340

340

2

-: HAND WRITTEN NOTES:-

OF

CIVIL ENGINEERING

1

-: SUBJECT:-

STRUCTURAL ANALYSIS

School: [redacted]

S. No.	Date	Title Theory of STRUCTURE	Page No.	Remarks
1	26-9-2011	Indeterminacy & stability of str.	3-34	
2	27-9-11	Influence line Diag.	41-71	
3	28-9-11	Arches	71-99	
4	30-9-11	Methods of Analysis		
5	2-10-11	(1) Strain Energy method	111-132	
6	5-10-11	(2) moment distribution mtd	133-26	
7	6-10-11	(3) Slope Deflection mtd.	211-239	
8	12-10-11	TRUSS	240-299	
9	17-10-11	Stiffness method	300-	

3

[redacted]

4

[redacted]

Structure Analysis

IES

GATE

5

8 to 10 marks

Obj

20-25 question

Conv.

50-60 Marks

Nature of Q^ns.   Easy to score

Type of subject   Conceptual

Hisstory of Teching   50

(6) Syllabus

→ Indeterminacy & Stability → obj

→ Influence line Dia & Effect of Rolling load → obj + conv

→ Arches → obj

→ Method of Analysis → Strain Energy
obj + conv.
→ Moment dist method
→ Slop Deflection Method
→ Matrix Method

→ Trusses → Determinate, Indeterminate } obj + conv.

Ref. Book

(1) Theory of St. — by S. Ramamurtham → Strain Ene..., Moment dist^n, Trusses

(2) Str. Analysis (L.S. Negi & Jandig) → Slop Defl^n, Strain Energy, ILD

(3) Str. Analysis (SSt) (Gupta & Pandit) → Indetermancy & stability, Matrix method

(4) Structure Analysis → R.C. Hibbler → Basic concept
2 - Truss

Indetermancy & Stability of Structure

(7)

Stability
- External → Related to the Support Condition
- Internal → Related to the Shap of Str.

External Stability :-

→ Large displacment of the entrie structure at the supports and joint is not permitted. However small elastic diformance may occur at free ends. To Provement Rigid body displacement there should be enough Reaction at the supports. At there arragement should be adquete.

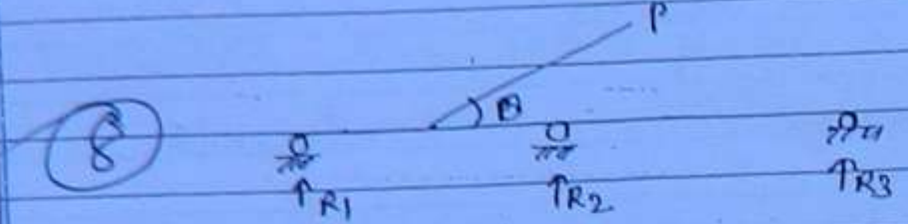
for plane structure (2-D) there should be min. of 3 independed Reactions at the support which should be

① Non Parrellel
② Non - Concernent
③ Non Trivial

if reaction is small which may permit large displacent that it is calledtrivall

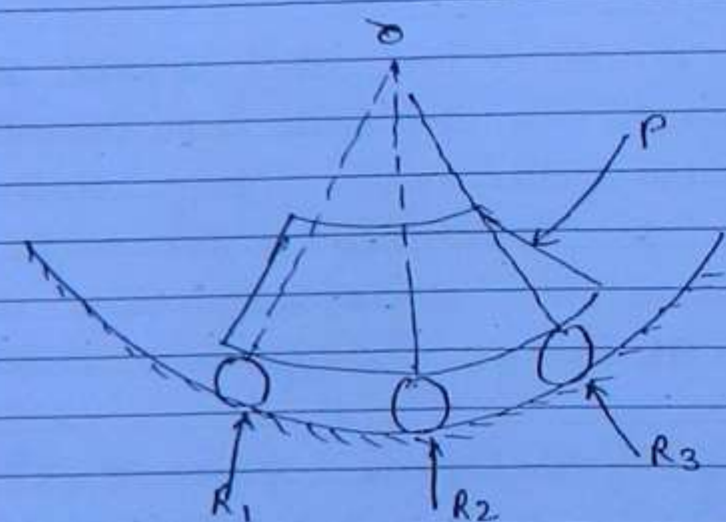
eg. A spring support with small stifness

Note → All the support reaction are (‖) than Linear unstability will occur Reaction are Parllel.



Linear unstabilty

→ If all reaction are concurent. than angular unStability will occur.



Angular | Rotational unstability

→ For stability of 2D body (Plane is x-y is loading Plane) there should be loading plane in
(1) x-direction ($\Delta x = 0$)
(2) y-direction ($\Delta y = 0$)
(3) Rotational about z-axis ($\theta_z = 0$)

To pravement above displacement following 3 condition of stable equilibrium should be staufied.

(1) $\Sigma F_x = 0$

(2) $\Sigma F_y = 0$

(3) $\Sigma M_z = 0$

9

For 3D structure (Space structure)

There should be no-displacement There should no-Rigid displacement in ffollowing dir^n.

$(\Delta x = 0 \quad \Delta y = 0 \quad \Delta z = 0 \quad \theta x = 0 \quad \theta y = 0 \quad \theta z = 0)$

In order to prevent rigid body displacement for static equlurum of space strucltere following 6-condition should be statified

$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \qquad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$

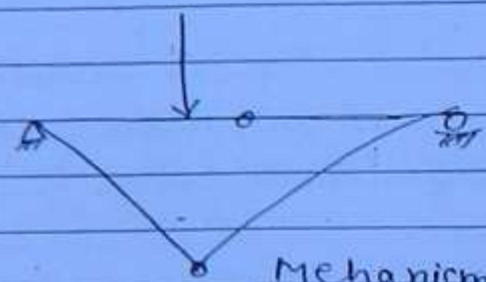
Internal Stability :-

No-Part of the str. should move Relative to the other part show as ~~to~~ other ~~pressure~~. ~~save of the~~ to presure shape of "Geometric" of Structure however small elastic deformation may occur to prerarence geometric enough no-of member & appropiate arragment is requried. There should be no formation of condition of mechanisum.

Machinesam Means Rotation Collapsle which may be complete str. or partial str.

Mechinesum may occur 3- coliner In a row there Rotation is permitted.

