MODULE 1

CLASSICAL ENCRYPTION TECHNIQUES

Symmetric encryption, also referred to as conventional encryption or single-key encryption, was the only type of encryption in use prior to the development of public-key encryption in the 1970s. It remains by far the most widely used of the two types of encryption. Part One examines a number of symmetric ciphers. In this chapter, we begin with a look at a general model for the symmetric encryption process; this will enable us to understand the context within which the algorithms are used. Next, we examine a variety of algorithms in use before the computer era. Finally, we look briefly at a different approach known as steganography. Chapter 3 examines the most widely used symmetric cipher: DES.

Before beginning, we define some terms. An original message is known as the **plaintext**, while the coded message is called the **ciphertext**. The process of converting from plaintext to ciphertext is known as **enciphering** or **encryption**; restoring the plaintext from the ciphertext is **deciphering** or **decryption**. The many schemes used for encryption constitute the area of study known as **cryptography**. Such a scheme is known as a **cryptographic system** or a **cipher**. Techniques used for deciphering a message without any knowledge of the enciphering details fall into the area of **cryptanalysis**. Cryptanalysis is what the layperson calls "breaking the code." The areas of cryptography and cryptanalysis together are called **cryptology**.

Symmetric Cipher Model

A symmetric encryption scheme has five ingredients (Figure 2.1):

Plaintext: This is the original intelligible message or data that is fed into the algorithm as input.

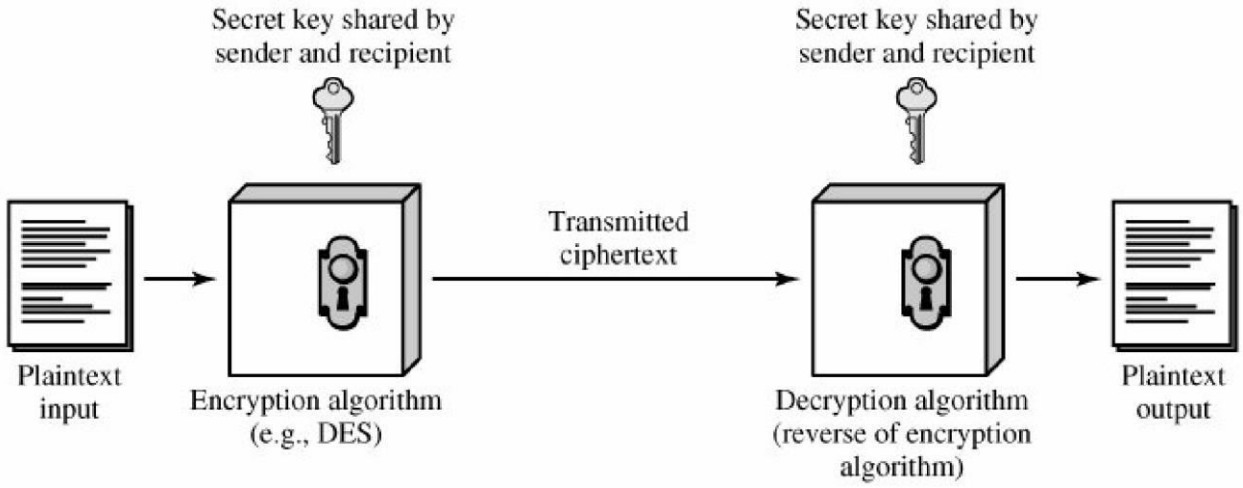
Encryption algorithm: The encryption algorithm performs various substitutions and transformations on the plaintext.

Secret key: The secret key is also input to the encryption algorithm. The key is a value independent of the plaintext and of the algorithm. The algorithm will produce a different output depending on the specific key being used at the time. The exact substitutions and transformations performed by the algorithm depend on the key.

Cipher text: This is the scrambled message produced as output. It depends on the plaintext and the secret key. For a given message, two different keys will produce two different cipher texts. The cipher text is an apparently random stream of data and, as it stands, is unintelligible.

Decryption algorithm: This is essentially the encryption algorithm run in reverse. It takes the cipher text and the secret key and produces the original plaintext.

Figure 2.1. Simplified Model of Conventional Encryption



There are two requirements for secure use of conventional encryption:

1. We need a strong encryption algorithm. At a minimum, we would like the algorithm to be such that an opponent who knows the algorithm and has access to one or more ciphertexts would be unable to decipher the ciphertext or figure out the key. This requirement is usually stated in a stronger form: The opponent should be unable to decrypt ciphertext or discover the key even if he or she is in possession of a number of ciphertexts together with the plaintext that produced each ciphertext.

2. Sender and receiver must have obtained copies of the secret key in a secure fashion and must keep the key secure. If someone can discover the key and knows the algorithm, all communication using this key is readable.

We assume that it is impractical to decrypt a message on the basis of the ciphertext *plus* knowledge of the encryption/decryption algorithm. In other words, we do not need to keep the algorithm secret; we need to keep only the key secret. This feature of symmetric encryption is

what makes it feasible for widespread use. The fact that the algorithm need not be kept secret means that manufacturers can and have developed low-cost chip implementations of data encryption algorithms. These chips are widely available and incorporated into a number of products. With the use of symmetric encryption, the principal security problem is maintaining the secrecy of the key.

Let us take a closer look at the essential elements of a symmetric encryption scheme, using Figure 2.2. A source produces a message in plaintext, $X = [X1, X2, ..., XM]$. The M elements of X are letters in some finite alphabet. Traditionally, the alphabet usually consisted of the 26 capital letters. Nowadays, the binary alphabet {0, 1} is typically used. For encryption, a key of the form $K = [K1, K2, ..., KJ]$ is generated. If the key is generated at the message source, then it must also be provided to the destination by means of some secure channel. Alternatively, a third party could generate the key and securely deliver it to both source and destination.

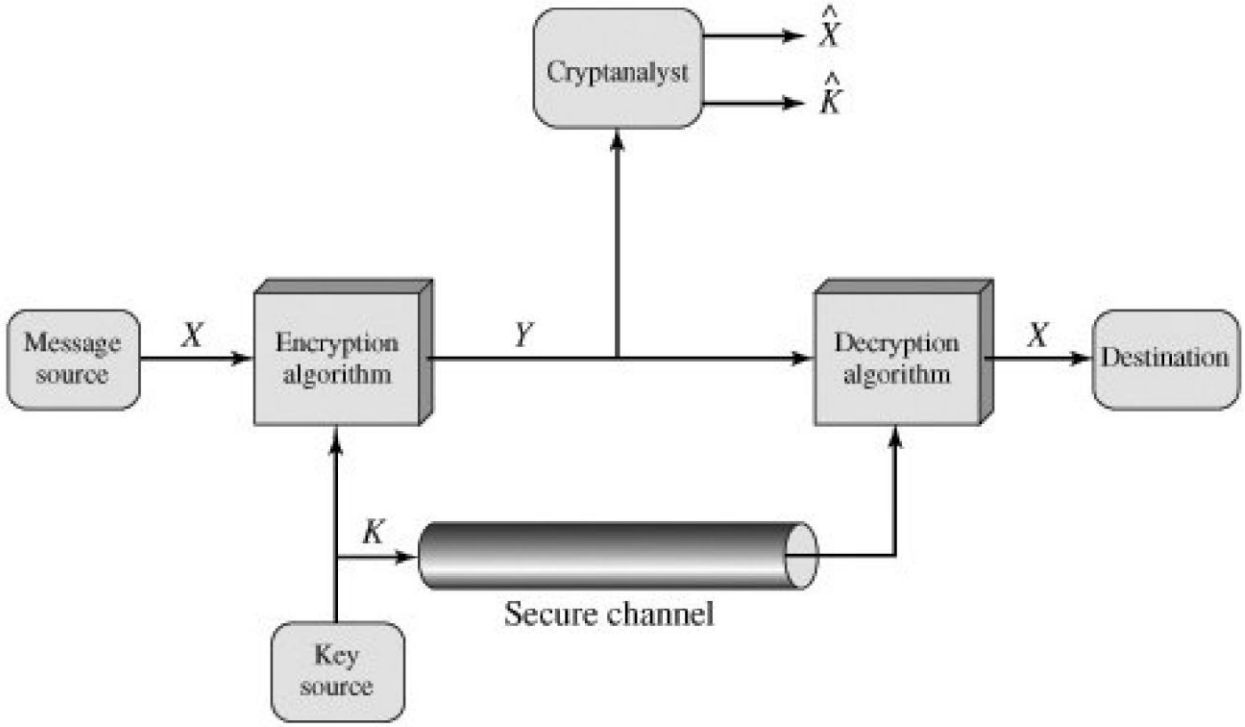


Figure 2.2. Model of Conventional Cryptosystem

With the message X and the encryption key K as input, the encryption algorithm forms the ciphertext $Y = [Y1, Y2, ..., YN]$. We can write this As $Y = E(K, X)$

This notation indicates that Y is produced by using encryption algorithm E as a function of the plaintex$X$t , with the specific function determined by the value of the key K.

The intended receiver, in possession of the key, is able to invert the transformation:

$X = D(K, Y)$

An opponent, observing Y but not having access to K or X, may attempt to recover X or K or both X and K. It is assumed that the opponent knows the encryption (E) and decryption (D) algorithms. If the opponent is interested in only this particular message, then the focus of the effort is to recover X by generating a plaintext estimate . Often, however, the opponent is interested in being able to read future messages as well, in which case an attempt is made to recover K by generating an estimate .

Cryptography

Cryptographic systems are characterized along three independent dimensions:

1. The type of operations used for transforming plaintext to ciphertext. All encryption algorithms are based on two general principles: substitution, in which each element in the plaintext (bit, letter, group of bits or letters) is mapped into another element, and transposition, in which elements in the plaintext are rearranged. The fundamental requirement is that no information be lost (that is, that all operations are reversible). Most systems, referred to as *product systems*, involve multiple stages of substitutions and transpositions.

2. The number of keys used. If both sender and receiver use the same key, the system is referred to as symmetric, single-key, secret-key, or conventional encryption. If the sender and receiver use different keys, the system is referred to as asymmetric, two-key, or public-key encryption.

3.The way in which the plaintext is processed. A *block cipher* processes the input one block of elements at a time, producing an output block for each input block. A *stream cipher* processes the input elements continuously, producing output one element at a time, as it goes along.

Cryptanalysis

Typically, the objective of attacking an encryption system is to recover the key in use rather then simply to recover the plaintext of a single ciphertext. There are two general approaches to attacking a conventional encryption scheme:

- **Cryptanalysis:** Cryptanalytic attacks rely on the nature of the algorithm plus perhaps some knowledge of the general characteristics of the plaintext or even some sample plaintext-ciphertext pairs. This type of attack exploits the characteristics of the algorithm to attempt to deduce a specific plaintext or to deduce the key being used.

- **Brute-force attack:** The attacker tries every possible key on a piece of ciphertext until an intelligible translation into plaintext is obtained. On average, half of all possible keys must be tried to achieve success. If either type of attack succeeds in deducing the key, the effect is catastrophic: All future and past messages encrypted with that key are compromised.

Table 2.1 summarizes the various types of **cryptanalytic attacks**, based on the amount of information known to the cryptanalyst. The most difficult problem is presented when all that is available is the *cipher text only*. In some cases, not even the encryption algorithm is known, but in general we can assume that the opponent does know the algorithm used for encryption. One possible attack under these circumstances is the brute-force approach of trying all possible keys. If the key space is very large, this becomes impractical. Thus, the opponent must rely on an analysis of the cipher text itself, generally applying various statistical tests to it.

Table 2.1. Types of Attacks on Encrypted Messages

Type of Attack Known to Cryptanalyst

Cipher text only

- Encryption algorithm
- Cipher text

Known plaintext

- Encryption algorithm
- Cipher text
- One or more plaintext-cipher text pairs formed with the secret key

Chosen plaintext

- Encryption algorithm
- Cipher text
- Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key

Chosen cipher text

- Encryption algorithm
- Cipher text

- Purported ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key

Chosen text

- Encryption algorithm
- Ciphertext
- Plaintext message chosen by cryptanalyst, together with its corresponding
- ciphertext generated with the secret key
- Purported ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key

A **brute-force attack** involves trying every possible key until an intelligible translation of the ciphertext into plaintext is obtained. On average, half of all possible keys must be tried to achieve success.

Results are shown for four binary key sizes. The 56-bit key size is used with the DES (Data Encryption Standard) algorithm, and the 168-bit key size is used for triple DES. The minimum key size specified for AES (Advanced Encryption Standard) is 128 bits. Results are also shown for what are called substitution codes that use a 26-character key (discussed later), in which all possible permutations of the 26 characters serve as keys. For each key size, the results are shown assuming that it takes 1 *m*s to perform a single decryption, which is a reasonable order of magnitude for today's machines. With the use of massively parallel organizations of microprocessors, it may be possible to achieve processing rates many orders of magnitude greater.

Substitution Techniques

In this section and the next, we examine a sampling of what might be called classical encryption techniques. A study of these techniques enables us to illustrate the basic approaches to symmetric encryption used today and the types of cryptanalytic attacks that must be anticipated.

