

END TERM EXAMINATION - 2021

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POWER SYSTEM-I (EE 222)

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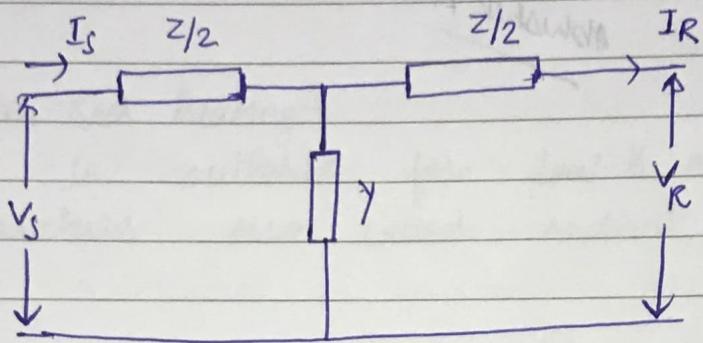
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4(a)

Mathematical Exp. (for M.T.L.)

i) T-model

Nominal T model of a Medium T.L has lumped shunt admittance at the middle, Net series impedance is divided into two equal values.



V_S : Sending end voltage V_R : Receiving end voltage

I_S : Sending end current I_R : Receiving end current

$$V_S = \left(V_R + \frac{Z}{2} I_R \right) + \left(\frac{Z}{2} I_S \right)$$

~~$$= V_R + \frac{Z}{2} I_S \quad I_S = I_R + Y \left(V_R + \frac{Z}{2} I_R \right)$$~~

$$\therefore V_S = \left(V_R + \frac{Z}{2} I_R \right) + \frac{Z}{2} \left(I_R + Y \left(V_R + \frac{Z}{2} I_R \right) \right)$$

$$= V_R \left[1 + \frac{YZ}{2} \right] + I_R \left[Z \left(1 + \frac{YZ}{4} \right) \right]$$

$$I_S = I_R + Y \left[V_R + \frac{Z}{2} I_R \right]$$

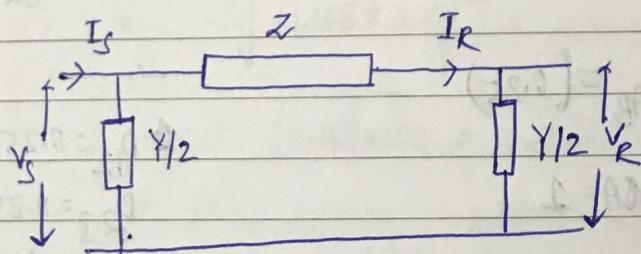
$$= V_R Y + I_R \left[1 + \frac{YZ}{2} \right]$$

$$V_S = V_R \left[1 + \frac{YZ}{2} \right] + I_R \left[Z \left(1 + \frac{YZ}{4} \right) \right], \quad I_S = V_R \left[Y \right] + I_R \left[1 + \frac{YZ}{2} \right]$$

$$\therefore \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1+YZ \\ \frac{Y}{2} \\ Y \\ 1+\frac{YZ}{2} \end{bmatrix} \begin{bmatrix} Z \\ 1+\frac{YZ}{4} \\ 1+\frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

ii) π -Model

It has lumped series impedance placed at middle of circuit whereas shunt admittances are at the end. The total lumped shunt admittance is divided into 2 values, $Y/2$ each.



$$I_S = \frac{Y}{2} \cdot V_S + \frac{Y}{2} \cdot V_R + I_R \quad \therefore I_S = \frac{Y}{2} V_S + \frac{Y}{2} V_R + I_R$$

$$\text{and } V_S = V_R + Z \left(I_R + \frac{Y}{2} V_R \right)$$

$$\therefore I_S = \frac{Y}{2} \cdot \left[V_R + Z \left(I_R + \frac{Y}{2} V_R \right) \right] + \frac{Y}{2} V_R + I_R$$

$$= V_R \left[Y + \frac{Y^2 Z}{4} \right] + I_R \left[1 + \frac{YZ}{2} \right]$$

$$\& V_S = V_R \left[1 + \frac{YZ}{2} \right] + I_R [Z]$$

$$\therefore \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1+YZ \\ \frac{Y}{2} \\ Y \left[1+\frac{YZ}{2} \right] \end{bmatrix} \begin{bmatrix} Z \\ 1+\frac{YZ}{4} \\ 1+\frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

