HMATH604 - Assignment 7 - Deepak Kumar,73217151

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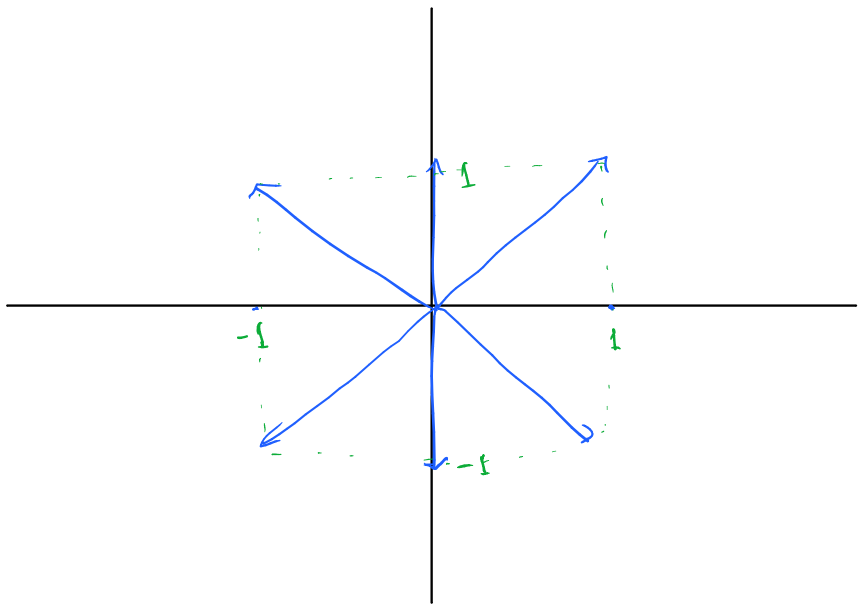
Q7.2→

$$D = \begin{bmatrix} 0 & 1 & -1 & 0 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 \end{bmatrix}$$

(a)



(b) Since, we need to find distinct positive spanning sets from column vectors of D, we can do this by

(i) removing zero vectors from D.

$D_1 = D$.

(ii) removing one vector from D.

$$D_2 = \begin{bmatrix} 0 & 1 & -1 & 0 & 1 \\ 1 & 1 & 1 & -1 & -1 \end{bmatrix}, D_3 = \begin{bmatrix} 0 & 1 & -1 & 0 & -1 \\ 1 & 1 & 1 & -1 & -1 \end{bmatrix} \ldots\ldots$$

hence by removing one vector at once and keeping rest five, number of distinct set = 6.

(iii) removing two vectors at once from D.

$$D_8 = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}, \quad D_9 = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}, \quad D_{10} = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

number of distinct sets = 3.

(iv) By removing three vectors at once from D.

$$D_{11} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

number of distinct sets = 2.

(v) By removing four or more vectors at once from D.

number of distinct vectors = 0.

Ans(b) Hence, number of distinct positive spanning sets = 12.

(c)

Ans (c) we can construct 5 distinct positive bases from column vectors of D. (D_8 to D_{12} in (b) part).

Q7.4>

(a) Let $Z = [z_1\ z_2] \in \mathbb{Z}^{3\times 2}$ and lets assume that $\mathrm{pspan}(Z) = \mathbb{R}^3$.

$(z_1 + z_2)$ as linear combination of

(a) Let $Z = [z_1\ z_2]$ ∈ Z and ...

So, we can write $z_3 = -\frac{1}{2}(z_1 + z_2)$ as linear combination of column vectors in Z i.e.

$$z_3 = \sum_{i=1}^{2} \lambda_i z_i$$

$$= \lambda_1 z_1 + \lambda_2 z_2$$

$$\Rightarrow -\frac{1}{2} z_1 + \left(-\frac{1}{2}\right) z_2 = \lambda_1 z_1 + \lambda_2 z_2$$

which inturn implies that $\lambda_1 = \lambda_2 = -1/2 < 0$. Hence, a contradiction. Similarly we can prove for $Z \in Z^{3\times3}$. This complies with the fact the smallest ~~p~~spanning set will always be the ~~p~~basis with $n+1$ number of vectors, where n is the dimension of the vector.

(b)

Sol.(b) for $p=4$,

$$D = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

for $p=5$,

$$D = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix}$$

For $p=6$,

$$D = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{bmatrix}$$

For $p=7$,

$$D = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 & 1 \end{bmatrix}$$

For $p=8$,

$$D = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

(c)

Ans (c). Since, Column vectors of Z form a p-spanning set, hence we always find a p-basis and add to the Column of Z linear Combinations of vectors from p-basis and still it would be p-spanning set. Now since we can form infinite number of such linear combinations, hence there is no upper limit on the dimensions of Z.

Q7.8 →

(d) Since $p_i = -a + bu$ and for part (b) $a = 3.1$, a well defined fixed decimal

Q1.6

(d) Since $p_i = -a + bu$ and for part (b) $a = 3.1$, a well defined fixed decimal value, so for any value of a and b in the given range, the distance between two grid points on p is exact while part (c) $u = 22/7$ is a repeating decimal value and, hence, the distance between grid points will not be exact and grid will not be consistent throughout.

When we programmed in matlab we found that the grids for part (b) is like what we expected from theory but for part (c) it is consistent and the distance between two adjacent grid points is same. This happened due to the fact that matlab approximated 22/7 as 3.1429 which is a well defined fixed decimal value.