

The Cuckoo Search Algorithm: A review.

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Abstract

Today's world is efficiency driven. All organizations irrespective of the type of industry they belong to, strive to achieve maximum efficiencies in their processes, this is where optimization comes into picture. It is mainly concerned with finding the optimum values for several decision variables to form a solution to an optimization problem. This paper aims to review the concept of Cuckoo Search Algorithm (CSA), which is a meta heuristic naturally inspired optimization algorithm. Further, the major improvements in the traditional CSA have also been reviewed. Finally the recent applications of the cuckoo search in optimization problems have also been presented in the form of a bibliographic review. This paper aims to be a one-stop article for researchers or readers who want to gain an overview of the concept of CSA and understand the concept thoroughly.

Keywords: Meta-heuristic, Literature Review, Optimization, Cuckoo Search Algorithm.

1. Introduction

Optimization is nothing but employing a maximising or minimising type decision making algorithm, adapted to methods of approximation[1]. The principle of decision making involves choosing between various alternatives. The result of this is to choose the best solution/decision from all the choices. These optimization algorithms are based on nature-derived concepts that deal with choosing the best alternative in the sense of the given objective function. An Optimization algorithms are mainly classified as: evolutionary algorithms (EAs), swarm-based algorithms, and trajectory- based algorithms. These algorithms emulate the principle called, The survival of the fittest. This starts with an initial group of individuals, called population[2]. At every generation, preferred characteristics of the current population are combined, and a new population, which is selected on the basis of the principle of natural selection[1]. On the other hand, swarm-based algorithms mimic the behaviour of a group of animals when searching for food. Solutions are constructed normally, based on previous data collected by previous generations. At each iteration, that solution will be moved to its neighbouring solution, which resides in the same search space region, using a specific neighbourhood structure. In this paper, we will be focusing on the Cuckoo search algorithm[3]. There are thousands of bird species today, but the most commonly observed trait in birds is the way of reproduction. Birds reproduce by laying eggs. Since these eggs are rich in protein, and are the ultimate source of nourishment for predators, hence it is of utmost importance for the parent bird to protect its egg. The cunning behaviour shown by some bird species to secure or increase the survival rate of their next generation, is known as brood parasitism. Cuckoos show this type of behaviour. They never make their own nests but lay their eggs in other bird's nests and thus if the eggs hatch, the host bird takes care of the cuckoo chicks. Cuckoo mothers show characteristics of stealth and speed. The mother cuckoo lays her egg in the host bird's nest and removes one host egg and flies off within a few seconds. This entire process is extremely fast, which allows cuckoos to parasitise hundreds of bird species. Cuckoos specialise in a particular type of bird species. They accurately mimic their egg size, shape and colour, making it difficult for the host bird to identify the cuckoo egg. Exactly how the cuckoos manage to mimic the host bird is not known and rather is one of nature's many unsolved mysteries. The host birds slowly learn to identify the cuckoo eggs and thus destroy them, hence the cuckoos have to continuously improve their strategy to lay their eggs in the host birds nests. Obligate brood species look for good environments where their chicks get well nourished. After these chicks grow into adults, they again carry on with the same life cycle. Hence this brood parasitism is passed on to the next generations.