4(b)

Length of cable, $l = 1000\text{m}$ $R = \text{cable insulation resistance}$

$$= 495 \text{ M}\Omega = 495 \times 10^6 \Omega$$

conductor radius $r_1 = 2.5/2 = 1.25\text{ cm}$

$$\begin{aligned} \text{Insulation } &= 4.5 \times 10^{14} \Omega \cdot \text{cm} \\ &= 4.5 \times 10^{12} \Omega \cdot \text{m} \end{aligned}$$

Let r_2 cm be inner radius,

$$\therefore R = \frac{\rho}{2\pi l} \ln \frac{r_2}{r_1}$$

$$\begin{aligned} \log_e \frac{r_2}{r_1} &= \frac{2\pi R \cdot l}{\rho} \\ &= \frac{2\pi \times 1000 \times 495 \times 10^6}{4.5 \times 10^{12}} = 0.69 \end{aligned}$$

$$2.3 \log_{10} \frac{r_2}{r_1} = 0.69$$

$$\begin{aligned} \frac{r_2}{r_1} &= \text{antilog } 0.69 / 2.3 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \therefore r_2 &= 2r_1 \\ &= 2.5 \text{ cm} \end{aligned}$$

Insulation thickness = $r_2 - r_1$

$$= 2.5 - 1.25$$

$$= 1.25 \text{ cm}$$

2(a)

String Efficiency :-

It is the ratio of voltage across the whole string to the product of number of discs and the voltage across the disc nearest to the conductor.

$$\text{String Efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across the disc nearest to conductor}}$$

n : number of discs in the string.

Methods to Improve String Efficiency.

i) By Using Longer cross-arms :-

The value of string efficiency depends upon the value of K i.e. the ratio of shunt capacitance to mutual capacitance. The lesser the value of K , more is the string efficiency & more uniform is voltage distribution.

The value of K can be decreased by reducing the shunt capacitance. In order to reduce shunt capacitance, the distance of conductor from tower must be increased i.e. longer cross-arms should be used.

ii) By Upgrading the insulators:-

Different insulators of different dimensions are used

such that each has a different capacitance. The insulators are capacitance graded i.e. they are assembled in the string in such a way that the top unit has the minimum capacitance increasing progressively as the bottom unit is reached. This method tends to equalize the potential distribution across the units in the string.

(iii) By using a Guard Ring :-

The potential across each unit in a string can be equalized by using a guard ring which is a metal ring electrically connected to the conductor and surrounding bottom insulator. The guard ring is contoured in such a way that shunt capacitance currents i_1, i_2 etc. are equal to metal fitting line capacitance currents i_1', i_2' etc. As a result same charging current I flows through each unit of string, so uniform potential distribution is formed across units.

2(b)

at Node 1,
using KCL,

$$\frac{V}{\frac{1}{wC}} + \frac{V}{\frac{1}{w(0.12C)}} + \frac{V-2V}{\frac{1}{wC_1}} = 0$$