10



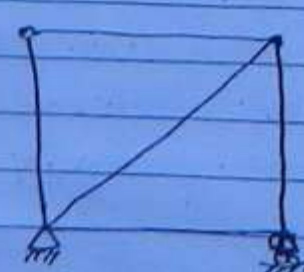
Mehanism/Collapse

Partial Collapse

Internal stability also called Geometric stability.

In Truss Geometric unstability occur due to ~~defence~~. defficiency ~~may~~ off member and inadequate arragement.

Unstable (internally)

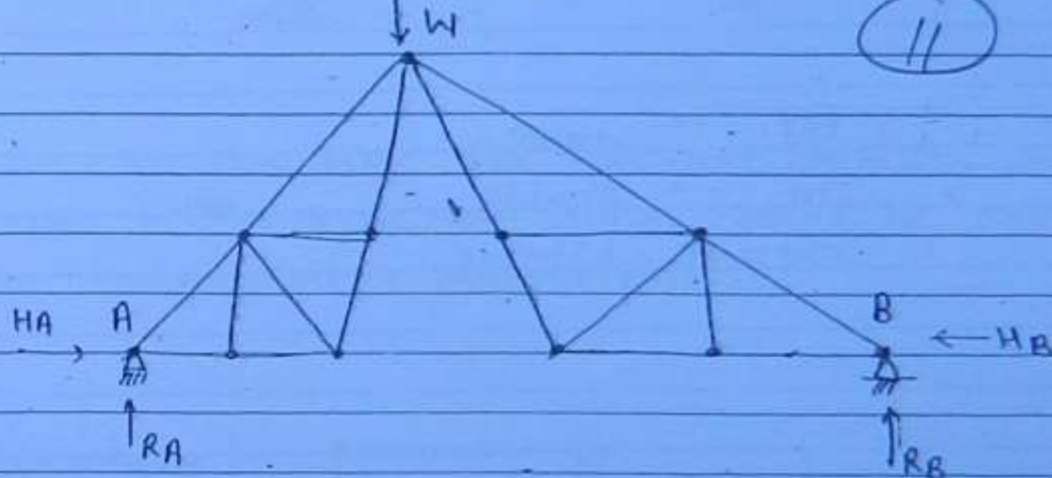


Stable (internally)

over-all stability:-

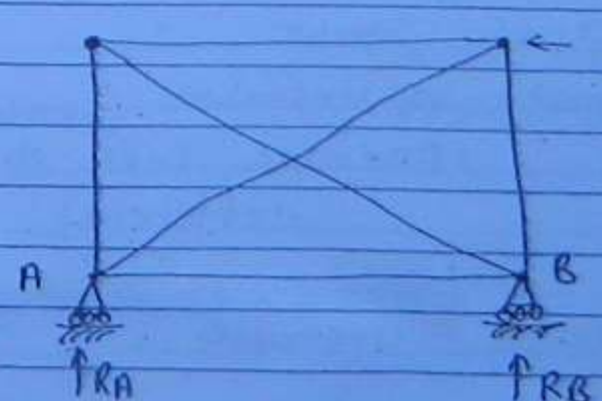
For overall stability external stability is complesery. in some cases over all stability may occur when structure is externaly is over stiff and Internaly is unstable ———— that is extra. External reaction may prevent Internal deformation.

(11)



No. of External Reactions = 4

Internaly it is defficent to 1st order } but overall
& Externally It is over stiff. to 1st order } it stable



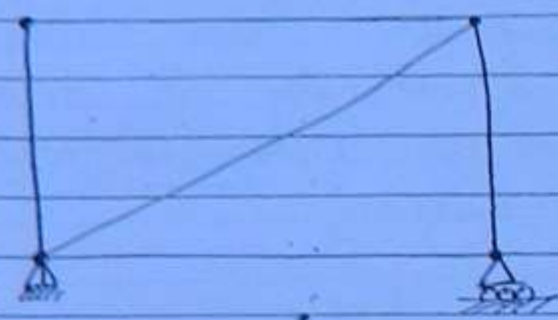
Externally defficient to 1st order but internally oven stiff to 1st order

↓

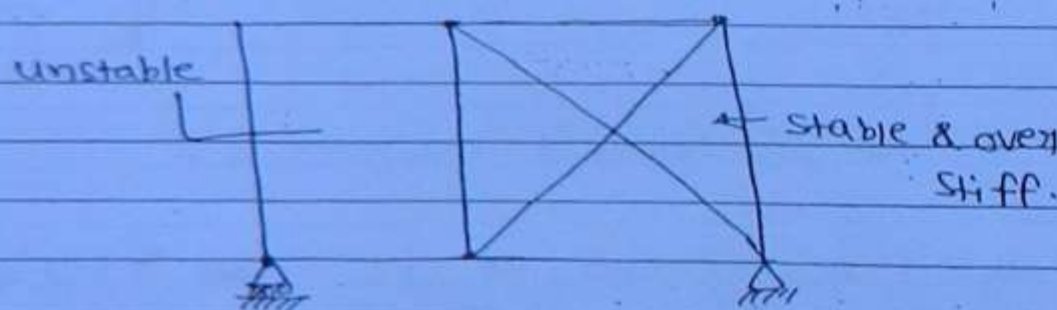
but over all is unstable

→ It is disirable to have internal and external stability both for overall stability.

(12)

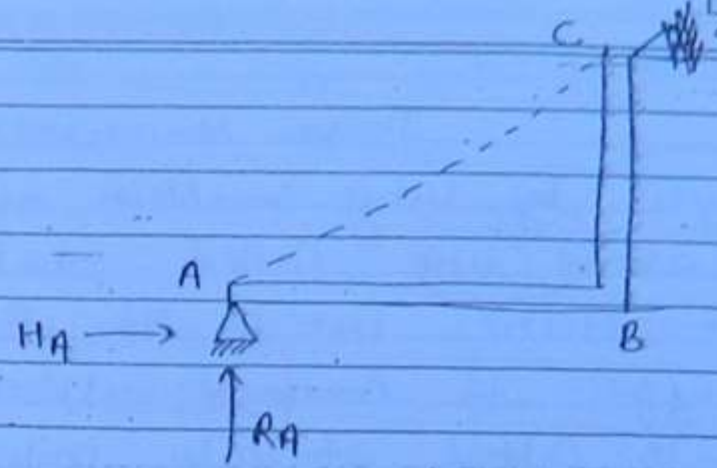


Ext. → stable
Int → stable
overall → stable



External – Stable
Internal – is unstable
(Because Left Panel is deficient)

Overall – unstable



13

unstable Because all reaction all concurrent

e →

In - 3D structure for external stability there should be ^{min} 6 - reaction which should be Non-Coplanar, Non-Concurrent, Non-trivial.

