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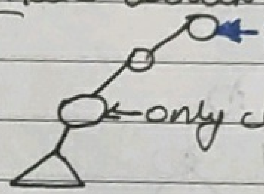
Homework 9.1

Ans => Proof by contradiction

Let there exist a non ~~empty~~ empty AVL tree T such that $LR(T) > \frac{1}{2}$.

i.e no. of nodes that are only children > $\frac{(\text{no. of nodes})}{2}$.

Now we know that only child node is always a leaf. (Otherwise its grand parent node ~~would~~ would not be ~~balanced~~ balanced.



Let number of only child = K.
Every node would have a parent ~~and every~~ i.e there will be at least K + K = 2K number of nodes.

~~thi~~ $\frac{\text{only child nodes}}{\text{total nodes}} \le \frac{1}{2}$ which contradicts the assumption.

Hence for every non empty AVL tree T, $LR(T) \le \frac{1}{2}$.

Hence Proved.

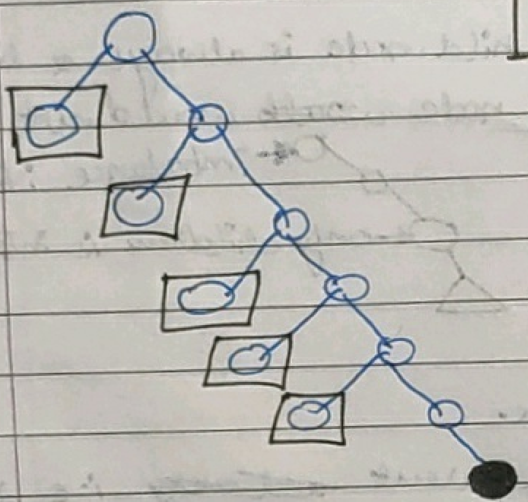
Homework 9.2

Ans => No. This is not true for all trees.

Consider the following tree as a counterexample

=> leaf node

=> only child node



Here, if $n \geq 3$ is number of nodes, then

$$height = \frac{n}{2} \quad \text{whereas}$$

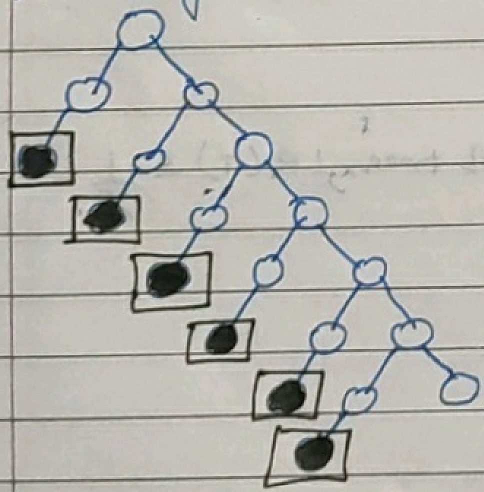
$$LR(T) = 1 \leq \frac{1}{2} \; [\text{for all } n \geq 3]$$

Therefore, above graph has $LR(T) \leq \frac{1}{2}$ but height is not $O(\log n)$

Homework 9.3

Proof by counter example

Ans =>



In such trees, ~~let~~ if n is the count for only-children, then the height of tree is $n+1$.

Therefore, $height(T) \neq O(\log n)$

=> leaf node

=> only child node