$$VL + 0.12VC - VC_1 = 0$$

$$0.12C = C_1$$

Thus

$$C_1 = 1.12C$$

At Node 2,

$$(2V-V)C_1 + 2V(0.12C) + (2V-3V)C_2 = 0$$

$$C_1 + 2 \times 0.12C - C_2 = 0$$

$$C_2 = 1.12C + 0.24C$$

$$C_2 = 1.36C$$

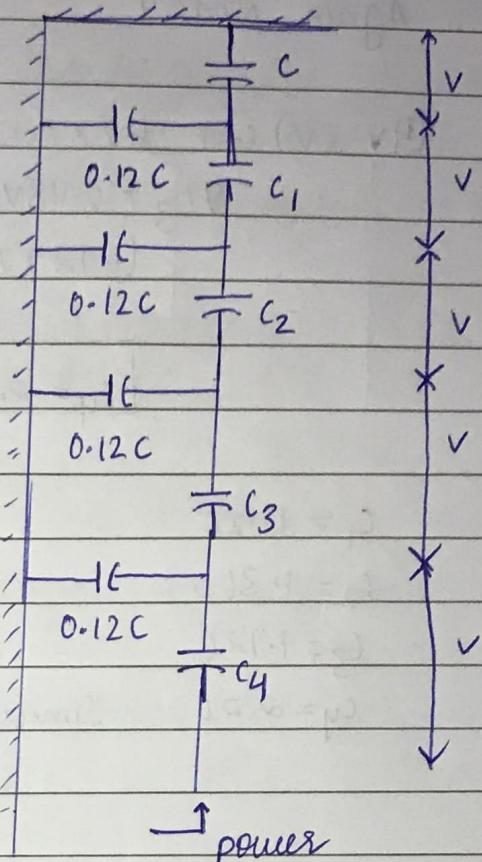
at Node 3,

$$(3V-2V)C_2 + (3V-4V)C_3 + 3V(0.12C) = 0$$

$$C_3 = C_2 + 0.72C$$

$$= 1.36C + 0.72C$$

$$C_3 = 2.08C$$



Again Node 4,

$$(4V - 3V) C_3 + 4V \times 0.12C + (4V - 5V) C_4 = 0$$

$$V C_3 + 0.48 V C = V C_4$$

$$(1.72 + 0.48) C = C_4$$

$$\therefore C_4 = 2.2C$$

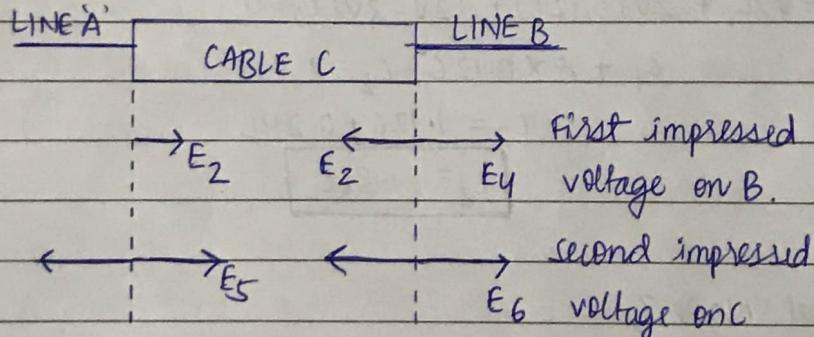
$$C_1 = 1.12C$$

$$C_2 = 1.36C$$

$$C_3 = 1.72C$$

$C_4 = 2.2C$ Similarly other capacitance can be calculated.

3(a)



The voltage wave of magnitude 40kV, is initiated in 'A' is partly reflected and partly transmitted on to cable C on reaching the junction of A and C.

$$\begin{aligned} \text{The transmitted wave } E_2 &= 40 \times \left[1 + \frac{70 - 500}{70 + 500} \right] \\ &= 40 \times \frac{140}{570} = 9.83 \text{ KV} \end{aligned}$$

This transmitted wave on reaching the junction of C and B again meet ~~an~~ irregularly so that a part of it is reflected and a part is transmitted on to B.

$$E_4 = 9.83 \times \left[1 + \frac{600-70}{600+70} \right] = 9.83 \times \frac{1200}{670}$$

$$= 17.61 \text{ KV}$$

This is also the first voltage impressed on B.