Indeterminacy :-

- Static (Ds)
  - External (Dse) (Related to support condition)
  - Internal (Dsi) (Shape and Member arrangement)
- Kinematic (Dk) (~~free of~~ Related to the degree of freedom and joints)

$$\left[D_s = D_{se} + D_{si} \right]$$

Static indeterminacy :-

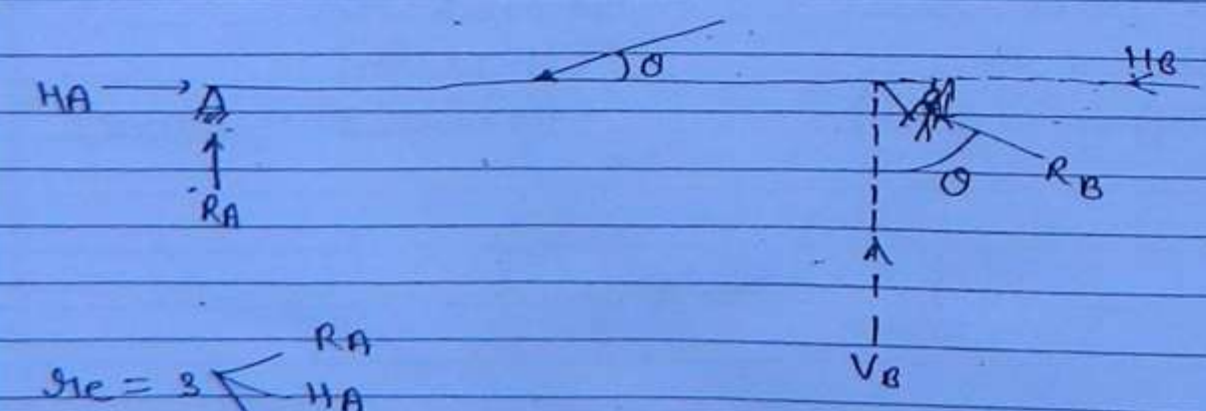
Those structures which can be analaysis by using condition of satic equb. alone are called staticaly derminate structure. but if No. of equcation are not enough to compute all the Reaction that is called staicaly indeterminate hyperstatic / Redudency structure.

14

① External Static indetermancy -

It is related to the support condition. If all the support reaction can be compute by using condition of static equilibrium alone than structure is statically determinate externaly.

Let r_e is total No. of external support Reaction of all joint (No. of independent Reaction)



$r_e = 3 \begin{cases} R_A \\ H_A \\ R_B \end{cases}$

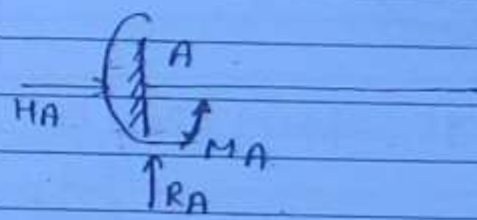
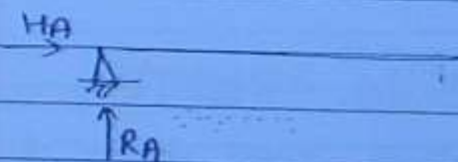
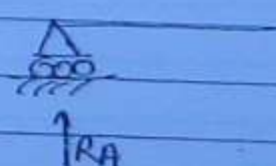
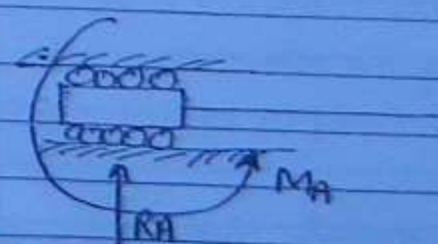
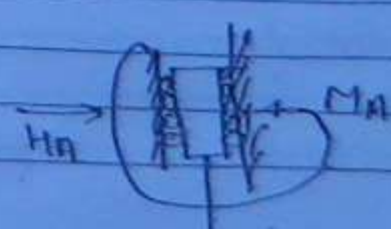
$V_B = R_B \cos\theta$

$H_B = R_B \sin\theta$

$\left(\frac{H_B}{V_B} = \tan\theta\right) \rightarrow$ depend.

* $D_{se} = r_e - 3$ — — —— , For 2 D Structure
$= r_e - 6$ — — —— For 3D Structure

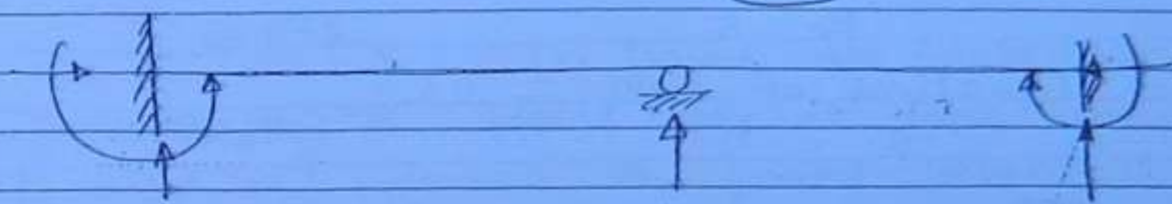
15

Type of support	No. of Reaction
2D - Plane	
(1) Fix support	 $r_e = 3$
(2) Hinge support	 $r_e = 2$
(3) Roller support	 $r_e = 1$
(4) Guided Roller support	 $r_e = 2$
	 $r_e = 2$

Types of support	No. of Reaction
3D Case	
① Fix support / Builtin support	r_e : R_x, R_y, R_z, M_x, M_y, M_z
6	
② Hinge support / Ball & Soket joint	$r_e = 3$: R_x, R_y, R_z
③ Roller support	$r_e = 1$ [Only Normal to Plane]

Ex:- Find external static indeterminacy for the Structure Shown in figure.

17



Total $r_e = 3+1+3 = 7$

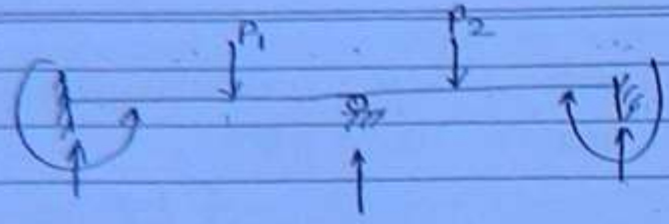
Total equ^m conditim $= 3$

$D_{se} = r_e - 3 = 7 - \cancel{4}3 = 4$

when loading is not mention then general loading is Consider which may have a horizontal and vertical Component both there-fore at the Support horizonal & vertical & moment rem all will be avaiable.

