NAME- DEEPANKAR MISHRA

ROLL NUMBER – CS22M041

TA- UTSAV

IMPORTANT NOTE- The data is taken from a csv file. Please set the path variable to appropriate value so that the code lifts the data and then runs on it.

```
path=open('D:\IITM MTech\FIrst Semester\Pattern Recognition and Machine Learning\Assignment\PRML Assignment 1\PRML Assignment 1/Dataset_test.csv')
data=np.loadtxt(path,delimiter=",",dtype='float')
```

The above snippet shows the path variable being set. Kindly set it before running the code. I have kept the Dataset_test.csv file in the zip folder.

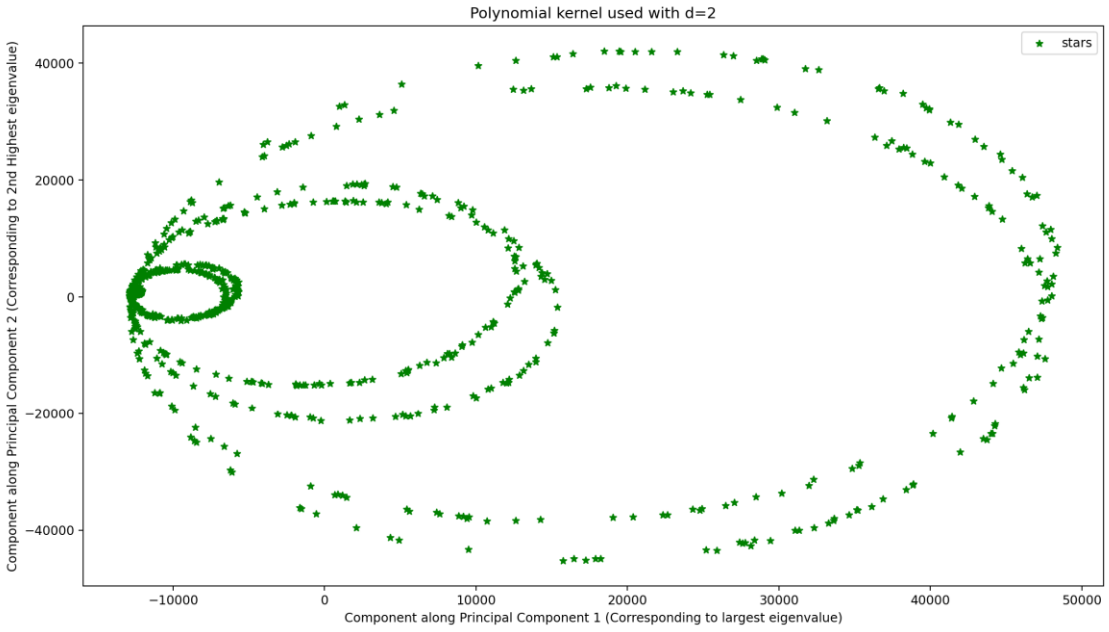
1)

i) The variance represented by Principal Component 1 is approximately 54.178 %.
The variance represented by Principal Component 1 is approximately 45.822 %

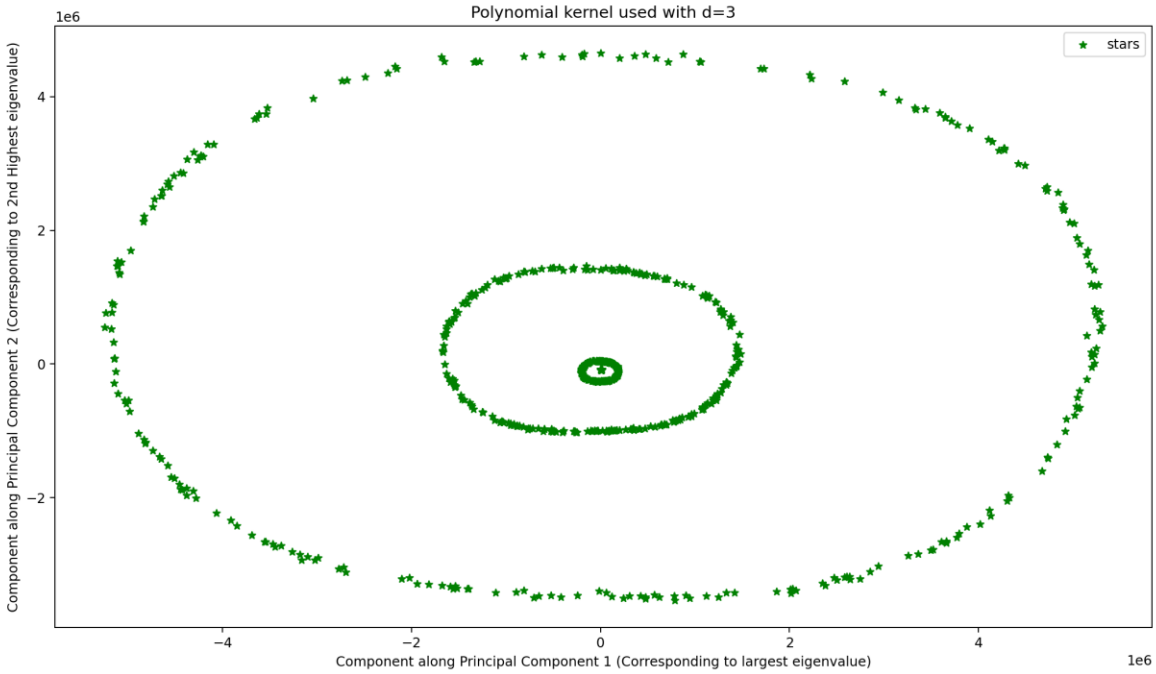
ii) The mean of the given dataset is very close to the origin (0,0) hence the effect upon centering is almost negligible.
After centering the new variances are-
Variance for principal component 1 = 54.17802452885223 (The difference compared to non centered data here is very very less).
Variance for principal component 2 = 45.821975471147766. (Again the difference here is very less).

iii)

A) The plots below are for polynomial kernels.

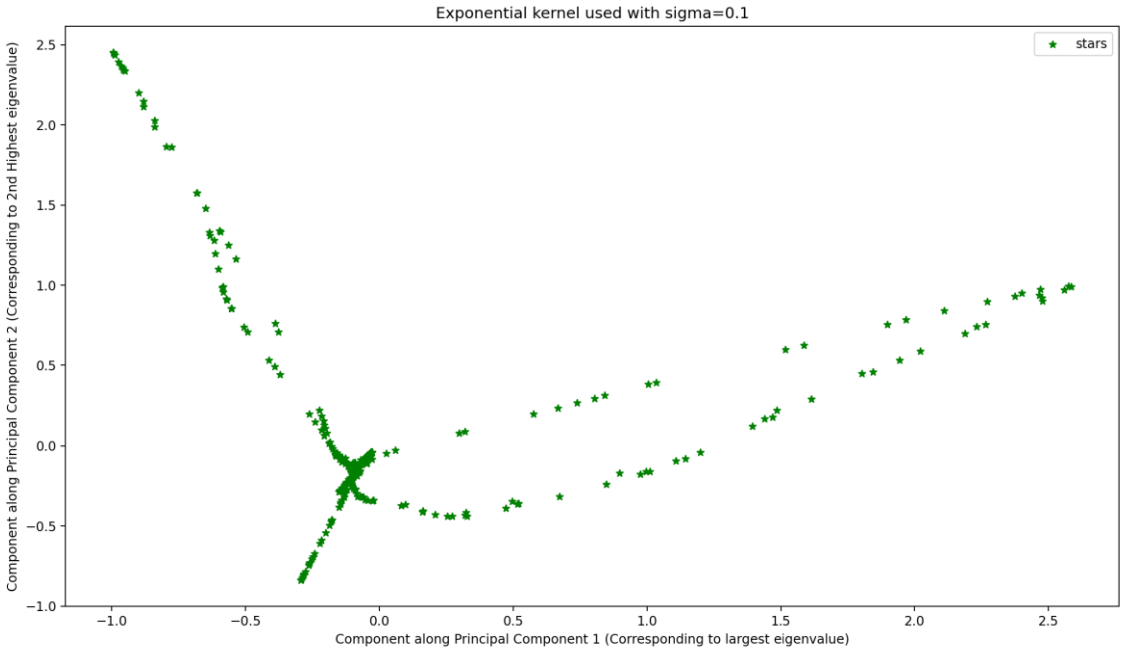


Kernel PCA with polynomial kernel , d=2

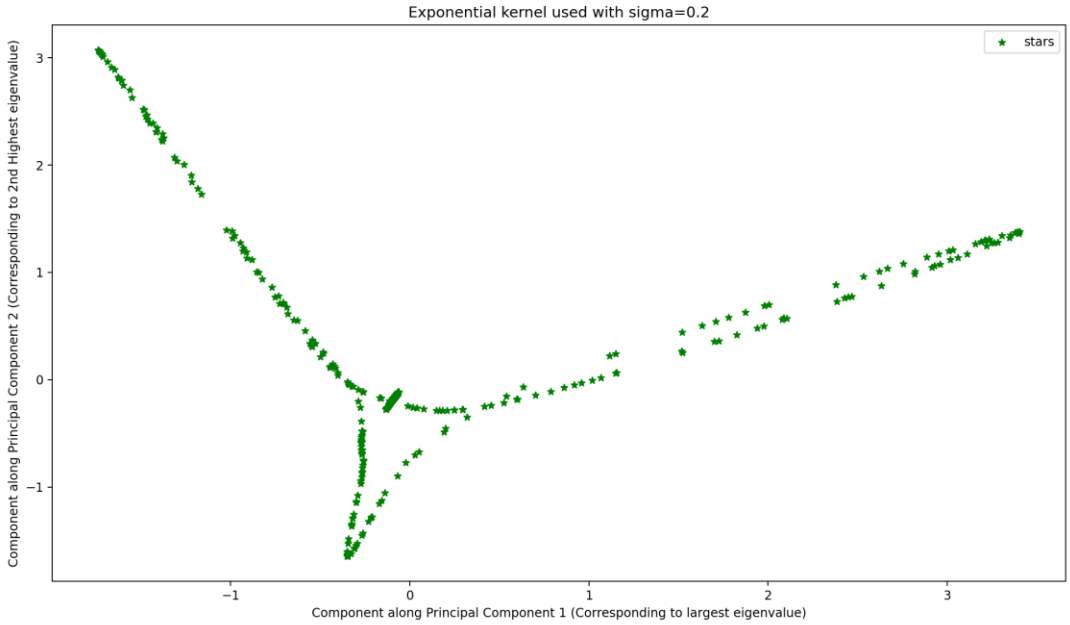


Kernel PCA with polynomial kernel , d=3

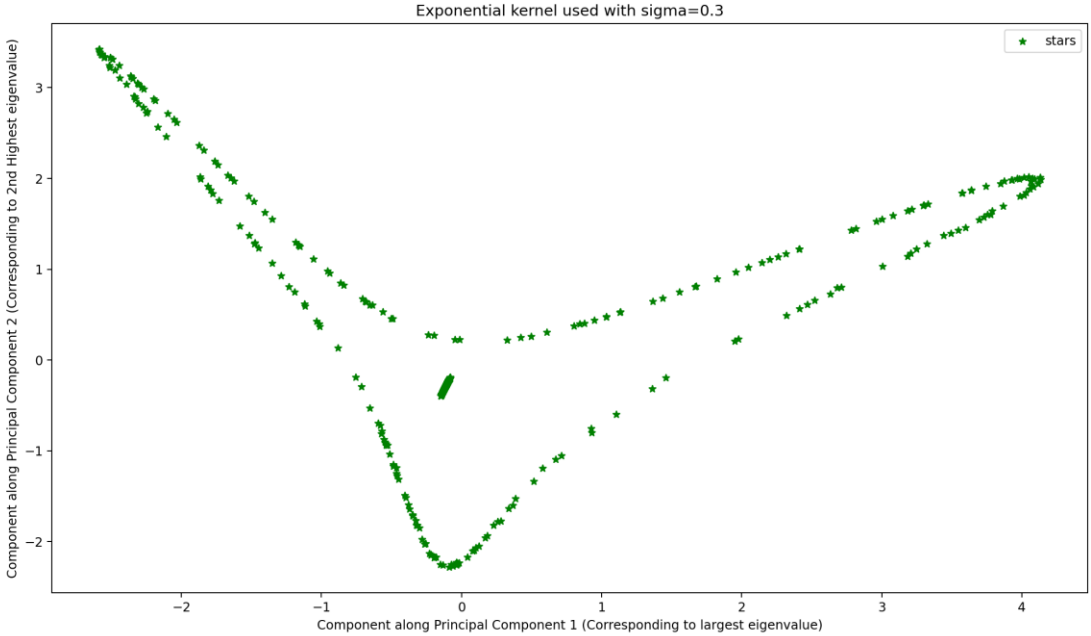
B) The plots below are for exponential kernels with sigma varying from 0.1 to 1.0 with 0.1 as increments.



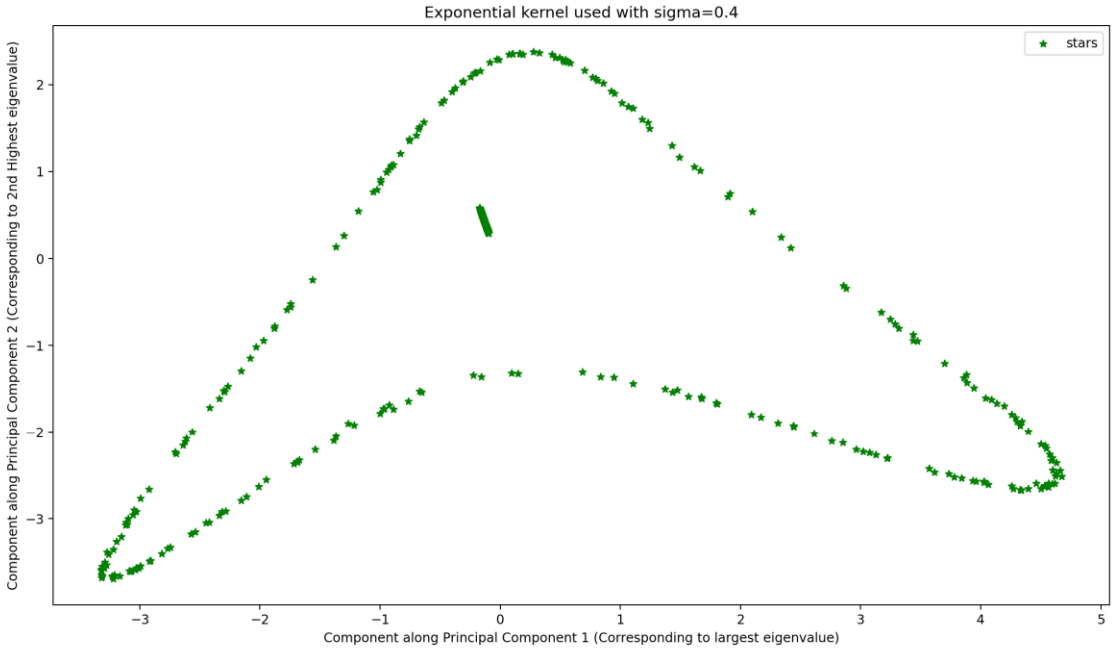
Kernel PCA with exponential kernel , sigma=0.1



