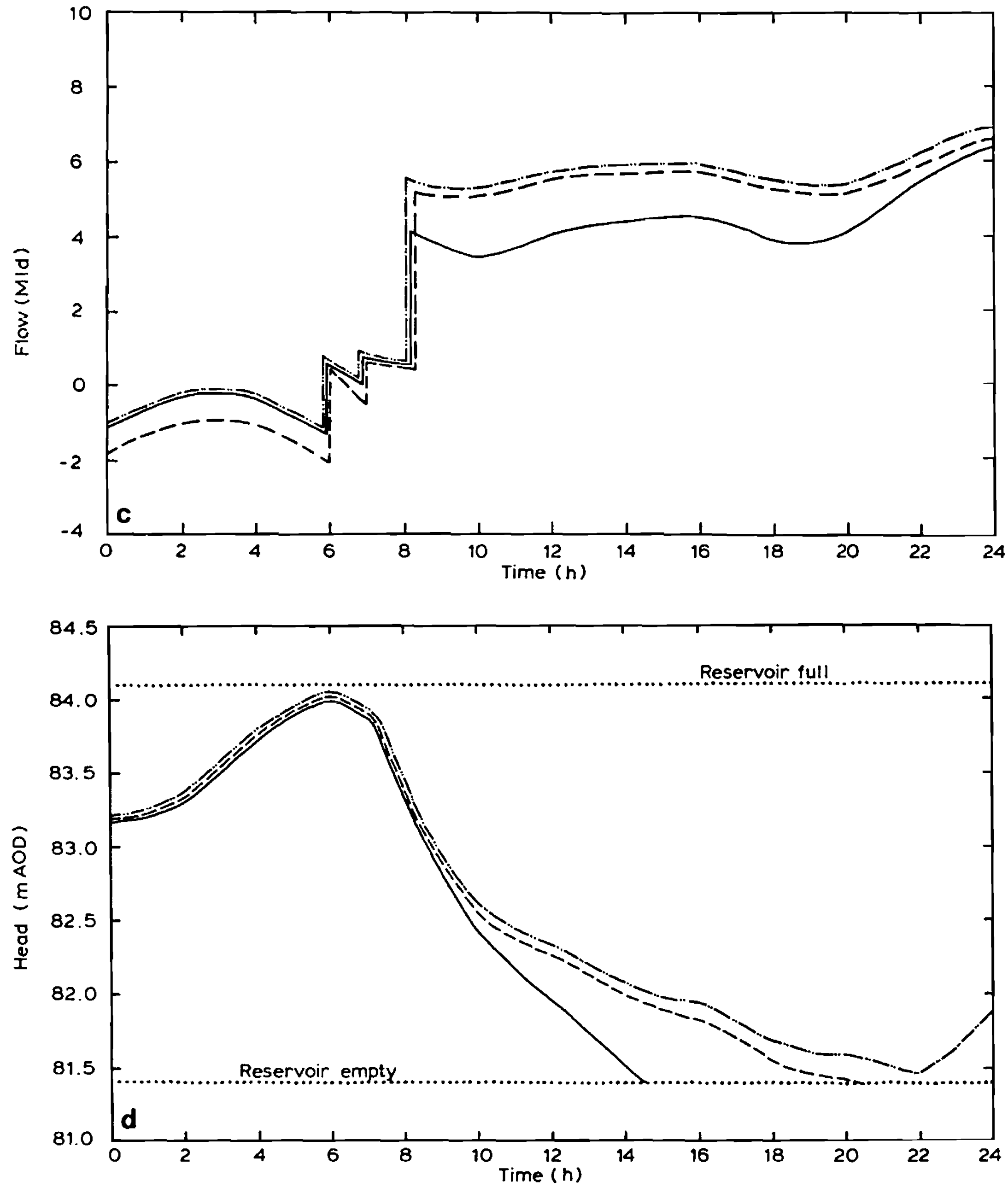


Fig 4 (*continued*) (c), flow from node 21 to 20. (d), head at node 34 (Service reservoir SR1)

pressure–leakage relations of Eqs (5) and (6). Useful information for calibrating the pressure–consumption relationship can be obtained by examining the hydraulic arrangement between network and consumers at each node, the topography of the area (ground levels etc), and information from the network operators on the minimum pressures required to fully satisfy consumer demands at different parts of the network. The age and material of the network components as well as an indication by the network operators on the estimated total quantity of leakage losses can be used to calibrate the pressure–leakage relationship. Obviously, field data on the variation of leakage losses with pressure for a particular undertaking would be particularly useful for this purpose.

On the whole, the pressure–consumption and pressure–leakage relationships proposed should be regarded as approximate. Their purpose is not to evaluate the consumer flows and leakage losses with great accuracy, but to give a network simulation model which can be used to approach more realistically certain problems in water distribution system operation. It is believed that the present study has contributed in identifying the problem of modelling the effect of pressure on the outflows and losses from a water supply network, as well as in proposing a solution without additional numerical difficulties or computational requirements.

Acknowledgement

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Nomenclature

a_i, b_i, P_i^*	constants for pressure–consumption relationship at node i
C_i	consumer outflow at node i
$\boldsymbol{C}$	vector of nodal consumer outflows
C_i^*	nominal consumer demand at node i
c_l	constant for network pressure–leakage relationship
$\boldsymbol{f}$	vector functional with elements corresponding to network nodal flow balance equations
$\boldsymbol{f}'$	Jacobian matrix of $\boldsymbol{f}$
H_i	piezometric head at node i
$\boldsymbol{H}$	vector of nodal piezometric heads
K_{ij}	conveyance of pipe connecting nodes i and j
$\boldsymbol{K}$	vector of pipe conveyances
L_{ij}	length of pipe connecting nodes i and j
n	number of nodes in network
P_i	pressure at node i
P_{ij}^{av}	average pressure along pipe connecting nodes i and j
Q_{ij}	flow in pipe connecting nodes i and j
Q_i^{out}	total outflow from node i
V_{ij}	leakage flow rate from pipe connecting nodes i and j
$\boldsymbol{X}$	vector of network unknowns

Units

Ml	megalitres
Mld	megalitres per day
m AOD	metres above ordnance datum

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