1(a) $KG(s)H(s) = \dfrac{K}{(s+1)(s+2)}$

No. of open loop poles $= 2 (=P)$ Poles $= -1, -2$

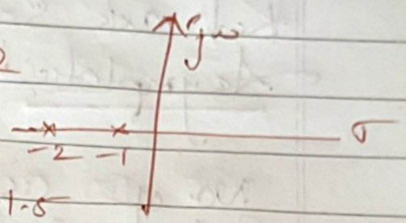
No. of open loop zeros $= 0 (=Z)$.

$\therefore$ No. of branches of root locus $= \max(P, Z) = 2$

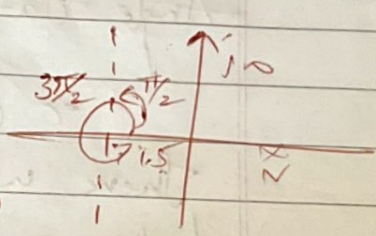
Real axis segment $\to (-2, -1)$

No. of asymptotes $= |P - Z| = 2$

Real axis intercept $\sigma_a = \dfrac{(-2)+(-1)}{2-0} = -1.5$



Asymptotes angles $\theta_a = \dfrac{(2r+1)\pi}{P-Z}$, $r = 0, 1 \Rightarrow \theta_a = \pi/2, 3\pi/2$



Breaking point

$\dfrac{dK}{ds} = 0 \Rightarrow \dfrac{d}{ds}\left(\dfrac{-1}{G(s)H(s)}\right) = 0$

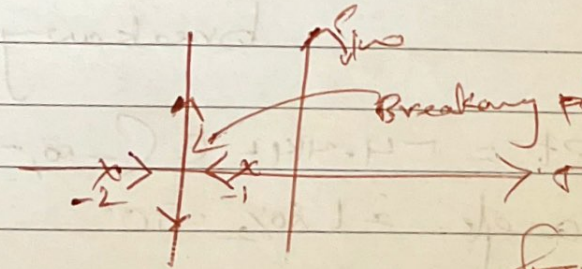
$\Rightarrow \dfrac{d}{ds}\left((s+1)(s+2)\right) = 0$

$\Rightarrow s+2+s+1 = 0 \therefore s = -3/2 = -1.5$

$-1.5 \in (-2, -1)$ so -1.5 is valid breaking point.

Breakaway angle $= \dfrac{180^\circ}{2} = 90^\circ$

Sketch.



No $j\omega$-crossing

For all values of $K \geq 0$

$\therefore$ the closed loop system is stable.

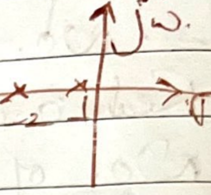
1(b) $KG(s)H(s) = \dfrac{K(s+3)}{(s+1)(s+2)}$

No. of open loop poles $= 2 (=P)$ Poles $-1, -2$

No. of open loop zeros $= 1 (=Z)$ Zeros $= -3$.

$\therefore$ No. of branches of root locus $= \text{Max}(P, Z) = 2$

Real axis segment $= (-\infty, -3) \cup (-2, -1)$



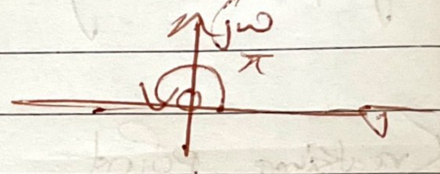
Asymptotes:

No. of asymptotes $= (P-Z) = 1$

Real axis intercept $\sigma_a = \dfrac{(-1)+(-2)-(-3)}{2-1} = 0$

Angle $\theta_a = \dfrac{(2r+1)\pi}{2-1}$, $r = 0$

$\Rightarrow \theta_a = \pi$



There will be a breakaway point between -2 and -1 and a break in point between $-\infty$ and -3

$\dfrac{dk}{ds} = 0 \Rightarrow \dfrac{d}{ds}\left(-\dfrac{(s+1)(s+2)}{(s+3)}\right) = 0$