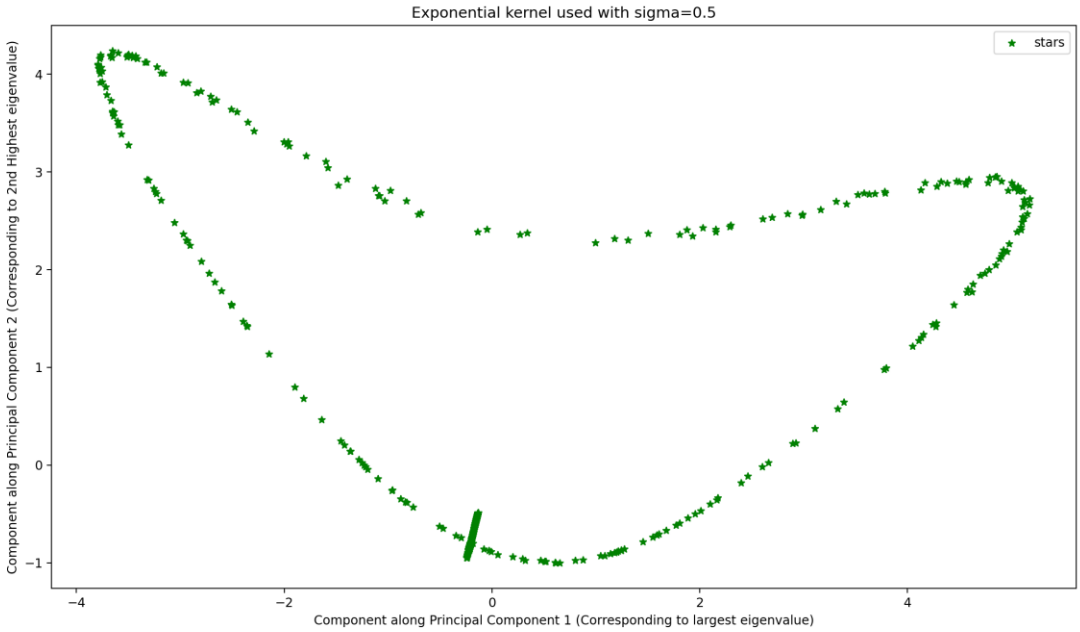
Kernel PCA with exponential kernel , sigma=0.2



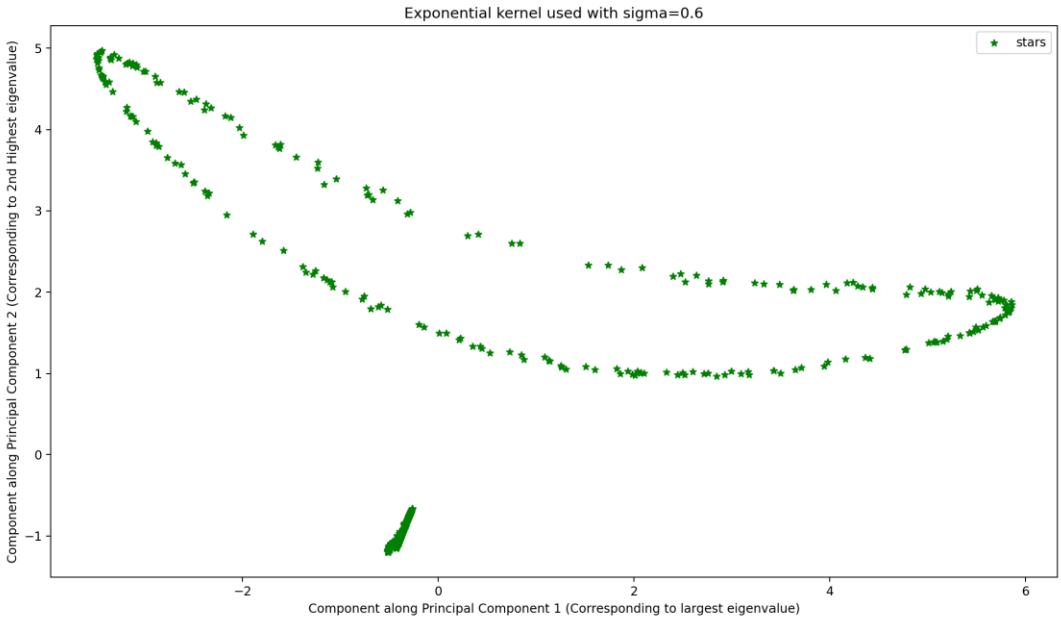
Kernel PCA with exponential kernel , sigma=0.3



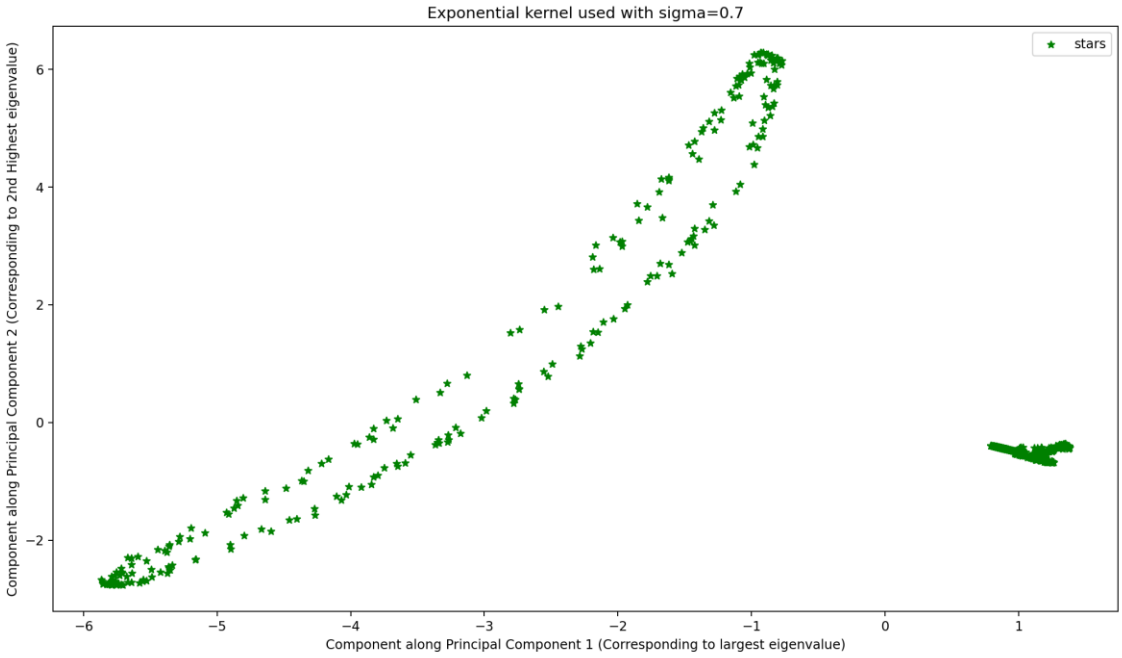
Kernel PCA with exponential kernel ，sigma=0.4



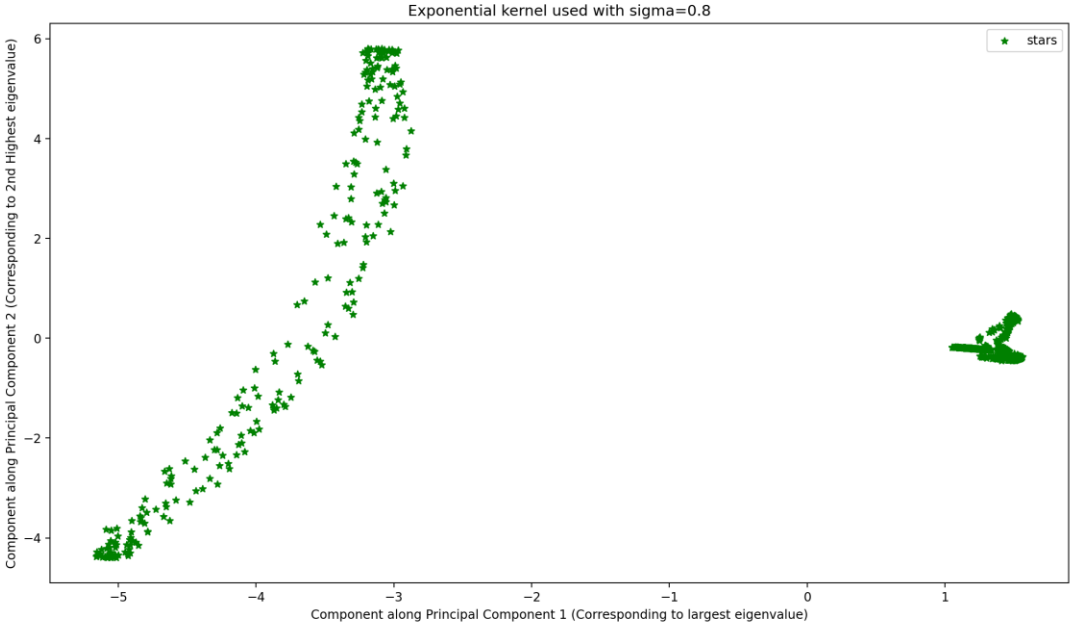
Kernel PCA with exponential kernel ，sigma=0.5



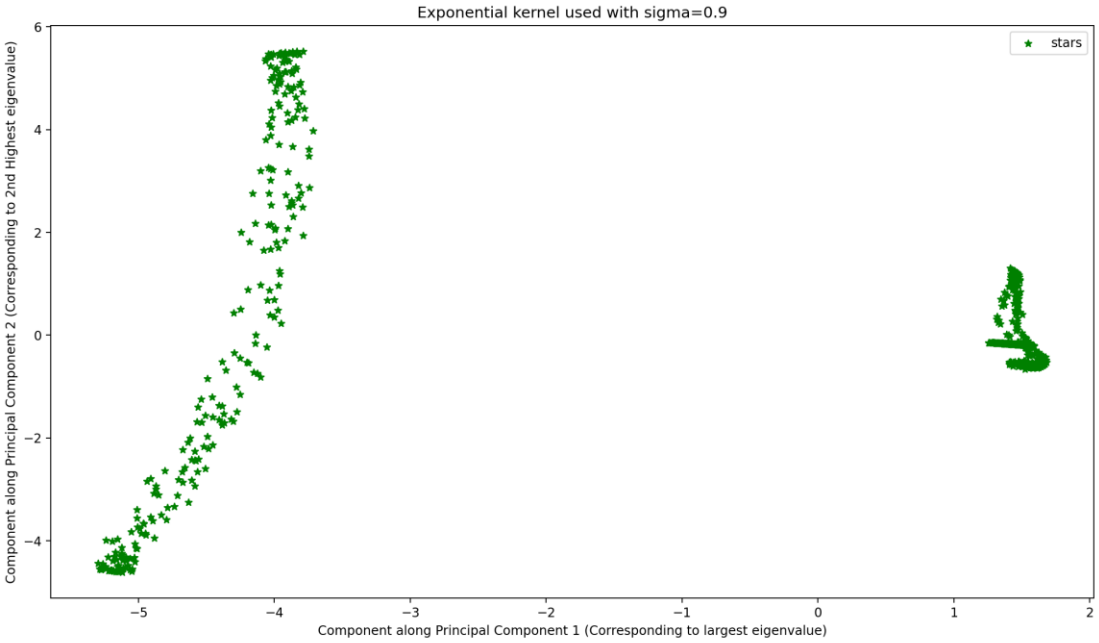
Kernel PCA with exponential kernel , sigma=0.6



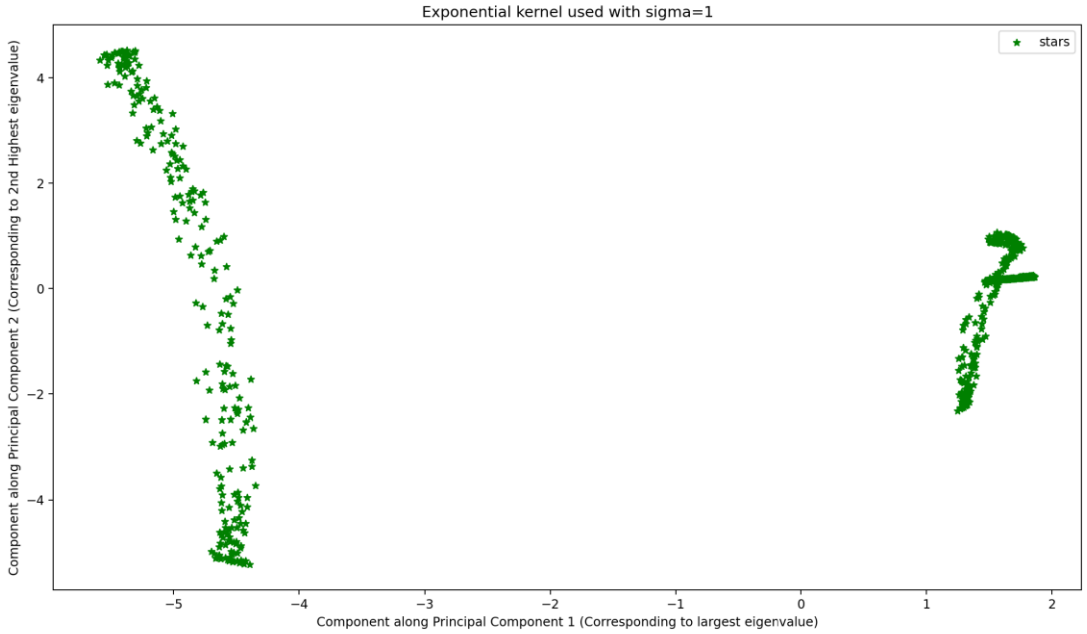
Kernel PCA with exponential kernel , sigma=0.7



Kernel PCA with exponential kernel , sigma=0.8



Kernel PCA with exponential kernel , sigma=0.9



Kernel PCA with exponential kernel , sigma=1.0

iv)

COMPARISON BETWEEN PERFORMANCE OF KERNELS

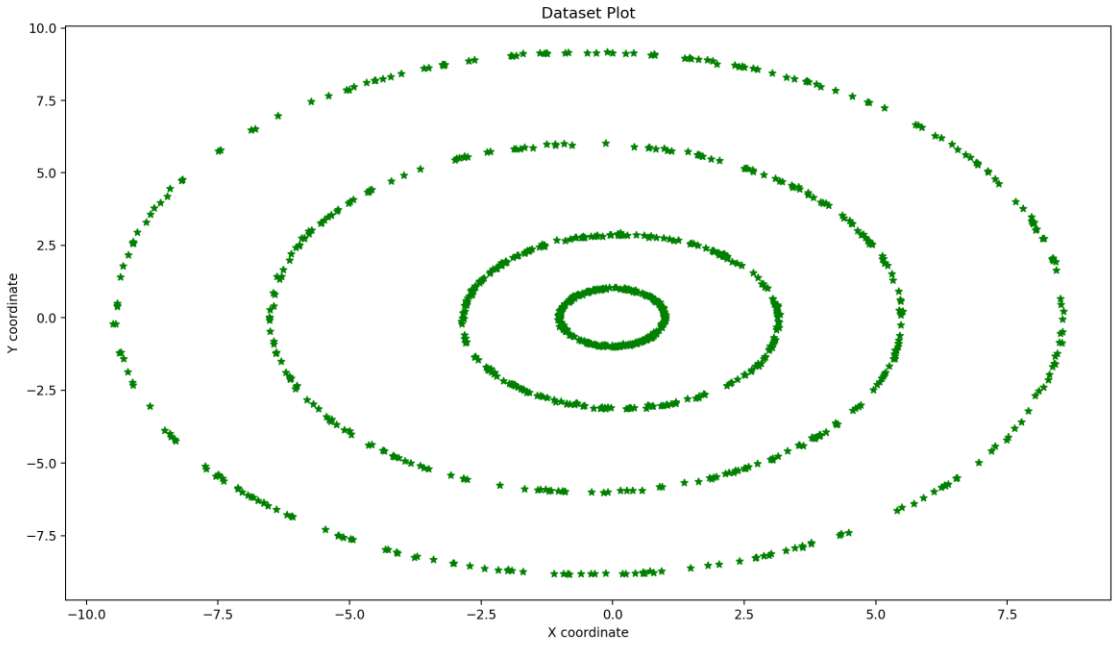
Kernel function	Variance represented by eigenvector corresponding to largest eigenvalue. (%)	Variance represented by eigenvector corresponding to 2nd largest eigenvalue. (%)	Variance represented by eigenvector corresponding to 3rd largest eigenvalue. (%)
Polynomial with d=2	36.296	32.235	27.414
Polynomial with d=3	41.16	32.252	12.789
Radial Basis Function, sigma=0.1	1.265	1.189	1.136
Radial Basis Function, sigma=0.2	2.393	2.169	1.973
Radial Basis Function, sigma=0.3	3.41	3.03	2.677
Radial Basis Function, sigma=0.4	4.327	3.782	3.432
Radial Basis Function, sigma=0.5	5.129	4.495	4.184
Radial Basis Function, sigma=0.6	5.775	5.318	4.748
Radial Basis Function, sigma=0.7	6.435	6.025	5.076
Radial Basis Function, sigma=0.8	7.405	6.243	5.209

Radial Basis Function, sigma=0.9	8.408	6.273	5.222
Radial Basis Function, sigma=1.0	9.369	6.276	5.236

TABLE 1: Variances for different choice of Kernel

For this data set how do we decide which kernel gives us good results? What we need to compare is the variance along the principal components. From Table 1 it is very clear that Radial basis function does not serve as a good kernel compared to polynomial kernels because the variance along first two principal components are lower in RBF compared to polynomial kernels.