If only vertical load is Present than in frame, Arches and Truss a hence vertical component of rem will be present. also moment depending upon support condition.

But in beams due to vertical loading alone horizonatal reactions will not be Present there fore. to determine ~~d~~ Dse · horizontal reaction may be Ingroed and equcation for horizonal forces may be ingreed.

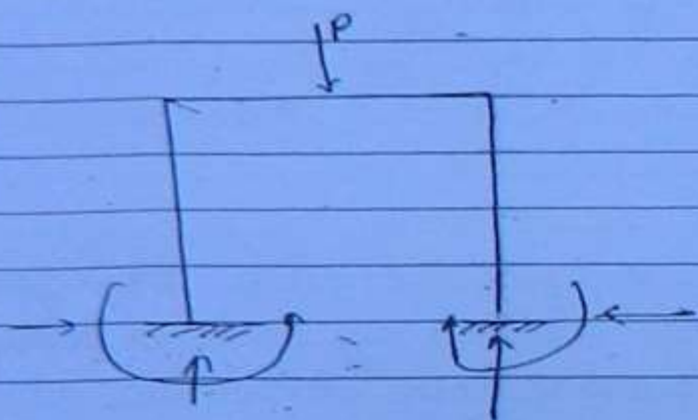


$r_e = 5$

Equations of Eq^m = 2 $\begin{cases} \Sigma F_y = 0 \\ \Sigma M_2 = 0 \end{cases}$

18 ~~re~~ $D_{se} = r_e - 2$

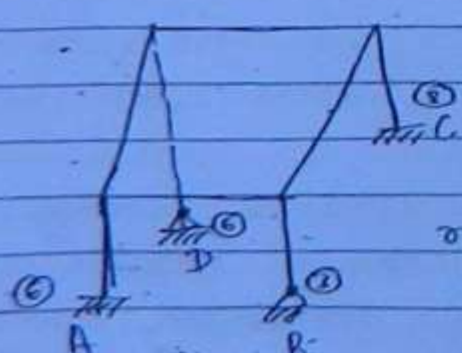
$\Rightarrow 5 - 2 = 3$



$r_e = 3+3 = 6$

$D_{se} = 6-3 = 3$

eg →



$r_e = 6+3+6+3 = 18$

$D_{se} = r_e - 6$

$\Rightarrow 18 - 6 = 12$

Internal static indetermancy :- (D_{si})

Case - A   2D Truss / Plane Truss   (19)

In Truss all joints are Hinge, loading is applied only at joints and self weight of member is ignored. Hence all member carries only axial forces there fore each member has only one internal reaction that is axial Force.

If there are m members than total no. of internal Reactions. let there are J no. of joints. at each joint there are 2 equation are equilibrium. $\left(\begin{array}{l}\Sigma F_x = 0 \\ \Sigma F_y = 0\end{array}\right)$

Total no. of equilibrium equation at all joint $\Rightarrow 2J$

Out of which, 3 equation are utilised to determine external support reaction Hence Net available equation two find Internal Forces $= (2J-3)$

D_{si} = Total No. of Internal Reaction − Total available equation for int. reaction.

$$\boxed{D_{si} = m - (2J-3)}$$

Truss is

if $D_{si} = 0$ {internally determinate}

$D_{si} > 0$ Truss is indeterminate | Redundant | over stiff

$D_{si} < 0$ internally unstable | Deficient

20

Case-2 3-D Truss | Space Truss →

→ In each member of Truss there is only one Internal force (axial force) but at each joint there are 3 equations ($\Sigma f_x = 0$, $\Sigma f_y = 0$ and $\Sigma f_z = 0$) Hence total equ^n of all joint 3J (3J)

out of which 6 equation are use for external support reaction Hence Net available equation to find internal reaction = (3J - 6)

$$D_{si} = m - (3J - 6)$$

If $D_{si} = 0$ - Truss is Internally determinate

Case-2. Rigid Frame (2-D, 3-D) →

In Rigid frames ~~internally~~ internal indermancy will not exist. If structure has open configuration like a tree. beams are ~~allow~~ always internaly determinate. To check Internal indeterminacy following thumb rule can be appled.

(21)

Rule-1 If structure is internaly determinate than it is not possible to provided a cut any where on the structure without spliting the structure in a two part.

eg.

Tree /open ~~space~~ structure

If structure is statically determinate it is impossible to return back at any point without ——— the path It mean in internmaly determinate structure there will be no. close loops if

internal hinges, links, shear joint extra

2-D Rigid frame →

in each member 2-D rigid member there will be 3 internal reaction (r_x, r_y and z) or axial force, Shear force and Bending moment.

22

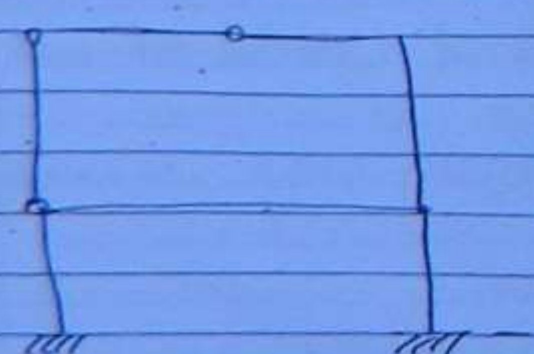
if there is no close loop all the above three internal reaction can be found by provideding it cut any section. at Stablice free body equilbriym for either left of Right Portion.

