

Kshatriyas Hall 2 IntraHall Takneek

Problem 2 [Astronomy]

1. A conjunction is defined as an apparent meeting or passing of two or more celestial objects. For ease, let us work with two planets (i.e. apart from the one you currently are on), since a triple conjunction is astronomically rare. You witnessed the conjunction of Jupiter and Saturn during December of 2020. What is the maximum number of conjunctions of these two outer planets that can occur in 365 consecutive days? [40 marks]

Problem-2
 Ans → Max. conjunctions = 3
 Next conjunction = 2040-2041
 Next constellation = Virgo-Libra
 Date

H3KMRB

First, eccentricities of orbits of Earth, Jupiter, Saturn are 0.017, 0.049, 0.057 (extremely small)

★ Revolution Direction of 3 Planets is same.
 ★ So, ~~that~~ I will assume them circular for calculation.
 ★ For a conjunction Earth should be exterior of Jupiter, and be collinear to Jup., Saturn.

Now radius of orbits of 3 Planets Earth = 149 mil.km
 Jupiter = 778 mil.km
 Saturn = 1.5 bil.km

Let them (R, SR, JR)
 angular speed of 3 Planets → Earth = $\frac{2\pi}{1}$ / year
 Jupiter = $\frac{2\pi}{12}$ / year
 Saturn = $\frac{2\pi}{29}$ / year

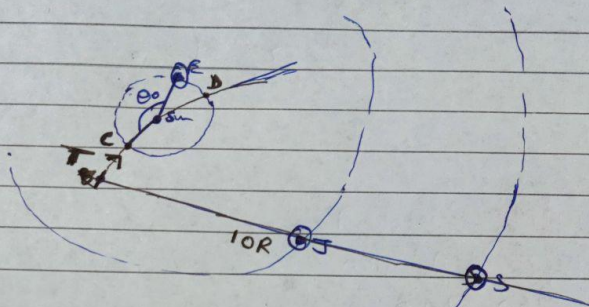
~~now for angular speed of saturn's spin~~

⇒ Now to find change in slope of JS line / year,
 • Definitely conjunction will happen when θ is small, else JS line will not cut Earth's orbit.
 arc length covered by Jupiter = $R_s(d\theta) \omega_J$
 " Saturn = $R_s(d\theta) \omega_S$
 this (ω_{JS}) will be max for $\theta \sim 0$ and will be $\Rightarrow \frac{R_J \omega_J - R_S \omega_S}{R_J - R_S}$
 $\omega_{JS} \Rightarrow (\omega_J - 2\omega_S)$ app. Spiral

$$W_{JS} = (W_J - 2W_S)$$

Date

Now analysing this line,



movement of T which is $10R$ from S is given

So, we can consider CD as straight line
(Approx)

$$\text{Speed of Point T} = (\text{Speed of S}) + W_{JS} \times (10R)$$

Now taking O_0 in Prev. diagram

to be 10^+ when

line just intersect earth's orbit;

\Rightarrow Earth can revolve $\approx (230^\circ)$
in (0.65 years)

$$V_T = V_S + W_{JS} \times 10R$$

$$\Rightarrow V_T = ((W_S) \times 10R) + (10R \times W_{JS})$$

Time taken to pass by Earth orbit

$$T_{\text{pass}} \Rightarrow \frac{2R}{V_T}$$

$$\Rightarrow \frac{2R}{V_T}$$

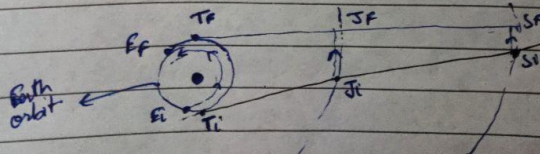
$$\left(\frac{2\pi}{25} + \frac{2\pi}{12} - \frac{2\pi}{25} \right) 10R$$