Now it is clear polynomial kernel with d=2 and d=3 are in consideration for deciding which one is the best kernel. The dataset when plotted looks like the below figure.



Dataset plotted in X-Y plane

From the data we can clearly see that only two components are enough to represent a point(Obviously since it it is XY plane) hence when we do kernelized PCA we can look at the top two components and their corresponding variances can be added up and this sum can be compared with sum we obtain on changing the kernel.

For d=2, variance(along Principal Component 1) + variance(along Principal Component 2)= 36.296+32.235= **68.531**% of the total variance.

For d=3, variance(along Principal Component 1) + variance(along Principal Component 2)= 41.16+32.252= **73.412**% of the total variance.

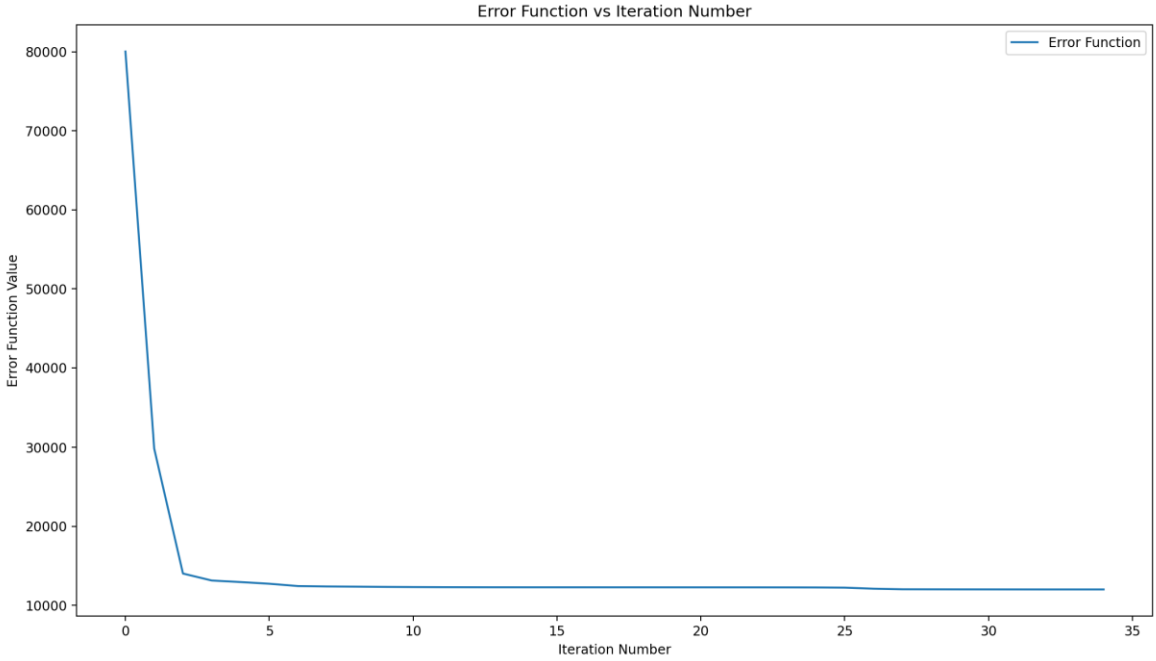
Also, from the plots of Kernel PCA with polynomial kernel , d=2 and Kernel PCA with polynomial kernel , d=3 we can clearly see the plot for d=3 somehow has more similarity with the actual data set from given above.

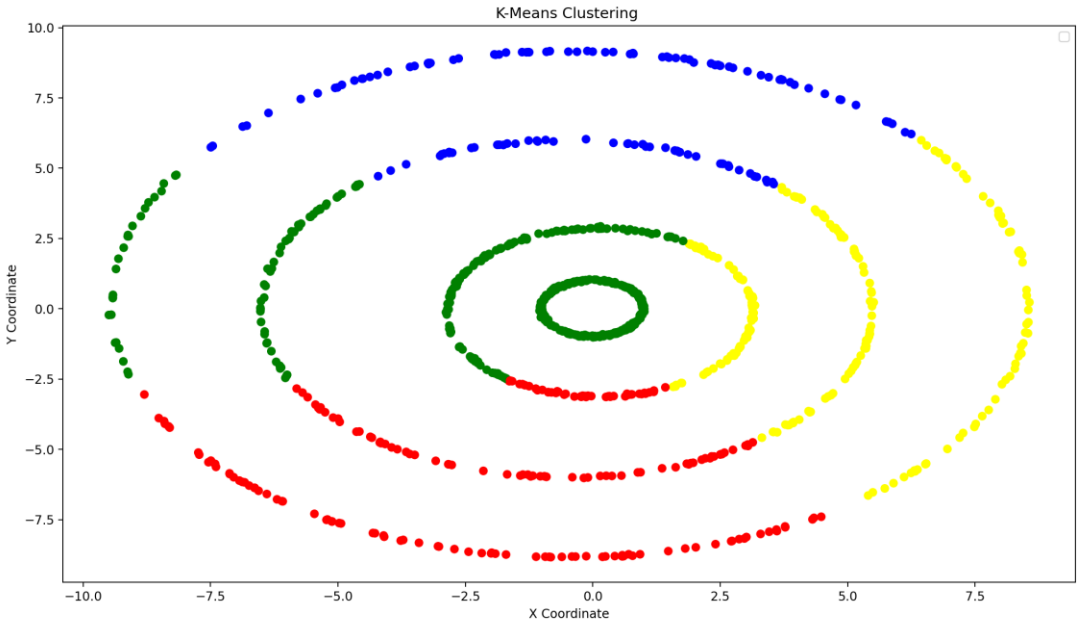
Hence polynomial kernel with d=3 is a better choice for this dataset.

2)

i) Below are 5 different random initialisations, and their final clustering we get. Number of clusters k=4.

1) INITIALLY THE VALUE OF ASSIGNMENT LIST: [1, 4, 1, 4, 1, 3, 3, 3, 1, 1, 2, 4, 1, 3, 2, 4, 1, 3, 2, 2, 1, 2, 1, 2, 1, 1, 3, 1, 3, 2, 4, 3, 3, 4, 3, 3, 2, 3, 2, 1, 1, 2, 1, 4, 1, 4, 3, 4, 1, 1, 4, 4, 2, 1, 2, 3, 3, 3, 4, 1, 4, 3, 2, 4, 2, 2, 3, 2, 3, 2, 2, 3, 4, 1, 4, 4, 2, 2, 4, 2, 3, 4, 1, 1, 3, 4, 1, 1, 3, 4, 4, 1, 4, 2, 3, 2, 3, 3, 1, 3, 4, 4, 3, 1, 3, 2, 1, 3, 3, 4, 3, 3, 2, 4, 1, 4, 4, 4, 2, 4, 1, 4, 1, 1, 2, 2, 4, 4, 2, 2, 3, 4, 3, 1, 2, 3, 3, 4, 2, 2, 2, 2, 1, 2, 4, 1, 2, 1, 1, 1, 3, 1, 2, 1, 1, 3, 4, 1, 1, 1, 2, 4, 1, 3, 2, 2, 3, 3, 2, 4, 2, 2, 3, 3, 4, 4, 2, 4, 3, 4, 2, 3, 2, 2, 4, 3, 1, 1, 3, 3, 2, 1, 4, 2, 1, 1, 2, 3, 4, 1, 2, 1, 1, 3, 2, 2, 3, 2, 3, 1, 1, 2, 4, 4, 1, 4, 1, 2, 4, 1, 1, 2, 2, 1, 3, 1, 2, 4, 2, 2, 2, 2, 3, 2, 3, 1, 2, 3, 2, 2, 3, 3, 2, 3, 2, 3, 4, 4, 3, 3, 3, 4, 4, 2, 1, 4, 4, 2, 1, 2, 3, 1, 1, 1, 4, 2, 4, 1, 3, 4, 2, 4, 1, 2, 2, 1, 1, 1, 4, 3, 3, 2, 3, 4, 2, 4, 4, 1, 2, 2, 2, 1, 1, 4, 1, 4, 3, 4, 2, 3, 3, 3, 2, 1, 2, 1, 3, 2, 1, 1, 4, 3, 1, 4, 1, 4, 1, 2, 2, 3, 4, 3, 4, 3, 4, 3, 1, 3, 4, 1, 3, 2, 3, 1, 1, 3, 4, 4, 1, 2, 1, 2, 1, 3, 3, 4, 1, 3, 2, 1, 3, 2, 3, 3, 1, 3, 2, 2, 4, 1, 3, 3, 4, 2, 2, 4, 4, 4, 3, 4, 3, 1, 2, 1, 1, 1, 2, 3, 1, 3, 2, 1, 4, 4, 1, 4, 1, 4, 4, 4, 3, 4, 1, 2, 4, 4, 3, 3, 1, 3, 2, 1, 3, 2, 1, 2, 4, 3, 4, 4, 3, 2, 1, 4, 4, 3, 3, 3, 1, 4, 2, 3, 2, 4, 4, 2, 4, 4, 3, 2, 4, 4, 3, 3, 4, 1, 2, 1, 3, 1, 2, 2, 4, 1, 3, 1, 2, 2, 1, 3, 2, 4, 2, 2, 3, 2, 2, 1, 3, 1, 1, 1, 3, 4, 3, 3, 4, 3, 2, 4, 1, 3, 4, 2, 1, 4, 2, 4, 1, 4, 1, 3, 2, 4, 4, 4, 3, 3, 2, 2, 4, 4, 3, 4, 4, 4, 4, 4, 3, 2, 4, 4, 1, 1, 1, 2, 1, 2, 1, 4, 2, 1, 2, 4, 3, 1, 1, 3, 2, 4, 1, 3, 3, 3, 2, 1, 4, 4, 2, 4, 4, 1, 4, 3, 1, 1, 4, 4, 3, 2, 3, 2, 1, 2, 2, 2, 3, 2, 1, 3, 3, 3, 3, 1, 4, 1, 3, 3, 1, 4, 1, 1, 1, 3, 4, 2, 2, 4, 3, 4, 3, 3, 4, 2, 2, 2, 3, 1, 1, 3, 4, 4, 2, 3, 4, 4, 2, 4, 2, 3, 1, 3, 3, 4, 4, 1, 4, 1, 1, 2, 1, 1, 1, 4, 4, 4, 4, 3, 3, 3, 1, 3, 3, 4, 4, 4, 3, 4, 4, 4, 2, 4, 3, 1, 4, 2, 4, 3, 3, 3, 4, 3, 1, 1, 4, 3, 2, 2, 4, 1, 4, 3, 3, 2, 4, 1, 2, 4, 2, 2, 3, 4, 2, 1, 2, 3, 3, 4, 2, 2, 4, 3, 3, 3, 1, 1, 3, 1, 3, 1, 1, 3, 3, 3, 2, 1, 4, 2, 4, 2, 4, 1, 1, 4, 1, 4, 2, 4, 2, 3, 3, 3, 4, 1, 1, 1, 1, 4, 3, 1, 4, 1, 2, 4, 1, 4, 4, 4, 4, 3, 4, 1, 2, 4, 2, 3, 4, 2, 1, 2, 1, 4, 4, 1, 1, 2, 3, 3, 4, 4, 4, 2, 3, 3, 1, 4, 4, 2, 3, 2, 3, 2, 3, 3, 3, 1, 1, 1, 4, 1, 1, 2, 4, 2, 1, 2, 1, 3, 2, 4, 2, 3, 2, 4, 1, 4, 2, 2, 2, 3, 4, 2, 3, 1, 2, 3, 2, 2, 3, 2, 2, 3, 1, 1, 1, 4, 3, 2, 2, 2, 4, 4, 3, 1, 2, 1, 3, 2, 2, 1, 4, 3, 1, 3, 1, 1, 4, 3, 3, 1, 3, 4, 2, 4, 3, 4, 2, 2, 4, 1, 2, 2, 1, 1, 4, 2, 4, 1, 4, 4, 3, 4, 3, 3, 4, 2, 2, 3, 3, 2, 3, 4, 3, 1, 4, 2, 3, 3, 4, 3, 2, 4, 2, 3, 2, 2, 2, 2, 4, 4, 1, 4, 4, 3, 4, 1, 3, 3, 4, 1, 4, 3, 1, 4, 2, 1, 1, 1, 4, 1, 3, 1, 1, 1, 2, 2, 4, 3, 1, 4, 1, 3, 1, 4, 2, 4, 1, 1, 4, 1, 4, 4, 3, 4, 1, 1, 2, 2, 1, 2, 3, 1, 3, 1, 4, 2, 2, 3, 3, 4, 1, 1, 1, 1, 1, 4, 4, 2, 2, 1, 2, 4, 4, 1, 3, 4, 1, 2, 2, 2, 2, 3, 2, 3, 1, 1, 1, 4, 3, 4, 1, 3, 1, 4, 2, 2, 4, 2, 2, 3, 1, 2, 1, 2, 3, 1, 2, 1, 3, 2, 2, 3, 4, 1, 4, 2, 1, 3, 3, 2, 1, 1, 4, 3, 2, 3, 2, 4, 2, 2, 4, 3, 3, 2, 4, 2, 4, 3, 3, 2]





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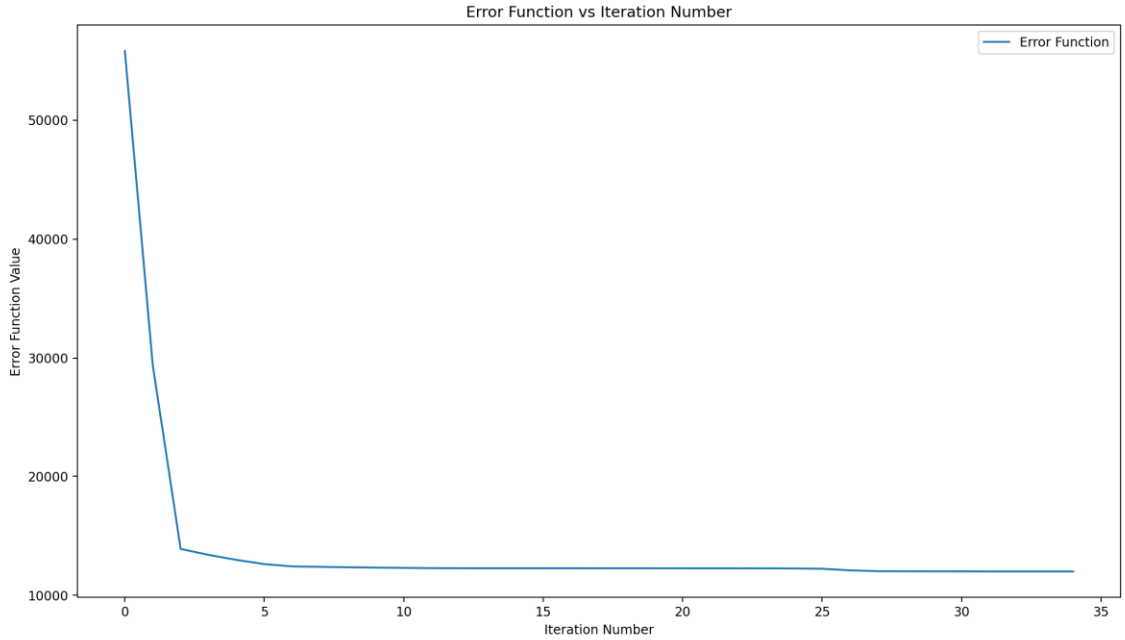