2. Concept of Cuckoo search algorithm

The cuckoo search was developed in recent times (in the year 2009) by Xin-she yang and Subhash Deb. The cuckoo optimization algorithm was later developed by Rajabioun in 2011. Before getting into the details, let us understand what the word 'meta-heuristic' means. Firstly a heuristic algorithm means algorithm is one that is designed to solve a problem in a faster and more efficient manner as compared to traditional methods[1]. Heuristic algorithms are most often used when approximate solutions are sufficient and exact solutions are necessarily computationally expensive. The Cuckoo optimization algorithm is based on the life cycle of the Cuckoo bird species. i.e. the characteristic brood parasitism of these birds. The cuckoo birds lay their eggs in nests made by other birds. The eggs that the cuckoos lay in the host nests, may or may not survive, this will happen when the host bird identifies the foreign cuckoo egg. Hosts may throw this egg out of its nest or may altogether abandon its nest and make a new one. To avoid this, cuckoos try to mimic the colour, size etc of the hosts eggs and place their eggs in the host nest very carefully so that the host won't recognise the eggs laid by the cuckoo. This aggressive reproduction strategy inspires the CS algorithm. Thus the key point to note here is that the cuckoo must be very accurate to mimic the hosts eggs and the host must be vigilant enough to identify a parasite egg this is the fight of survival. We can very meaningfully compare this system to an optimization problem. The eggs in the nest represent solutions and the cuckoo eggs represent new solutions. The aim here is to replace average/not as good solutions with better solutions. The probability that the host will recognise and throw away the cuckoo birds egg is given by $p_a \in [0,1]$. If the host bird is unable to identify the cuckoo eggs, then the cuckoo eggs tend to hatch early as compared to the host eggs. When the chicks hatch, the host destroys its own eggs. This increases the cuckoo birds chances of survival by getting more share of food. Following are certain basic concepts used in the CS algorithm:

2.1 Basic concepts.

Optimization in simple words means betterment or improvement of a process, achieved by tweaking or changing the input parameters of a process, mathematical equation, experiment etc to get the output as maximum or minimum. The input comprises of variables, where the process as a whole is known as a function, also known as cost function/objective function/fitness function. Similarly, the output is called as cost or fitness. There are various methods which can be used to solve optimization problems. The most common of all of these methods are nature-inspired algorithms. For example, PSO or Particle Swarm Optimization. This is inspired by bird flocking or fish schooling. The Genetic Algorithm (GA) is another very popularly used method to solve optimization problems[5]. It uses operators similar to the natural genetic variation and natural selection. Other examples include Ant Colony Optimization (ACO) which is an evolutionary optimization algorithm. Since we will only be focusing on the Cuckoo Search Algorithm, (CSA), we need to understand that the main goal of the cuckoo mother bird is to place her egg in only those nests in which her eggs will hatch, thus in optimization terms the profitability of that nest must be high. Cuckoos have a cunning strategy when it comes to reproduction. After the cuckoo egg is placed, one host egg is thrown off from the host nest by the cuckoo so that the host cannot make out the difference in the number of eggs. Also when the cuckoo egg hatches, the chick is a bit larger than the host chick. Hence, they consume a large portion of the food brought in by the host bird. As a result, the host bird's chicks might die of insufficient food. Also cuckoo chicks try to imitate the other host birds chirps/sounds to attract the mother host bird to get more food. Thus the cunning trait in cuckoos is passed on from one generation to the next. Cuckoo Search Algorithm is generally used in combination with Levy's flight.

Levy Flights- 'when animals go out in search of food or other resources, they walk randomly. Their walk is random in nature because their next step depends upon their current location/position and the probability of transition to the next position. This random walk can be modelled mathematically, almost all insects/birds/animals follow the levy flights principle. So we can say that the Levy flights is a random walk shown by (in this context) Cuckoos in which the step length can be determined by using a heavy tailed probability distribution. Heavy-tailed probability distributions are the ones in which their tails are not bounded exponentially hence having heavy tails than the rest of the distribution.'

3. Cuckoo Search Algorithm framework.

Metaheuristics exhibit the characteristic of imitating the best features of the nature, that is the biological systems that have evolved over a long period of time due to natural selection. These systems show two main points of interests. These are- adaptation to the environment and survival/selection of the fittest. In modern metaheuristics, these features can be utilised into defining the terms intensification and diversification. Diversification enables the algorithm to search

the entire space efficiently. On the other hand, intensification deals with the search around current best solutions and the selection of the best solutions amongst them. The CSA is based on three simple rules or assumptions. These are:

- 1) Each cuckoo lays only one egg and dumps it in a randomly selected nest.
- 2) The best nests with high quality of eggs will pass on or carry on to the next generation of cuckoos.
- 3) The number of host nests are fixed, hence the probability that the host identifies the cuckoo egg is given by the probability of $P_a \in [0,1]$. Hence as mentioned before, the host can throw away the cuckoo egg or simply abandon the nest and build a new one.

