EE23BTECH11047 - Deepakreddy P

17 If a, b, c, d are in G.P, prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P

Solution:

TABLE I
INPUT PARAMETERS

Symbol	Input value
$x(0)$	a
$x(1)$	b
$x(2)$	c
$x(3)$	d
r	$\frac{b}{a}$

$$r = \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \tag{1}$$

From eq(1)

$$= \frac{b^n + c^n}{a^n + b^n} \tag{2}$$

$$= \frac{\left(ar^1\right)^n + \left(ar^2\right)^n}{\left(ar^0\right)^n + \left(ar^1\right)^n} \tag{3}$$

$$= \frac{r^n\left(\left(ar^1\right)^n + \left(ar^2\right)^n\right)}{r^n\left(\left(ar^0\right)^n + \left(ar^1\right)^n\right)} \tag{4}$$

$$= \frac{\left(ar^2\right)^n + \left(ar^3\right)^n}{\left(ar^1\right)^n + \left(ar^2\right)^n} \tag{5}$$

$$= \frac{c^n + d^n}{b^n + c^n} \tag{6}$$

$$\implies \frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n} \tag{7}$$

$$\implies \frac{x(1)^n + x(2)^n}{x(0)^n + x(1)^n} = \frac{x(2)^n + x(3)^n}{x(1)^n + x(2)^n} \tag{8}$$

$$\tag{9}$$

Hence proved they are in in G.P

$$x(n) = ar^n u(n) \tag{10}$$

$$X(z) = \frac{a}{1 - rz^{-1}}, \quad |z| > |r| \tag{11}$$