Assignment 12

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Q 10.5 →

Sol. 10.5 → let $f(x) = x^2$. $\forall x \in \mathbb{R}^2$.

Then, $\nabla f(x) = 2x$. , So $g^k = 2x^k$ and, let $d^k = -g^k$.

So,

$$f(x^k + t^k d^k) < f(x^k) + \eta t^k (d^k)^T \nabla f(x^k)$$

$$\Rightarrow (x^k - 2t^k x^k)^2 < (x^k)^2 + \eta t^k - 4\langle x^k, x^k \rangle$$

$$\Rightarrow (x^k)^2 (1 - 2t^k)^2 < (x^k)^2 (1 - \eta 4 t^k) \quad \text{where } (x^k)^2 = \langle x^k, x^k \rangle$$

$$\Rightarrow 1 + 4(t^k)^2 - 4t^k < 1 - \eta 4 t^k.$$

let $\eta = \frac{1}{2}$.

let $\eta = 1$.

$\Rightarrow \quad 1 + (4t^k)^2 < 1$

$\Rightarrow \quad (4t^k)^2 < 0$

$\Rightarrow \quad (t^k)^2 < 0$ $\quad \Longrightarrow$ not feasible.

So, for $f(x) = x^2$, $x \in \mathbb{R}^2$. Any value of x with steepest descent, d^k, the inequility doesn't hold at $\eta = 1$.

Q 10.11

Sol. 10.11 $\Rightarrow$ $f(x) = (x_1)^2 + (x_2)^2$, $\eta = 1/2$, $\bar{x} = (1,1)$. $\tilde{g}_\Delta(x) = [2x_1 + b, 2x_2 - b]^T$

and, $d = -\hat{g}_\Delta(x)$

By substituting the above values in the given equation, we get

$$(1 + t(\Delta+2))^2 + (1 + t(2-\Delta))^2 \le (1)^2 + (1)^2 - \frac{t}{2}\left((\Delta+2)^2 + (2-\Delta)^2\right)$$

$$\Rightarrow \underline{1} + t^2(\Delta+2)^2 + 2t(\Delta+2) + \underline{1} + t^2(2-\Delta)^2 + 2t(2-\Delta)$$

$$\le \underline{2} - \frac{t}{2}\left((\Delta+2)^2 + (2-\Delta)^2\right)$$

$$\Rightarrow t^{\cancel{2}}\left((\Delta+2)^2 + (2-\Delta)^2\right) + 8\cancel{t} \le \cancel{-\frac{t}{2}}\left((\Delta+2)^2 + (2-\Delta)^2\right)$$

$$\Rightarrow t\left((\Delta+2)^2 + (2-\Delta)^2\right) \le 8 - \frac{t}{2}\left((\Delta+2)^2 + (2-\Delta)^2\right)$$

$$\Rightarrow t \le \frac{8}{(\Delta+2)^2 + (2-\Delta)^2}$$