Now, in case of maximization problems, the quality of the solution is proportional to the objective function. The cuckoo search algorithm can also be applied to cases with more than one cuckoo egg in the host nest i.e., multiple eggs per nest. But we will consider the simplest form of this algorithm, which is according to the above mentioned rules. Each cuckoo only lays one egg. In order to generate a new function, denoted by: $x^{(t+1)}$ for a cuckoo 'i'. then the Levy flights is performed as:

$$x^{(t+1)} = x^{(t)} + \alpha \oplus \text{Levy}(\lambda), \quad [1]$$

In the above equation, α denotes the step length which is greater than 0. Generally or in most of the cases, the value of step size is taken as unity. i.e. $\alpha=1$. A stochastic equation is the one in which one or more terms are stochastic and the resulting solution is also a process which is stochastic in nature. A random walk is a Markov chain. Hence the next location or the status depends on the current position and the probability of transition. The first term in equation [1] denotes current position and the second term of the equation denotes the probability of transition. \oplus denotes entry wise multiplication. The Levy flights is used to obtain the random walk, but the random step length is given by Levy distribution.

$$\text{Levy} \sim u = t^{-\lambda}, \quad (1 < \lambda \leq 3), \quad [2]$$

This has infinite mean and variance. These steps essentially create a random walk, as mentioned earlier, the random walk process is generated with a power-law step-length distribution with a heavy tail. Now we can look at a very simple flow chart depicting the cuckoo algorithm.

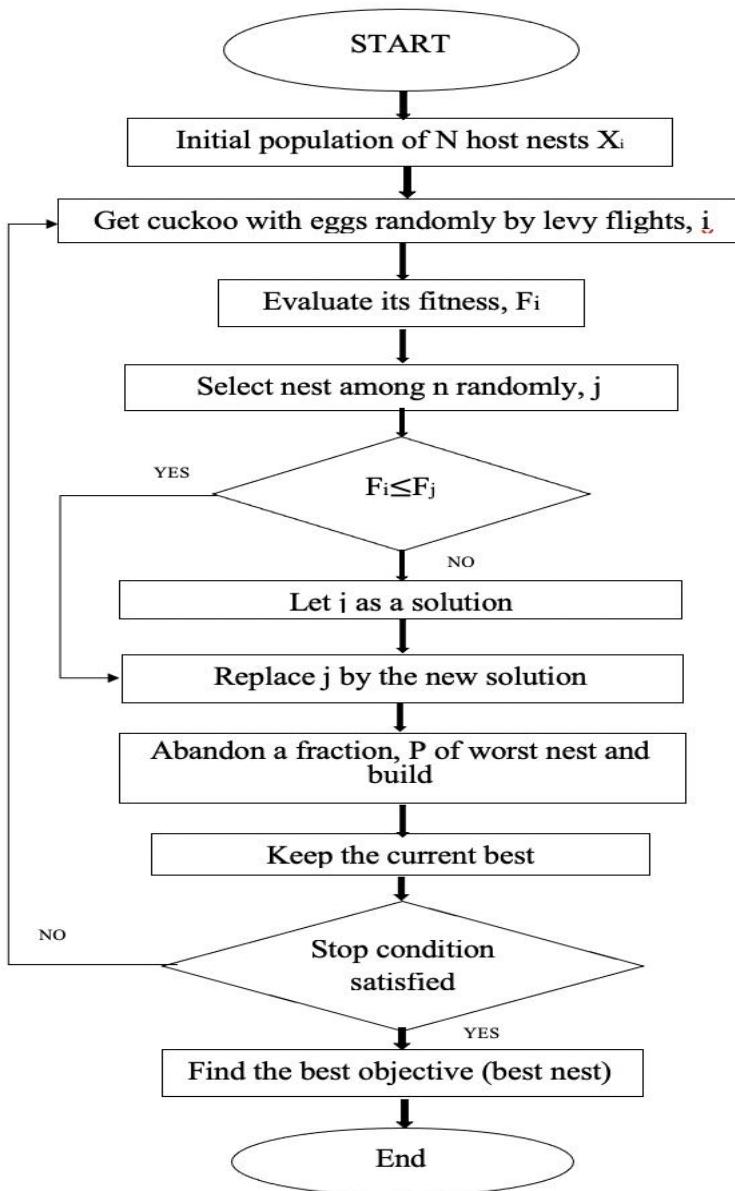


Fig.1 Cuckoo algorithm flowchart [Shehab et.al. 2017]

