Assignment 9

Thursday, March 16, 2017 10:57 AM

Q 8.2 →

$$D = \begin{bmatrix} 1 & 0 & 1 & 1 & -1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 & 0 & -1 & -1 & -1 \end{bmatrix}$$

$$\Delta^k = 2^{-p}, \quad \delta^k = 4^{-p}$$

Sol. (a) Since, $\delta^k \|d\|_\infty \leq \Delta^k \max\{\|d'\|_\infty : d' \in D\}$

and, $\|d\|_\infty = 1$.

hence, number of points in one dimension and on one side of x^k will be,

$$n_p = \frac{\Delta^k}{\delta^k}$$

$$= \frac{2^{-p}}{4^{-p}} = \frac{4^p}{2^p} = 2^p.$$

Hence, total number of points in one dimension will be,

$$N = 2 \times 2^p + 1$$

$$= 2^{p+1} + 1$$

Now, Since, the column vectors of D are in two dimensions, Now, since, the column vectors of D are in

Hence, total number of points $= (2^{p+1}+1)^2$

Now, polling directions can be computed as,

$$\frac{x^k - f^k}{\|x^k - f^k\|} \quad \text{where } f^k \in F^k \setminus x^k.$$

and since there could be $(2^{p+1}+1)^2 - 1$ such f^k_s,

hence, Total Number of polling directions $= (2^{p+1}+1)^2 - 1$.

(b)

Sol (b) for $\Delta^k = 2^{-p}$ & $\delta^k = 4^{-p}$,

Number of potential polling direction $= (2^{p+1}+1)^n - 1$

Q8.6→ Let $\Omega \subseteq \mathbb{R}^N$ be bounded.

(a)

Sol. (a) Since, Ω is bounded, lets say by B i.e.

$$\|x\| \leq B \quad \forall\, x \in \Omega$$

hence,

$$\|x_i\| \leq B \quad \forall\, x_i \in \mathbb{Z},$$

Hence, number of points in each dimension $= 2 \times \lfloor B \rfloor + 1$

and hence, the set is finite.

(b)

Sol. (b) from above part, since,

$$\|x\| \le B \quad \forall\ x \in \Omega$$

hence, $\|10^m x_i\| \le B \quad \forall\ 10^m x_i \in \mathbb{Z}$, $m \in \mathbb{N}$

Hence, number of point in each dimension $= 2 \times \lfloor 10^m B \rfloor + 1$

and, the set is finite.

(c)

Sol. (c) Since for $\mathbb{R}^N$, the total number of point in part b

$$= (2 \times \lfloor 10^m B \rfloor + 1)^N$$

and, as we increase m, the number of points increase by a factor of 10^{mN}.

and, as we increase m, we can create a set that is

Hence, by choosing large values of m, we can create a set that is asymptotically dense set of int (Ω).

Q 8.9 →

(a) $v = (-4, -3)^T$

Sol. (a) $H = I - 2\left(\frac{v}{\|v\|} \cdot \frac{v^T}{\|v\|}\right)$

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2 \times \frac{1}{25}\left(\begin{bmatrix} -4 \\ -3 \end{bmatrix} [-4 \ \ -3]\right)$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \frac{2}{} \begin{bmatrix} 16 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{2}{25}\begin{bmatrix} 16 & 12 \\ 12 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \frac{32}{25} & -\frac{12}{25} \\ -\frac{12}{25} & 1 - \frac{18}{25} \end{bmatrix} = \frac{1}{25} \times \begin{bmatrix} -7 & -12 \\ -12 & 7 \end{bmatrix}$$

Now,

$$\left\| \frac{1}{25}\begin{bmatrix} -7 \\ -12 \end{bmatrix} \right\| = \frac{1}{25} \times 25 = 1 \quad \text{and,} \quad \left\| \frac{1}{25}\begin{bmatrix} -12 \\ 7 \end{bmatrix} \right\| = \frac{1}{25} \times 25 = 1$$

and, $[-7 \quad -12]\begin{bmatrix} -12 \\ 7 \end{bmatrix} = [84 - 84) = 0.$

Q 8.10 →

Sol. Please run the "generateNNmatrixTest" test to verify the program.

Test 2, 3 & 4 Corresponds respectively to (a), (b) & (C) part of 8.9.

Q 8.1 →

(a) Prove the $P^k_{GPS} \subseteq F^k$.

Sol. (a) Since, $P^k_{GPS} = \{ x^k + \delta^k d : d \in D^k = D \}$

while, $F^k = \{ x^k + \delta^k d : d = Dy, y \in N^p, \delta^k \| d \|_\infty \leq \Delta^k \max \{ \| d' \|_\infty : d' \in D \}$

Now,

Here, in P^k $d \in D$ while in F^k $d \in Dy$ with the condition that

Now,

Since, in P^k_{GPS}, $d \in D$ while in F^k, $d \in D_y$ with the condition that

$\delta^k \|d\|_\infty \le \Delta^k \max\{\|d'\|_\infty : d' \in D\}$ with $\delta^k \le \Delta^k$

hence, $\{d_{GPS}\} \subseteq \{d_F\}$, as $\delta^k \le \Delta^k$ and $y \in N^p$

and,

so, $P^k_{GPS} \subseteq F^k$

(b) Prove $F^k \subseteq M^k$

Sol.(b) Now, Since $M^k = \{x^k + \delta^k D_y : y \in N^p\}$

and, there is no restriction on $d_M \in D_y$ where D_y = items in the pSpan of the Columns of D, Hence, M is set of all the points that could be found through repeated "poll" steps in directions from D, while for F^k, $d_{F^k} \in D_y$ Satisfying the restrictive condition defined in part (a), so

$\{d_{F^k}\} \subseteq \{d_{M^k}\}$

and, hence,

$F^k \subseteq M^k$.

Q8.6

(d) for set $\Omega = \{\sqrt{2}\} \in \mathbb{R}$.

Sol.(d) Since, $\sqrt{2}$ is an irrational number, hence it is not well defined and hence, we cannot find any m such that $10^m x \in \mathbb{Z}$ where $m \in \mathbb{N}$. Even we cannot find any rational value α such that $\alpha x \in \mathbb{Z}$.