Model Question Paper-I with effect from 2022

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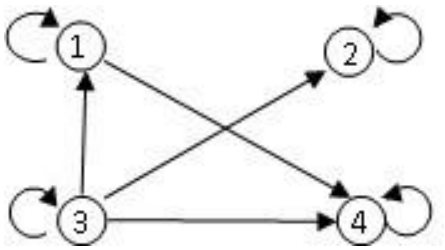
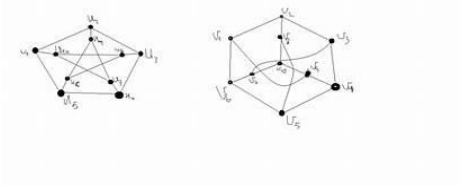
Fourth Semester B.E Degree Examination

Mathematical Foundations for Computing, Probability & Statistics (Computer Science & Allied Engg. branches)-21MATCS41

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

Q.No.		Question	M	L	CO
		Module -1			
01	a	Define tautology. Determine whether the following compound statement is a tautology or not. $\{(p \vee q) \rightarrow r\} \Leftrightarrow \{\neg r \rightarrow \neg(p \vee q)\}$	**06**	**L2**	**CO1**
	b	Using the laws of logic, prove the following logical equivalence $[(\neg p \vee \neg q) \wedge (F_0 \vee p) \wedge p] \Leftrightarrow p \wedge \neg q.$	**07**	**L3**	**CO1**
	c	Give direct proof and proof by contradiction for the statement "If n is an odd integer then $n+9$ is an even integer"	**07**	**L2**	**CO1**
		OR			
02	a	Test the validity of the arguments using rules of inference. $(\neg p \vee q) \rightarrow r$ $r \rightarrow (s \vee t)$ $\neg s \wedge \neg u$ $\underline{\neg u \rightarrow \neg t}$ $\therefore p$	**06**	**L3**	**CO1**
	b	Find whether the following arguments are valid or not for which the universe is the set of all triangles. In triangle XYZ, there is no pair of angles of equal measure. If the triangle has two sides of equal length, then it is isosceles. If the triangle is isosceles, then it has two angles of equal measure. Therefore Triangle XYZ has no two sides of equal length.	**07**	**L3**	**CO1**
	c	If $p(x): x \geq 0, q(x): x^2 \geq 0, r(x): x^2 - 3x - 4 = 0, s(x): x^2 - 3 > 0$ Determine the truth or falsity of the following statement: *i)* $\exists x[p(x) \wedge q(x)]$ *ii)* $\forall x[p(x) \rightarrow q(x)]$ *iii)* $\forall x[q(x) \rightarrow s(x)]$ *iv)* $\forall x[r(x) \wedge s(x)]$ *v)* $\exists x[p(x) \wedge r(x)]$ *vi)* $\forall x[r(x) \rightarrow p(x)]$ *vii)* $\exists x[r(x) \rightarrow \neg p(x)]$	**07**	**L2**	**CO1**
		Module-2			
03	a	Let f and g be functions from R to R defined by $f(x) = ax + b$ *and* $g(x) = 1 - x + x^2$, If $(g \circ f)(x) = 9x^2 - 9x + 3$ determine *a* and *b*.	**06**	**L2**	**CO2**

	b	Let $A=\{1,2,3,4,6\}$ and R be a relation on A defined by aRb if and only if " a is a multiple of b". Write down the relation R, relation matrix $M(R)$ and draw its digraph.	**07**	**L2**	**CO2**
	c	Prove that in every graph the number of vertices of odd degree is even.	07	**L2**	**CO2**
		OR			
4	a	The digraph of a relation R defined on the set $A=\{1,2,3,4\}$ is shown below. Verify that $(A,\ R)$ is a poset and construct the corresponding Hasse diagram. 	**06**	**L2**	**CO2**
	b	Let $A=B=C=R$, and $f:A\to B$ *and* $g:B\to C$ be defined by $f(a)=2a+1, g(b)=\frac{1}{3}b, \forall a\in A, \forall b\in B.$ Compute $g\circ f$ and show that $g\circ f$ is invertible. What is $(g\circ f)^{-1}$?	**07**	**L2**	**CO2**
	c	Define Graph isomorphism. Determine whether the following graphs are isomorphic or not. 	**07**	**L2**	**CO2**
		Module-3			
5	a	Ten competitors in a beauty contest are ranked by two judges A and B in the following order:	**06**	**L2**	**CO3**

ID No. of competitors	1	2	3	4	5	6	7	8	9	10
Judge A	1	6	5	10	3	2	4	9	7	8
Judge B	6	4	9	8	1	2	3	10	5	7

Calculate the rank correlation coefficient.

	b	In a partially destroyed laboratory record, the lines of regression of y on x and x on y are available as $4x-5y+33=0$ *and* $20x-9y=107$. Calculate $\bar{x}$ *and* $\bar{y}$ and the coefficient of correlation between x and y.	**07**	**L2**	**CO3**
	c	An experiment gave the following values:	**07**	**L2**	**CO3**

v(ft/min)	350	400	500	600
t(min.)	61	26	7	26

It is known that v and t are connected by the relation $v=at^b$. Find the best possible values of a and b.