The Cuckoo search algorithm can be represented using the following pseudo code:

```

begin
    Objective function f(x), x = (x1,...,xd)T

```

Generate initial population of

n host nests x_i ($i = 1, 2, \dots, n$)

```

while (t < MaxGeneration) or (stop criterion)

```

Get a cuckoo randomly by Levy flights

evaluate its quality/fitness F_i

Choose a nest among n (say, j) randomly

```
if (Fi > Fj),  
    replace j by the new solution;  
end  
A fraction (pa) of worse nests  
    are abandoned and new ones are built;  
Keep the best solutions  
    (or nests with quality solutions);  
Rank the solutions and find the current best  
end while  
Postprocess results and visualization  
end
```

The most important advantage of the cuckoo search algorithm is that it uses very few control parameters. The following table shows these parameters and the commonly used values for them.

| PARAMETER | SYMBOL | RANGE | COMMONLY USED |
|-----------|--------|-----------|---------------|
| NEST | N | [15 , 50] | N= 15 |
| FRACTION | Pa | [0,1] | Pa= 0.25 |
| STEP SIZE | a | a >0 | a=1 |

Table.1. CSA parameters and common values.

4. CS algorithm suggested by Rajabioun:

A better Cuckoo algorithm approach was suggested by Rajabioun in 2011. Initially, the algorithm starts with an initial population of cuckoos. These cuckoos have eggs that they will lay in some other bird's nest. The eggs which are more similar to those of the host bird's eggs, will have a greater chance of survival. That is, they will hatch and become adult cuckoos. Other eggs that the host identifies, will be thrown away and killed. The eggs that successfully hatch, depict the suitability of that area for cuckoo breeding. Areas where large number of eggs survive, are more profitable outcome wise. The Cuckoo algorithm will optimize the positions where more eggs survive. Cuckoos will aim to search for the best areas to lay eggs, so as to , maximise the survival rate of their eggs. The most suitable or appropriate area is what cuckoo birds search for, so as to maximise the survival rate of their eggs. Once the eggs that survive grow, and turn into a fully grown cuckoo, they start making societies. Here, every such society has its habitat area to live in. Finally, the best habitat amongst all these, will be the one which will be aimed for by all cuckoos from other societies. Then they immigrate toward this best habitat. The number of eggs laid by cuckoos, and its distance to the goal point, the egg laying radius is decided. Accordingly, the bird starts to lay eggs in totally random nests which are within the egg laying radius. This is continued until the best target with the highest profit value is reached[1].

According to the theory explained in cuckoo algorithm this is stochastic algorithm, means it has random probability, anyone cannot predict further step. This type of algorithm can be analysed statistically but may not be predicted precisely. The cuckoo algorithm is Immune Evolutionary Algorithm.

Immune Evolutionary algorithm is based on immune system inspired by defence process of biological immune system and evolutionary means continuous evolutions or alterations are made in current product to obtain most approximate or fittest solution for problem. This goes according to Darwin's theory SURVIVAL OF FITTEST. Some of the advantages of evolutionary algorithm are:

- 1) Being robust to dynamic changes

- 2) Broad applicability
- 3) Hybridization with other method is possible
- 4) Solve problems that have no solutions (no human expertise required)

We can observe wide range application of evolutionary algorithms in following regions of study:

Power system operation and control, NP-hard combinational problem, Chemical Process, Job scheduling problems, Vehicle routing, Mobile networking, Batch process scheduling, Multi objective optimization problem, Modelling optimized parameters, Image processing and pattern recognition problem

