

Assignment-2 SequenceAndSeries

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EXERCISE 9.1

Question:

Write the first five terms of each of the sequences in Exercises 1 to 6 whose n^{th} terms are:

4. $a_n = \frac{2n-3}{6}$

Solution:

n^{th} term of the sequence is given by the above expression

we use $n=0$, we get

$$a_0 = \frac{2 \times 0 - 3}{6} = \frac{-1}{2} \quad (1)$$

we use $n=1$, we get First Term,

$$a_1 = \frac{2 \times 1 - 3}{6} = \frac{-1}{6} \quad (2)$$

Similarly we will find other terms like this

we use $n=2$, we get Second Term,

$$a_2 = \frac{2 \times 2 - 3}{6} = \frac{1}{6} \quad (3)$$

we use $n=3$, we get Third Term,

$$a_3 = \frac{2 \times 3 - 3}{6} = \frac{1}{2} \quad (4)$$

we use $n=4$, we get Fourth Term,

$$a_4 = \frac{2 \times 4 - 3}{6} = \frac{5}{6} \quad (5)$$

Therefore, The first five terms of the given sequence are $a_0 = \frac{-1}{2}, a_1 = \frac{-1}{6}, a_2 = \frac{1}{6}, a_3 = \frac{1}{2}, a_4 = \frac{5}{6}$.

Question 2

Express $x(n) = \frac{2n-3}{6}$ in terms of $u(n)$ and find its Z transform

Solution: The $u(n)$ unit step function, which is defined as

$$u(n) = 0 \text{ if } n < 0$$

$$u(n) = 1 \text{ if } n \geq 0$$

Now if we want to express $x(n)$ in terms of $u(n)$

we can break it into two parts: one for $n < 0$ and one for $n \geq 0$

$$x(n) = \frac{2n-3}{6} = \frac{n}{3} - \frac{1}{2} \cdot u(n) \quad (6)$$

Now if we want to find Z transform we can use

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n) \cdot Z^{-n} \quad (7)$$

Now we substitute $x(n) = \frac{n}{3} - \frac{1}{2} \cdot u(n)$ in the above equation we get

$$X(Z) = \sum_{n=0}^{\infty} \left(\frac{n}{3} - \frac{1}{2} \cdot u(n) \right) \cdot Z^{-n} \quad (8)$$

Now we can write it as

$$X(Z) = \sum_{n=0}^{\infty} \frac{n}{3} \cdot Z^{-n} - \sum_{n=0}^{\infty} \frac{1}{2} \cdot Z^{-n} \quad (9)$$

Now we will find the both summations

$$\sum_{n=0}^{\infty} \frac{n}{3} \cdot Z^{-n} = \frac{1}{3} \cdot (0 + 1 \cdot Z^{-1} + 2 \cdot Z^{-2} + \dots) \quad (10)$$

Now if we divide the above equation by Z^{-1} we get

$$Z^{-1} \cdot \sum_{n=0}^{\infty} \frac{n}{3} \cdot Z^{-n} = \frac{1}{3} \cdot (1 \cdot Z^{-2} + 2 \cdot Z^{-3} + \dots) \quad (11)$$

Now if we subtract the above two equations we get

$$\sum_{n=0}^{\infty} \frac{n}{3} \cdot Z^{-n} = \frac{Z}{Z-1} \cdot \frac{1}{3} \cdot (Z^{-1} + Z^{-2} + Z^{-3} + \dots) \quad (12)$$

$$\sum_{n=0}^{\infty} \frac{n}{3} \cdot Z^{-n} = \frac{1}{3} \cdot \left(\frac{Z}{(Z-1)^2} \right) \quad (13)$$

$$\sum_{n=0}^{\infty} \frac{1}{2} \cdot Z^{-n} = \frac{1}{2} \cdot (1 + Z^{-1} + Z^{-2} + \dots) = \frac{1}{2} \cdot \frac{1}{1 - Z^{-1}} \quad (14)$$

So now Z transform of the $x(n)$ is

$$X(Z) = \frac{1}{3} \cdot \frac{Z}{(Z - 1)^2} - \frac{1}{2} \cdot \frac{1}{1 - Z^{-1}} \quad (15)$$

$$X(Z) = \frac{5Z - 3Z^2}{6(Z - 1)^2} \quad (16)$$