The reflected wave E_3 is

$$E_3 = 9.83 \times \frac{600-70}{600+70} = 9.83 \times \frac{530}{670}$$

$$= 7.78 \text{ KV}$$

E_3 on reaching the junction of A and C, gets partially reflected in C and partially transmitted on to A.

Let E_5 be the reflected wave proceeding through A

Then,

$$E_5 = 7.78 \times \frac{500-70}{500+70} = 7.78 \times \frac{430}{570} = 5.87 \text{ KV}$$

E_5 on reaching the junction of C and B, gets partially transmitted on to BC.

Let this be E_6 .

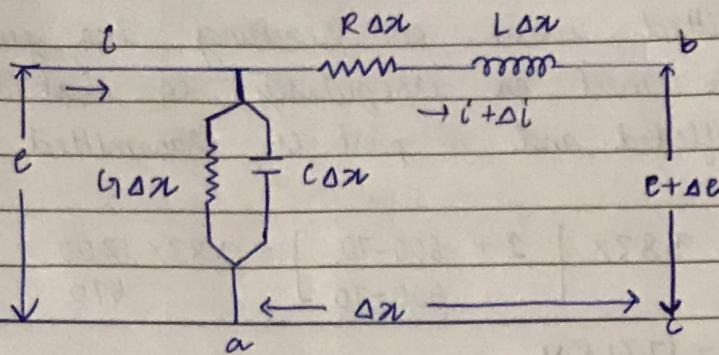
$$E_6 = 5.87 \times \frac{1200}{670} = 10.51 \text{ KV}$$

$$\therefore \text{Second Impressed Voltage} = E_4 + E_6$$

$$= 17.61 + 10.51$$

$$= 28.12 \text{ KV}$$

3(b)



Applying KCL around loop ~~around~~:

$$(i + \Delta i)(R\Delta x) + L\Delta x \frac{di}{dt} + e + \Delta e = e$$

$$iR\Delta x + L\Delta x \frac{di}{dt} + \Delta e = 0$$

Neglecting differential quantities of higher orders

$$(Ri + L\frac{di}{dt})\Delta x + \Delta e = 0$$

$$\Delta e = -\left(Ri + L\frac{di}{dt}\right)\Delta x$$

$$\frac{\Delta e}{\Delta x} = -Ri - L\frac{di}{dt}$$

$$\Delta x \rightarrow 0 \Rightarrow \frac{\Delta e}{\Delta x} = -Ri - L\frac{di}{dt} \quad \text{--- (1)}$$

Similarly applying KCL in above eqt

$$i - \left(G\Delta x e + C\Delta x \frac{\partial e}{\partial t}\right) - (i + \Delta i) = 0$$

$$\Delta i = -\left(G\Delta x e + C\Delta x \frac{\partial e}{\partial t}\right)$$

$$\frac{\partial i}{\partial x} = - \left(G_e + C \frac{\partial e}{\partial t} \right)$$

$$\Delta x \rightarrow 0 \Rightarrow \frac{\partial i}{\partial x} = - G_e - C \frac{\partial e}{\partial t} \quad \text{--- (2)}$$

Eqⁿ ① and ② are two simultaneous partial diff. equation which can be solved for e & i .

$$\text{Eq}^n ① \quad \frac{\partial e}{\partial x} = -Ri - L \frac{\partial i}{\partial t} \quad \text{Eq}^n ② \quad \frac{\partial i}{\partial x} = -G_e - C \frac{\partial e}{\partial t}$$

$$\frac{\partial^2 e}{\partial x^2} = -R \frac{\partial i}{\partial x} - L \frac{\partial^2 i}{\partial t \partial x} \quad \text{--- (3)}$$

$$\frac{\partial^2 i}{\partial x \partial t^2} = -G_e \frac{\partial e}{\partial t} - C \frac{\partial^2 e}{\partial t^2} \quad \text{--- (4)}$$

Substituting values of $\frac{\partial i}{\partial x}$ and $\frac{\partial^2 i}{\partial t \partial x}$ in eqⁿ ③ from eqⁿ ② and eqⁿ ④.

