

Discrete Assignment

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Question (11.9.3.14) The sum of first three terms of a G.P. is 16 and the sum of next three terms is 128. Determine the first term, the common ratio, and the sum to n terms of the G.P.

Answer

Parameter	Description	Value
$x(0)$	First term of GP	–
r	Common ratio	–
$x(n)$	General term of given GP	$x(0)r^n u(n)$
$x(0) + x(1) + x(2)$	sum of 1st, 2nd, and 3rd terms	16
$x(3) + x(4) + x(5)$	sum of 3rd, 4th, and 5th terms	128

$$y(n) = x(n) * u(n) \quad (1)$$

$$Y(z) = X(z)U(z) \quad (2)$$

$$X(z) = A \cdot \frac{1}{1 - r^{-1}} \quad (3)$$

$$Y(z) = A \cdot \frac{1}{(1 - r^{-1})^2} \quad (4)$$

Applying inverse Z-transform:

$$y(n) = x(0) \left[\frac{r^n - 1}{r - 1} \right] \quad (5)$$

For $y(3)$:

$$y(3) = x(0) \left[\frac{r^3 - 1}{r - 1} \right] \quad (6)$$

For $y(6) - y(3)$:

$$y(6) - y(3) = x(0) \left[\frac{r^6 - 1}{r - 1} - \frac{r^3 - 1}{r - 1} \right] \quad (7)$$

$$128 = x(0) \left[\frac{r^6 - 1}{r - 1} - \frac{r^3 - 1}{r - 1} \right] \quad (8)$$

$$128 = x(0)r^3 \left[\frac{r^3 - 1}{r - 1} \right] \quad (9)$$

$$16 = x(0) \left[\frac{r^3 - 1}{r - 1} \right] \quad (10)$$

Divide equation (??) by equation (??):

$$\frac{128}{16} = r^3 \quad (11)$$

$$r^3 = 8 \quad (12)$$

$$r = 2 \quad (13)$$

So, the solution for r is 2. Substituting this value back into the expression for $x(0)$, we get:

$$x(0) = \frac{16}{2^2 + 2 + 1} \quad (14)$$

$$= \frac{16}{7} \quad (15)$$

So, $r = 2$ and $x(0) = \frac{16}{7}$.

