EE23BTECH11047 - Deepakreddy P

Exercise 9.1

4 Write the first five terms of the sequence whose nth term is $\frac{2n-3}{6}$ and obtain the Z transform of the series

Solution:

$$x(0) = \frac{2\times 0-3}{6} = \frac{-1}{2} \tag{1}$$
$$x(1) = \frac{2\times 1-3}{6} = \frac{-1}{6} \tag{2}$$
$$x(2) = \frac{2\times 2-3}{6} = \frac{1}{6} \tag{3}$$
$$x(3) = \frac{2\times 3-3}{6} = \frac{1}{2} \tag{4}$$
$$x(4) = \frac{2\times 4-3}{6} = \frac{5}{6} \tag{5}$$
$$x(n) = \frac{2n-3}{6}\left(u(n)\right) \tag{6}$$

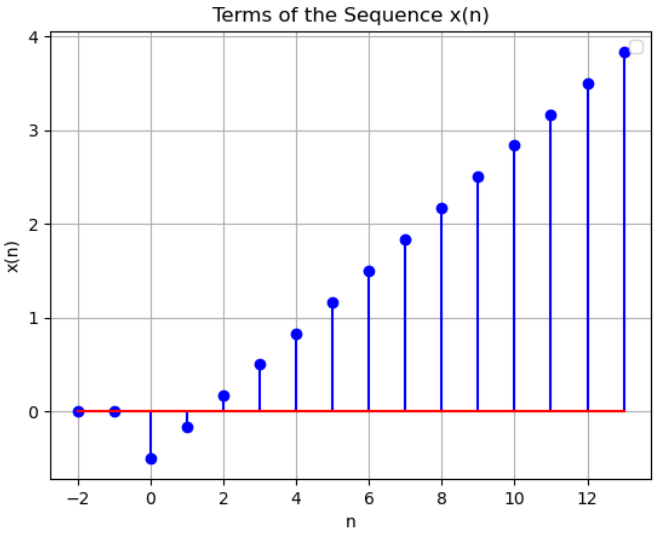

Fig. 1. Plot of x(n) vs n

$$x(n)\, ZX(z) \tag{7}$$

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n)Z^{-n} \tag{8}$$
$$= \sum_{n=-\infty}^{\infty} \left(\frac{n}{3} - \frac{1}{2}\right)u(n)z^{-n} \tag{9}$$
$$= \sum_{n=0}^{\infty} \frac{n}{3}z^{-n} - \sum_{n=0}^{\infty} \frac{1}{2}z^{-n} \tag{10}$$

$$\sum_{n=0}^{\infty} \frac{n}{3}z^{-n} = \frac{1}{3}\left(\frac{Z}{(z-1)^2}\right) \tag{11}$$
$$\sum_{n=0}^{\infty} \frac{1}{2}z^{-n} = \frac{1}{2}\left(\frac{1}{1-z^{-1}}\right) \tag{12}$$

$$X(z) = \frac{1}{3}\frac{z}{(z-1)^2} - \frac{1}{2}\frac{1}{1-z^{-1}} \tag{13}$$
$$X(z) = \frac{5z-3z^2}{6(z-1)^2} \qquad \{z \in \mathbb{C} : z \neq 1\} \tag{14}$$

Symbol	Parameters
x(n)	general term of the series
X(z)	Z-transform of x(n)
u(n)	unit step function

TABLE I
Parameters