Page :
Date :

Deepanshu 2019CS50427

Homework 14.1

Whatever be the partition value, a total of n-1 comparisons are required. The partition key will be compared to every other value in the array.

II

- big-O(n)
- big-omega(n)
- theta (n)

Page :

Date :

Deepanshu 2019CS50427

Ans => In every possibility, total comparisons would be $n-1$ as we have to compare every element with the ~~pa~~ key and decide its place relative to key.

(i) → following possibilities are equally probable -

1 in left of key and rest in right

2 in left of key and rest in right

$$T(n) = \frac{1}{n}\sum_{j=0}^{n-1}\left(T(j-1) + T(n-j) + (n-1)\right).$$

$$T(n) = \frac{1}{n}\sum_{j=0}^{n-1}\left(T(j-1) + T(n-j)\right) + \frac{1}{n}\sum_{j=1}^{n}(n-1)$$

$$T(n) = \frac{2}{n}\sum_{j=0}^{n-1}T(j) + \frac{\cancel{n}}{\cancel{n}}(n-1)$$

$$\boxed{T(n) = \frac{2}{n}\sum_{j=0}^{n-1}T(j) + (n-1)}$$ The required recurrence relation

(ii) Since quicksort is a sorting algorithm, it is very natural to assume the solution to be $O(n\log n)$.

LHS $T(n) = O(n\log n)$

RHS $= \frac{2}{n}\left[1\log 1 + 2\log 2 \ldots n\log n\right] + (n-1)$

$$\frac{2}{n}\sum i\log i \le \frac{2}{n}\sum i\log n \le \frac{2}{\cancel{n}}\log n \times \cancel{n} \times \frac{(n-1)}{\cancel{2}}$$

RHS $\le n\log n - \log n + n - 1$

which is $O(n\log n)$

Note similar proof by assumption was used by naveen sir earlier (which he referred to during the lecture).