The two basic building blocks of all encryption techniques are substitution and transposition. We examine these in the next two sections. Finally, we discuss a system that combines both substitution and transposition. A substitution technique is one in which the letters of plaintext are replaced by other letters or by numbers or symbols. If the plaintext is viewed as a sequence of bits, then substitution involves replacing plaintext bit patterns with ciphertext bit patterns

Caesar Cipher

The earliest known use of a substitution cipher, and the simplest, was by Julius Caesar. The Caesar cipher involves replacing each letter of the alphabet with the letter standing three places further down the alphabet. For example,

plain: meet me after the toga party

Cipher: PHHW PH DIWHU WKH WRJD SDUWB

Note that the alphabet is wrapped around, so that the letter following Z is A. We can define the transformation by listing all possibilities, as follows:

plain: a b c d e f g h i j k l m n o p q r s t u v w x y z

cipher: D E F G H I J K L M N O P Q R S T U V W X Y Z A B C

Let us assign a numerical equivalent to each letter:

a	b	c	d	e	f	g	h	i	j	k	l	m
0	1	2	3	4	5	6	7	8	9	10	11	12
n	o	p	q	r	s	t	u	v	w	x	y	z
13	14	15	16	17	18	19	20	21	22	23	24	25

Then the algorithm can be expressed as follows. For each plaintext letter p, substitute the ciphertext letter C:

$C = \mathrm{E}(3, p) = (p + 3) \bmod 26$

A shift may be of any amount, so that the general Caesar algorithm is

$C = \mathrm{E}(k, p) = (p + k) \bmod 26$

where k takes on a value in the range 1 to 25. The decryption algorithm is simply

$p = \mathrm{D}(k, C) = (C\ k) \bmod 26$

If it is known that a given ciphertext is a Caesar cipher, then a brute-force cryptanalysis is easily performed: Simply try all the 25 possible keys. Figure 2.3 shows the results of applying this strategy to the example ciphertext. In this case, the plaintext leaps out as occupying the third line.

KEY	PHHW PH DIWHU WKH WRJD SDUWB
1	oggv og chvgt vjg vqic rctva
2	nffu nf bgufs uif uphb qbsuz
3	meet me after the toga party
4	ldds ld zesdq sgd snfz ozqsx
5	kccr kc ydrcp rfc rmey nyprw
6	jbbq jb xcqbo qeb qldx mxoqv
7	iaap ia wbpan pda pkcw lwnpu
8	hzzo hz vaozm ocz ojbv kvmot
9	gyyn gy uznyl nby niau julns
10	fxxm fx tymxk max mhzt itkmr
11	ewwl ew sxlwj lzw lgys hsjlq
12	dvvk dv rwkvi kyv kfxr grikp
13	cuuj cu qvjuh jxu jewq fqhjo
14	btti bt puitg iwt idvp epgin
15	assh as othsf hvs hcuo dofhm
16	zrrg zr nsgre gur gbtn cnegl
17	yqqf yq mrfqd ftq fasm bmdfk
18	xppe xp lqepc esp ezrl alcej
19	wood wo kpdob dro dyqk zkbdi
20	vnnc vn jocna cqn cxpj yjach
21	ummb um inbmz bpm bwoi xizbg
22	tlla tl hmaly aol avnh whyaf
23	skkz sk glzkx znk zumg vgxze
24	rjjy rj fkyjw ymj ytlf ufwyd
25	qiix qi ejxiv xli xske tevxc

Three important characteristics of this problem enabled us to use a brute-force cryptanalysis:

1. The encryption and decryption algorithms are known.

2. There are only 25 keys to try.

3. The language of the plaintext is known and easily recognizable

In most networking situations, we can assume that the algorithms are known. What generally makes brute-force cryptanalysis impractical is the use of an algorithm that employs a large number of keys. For example, the triple DES algorithm, examined in Chapter 6, makes use of a 168-bit key, giving a key space of 2 168 or greater than 3.7 x 10^5 possible keys. The third characteristic is also significant. If the language of the plaintext is unknown, then plaintext output may not be recognizable. Furthermore, the input may be abbreviated or compressed in some fashion, again making recognition difficult.

Monoalphabetic Ciphers

With only 25 possible keys, the Caesar cipher is far from secure. A dramatic increase in the key space can be achieved by allowing an arbitrary substitution. Recall the assignment for the Caesar cipher:

plain: a b c d e f g h i j k l m n o p q r s t u v w x y z

cipher: D E F G H I J K L M N O P Q R S T U V W X Y Z A B C

If, instead, the "cipher" line can be any permutation of the 26 alphabetic characters, then there are 26! or greater than 4 x 10^26 possible keys. This is 10 orders of magnitude greater than the key space for DES and would seem to eliminate brute-force techniques for cryptanalysis. Such an approach is referred to as a **monoalphabetic substitution cipher**, because a single cipher alphabet (mapping from plain alphabet to cipher alphabet) is used per message. There is, however, another line of attack. If the cryptanalyst knows the nature of the plaintext (e.g., noncompressed English text), then the analyst can exploit the regularities of the language. To see how such a cryptanalysis might proceed. The ciphertext to be solved is

UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAIZ

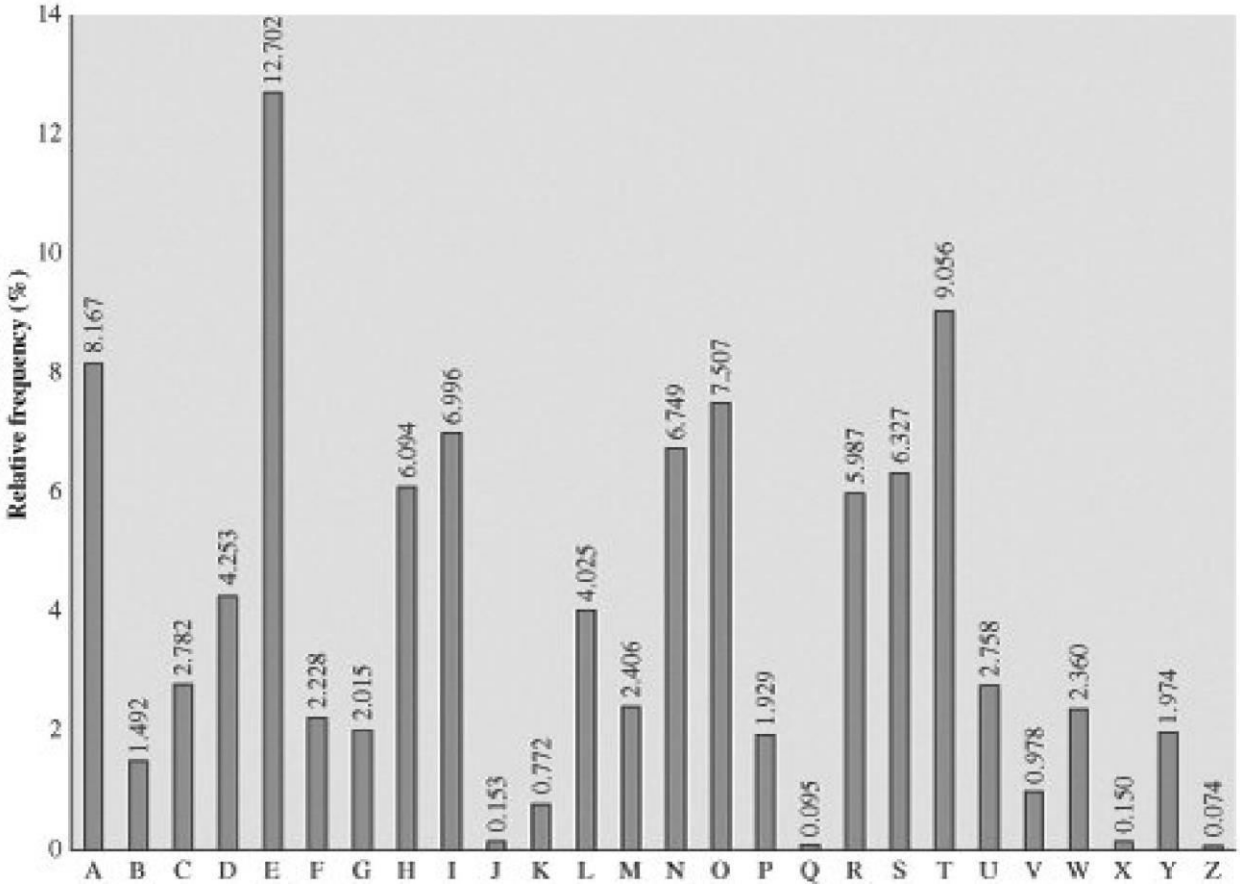
VUEPHZHMDZSHZOWSFPAPPDTSVPQUZWYMXUZUHSX

EPYEPOPDZSZUFPOMBZWPFUPZHMDJUDTMOHMQ

As a first step, the relative frequency of the letters can be determined and compared to a standard frequency distribution for English, such as is shown in Figure 2.5. If the message were long enough, this technique alone might be sufficient, but because this is a relatively short message, we cannot expect an exact match. In any case, the relative frequencies of the letters in the ciphertext (in percentages) are as follows:

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	T 2.50	I 0.83	N 0.00
O 7.50	X 4.17	A 1.67	J 0.83	R 0.00
M 6.67				

Figure 2.5. Relative Frequency of Letters in English Text



Comparing this breakdown with Figure 2.5, it seems likely that cipher letters P and Z are the equivalents of plain letters e and t, but it is not certain which is which. The letters S, U, O, M, and H are all of relatively high frequency and probably correspond to plain letters from the set {a, h, i, n, o, r, s}.The letters with the lowest frequencies (namely, A, B, G, Y, I, J) are likely included in the set {b, j, k, q, v, x, z}. There are a number of ways to proceed at this point. We could make some tentative assignments and start to fill in the plaintext to see if it looks like a

reasonable "skeleton" of a message. A more systematic approach is to look for other regularities. For example, certain words may be known to be in the text. Or we could look for repeating sequences of cipher letters and try to deduce their plaintext equivalents.

A powerful tool is to look at the frequency of two-letter combinations, known as digrams. A table similar to Figure 2.5 could be drawn up showing the relative frequency of digrams. The most common such digram is th. In our cipher text, the most common digram is ZW, which appears three times. So we make the correspondence of Z with t and W with h. Then, by our earlier hypothesis, we can equate P with e. Now notice that the sequence ZWP appears in the cipher text, and we can translate that sequence as "the." This is the most frequent trigram (three-letter combination) in English, which seems to indicate that we are on the right track. Next, notice the sequence ZWSZ in the first line. We do not know that these four letters form a complete word, but if they do, it is of the form th_t. If so, S equates with a. So far, then, we have

UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAIZ

t a e e te a that e e a a

VUEPHZHMDZSHZOWSFPAPPDTSVPQUZWYMXUZUHSX

e t ta t ha e ee a e th t a

EPYEPOPDZSZUFPOMBZWPFUPZHMDJUDTMOHMQ

e e e tat e the t

Only four letters have been identified, but already we have quite a bit of the message. Continued analysis of frequencies plus trial and error should easily yield a solution from this point. The complete plaintext, with spaces added between words, follows: it was disclosed yesterday that several informal but direct contacts have been made with political representatives of the viet cong in Moscow Monoalphabetic ciphers are easy to break because they reflect the frequency data of the original alphabet. A countermeasure is to provide multiple substitutes, known as homophones, for a single letter. For example, the letter e could be assigned a number of different cipher symbols, such as 16, 74, 35, and 21, with each homophone used in rotation, or randomly. If the number of symbols assigned to each letter is proportional to the relative frequency of that letter, then single-letter frequency information is completely obliterated. The great mathematician Carl Friedrich Gauss believed that he had devised an unbreakable cipher using homophones. However, even with homophones, each element of plaintext affects only one element of ciphertext, and multiple-letter patterns (e.g., digram frequencies) still survive in the

ciphertext, making cryptanalysis relatively straightforward. Two principal methods are used in substitution ciphers to lessen the extent to which the structure of the plaintext survives in the ciphertext: One approach is to encrypt multiple letters of plaintext, and the other is to use multiple cipher alphabets. We briefly examine each.

Playfair Cipher

The best-known multiple-letter encryption cipher is the Playfair, which treats digrams in the plaintext as single units and translates these units into ciphertext digrams.

The Playfair algorithm is based on the use of a 5 x 5 matrix of letters constructed using a keyword. Here is an example, solved by Lord Peter Wimsey in Dorothy Sayers's *Have His Carcase*.

M O N A R

C H Y B D

E F G I/J K

L P Q S T

U V W X Z

In this case, the keyword is *monarchy*. The matrix is constructed by filling in the letters of the keyword (minus duplicates) from left to right and from top to bottom, and then filling in the remainder of the matrix with the remaining letters in alphabetic order. The letters I and J count as one letter. Plaintext is encrypted two letters at a time, according to the following rules:

1. Repeating plaintext letters that are in the same pair are separated with a filler letter, such as x, so that balloon would be treated as ba lx lo on.

2. Two plaintext letters that fall in the same row of the matrix are each replaced by the letter to the right, with the first element of the row circularly following the last. For example, ar is encrypted as RM.