Cuckoo species uses STEALTH, SURPRISE AND SPEED strategy for its survival. Cuckoo's majority species occur in forests and woodland and in evergreen rain forests. Most species of cuckoo are sedentary, but several species undertake partial migration over complete range. For species breeding at higher latitudes food availability dictates that they migrate to warmer climates during the winter, and all do so. Long migration flights which are also observed, include the Lesser Cuckoo which takes its flight journey from India to Kenya across the Indian ocean. Whereas the common cuckoo birds mainly the European ones, fly nonstop over the Mediterranean Sea and the Saharan Desert over to South Africa[1,2,6]

Cuckoo optimization algorithm is going to optimize where more cuckoo eggs are grown. Cuckoo's search is based on to lay eggs in order to maximise egg survival rate. If baby cuckoo grows and it hatches the egg and become mature cuckoo, then they create their society in that respective area. In this process cuckoo make certain habitat where probability of survival of cuckoo is highest. In other words, cuckoo makes that certain region CUCKOOPRONE OR FIT FOR CUCKOO SURVIVAL. Accordingly, other cuckoos inhabit near the best habitat. According to number of cuckoo eggs each cuckoo has cuckoo lay eggs in some radius around best habitat (which is created by some other cuckoos). This process continues till best region with maximum profit outcome is obtained.

Cuckoo algorithm pseudo code [1]

1. Initialise cuckoo habitats with some random points on the profit function.
2. Dedicate some eggs to each cuckoo.
3. Define ELR for each cuckoo.
4. Let the cuckoos lay eggs inside their corresponding ELR.
5. Kill eggs which are recognised by host birds.
6. Let the eggs hatch and the chicks grow.
7. Evaluate habitat of each newly grown cuckoo.
8. Limit cuckoos maximum number in environment and kill those who live in the worst habitats.
9. Cluster cuckoos and find best group and select goal habitat.
10. Let new cuckoo population immigrate toward goal habitat.
11. If stop condition is satisfied, then stop, if not go to step 2.

One interesting and specific fact about cuckoo is, they lay their eggs in random bird's nest in their respective ELR, but eggs which can't match with characteristics of host bird's eggs are killed by host bird [1]. From the remaining eggs of cuckoo only one egg has chance to grow. As if one cuckoo chick comes out of egg, first it will throw all other eggs outside the nest. And if cuckoo comes outside the later then it eats all the food given by host bird, it results in death of remaining chick due to hunger.

Once the cuckoo chick is outside the they live in that society, feed themselves in that society until egg laying period. AS egg laying period approaches they find better place or area where there will be sufficient food for new youngsters. After these cuckoo groups are formed in different societies the one society with maximum profit value is set as goal for other cuckoos to immigrate. While migrating all cuckoos do not fly all the way but they get deviated or some fly half the way.

It is observed each cuckoo fly Only l% distance towards the goal and has $\Phi\%$ deviation. These two parameters l and Φ help the cuckoos to find much more new positions in all environment

For each cuckoo $l \sim U(0,1)$ = it is random number between 0 and 1

$\Phi \sim U(-w, w)$ = it limits the deviation cuckoos

As it is known nature always has equilibrium maximum number of cuckoos are always restricted in the environment. After some iteration and trials all cuckoos will move to one best habitat with good food resource and having maximum egg resemblance with host bird eggs. This habitat will produce maximum output. Convergence of more than 95% of all cuckoos in same habitat give you the end of cuckoo algorithm.