$$\begin{aligned} \frac{\partial^2 e}{\partial x^2} &= -R \left(-G_e - C \frac{\partial e}{\partial t} \right) - L \left(-G_e \frac{\partial e}{\partial t} - C \frac{\partial^2 e}{\partial t^2} \right) \\ &= R G_e + (R C + L G_e) \frac{\partial e}{\partial t} + L C \frac{\partial^2 e}{\partial t^2} \quad \text{--- (5)} \end{aligned}$$

Similarly eliminating ' e ', we get ' i ' :

$$\frac{\partial^2 i}{\partial x^2} = R G_i + (R G + L G) \frac{\partial i}{\partial t} + L C \frac{\partial^2 i}{\partial t^2} \quad \text{--- (6)}$$

Equation ⑤ and ⑥ are called Telegrapher's Eqⁿ.

let us now consider the case of line without loss for which $R=0$, $G=0$.

substituting these values in eqn ⑤ & ⑥

$$\frac{\partial^2 i}{\partial x^2} = LC \frac{\partial^2 e}{\partial t^2} \quad \text{--- ⑦}$$

$$\frac{\partial^2 i}{\partial x^2} = LC \frac{\partial^2 i}{\partial t^2} \quad \text{--- ⑧}$$

The general solution for above eqn is

$$e = f(\sqrt{LC}x - t)$$

Similarly, it can be shown that,

$$e = \phi(\sqrt{LC}x + t)$$

$$\therefore e = f(\sqrt{LC}x - t) + \phi(\sqrt{LC}x + t)$$

$$e = e' + e''$$

e' : incident wave, \rightarrow travelling in forward direction

e'' : reflected wave. \rightarrow " " backward "

$$i' = e'/R_o \quad i'' = e''/R_o$$

R_o : surge impedance

$$\therefore \boxed{i = i' + i''}$$

5(a)

we know, total corona loss (Peck's formula)

$$P_c = 244 \frac{(f+25)}{\delta} \sqrt{\frac{Y}{\delta}} (E - E_0)^2 \times 10^{-5} \text{ kW per phase per km.}$$

$$P_c \propto (E - E_0)^2 \quad \text{--- (1)}$$

$$\therefore \frac{P_1}{P_2} = \frac{(E_1 - E_0)^2}{(E_2 - E_0)^2} \quad \text{--- (11)}$$

where P_1 and P_2 are the power losses in two cases

we g are given,

$$P_1 = 48 \text{ kW}, E_1 = \frac{105 \text{ kV}}{\sqrt{3}} = 60.621 \text{ kV}$$

$$P_2 = 12.5 \text{ kW}, E_2 = \frac{12.5 \text{ kV}}{\sqrt{3}} = 72.168 \text{ kV}$$

$$P_3 = ? \quad E_3 = \frac{110 \text{ kV}}{\sqrt{3}} = 63.508 \text{ kV}$$

if E_0 : disruptive critical voltage

$$\sqrt{\frac{48}{125}} = \frac{60.621 - E_0}{72.168 - E_0}$$

$$44.720 - 0.6197 E_0 = 60.621 - E_0$$

$$0.3803 E_0 = 15.9$$

$$E_0 = 41.80 \text{ kV}$$

$$\begin{aligned} \text{Now, } P_3 &= P_2 \times \left(\frac{E_3 - E_0}{E_2 - E_0} \right)^2 = 12.5 \times \left(\frac{63.508 - 41.8}{72.168 - 41.8} \right)^2 \\ &= 63.872 \text{ kW.} \end{aligned}$$

\therefore the disruptive critical voltage $E_0 = 41.80 \text{ kV}$
 and corona loss at 110 kV = 63.872 kW.

5(b)

Factors affecting corona loss:-

1) Effect of frequency:-

corona loss is directly proportional to system frequency.