3. Two plaintext letters that fall in the same column are each replaced by the letter beneath, with the top element of the column circularly following the last. For example, mu is encrypted as CM.

4. Otherwise, each plaintext letter in a pair is replaced by the letter that lies in its own row and the column occupied by the other plaintext letter. Thus, hs becomes BP and ea becomes IM (or JM, as the encipherer wishes).

The Playfair cipher is a great advance over simple monoalphabetic ciphers. For one thing, whereas there are only 26 letters, there are 26 x 26 = 676 digrams, so that identification of

individual digrams is more difficult. Furthermore, the relative frequencies of individual letters exhibit a much greater range than that of digrams, making frequency analysis much more difficult. For these reasons, the Playfair cipher was for a long time considered unbreakable. It was used as the standard field system by the British Army in World War I and still enjoyed considerable use by the U.S. Army and other Allied forces during World War II. Despite this level of confidence in its security, the Playfair cipher is relatively easy to break because it still leaves much of the structure of the plaintext language intact. A few hundred letters of ciphertext are generally sufficient.

Hill Cipher

Another interesting multiletter cipher is the Hill cipher, developed by the mathematician Lester Hill in 1929. The encryption algorithm takes *m* successive plaintext letters and substitutes for them *m* ciphertext letters. The substitution is determined by *m* linear equations in which each character is assigned a numerical value (a = 0, b = 1 ... z = 25). For *m* = 3, the system can be described as follows:

c1 = ($k11P1 + k12P2 + k13P3$) mod 26

c2 = ($k21P1 + k22P2 + k23P3$) mod 26

c3 = ($k31P1 + k32P2 + k33P3$) mod 26

This can be expressed in term of column vectors and matrices:

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \bmod 26$$

or

C = **KP** mod 26

where **C** and **P** are column vectors of length 3, representing the plaintext and ciphertext, and **K** is a 3 x 3 matrix, representing the encryption key. Operations are performed mod 26. For example, consider the plaintext "paymoremoney" and use the encryption key

$$\mathbf{K} = \begin{pmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{pmatrix}$$

The first three letters of the plaintext are represented by the vector $\begin{pmatrix} 15 \\ 0 \\ 24 \end{pmatrix}$. Then $\mathbf{K}\begin{pmatrix} 15 \\ 0 \\ 24 \end{pmatrix} = \begin{pmatrix} 375 \\ 819 \\ 486 \end{pmatrix} \bmod 26 = \begin{pmatrix} 11 \\ 13 \\ 18 \end{pmatrix} =$ LNS. Continuing in this fashion, the ciphertext for the entire plaintext is LNSHDLEWMTRW.

Decryption requires using the inverse of the matrix **K**. The inverse **K-**1 of a matrix **K** is defined by the equation **KK-**1 = **K-**1**K** = **I**, where **I** is the matrix that is all zeros except for ones along the main diagonal from upper left to lower right. The inverse of a matrix does not always exist, but when it does, it satisfies the preceding equation. In this case, the inverse

$$\mathbf{K}^{-1} = \begin{pmatrix} 4 & 9 & 15 \\ 15 & 17 & 6 \\ 24 & 0 & 17 \end{pmatrix}$$

This is demonstrated as follows:

$$\begin{pmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{pmatrix}\begin{pmatrix} 4 & 9 & 15 \\ 15 & 17 & 6 \\ 24 & 0 & 17 \end{pmatrix} = \begin{pmatrix} 443 & 442 & 442 \\ 858 & 495 & 780 \\ 494 & 52 & 365 \end{pmatrix} \bmod 26 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It is easily seen that if the matrix **K-**1 is applied to the ciphertext, then the plaintext is recovered. To explain how the inverse of a matrix is determined, we make an exceedingly brief excursion into linear algebra. For any square matrix (*m* x *m*) the **determinant** equals the sum of all the products that can be formed by taking exactly one element from each row and exactly one element from each column, with certain of the product terms preceded by a minus sign. For a 2*2 matrix,

$$\begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}$$

the determinant is $k11k22$ $k12k21$. For a 3 x 3 matrix, the value of the determinant is$k$ $11k22k33$ + $k21k32k13$ + $k31k12k23$ $k31k22k13$

$k21k12k33$ $k11k32k23$. If a square matrix**A** has a nonzero determinant, then the inverse of the matrix is computed as$\boldsymbol{A^1}ij$ =$(1)^{I+J}$(Dij)/ded(**A**), where (Dij) is the subdeterminant formed by deleting the ith row and the jth column of **A** and det(**A**) is the determinant of **A**. For our purposes, all arithmetic is done mod 26.

In general terms, the Hill system can be expressed as follows:

C = E(**K, P**) = **KP** mod 26

P = D(**K, P**) = $\boldsymbol{K^1}$**C** mod 26 = $\boldsymbol{K^1}$ **KP** = **P**

As with Playfair, the strength of the Hill cipher is that it completely hides single-letter frequencies. Indeed, with Hill, the use of a larger

matrix hides more frequency information. Thus a 3 x 3 Hill cipher hides not only single-letter but also two-letter frequency information.

Although the Hill cipher is strong against a cipher text-only attack, it is easily broken with a known plaintext attack. For an m x m Hill cipher,

suppose we have m plaintext-ciphe r text pairs, each of length m. We label the pairs

$$\mathbf{P}_j = \begin{pmatrix} p_{1j} \\ p_{2j} \\ \vdots \\ p_{mj} \end{pmatrix} \text{ and } \mathbf{C}_j = \begin{pmatrix} c_{1j} \\ c_{2j} \\ \vdots \\ c_{mj} \end{pmatrix} \text{ such that } \mathbf{C}_j = \mathbf{KP}_j \text{ for } 1 \le j \le m \text{ and for some}$$

unknown key matrix **K**. Now define two m x m matrices **X** = (Pij) and **Y** = (Cij). Then we can form the matrix equation **Y** = **KX**. If **X** has an inverse, then we can determine **K** = **YX**1.If **X** is not invertible, then a new version of **X** can be formed with additional plaintext-ciphertext pairs until an invertible **X** is obtained. Suppose that the plaintext "friday" is encrypted using a 2 x 2 Hill cipher to yield the ciphertext PQCFKU. Thus, we know that

$$\mathbf{K}\begin{pmatrix} 5 \\ 17 \end{pmatrix} \bmod 26 = \begin{pmatrix} 15 \\ 16 \end{pmatrix}; \mathbf{K}\begin{pmatrix} 8 \\ 3 \end{pmatrix} \bmod 26 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}; \quad \text{and} \quad \mathbf{K}\begin{pmatrix} 0 \\ 24 \end{pmatrix} \bmod 26 = \begin{pmatrix} 10 \\ 20 \end{pmatrix}$$

Using the first two plaintext-ciphertext pairs, we have

$$\begin{pmatrix} 15 & 2 \\ 16 & 5 \end{pmatrix} = \mathbf{K}\begin{pmatrix} 5 & 8 \\ 17 & 3 \end{pmatrix} \bmod 26$$

The inverse of **X** can be computed:

$$\mathbf{K} = \begin{pmatrix} 15 & 2 \\ 16 & 5 \end{pmatrix}\begin{pmatrix} 9 & 2 \\ 1 & 15 \end{pmatrix} = \begin{pmatrix} 137 & 60 \\ 149 & 107 \end{pmatrix} \bmod 26 = \begin{pmatrix} 7 & 8 \\ 19 & 3 \end{pmatrix}$$

Polyalphabetic Ciphers

Another way to improve on the simple monoalphabetic technique is to use different monoalphabetic substitutions as one proceeds through the plaintext message. The general name for this approach is **polyalphabetic substitution cipher**. All these techniques have the following features in common:

1. A set of related monoalphabetic substitution rules is used.

2. A key determines which particular rule is chosen for a given transformation.

The best known, and one of the simplest, such algorithm is referred to as the Vigenère cipher. In this scheme, the set of related monoalphabetic substitution rules consists of the 26 Caesar ciphers, with shifts of 0 through 25. Each cipher is denoted by a key letter,Mwhich is the ciphertext letter that substitutes for the plaintext letter a. Thus, a Caesar cipher with a shift of 3 is denoted by the key value *d*. To aid in understanding the scheme and to aid in its use, a matrix known as the Vigenère tableau is constructed (Table 2.3). Each of the 26 ciphers is laid out horizontally, with the key letter for each cipher to its left. A normal alphabet for the plaintext runs across the top. The process of encryption is simple: Given a key letter *x* and a plaintext letter y, the ciphertext letter is at the intersection of the row labeled *x* and the column labeled y; in this case the ciphertext is V.

Key	Plaintext																									
	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
a	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
b	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
c	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
d	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
e	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
f	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
g	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
h	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
i	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
j	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
k	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
l	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
m	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
n	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
o	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
p	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
r	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
s	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
t	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
u	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
v	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
w	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
x	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Table 2.3. The Modern Vigenère Tableau

To encrypt a message, a key is needed that is as long as the message. Usually, the key is a repeating keyword. For example, if the keyword is *deceptive*, the message "we are discovered save yourself" is encrypted as follows:

key: *deceptivedeceptivedeceptive*

plaintext: wearediscoveredsaveyourself

ciphertext: ZICVTWQNGRZGVTWAVZHCQYGLMGJ

Decryption is equally simple. The key letter again identifies the row. The position of the ciphertext letter in that row determines the column, and the plaintext letter is at the top of that column. The strength of this cipher is that there are multiple ciphertext letters for each plaintext letter, one for each unique letter of the keyword. Thus, the letter frequency information is obscured. However, not all knowledge of the plaintext structure is lost. For example, Figure 2.6 shows the frequency distribution for a Vigenère cipher with a keyword of length 9. An improvement is achieved over the Playfair cipher, but considerable frequency information remains.

It is instructive to sketch a method of breaking this cipher, because the method reveals some of the mathematical principles that apply in cryptanalysis. First, suppose that the opponent believes that the ciphertext was encrypted using either monoalphabetic substitution or a Vigenère cipher.

A simple test can be made to make a determination. If a monoalphabetic substitution is used, then the statistical properties of the ciphertext should be the same as that of the language of the plaintext. Thus, referring to Figure 2.5, there should be one cipher letter with a relative frequency of occurrence of about 12.7%, one with about 9.06%, and so on. If only a single message is available for analysis, we would not expect an exact match of this small sample with the statistical profile of the plaintext language. Nevertheless, if the correspondence is close, we can assume a monoalphabetic substitution. If, on the other hand, a Vigenère cipher is suspected, then progress depends on determining the length of the keyword, as will be seen in a moment. For now, let us concentrate on how the keyword length can be determined. The important insight that leads to a solution is the following: If two identical sequences of plaintext letters occur at a distance that is an integer multiple of the keyword length, they will generate identical ciphertext sequences. In the foregoing example, two instances of the sequence "red" are separated by nine character positions. Consequently, in both cases, r is encrypted using key letter *e*, e is encrypted using key letter *p*, and d is encrypted using keyM letter *t*. Thus, in both cases the ciphertext sequence is VTW. An analyst looking at only the ciphertext would detect the repeated sequences VTW at a displacement of 9 and make the assumption that the keyword is either three or nine letters in length. The appearance of VTW twice could be by chance and not reflect identical plaintext letters encrypted with identical key letters. However, if the message is long enough, there will be a number of such repeated cipher text sequences. By looking for common factors in the displacements of the various sequences, the analyst should be able to make a good guess of the keyword length. Solution of the cipher now depends on an important insight. If the keyword length is *N*, then the cipher, in effect, consists of *N* monoalphabetic substitution ciphers. For example, with the keyword DECEPTIVE, the letters in positions 1, 10, 19, and so on are all encrypted with the same monoalphabetic cipher. Thus, we can use the known frequency characteristics of the plaintext language to attack each of the monoalphabetic ciphers separately.

The periodic nature of the keyword can be eliminated by using a nonrepeating keyword that is as long as the message itself. Vigenère proposed what is referred to as an **autokey system**, in which a keyword is concatenated with the plaintext itself to provide a running key.

For our example,

key: *deceptivewearediscoveredsav*

plaintext: wearediscoveredsaveyourself

ciphertext: ZICVTWQNGKZEIIGASXSTSLVVWLA

The ultimate defense against such a cryptanalysis is to choose a keyword that is as long as the plaintext and has no statistical relationship to it. Such a system was introduced by an AT&T engineer named Gilbert Vernam in 1918. His system works on binary data rather than letters. The system can be expressed succinctly as follows:

$ci = pi + ki$

where

pi = ith binary digit of plaintext

ki = ith binary digit of key

ci = ith binary digit of cipher text

'+'= exclusive-or (XOR) operation

Thus, the ciphertext is generated by performing the bitwise XOR of the plaintext and the key. Because of the properties of the XOR, decryption simply involves the same bitwise operation:

$pi = ci + ki$

The essence of this technique is the means of construction of the key. Vernam proposed the use of a running loop of tape that eventually repeated the key, so that in fact the system worked with a very long but repeating keyword. Although such a scheme, with a long key, presents formidable cryptanalytic difficulties, it can be broken with sufficient ciphertext, the use of known or probable plaintext sequences, or both.