$\Rightarrow (s+2)(s+3) + (s+1)(s+3) - (s+1)(s+2) = 0$

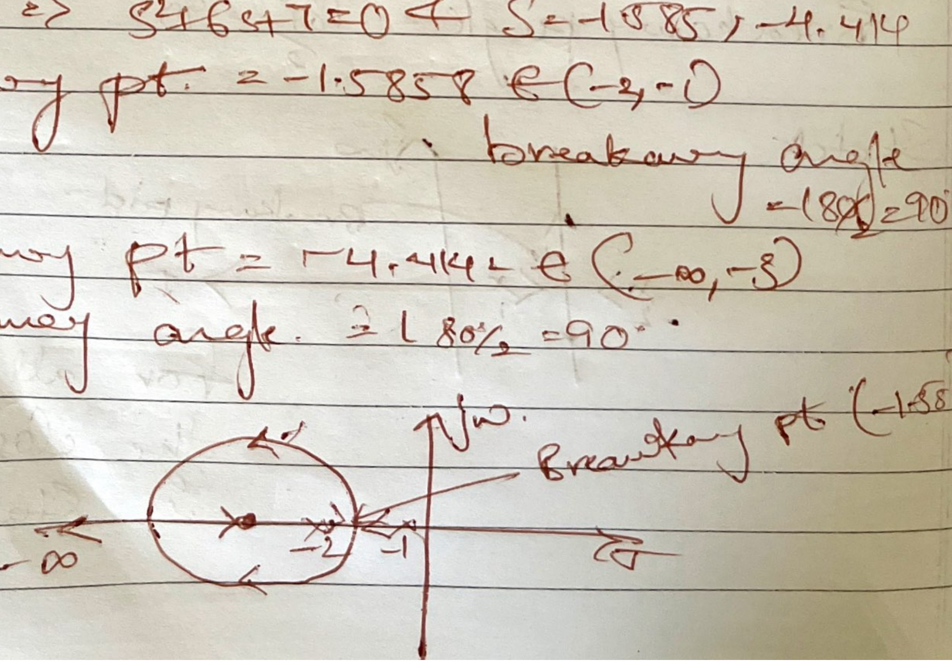
$\Rightarrow s^2+6s+7 = 0 \Rightarrow s = -1.585, -4.414$

Breakaway pt. $= -1.5858 \in (-2, -1)$

$\therefore$ breakaway angle $= 180/2 = 90°$

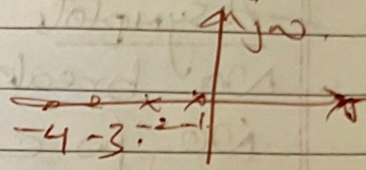
Breakaway pt $= -4.414 \in (-\infty, -3)$

Breakaway angle $= 180/2 = 90°$.



4(c) $KG(s)H(s) = \frac{K(s+3)(s+4)}{(s+1)(s+2)}$

No. of open loop poles = 2 (= P) Poles = -1, -2
No. of open loop zeros = 2 (= Z) Zeros = -3, -4.
∴ No. of Branches of root locus = max(P, Z) = 2
Real axis Segment = (-4, -3) ∪ (-2, -1)



Asymptotes

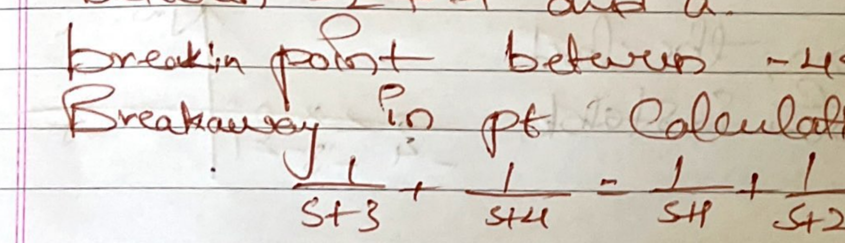
No. of asymptotes = |P - Z| = 0 → no asymptote
There will be a breakaway point between -2 & -1 and a breakin point between -4 & -3
Breakaway in pt Calculation (another method)

$$\frac{1}{s+3} + \frac{1}{s+4} = \frac{1}{s+1} + \frac{1}{s+2} \Rightarrow \frac{2s+7}{(s^2+7s+12)} = \frac{2(s+1.5)}{(s^2+3s+2)}$$

$$\Rightarrow 13s^2+25s+14 = 17s^2+45s+36$$
$$\Rightarrow 4s^2+20s+22=0$$
$$\Rightarrow s = -1.634, -3.3660$$