$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_z = 0$

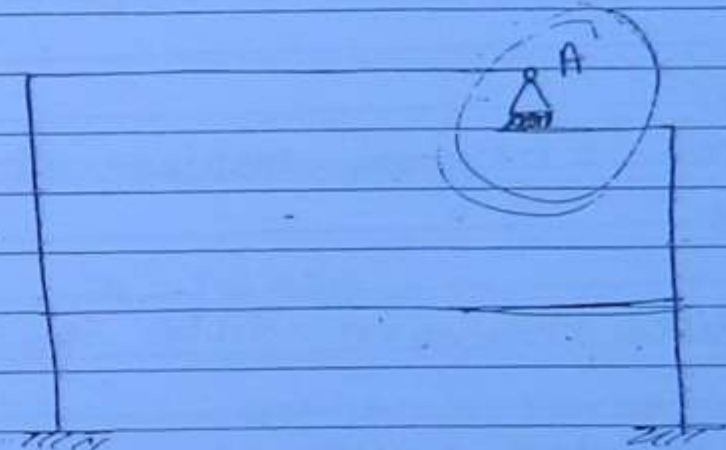
but if there are close loop than each close loop will have three Internal indetermancy Hence if No. of close loops are c than total degree of internal indetermancy $D_{si} = 3c$.

$$\boxed{D_{si} = m - (3J - 6)}$$

(c No. of close loops / No. of cuts required to convert close structure into an open structure)

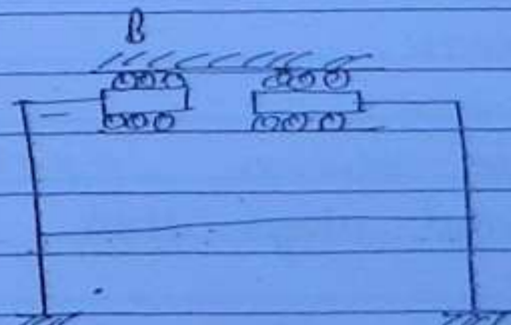


Note → If all joints are not rigid that is sum of the joint are hybride due to presents of internal hinge, internal rollar, shear rollar than some ~~mole~~ of the internal reaction are relased. No. of internal reaction released upon. Type of joint and No. of member metting at the joint.

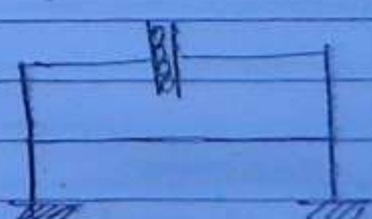


(23)

No. of int. Reaction replaced at A ⇒ $(r_r) = 2$ → A.F. = 0, B.M. = 0

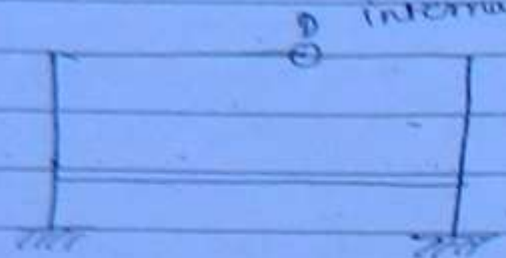


r_r at B = 1
i.e. A.F. at B = 0



r_r at C = 1
(S.F. at C = 0)

internal Hings



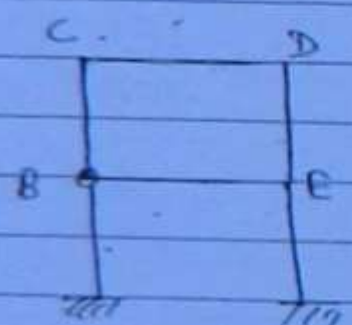
24 at D. $B.M = 0$

$r_r = 1$

If more than 2 member metting at internal hinge.

No. of released reacetion will be $\Rightarrow r_r = (m' - 1)$

Imp m' = No. of member metting at in Hings.



$r_r = (m' - 1)$

$r_{r_B} = (3-1) = 2$

if 3D structure is internal Hings than r_r is.

$$r_r = 3(m' - 1)$$

main formula → $D_{SI} = 3C - r_r$

r_r = Total No. of Internal Reaction released at Hybrid Joint.

3D - Rigid Frame :-

At each close loops there are 6 internal if No. of close loop then total No. of internal indetermancy $DSi = 6C$ and if some of the joints are Hybrid $\boxed{DSi = 6C - r_r}$. for 3D Internal Hinge. $r_r = 3(m' - 1)$

m' = No. of members meeting at Hybrid joints.

25

overall static indetermancy :-

1st method $D_s = D_{se} + D_{si}$

$D_s = 0$ Structure is Determinate Provided are Non Parallel, Non concurrent, Nontrival and There no condition of mechanism

$D_s > 0$ Stable & indeterminate / overstiff & Desirable

indeterminate str. are more economical than Determinate because thinner Section is required.

$D_s < 0$ unstable structure.

Method 2 (2-D Truss) :-

① D_s = Total reaction (Internal reaction + external reaction) - Total Eqn equation.

$$D_s = m + r_e - 2j$$

② 3D - Truss :-

D6

$$D_s = m + r_e - 3J$$

③ 2D rigid Frame/Beams :-

D_s = Total reaction (internal+external) − Total equation at All Joints

$$D_s = (3m + r_e - 3J)$$

when all joints are rigid.

$$D_s = (3m + r_e - 3J) - r_r$$

when of the joints are Hybrid

④ 3D Rigid frames/ Beams →

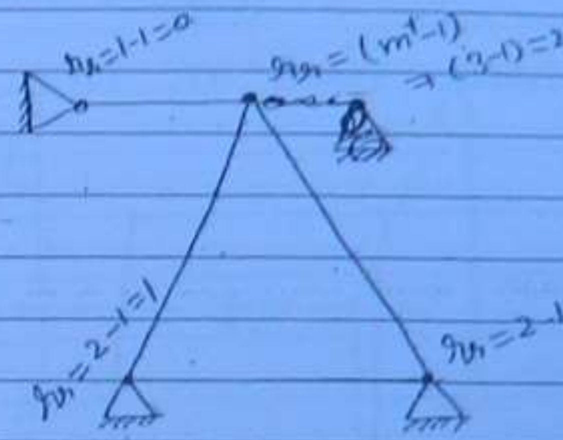
$D_s = (6m + r_e - 6J)$ -- when all joint are rigid

$$D_s = (6m + r_e - 6J) - r_r$$

when some joint are Hybrid.

r_r ⇒ Total No. of Released reaction.

