Assignment 10

Thursday, March 23, 2017 2:07 PM

Q 1 → $f(x,y,z) = \sin x + y^2 z^2$ at $(x,y,z) = (0,1,1)$

Sol. 1 (a) Since, $L(x) = f(\bar{p}) + (\nabla f(\bar{p}))^T (x - \bar{p})$ where $\bar{p} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Now,

$$\nabla f(\bar{x}) = \begin{bmatrix} \cos x \\ 2yz^2 \\ 2y^2 z \end{bmatrix} \qquad \text{where } \bar{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

hence,

$$L(\bar{x}) = 1 + \begin{bmatrix} \cos x \\ 2yz^2 \\ 2y^2 z \end{bmatrix}^T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right)$$

$$= 1 + \begin{bmatrix} \cos x \\ 2yz^2 \\ 2y^2 z \end{bmatrix}^T \begin{bmatrix} x \\ y-1 \\ z-1 \end{bmatrix}$$

Now,

$$Q(\bar{x}) = L(\bar{x}) + \frac{1}{2} (\bar{x} - \bar{p})^T (\nabla^2 f(\bar{p})) (\bar{x} - \bar{p})$$

and,

$$\nabla^2 f(\bar{x}) = \begin{bmatrix} -\sin x & 0 & 0 \\ 0 & 2z^2 & 4yz \\ 0 & 4yz & 2y^2 \end{bmatrix}$$

Hence,

$$Q(\bar{x}) = 1 + \begin{bmatrix} \cos x \\ 2yz^2 \\ 2y^2 z \end{bmatrix}^T \begin{bmatrix} x \\ y-1 \\ z-1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x \\ y-1 \\ z-1 \end{bmatrix}^T \begin{bmatrix} -\sin x & 0 & 0 \\ 0 & 2z^2 & 4yz \\ 0 & 4yz & 2y^2 \end{bmatrix} \begin{bmatrix} x \\ y-1 \\ z-1 \end{bmatrix}$$

(ii)

Sol. $f(x,y,z) = \sin(x) + y^2 z^2$

hence at $p = \begin{bmatrix} -0.1 \\ 1.1 \\ 0.9 \end{bmatrix}$

$f(p) = 0.8803$, $L_{err} = |f(p) - L(p)| = 1.97 \times 10^{-2}$

$L(p) = 0.9000$

and,

$Q(p) = 0.8800$

hence, $Q_{err} = 3 \times 10^{-4}$

Q4.3 → $f(x) = 3 + x_1 + 4x_2 + x_3 + 5x_4 + 9x_5$ and $X = \{0, e_1, e_2, \ldots e_5\}$.

Sol. (a) $L = \begin{bmatrix} 1 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 1 \end{bmatrix} = L^{-1}$

and, $\delta^{f(x)} = \begin{bmatrix} f(y^1) - f(y^0) \\ \vdots \\ f(y^n) - f(y^0) \end{bmatrix} = \begin{bmatrix} 4-3 \\ 7-3 \\ 4-3 \\ 8-3 \\ 12-3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 1 \\ 5 \\ 9 \end{bmatrix}$

and, $\nabla_s f(y) = L^{-1} \cdot \delta^{f(y)}$

$= I \cdot \begin{bmatrix} 1 \\ 4 \\ 1 \\ 5 \\ 9 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 4 \\ 1 \\ 5 \\ 9 \end{bmatrix}$

Sol. (b) Now, $\nabla f(x) = \begin{bmatrix} 1 \\ 4 \\ 1 \\ 5 \\ 9 \end{bmatrix}$

$\nabla f(x) = \nabla_s f(x)$

Both the above values are same because the function itself is linear and, Y consists of unit co-ordinate vectors.

Q9.5 → $f(x) = e^x$.

(a)

Sol. (a) let d be such that $x+d \in B_{\bar{\Delta}}(x)$, $d \neq 0$, then

$$\frac{|e^{x+d} - e^x|}{\|x+d-x\|} = \frac{e^x |e^d - 1|}{\|d\|} \leq \frac{e^x |e^{\bar{\Delta}} - 1|}{\bar{\Delta}}$$ [as $\|d\| \leq \bar{\Delta}$]

Hence, for $\bar{\Delta} > 0$,

$$f \in C^{1+} \text{ with } k = \frac{e^x |e^{\bar{\Delta}} - 1|}{\bar{\Delta}}$$

(b)

Sol. (b) $f \notin C^{1+}$ with constant k for any k because if $\bar{\Delta}$ goes to $+\infty$, k is undefined.

(c)

Sol. (c) This relates to the importance of $\bar{\Delta}$ in Theorems 9.10, 9.15, 9.18 and 9.21, because the error bounds on the models in above theorems depend upon $\bar{\Delta}$ and, if $\bar{\Delta}$ goes to infinity, the models makes no sense as the error bound between $\tilde{f}_\Delta$ and f also go to infinity.