5. A bibliographic review of major applications of CSA.

Major research and improvement is done on CS algorithm as improvised and superior results obtained after application of CS algorithm to various fields of engineering. In 2010 design of spring and welded beam were improvised after application of CS algorithm to design process by Yang and Deb. Yang and Deb (2010)[25] applied CS algorithm to solve various problems in field of engineering. Objective was to reduce weight and to reduce overall cost of fabrication. CS algorithm was proved to be very efficient among Genetic algorithm and particle swarm optimization. Model with CS algorithm of accuracy measurement for spiking neuron in pattern recognition was proved better than same model with Differential evolution algorithm. This comparison was done by Vazquez (2011). Burnwal and Deb (2012)[8] tested CS algorithm for scheduling optimization of flexible manufacturing system by minimizing penalty cost and maximizing machine utilization time. CS was proved to be better than other algorithms. Enhanced CS algorithm was proposed for optimization of bloom filter in spam filtering by Natarajan and Subramanian (2012)[9]. Enhanced cuckoo algorithm was employed to minimize the total member ship invalidation cost of bloom filters by finding optimal false positive rates and number of elements stored in every bin. CS was implemented in object oriented software for unconstrained optimization problems. In proposed test this software performed well. Cuckoo based particle approach was applied to achieve energy efficient and wireless sensor networks. This implementation was done by Dhivhya, Sundarambal and Anand (2011)[7]. Results obtained were comparable with LEACH and HEED protocols. In the paper authored by Sang Dang Ho, Ve Song, Toan Minh Le and Thang Trung Nguyen [12] two modified versions of CSA were proposed, where new solutions were obtained using two distributions including Gaussian and Cauchy distributions which were proposed for economic emission load dispatch (EELD) problem with multiple fuel options. The advantages of Cuckoo search algorithm with Gaussian distribution (CSA-Gauss) and Cuckoo search algorithm with Cauchy distribution (CSA-Cauchy) over CSA with Lévy distribution are fewer parameters and fewer equations and shorter computational process. The proposed method was tested on one test system consisting of ten generating units with various load demands and compared to other methods. Similarly, Mareli et.al.[13] proposed the Adaptive Cuckoo search algorithm for optimisation (2017). This paper also emphasizes on dynamic parameter switching in cuckoo algorithm and 3 new models of cuckoo algorithm. These new models with dynamic switching parameters are compared with CS algorithm with constant parameters. Many other papers also compare Levy flight technique with some traditional techniques to get better one for optimisation of the process.

6. Conclusions.

In this paper, the concept of cuckoo search algorithm was reviewed, along with it, significant improvements over the traditional CSA were also seen. These methods aimed to increase the convergence rate of the CSA to get more accurate and efficient results. After understanding the concept of CSA theoretically, it is of great importance that the readers of this article get insights about the applications of CSA in real world optimization problems. This was achieved by providing an in depth but apt bibliographic review about the applications and fruitful outcomes of CSA. Optimization is the key to efficiency in any process or task which needs to be executed. Meta-heuristic algorithms do prove to be a solution to increase efficiency, and still have a lot of scope for improvements, which should be focused upon.

