Homework 15.1

Ans $\Rightarrow$ $T(n) = 1 + \underbrace{2T(n/2)}_{\downarrow}$

This term suggest the answer would be linear.

In Dr. Naveen's example, 8 elements called Mergesort 15 times $2n-1$ can be the solution.

LHS $\quad 2n-1$

RHS $\quad 1 + 2(n-1) = \quad 2n-1$

Hence solution is $\quad T(n) = 2n-1$

Merge Sort (n) —————— ① call
Merge sort (n/2) $\rightarrow T(n/2)$
Merge sort (n/2) $\rightarrow T(n/2)$
Merge

Homework 15.2

Ans $\Rightarrow$ $T(n) = \begin{cases} c_1 & \text{if } n=1 \\ 2T(n/2) + c_2 n & \text{if } n > 1 \end{cases}$

$T(n) = 2T(n/2) + c_2 n$

$T(n) = 2[2T(n/4) + c_2 n/2] + c_2 n$

$T(n) = 2^2 T(n/4) + 2c_2 n$

$T(n) = 2^3 T(n/8) + 3c_2 n$

$\vdots$

$T(n) = 2^i T(n/2^i) + i c_2 n$

Put $i = \log n$

$T(n) = 2^{\log n} T(n/n) + \log n \, c_2 n$

$T(n) = n T(1) + c_2 n \log n$

$\boxed{T(n) = c_1 n + c_2 n \log n \quad \text{or} \quad n(c_1 + c_2 \log n)}$