2) Effect of conductivity of air:-

If the number of ions in ~~air~~ increases, the conductivity of air also increases. During rain and thunderstorm, ion content in the air enhances & thus corona loss becomes high during bad weather conditions.

3) Effect of load current:-

The flow of load current increases with temperature of conductor. It prevents deposition of dew or snow on the conductor's surface. This reduces corona loss.

4) Effect of conductor's surface:-

Roughness of the conductor surface results in field distortion and gives rise to high potential gradient, causing high corona loss.

5) Effect of system voltage:-

With increase in system voltage, the electric field increases & hence corona loss also increases in a steep way.

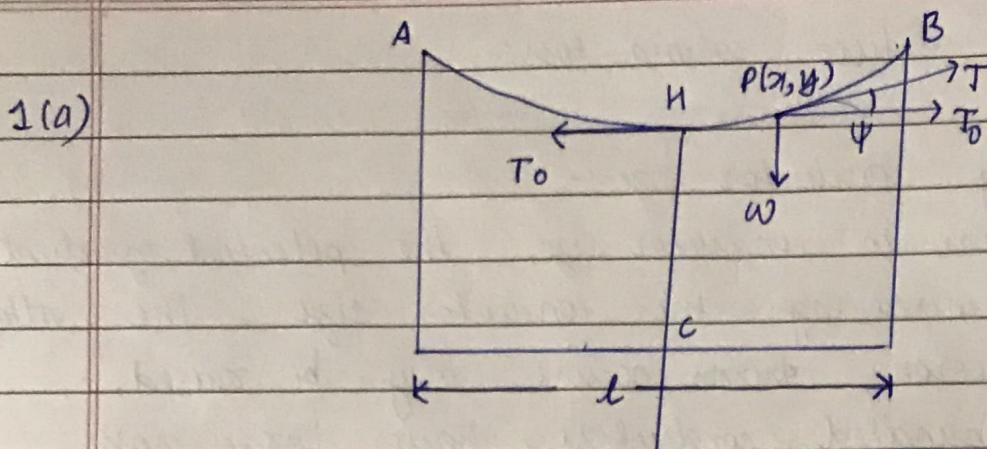
Method to reduce corona loss:-

1) By increasing conductor size:-

With increase in conductor size, the potential gradient decreases. By increasing the conductor size, the voltage at which corona occurs may be raised, because of bundled conductors have large cross sectional area & are widely used in transmission line.

2) By increasing conductor spacing:-

With increase in conductor spacing, the voltage at which corona is increased & hence corona effects can be eliminated.



consider a transmission line of equal heights and

i: length distance between supports

w: weight / length of wire

T_0 : Tension in the lowest point

H: Height of lowest point

ϕ : angle subtended by T with horizontal axis

consider equilibrium of a small length s of wire upto point P(x, y)

i) horizontal Tension $T_0 = ws$

ii) vertical weight ws

iii) Tension T

$$T \cos \phi = T_0 = ws \quad \text{--- (i)}$$

$$T \sin \phi = ws \quad \text{--- (ii)}$$

$$\tan \phi = \frac{dy}{dx} = \frac{ws}{ws} = \frac{s}{l} \quad \text{--- (iii)}$$

for differential length :-

$$ds = \sqrt{dx^2 + dy^2}$$

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\frac{ds}{dx} = \sqrt{1 + \frac{s^2}{c^2}} = \sqrt{\frac{c^2 + s^2}{c^2}}$$

$$\text{or } \frac{c ds}{\sqrt{c^2 + s^2}} = dx$$

$$\text{let } s = c \sinh \theta$$

$$ds = c \cosh \theta d\theta \quad \text{--- (vi)}$$

$$\frac{c \cosh \theta d\theta}{c \cosh \theta} = dx$$

$$c d\theta = dx$$

$$\text{or } c\theta = x + A$$

$$\text{Now for } x=0, s=0 \quad \therefore \theta=0$$

$$\therefore \theta = \frac{x}{c}$$

$$\text{from (vi)} \quad \theta = \sinh^{-1} \frac{s}{c} \Rightarrow \frac{s}{c} = \sinh \frac{x}{c} \quad \text{--- (vii)}$$

