EE23BTECH11047 - Deepakreddy P

17 If a, b, c, d are in G.P, prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P

Solution:

TABLE I
INPUT PARAMETERS

Symbol	Remarks
$x(0)$	a
$x(1)$	b
$x(2)$	c
$x(3)$	d
r	ratio of G.P a,b,c....
r_1	ratio of G.P $a^n + b^n, b^n + c^n, ...$
$X(z)$	z transform of G.P a,b,c....
$Y(z)$	z transform of G.P $a^n + b^n, b^n + c^n, ...$

$$r = \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \tag{1}$$

From eq(1)

$$\frac{b^n + c^n}{a^n + b^n} = \frac{(ar)^n + \left(ar^2\right)^n}{(a)^n + (ar)^n} \tag{2}$$
$$= \frac{a^n r^n (1 + r^n)}{a^n (1 + r^n)} \tag{3}$$
$$= r^n \tag{4}$$
$$\frac{c^n + d^n}{b^n + c^n} = \frac{\left(ar^2\right)^n + \left(ar^3\right)^n}{(ar)^n + (ar^2)^n} \tag{5}$$
$$= \frac{a^n r^{2n} (1 + r^n)}{a^n r^n (1 + r^n)} \tag{6}$$
$$= r^n \tag{7}$$
$$\frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n} \tag{8}$$

Hence proved they are in in G.P

$$x(n) = a\left(\frac{b}{a}\right)^n u(n) \tag{9}$$
$$X(z) = \frac{a}{1 - \frac{b}{a}z^{-1}}, \quad |z| > \left|\frac{b}{a}\right| \tag{10}$$

$$r_1 = \frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n} \tag{11}$$

From eq(11)

$$y(n) = (a^n + b^n)\left(\frac{b^n + c^n}{a^n + b^n}\right)^n u(n) \tag{12}$$
$$Y(z) = \frac{a^n + b^n}{1 - \left(\frac{b^n + c^n}{a^n + b^n}\right)z^{-1}}, \quad |z| > \left|\frac{b^n + c^n}{a^n + b^n}\right| \tag{13}$$