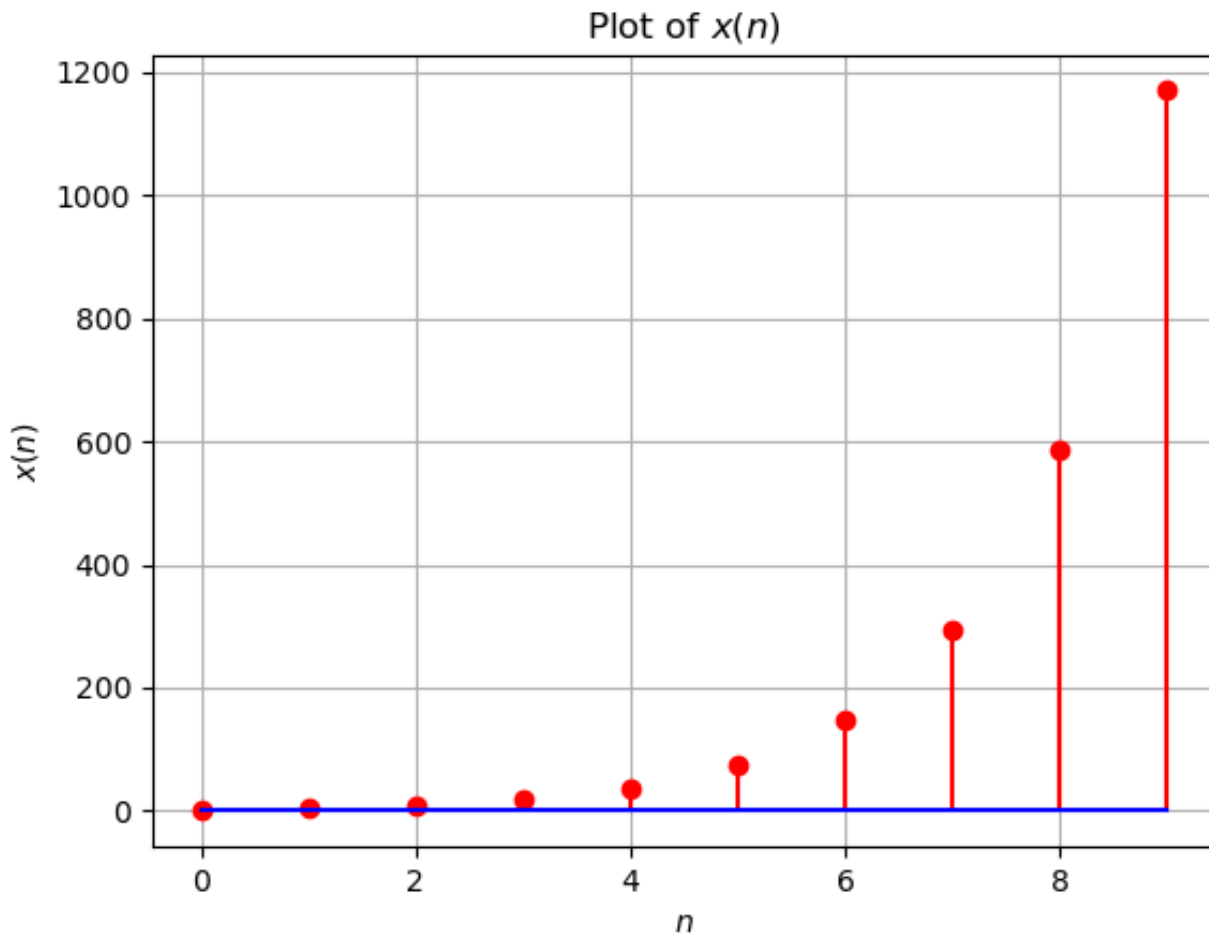


Fig. 1. Stem plots of $x(n)$

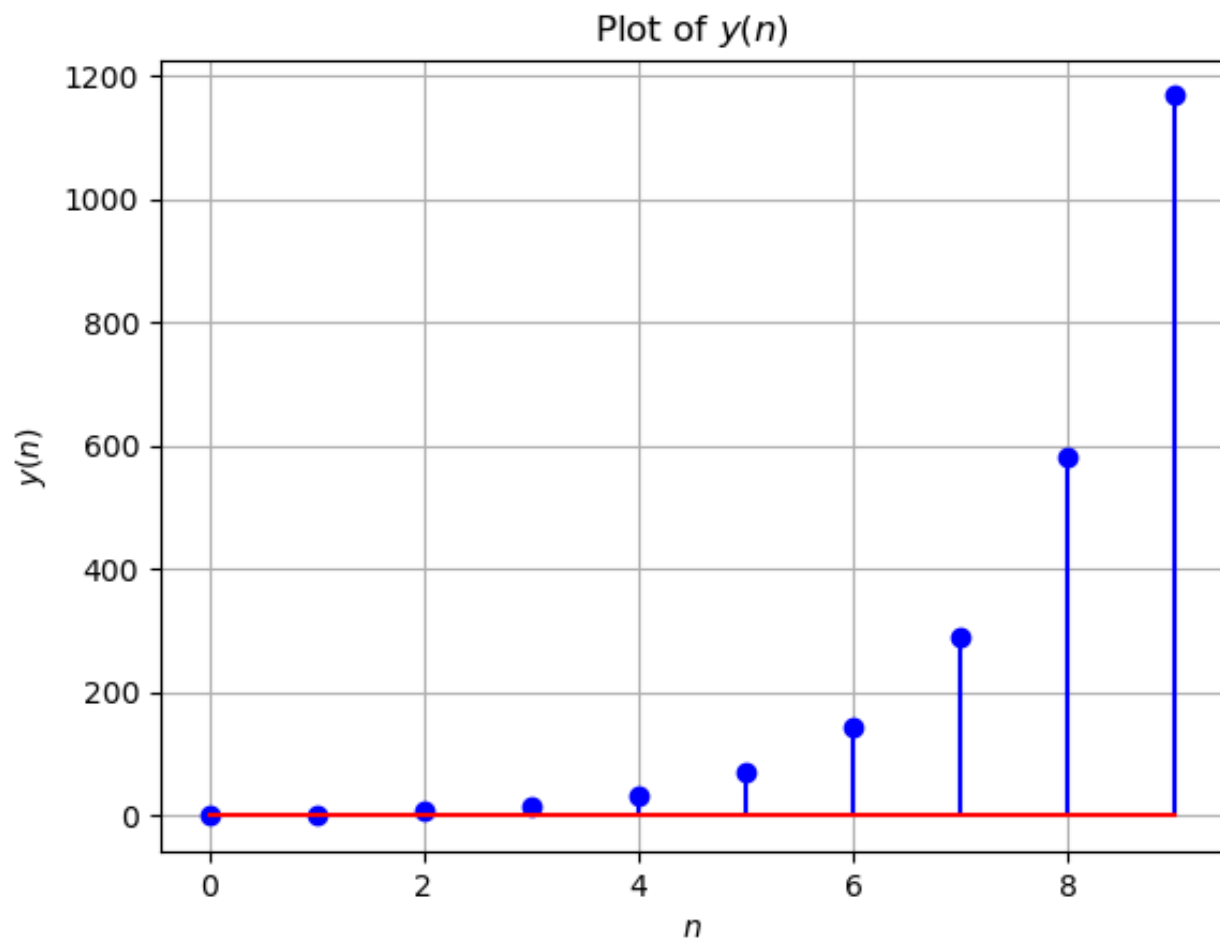


Fig. 2. Stem plots of $y(n)$