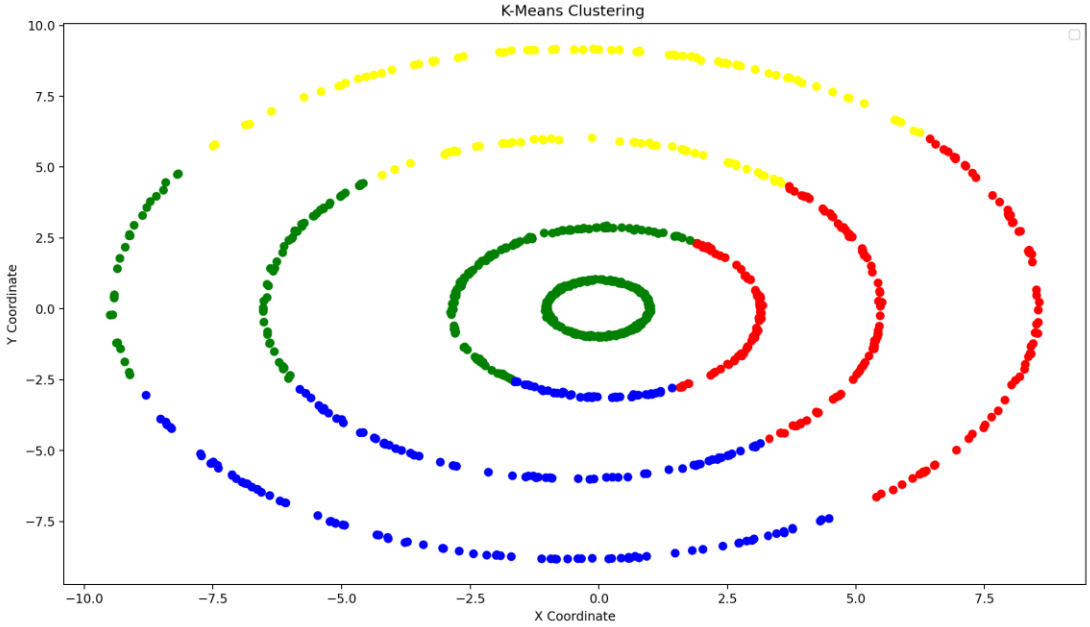

3) INITIALLY THE VALUE OF ASSIGNMENT LIST: [4, 4, 2, 2, 1, 1, 2, 3, 4, 2, 3, 2, 2, 2, 1, 4, 3, 3, 3, 1, 1, 2, 4, 4, 1, 1, 2, 1, 4, 3, 4, 4, 4, 4, 2, 3, 2, 3, 2, 1, 3, 4, 1, 3, 1, 4, 2, 4, 1, 2, 4, 3, 3, 2, 1, 1, 3, 1, 4, 1, 4, 2, 3, 3, 3, 2, 4, 3, 1, 4, 1, 3, 1, 4, 4, 2, 4, 3, 3, 2, 2, 1, 1, 4, 1, 1, 3, 3, 4, 3, 4, 2, 3, 4, 1, 1, 4, 3, 1, 4, 3, 3, 2, 3, 4, 4, 3, 4, 1, 3, 1, 1, 4, 2, 2, 1, 3, 3, 1, 1, 4, 4, 2, 3, 4, 3, 4, 2, 2, 2, 4, 4, 2, 3, 3, 2, 1, 2, 2, 4, 2, 3, 1, 2, 3, 1, 3, 3, 3, 1, 3, 2, 2, 3, 2, 2, 4, 4, 2, 3, 4, 4, 1, 3, 4, 1, 3, 2, 1, 1, 3, 4, 4, 2, 2, 3, 4, 3, 3, 2, 3, 2, 3, 3, 4, 4, 1, 1, 3, 3, 1, 3, 3, 4, 2, 2, 3, 4, 3, 2, 1, 3, 1, 1, 2, 3, 2, 1, 2, 3, 2, 2, 3, 2, 3, 3, 2, 2, 4, 3, 2, 4, 2, 3, 4, 1, 1, 2, 1, 2, 1, 3, 2, 2, 4, 2, 2, 2, 3, 1, 3, 3, 1, 2, 2, 4, 2, 4, 4, 1, 2, 2, 4, 2, 1, 3, 3, 4, 3, 3, 3, 4, 1, 3, 3, 1, 2, 2, 3, 1, 1, 1, 1, 1, 4, 2, 2, 4, 2, 3, 4, 1, 3, 3, 1, 1, 2, 4, 2, 4, 1, 3, 3, 2, 3, 4, 3, 2, 4, 3, 2, 2, 2, 3, 1, 2, 3, 4, 4, 4, 2, 4, 2, 4, 1, 4, 1, 4, 3, 2, 3, 4, 1, 4, 4, 3, 4, 1, 1, 1, 2, 3, 2, 4, 2, 4, 4, 2, 3, 3, 2, 1, 1, 1, 2, 3, 2, 3, 3, 2, 4, 1, 1, 2, 4, 4, 4, 4, 2, 3, 2, 4, 4, 4, 4, 2, 1, 2, 4, 2, 2, 3, 4, 4, 1, 1, 3, 1, 3, 2, 3, 4, 1, 2, 2, 2, 2, 2, 1, 4, 4, 1, 4, 3, 3, 1, 3, 1, 1, 1, 4, 2, 3, 3, 4, 4, 4, 2, 1, 4, 4, 3, 2, 2, 4, 1, 3, 1, 1, 3, 4, 3, 4, 4, 2, 3, 4, 1, 4, 4, 2, 1, 2, 1, 3, 1, 2, 3, 3, 2, 1, 3, 2, 4, 1, 2, 3, 3, 3, 4, 2, 4, 3, 3, 2, 1, 1, 1, 1, 4, 1, 1, 2, 4, 4, 2, 1, 2, 1, 3, 2, 1, 2, 3, 2, 2, 4, 4, 1, 3, 1, 3, 3, 3, 1, 1, 4, 2, 3, 3, 3, 3, 2, 4, 2, 2, 1, 3, 3, 3, 1, 3, 3, 1, 2, 2, 1, 3, 3, 1, 1, 3, 3, 3, 1, 1, 4, 2, 4, 1, 1, 4, 4, 1, 2, 3, 2, 3, 3, 2, 3, 4, 4, 4, 3, 4, 2, 1, 3, 4, 4, 1, 1, 1, 1, 4, 4, 3, 1, 1, 4, 1, 2, 4, 1, 2, 1, 3, 1, 1, 1, 1, 4, 2, 2, 4, 2, 1, 1, 3, 1, 2, 4, 2, 2, 3, 1, 4, 2, 2, 4, 3, 2, 3, 2, 1, 4, 2, 1, 2, 1, 1, 3, 4, 1, 1, 4, 2, 3, 2, 3, 1, 1, 4, 2, 3, 1, 1, 1, 1, 4, 2, 1, 4, 2, 3, 4, 1, 2, 2, 1, 4, 1, 2, 2, 3, 4, 1, 2, 3, 4, 2, 2, 1, 4, 1, 1, 2, 1, 3, 2, 2, 4, 2, 4, 2, 1, 3, 4, 2, 1, 4, 3, 1, 3, 2, 1, 4, 3, 4, 2, 2, 2, 3, 1, 2, 2, 3, 4, 1, 3, 3, 3, 4, 3, 2, 1, 2, 2, 4, 1, 3, 3, 1, 4, 3, 1, 2, 3, 4, 3, 1, 2, 3, 3, 2, 1, 1, 4, 4, 3, 2, 3, 3, 3, 2, 1, 3, 2, 4, 1, 3, 1, 3, 1, 3, 2, 1, 2, 2, 2, 1, 2, 1, 1, 4, 2, 1, 3, 3, 1, 2, 1, 4, 2, 4, 1, 3, 1, 1, 1, 4, 2, 3, 1, 2, 1, 3, 4, 3, 4, 1, 2, 3, 2, 4, 3, 4, 3, 2, 4, 3, 2, 2, 1, 1, 1, 3, 2, 1, 3, 1, 4, 1, 3, 4, 4, 2, 4, 2, 2, 3, 2, 3, 3, 4, 1, 1, 4, 1, 1, 2, 2, 3, 3, 4, 1, 3, 2, 1, 4, 2, 3, 2, 4, 2, 2, 1, 1, 1, 2, 4, 2, 4, 2, 1, 4, 4, 3, 3, 2, 2, 3, 4, 3, 1, 2, 4, 4, 3, 1, 1, 3, 4, 3, 3, 3, 4, 3, 4, 1, 4, 3, 3, 4, 1, 1, 4, 4, 3, 1, 2, 3, 4, 1, 4, 3, 3, 3, 2, 3, 1, 2, 1, 4, 3, 3, 3, 2, 2, 4, 2, 2, 4, 3, 2, 1, 3, 2, 2, 1, 2, 4, 3, 1, 3, 1, 2, 2, 2, 2, 3, 4, 3, 1, 1, 2, 1, 3, 1, 3, 4, 3, 2, 2, 2, 4, 2, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 3, 4, 3, 3, 4, 4, 2, 2, 4, 2, 4, 2, 3, 1, 4, 3, 4, 2, 4, 1, 4, 1, 2, 1, 3, 2, 4, 1, 4, 3, 1, 2, 1, 2, 4, 3, 4, 2, 4, 4, 2, 3,

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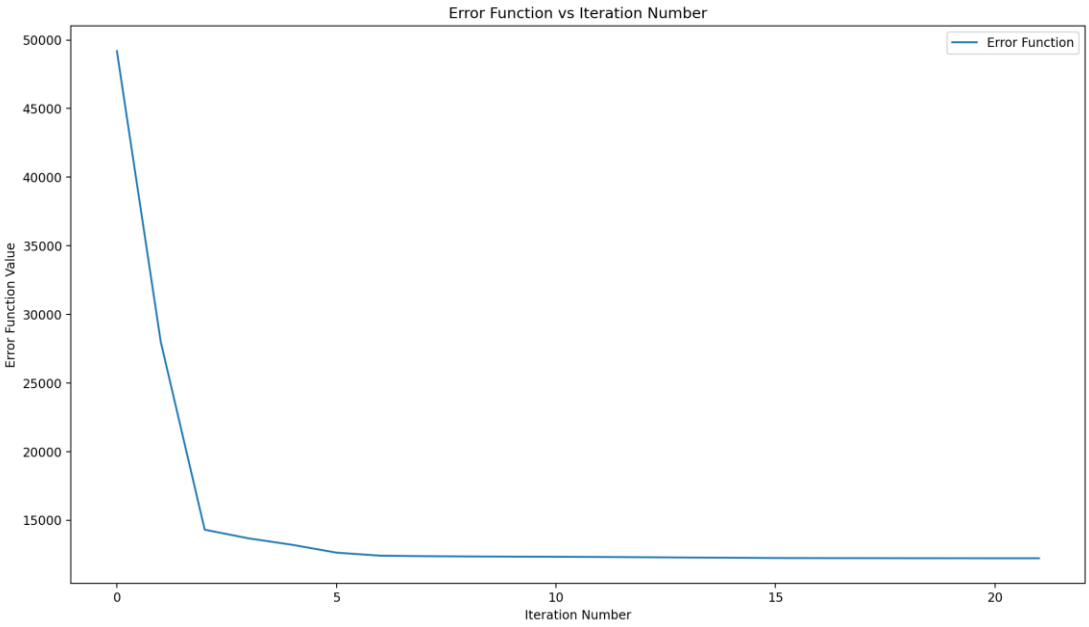

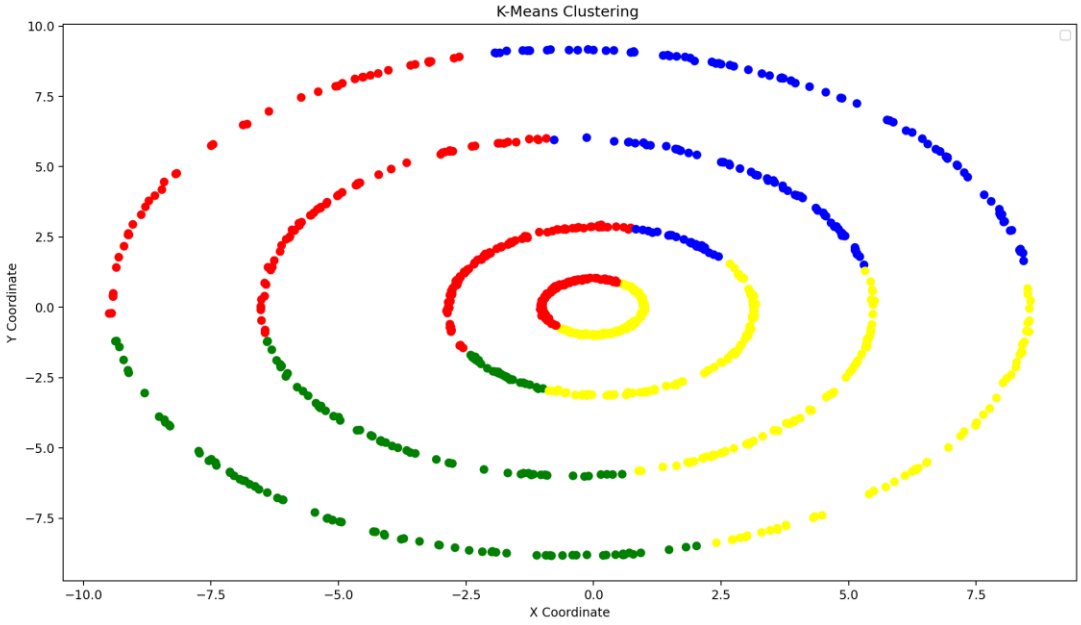

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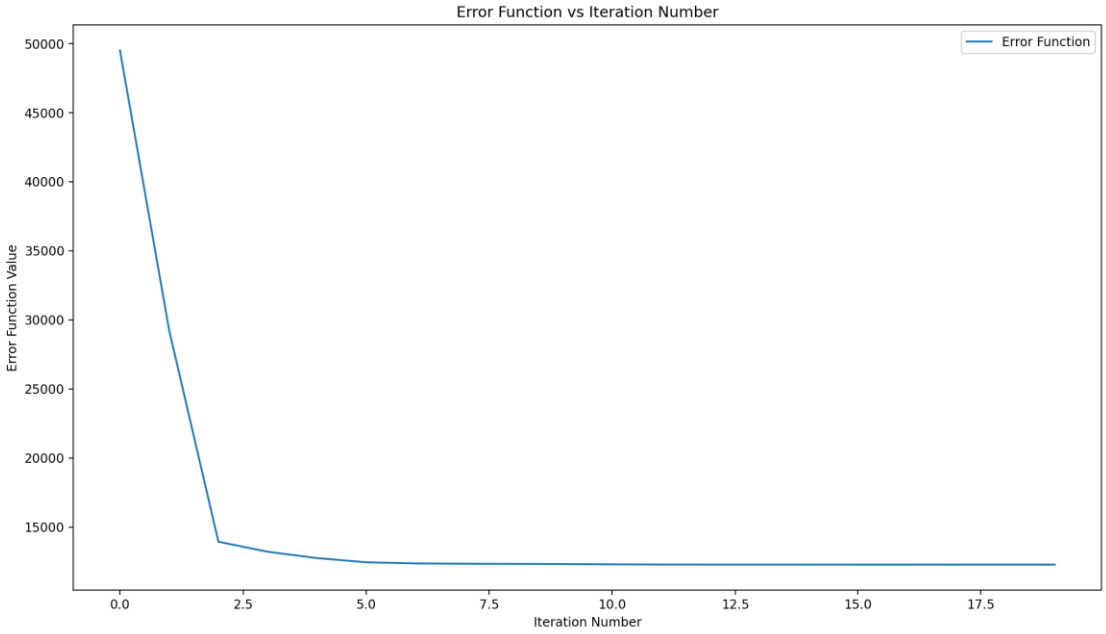

