EE23BTECH11047 - Deepakreddy P

17 If a, b, c, d are in G.P, prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P

Solution:

TABLE I
INPUT PARAMETERS

Symbol	Input value
a	ar^0
b	ar^1
c	ar^2
d	ar^3

$$r = \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \tag{1}$$

$$= \frac{b^n + c^n}{a^n + b^n} \tag{2}$$

From eq(1)

$$\implies \frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n} \tag{3}$$

Hence proved they are in in G.P

TABLE II
INPUT PARAMETERS

Symbol	Input value
a	0.25
b	0.25 (2)
c	0.25 (4)
d	0.25 (8)

$$\frac{b^n + c^n}{a^n + b^n} = \frac{0.25^n (2^n + 4^n)}{0.25^n (1^n + 2^n)} \tag{4}$$

$$= \frac{0.25^n (2)^n (2^n + 4^n)}{0.25^n (2)^n (1^n + 2^n)} \tag{5}$$

$$= \frac{0.25^n (4^n + 8^n)}{0.25^n (2^n + 4^n)} \tag{6}$$

$$\implies \frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n} \tag{7}$$

$$x(n) = x(0) r^n u(n) \tag{8}$$
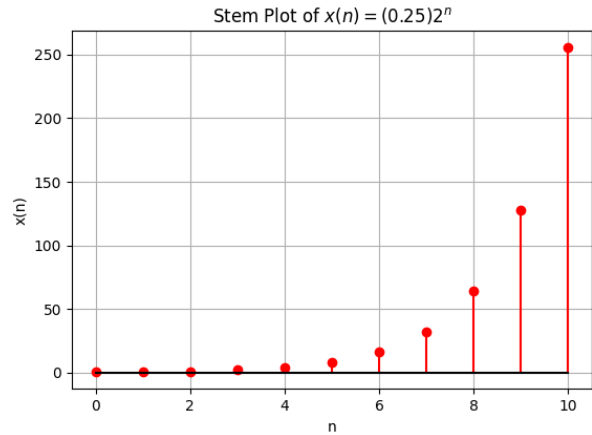
$$X(z) = \frac{x(0)}{1 - rz^{-1}}, \quad |z| > |r| \tag{9}$$

Fig. 1. Plot of x(n) vs n where x(0)= 0.25 and r = 2