One-Time Pad

An Army Signal Corp officer, Joseph Mauborgne, proposed an improvement to the Vernam cipher that yields the ultimate in security. Mauborgne suggested using a random key that is as long as the message, so that the key need not be repeated. In addition, the key is to

be used to encrypt and decrypt a single message, and then is discarded. Each new message requires a new key of the same length as the new message. Such a scheme, known as a **one-time pad**, is unbreakable. It produces random output that bears no statistical relationship to the plaintext. Because the ciphertext contains no information whatsoever about the plaintext, there is simply no way to break the code. An example should illustrate our point. Suppose that we are

using a Vigenère scheme with 27 characters in which the twenty-seventh character is the space character, but with a one-time key that is as long as the message. Thus, the tableau of Table 2.3 must be expanded to 27 x 27. Consider the ciphertext

ANKYODKYUREPFJBYOJDSPLREYIUNOFDOIUERFPLUYTS

We now show two different decryptions using two different keys:

ciphertext: ANKYODKYUREPFJBYOJDSPLREYIUNOFDOIUERFPLUYTS

Key: *pxlmvmsydofuyrvzwc tnlebnecvgdupahfzzlmnyih*

Plaintext: mr mustard with the candlestick in the hall

Cipher text: ANKYODKYUREPFJBYOJDSPLREYIUNOFDOIUERFPLUYTS

Key: *mfugpmiydgaxgoufhklllmhsqdqogtewbqfgyovuhwt*

Plaintext: miss scarlet with the knife in the library

Suppose that a cryptanalyst had managed to find these two keys. Two plausible plaintexts are produced. How is the cryptanalyst to decide which is the correct decryption (i.e., which is the correct key)? If the actual key were produced in a truly random fashion, then them cryptanalyst cannot say that one of these two keys is more likely than the other. Thus, there is no way to decide which key is correct and therefore which plaintext is correct.

In fact, given any plaintext of equal length to the ciphertext, there is a key that produces that plaintext. Therefore, if you did an exhaustive search of all possible keys, you would end up with many legible plaintexts, with no way of knowing which was the intended plaintext.

Therefore, the code is unbreakable. The security of the one-time pad is entirely due to the randomness of the key. If the stream of characters that constitute the key is truly random, then the stream of characters that constitute the ciphertext will be truly random. Thus, there are no patterns or regularities that a cryptanalyst can use to attack the ciphertext. In theory, we need look no further for a cipher. The one-time pad offers complete security but, in practice, has two fundamental difficulties:

1. There is the practical problem of making large quantities of random keys. Any heavily used system might require millions of random characters on a regular basis. Supplying truly random characters in this volume is a significant task.

2. Even more daunting is the problem of key distribution and protection. For every message to be sent, a key of equal length is needed by both sender and receiver. Thus, a mammoth key distribution problem exists.

Because of these difficulties, the one-time pad is of limited utility, and is useful primarily for low-bandwidth channels requiring very high security.

Transposition Techniques

All the techniques examined so far involve the substitution of a ciphertext symbol for a plaintext symbol. A very different kind of mapping is achieved by performing some sort of permutation on the plaintext letters. This technique is referred to as a transposition cipher. The simplest such cipher is the rail fence technique, in which the plaintext is written down as a sequence of diagonals and then read off as a sequence of rows. For example, to encipher the message "meet me after the toga party" with a rail fence of depth 2, we write the following:

m e m a t r h t g p r y

e t e f e t e o a a t

The encrypted message is

MEMATRHTGPRYETEFETEOAAT

This sort of thing would be trivial to cryptanalyze. A more complex scheme is to write the message in a rectangle, row by row, and read the message off, column by column, but permute the order of the columns. The order of the columns then becomes the key to the algorithm. For example,

Key: 4 3 1 2 5 6 7

Plaintext: a t t a c k p

o s t p o n e

d u n t i l t

w o a m x y z

Ciphertext: TTNAAPTMTSUOAODWCOIXKNLYPETZ

A pure transposition cipher is easily recognized because it has the same letter frequencies as the original plaintext. For the type of columnar transposition just shown, cryptanalysis is fairly straightforward and involves laying out the ciphertext in a matrix and playing around with

column positions. Digram and trigram frequency tables can be useful. The transposition cipher can be made significantly more secure by performing more than one stage of transposition. The result is a more complex permutation that is not easily reconstructed. Thus, if the foregoing message is reencrypted using the same algorithm,

Key: 4 3 1 2 5 6 7

Input: t t n a a p t

m t s u o a o

d w c o i x k

n l y p e t z

Output: NSCYAUOPTTWLTMDNAOIEPAXTTOKZ

To visualize the result of this double transposition, designate the letters in the original plaintext message by the numbers designating their position. Thus, with 28 letters in the message, the original sequence of letters is

01 02 03 04 05 06 07 08 09 10 11 12 13 14

15 16 17 18 19 20 21 22 23 24 25 26 27 28

After the first transposition we have

03 10 17 24 04 11 18 25 02 09 16 23 01 08

15 22 05 12 19 26 06 13 20 27 07 14 21 28

which has a somewhat regular structure. But after the second transposition, we have

17 09 05 27 24 16 12 07 10 02 22 20 03 25

15 13 04 23 19 14 11 01 26 21 18 08 06 28

This is a much less structured permutation and is much more difficult to cryptanalyze.

Rotor Machines

The example just given suggests that multiple stages of encryption can produce an algorithm that is significantly more difficult to cryptanalyze. This is as true of substitution ciphers as it is of transposition ciphers. Before the introduction of DES, the most important application of the principle of multiple stages of encryption was a class of systems known as rotor machines.

The basic principle of the rotor machine is illustrated in Figure 2.7. The machine consists of a set of independently rotating cylinders through which electrical pulses can flow. Each cylinder has 26 input pins and 26 output pins, with internal wiring that connects each input pin to a unique output pin. For simplicity, only three of the internal connections in each cylinder are shown.

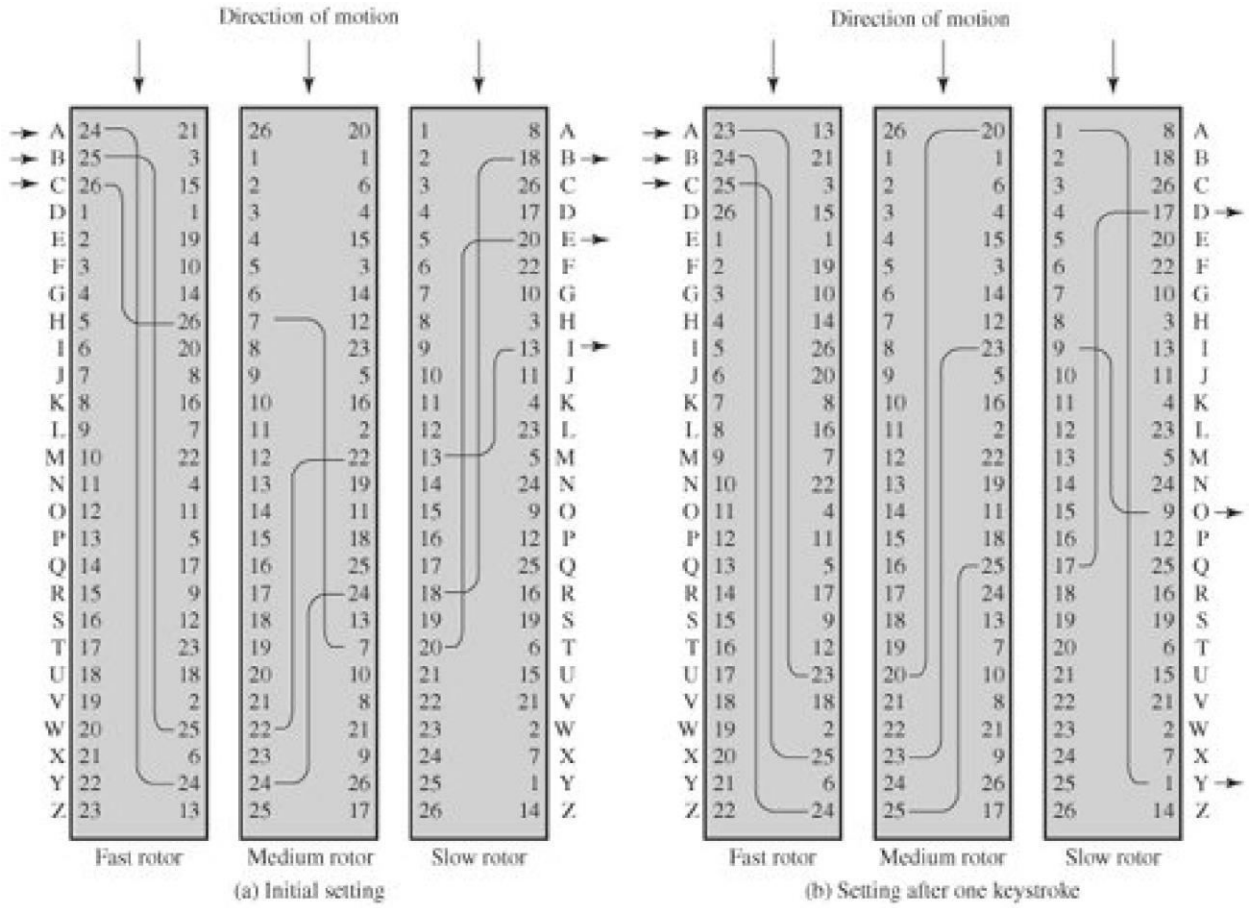


Figure 2.7. Three-Rotor Machine with Wiring Represented by Numbered Contacts

If we associate each input and output pin with a letter of the alphabet, then a single cylinder defines a monoalphabetic substitution. For example, in Figure 2.7, if an operator depresses the key for the letter A, an electric signal is applied to the first pin of the first cylinder and flows through the internal connection to the twenty-fifth output pin. Consider a machine with a single cylinder. After each input key is depressed, the cylinder rotates one position, so that the internal connections are shifted accordingly. Thus, a different monoalphabetic substitution cipher is defined. After 26 letters of plaintext, the cylinder would be back to the initial position. Thus, we have a polyalphabetic substitution algorithm with a period of 26. A single-cylinder system is trivial and does not present a formidable cryptanalytic task. The power of the rotor machine is in the use of multiple cylinders, in which the output pins of one cylinder are connected to the input pins of the next. Figure 2.7 shows a three-cylinder system. The left half of the figure shows a position in which the input from the operator to the first pin (plaintext letter a) is routed through the three cylinders to appear at the output of the second pin (ciphertext letter B).

With multiple cylinders, the one closest to the operator input rotates one pin position with each keystroke. The right half of Figure 2.7 shows the system's configuration after a single keystroke. For every complete rotation of the inner cylinder, the middle cylinder rotates one pin position. Finally, for every complete rotation of the middle cylinder, the outer cylinder rotates one pin position. This is the same type of operation seen with an odometer. The result is that there are 26 x 26 x 26 = 17,576 different substitution alphabets used before the system repeats. The addition of fourth and fifth rotors results in periods of 456,976 and 11,881,376 letters, respectively. As David Kahn eloquently put it, referring to a five-rotor machine [KAHN96, page 413]:

A period of that length thwarts any practical possibility of a straightforward solution on the basis of letter frequency. This general solution would need about 50 letters per cipher alphabet, meaning that all five rotors would have to go through their combined cycle 50 times. The ciphertext would have to be as long as all the speeches made on the floor of the Senate and the House of Representatives in three successive sessions of Congress. No cryptanalyst is likely to bag that kind of trophy in his lifetime; even diplomats, who can be as verbose as politicians, rarely scale those heights of loquacity.

Steganography

We conclude with a discussion of a technique that is, strictly speaking, not encryption, namely, steganography. A plaintext message may be hidden in one of two ways. The methods of steganography conceal the existence of the message, whereas the methods of cryptography render the message unintelligible to outsiders by various transformations of the text.

Steganography was an obsolete word that was revived by David Kahn. A simple form of steganography, but one that is time-consuming to construct, is one in which an arrangement of words or letters within an apparently innocuous text spells out the real message. For example, the sequence of first letters of each word of the overall message spells out the hidden message.

Various other techniques have been used historically; some examples are the following

Character marking: Selected letters of printed or typewritten text are overwritten in pencil. The marks are ordinarily not visible unless the paper is held at an angle to bright light.

Invisible ink: A number of substances can be used for writing but leave no visible trace until heat or some chemical is applied to the paper.

Pin punctures: Small pin punctures on selected letters are ordinarily not visible unless the paper is held up in front of a light.

Typewriter correction ribbon: Used between lines typed with a black ribbon, the results of typing with the correction tape are visible only under a strong light.