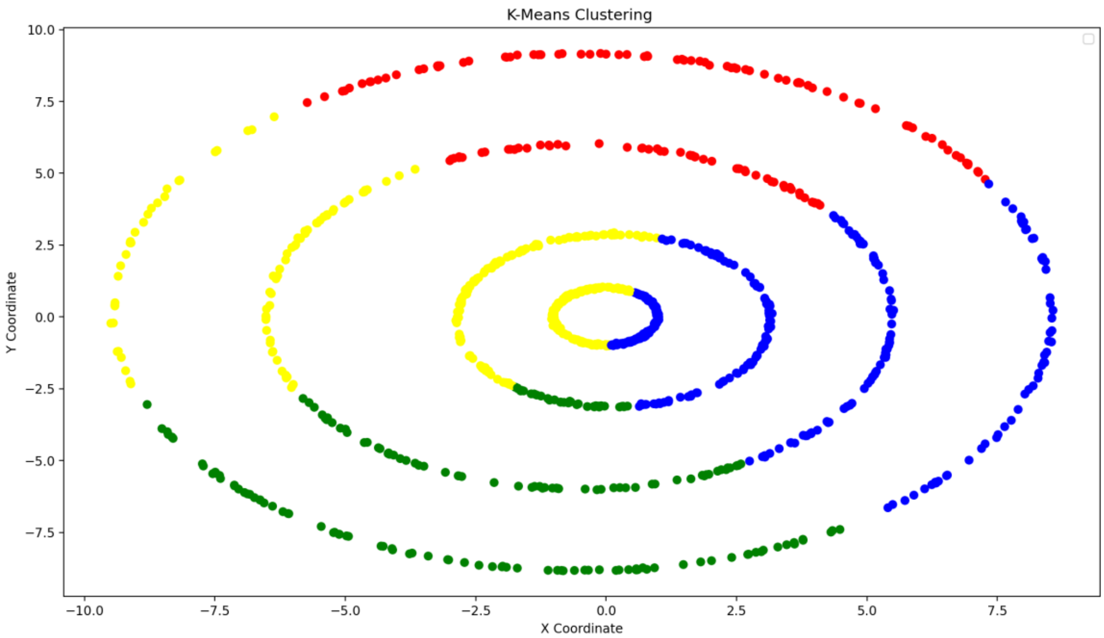

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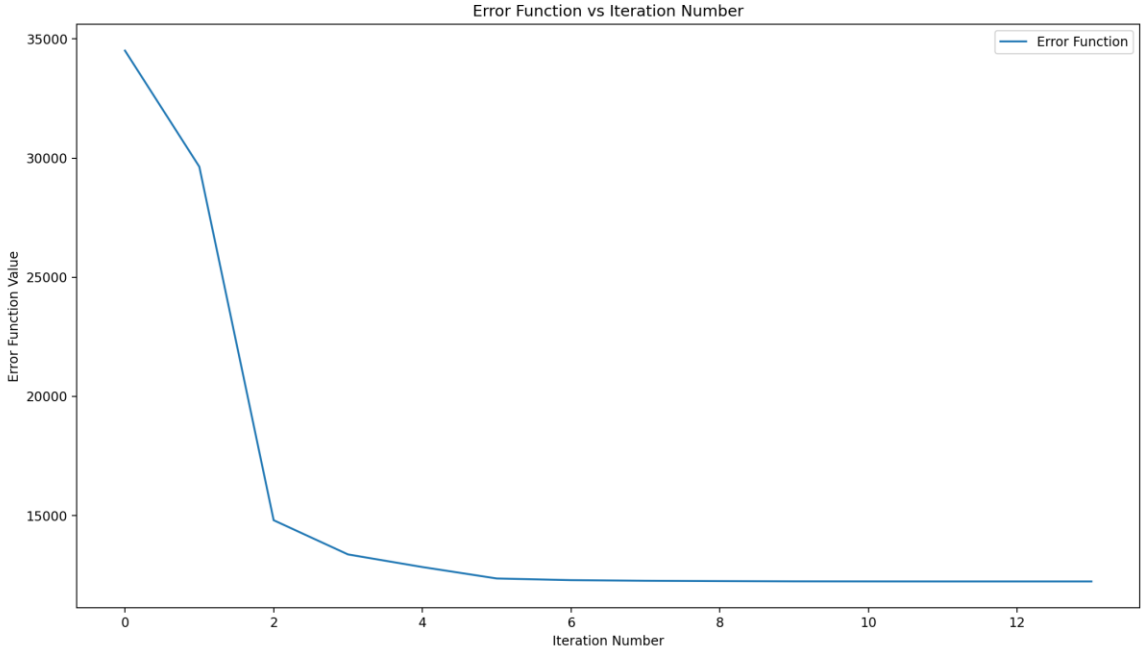
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Error Function Value
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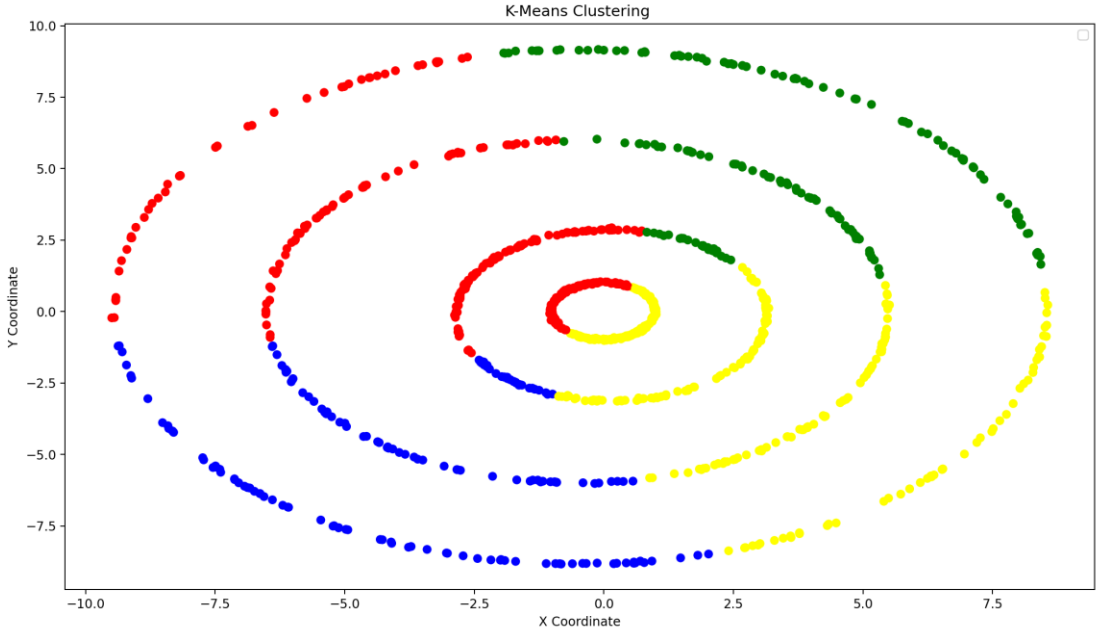
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−7.5
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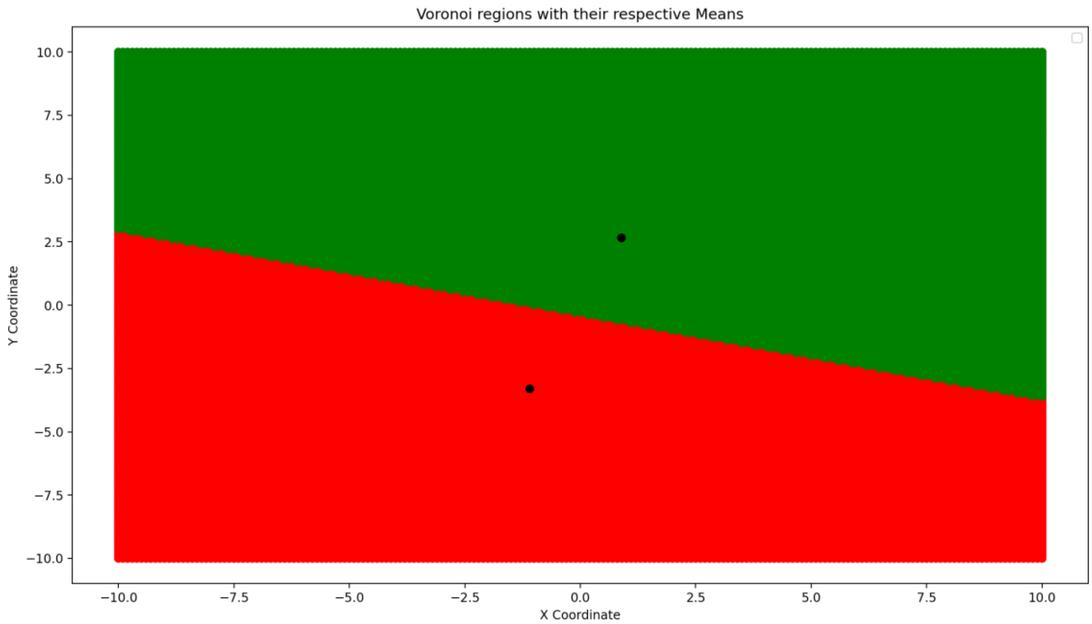
−5.0
−2.5
0.0
2.5
5.0
7.5

ii) We have to obtain cluster center for a fixed assignment using K means algorithm , For k=2,3,4,5, and also plot Voronoi regions.

1) K=2, (2 clusters)

INITIALLY THE VALUE OF ASSIGNMENT LIST: [1, 2, 2, 2, 2, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 2, 2, 2, 2, 1, 2, 1, 1, 2, 1, 2, 1, 1, 1, 2, 1, 1, 2, 2, 1, 2, 1, 1, 1, 2, 2, 1, 2, 2, 2, 2, 2, 1, 1, 2, 1, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 1, 1, 1, 2, 2, 2, 1, 2, 1, 2, 1, 2, 2, 2, 2, 1, 1, 2, 1, 1, 2, 1, 2, 2, 2, 2, 2, 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 1, 2, 1, 2, 2, 2, 2, 1, 1, 1, 2, 1, 1, 2, 2, 2, 2, 1, 1, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 1, 1, 1, 2, 2, 1, 2, 2, 2, 1, 2, 1, 2, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 2, 2, 1, 2, 1, 1, 1, 2, 1, 2, 2, 1, 1, 1, 2, 1, 2, 1, 1, 2, 1, 2, 1, 1, 2, 1, 2, 2, 1, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 2, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 1, 2, 2, 1, 1, 1, 2, 1, 1, 1, 2, 2, 1, 1, 1, 2, 2, 1, 2, 2, 2, 2, 2, 1, 1, 1, 2, 2, 1, 1, 2, 2, 2, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 2, 2, 1, 1, 1, 2, 2, 1, 1, 2, 1, 2, 1, 1, 2, 1, 2, 2, 1, 1, 2, 2, 1, 1, 2, 1, 1, 2, 2, 2, 1, 2, 1, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 2, 2, 1, 2, 2, 1, 2, 2, 2, 2, 1, 2, 1, 2, 2, 1, 2, 1, 1, 2, 1, 2, 2, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 2, 1, 2, 1, 2, 2, 1, 1, 2, 2, 1, 1, 1, 2, 1, 2, 1, 1, 2, 2, 2, 2, 1, 2, 2, 2, 2, 1, 1, 2, 1, 2, 2, 2, 1, 1, 1, 1, 1, 2, 1, 1, 2, 2, 2, 1, 2, 2, 2, 1, 1, 1, 2, 1, 2, 2, 2, 1, 1, 2, 1, 2, 2, 2, 2, 1, 1, 1, 2, 2, 2, 2, 1, 1, 2, 2, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 2, 2, 1, 1, 1, 1, 2, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 1, 2, 2, 1, 1, 2, 1, 1, 1, 2, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, 2, 2, 1, 1, 2, 2, 1, 2, 2, 2, 2, 1, 1, 1, 1, 2, 1, 2, 1, 1, 1, 2, 1, 1, 2, 2, 2, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 2, 1, 2, 2, 1, 2, 1, 1, 1, 1, 2, 2, 1, 2, 2, 2, 2, 2, 2, 1, 2, 1, 1, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 1, 1, 2, 1, 1, 1, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 1, 2, 2, 1, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, 1, 2, 1, 2, 2, 2, 2, 1, 1, 1, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 1, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 2, 2, 2, 1, 1, 1, 1, 1, 1, 2, 2, 1, 2, 2, 1, 1, 2, 2, 1, 1, 2, 1, 2, 2, 1, 2, 2, 1, 2, 1, 1, 1, 2, 2, 1, 1, 2, 1, 1, 1, 2, 2, 1, 2, 1, 1, 2, 2, 2, 1, 2, 2, 2, 1, 2, 2, 1, 2, 1, 2, 1, 2, 1, 1, 1, 1, 2, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 2, 2, 2, 1, 2, 1, 1, 2, 2, 1, 1, 1, 1, 1, 2, 2, 2, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 2, 1, 1, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 2, 2, 1, 1, 2, 1, 1, 1, 2, 1, 2, 1, 1, 2, 1, 1, 2, 2, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 1, 2, 2, 2, 1, 2, 2, 1, 1, 2, 2, 1, 2, 2, 2, 1, 2, 1, 2, 2, 1, 1, 1, 1, 2, 2, 1, 2, 2, 2, 2, 1, 1, 2, 1, 2, 1, 2, 2, 2, 2, 1, 1, 2, 2, 1, 1, 2, 2, 1, 2, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 2, 2, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 2, 1, 1, 2, 1, 2, 1, 2, 1, 1, 1, 1, 2, 1, 1, 2, 2, 2, 2, 1, 1, 2, 1, 2, 2, 1, 2, 1, 1, 1, 2, 2, 2, 2, 2, 2, 1, 2, 2, 2, 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, 1, 1, 2, 2, 1, 2, 1, 1, 2, 2, 2, 1, 1, 2]

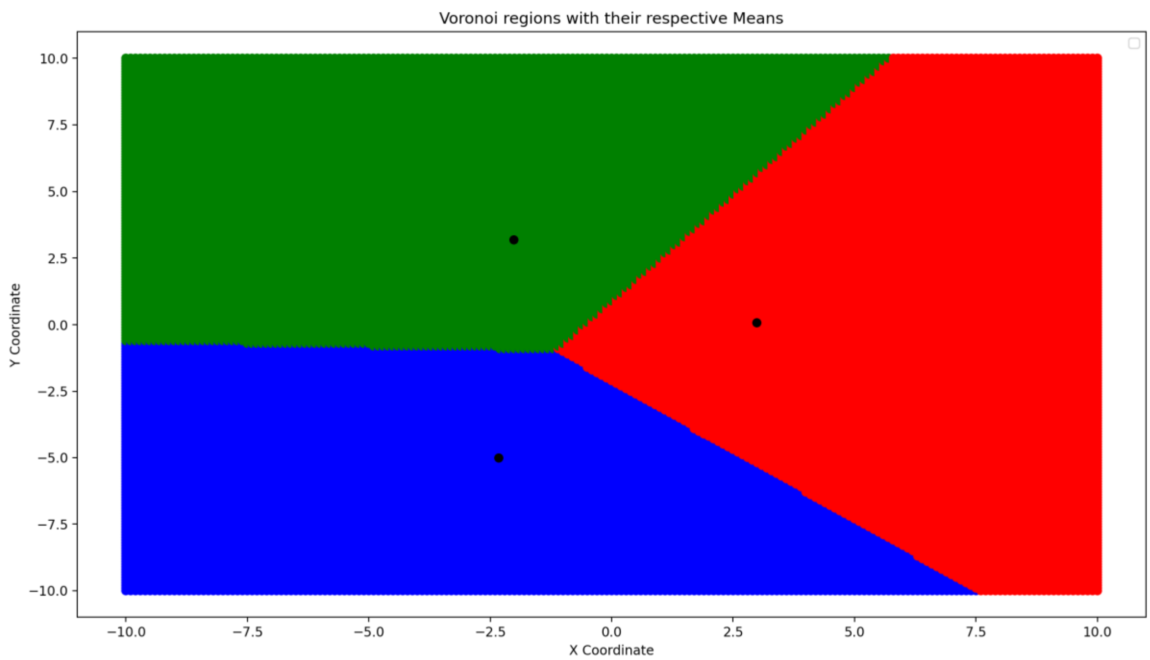


Mean of cluster number 1 is (0.8915593520871145 , 2.6746856871143394) - green colour

