Problem Description :

The problem we are solving is a classic Facility Location Problem (FLP). The goal is to determine the optimal locations for facilities to minimize the total cost of setting up the facilities and serving a set of customers. Each facility has a setup cost, a capacity, and a fixed location. Each customer has a demand and a fixed location. The objective is to assign each customer to a facility such that the total cost is minimized, and the following constraints are satisfied:

1. Each customer is assigned to exactly one facility.
2. The total demand assigned to a facility does not exceed its capacity.
3. If a customer is assigned to a facility, that facility must be open.

Formulation:

F : Set of facilities, indexed by fff.
C : Set of customers, indexed by ccc.
s_f : Setup cost for facility fff.
K_f : Capacity of facility fff.
d_c : Demand of customer ccc.
dist(f,c): Distance between facility fff and customer ccc, calculated as the Euclidean distance between their locations.
y_f: Binary variable, yf=1y_f = 1yf=1 if facility fff is open, 0 otherwise.
X_fc : Binary variable, xfc=1x_{fc} = 1xfc=1 if customer ccc is assigned to facility fff, 0 otherwise.

The objective is to minimize the total cost, which is the sum of the setup costs and the assignment costs. The problem can be formulated as:

$$\text{Minimize} \quad \sum_{f \in F} s_f y_f + \sum_{f \in F} \sum_{c \in C} \text{dist}(f, c) x_{fc}$$

Subject to the following constraints:

1. Each customer is assigned to exactly one facility:

$$\sum_{f \in F} x_{fc} = 1 \quad \forall c \in C$$

2. The total demand assigned to a facility does not exceed its capacity:

$$\sum_{c \in C} d_c x_{fc} \leq K_f y_f \quad \forall f \in F$$

3. A customer can only be assigned to an open facility:

$$x_{fc} \leq y_f \quad \forall f \in F, \forall c \in C$$