Although these techniques may seem archaic, they have contemporary equivalents. [WAYN93] proposes hiding a message by using the least significant bits of frames on a CD. For example, the Kodak Photo CD format's maximum resolution is 2048 by 3072 pixels, with each pixel containing 24 bits of RGB color information. The least significant bit of each 24-bit pixel can be changed without greatly affecting the quality of the image. The result is that you can hide a 2.3-megabyte message in a single digital snapshot. There are now a number of software packages available that take this type of approach to steganography. Steganography has a number of drawbacks when compared to encryption. It requires a lot of overhead to hide a relatively few bits of information, although using some scheme like that proposed in the preceding paragraph may make it more effective. Also, once the system is discovered, it becomes virtually worthless. This problem, too, can be overcome if the insertion method depends on some sort of key (e.g., see Problem 2.11). Alternatively, a message can be first encrypted and then hidden using steganography. The advantage of steganography is that it can be employed by parties who have something to lose should the fact of their secret communication (not necessarily the content) be discovered. Encryption flags traffic as important or secret or may identify the sender or receiver as someone with something to hide.

Block Ciphers and the Data Encryption Standard

Block Cipher Principles

Most symmetric block encryption algorithms in current use are based on a structure referred to as a Feistel block cipher. For that reason, it is important to examine the design principles of the Feistel cipher. We begin with a comparison of stream ciphers and block ciphers. Then we discuss the motivation for the Feistel block cipher structure. Finally, we discuss some of its implications.

Stream Ciphers and Block Ciphers

A **stream cipher** is one that encrypts a digital data stream one bit or one byte at a time. Examples of classical stream ciphers are the autokeyed Vigenère cipher and the Vernam cipher.

A **block cipher** is one in which a block of plaintext is treated as a whole and used to produce a ciphertext block of equal length. Typically, a block size of 64 or 128 bits is used. Using some of the modes of operation, a block cipher can be used to achieve the same effect as a stream cipher. Far more effort has gone into analyzing block ciphers. In general, they seem applicable to a broader range of applications than stream ciphers. The vast majority of network-based symmetric cryptographic applications make use of block ciphers.

The Feistel Cipher

Feistel proposed that we can approximate the ideal block cipher by utilizing the concept of a product cipher, which is the execution of two or more simple ciphers in sequence in such a way that the final result or product is cryptographically stronger than any of the component ciphers. The essence of the approach is to develop a block cipher with a key length of k bits and a block length of n bits, allowing a total of 2^k possible transformations, rather than the $2^n!$ transformations available with the ideal block cipher. In particular, Feistel proposed the use of a cipher that alternates substitutions and permutations. In fact, this is a practical application a proposal by Claude Shannon to develop a product cipher that alternates *confusion* and *diffusion* functions. We look next at these concepts of diffusion and confusion and then present the Feistel cipher.

Diffusion and Confusion

The terms *diffusion* and *confusion* were introduced by Claude Shannon to capture the two basic building blocks for any cryptographic system.Shannon's concern was to thwart cryptanalysis based on statistical analysis. The reasoning is as follows. Assume the attacker has some knowledge of the statistical characteristics of the plaintext. For example, in a human-readable message in some language, the frequency distribution of the various letters may be known. Or there may be words or phrases likely to appear in the message (probable words). If these statistics are in any way reflected in the ciphertext, the cryptanalyst may be able to deduce the encryption key, or part of the key, or at least a set of keys likely to contain the exact key.

Other than recourse to ideal systems, Shannon suggests two methods for frustrating statistical cryptanalysis: diffusion and confusion. In **diffusion**, the statistical structure of the plaintext is

dissipated into long-range statistics of the ciphertext. This is achieved by having each plaintext digit affect the value of many ciphertext digits; generally this is equivalent to having each ciphertext digit be affected by many plaintext digits. An example of diffusion is to encrypt a message $M = m1, m2, m3,...$ of characters with an averaging operation: adding k successive letters to get a ciphertext letter yn. One can show that the statistical structure of the plaintext has been dissipated. Thus, the letter frequencies in the ciphertext will be more nearly equal than in the plaintext; the digram frequencies will also be more nearly equal, and so on. In a binary block cipher, diffusion can be achieved by repeatedly performing some permutation on the data followed by applying a function to that permutation; the effect is that bits from different positions in the original plaintext contribute to a single bit of cipher text.

$$y_n = \left(\sum_{i=1}^{k} m_{n+i} \right) \bmod 26$$

adding k successive letters to get a ciphertext letter yn. One can show that the statistical structure of the plaintext has been dissipated.

Thus, the letter frequencies in the ciphertext will be more nearly equal than in the plaintext; the digram frequencies will also be more nearly equal, and so on. In a binary block cipher, diffusion can be achieved by repeatedly performing some permutation on the data followed by applying a function to that permutation; the effect is that bits from different positions in the original plaintext contribute to a single bit of cipher text Every block cipher involves a transformation of a block of plaintext into a block of ciphertext, where the transformation depends on the key. The mechanism of diffusion seeks to make the statistical relationship between the plaintext and ciphertext as complex as possible in order to thwart attempts to deduce the key. On the other hand, **confusion** seeks to make the relationship between the statistics of the ciphertext and the value of the encryption key as complex as possible, again to thwart attempts to discover the key. Thus, even if the attacker can get some handle on the statistics of the ciphertext, the way in which the key was used to produce that ciphertext is so complex as to make it difficult to deduce the key. This is achieved by the use of a complex substitution algorithm. In contrast, a simple linear substitution function would add little confusion.

Feistel Cipher Structure

Figure 3.2 depicts the structure proposed by Feistel. The inputs to the encryption algorithm are a plaintext block of length 2*w* bits and a key *K*. The plaintext block is divided into two halves, *L*0 and *R*0. The two halves of the data pass through *n* rounds of processing and then combine to produce the ciphertext block. Each round *i* has as inputs *Li-1* and *Ri-1*, derived from the previous round, as well as a subkey *Ki*, derived from the overall *K*. In general, the subkeys *Ki* are different from *K* and from each other.

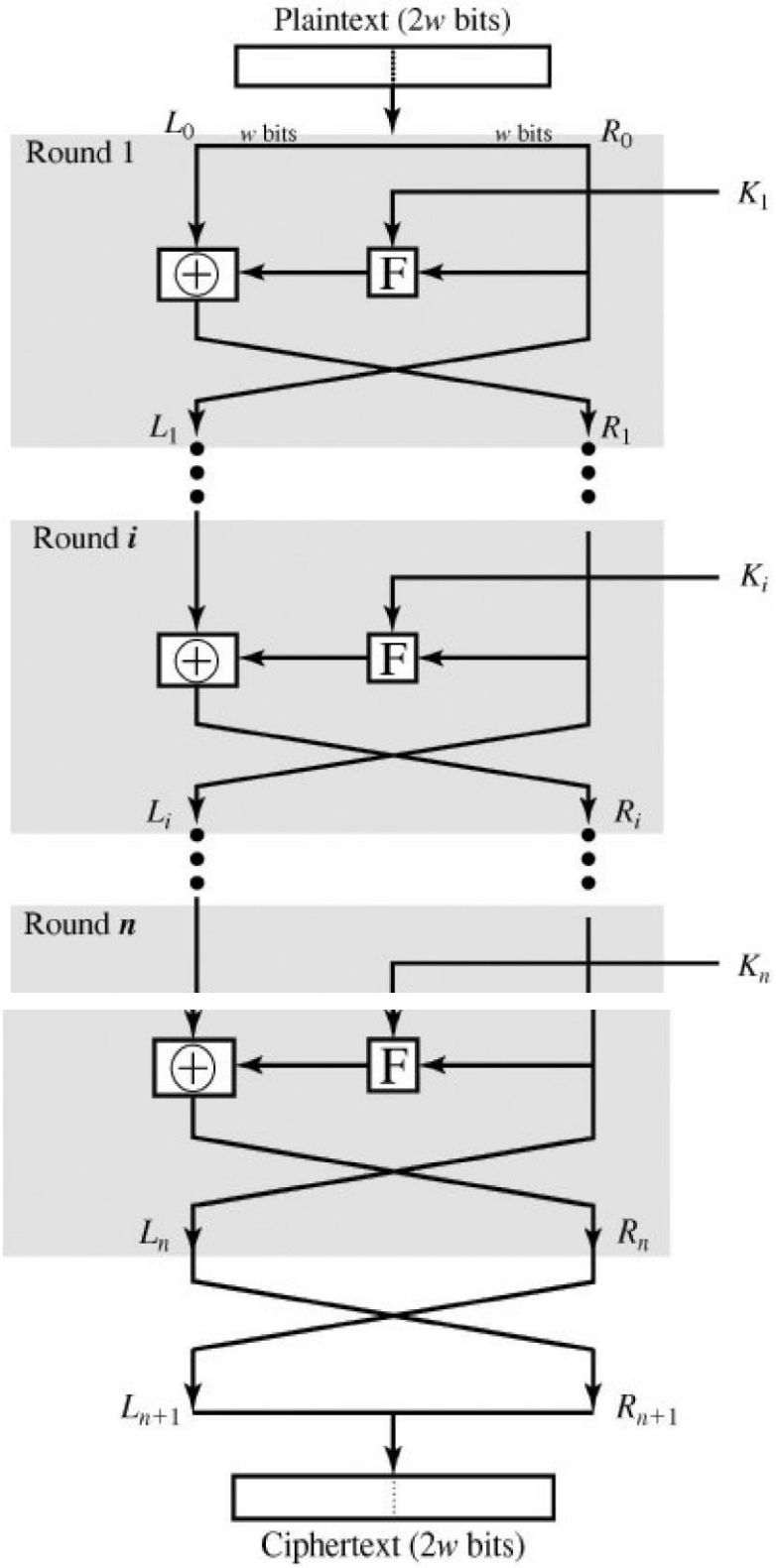


Figure 3.2. Classical Feistel Network

The exact realization of a Feistel network depends on the choice of the following parameters and design features:

Block size: Larger block sizes mean greater security (all other things being equal) but reduced encryption/decryption speed for a given algorithm. The greater security is achieved by greater diffusion Traditionally, a block size of 64 bits has been considered a reasonable tradeoff and was nearly universal in block cipher design. However, the new AES uses a 128-bit block size.

Key size: Larger key size means greater security but may decrease encryption/decryption speed. The greater security is achieved by greater resistance to brute-force attacks and greater confusion. Key sizes of 64 bits or less are now widely considered to be inadequate, and 128 bits has become a common size.

Number of rounds: The essence of the Feistel cipher is that a single round offers inadequate security but that multiple rounds offer increasing security. A typical size is 16 rounds.

Subkey generation algorithm: Greater complexity in this algorithm should lead to greater difficulty of cryptanalysis.

Round function: Again, greater complexity generally means greater resistance to cryptanalysis.

There are two other considerations in the design of a Feistel cipher:

Fast software encryption/decryption: In many cases, encryption is embedded in applications or utility functions in such a way as to preclude a hardware implementation. Accordingly, the speed of execution of the algorithm becomes a concern.

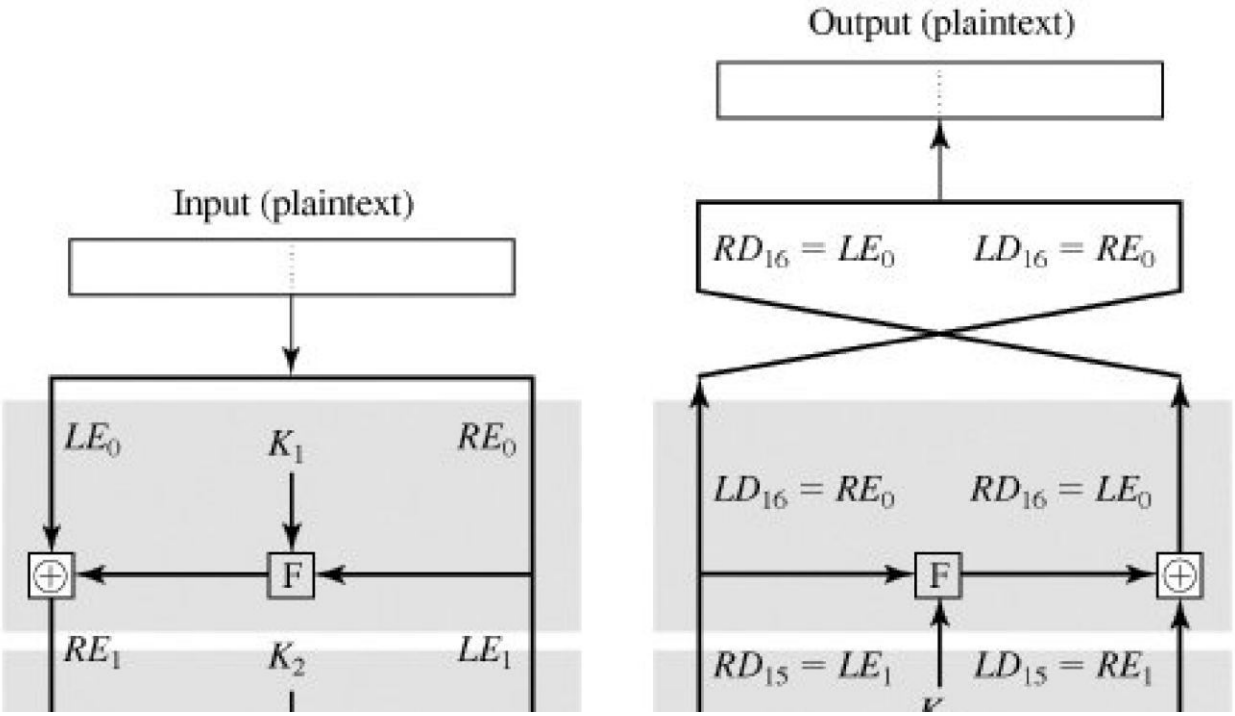
Ease of analysis: Although we would like to make our algorithm as difficult as possible to cryptanalyze, there is great benefit in making the algorithm easy to analyze. That is, if the algorithm can be concisely and clearly explained, it is easier to analyze that algorithm for cryptanalytic vulnerabilities and therefore develop a higher level of assurance as to its strength. DES, for example, does not have an easily analyzed functionality.

