ELL715 : Assignment 1

Deepali Gupta, 2013MT60079

January 26, 2017

All questions use the following original image:

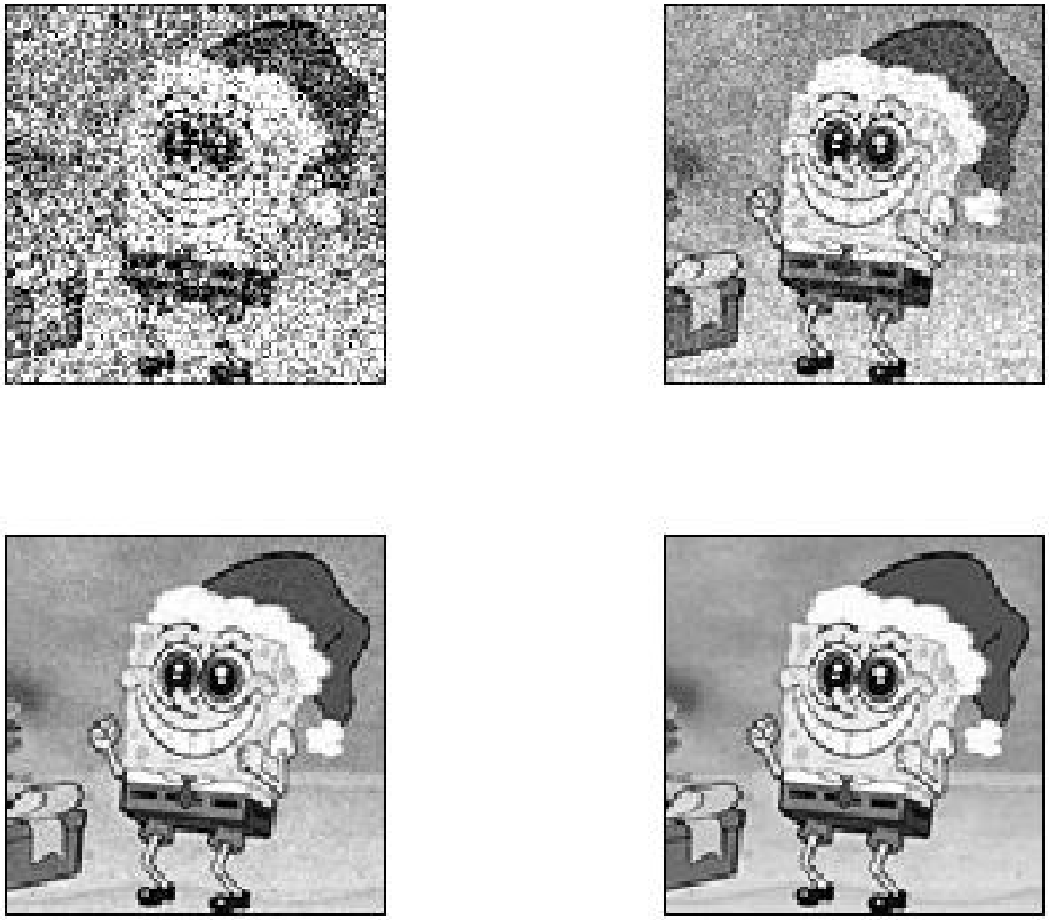
Original Image

1 Answer 1

(Code in q1.m)

