EE23BTECH11047 - Deepakreddy P

17 If a, b, c, d are in G.P, prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P and find the Z transform of General term of G.P.

Solution:

TABLE I
INPUT PARAMETERS

Symbol	Input value
$x(0)$	$x(0)r^0$
$x(1)$	$x(0)r^1$
$x(2)$	$x(0)r^2$
$x(3)$	$x(0)r^3$

$$r = \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \tag{1}$$

$$= \frac{b^n + c^n}{a^n + b^n} \tag{2}$$

$$= \frac{b^n\left(1 + \frac{c^n}{b^n}\right)}{a^n\left(1 + \frac{b^n}{a^n}\right)} \tag{3}$$

$$= \frac{c^n\left(1 + \left(\frac{d}{c}\right)^n\right)}{b^n\left(1 + \left(\frac{c}{b}\right)^n\right)} \tag{4}$$

$$\implies \frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n} \tag{5}$$

Hence proved they are in in G.P

TABLE II
INPUT PARAMETERS

Symbol	Input value
a	0.25
b	0.25 (2)
c	0.25 (4)
d	0.25 (8)

$$\frac{b^n + c^n}{a^n + b^n} = \frac{0.25^n\left(2^n + 4^n\right)}{0.25^n\left(1^n + 2^n\right)} \tag{6}$$

$$= \frac{0.25^n\left(2\right)^n\left(2^n + 4^n\right)}{0.25^n\left(2\right)^n\left(1^n + 2^n\right)} \tag{7}$$

$$= \frac{0.25^n\left(4^n + 8^n\right)}{0.25^n\left(2^n + 4^n\right)} \tag{8}$$

$$\implies \frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n} \tag{9}$$

$$x\left(n\right) = x\left(0\right) r^n u\left(n\right) \tag{10}$$

$$X\left(z\right) = \frac{x\left(0\right)}{1 - rz^{-1}}, \quad |z| > |r| \tag{11}$$

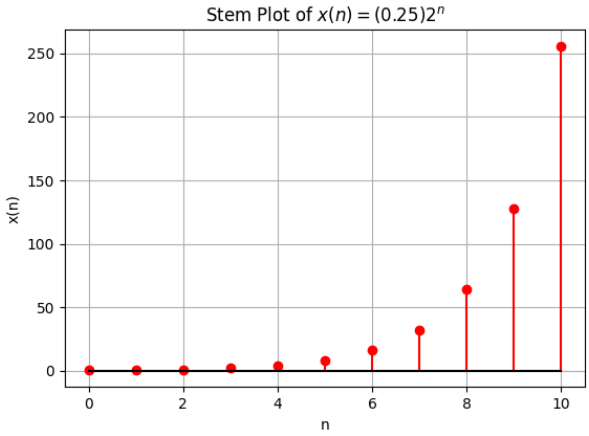

Fig. 1. Plot of x(n) vs n where x(0)= 0.25 and r = 2