7. References.

- [1] Rajabioun, Ramin. (2011). Cuckoo Optimization Algorithm. *Applied Soft Computing*. 11. 5508-5518. doi:10.1016/j.asoc.2011.05.008.
- [2] Shehab, M. (2020). *Artificial Intelligence in Diffusion MRI. Studies in Computational Intelligence*. doi:10.1007/978-3-030-36083-2
- [3] Shehab, M., Khader, A. T., & Al-Betar, M. A. (2017). *A survey on applications and variants of the cuckoo search algorithm*. *Applied Soft Computing*, 61, 1041–1059. doi:10.1016/j.asoc.2017.02.034
- [4] Lu Hong, A novel particle swarm optimization method using clonal selection algorithm, in: International Conference on Measuring Technology and Mechatronics Automation, vol. 2, 2009, pp. 471–474.
- [5] X.-S. Yang, S. Deb, “Cuckoo search via L’evy flights”, in: Proc. of World Congress on Nature & Biologically Inspired Computing (NaBIC 2009), December 2009, India. IEEE Publications, USA, pp. 210-214 (2009).
- [6] Barthelemy P., Bertolotti J., Wiersma D. S., A L’evy flight for light, *Nature*, 453, 495-498 (2008).
- [7] Marichelvam, M. K. (2012). *An improved hybrid Cuckoo Search (IHCS) metaheuristics algorithm for permutation flow shop scheduling problems*. *International Journal of Bio-Inspired Computation*, 4(4), 200. doi:10.1504/ijbic.2012.048061
- [8] Burnwal, S., & Deb, S. (2012). *Scheduling optimization of flexible manufacturing system using cuckoo search-based approach*. *The International Journal of Advanced Manufacturing Technology*, 64(5-8), 951–959. doi:10.1007/s00170-012-4061-z
- [9] Natarajan, A., Subramanian, S., & Premalatha, K. (2012). *A comparative study of cuckoo search and bat algorithm for Bloom filter optimisation in spam filtering*. *International Journal of Bio-Inspired Computation*, 4(2), 89. doi:10.1504/ijbic.2012.047179
- [10] Deb. K., Optimisation for Engineering Design, Prentice-Hall, New Delhi, (1995).
- [11] A survey on applications and variants of the cuckoo search algorithm S1568-4946(17)30127-8 <http://dx.doi.org/doi:10.1016/j.asoc.2017.02.034>.
- [12] Sang Dang Ho et.al., “Economic Emission Load Dispatch with Multiple Fuel Options Using Cuckoo Search Algorithm with Gaussian and Cauchy distributions”, *International Journal of Energy, Information and Communications* Vol.5, Issue 5 (2014), pp.39-54 <http://dx.doi.org/10.14257/ijeic.2014.5.5.04>.
- [13] S. Pare, A. Kumar, V. Bajaj, G. Singh, A multilevel color image segmentation technique based on cuckoo search algorithm and energy curve, *Applied Soft Computing* 47 (2016) 76–102.
- [14] A. K. Bhandari, V. K. Singh, A. Kumar, G. K. Singh, Cuckoo search algorithm and wind driven optimization based study of satellite image segmentation for multilevel thresholding using kapurs entropy, *Expert Systems with Applications* 41 (7) (2014) 3538–3560.
- [15] X. Liu, H. Fu, Pso-based support vector machine with cuckoo search technique for clinical disease diagnoses, *The Scientific World Journal* 2014 (2014) .
- [16] S. Goel, A. Sharma, P. Bedi, Cuckoo search clustering algorithm: A novel strategy of biomimicry, in: *Information and Communication Technologies (WICT)*, 2011 World Congress on, IEEE, 2011, pp. 916–921.
- [17] A. Kaveh, T. Bakhshpoori, An efficient multi-objective cuckoo search algorithm for design optimization, *Advances in Computational Design* 1 (1) (2016) 87–103.
- [18] Mareli, M., Twala, B., An adaptive Cuckoo search algorithm for optimisation, *Applied Computing and Informatics* (2017), doi: <http://dx.doi.org/10.1016/j.aci.2017.09.001>

- [19] S. Roy, A. Mallick, S. S. Chowdhury and S. Roy, "A novel approach on cuckoo search algorithm using Gamma distribution," in Second International Conference on Electronics and Communication systems, 2015.
- [20] M. Tuba, M. Subotic and N. Stanarevic, "Modified Cuckoo search algorithm for unconstrained optimization problems," in Proceedings of the European Computing Conference, 2011.
- [21] Walton, S., Hassan, O. and Morgan, K. (2012), "Reduced order mesh optimisation using proper orthogonal decomposition and a modified cuckoo search". Int. J. Numer. Meth(2010).
- [22] Mustafa İLARSLAN, Salih DEMIREL, Hamid TORPI, A. Kenan KESKIN, M. Fatih ÇAĞLAR, "Optimization Of Filter By Using Support Vector Regression Machine With Cuckoo Search Algorithm", Radioengineering, 23, no. 3(2014):790-797
- [23] Iztok Fister Jr.a,Iztok Fistera,Xin-She Yangb, "A short discussion about Economic optimization design of shell-and-tube heat exchangers by a cuckoo-search-algorithm", International Journal of Applied Thermal Engineering 76 (2015) :535-537.
- [24] Manjeet Kumar, Tarun Kumar Rawat, " Optimal design of FIR fractional order differentiator using cuckoo search 4 algorithm",ScienceDirect:1-17
- [25] Yang, X. S., & Deb, S. (2010). *Engineering optimisation by cuckoo search*. *International Journal of Mathematical Modelling and Numerical Optimisation*, 1(4), 330.doi:10.1504/ijmmno.2010.035430