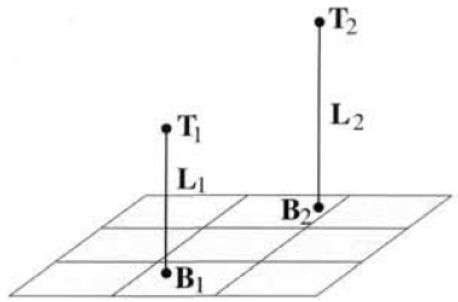
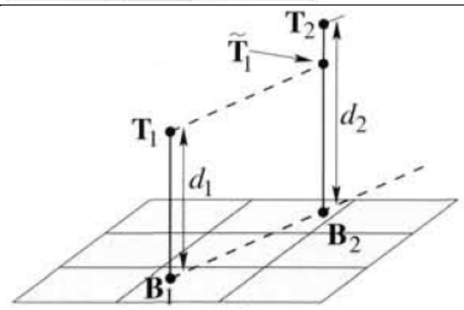
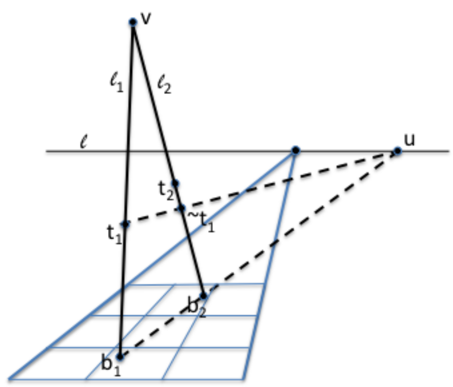
2. MATLAB Task: compute the length ratios of parallel scene lines (45 Points)

a) Read and understand the table below, parts (a),..,(d)! What happens between steps (c)->(d) and why is this necessary?
b) How to find a line ℓ going through two points $\mathbf{x_1}, \mathbf{x_2}$ and how to determine the intersection point $\mathbf{x}$ of two lines ℓ_1 and ℓ_2?
c) Then write a MATLAB function for the following objective: Given the vanishing line of the ground plane ℓ and the vertical vanishing point **v** and the top ($\mathbf{t_1},\mathbf{t_2}$) and base ($\mathbf{b_1},\mathbf{b_2}$) points of two line segments as in the table, compute the ratio of lengths of the line segments in the scene.

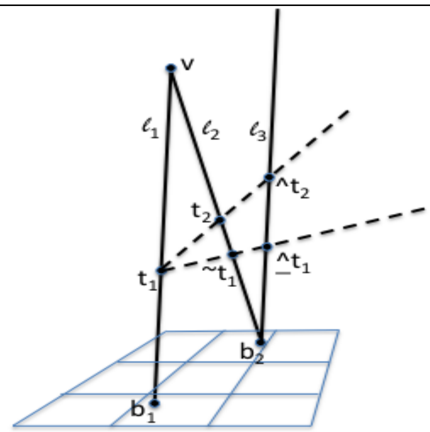
Hint:

(i) Compute the vanishing point **u** as intersection of ℓ with the line through $\mathbf{b_1},\mathbf{b_2}$
(ii) Compute the transferred point $\tilde{\mathbf{t}}_1$ as intersection of the lines $\mathbf{t_1},\mathbf{u}$ and $\mathbf{v},\mathbf{b_2}$
(iii) Represent the four points $\mathbf{b_2}, \tilde{\mathbf{t}}_1, \mathbf{t_2}$ and **v** on the image line ℓ_2 by their distance from $\mathbf{b_2}$, as 0, $\tilde{t}_1$, t_2 and v respectively
(iv) Compute an 1D projective $\mathbf{H}_{2x2}$ mapping homogeneous coordinates (0,1) —> (0,1) and (v, 1) —> (1,0) (which maps the vanishing point v to infinity).
(v) The (scaled) distance of the scene points $\tilde{\mathbf{T}}_1$ and $\mathbf{T_2}$ from $\mathbf{B_2}$ on $\mathbf{L_2}$ may then be obtained from the position of the points $\mathbf{H}_{2x2}(\tilde{t}_1, 1)^T$ and $\mathbf{H}_{2x2}(t_2,1)^T$. Their distance ratio is then given by

$$\frac{d_1}{d_2} = \frac{\tilde{t_1}(v - t_2)}{t_2(v - \tilde{t_1})}$$

(a) Example in 3D Euclidian geometry: The vertical line segments $\mathbf{L_1} = <\mathbf{B_1},\mathbf{T_1}>$ and $\mathbf{L_2} = <\mathbf{B_2},\mathbf{T_2}>$ have length d_1 and d_2 respectively. The base points $\mathbf{B_1}, \mathbf{B_2}$ are on the ground plane. We wish to compute the scene length ratio d_1:d_2 from the imaged configuration	
(b) Idea (in 3D Euclidian geometry): In the scene the length of the line segment $\mathbf{L_1}$ may be transferred to $\mathbf{L_2}$ by constructing a line parallel to the ground plane to generate the point $\tilde{\mathbf{T}}_1$.	
(c) in projective image geometry: $\boldsymbol{\ell}$ is the ground plane vanishing line, and **v** the vertical vanishing point. A corresponding parallel line construction in the image requires first determining the vanishing point **u** from the images $\mathbf{b_i}$ of $\mathbf{B_i}$, and then determining $\tilde{\mathbf{t}}_1$ (the image of $\mathbf{t_1}$) by the intersection of $\boldsymbol{\ell_2}$ and the line $<\mathbf{t_1},\mathbf{u}>$.	

(d) using rectified, parallel lines:
The line $\boldsymbol{\ell}_3$ is parallel to $\boldsymbol{\ell}_1$ in the image. The points $\hat{\mathbf{t}}_1$ and $\hat{\mathbf{t}}_2$ are constructed by intersecting $\boldsymbol{\ell}_3$ with the lines $<\mathbf{t}_1, \tilde{\mathbf{t}}_1>$ and $<\mathbf{t}_1, \mathbf{t}_2>$ respectively. The distance ratio $d(\mathbf{b}_2, \hat{\mathbf{t}}_1) : d(\mathbf{b}_2, \hat{\mathbf{t}}_2)$ is the computed estimate of $d_1:d_2$.



Use this Test data

```
null null           0          0 at lower left ground grid
b1                222         43
b2                389        168
t1                232        311
~t1               344        340
t2                331        391
v                 247        713
u'                590        451
u                 767        451
```