EE23BTECH11047 - Deepakreddy P

17 If a, b, c, d are in G.P, prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P

Solution:

TABLE I Input Parameters

Symbol	Remarks
x(0)	a
x(1)	b
x(2)	С
x(3)	d
r	ratio of G.P a,b,c
r_1	ratio of G.P $a^n + b^n, b^n + c^n, \dots$
X(z)	z transform of G.P a,b,c
Y(z)	z transform of G.P $a^n + b^n, b^n + c^n,$

$$r = \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \tag{1}$$

From eq(1)

$$\frac{b^n + c^n}{a^n + b^n} = \frac{(ar)^n + (ar^2)^n}{(a)^n + (ar)^n}$$
(2)

$$= \frac{a^n r^n (1 + r^n)}{a^n (1 + r^n)}$$
 (3)

$$=r^{n} \tag{4}$$

$$\frac{c^{n} + d^{n}}{b^{n} + c^{n}} = \frac{\left(ar^{2}\right)^{n} + \left(ar^{3}\right)^{n}}{\left(ar\right)^{n} + \left(ar^{2}\right)^{n}}$$
 (5)

$$=\frac{a^{n}r^{2n}(1+r^{n})}{a^{n}r^{n}(1+r^{n})}$$
 (6)

$$=r^{n} \tag{7}$$

$$\frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n}$$
 (8)

$$x(n) = a\left(\frac{b}{a}\right)^n u(n) \tag{9}$$

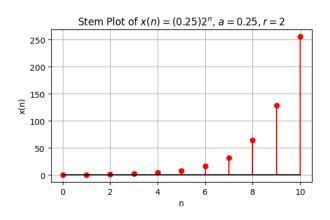
$$X(z) = \frac{a}{1 - \frac{b}{a}z^{-1}}, \quad |z| > \left|\frac{b}{a}\right|$$
 (10)

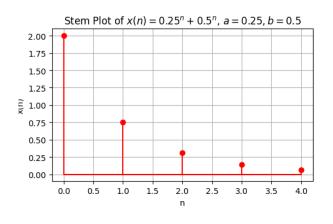
$$r_1 = \frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n}$$
 (11)

From eq(11)

$$y(n) = (a^n + b^n) \left(\frac{b^n + c^n}{a^n + b^n}\right)^n u(n)$$
 (12)

$$Y(z) = \frac{a^n + b^n}{1 - \left(\frac{b^n + c^n}{a^n + b^n}\right)z^{-1}}, \quad |z| > \left|\frac{b^n + c^n}{a^n + b^n}\right|$$
(13)





Hence proved they are in in G.P