$$T_{\text{pass}} \Rightarrow \frac{7R}{(2\pi)(10R)(17)}$$

\Rightarrow For this initial position of Earth, Jupiter, Saturn

~~Conjunction~~ 3 conjunctions will occur

$$T_{\text{pass}} = \frac{348}{170\pi} \approx 0.65 \text{ years}$$

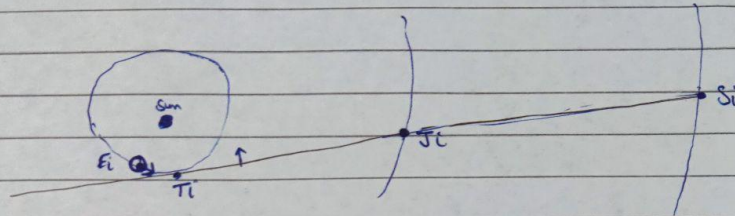


means This line can intersect
earth's orbit for ~~at least~~ (0.65 years)
for some position of J, S

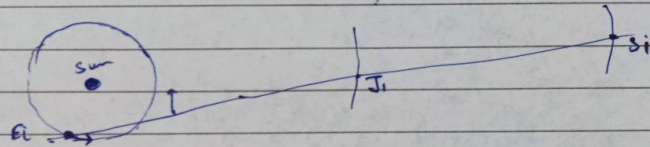
Spiral

Date

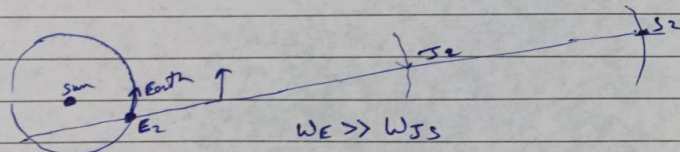
Initial Position.



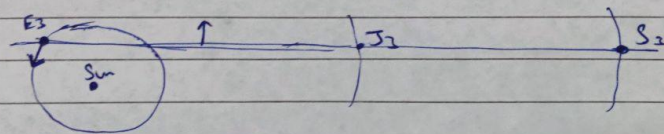
Intersection (1) \Rightarrow



Intersection (2) \Rightarrow



Intersection (3) \Rightarrow



★ They may not be seen from a single point of earth.

Spiral

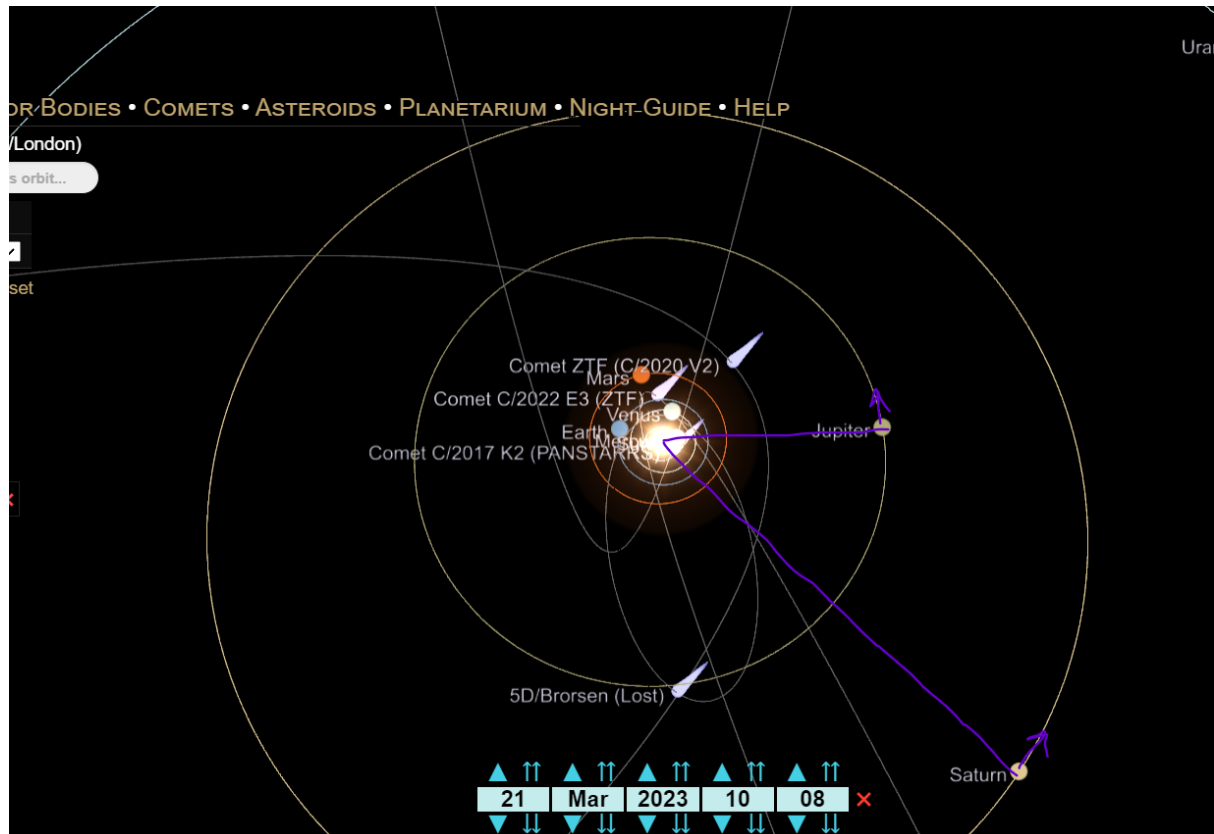
Answer= 3 conjunctions can happen max in consecutive 365 days

