Deepanshu 2019CS50427

Homework 18.1

Page :
Date :

Claim → Proof by cases

Given the fact that u is inserted before v, following two cases are possible.

Case 1: Both u and v are inserted in same iteration of outer while loop.

This means that both of them have the same ancestor and hence their distance is equal (which is one more than the distance of ancestor).

Case 2: Both u and v are inserted in different iterations of while loop

Here two subcases are possible:

Case 2.1: The ancestor of u and v were at same distance from source. In this case again (by above argument), u and v have same distance and is equal to one more than the ancestor's distance.

Case 2.2: The ancestor of u and v were at different distance from source. Here since u is inserted before v and BFS covers all the successor for a given vertex, there is an ancestor of v which ~~is at~~ was inserted at same level as that of u. Hence, dist(u) = dist of one of ancestor of v.
Also, for every vertex y, dist(v) > distance of any ancestor
And hence dist(u) ≤ dist(v).

All the above cases are exhaustive and cover all possibilities. Also dist(u) ≤ dist(v) holds for every case. Hence it holds in general.

Hence Proved

Page :
Date :

Deepanshu 2019CS50427

Ans⇒ **Proof by induction**

Inducting on distance of the vertex.

Predicate: P(d) : If the distance is d, then it is the shortest distance of that vertex from source s.

Base Case

d = 0

Possible for the source itself. and 0 is indeed the shortest distance of source to itself.

Hence, P(0) is true.

Induction Hypothesis

Assume P(d) is true for d = k (weak Induction)

for d = k+1

Consider any vertex with distance equal to k+1 (call it v)

Let k' be the distance of a vertex adjacent to v.

Since they are adjacent, $|dist(v) - k'| = 1$

$|k+1 - k'| = 1$

$k' = k$ or $k' = k+2$.

We have to look for path having shortest distance.

Therefore, It must have the vertex with distance equal to k and not k+2.

Now from our Induction hypothesis P(k) is true. which means k is the shortest distance possible for any vertex having distance of k.

Hence, shortest distance of v from source is precisely one more than k. i.e k+1.

Hence P(k+1) is true whenever P(k) is true.

Therefore, by principle of mathematical induction, P(d) is true for all d.

Hence Proved