Exe Lect [1] Intro

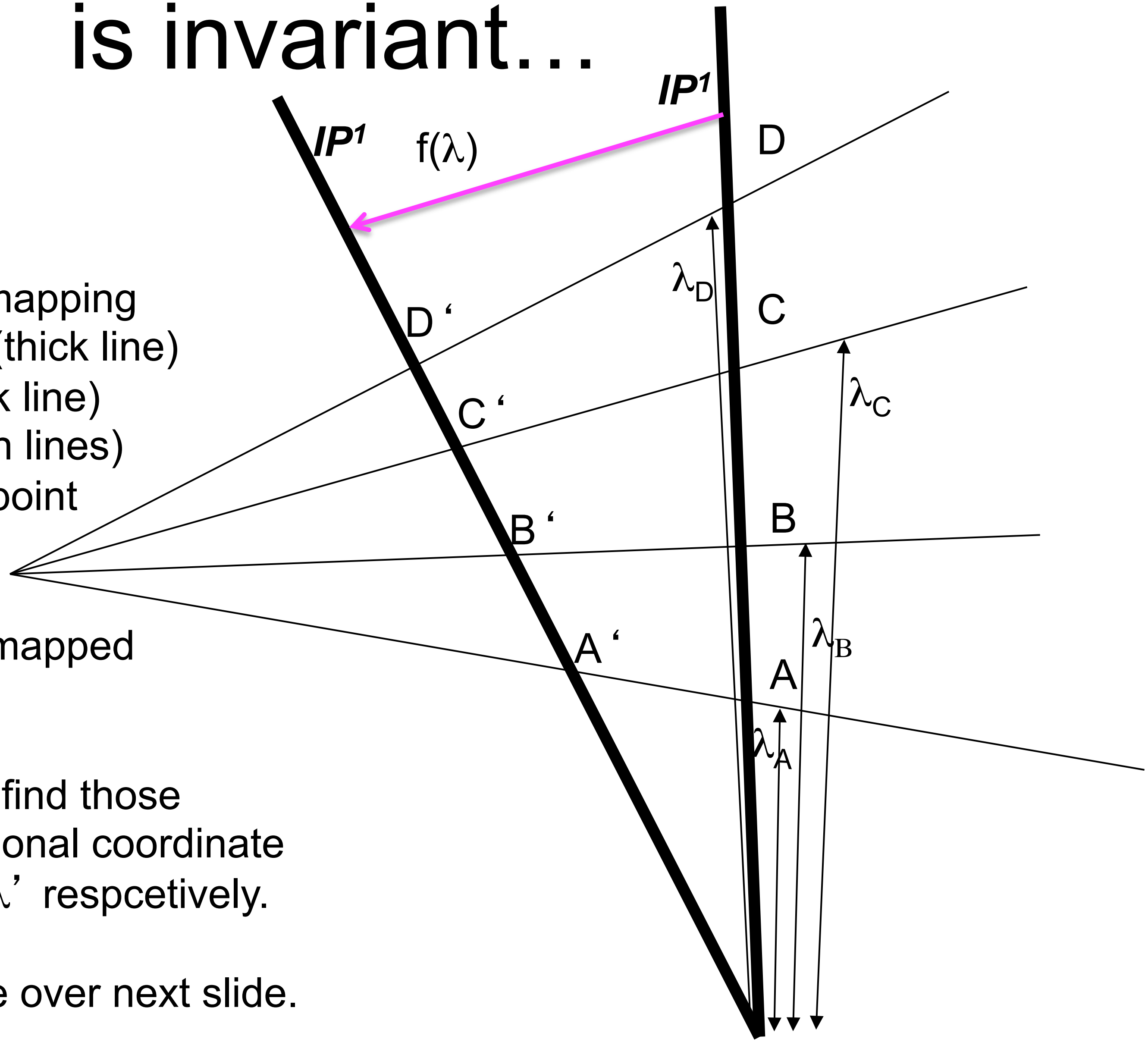
1. Pose 3 questions to last lecture
2. When defining projective space $\mathbb{P}^2$ what is the difference between

   a) $[x,y,z]^T$ $[u,v,w]^T$ are called equivalent iff $\exists\, \lambda > 0 : \lambda \cdot [x,y,z]^T = [u,v,w]^T$ and

   b) $[x,y,z]^T$ $[u,v,w]^T$ are called equivalent iff $\exists\, \lambda \neq 0 : \lambda \cdot [x,y,z]^T = [u,v,w]^T$

   What is correct?

3) The Cross Ratio under projections is invariant…



This figure illustrates a mapping from domain space IP1 (thick line) to target space IP1 (thick line) via a projective map (thin lines) intersecting in the focal point on the left.

The points A,B,C,D get mapped to A‘,B‘,C‘,D‘

In the domain space we find those Points via a one dimensional coordinate called λ in taget space λ' respcetively.

For the definition of f see over next slide.

