MATH604 - Assignment 3 - Deepak Kumar,73217151

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Q4.4) Suppose P^k has 5 individuals with fitness levels

$$f^1 = 2, \; f^2 = 2, \; f^3 = 3, \; f^4 = 5, \; f^5 = 8.$$

Sol. 4.4(a) for $N=1$,

Number of tournaments $= {}^5C_1 = 5$

$Prob(f^2) = 1/5$

for $N=2$,

Number of tournaments $= {}^5C_2 = \frac{5!}{3!\,2!} = \frac{4 \times 5}{2} = 10$

$$Prob(f^2) = Prob(f^2 \mid T = \{f^1, f^2\}) \cdot Prob(T = \{f^1, f^2\}) + \dots$$

$$\dots + Prob(f^2 \mid f^2 \notin T) \cdot Prob(f^2 \notin T)$$

$$= \frac{1}{10} \times \frac{2}{4} + \frac{1}{10} \times \frac{2}{3} + \frac{1}{10} \times \frac{2}{7} + \frac{1}{10} \times \frac{2}{10}$$

$$= \frac{2}{10}\left(\frac{1}{4} + \frac{1}{3} + \frac{1}{7} + \frac{1}{10}\right)$$

for $N=5$,

$$Prob(f^2) = Prob(f^2 \mid T = \{f^1, f^2, f^3, f^4, f^5\}) \cdot Prob\{T = \{f^1, f^2, f^3, f^4, f^5\})$$

$$= \frac{2}{20}$$

$$= \frac{1}{10}.$$

(b)

Sol.(b) for $N=1$,

Number of tournaments $= 5$, $Prob(f^2) = 1/5$

for $N=2$,

No. of tournaments $= {}^5C_2 = 10$

hence, $Prob(f^2) = 1/10$.

for $N=5$,

No. of tournaments $= 5$, $Prob(f^2) = 0/5 = 0$.

Q4.6

(a)

Sol.(a) $Prob(c=[1,1,1,0,1,1,0,1]) = (1-\theta).1.\theta.1.\theta.(1-\theta).1.(1-\theta)$

$= 0.6 \times 0.4 \times 0.4 \times 0.6 \times 0.6$

$= 0.216 \times 0.16$

(b)

Sol.(b) $Prob(m_1 \leq 0) = Prob(m_1 \leq 0 \mid mutation). P(mutation) + Prob(m \leq 0 \mid$ not mutation). P(not mutation)

$= \frac{0.1}{} \times \frac{1}{} + \frac{19}{} \times 1$

$$= \frac{0.1}{0.2} \times \frac{1}{20} + \frac{19}{20} \times 1$$

$$= \frac{1}{40} + \frac{19}{20} = \frac{39}{40}$$

Q 4.8 → C is encoded using 6 binary bits.

$p^1 = [0,0,1,0,1,1]$ and $p^2 = [1,0,0,1,1,1]$

(a) Suppose $p^1 = [0,0,1,0,1,1]$ and $p^2 = [1,0,0,1,1,1]$

If $x = 1, 2$, $c = [0,0,0,1,1,1] = 2/6$

If $x = 3$, $C = [p^1_1, p^1_2, p^1_3, p^2_4, p^2_5, p^2_6] = [0,0,1,1,1,1] = 1/6$

If $x = 4,5,6$, $C = [0,0,1,0,1,1] = 3/6 = 1/2$

(b) $\theta = 1/2$

Sol.(b) $P(c = [1,0,1,0,1,1] = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$P(C = [1,0,0,0,1,1] = 1/8$

$P(C = [0,0,0,0,1,1] = 1/8$

$P(C = [0,0,1,1,1,1] = 1/8$

$P(C = [0,0,0,1,1,1] = 1/8$

$P(C = [1,0,1,1,1,1] = 1/8$

$P(C=[1,0,1,1,1,1]) = 1/8$

$P(C=[0,0,1,0,1,1]) = 1/8$

$P(C=[1,0,0,1,1,1]) = 1/8$

Q4.7:

Sol. 4.7: P^k has $\bar{p}$ individuals with fitness levels

$f^1 < f^2 < f^3 < f^4 \ldots < f^{\bar{p}}$

Let us choose a tournament of size N then,

$$\text{Prob}(f^1_{\bar{p}}) = \text{Prob}(f^1 \mid T=\{f^1, f^2, \ldots f^N\}).\text{Prob}(T) + \text{Prob}(f^1 \mid T=\{f^1, f^3, \ldots f^{N+1}\})$$

$$\ldots + \text{Prob}(f^1 \mid f^1 \notin T).\text{Prob}(f^1 \notin T)$$

$$= \frac{1}{\bar{p}} \times \frac{f^1}{\sum_{i=1}^{N} f^i} + \frac{1}{\bar{p}} \times \frac{f^1}{f^1 + \sum_{i=3}^{N+1} f^i} + \ldots + 0$$

$$= \frac{1}{\bar{p}} \times (C) \qquad \text{, where } C = \frac{f^1}{\sum_{i=1}^{N} f^i} + \frac{f^1}{f^1 + \sum_{i=3}^{N+1} f^i} + \ldots$$

Now,

$$\text{Prob}(f^1_{\bar{p}+1}) = \frac{1}{\bar{p}+1} \times C + \frac{1}{\bar{p}+1} \times \frac{f^1}{f^1 + \sum_{\bar{p}+1-N-2}^{\bar{p}+1} f^i}$$

$$= \frac{1}{\bar{p}+1}(C + \varepsilon) \qquad \text{, where } \varepsilon = \frac{f^1}{f^1 + \sum^{\bar{p}+1} f^i}$$

$\bar{p}+1$ $\qquad\qquad\qquad\qquad$ $f^1 + \sum_{\bar{p}+1-N-2}^{\bar{p}+1} f^i$

Now, lets assume that $\text{Prob}(f^1)$ is non decreasing, hence

$$\text{Prob}(f^1_{\bar{p}+1}) \geq \text{Prob}(f^1_{\bar{p}})$$

$$\Rightarrow \frac{\varepsilon + c}{\bar{p}+1} \geq \frac{c}{\bar{p}}$$

$$\Rightarrow \bar{p}c + \bar{p}\varepsilon \geq \bar{p}c + c$$

$$\Rightarrow \bar{p}\varepsilon \geq c$$

$$\Rightarrow \varepsilon \geq \frac{c}{\bar{p}}$$

but since,

$$\text{Prob}(f^1_{\bar{p}}) = \frac{c}{\bar{p}} > \frac{1}{\bar{p}} \cdot \frac{{}^{\bar{p}}C_N \cdot f^1}{f^1 + \sum_{\bar{p}+1-N-2}^{\bar{p}+1} f^i} > \frac{1}{\bar{p}} \cdot {}^{\bar{p}}C_N \cdot \varepsilon.$$

hence, using this, we get

$$\Rightarrow \varepsilon > \frac{1}{\bar{p}} \cdot {}^{\bar{p}}C_N \cdot \varepsilon$$

$$\Rightarrow \bar{p} > {}^{\bar{p}}C_N.$$

hence a contradiction.

So, $Prob(f_{\bar{p}+1}) < Prob(f_{\bar{p}})$.

Q 3.7,

Ans 3.7, The matlab and Java implementation is with the attachment of the mail. After running the codes for (0,0) as initial point, $\delta = 1$, and number of iterations as 20, we observed that none of the techniques has been converged but opportunistic polling and dynamic polling both after 20 iterations reach point (20,0) with minimum value 2.06115×10^{-9} whereas complete polling displayed result at point (0,-20) with value -19.0 at iteration 20 and since the value is decreasing linearly, hence it seems to converge at $(0,-\infty)$ with value $-\infty$, while at the same time, if we choose the tolerance value as 10^{-8}, both opportunistic polling and dynamic polling had already been converged at local minima.

Even though when we increase the delta value to be 10, we see the same trend because as soon as opportunistic polling and dynamic polling find a value less than current minima, they stop looking for other directions and hence, they converge to local minima while complete polling looking for values in all the four directions have better chance of finding the all the four directions have been [illegible]

minimum value. Had the direction order been $(-e_2, -e_1, e_2, e_1)$ all the three techniques would have been tending to converge towards $-\infty$ with Dynamic and opportunistic polling achieving that with less number of function evaluation.