Now from (iii) & (vii)

$$\frac{dy}{dx} = \sinh \frac{x}{c}$$

$$dy = \sinh \frac{x}{c} \cdot dx$$

$$y = c \cosh \frac{x}{c} + B \quad \text{--- (viii)}$$

but from the figure :-

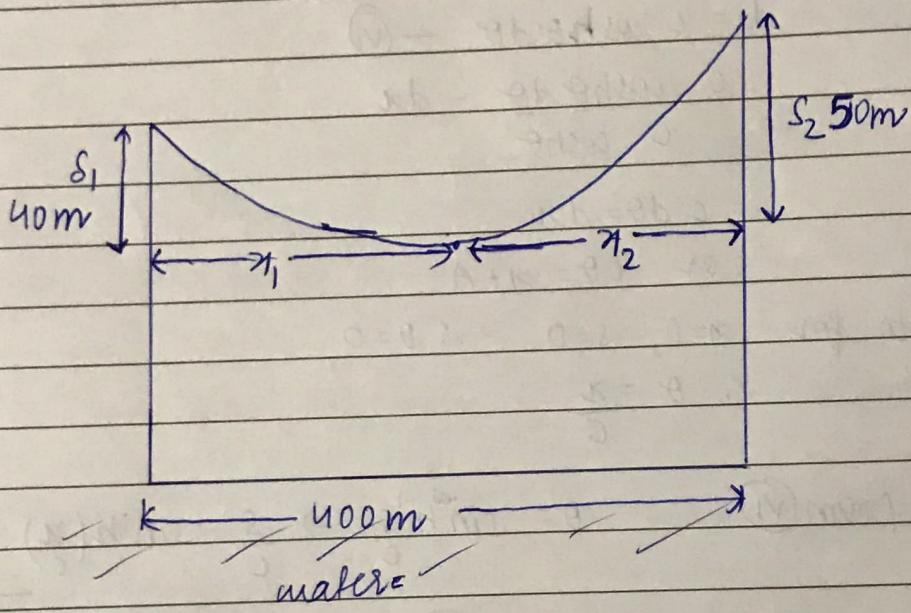
$$x=0 \Rightarrow y=c \quad \text{--- (ix)}$$

$$\therefore c = c + B \text{ hence } B=0$$

$$\therefore y = c \cosh \left(\frac{x}{c} \right)$$

The final equation represents catenary and gives the relationship between length of wire measured from lowest point H and vertical height of any point P(x, y).

1(b)



Given $l = 400\text{m}$

$H = 10\text{m}$

$w = 0.925 \text{ kg/m}$

$T = 2400 \text{ N}$

$$x_1 = \frac{l}{2} - \frac{TH}{wl}$$

$$x_2 = \frac{l}{2} + \frac{TH}{wl}$$

$$x_1 = 135.135 \text{ m}$$

$$\text{hence, } x_2 = 400 - x_1 = 264.865 \text{ m}$$

Now that we have x_1 , we can calculate s_1

$$s_1 = \frac{w x_1^2}{2T} = 3.52 \text{ m}$$

$$\begin{aligned} \text{The minimum clearance} &= 40 - s_1 \\ &= 40 - 3.52 = 36.48 \text{ m} = OP \end{aligned}$$

Now, at midway i.e. at 200m,
we have,

$$s_2 = \frac{w x_2^2}{2T}$$

$$\begin{aligned} x_2 &\text{: distance from point O to midpoint} \\ &= 200 - x_1 \\ &= 200 - 135.135 \\ &= 64.865 \text{ m} \end{aligned}$$

$$s_2 = \frac{w x_2^2}{2T} = \frac{0.925 \times 64.865^2}{2 \times 2400}$$

$$s_2 = 0.8108 \text{ m}$$