256 scale 100x100 grayscale image

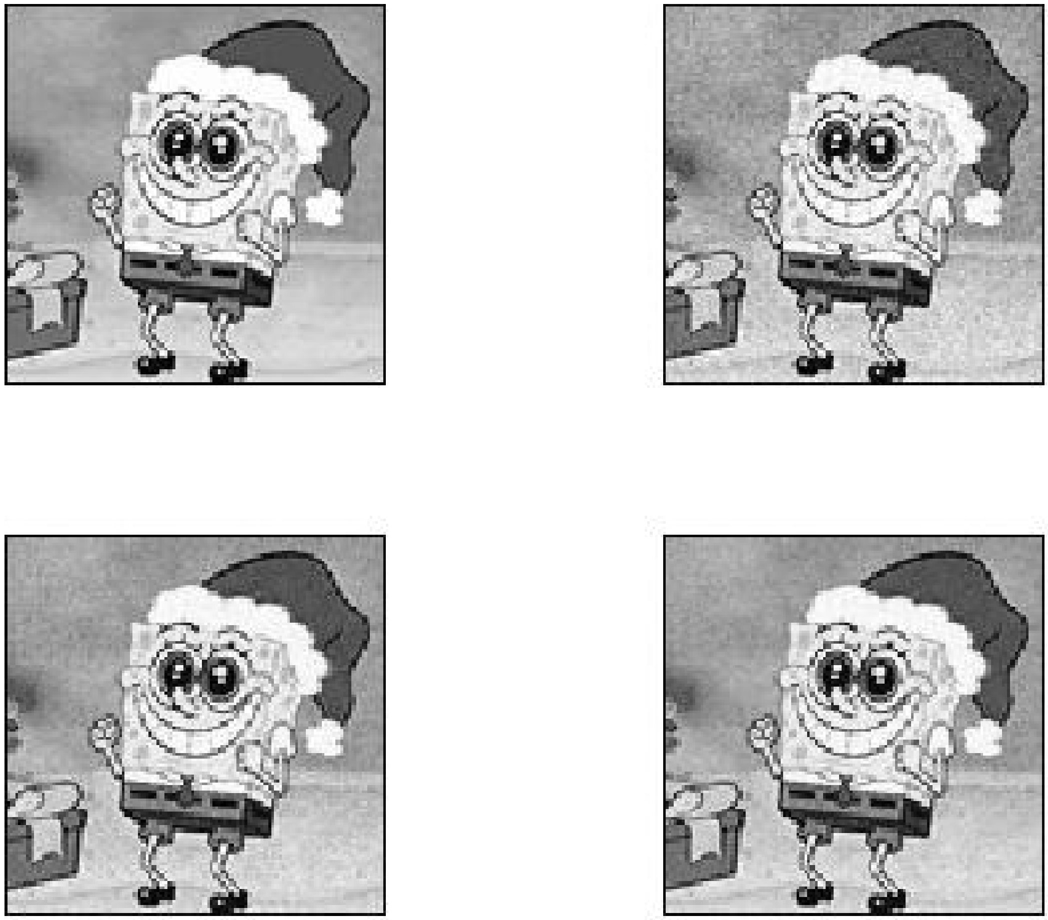
1.1 Adding Gaussian Noise



Images after applying noise with SNR 0, 10, 20, 30dB respectively

1.2 Applying Smoothing Operation

No of images used	Mean Square Error
5	0.0013
10	6.8139e-04
15	4.9275e-04



After smoothing operation using 5, 10, 15 images respectively

2 Answer 2

(Code in q2.m)

2.1 Applying 2D affine transform

The following matrices were sequentially multiplied to the transformed image.

- Resizing :

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Resizing :

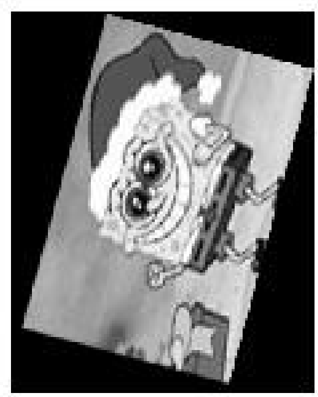
$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 6 & 7 & 1 \end{bmatrix}$$

- Resizing :

$$C = \begin{bmatrix} cos(r) & sin(r) & 0 \\ -sin(r) & cos(r) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Final Transformation Matrix :

$$M = \begin{bmatrix} 0.7784 & -1.9315 & 0 \\ 2.8973 & 0.5189 & 0 \\ 18 & 14 & 1 \end{bmatrix}$$



