Assignment-2 SequenceAndSeries

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Exercise 9.1

Question:
Write the first five terms of each of the sequences in Exercises 1 to 6 whose n^{th} terms are:
4. $x(n) = \frac{2n-3}{6}$
Solution:
n^{th} term of the sequence is given by the above expression
we use n=0, we get

$$x(0) = \frac{2 \times 0 - 3}{6} = \frac{-1}{2} \tag{1}$$

we use n=1, we get First Term,

$$x(1) = \frac{2 \times 1 - 3}{6} = \frac{-1}{6} \tag{2}$$

Similarly we will find other terms like this
we use n=2, we get Second Term,

$$x(2) = \frac{2 \times 2 - 3}{6} = \frac{1}{6} \tag{3}$$

we use n=3, we get Third Term,

$$x(3) = \frac{2 \times 3 - 3}{6} = \frac{1}{2} \tag{4}$$

we use n=4, we get Fourth Term,

$$x(4) = \frac{2 \times 4 - 3}{6} = \frac{5}{6} \tag{5}$$

Therefore, The first five terms of the given sequence are $x(0) = \frac{-1}{2}, x(1) = \frac{-1}{6}, x(2) = \frac{1}{6}, x(3) = \frac{1}{2}, x(4) = \frac{5}{6}$.

To express x(n) in terms of u(n) we can express it as

$$x(n) = \frac{2n-3}{6} = \left(\frac{n}{3} - \frac{1}{2}\right) \cdot u(n) \tag{6}$$

Now lets find Z transform of x(n)

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n) \cdot Z^{-n} \tag{7}$$

Now substitute the expression of $x(n) = \left(\frac{n}{3} - \frac{1}{2}\right) \cdot u(n)$ into the Z-Transform Formula:

$$X(Z) = \sum_{n=-\infty}^{\infty} (\frac{n}{3} - \frac{1}{2}) \cdot u(n) \cdot Z^{-n} \tag{8}$$

Now we can write it as

$$X(Z) = \sum_{n=0}^{\infty} \frac{n}{3} \cdot Z^{-n} - \sum_{n=0}^{\infty} \frac{1}{2} \cdot Z^{-n} \tag{9}$$

Now we will find the both summations

$$\sum_{n=0}^{\infty} \frac{n}{3} \cdot Z^{-n} = \frac{1}{3} \cdot (0 + 1.Z^{-1} + 2.Z^{-2} + \ldots) \tag{10}$$

Now if we multiply the above equation by Z^{-1} we get

$$Z^{-1} \cdot \sum_{n=0}^{\infty} \frac{n}{3} \cdot Z^{-n} = \frac{1}{3} \cdot (1.Z^{-2} + 2.Z^{-3} + \ldots) \tag{11}$$

Now if we subtract the above two equations we get

$$\sum_{n=0}^{\infty} \frac{n}{3} \cdot Z^{-n} = \frac{Z}{Z-1} \cdot \frac{1}{3} \cdot (Z^{-1} + Z^{-2} + Z^{-3} + \ldots) \tag{12}$$

$$\sum_{n=0}^{\infty} \frac{n}{3} \cdot Z^{-n} = \frac{1}{3} \cdot \left(\frac{Z}{(Z-1)^2}\right) \tag{13}$$

$$\sum_{n=0}^{\infty} \frac{1}{2} \cdot Z^{-n} = \frac{1}{2} \cdot (1 + Z^{-1} + Z^{-2} + \ldots) = \frac{1}{2} \cdot \frac{1}{1 - Z^{-1}} \tag{14}$$

So now Z transform of the x(n) is

$$X(Z) = \frac{1}{3} \cdot \frac{Z}{(Z-1)^2} - \frac{1}{2} \cdot \frac{1}{1-Z^{-1}} \tag{15}$$

$$X(Z) = \frac{5Z - 3Z^2}{6(Z-1)^2} \tag{16}$$