EE23BTECH11047 - Deepakreddy P

17 If a, b, c, d are in G.P, prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P

Solution:

TABLE I
INPUT PARAMETERS

Symbol	Input value
$x(0)$	ar^0
$x(1)$	ar^1
$x(2)$	ar^2
$x(3)$	ar^3

$$r = \frac{x(1)}{x(0)} = \frac{x(2)}{x(1)} = \frac{x(3)}{x(2)} \tag{1}$$

$$= \frac{x(1)^n + x(2)^n}{x(0)^n + x(1)^n} \tag{2}$$

From eq(1)

$$\implies \frac{x(1)^n + x(2)^n}{x(0)^n + x(1)^n} = \frac{x(2)^n + x(3)^n}{x(1)^n + x(2)^n} \tag{3}$$

Hence proved they are in in G.P

$$x(n) = x(0)\, r^n u(n) \tag{4}$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}}, \quad |z| > |r| \tag{5}$$

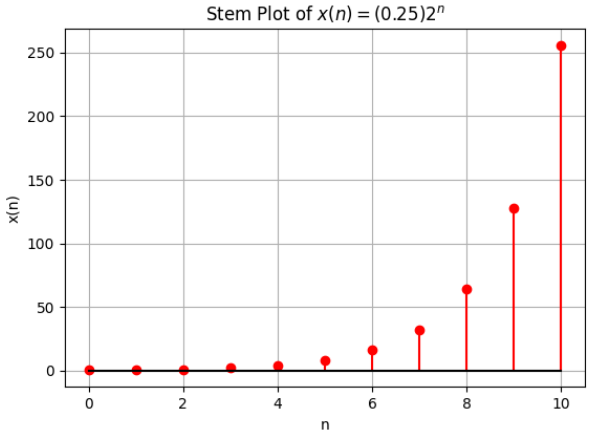

Fig. 1. Plot of x(n) vs n where x(0)= 0.25 and r = 2

$$\frac{x(1)^n + x(2)^n}{x(0)^n + x(1)^n} = \frac{0.25^n\,(2^n + 4^n)}{0.25^n\,(1^n + 2^n)} \tag{6}$$

$$= \frac{0.25^n\,(2)^n\,(2^n + 4^n)}{0.25^n\,(2)^n\,(1^n + 2^n)} \tag{7}$$

$$= \frac{0.25^n\,(4^n + 8^n)}{0.25^n\,(2^n + 4^n)} \tag{8}$$

$$\implies \frac{x(1)^n + x(2)^n}{x(0)^n + x(1)^n} = \frac{x(2)^n + x(3)^n}{x(1)^n + x(2)^n} \tag{9}$$

$$X(z) = \frac{0.25}{1 - 2z^{-1}}, \quad |z| > |2| \tag{10}$$