Mean of cluster number 2 is (-1.094095312917595 , -3.282297529844097) - red colour

2) K=3,(3 Clusters)

INITIALLY THE VALUE OF ASSIGNMENT LIST: [3, 1, 3, 1, 1, 3, 2, 2, 3, 2, 2, 3, 2, 3, 3, 3, 3, 3, 2, 3, 3, 1, 2, 1, 3, 1, 2, 3, 1, 2, 2, 3, 3, 1, 2, 1, 2, 1, 1, 2, 2, 2, 2, 2, 2, 1, 3, 3, 2, 2, 2, 3, 1, 3, 2, 1, 2, 2, 1, 2, 2, 2, 3, 1, 1, 1, 3, 1, 2, 1, 1, 1, 3, 1, 3, 3, 2, 3, 1, 1, 2, 3, 2, 1, 2, 1, 1, 2, 3, 1, 2, 1, 3, 1, 2, 2, 2, 2, 2, 1, 1, 3, 1, 1, 1, 2, 2, 1, 3, 3, 1, 2, 2, 3, 2, 2, 3, 3, 2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 1, 3, 3, 2, 3, 2, 3, 1, 3, 3, 1, 3, 3, 1, 3, 1, 2, 1, 3, 1, 1, 1, 3, 1, 3, 2, 2, 2, 2, 1, 3, 2, 3, 1, 1, 3, 1, 1, 2, 2, 2, 1, 3, 2, 2, 3, 2, 2, 1, 3, 3, 2, 1, 3, 1, 1, 2, 3, 1, 2, 2, 3, 2, 3, 2, 3, 3, 1, 1, 2, 1, 3, 3, 1, 2, 2, 2, 1, 3, 1, 1, 2, 2, 3, 2, 1, 1, 1, 3, 2, 2, 2, 1, 1, 2, 2, 2, 3, 2, 3, 1, 2, 1, 2, 2, 2, 1, 2, 1, 1, 2, 1, 2, 1, 1, 3, 1, 3, 1, 3, 3, 3, 1, 2, 3, 2, 1, 1, 1, 3, 2, 3, 2, 1, 2, 1, 3, 2, 3, 1, 2, 1, 2, 3, 3, 1, 2, 3, 1, 2, 1, 1, 3, 3, 3, 2, 2, 3, 2, 3, 3, 2, 2, 3, 1, 3, 2, 3, 2, 1, 2, 2, 2, 3, 1, 1, 2, 3, 1, 3, 3, 1, 1, 2, 3, 2, 3, 3, 1, 1, 3, 1, 2, 1, 1, 2, 1, 3, 3, 3, 1, 1, 1, 3, 3, 3, 3, 3, 2, 2, 3, 2, 1, 2, 1, 3, 3, 3, 2, 3, 2, 3, 2, 3, 2, 1, 3, 1, 1, 1, 1, 3, 1, 2, 3, 1, 2, 2, 1, 2, 1, 1, 3, 1, 1, 1, 3, 3, 1, 1, 1, 1, 2, 1, 3, 1, 3, 2, 3, 1, 1, 3, 3, 2, 1, 3, 1, 2, 2, 3, 2, 2, 1, 3, 1, 3, 1, 1, 3, 1, 2, 2, 1, 2, 1, 2, 1, 3, 3, 1, 2, 3, 2, 3, 3, 2, 3, 1, 2, 3, 3, 3, 2, 2, 1, 2, 2, 1, 1, 1, 3, 2, 3, 1, 3, 1, 1, 3, 2, 1, 2, 1, 3, 2, 3, 3, 3, 3, 2, 2, 2, 1, 1, 2, 1, 2, 3, 1, 2, 1, 2, 3, 2, 1, 3, 1, 2, 1, 1, 2, 2, 3, 1, 3, 1, 3, 2, 3, 1, 2, 2, 1, 2, 2, 3, 2, 3, 2, 1, 2, 3, 2, 3, 2, 2, 2, 3, 2, 3, 1, 2, 3, 2, 3, 1, 1, 3, 3, 1, 3, 2, 3, 1, 2, 1, 2, 2, 2, 2, 2, 3, 3, 3, 1, 3, 1, 2, 2, 1, 2, 1, 1, 1, 3, 3, 2, 2, 2, 3, 2, 1, 1, 2, 2, 2, 2, 3, 2, 1, 1, 3, 3, 3, 3, 3, 3, 3, 1, 3, 1, 1, 2, 1, 3, 3, 1, 2, 1, 3, 3, 1, 2, 3, 1, 2, 3, 3, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 2, 2, 3, 2, 2, 3, 1, 2, 2, 2, 3, 3, 1, 2, 2, 2, 1, 3, 1, 1, 1, 1, 2, 3, 1, 1, 3, 2, 2, 3, 1, 3, 1, 3, 1, 2, 3, 3, 3, 1, 3, 3, 1, 2, 1, 3, 1, 1, 2, 1, 2, 3, 2, 2, 2, 1, 1, 1, 2, 2, 3, 2, 2, 2, 1, 2, 1, 2, 3, 1, 2, 3, 2, 3, 2, 2, 3, 1, 3, 1, 2, 3, 2, 1, 1, 2, 1, 3, 2, 1, 2, 3, 1, 2, 3, 2, 3, 2, 3, 1, 1, 1, 1, 3, 1, 1, 1, 3, 2, 1, 1, 2, 3, 3, 1, 3, 2, 3, 1, 2, 2, 3, 3, 3, 3, 2, 1, 1, 3, 2, 3, 2, 2, 1, 2, 2, 1, 3, 1, 2, 2, 3, 1, 1, 1, 3, 1, 3, 1, 1, 2, 1, 1, 1, 2, 2, 1, 2, 3, 1, 2, 3, 2, 3, 2, 2, 3, 3, 2, 1, 1, 3, 3, 3, 2, 3, 3, 2, 1, 1, 1, 1, 2, 1, 1, 3, 1, 1, 3, 2, 1, 3, 3, 2, 1, 1, 1, 3, 3, 2, 2, 2, 2, 3, 1, 2, 3, 2, 3, 2, 2, 3, 1, 1, 3, 3, 3, 3, 2, 3, 1, 1, 3, 2, 3, 2, 2, 3, 1, 2, 1, 1, 3, 1, 3, 3, 3, 3, 1, 3, 2, 1, 1, 1, 3, 1, 2, 3, 1, 3, 3, 2, 3, 3, 1, 1, 1, 3, 3, 3, 2, 1, 2, 3, 1, 1, 1, 1, 3, 3, 3, 2, 3, 3, 1, 2, 2, 1, 2, 3, 3, 1, 2, 2, 2, 1, 1, 3, 3, 1, 3, 2, 3, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 3, 2, 2, 1, 2, 1, 2, 3, 1, 1, 1, 1, 3, 2, 2, 1, 3, 1, 3, 3, 1, 3, 2, 2, 1, 1, 3, 1, 1, 1, 2, 2, 3, 2, 3, 2, 1, 3, 1, 3, 3, 2, 2, 3, 1, 2, 1, 2, 3, 2, 2, 1, 1, 2, 2, 3, 2, 2, 2, 1, 2, 2, 3, 3, 1, 1, 2, 3, 2, 3, 2, 2, 1, 3, 2, 3, 1, 3, 1, 3, 2, 3, 2, 1, 1, 2, 1, 1, 1, 1, 2, 3, 3, 3, 3, 1, 3, 2, 2, 2, 2]



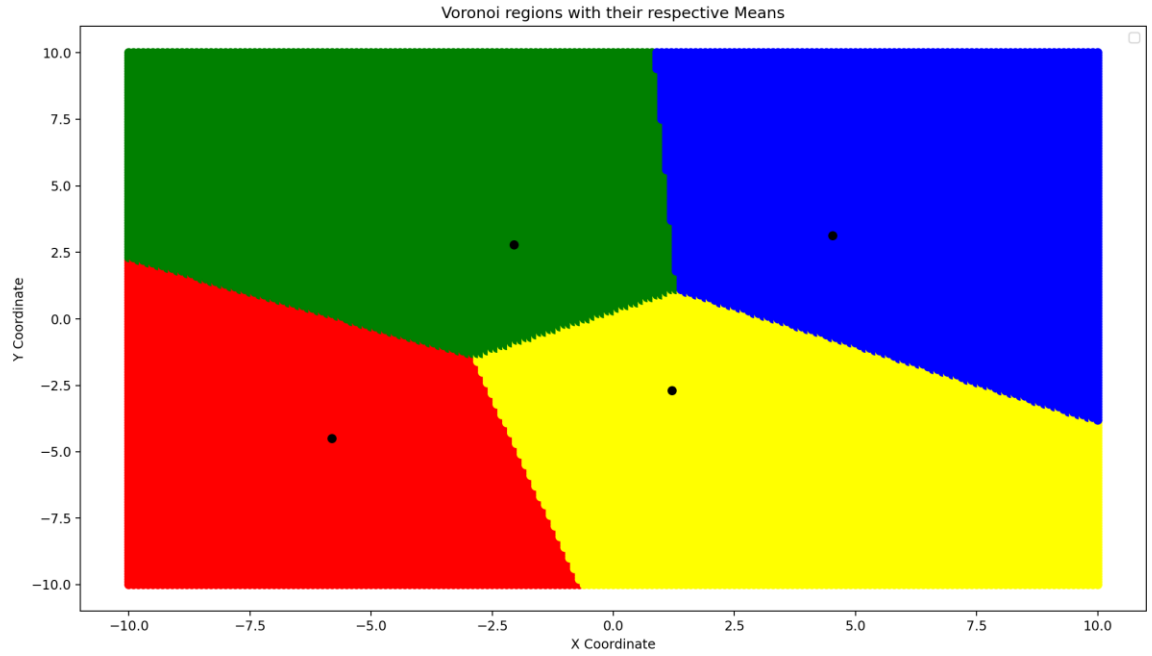
Mean of cluster number 1 is (-2.0137166960227275 , 3.203643878125004) - green colour

Mean of cluster number 2 is (2.9881975311750595 , 0.0735895386091127) - red colour

Mean of cluster number 3 is (-2.325756216450217 , -5.014585541125541) - blue colour

3) K=4 (4 Clusters)

INITIALLY THE VALUE OF ASSIGNMENT LIST: [1, 1, 3, 1, 3, 4, 1, 3, 3, 2, 2, 3, 4, 3, 1, 1, 2, 1, 4, 4, 1, 3, 1,
2, 3, 4, 2, 3, 2, 4, 2, 1, 4, 3, 3, 1, 1, 2, 2, 4, 2, 4, 3, 3, 1, 1, 2, 2, 3, 4, 2, 3, 3, 2, 2, 3, 3, 4, 4, 1, 3, 2, 2, 2,
2, 4, 1, 1, 1, 2, 1, 4, 1, 1, 4, 3, 2, 2, 3, 3, 3, 2, 1, 4, 1, 1, 2, 1, 3, 4, 3, 3, 2, 1, 1, 1, 4, 2, 4, 1, 3, 1, 1, 1, 1,
3, 1, 3, 3, 1, 2, 2, 1, 3, 1, 1, 3, 1, 3, 2, 3, 4, 2, 4, 4, 1, 3, 1, 3, 4, 4, 1, 4, 3, 4, 1, 4, 2, 3, 1, 2, 4, 4, 1, 1, 4,
1, 3, 2, 3, 3, 3, 3, 4, 1, 1, 2, 2, 3, 1, 3, 2, 3, 2, 1, 3, 1, 2, 4, 2, 2, 3, 1, 2, 4, 2, 1, 3, 3, 2, 3, 4, 2, 4, 3, 1, 2,
2, 4, 1, 4, 3, 3, 3, 4, 1, 4, 3, 3, 3, 3, 4, 3, 2, 1, 3, 1, 2, 2, 3, 3, 3, 4, 3, 2, 2, 4, 4, 4, 1, 2, 3, 1, 1, 3, 3, 4, 3,
2, 1, 3, 2, 1, 2, 4, 2, 4, 1, 2, 2, 2, 1, 2, 4, 4, 4, 4, 4, 1, 4, 2, 4, 3, 4, 1, 1, 3, 2, 2, 3, 4, 2, 4, 2, 3, 3, 2, 2, 4,
3, 2, 2, 3, 2, 4, 4, 3, 3, 2, 2, 1, 1, 3, 2, 4, 3, 1, 4, 3, 2, 3, 1, 2, 3, 3, 4, 1, 3, 1, 1, 1, 3, 3, 1, 1, 3, 4, 4, 1, 1,
4, 3, 2, 1, 2, 1, 3, 1, 2, 3, 1, 4, 4, 3, 4, 1, 2, 2, 3, 4, 3, 4, 1, 4, 3, 3, 4, 1, 2, 1, 4, 4, 3, 3, 4, 4, 1, 2, 4, 1, 4,
4, 3, 2, 2, 2, 3, 3, 2, 1, 3, 3, 1, 2, 2, 4, 4, 4, 4, 2, 4, 1, 2, 1, 3, 1, 1, 3, 2, 1, 4, 4, 1, 3, 3, 1, 3, 4, 4, 1, 1, 2,
3, 1, 1, 1, 4, 1, 4, 3, 4, 2, 3, 1, 2, 3, 4, 1, 2, 1, 3, 2, 4, 4, 3, 3, 1, 1, 3, 3, 4, 4, 4, 1, 2, 1, 4, 4, 4, 4, 1, 4, 1,
3, 1, 4, 3, 1, 3, 4, 2, 4, 1, 1, 3, 2, 4, 4, 3, 4, 4, 1, 4, 2, 3, 3, 1, 4, 4, 4, 2, 1, 4, 3, 3, 1, 2, 4, 2, 4, 3, 1, 1, 3,
3, 1, 4, 3, 2, 4, 3, 3, 1, 4, 4, 1, 1, 3, 1, 2, 1, 2, 2, 4, 1, 3, 3, 2, 1, 3, 4, 1, 2, 3, 3, 4, 3, 4, 1, 3, 4, 1, 4, 1, 2,
4, 1, 3, 4, 4, 3, 1, 4, 4, 2, 3, 1, 2, 3, 1, 2, 2, 1, 4, 4, 2, 2, 2, 1, 3, 2, 2, 2, 3, 4, 2, 4, 3, 1, 1, 3, 2, 4, 1, 2, 1,
4, 3, 4, 1, 3, 3, 2, 3, 4, 3, 3, 3, 2, 3, 4, 4, 2, 1, 4, 4, 1, 3, 1, 2, 4, 3, 2, 3, 2, 1, 1, 1, 4, 3, 2, 4, 4, 4, 4, 1, 2,
2, 4, 2, 2, 3, 2, 4, 1, 3, 1, 3, 1, 2, 2, 2, 1, 3, 2, 2, 3, 2, 2, 3, 2, 2, 1, 2, 2, 1, 4, 3, 2, 4, 1, 4, 2, 1, 3, 1, 2, 4,
1, 1, 4, 4, 1, 2, 1, 2, 1, 1, 3, 4, 4, 4, 2, 3, 4, 2, 1, 1, 1, 1, 4, 2, 2, 3, 1, 1, 2, 2, 2, 2, 2, 3, 4, 2, 3, 1, 1, 2, 1,
3, 4, 4, 1, 3, 3, 4, 1, 3, 1, 4, 3, 4, 1, 3, 2, 3, 2, 3, 3, 3, 3, 4, 1, 3, 4, 4, 1, 1, 3, 4, 1, 2, 4, 3, 2, 4, 4, 1, 1, 2,
2, 1, 2, 3, 3, 4, 2, 2, 2, 4, 3, 3, 3, 2, 1, 3, 4, 3, 2, 2, 3, 4, 2, 4, 1, 4, 1, 2, 3, 1, 1, 1, 3, 2, 2, 3, 4, 3, 2, 2, 3,
1, 1, 4, 2, 2, 4, 4, 1, 1, 1, 2, 2, 1, 2, 1, 1, 3, 4, 4, 3, 1, 3, 3, 4, 2, 3, 4, 1, 2, 3, 1, 3, 4, 2, 4, 4, 3, 3, 2, 4, 3,
2, 3, 3, 4, 3, 4, 2, 1, 1, 3, 2, 3, 3, 2, 3, 1, 1, 1, 1, 4, 1, 2, 2, 3, 2, 4, 3, 2, 2, 4, 3, 3, 1, 1, 1, 4, 2, 3, 4, 4, 3,
4, 4, 3, 2, 2, 4, 4, 1, 2, 4, 2, 1, 4, 4, 4, 4, 2, 2, 1, 3, 1, 3, 1, 4, 1, 3, 2, 4, 2, 3, 2, 2, 2, 4, 2, 2, 3, 3, 3, 3, 3,
1, 1, 2, 4, 1, 4, 2, 2, 2, 3, 2, 4, 1, 4, 4, 3, 2, 3, 1, 3, 2, 2, 2, 1, 1, 2, 1, 4, 1, 1, 3, 1, 3, 1, 4, 1, 2, 4, 4, 4, 3,
2, 3, 2, 2, 3, 2, 2, 2, 2, 4, 3, 3, 4, 3, 1, 4, 1, 3, 2, 2, 2, 4, 4, 4, 3, 3, 2, 1, 3, 2, 2, 2, 2, 2, 2, 3, 1, 4, 1, 3, 2,
2, 4, 3, 4, 4, 4, 2, 1, 2, 1, 1, 2, 2, 4, 4, 2, 4, 3, 1, 4, 4, 3, 2, 3, 1, 4, 3, 3, 3, 1, 4, 4, 1, 4]