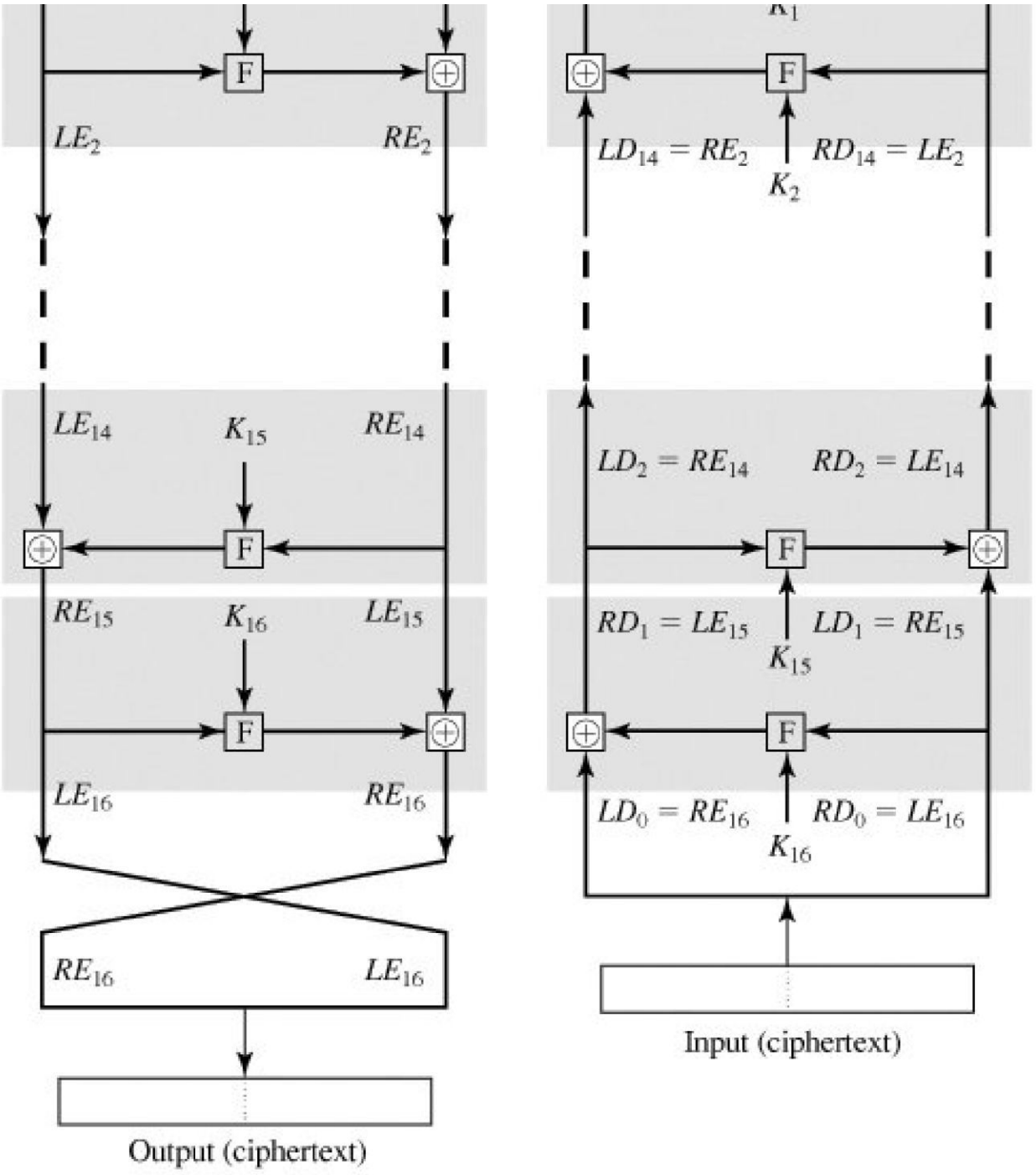
Feistel Decryption Algorithm

The process of decryption with a Feistel cipher is essentially the same as the encryption process. The rule is as follows: Use the cipher text as input to the algorithm, but use the subkeys *Ki* in reverse order. That is, use *Kn* in the first round, *Kn-1* in the second round, and so on until *K*1 is used in the last round. This is a nice feature because it means we need not implement two different algorithms, one for encryption and one for decryption. To see that the same algorithm with a reversed key order produces the correct result, consider Figure 3.3, which shows the

encryption process going down the left-hand side and the decryption process going up the right-hand side for a 16-round algorithm (the result would be the same for any number of rounds). For clarity, we use the notation LE*i* and RE*i* for data traveling through the encryption algorithm and *LDi* and RD*i* for data traveling through the decryption algorithm. The diagram indicates that, at every round, the intermediate value of the decryption process is equal to the corresponding value of the encryption process with the two halves of the value swapped. To put this another way, let the output of the *i*th encryption round be LE*i*||RE*i* (*Li* concatenated with *Ri*). Then the corresponding input to the (16*i*)th decryption round is RE*i*||LE*i* or, equivalently, RD16-*i*||*LD*16-*i*.

Figure 3.3. Feistel Encryption and Decryption





After the last iteration of the encryption process,

the two halves of the output are swapped, so that the ciphertext is RE16||LE16. The output of that round is the ciphertext. Now take that ciphertext and use it as input to the same algorithm. The input to the first round is RE16||LE16, which is equal to the 32-bit swap of the output of the sixteenth round of the encryption process.

Now we would like to show that the output of the first round of the decryption process is equal to a 32-bit swap of the input to the sixteenth round of the encryption process. First, consider the encryption process. We see that

LE16 = RE15

RE16 = LE15 x F(RE15, *K*16)

On the decryption side,

*LD*1 = RD0 = LE16 = RE15

RD1 = LD0 x F(RD0, *K*16)

= RE16 x F(RE15, *K*16)

= [LE15 x F(RE15, *K*16)] x F(RE15, *K*16)

The XOR has the following properties:

[*A* x *B*] x *C* = *A* x [*B* x *C*]

D x *D* = 0

E x 0 = E

Thus, we have *LD*1 = RE15 and RD1 = LE15. Therefore, the output of the first round of the decryption process is LE15||RE15, which is the 32-bit swap of the input to the sixteenth round of the encryption. This correspondence holds all the way through the 16 iterations, as is easily shown. We can cast this process in general terms. For the *i*th iteration of the encryption algorithm,

LE*i* = RE*i-1*

RE*i* =LE*i*-1 x F(RE*i*-1, *Ki*)

Rearranging terms,

RE*i*-1 = LE*i*

LE*i*-1 = RE*i* x F(RE*i*-1, *Ki*2 = RE*i* x F(LE*i*, *Ki*)

The Data Encryption Standard

The most widely used encryption scheme is based on the Data Encryption Standard (DES) adopted in 1977 by the National Bureau of Standards, now the National Institute of Standards and Technology (NIST), as Federal Information Processing Standard 46 (FIPS PUB 46). The algorithm itself is referred to as the Data Encryption Algorithm (DEA).

For DES, data are encrypted in 64-bit blocks using a 56-bit key. The algorithm transforms 64-bit input in a series of steps into a 64-bit output. The same steps, with the same key, are used to reverse the encryption.

DES Encryption

The overall scheme for DES encryption is illustrated in Figure 3.4. As with any encryption scheme, there are two inputs to the encryption function: the plaintext to be encrypted and the key. In this case, the plaintext must be 64 bits in length and the key is 56 bits in length.

Looking at the left-hand side of the figure, we can see that the processing of the plaintext proceeds in three phases. First, the 64-bit plaintext passes through an initial permutation (IP) that rearranges the bits to produce the *permuted input*. This is followed by a phase consisting of 16 rounds of the same function, which involves both permutation and substitution functions. The output of the last (sixteenth) round consists of 64 bits that are a function of the input plaintext and the key. The left and right halves of the output are swapped to produce the **preoutput**. Finally, the preoutput is passed through a permutation (IP-1) that is the inverse of the initial permutation function, to produce the 64-bit ciphertext. With the exception of the initial and final permutations, DES has the exact structure of a Feistel cipher, as shown in Figure 3.2.