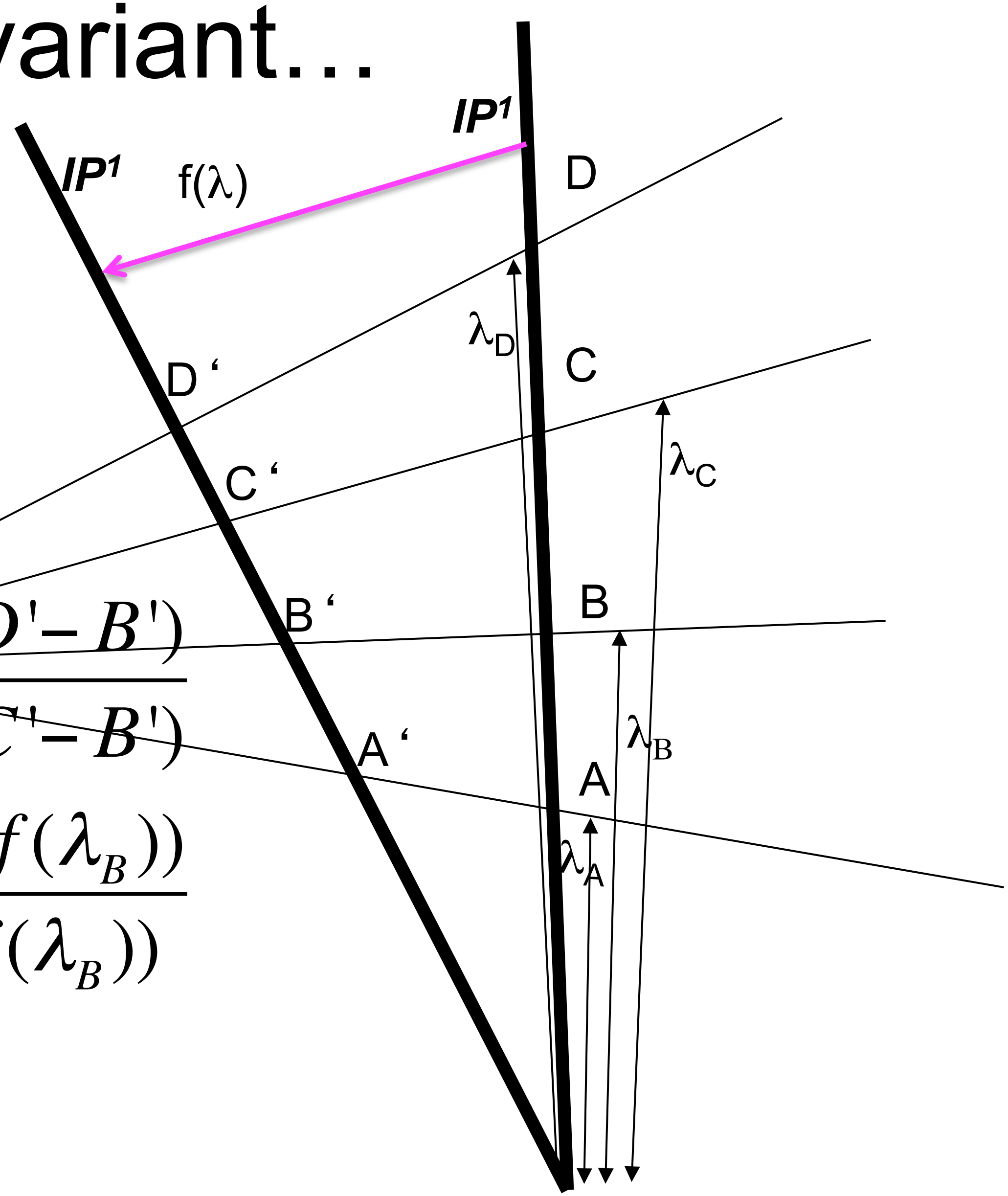
3) The Cross Ratio under projections is invariant…

Prove this equality!

$$\frac{(\lambda_C - \lambda_A)(\lambda_D - \lambda_B)}{(\lambda_D - \lambda_A)(\lambda_C - \lambda_B)} =$$

$$\frac{(C - A)(D - B)}{(D - A)(C - B)} \overset{!}{=} \frac{(C' - A')(D' - B')}{(D' - A')(C' - B')}$$

$$= \frac{(f(\lambda_C) - f(\lambda_A))(f(\lambda_D) - f(\lambda_B))}{(f(\lambda_D) - f(\lambda_A)(f(\lambda_C - f(\lambda_B))}$$



Finding ***f(λ)***: ***IP¹ -> IP¹***

- From lecture slide 35 a projective map from ***IP¹->IP¹*** is a matrix H=[a b;c d] (in 2D):

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix} \textit{so in homogenous coord.}$$

$$\begin{pmatrix} x \\ 1 \end{pmatrix} \textit{is mapped to} \mapsto \begin{pmatrix} ax+b \\ cx+d \end{pmatrix} \sim \begin{pmatrix} \frac{ax+b}{cx+d} \\ 1 \end{pmatrix}$$

$$\textit{when looking at the first coordinate only:}$$

$$f(x) = \tfrac{ax+b}{cx+d}$$

4) Read

- Read and sum up (mindmap or alike):

  Appendix 23: Projective Geometry for Machine Vision (1992), by Joseph L. Mundy , Andrew Zisserman 23.1.-23.5