Q. determine Ds for Plane truss shown in figure.



21

→ I[st] method →

$$D_{se} = r_e - 3$$
$$\Rightarrow (2+2+2) - 3$$
$$D_{se} \Rightarrow 6 - 3 = 3$$

$$D_{si} = m - (2J - 3)$$
$$\Rightarrow 4 - (2\times4 - 3)$$
$$\Rightarrow 4 - 5 \Rightarrow -1$$

$$D_s = D_{se} + D_{si}$$
$$\Rightarrow 3 - 1 = 2$$

2nd method :-

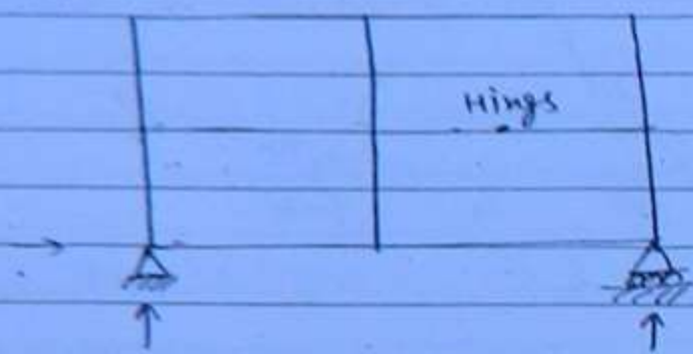
$$D_s = m + r_e - 2j$$
$$\Rightarrow 4 + 6 - 2\times4$$
$$\Rightarrow 10 - 8 = 2$$

3rd method 2D

Case

28

Q → For 2D frame shown in fig. find D_s



$c = 4$

$r_r = (2-1) = 1$

1st method :-

$r_e = 2+1 = 3$

$D_{se} = r_e - 3$

$\Rightarrow 3 - 3 = 0$

$D_{si} = 3c - r_r \Rightarrow 3\times4 - 1 \Rightarrow 12 - 1 = 11$

$D_s = D_{se} + D_{si}$

$\Rightarrow 0 + 11 \Rightarrow 11$

2nd Method :-

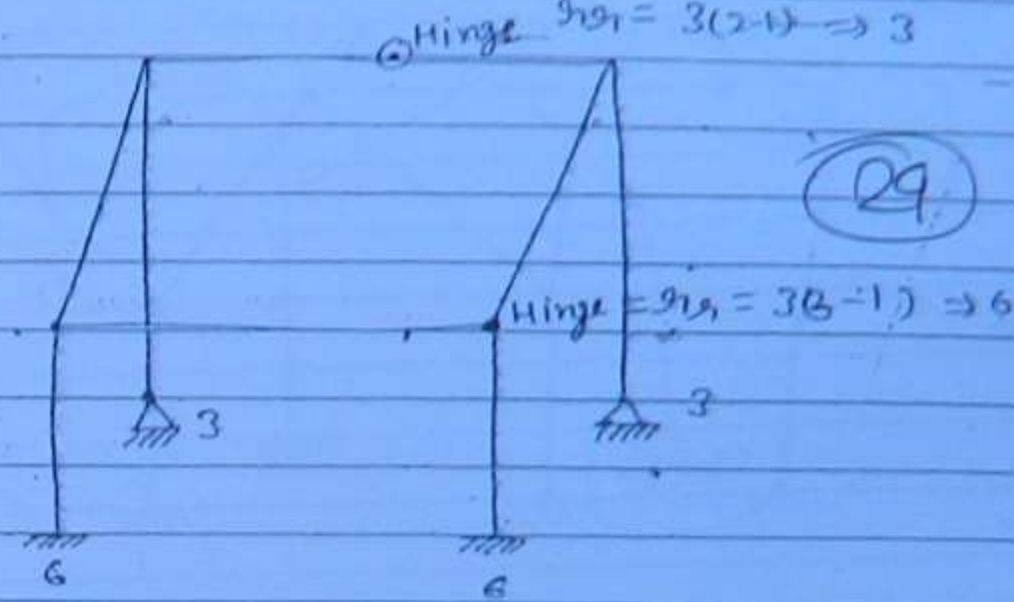
$D_s = 3m + r_e - 3J - r_r$

$\Rightarrow 3\times13 + 3 - 3\times10 - 1$

$\Rightarrow 39 + 3 - 30 - 1$

$\Rightarrow 11$

Q. For 2D rigid frame shown in fig. find D_s



29

Method -1

$$D_{se} = r_e - 6$$
$$= 18 - 6 = 12$$

$$D_{si} = 6C - r_r$$
$$\Rightarrow 6 \times 1 - (3+6)$$
$$\Rightarrow 6 - 9 = -3$$

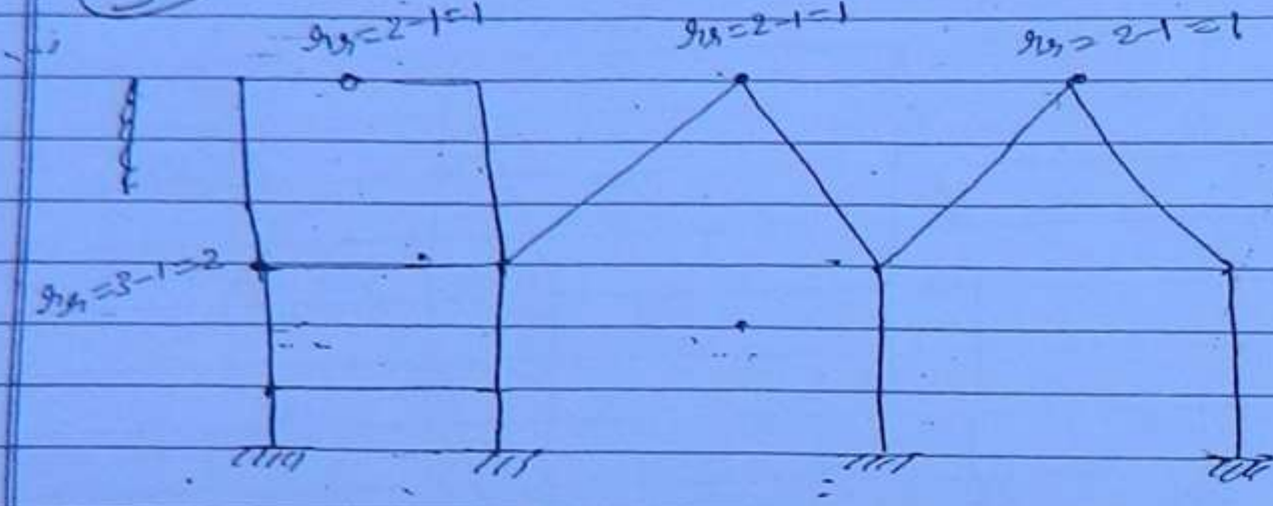
$$D_s = D_{se} + D_{si} \Rightarrow 12 - 3 \Rightarrow 9$$