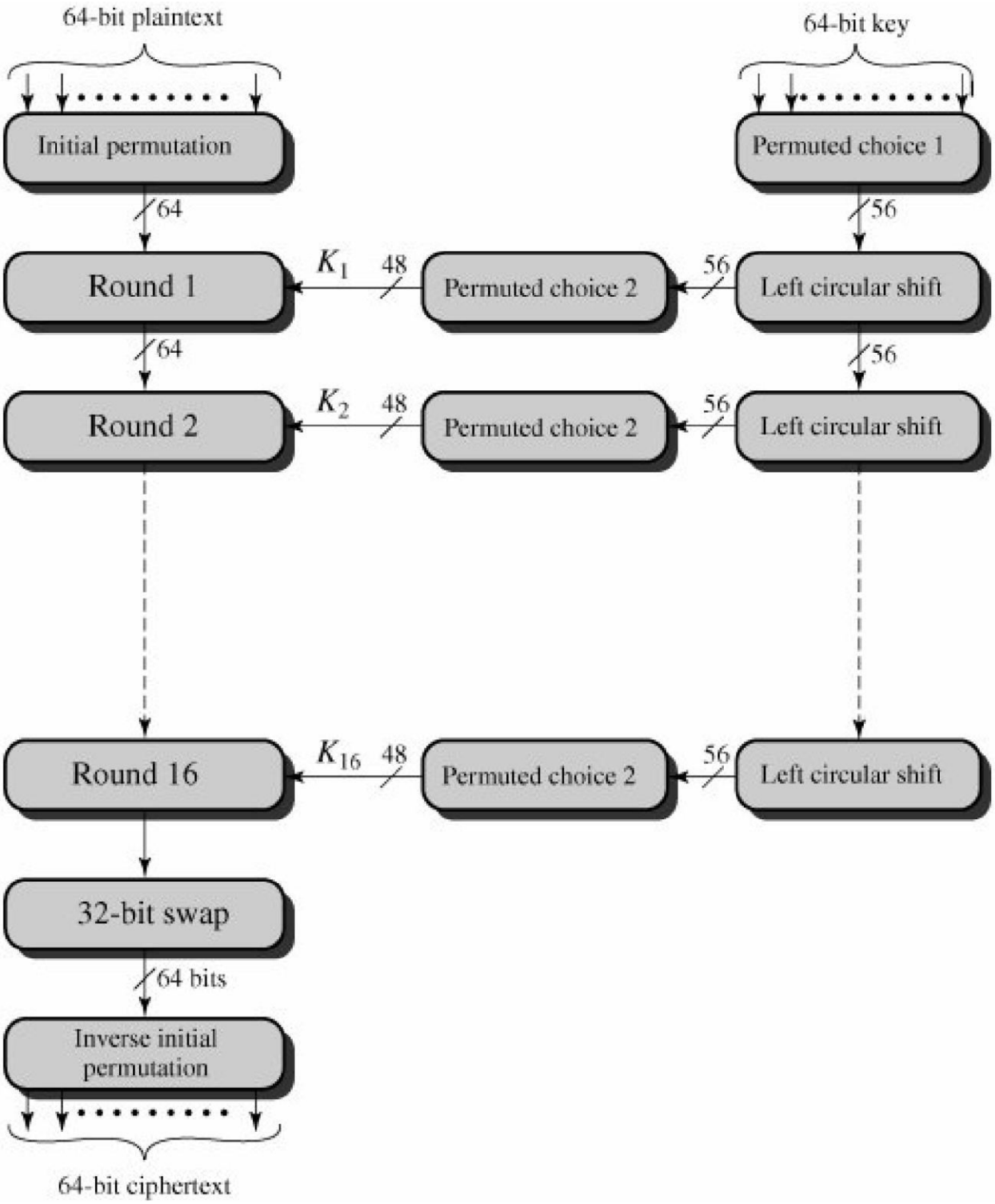


Figure 3.4. General Depiction of DES Encryption Algorithm

Details of Single Round

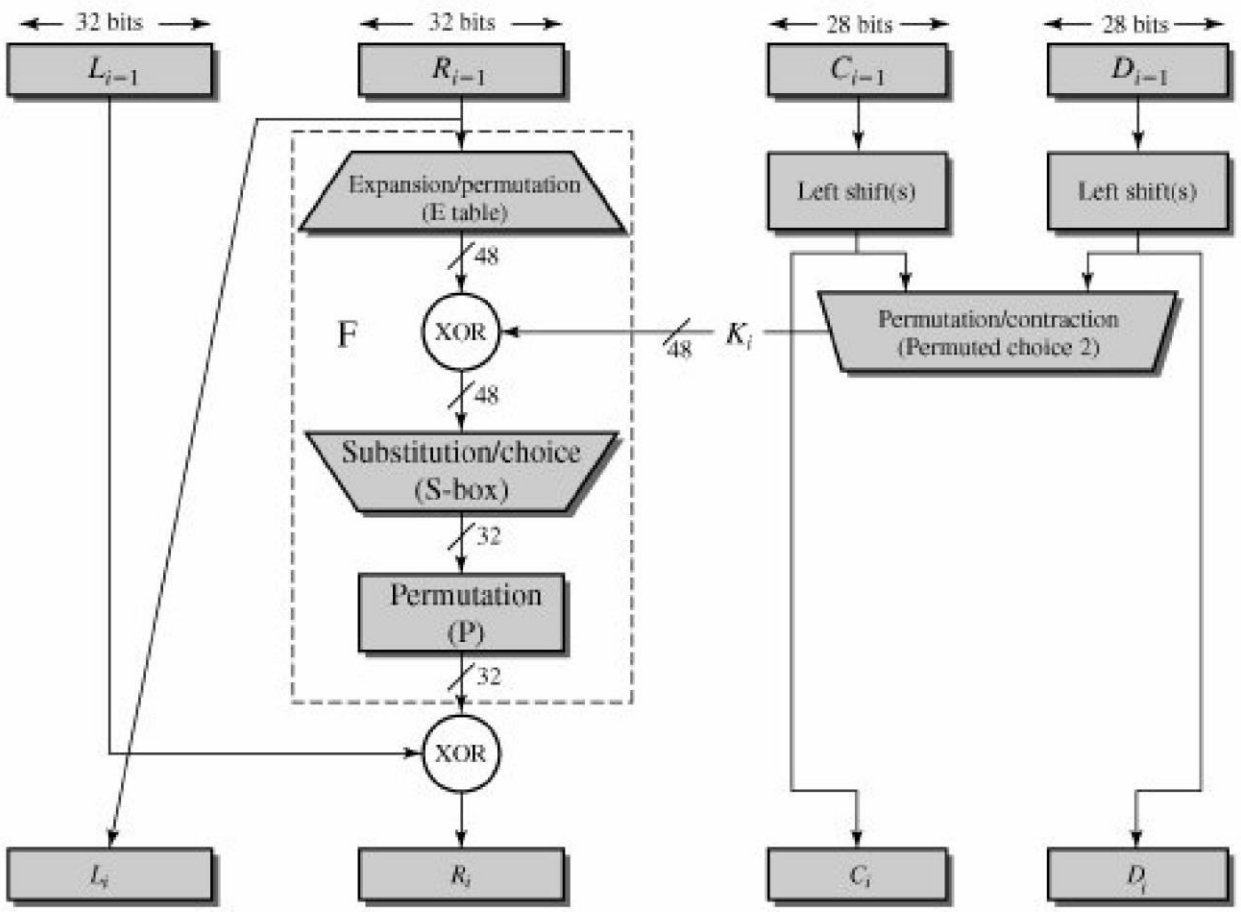
Figure 3.5 shows the internal structure of a single round. Again, begin by focusing on the left-hand side of the diagram. The left and right halves of each 64-bit intermediate value are treated as separate 32-bit quantities, labeled L (left) and R (right). As in any classic Feistel cipher, the overall processing at each round can be summarized in the following formulas:

$L_i = R_{i-1}$

$R_i = L_{i-1} \times F(R_{i-1}, K_i)$

[Page 77]

Figure 3.5. Single Round of DES Algorithm



The round key *Ki* is 48 bits. The *R* input is 32 bits. This *R* input is first expanded to 48 bits by using a table that defines a permutation plus an expansion that involves duplication of 16 of the *R* bits (Table 3.2c). The resulting 48 bits are XORed with *Ki*. This 48-bit result passes through a substitution function that produces a 32-bit output, which is permuted as defined by Table 3.2d.

The role of the S-boxes in the function F is illustrated in Figure 3.6. The substitution consists of a set of eight S-boxes, each of which accepts 6 bits as input and produces 4 bits as output. These transformations are defined in Table 3.3, which is interpreted as follows: The first and last bits of the input to box S*i* form a 2-bit binary number to select one of four substitutions defined by the four rows in the table for S*i*. The middle four bits select one of the sixteen columns. The decimal value in the cell selected by the row and column is then converted to its 4-bit representation to produce the output. For example, in S1 for input 011001, the row is 01 (row 1) and the column is 1100 (column 12). The value in row 1, column 12 is 9, so the output is 1001.

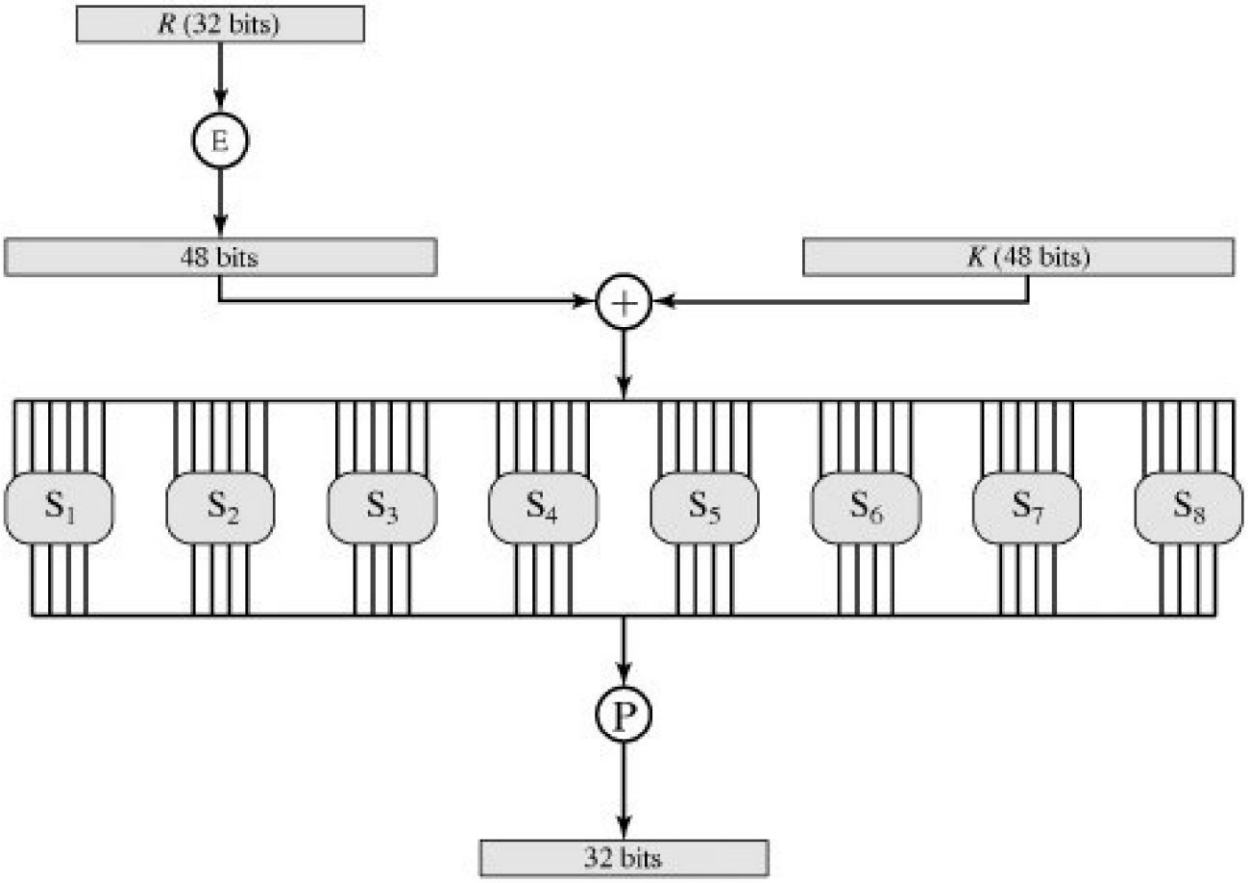


Figure 3.6. Calculation of F(R, K)

S_1

14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

S_2

15	1	8	14	6	11	3	4	9	7	2	13	12	0	5	10
3	13	4	7	15	2	8	14	12	0	1	10	6	9	11	5
0	14	7	11	10	4	13	1	5	8	12	6	9	3	2	15
13	8	10	1	3	15	4	2	11	6	7	12	0	5	14	9

S_3

10	0	9	14	6	3	15	5	1	13	12	7	11	4	2	8
13	7	0	9	3	4	6	10	2	8	5	14	12	11	15	1
13	6	4	9	8	15	3	0	11	1	2	12	5	10	14	7
1	10	13	0	6	9	8	7	4	15	14	3	11	5	2	12

S_4

7	13	14	3	0	6	9	10	1	2	8	5	11	12	4	15
13	8	11	5	6	15	0	3	4	7	2	12	1	10	14	9
10	6	9	0	12	11	7	13	15	1	3	14	5	2	8	4
3	15	0	6	10	1	13	8	9	4	5	11	12	7	2	14

S_5

2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3

S_6

12	1	10	15	9	2	6	8	0	13	3	4	14	7	5	11
10	15	4	2	7	12	9	5	6	1	13	14	0	11	3	8
9	14	15	5	2	8	12	3	7	0	4	10	1	13	11	6
4	3	2	12	9	5	15	10	11	14	1	7	6	0	8	13

S_7

4	11	2	14	15	0	8	13	3	12	9	7	5	10	6	1
13	0	11	7	4	9	1	10	14	3	5	12	2	15	8	6
1	4	11	13	12	3	7	14	10	15	6	8	0	5	9	2
6	11	13	8	1	4	10	7	9	5	0	15	14	2	3	12

S_8

13	2	8	4	6	15	11	1	10	9	3	14	5	0	12	7
1	15	13	8	10	3	7	4	12	5	6	11	0	14	9	2
7	11	4	1	9	12	14	2	0	6	10	13	15	3	5	8
2	1	14	7	4	10	8	13	15	12	9	0	3	5	6	11

Table 3.3. Definition of DES S-Boxes

Key Generation

we see that a 64-bit key is used as input to the algorithm. The bits of the key are numbered from 1 through 64; every eighth bit is ignored.The key is first subjected to a permutation governed by a table labeled Permuted Choice One. The resulting 56-bit key is then treated as two 28-bit quantities, labeled*C* 0 and *D*0. At each round, *Ci*-1 and *Di*-1 are separately subjected to a circular left shift, or rotation, of 1 or 2 bits. These shifted values serve as input to the next round. They also serve as input to Permuted Choice Two, which produces a 48-bit output that serves as input to the function F(*Ri*-1, *Ki*).

The Strength of DES

Since its adoption as a federal standard, there have been lingering concerns about the level of security provided by DES. These concerns, by and large, fall into two areas: key size and the nature of the algorithm.

The Use of 56-Bit Keys

With a key length of 56 bits, there are 2 56 possible keys, which is approximately 7.2 x 10 16Thus, on the face of it, a brute-force attack appears impractical. Assuming that, on average, half the key space has to be searched, a single machine performing one DES encryption per microsecond would take more than a thousand years (see Table 2.2) to break the cipher. however, the assumption of one encryption per microsecond is overly conservative. As far back as 1977, Diffie and Hellman postulated that the technology existed to build a parallel machine with 1 million encryption devices, each of which could perform one encryption per microsecond [DIFF77]. This would bring the average search time down to about 10 hours. The authors estimated that the cost would be about $20 million in 1977 dollars. It is important to note that there is more to a key-search attack than simply running through all possible keys. Unless known plaintext is provided, the analyst must be able to recognize plaintext as plaintext. If the message is just plain text in English, then the result pops out easily, although the task of recognizing English would have to be automated. If the text message has been compressed before encryption, then recognition is more difficult. And if the message is some more general type of data, such as a numerical file, and this has been compressed, the problem becomes even more difficult to automate. Thus, to supplement the brute-force approach, some degree of knowledge about the expected plaintext is needed, and some means of automatically distinguishing plaintext from garble is also needed. The EFF approach addresses this issue as well and introduces some

automated techniques that would be effective in many contexts. Fortunately, there are a number of alternatives to DES, the most important of which are AES and triple DES, discussed in Chapters 5 and 6, respectively.

The Nature of the DES Algorithm

Another concern is the possibility that cryptanalysis is possible by exploiting the characteristics of the DES algorithm. The focus of concern has been on the eight substitution tables, or S-boxes, that are used in each iteration. Because the design criteria for these boxes, and indeed for the entire algorithm, were not made public, there is a suspicion that the boxes were constructed in such a way that cryptanalysis is possible for an opponent who knows the weaknesses in the S-boxes. This assertion is tantalizing, and over the years a number of regularities and unexpected behaviors of the S-boxes have been discovered. Despite this, no one has so far succeeded in discovering the supposed fatal weaknesses in the S-boxes.