Mean of cluster number 1 is (-2.049660083333333 , 2.784651092901236) - green colour

Mean of cluster number 2 is (-5.812258407079645 , -4.506580522123896) - red colour

Mean of cluster number 3 is (4.527171502590671 , 3.131083269430052) - blue colour

Mean of cluster number 4 is (1.2084631743243237 , -2.695357306486485) - yellow colour

4) K=5 (5 Clusters)

INITIALLY THE VALUE OF ASSIGNMENT LIST: [4, 3, 1, 4, 5, 2, 5, 3, 5, 3, 3, 4, 4, 1, 2, 5, 5, 1, 2, 5, 2, 5, 1,
4, 4, 4, 2, 1, 2, 1, 4, 5, 5, 4, 2, 1, 5, 1, 4, 3, 1, 3, 5, 4, 3, 3, 4, 1, 2, 4, 4, 1, 3, 1, 2, 2, 4, 1, 2, 5, 4, 4, 5, 4,
4, 5, 3, 2, 3, 3, 2, 3, 2, 2, 5, 4, 2, 1, 3, 4, 3, 5, 1, 3, 4, 1, 2, 4, 4, 3, 2, 2, 1, 1, 5, 2, 1, 3, 4, 1, 5, 2, 1, 1, 1,
1, 4, 4, 5, 5, 3, 3, 3, 5, 2, 5, 5, 1, 1, 2, 3, 5, 2, 2, 3, 4, 3, 3, 1, 5, 3, 3, 3, 4, 3, 1, 3, 4, 5, 3, 1, 5, 2, 2, 3, 4,
1, 4, 1, 1, 3, 3, 4, 2, 3, 5, 5, 2, 1, 3, 3, 4, 4, 2, 1, 2, 2, 3, 5, 4, 1, 1, 3, 1, 2, 4, 2, 5, 4, 5, 2, 3, 5, 4, 5, 4, 3,
1, 1, 4, 2, 4, 1, 4, 5, 2, 2, 4, 5, 4, 4, 5, 5, 5, 2, 2, 4, 1, 4, 5, 4, 5, 1, 1, 5, 5, 1, 2, 2, 1, 3, 5, 1, 3, 2, 3, 2, 1,
3, 1, 1, 3, 2, 1, 2, 5, 3, 4, 1, 1, 1, 4, 5, 2, 1, 5, 4, 3, 1, 4, 1, 1, 1, 1, 1, 5, 4, 2, 3, 5, 3, 3, 5, 1, 4, 5, 2, 1, 5,
2, 2, 5, 4, 4, 5, 5, 3, 2, 1, 2, 4, 2, 3, 3, 4, 4, 1, 4, 2, 1, 2, 4, 2, 3, 1, 1, 5, 3, 2, 5, 4, 2, 4, 1, 4, 4, 4, 1, 1, 3,
2, 3, 1, 3, 4, 1, 5, 5, 2, 4, 4, 2, 3, 1, 5, 4, 3, 2, 5, 2, 2, 3, 4, 1, 2, 1, 5, 3, 4, 2, 3, 4, 4, 3, 3, 2, 1, 3, 5, 1, 5,
2, 2, 5, 1, 4, 3, 4, 1, 2, 4, 1, 4, 2, 2, 2, 4, 4, 4, 1, 4, 2, 3, 3, 4, 5, 4, 2, 5, 1, 4, 5, 4, 2, 5, 2, 2, 4, 3, 3, 3, 3,
1, 2, 4, 5, 4, 3, 5, 1, 5, 3, 2, 3, 5, 2, 3, 3, 4, 2, 1, 5, 1, 2, 2, 1, 5, 2, 5, 5, 5, 4, 3, 2, 5, 3, 4, 5, 3, 4, 2, 1, 4,
2, 5, 3, 1, 3, 1, 1, 1, 2, 3, 4, 2, 1, 3, 2, 3, 4, 2, 5, 5, 4, 5, 1, 3, 5, 1, 4, 1, 5, 1, 4, 3, 3, 4, 4, 1, 2, 4, 3, 1, 4,
5, 1, 5, 3, 5, 2, 1, 5, 4, 5, 5, 4, 4, 4, 2, 2, 2, 5, 3, 3, 2, 5, 4, 4, 1, 2, 4, 4, 3, 5, 2, 2, 4, 2, 4, 1, 4, 5, 1, 3, 1,
1, 1, 1, 4, 3, 3, 2, 4, 2, 4, 2, 5, 1, 4, 5, 3, 5, 1, 5, 2, 1, 2, 3, 5, 2, 4, 1, 1, 4, 3, 5, 2, 2, 2, 2, 3, 4, 5, 4, 5, 5,
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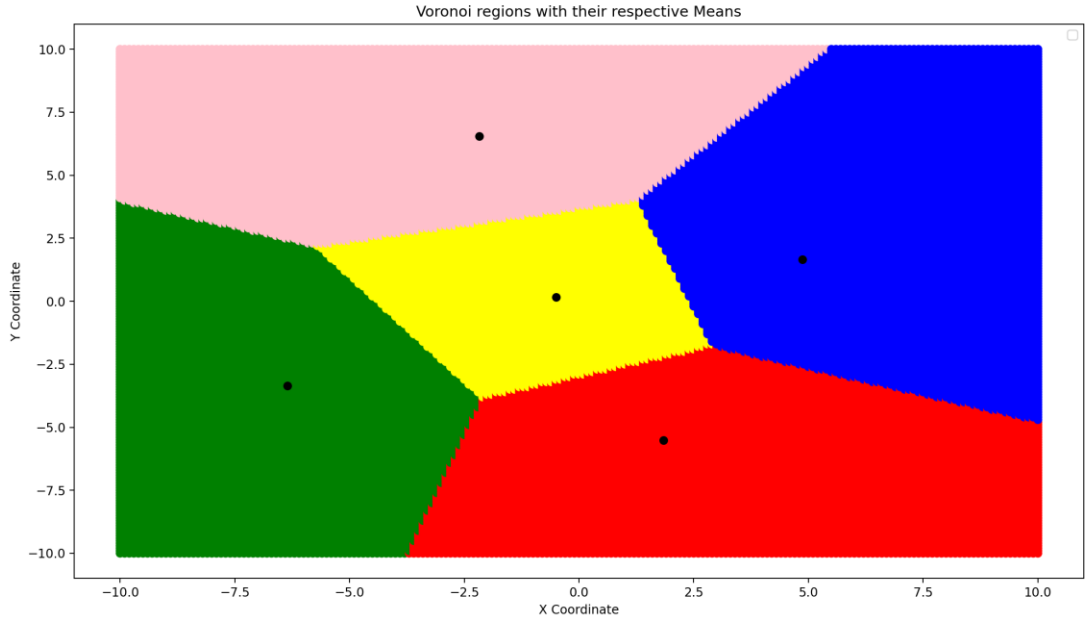


Mean of cluster number 1 is (-6.356763793103447 , -3.3544578362068993) - green colour

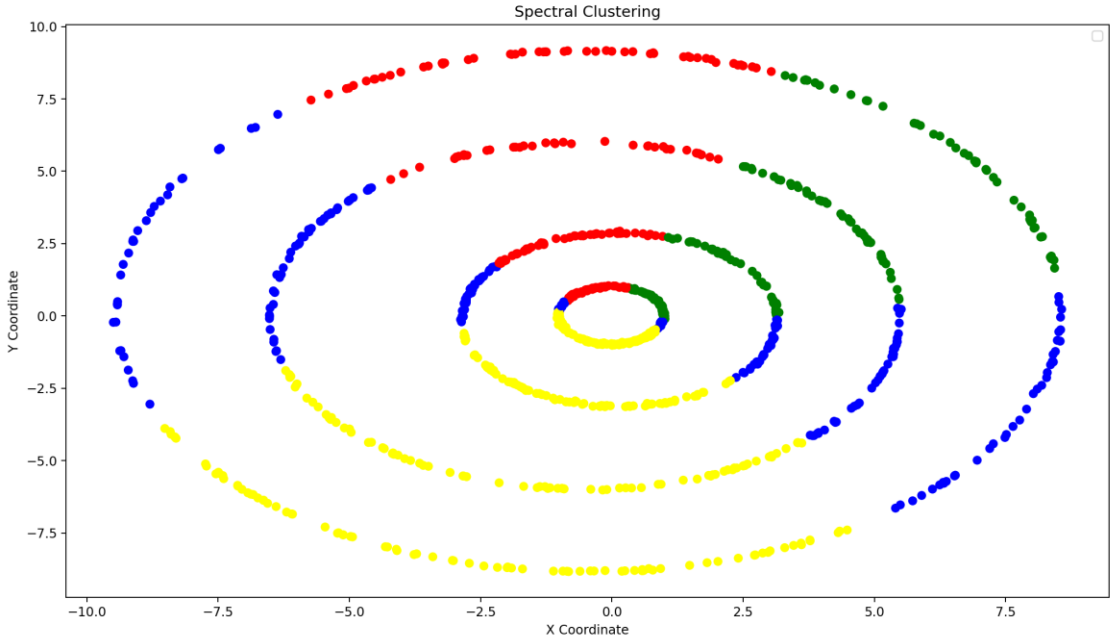
Mean of cluster number 2 is (1.8430142292993634 , -5.515724203821654) - red colour

Mean of cluster number 3 is (4.862576683937822 , 1.6551438756476684) - blue colour

Mean of cluster number 4 is (-0.5011019563591022 , 0.16425427356608477) - yellow colour

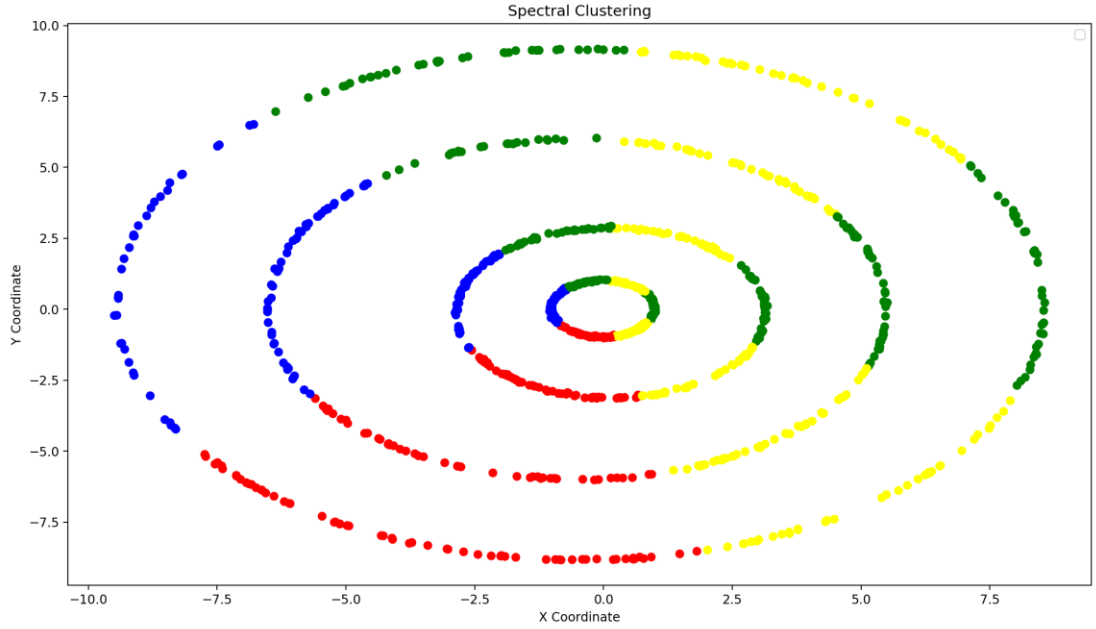
Mean of cluster number 5 is (-2.1767191127819556 , 6.539678947368422) - pink colour

iii) Spectral Clustering, Using Polynomial Kernel, with d=2.

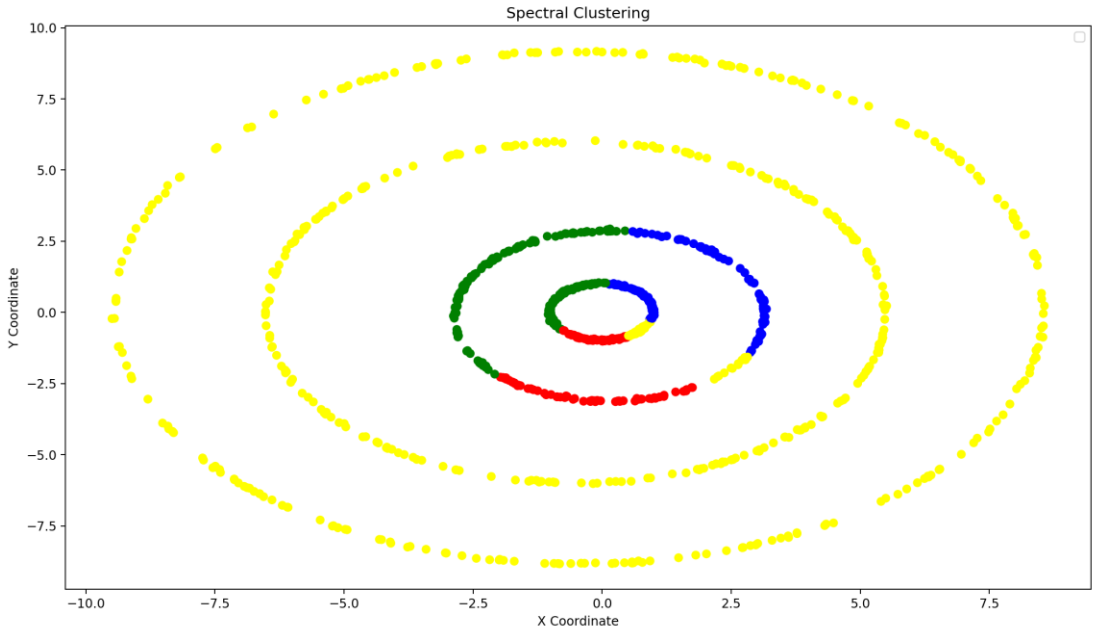


The above figure shows the clustering, when Spectral Relaxation of K-means is done using Kernel PCA.