2nd Method

$$D_s = 6m + r_e - 6J - r_r$$
$$\Rightarrow 6 \times 9 + 18 - 6 \times 9 - 9$$
$$\Rightarrow 9$$

Q. For 2D rigid fram find Ds?

30



Ist method :-

$r_e = 3+3+3+3 = 12$

$D_{se} = r_e - 3$

$\Rightarrow 12 - 3 = 9$

$D_{si} = 3C - r_r$

$\Rightarrow 3 \times 2 - 5 \Rightarrow 1$

$D_s \Rightarrow D_{se} + D_{si} \Rightarrow 9 + 1 \Rightarrow 10$

2nd method $\rightarrow$

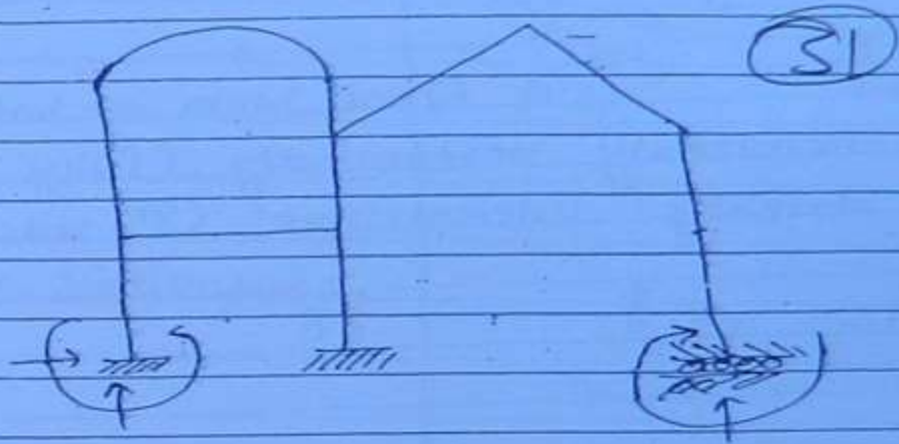
$D_s = 3m + r_e - 3J - r_r$

$\Rightarrow 3 \times 16 + 12 - 3 \times 15 - 5$

$\Rightarrow 48 + 12 - 45 - 5$

$\Rightarrow 10$

Q. Find Ds Shown in Figure.



Ist method →

$re = 3+1+2 = 6$

$Dse = re - 3 \Rightarrow 6-3 = 3$

$Dsi = 3C - rr$

$\Rightarrow 3 \times 1 - 0 \Rightarrow 3$

$Ds = Dse + Dsi \Rightarrow 3+3 \Rightarrow 6$

2nd Method →

$Ds = 3m + re - 3J$

$\Rightarrow 3 \times 8 + 6 - 3 \times 8$

$\Rightarrow 18 + 6 - 24$

$\Rightarrow 6$

Kinematic indetermency :- "Kinematic Indetermancy (Dk) is refers to the total No. of avaiable degree of freedom at all joint"

or

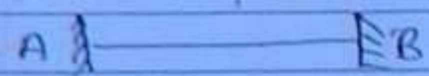
Kinematic indetermency (Dk)

Dk total No. of unrestained displacement Component at all joints.

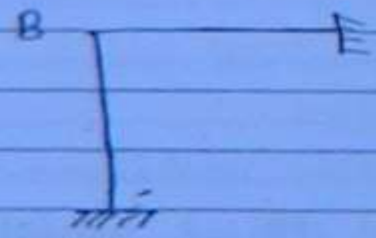
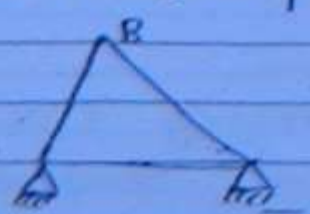
A fully locked structure all joint kinematically determinate because it has no degree of freedom at any joints.

(32)

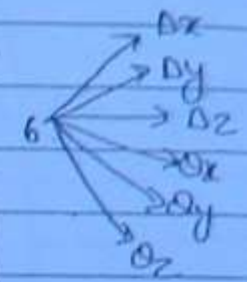
A fixed beam at both end ~~B~~ is kinematicaly determinaty (fully locked) but statically indeterminate (3rd order)



Note → The displacement at joints are elastic displacement. if a member ~~is~~ in ~~exten~~ inextinable. ~~than~~ in any direction, then displacement in that dirn will not be avaiable at the joint.

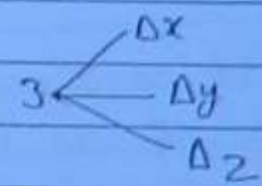
(Restraind Joint) Types of Joint	No. of displacement
① 2D rigid joint	3 < Δx, Δy, θ_z at joint B
	
(2) 2D Truss joint / Pin joint	at joint B
	2 < Δx, Δy

(3) 3D Rigid Joint



33

(4) 3D Truss Joint/Pin joint :-



Kinematic degree of determinacy :-

(1) For 2D plane Truss :-

$$D_k = 2J - r_e$$

At each ~~jo~~ joints there may ~~b~~ be two displacement $\Delta x, \Delta y$ out of which some of the joints are supported in the direction of support reaction displacement are not permitted Hence net avaiable displace = avaiable degree of freedom

$$2J - r_e$$

Page

For 3D Truss:-

$$D_K = 3J - r_e$$

(34)

2D Rigid Frame:-

$$D_K = 3J - r_e$$

Note → If some of the joint are hybrid. Some of the internal reaction are released degree of freedom will increased Hence D_K will be

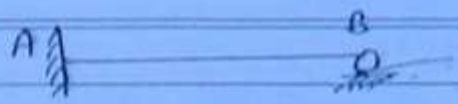
$$\boxed{D_K = 3j - r_e + r_r}$$

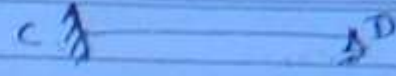
r_r = released re^n at hybrid joint

if some of member of the member are axial rigid than due to axialy rigidity linear axial displacement in the direction of member at some of the joint will not be available, Hence degree of freedome will get Redused

let m → no. of axial displacement are not available, at joint due to axial Ridigid

$$D_K = 3J - r_e + r_r - m$$





$D_K = 3J - r_e$

$\Rightarrow 3 \times 2 - 4 = 2$ — θ_B, Δ_{Bx}

$D_K = 3J - r_e$

$\Rightarrow 3 \times 2 - 5 \Rightarrow 1 \rightarrow \theta_D$

If AB is axially Rigid

$\Delta_{Bx} = 0$

$D_K = 3J - r_e - m$

$\Rightarrow 3 \times 2 - 4 - 1$

$\Rightarrow 1 \rightarrow \theta_B$

If CD is Axially Rigid than also

$D_K = 1 \rightarrow (\theta_D)$

Because in this case Δ_x is not available Hence D_K will remain 1 inspite ~~angle~~ member is axily rigid

3D Rigid frame

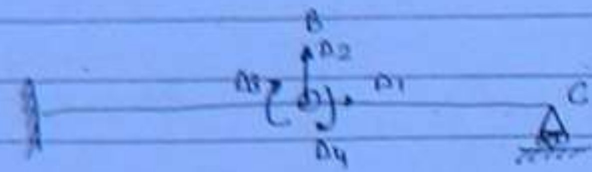
$D_K = 6J - r_e$ ------ when all joint are Rigid

$D_K \Rightarrow 6J - r_e + r_r$ ---- Some of the joints are hybrids.