Breakaway pt. = -1.634 ∈ (-2, -1). Breaking angle = 90°
Breakin pt = -3.3660 ∈ (-4, -3) Breaking angle = 90°

Sketch



No jω crossing
For all K > 0,
the closed loop
System is stable

1(d) $KG(s)H(s) = \frac{K(s+2)(s+4)}{(s+1)(s+3)}$

No. of open loop poles $= 2 (=P)$ poles $= -1, -3$

No. of loop zero $= 2 (=Z)$ zeros $= -2, -4$.

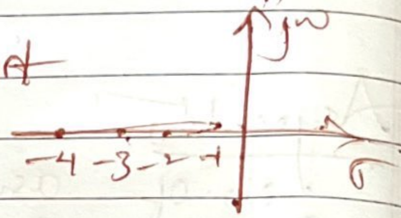
$\therefore$ No. of branches of root locus $= max(P, Z)$
$= 2$

Real axis Segment $= (-4, -3) \cup (-3, -1)$.

Asymptots $\longrightarrow$ None.

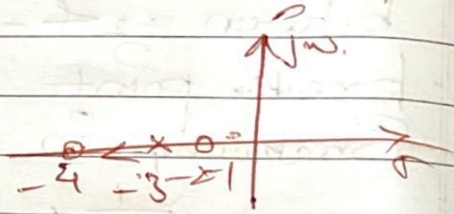
No breakaway / break in point

No $j\omega$ crossing



Sketch

For all $k > 0$, the closed loop System is Stable



1(e) $KG(s)H(s) = \frac{K}{s^2+2s+5} = \frac{K}{(s+1+2j)(s+1-2j)}$

open loop poles $= -1 \pm 2j$, $P = 2$

open loop zeros $=$ none $Z = 0$.

No. of branches of root locus $= max(P, Z)$

Real axis Segment $\longrightarrow$ none.

Asymptots

Number $= (P - Z) = 2$

$\sigma_a = \frac{(-1+2j) + (-1-2j)}{2} = -1$

$\theta_a = \frac{(2r+1)}{2}\pi$ $r = 0, 1$ $\theta_a = \pi/2, 3\pi/2$

No breakaway / breaking point

No $j\omega$ Crossing.

Angle of departure:

For $-1+2j$

$-(\theta_{-1+2j} + \theta_{-1-2j}) = 180°$

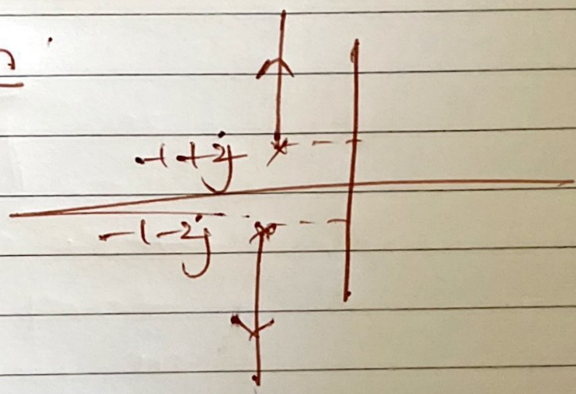
$\Rightarrow -\theta_{-1+2j} - (-1+2j+1+2j) = 180°$

$\Rightarrow -\theta_{-1+2j} - 90° = 180°$ & $\theta_{-1+2j} = -270° = 90°$

Similarly angle of departure for θ_{-1-2j}

$= -\theta_{-1+2j} = 90°$

Sketch:



For all $k>0$, the closed loop system is stable