

Homework 2.4.1

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- Ans \Rightarrow • From implementation of union, we just have to update the links and thus union takes constant time i.e. $O(1)$
- for find, in worst case analysis all the links could be linear i.e. in form of a single list where reaching to the node for establishing link can take linear time because we would have to traverse the whole list. Thus find takes linear time i.e. $O(n)$ in worst case.

b. Consider the example

① ② ————— ⑦

1. $\text{Union}(\text{find}(1), \text{find}(2)) : ① \rightarrow ② \rightarrow ③ \rightarrow ④ \rightarrow \dots \rightarrow ⑦$

2. $\text{Union}(\text{find}(1), \text{find}(3)) : ① \rightarrow ② \rightarrow ③ \rightarrow ④ \rightarrow ⑤ \rightarrow \dots \rightarrow ⑦$
and so on

Here, we are traversing whole list first to join two sets ~~that~~ that takes $O(n)$ time.

c. If ~~we~~ we directly keep pointer to the element to be linked then, it would be $O(1)$ but but but

But then to update that pointer, we have to traverse each node. So, for each set only, we can maintain a pointer ~~keep a pointer~~ which might work in cases with more components with user nodes in each component.

We may also perform rotation, or maybe decide a pattern of assigning the 'leader of set' to make its access faster.

Homework 24-2

Ans 2 After using union-by-height method, the height of the trees can be limited to $O(\log n)$ and ~~not~~ ~~less~~.

- Union still takes constant time i.e $O(1)$ since it still require updating links and some variable values that might be storing the height.
- Find would now take $O(\log n)$ as reaching the leader of set (root here) can be achieved in traversing up a tree of height $O(\log n)$.
- for updating this information, we basically need information about the size of tree which we can store in the root of each tree.

while comparing the two heights (say h_1 & h_2) final height of combined tree will be either h_2 or h_1+1 depending on the situation.

So, we will update that value everytime we merge two trees into one.