Original, Enlarged, Translated, Rotated(final) operations applied successively

3 Answer 3

(Code in q3.m)

3.1 Bilinear Interpolation

Suppose that we want to find the value of the unknown function f at the point (x, y). It is assumed that we know the value of f at the four points $Q_{11} = (x_1, y_1), Q_{12} = (x_1, y_2), Q_{21} = (x_2, y_1), and Q_{22} = (x_2, y_2)$.

$$
\begin{aligned}
f(x,y) &\approx \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2) \\
&= \frac{y_2 - y}{y_2 - y_1}\left(\frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21})\right) + \frac{y - y_1}{y_2 - y_1}\left(\frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22})\right) \\
&= \frac{1}{(x_2 - x_1)(y_2 - y_1)} \left(f(Q_{11})(x_2 - x)(y_2 - y) + f(Q_{21})(x - x_1)(y_2 - y) + f(Q_{12})(x_2 - x)(y - y_1) + f(Q_{22})(x - x_1)(y - y_1)\right) \\
&= \frac{1}{(x_2 - x_1)(y_2 - y_1)} \begin{bmatrix} x_2 - x & x - x_1 \end{bmatrix} \begin{bmatrix} f(Q_{11}) & f(Q_{12}) \\ f(Q_{21}) & f(Q_{22}) \end{bmatrix} \begin{bmatrix} y_2 - y \\ y - y_1 \end{bmatrix}
\end{aligned}
$$

3.2 Bicubic Interpolation

Suppose the function values f and the derivatives f_x, f_y and f_{xy} are known at the four corners $(0, 0), (1, 0), (0, 1), and (1, 1)$ of the unit square. The interpolated surface can then be written

$$p(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j.$$

The interpolation problem consists of determining the 16 coefficients a_{ij}. Matching $p(x, y)$ with the function values yields four equations,

1. $f(0,0) = p(0,0) = a_{00}$
2. $f(1,0) = p(1,0) = a_{00} + a_{10} + a_{20} + a_{30}$
3. $f(0,1) = p(0,1) = a_{00} + a_{01} + a_{02} + a_{03}$
4. $f(1,1) = p(1,1) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij}$

Likewise, eight equations for the derivatives in the x-direction and the y-direction

1. $f_x(0,0) = p_x(0,0) = a_{10}$
2. $f_x(1,0) = p_x(1,0) = a_{10} + 2a_{20} + 3a_{30}$
3. $f_x(0,1) = p_x(0,1) = a_{10} + a_{11} + a_{12} + a_{13}$
4. $f_x(1,1) = p_x(1,1) = \sum_{i=1}^{3}\sum_{j=0}^{3} a_{ij} i$
5. $f_y(0,0) = p_y(0,0) = a_{01}$
6. $f_y(1,0) = p_y(1,0) = a_{01} + a_{11} + a_{21} + a_{31}$
7. $f_y(0,1) = p_y(0,1) = a_{01} + 2a_{02} + 3a_{03}$
8. $f_y(1,1) = p_y(1,1) = \sum_{i=0}^{3}\sum_{j=1}^{3} a_{ij} j$

And four equations for the cross derivative xy.