Timing Attacks

We discuss timing attacks in more detail in Part Two, as they relate to public-key algorithms. However, the issue may also be relevant for symmetric ciphers. In essence, a timing attack is one in which information about the key or the plaintext is obtained by observing how long it takes a given implementation to perform decryptions on various ciphertexts. A timing attack exploits the fact that an encryption or decryption algorithm often takes slightly different amounts of time on different inputs on an approach that yields the Hamming weight (number of bits equal to one) of the secret key. This is a long way from knowing the actual key, but it is an intriguing first step. The authors conclude that DES appears to be fairly resistant to a successful timing attack but suggest some avenues to explore. Although this is an interesting line of attack, it so far appears unlikely that this technique will ever be successful against DES or more powerful symmetric ciphers such as triple DES and AES.

DES Design Criteria

The criteria used in the design of DES,focused on the design of the S-boxes and on the P function that takes

the output of the S boxes (Figure 3.6). The criteria for the S-boxes are as follows:

1. No output bit of any S-box should be too close a linear function of the input bits. Specifically, if we select any output bit and any subset of the six input bits, the fraction of inputs for which

this output bit equals the XOR of these input bits should not be close to 0 or 1, but rather should be near 1/2.

2. Each row of an S-box (determined by a fixed value of the leftmost and rightmost input bits) should include all 16 possible output bit combinations.

3. If two inputs to an S-box differ in exactly one bit, the outputs must differ in at least two bits.

4. If two inputs to an S-box differ in the two middle bits exactly, the outputs must differ in at least two bits.

5. If two inputs to an S-box differ in their first two bits and are identical in their last two bits, the two outputs must not be the same.

6. For any nonzero 6-bit difference between inputs, no more than 8 of the 32 pairs of inputs exhibiting that difference may result in the same output difference.

7. This is a criterion similar to the previous one, but for the case of three S-boxes. Coppersmith pointed out that the first criterion in the preceding list was needed because the S-boxes are the only nonlinear part of DES.

If the S-boxes were linear (i.e., each output bit is a linear combination of the input bits), the entire algorithm would be linear and easily broken. We have seen this phenomenon with the Hill cipher, which is linear. The remaining criteria were primarily aimed at thwarting differential cryptanalysis and at providing good confusion properties. The criteria for the permutation P are as follows:

1. The four output bits from each S-box at round i are distributed so that two of them affect (provide input for) "middle bits" of round $(i + 1)$ and the other two affect end bits. The two middle bits of input to an S-box are not shared with adjacent S-boxes. The end bits are the two left-hand bits and the two right-hand bits, which are shared with adjacent S-boxes.

2. The four output bits from each S-box affect six different S-boxes on the next round, and no two affect the same S-box.

3. For two S-boxes j, k, if an output bit from Sj affects a middle bit of Sk on the next round, then an output bit from Sk cannot affect a middle bit of Sj. This implies that for $j = k$, an output bit from Sj must not affect a middle bit of Sj.

These criteria are intended to increase the diffusion of the algorithm.

Number of Rounds

The cryptographic strength of a Feistel cipher derives from three aspects of the design: the number of rounds, the function F, and the key schedule algorithm. Let us look first at the choice of the number of rounds. The greater the number of rounds, the more difficult it is to perform cryptanalysis, even for a relatively weak F. In general, the criterion should be that the number of rounds is chosen so that known cryptanalytic efforts require greater effort than a simple brute-force key search attack. This criterion was certainly used in the design of DES. Schneier observes that for 16-round DES, a differential cryptanalysis attack is slightly less efficient than brute force: the differential cryptanalysis attack requires perations,whereasbrute force requires 2^ 55. If DES had 15 or fewer rounds, differential cryptanalysis would require less effort than brute-force key search.

Design of Function F

The heart of a Feistel block cipher is the function F. As we have seen, in DES, this function relies on the use of S-boxes. This is also the case for most other symmetric block ciphers, as we shall see in Chapter 4. However, we can make some general comments about the criteria for designing F. After that, we look specifically at S-box design.

Design Criteria for F

The function F provides the element of confusion in a Feistel cipher. Thus, it must be difficult to "unscramble" the substitution performed by F. One obvious criterion is that F be nonlinear, as we discussed previously. The more nonlinear F, the more difficult any type of cryptanalysis will be. There are several measures of nonlinearity, which are beyond the scope of this book. In rough terms, the more difficult it is to approximate F by a set of linear equations, the more nonlinear F is.

S-Box Design

One of the most intense areas of research in the field of symmetric block ciphers is that of S-box design. The papers are almost too numerous to count. Here we mention some general principles. In essence, we would like any change to the input vector to an S-box to result in random-looking changes to the output. The relationship should be nonlinear and difficult to approximate with linear functions. Obvious characteristic of the S-box is its size. An n x m S-box has n input bits and m output bits. DES has 6 x 4 S-boxes. Blowfish, has 8 x 32 S-boxes. Larger S-boxes, by and

large, are more resistant to differential and linear cryptanalysis. On the other hand, the larger the dimension n , the (exponentially) larger the lookup table. Thus, for practical reasons, a limit on f Equal to about 8 to 10 is usually imposed. Another practical consideration is that the larger the S-box, the more difficult it is to design it properly. S-boxes are typically organized in a different manner than used in DES. An n x m S-box typically consists of 2n rows of m bits each. The n bits of input select one of the rows of the S-box, and the m bits in that row are the output. For example, in an 8 x 32 S-box, if the input is 00001001, the output consists of the 32 bits in row 9 (the first row is labeled row 0).

Differential and Linear Cryptanalysis

For most of its life, the prime concern with DES has been its vulnerability to brute-force attack because of its relatively short (56 bits) key length. However, there has also been interest in finding cryptanalytic attacks on DES. With the increasing popularity of block ciphers with longer key lengths, including triple DES, brute-force attacks have become increasingly impractical. Thus, there has been increased emphasis on cryptanalytic attacks on DES and other symmetric block ciphers. In this section, we provide a brief overview of the two most powerful and promising approaches: differential cryptanalysis and linear cryptanalysis.

Differential Cryptanalysis

One of the most significant advances in cryptanalysis in recent years is differential cryptanalysis. In this section, we discuss the technique and its applicability to DES.

History

Although differential cryptanalysis is a powerful tool, it does not do very well against DES. The reason, according to a member of the IBM team that designed DES [COPP94], is that differential cryptanalysis was known to the team as early as 1974. The need to strengthen DES against attacks using differential cryptanalysis played a large part in the design of the S-boxes and the permutation P. As evidence of the impact of these changes, consider these comparable results reported in [BIHA93]. Differential cryptanalysis of an eight-round LUCIFER algorithm requires

only 256 chosen plaintexts, whereas an attack on an eight-round version of DES requires 2 14 chosen plaintexts.

Differential Cryptanalysis Attack

The differential cryptanalysis attack is complex; [BIHA93] provides a complete description. The rationale behind differential cryptanalysis is to observe the behavior of pairs of text blocks evolving along each round of the cipher, instead of observing the evolution of a single text block. Here, we provide a brief overview so that you can get the flavor of the attack.

We begin with a change in notation for DES. Consider the original plaintext block *m* to consist of two halves *m*0,*m*1. Each round of DES maps the right-hand input into the left-hand output and sets the right-hand output to be a function of the left-hand input and the subkey for this round. So, at each round, only one new 32-bit block is created. If we label each new block *m*1 17), then the intermediate message halves are related as follows:

mi+1 = *mi*-1 Xor f(*mi*, *Ki*), *i* = 1, 2, ..., 16

In differential cryptanalysis, we start with two messages, *m* and *m*', with a known XOR difference D*m* = *m* Xor *m*', and consider the difference between the intermediate message halves: *mi* = *mi* Xor *mi*' Then we have:

$$\begin{aligned}\Delta m_{i+1} &= m_{i+1} \oplus m'_{i+1} \\ &= [m_{i-1} \oplus \mathrm{f}(m_i, K_i)] \oplus [m'_{i-1} \oplus \mathrm{f}(m'_i, K_i)] \\ &= \Delta m_{i-1} \oplus [\mathrm{f}(m_i, K_i) \oplus \mathrm{f}(m'_i, K_i)]\end{aligned}$$

Now, suppose that many pairs of inputs to f with the same difference yield the same output difference if the same subkey is used. To put this more precisely, let us say that *X may cause Y with probability p*, if for a fraction *p* of the pairs in which the input XOR is *X*, the output XOR equals *Y*. We want to suppose that there are a number of values of*X* that have high probability of causing a particular output difference. Therefore, if we know D*mi*-1 and D*mi* with high

probability, then we know D*mi*+1 with high probability. Furthermore, if a number of such differences are determined, it is feasible to determine the subkey used in the function f.

The overall strategy of differential cryptanalysis is based on these considerations for a single round. The procedure is to begin with two plaintext messages *m* and *m*' with a given difference and trace through a probable pattern of differences after each round to yield a probable difference for the ciphertext. Actually, there are two probable patterns of differences for the two 32-bit halves: (D*m*17||*m*16). Next, we submit *m* and *m*' for encryption to determine the actual difference under the unknown key and compare the result to the probable difference. If there is a match,

E(*K*, *m*)Xor E(*K*, *m*') = (D*m*17||*m*16)

then we suspect that all the probable patterns at all the intermediate rounds are correct. With that assumption, we can make some deductions about the key bits. This procedure must be repeated many times to determine all the key bits.

Linear Cryptanalysis

A more recent development is linear cryptanalysis, described in [MATS93]. This attack is based on finding linear approximations to describe the transformations performed in DES. This method can find a DES key given 2 43 known plaintexts, as compared to 2 47 chosen plaintexts for differential cryptanalysis. Although this is a minor improvement, because it may be easier to acquire known plaintext rather than chosen plaintext, it still leaves linear cryptanalysis infeasible as an attack on DES. So far, little work has been done by other groups to validate the linear cryptanalytic approach.

We now give a brief summary of the principle on which linear cryptanalysis is based. For a cipher with n-bit plaintext and ciphertext blocks

and an *m*-bit key, let the plaintext block be labeled P[1], ... P[*n*], the cipher text block C[1], ... C[*n*], and the key K[1], ... K[*m*]. Then define

A[*i, j, ..., k*] = *A*[*i*] xor *A*[*j*]xor ... *A*[*k*]

The objective of linear cryptanalysis is to find an effective *linear* equation of the form:

P[a1, a2, ..., aa] XOR C[b1, b2, ..., bb] = K[g1, g2, ..., gc]

(where *x* = 0 or 1; 1 ≤*a, b*≤ *n*, 1 ≤*c* ≤*m*, and where the a, b and g terms represent fixed, unique bit locations) that holds with probability *p* 0.5. The further *p* is from 0.5, the more effective the

equation. Once a proposed relation is determined, the procedure is to compute the results of the left-hand side of the preceding equation for a large number of plaintext-ciphertext pairs. If the result is 0 more than half the time, assume K[g1, g2, ..., gc] = 0. If it is 1 most of the time, assume Kg[1, g2, ..., gc] = 1. This gives us a linear equation on the key bits. Try to get more such relations so that we can solve for the key bits. Because we are dealing with linear equations, the problem can be approached one round of the cipher at a time, with the results combined.

Block Cipher Design Principles

Although much progress has been made in designing block ciphers that are cryptographically strong, the basic principles have not changed all that much since the work of Feistel and the DES design team in the early 1970s. It is useful to begin this discussion by looking at the published design criteria used in the DES effort. Then we look at three critical aspects of block cipher design: the number of rounds, design of the function F, and key scheduling.

DES Design Criteria

The criteria used in the design of DES, as reported in [COPP94], focused on the design of the S-boxes and on the P function that takes the output of the S boxes (Figure 3.6). The criteria for the S-boxes are as follows:

1. No output bit of any S-box should be too close a linear function of the input bits. Specifically, if we select any output bit and any subset of the six input bits, the fraction of inputs for which this output bit equals the XOR of these input bits should not be close to 0 or 1, but rather should be near 1/2.

2. Each row of an S-box (determined by a fixed value of the leftmost and rightmost input bits) should include all 16 possible output bit combinations.

3. If two inputs to an S-box differ in exactly one bit, the outputs must differ in at least two bits.

4. If two inputs to an S-box differ in the two middle bits exactly, the outputs must differ in at least two bits.

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The criteria for the permutation P are as follows:

1. The four output bits from each S-box at round i are distributed so that two of them affect (provide input for) "middle bits" of round $(i + 1)$ and the other two affect end bits. The two middle bits of input to an S-box are not shared with adjacent S-boxes.

The end bits are the two left-hand bits and the two right-hand bits, which are shared with adjacent S-boxes.

2. The four output bits from each S-box affect six different S-boxes on the next round, and no two affect the same S-box.

3. For two S-boxes j, k, if an output bit from Sj affects a middle bit of Sk on the next round, then an output bit from Sk cannot affect a middle bit of Sj. This implies that for $j = k$, an output bit from Sj must not affect a middle bit of Sj.

These criteria are intended to increase the diffusion of the algorithm.