OR

6	a	The following table gives the heights of fathers(x) and sons (y): x: 65, 66, 67, 67, 68, 69, 70, 72 y: 67, 68, 65, 68, 72, 72, 69, 71 Find the lines of regression and Calculate the coefficient of correlation.	**06**	**L2**	**CO3**
	b	Fit a parabola $y = ax^2 + bx + c$ for the data x: 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0 y: 1.1, 1.3, 1.6, 2.0, 2.7, 3.4, 4.1	**07**	**L2**	**CO3**
	c	With usual notation, compute means $\bar{x}, \bar{y}$ and correlation coefficient r from the following lines of regression: $2x + 3y + 1 = 0$ *and* $x + 6y - 4 = 0$.	**07**	**L2**	**CO3**
		Module-4			
7	a	A random variable X has the following probability function: x: -2, -1, 0, 1, 2, 3 $P(x)$: 0.1, k, 0.2, 2k, 0.3, k Find the value of k and calculate the mean and variance	**06**	**L2**	**CO4**
	b	Find the mean and standard deviation of the Binomial distribution	**07**	**L2**	**CO4**
	c	In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and Standard deviation of 60 hours. Estimate the number of bulbs likely to burn for i. More than 2150 hours ii. Less than 1950 hours iii. Between 1920 and 2160 hours	**07**	**L3**	**CO4**
		OR			
8	a	Find the constant k such that $f(x) = \begin{cases} kx^2 & 0 < x < 3 \\ 0 & otherwise \end{cases}$ is a p.d.f. Also, compute i) $P(1 < x < 2)$ ii) $P(x \le 1)$ iii) $P(x > 1)$	**06**	**L2**	**CO4**
	b	2% of fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains (i) no defective fuses (ii) 3 or more defective fuses (iii) at least one defective fuse.	**07**	**L2**	**CO4**
	c	In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and S.D of the distribution.	**07**	**L2**	**CO4**
		Module-5			
9	a	The joint distribution of two random variables X and Y is as follows X \ Y: -4, 2, 7 1: 1/8, 1/4, 1/8 5: 1/4, 1/8, 1/8 Compute the following. (i) E(X) and E(Y) (ii) E(XY) (iii) σ_X and σ_Y (iv) COV(X,Y) (v) $\rho(X,Y)$	**06**	L2	**CO5**

<table>
<tr><td></td><td>b</td><td>A coin was tossed 400 times and head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance.</td><td>07</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, and 4. Can it be concluded that the stimulus will increase the blood pressure? ($t_{.05}$ for 11 d.f = 2.201)</td><td>07</td><td>L3</td><td>CO5</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>10</td><td>a</td><td>Explain the terms: (i) Null hypothesis (ii) Confidence intervals (iii) Type-I and Type-II errors.</td><td>06</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b</td><td>The mean life of 100 fluorescent tube lights manufactured by a company is found to be 1570 hrs with a standard deviation of 120 hrs. Test the hypothesis that the mean lifetime of the lights produced by the company is 1600 hrs at 0.01 level of significance.</td><td>07</td><td>L3</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>A die is thrown 264 times and the number appearing on the face(x) follows the following frequency distribution.
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>y</td><td>40</td><td>32</td><td>28</td><td>58</td><td>54</td><td>60</td></tr>
</table>
Calculate the value of χ^2.</td><td>07</td><td>L3</td><td>CO5</td></tr>
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<tr><td rowspan="3">Bloom's Taxonomy Levels</td><td colspan="3">Lower-order thinking skills</td></tr>
<tr><td>Remembering (knowledge): L_1</td><td>Understanding (Comprehension): L_2</td><td>Applying (Application): L_3</td></tr>
<tr><td colspan="3">Higher-order thinking skills</td></tr>
<tr><td></td><td>Analyzing (Analysis): L_4</td><td>Valuating (Evaluation): L_5</td><td>Creating (Synthesis): L_6</td></tr>
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