1. $f_{xy}(0,0) = p_{xy}(0,0) = a_{11}$
2. $f_{xy}(1,0) = p_{xy}(1,0) = a_{11} + 2a_{21} + 3a_{31}$
3. $f_{xy}(0,1) = p_{xy}(0,1) = a_{11} + 2a_{12} + 3a_{13}$
4. $f_{xy}(1,1) = p_{xy}(1,1) = \sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} ij$

where the expressions above have used the following identities,

$$p_x(x,y) = \sum_{i=1}^{3}\sum_{j=0}^{3} a_{ij} i x^{i-1} y^j$$

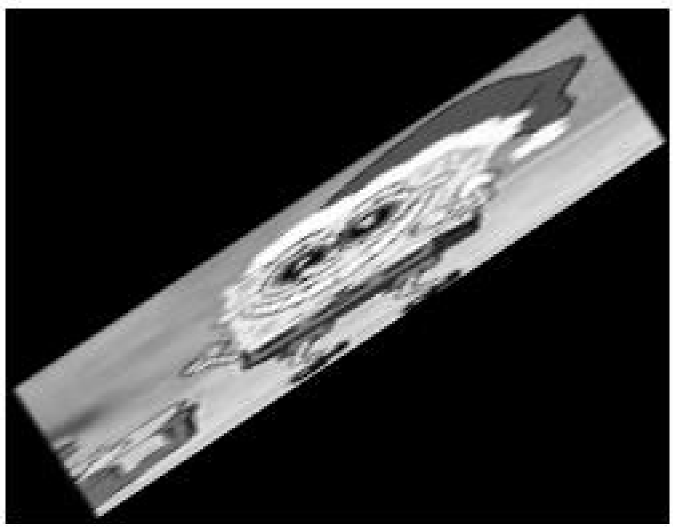
$$p_y(x,y) = \sum_{i=0}^{3}\sum_{j=1}^{3} a_{ij} x^i j y^{j-1}$$

$$p_{xy}(x,y) = \sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} i x^{i-1} j y^{j-1} .$$

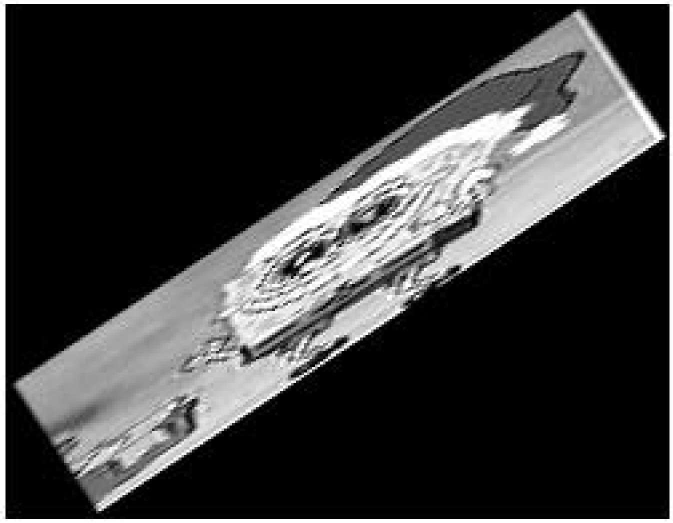
This procedure yields a surface $p(x, y)$ on the unit square $[0, 1] \times [0, 1]$ which is continuous and with continuous derivatives. Bicubic interpolation on an arbitrarily sized regular grid can then be accomplished by patching together such bicubic surfaces, ensuring that the derivatives match on the boundaries.

Results after enlarging 2.6 times along rows, 1.7 times along columns and rotating 33.5° clockwise:

Original Image



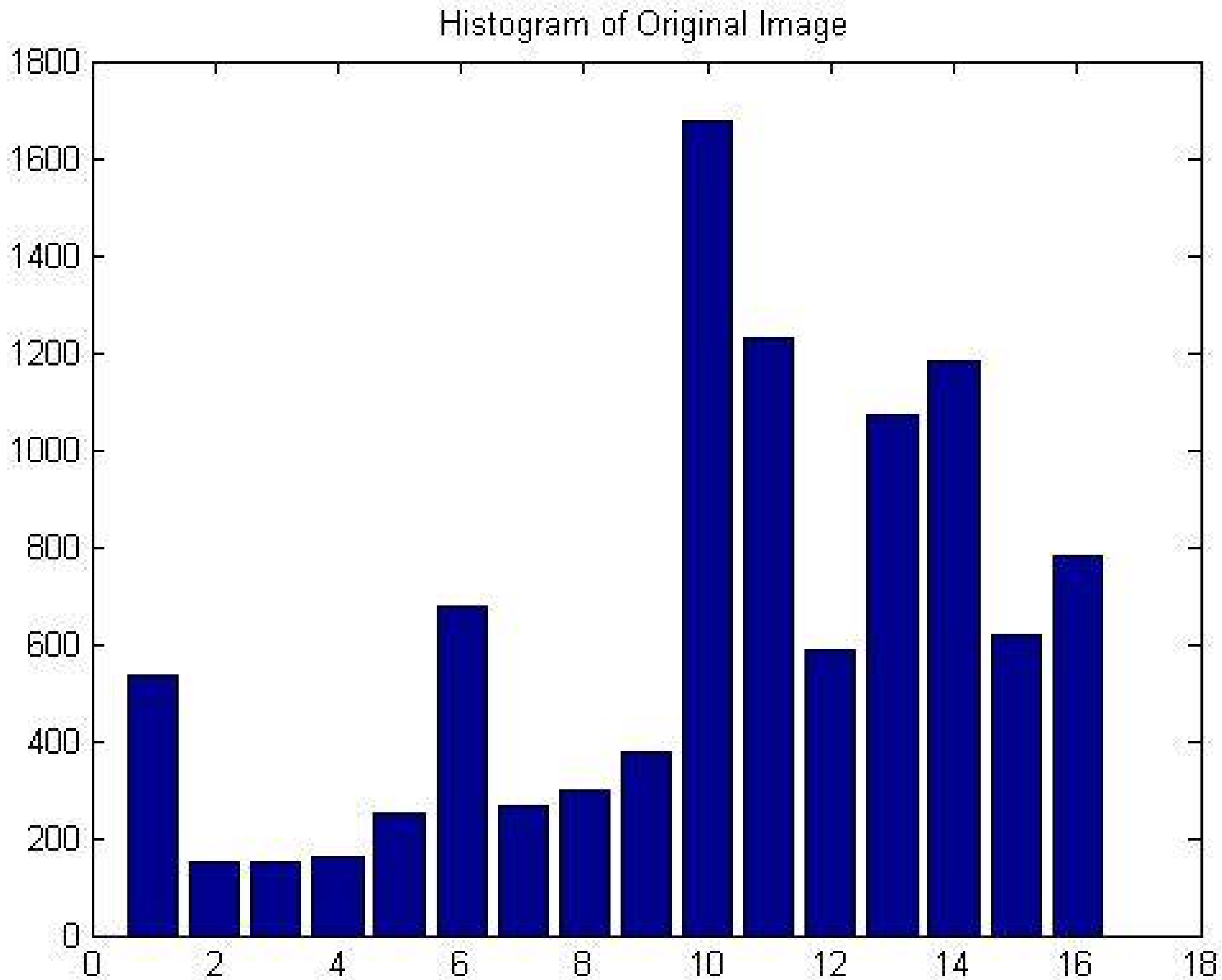
Bilinear Interpolation



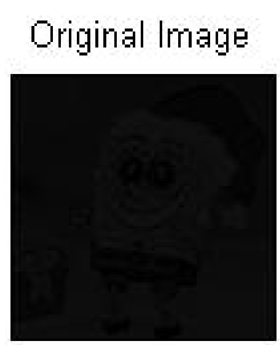
Bicubic Interpolation

4 Asnwer 4

(Code in q4.m)
Histogram of the original image plotted using MATLAB Code:



Histogram



Original 16-level Image

Image after histogram equalisation

Image with histogram uniform from 8 to 15 only

Image	Mean	Standard Deviation
Original	9.4348	4.0113
Histogram Equalised	8.3135	4.4275
Histogram Uniform(8-15)	11.1217	3.3515