Using Polynomial Kernel with d=3

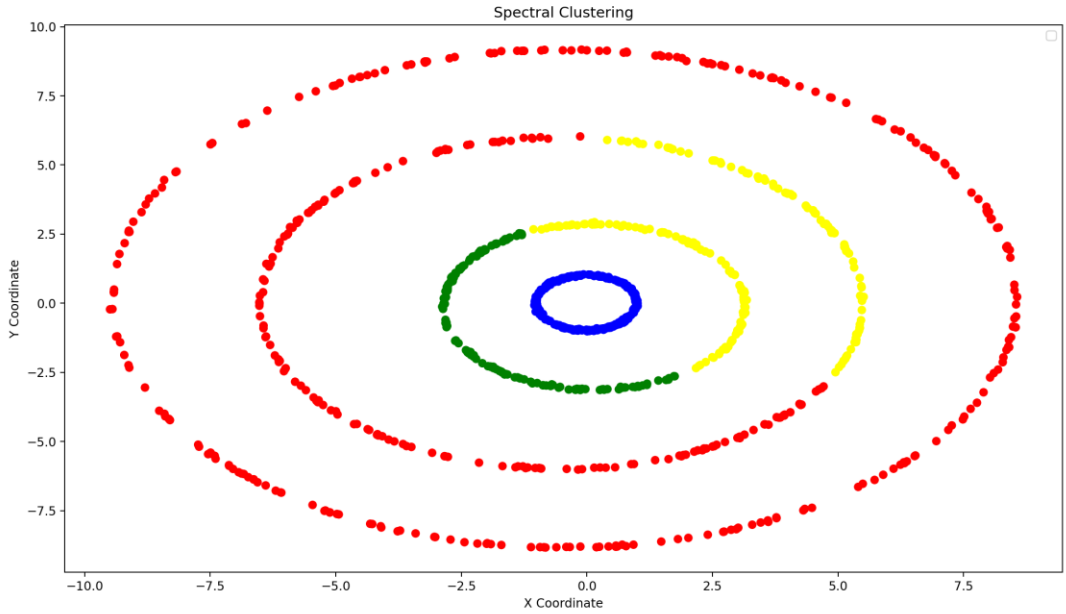


Using Radial basis function as kernel, with sigma=0.1



Using Radial basis function as kernel, with sigma=2

INITIALLY THE VALUE OF ASSIGNMENT LIST: [4, 4, 3, 2, 1, 2, 4, 2, 3, 1, 3, 2, 4, 2, 3, 3, 2, 3, 3, 2, 3, 3, 3, 3, 3, 2, 4, 3, 3, 2, 2, 3, 1, 2, 1, 1, 4, 1, 2, 3, 4, 3, 1, 1, 3, 1, 4, 3, 3, 1, 1, 4, 1, 4, 3, 2, 4, 3, 2, 4, 1, 2, 2, 4, 4, 2, 1, 3, 4, 1, 1, 3, 4, 1, 3, 1, 3, 1, 1, 2, 3, 4, 2, 3, 2, 4, 4, 2, 4, 3, 1, 1, 1, 2, 4, 3, 3, 2, 3, 3, 4, 2, 3, 2, 2, 4, 4, 1, 2, 3, 4, 4, 2, 1, 3, 3, 4, 1, 3, 3, 3, 2, 2, 1, 2, 2, 2, 3, 4, 1, 2, 1, 3, 4, 2, 3, 4, 1, 3, 1, 4, 2, 2, 3, 4, 1, 1, 4, 4, 1, 2, 4, 1, 4, 3, 4, 3, 3, 4, 1, 3, 1, 2, 4, 3, 3, 2, 2, 1, 1, 2, 1, 4, 3, 3, 3, 1, 3, 1, 4, 1, 2, 1, 1, 4, 2, 1, 3, 2, 4, 4, 4, 1, 4, 3, 2, 4, 3, 2, 2, 3, 1, 4, 3, 3, 2, 2, 1, 2, 3, 2, 1, 4, 4, 2, 1, 2, 3, 3, 3, 2, 1, 3, 3, 4, 2, 4, 1, 4, 2, 3, 3, 3, 4, 1, 3, 2, 1, 3, 1, 3, 1, 4, 1, 4, 2, 4, 1, 2, 3, 4, 4, 1, 1, 1, 1, 1, 1, 2, 2, 1, 3, 3, 1, 3, 3, 3, 3, 3, 3, 4, 4, 3, 2, 4, 3, 3, 3, 2, 2, 3, 2, 4, 1, 1, 1, 4, 2, 3, 4, 4, 4, 4, 2, 4, 1, 3, 3, 2, 4, 3, 3, 1, 3, 3, 3, 3, 4, 1, 1, 4, 1, 4, 3, 2, 1, 4, 3, 1, 4, 3, 1, 4, 1, 2, 1, 2, 2, 4, 4, 3, 4, 1, 2, 3, 2, 4, 3, 4, 1, 3, 3, 3, 1, 2, 2, 2, 3, 1, 1, 3, 4, 4, 1, 4, 3, 1, 2, 3, 4, 1, 1, 3, 3, 2, 1, 3, 4, 1, 1, 2, 1, 1, 3, 4, 4, 4, 2, 1, 4, 1, 1, 3, 1, 2, 3, 2, 2, 1, 3, 2, 3, 3, 4, 2, 3, 2, 3, 1, 4, 2, 3, 1, 3, 1, 2, 3, 2, 3, 1, 3, 1, 4, 4, 1, 4, 1, 4, 3, 4, 2, 3, 3, 3, 1, 4, 3, 4, 4, 1, 3, 1, 3, 4, 2, 3, 4, 4, 1, 3, 2, 4, 4, 4, 3, 2, 4, 1, 4, 2, 2, 4, 3, 1, 3, 2, 1, 3, 4, 2, 1, 1, 2, 3, 1, 4, 4, 4, 3, 3, 3, 3, 1, 4, 4, 1, 1, 4, 2, 4, 1, 2, 3, 2, 2, 4, 1, 1, 2, 4, 3, 4, 1, 4, 4, 2, 1, 3, 1, 2, 2, 3, 1, 2, 2, 4, 2, 4, 3, 4, 4, 2, 4, 3, 3, 2, 4, 3, 4, 2, 2, 1, 2, 3, 1, 3, 3, 4, 2, 4, 4, 4, 4, 3, 4, 2, 2, 4, 1, 1, 3, 1, 2, 2, 4, 4, 1, 3, 1, 2, 3, 3, 1, 1, 4, 2, 4, 4, 3, 4, 1, 3, 3, 4, 1, 4, 2, 3, 1, 2, 1, 2, 2, 4, 4, 1, 4, 4, 2, 1, 4, 1, 4, 3, 2, 2, 1, 3, 3, 3, 3, 2, 1, 3, 4, 3, 4, 4, 3, 1, 2, 2, 2, 2, 2, 2, 4, 4, 3, 4, 4, 2, 2, 4, 3, 1, 3, 1, 4, 2, 3, 4, 2, 3, 4, 3, 2, 1, 3, 2, 2, 2, 3, 1, 4, 1, 4, 1, 1, 1, 3, 1, 3, 1, 1, 2, 3, 4, 4, 3, 2, 3, 4, 1, 2, 1, 4, 3, 1, 1, 4, 3, 1, 1, 2, 3, 4, 3, 3, 4, 4, 2, 3, 1, 1, 2, 2, 3, 3, 1, 3, 1, 1, 1, 4, 3, 2, 3, 4, 1, 4, 2, 4, 3, 4, 1, 2, 2, 3, 1, 1, 2, 2, 4, 2, 1, 1, 4, 4, 2, 3, 2, 1, 1, 4, 4, 3, 1, 2, 4, 4, 3, 1, 4, 1, 2, 4, 4, 1, 2, 4, 3, 4, 1, 2, 1, 2, 1, 4, 3, 2, 4, 4, 2, 4, 3, 1, 4, 3, 2, 3, 4, 1, 3, 1, 2, 3, 1, 4, 3, 4, 2, 4, 3, 4, 2, 4, 2, 4, 4, 2, 2, 3, 3, 1, 1, 3, 1, 3, 4, 1, 3, 3, 2, 4, 4, 4, 3, 1, 3, 2, 2, 1, 2, 3, 2, 2, 1, 4, 1, 2, 3, 2, 2, 2, 1, 2, 3, 3, 3, 2, 3, 4, 3, 4, 2, 2, 2, 2, 3, 3, 4, 1, 2, 2, 3, 1, 1, 4, 1, 3, 2, 3, 3, 2, 4, 2, 1, 4, 1, 2, 4, 3, 4, 4, 4, 4, 4, 1, 1, 4, 1, 2, 2, 3, 4, 2, 4, 3, 2, 1, 3, 2, 4, 2, 1, 4, 2, 2, 3, 3, 2, 1, 3, 4, 3, 1, 2, 1, 1, 3, 4, 3, 3, 2, 4, 4, 3, 2, 2, 3, 4, 3, 3, 4, 2, 1, 1, 4, 3, 1, 1, 2, 4, 2, 1, 2, 3, 4, 3, 2, 2, 2, 3, 4, 2, 4, 3, 2, 3, 1, 2, 3, 3, 3, 2, 3, 3, 3, 1, 3, 1, 4, 1, 1, 1, 4, 3, 4, 2, 3, 2, 4, 2, 3, 1, 2, 1, 3, 4, 1, 2, 3, 4, 4, 1, 3, 1, 2, 1, 2, 3, 1, 1, 4, 4, 3, 3, 1, 4, 1, 3, 2, 3, 2, 1, 2, 1, 1, 3, 2, 4, 3, 3, 3, 3, 4, 3, 2, 4, 1, 4, 2, 1, 2, 1, 1, 2, 4, 2]



From these four plots, i.e,

1) Spectral Clustering using Polynomial Kernel with d=2.
2) Spectral Clustering using Polynomial Kernel with d=3.
3) Spectral Clustering using Radial Basis Function Kernel with sigma=0.1.
4) Spectral Clustering using Radial Basis Function Kernel with sigma=2.0

It is clear that polynomial Kernel functions are unable to catch the ring structure of data, whereas Exponential kernels are able to capture the ring structure, but there is error in all four plots, the ideal kernel selection would be such that all four rings be coloured in different colours. Hence an exponential function would be better an appropriate Kernel.

In case 4) with sigma=2 we can clearly see that the first ring is in one cluster, Perfect! This is what we needed! And then Fourth ring is also in one colour, perfect! We needed this (But ring 2 and ring 3 are not captured with perfection though). But in the case of any polynomial kernel, we are getting Voronoi type clustering once again, which is not desirable.

iv) Given method of clustering says assign data point i to cluster l whenever ,

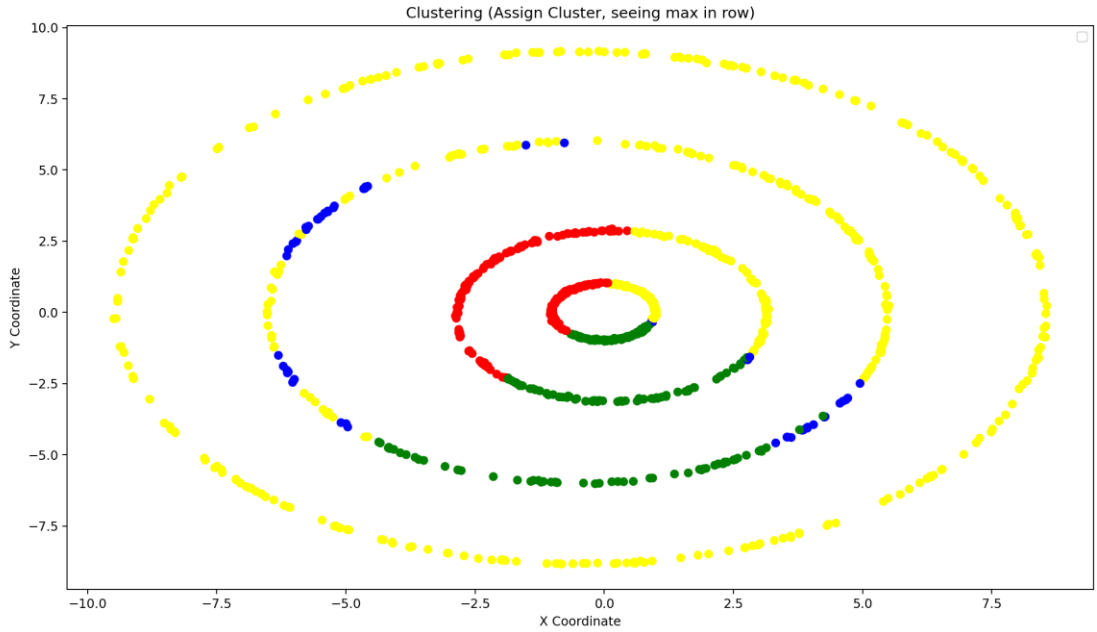
$$\ell = \arg \max_{j=1,\ldots,k} v_i^j$$

Where v^j belongs to R^n is the eigenvector of the Kernel Matrix associated with the j-th largest eigenvalue. So what does this mean? It means when we calculate the H Matrix (By taking top k (Where k is the number of clusters given) eigenvectors of the Kernel matrix aligning them column wise, we have to simply traverse each row of the H matrix and then find at what place we are finding the maximum value in that row. In question K=4.

So the H matrix would be 1000 rows (1 for each data point) X 4 Columns (One for each Column).

For deciding which Cluster should be assigned to data point 1, we simply see the first four values in Row1. If we find the max at 3rd position, we say the the data point 1 belongs to 3rd cluster.

We found out that exponential Kernel function was better to find out the the natural clustering visible to human eye, i.e, detect the rings of data. Below is the plot using the given algorithm, we can clearly see red erratic behaviour in the third ring with red dots amidst blue dots. Only one ring could be detected.



FINAL MEANS-

[0.09395097 -2.68240571] Cluster: 1 green

[-1.3194817 0.74576716] Cluster: 2 red

[-1.89957091 -0.38495927] Cluster: 3 blue

[0.6627855 0.81407415] Cluster: 4 yellow

Using this assignment list as the start for Llyod Algorithm we clearly see the objective_function value decreasing even more , hence this Algorithm is not a good approach to clustering.

Initial Assignment List=

[4, 2, 1, 4, 2, 4, 1, 1, 4, 2, 2, 4, 1, 4, 4, 4, 2, 1, 2, 1, 4, 4, 4, 2, 4, 1, 4, 1, 1, 1, 2, 2, 4, 4, 1, 1, 1, 2, 1, 2, 2, 4, 2, 2, 2, 4, 2, 1, 1, 1, 2, 4, 4, 4, 2, 2, 1, 1, 2, 1, 1, 2, 1, 2, 4, 3, 1, 1, 2, 4, 1, 1, 2, 1, 1, 1, 1, 2, 1, 4, 1, 1, 1, 1, 2, 2, 1, 4, 2, 4, 4, 1, 2, 2, 1, 1, 2, 1, 2, 4, 2, 3, 2, 2, 2, 1, 2, 2, 1, 2, 1, 1, 4, 2, 2, 4, 1, 4, 2, 2, 1, 4, 2, 4, 1, 2, 2, 4, 4, 1, 1, 2, 1, 2, 4, 4, 2, 4, 2, 4, 1, 2, 1, 1, 1, 2, 2, 1, 1, 4, 4, 4, 1, 4, 4, 1, 2, 1, 4, 2, 4, 2, 4, 4, 4, 4, 2, 2, 4, 1, 2, 2, 2, 1, 2, 1, 4, 4, 1, 2, 2, 2, 2, 1, 4, 1, 2, 1, 4, 2, 1, 4, 1, 2, 2, 2, 4, 2, 2, 1, 2, 2, 1, 4, 1, 1, 2, 4, 1, 2, 2, 2, 4, 2, 1, 1, 2, 2, 1, 1, 2, 1, 4, 4, 4, 1, 2, 2, 2, 2, 2, 2, 4, 1, 2, 4, 4, 2, 2, 1, 2, 2, 2, 1, 2, 1, 4, 1, 4, 1, 1, 2, 1, 4, 4, 2, 4, 4, 1, 2, 1, 2, 1, 1, 2, 1, 1, 2, 4, 2, 2, 2, 2, 4, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 1,

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4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4]

Running LLyods using Code of Q2 Part 1, we see the decrease in Objective value function.