

# NCERT Physics 12.7 Q6

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**Question:** Obtain the resonance frequency of a series LCR circuit with  $L = 2.0\text{ H}$ ,  $C = 32\text{ }\mu\text{F}$ , and  $R = 10\text{ }\Omega$ . What is the Q-value of the circuit.

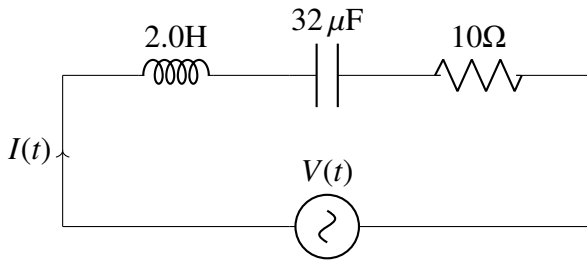


Fig. 1. LCR Circuit

**Solution:**

Symbol	Value	Description
$L$	$2.0\text{ H}$	Inductance
$C$	$32\text{ }\mu\text{F}$	Capacitance
$R$	$10\text{ }\Omega$	Resistance
$Q$	$\frac{V_L}{V_R}$	Quality Factor
$V_L$	$sLI(j\omega)$	Voltage across inductance
$V_C$	$RI(j\omega)$	Voltage across capacitor
$\omega_0$	$\frac{1}{\sqrt{LC}}$	Resonant frequency

TABLE I  
PARAMETERS

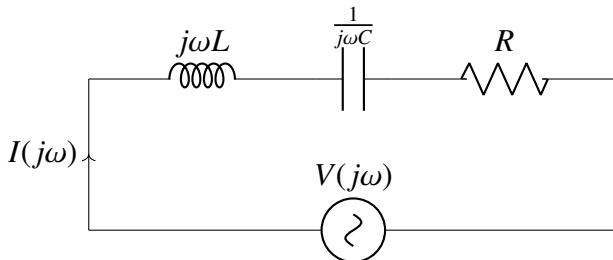


Fig. 2. LCR Circuit

1) Frequency Response of the Circuit

$$V(j\omega) = I(j\omega) \left( R + Lj\omega + \frac{1}{j\omega C} \right) \quad (1)$$

$$\Rightarrow I(j\omega) = \frac{V(s)}{\left( R + Lj\omega + \frac{1}{j\omega C} \right)} \quad (2)$$

At resonance,

$$Lj\omega + \frac{1}{j\omega C} = 0 \quad (3)$$

$$\omega = \frac{1}{\sqrt{LC}} \quad (4)$$

At resonance, Resonant frequency( $\omega_0$ ) =  $\frac{1}{\sqrt{LC}}$

2) Quality Factor

a) voltage across inductor,

$$Q = \left( \frac{V_L}{V_R} \right)_{\omega_0} = \frac{|j\omega_0 LI(j\omega)|}{|RI(j\omega)|} \quad (5)$$

$$= \frac{1}{\sqrt{LC}} \frac{L}{R} \quad (6)$$

$$= \frac{1}{R} \sqrt{\frac{L}{C}} \quad (7)$$

b) Using voltage across capacitor,

$$Q = \left( \frac{V_C}{V_R} \right)_{\omega_0} = \frac{\left| \frac{I(j\omega)}{j\omega_0 C} \right|}{|RI(j\omega)|} \quad (8)$$

$$= \frac{\sqrt{LC}}{RC} \quad (9)$$

$$= \frac{1}{R} \sqrt{\frac{L}{C}} \quad (10)$$

3) Plot of Impedance vs Angular Frequency

$$H(j\omega) = \frac{V(j\omega)}{I(j\omega)} \quad (11)$$

$$H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \quad (12)$$

$$\Rightarrow |H(j\omega)| = \sqrt{R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2} \quad (13)$$

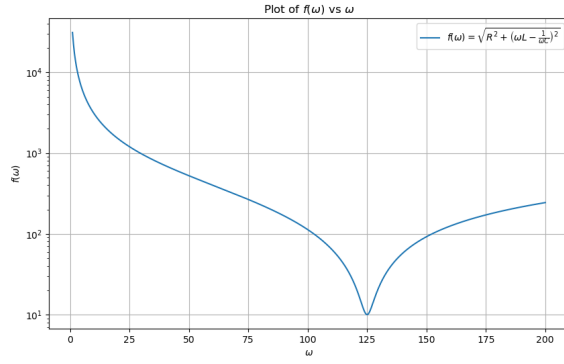


Fig. 3. Impedance vs  $\omega$

Substituting values,

$$\omega_0 = \frac{1}{\sqrt{(2.0)(32 \times 10^{-6})}} \quad (14)$$

$$\omega_0 = 125 \text{ Hz} \quad (15)$$

$$Q = \frac{1}{10} \sqrt{\frac{2}{32 \times 10^{-6}}} \quad (16)$$

$$Q = 25 \quad (17)$$