

Homework 23.1

Ans 1 \rightarrow For Kruskal's algorithm, we need edge weights at different time ~~to~~ from smallest cost and in increasing order. So, we will sort the edges beforehand.

Algorithm $\text{kruskal}(G)$

Set $\text{selectedEdges} \leftarrow T$

$T \leftarrow \emptyset$

while ($|T| < |V(G)| - 1$)

let (u, v) be next edge in consideration

if (u and v are in different component)

join the components of u & v

$T \leftarrow T \cup \{(u, v)\}$

return T

Note the following :-

1. We have used a data structure called set and also many of its operation like $|T|$ (size), union etc.

As discussed in the lecture, for Kruskal's algorithm we will make use of set data structure for fast implementation.

2. Note that I have assumed that we know how to find whether u & v are in same component or not.

In other words, adding (u, v) to T will lead to formation of cycle or not.

However, this has to be handled separately and effectively.

Homework 2.3.2

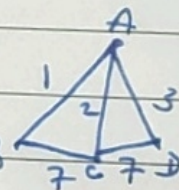
Ans \Rightarrow

The sufficient condition would be that if "graph has distinct edges" because clearly then the lowest $n-1$ edges not forming a cycle would be part of MST.

But what if edges are same?

Then if the edges of tree are all distinct, then MST is unique. consider the following example

Here, we have 2 vertices of cost 7 but none of them is part of MST so - "chill hai".



As long as edges ~~are~~ ~~are~~ ~~are~~ ~~are~~ distinct, the MST is unique.

If we encounter a case when edges are not distinct, we will use the proof of correctness of Kruskal's algo by Dr. Naorem.

By doing that technique, we end up claiming that the two trees are identical.