2. Let's say you get the number as k , with $k \in \mathbb{Z}^+$. During which year(s) will it happen next and in what constellation(s) will this (these) conjunction(s) happen in during that time? [40 + 20 marks]

NOTE: You are free to consult the internet for the information regarding the details such

as orbital radius of Jupiter and Saturn, etc.

Current planetary situation:

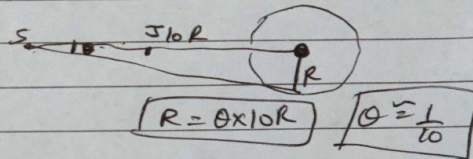


Date

Analysing current situation of Jupiter, Saturn, Earth.

angular separation between them ^{Dec} in 2020
 $\approx \left[-\frac{1}{10}, +\frac{1}{10} \text{ Radian} \right]$
 \Rightarrow Jupiter was ahead of Saturn

$\left(\frac{1}{10} \right)$ kaise nikale?



now to get this angular sep ($\Delta\theta$) again in range of $\left[-\frac{1}{10}, \frac{1}{10} \right]$
 so that conjunction happen,

Relative angular speed of Planets J, S \Rightarrow

\Rightarrow Previous constellation was
 happened in between
 (Capricornus - Aquarius)

$$\Delta\omega \Rightarrow \left(\frac{2\pi}{12} - \frac{2\pi}{29} \right)$$

Time req. to get $\Delta\theta$ in $\left[-\frac{1}{10}, 0 \right] \Rightarrow$

next conjunction will happen

$$\left[(2\pi) \times \left(\frac{\frac{2\pi}{12}}{\frac{2\pi}{12} - \frac{2\pi}{29}} \right) \right] \bmod 2\pi$$

$$\frac{\Delta(\Delta\theta)}{\Delta\omega} = \frac{2\pi - \frac{1}{10}}{\frac{2\pi}{12} - \frac{2\pi}{29}} \Rightarrow \frac{1 - \frac{1}{20\pi}}{\frac{17}{348}}$$

$$\Rightarrow \left[\frac{(20\pi - 1)348}{17 \times 20\pi} \right]$$

ahead of Aquarius
 $\Rightarrow (254^\circ)$ ahead of Aquarius

$$\Rightarrow \boxed{20 \text{ years Aft.}}$$

in $\boxed{\text{Virgo - Libra}}$

So, ^{next} conjunction may happen
 in $\boxed{(2040 - 2041)}$

Spiral

Answer a/ 2040-2041

Answer b/ Virgo-Libra