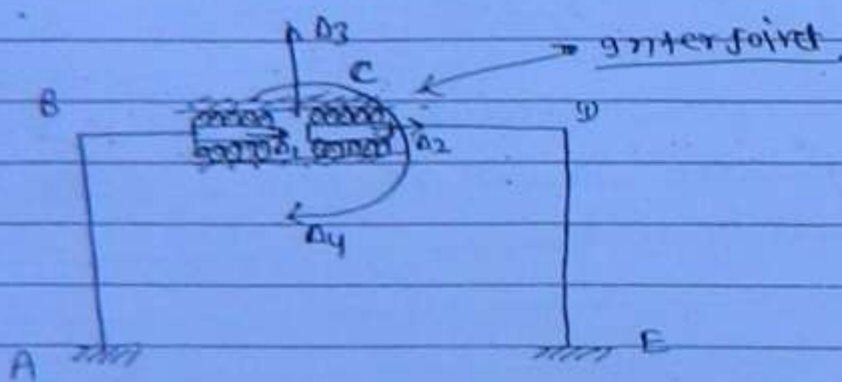
$D_K = 6J - r_e + r_r - m$ if some of the members are axially Rigid

m = No. of Axial displacement prevent due to axial rigidity.

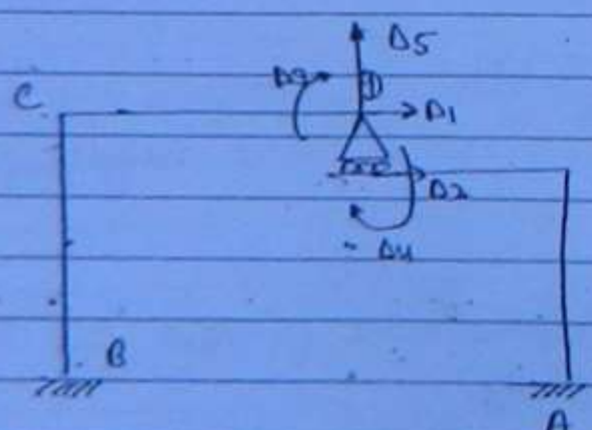
degree of freedom at hybrids joint:-



Δ_1 = Axial displacement
Δ_2 = Shear displacement
$\Delta_3 = \theta_{B_1}$ for BA at B
$\Delta_4 = \theta_{B_2}$ for BC at B

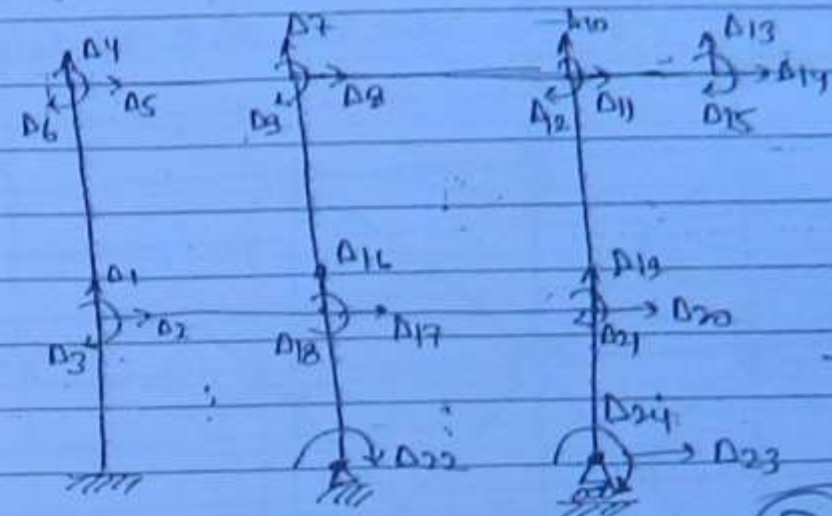


Δ_1 & Δ_2 are Axial displacement
Δ_3 = Shear displacement
Δ_4 = θ Angle displacement



Δ_1 & Δ_2 = Axial displacement

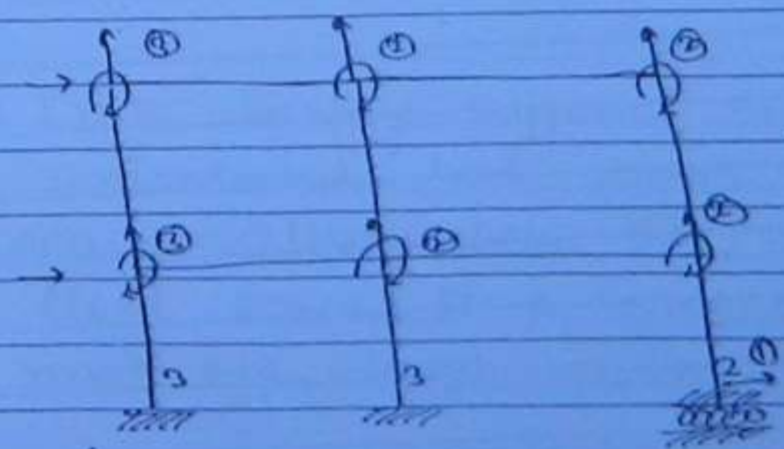
Q. Find degree of kinematic Indetermancy & represent all displacement at joints.



$D_K \Rightarrow 3 \cdot J - r_e$

$3 \times 10 - 6 = 24$

Q. For the rigid frame shown in figure find D_K of indetermancy assuming beams as axial Rigid.



$r_e = 3+3+2 = 8$

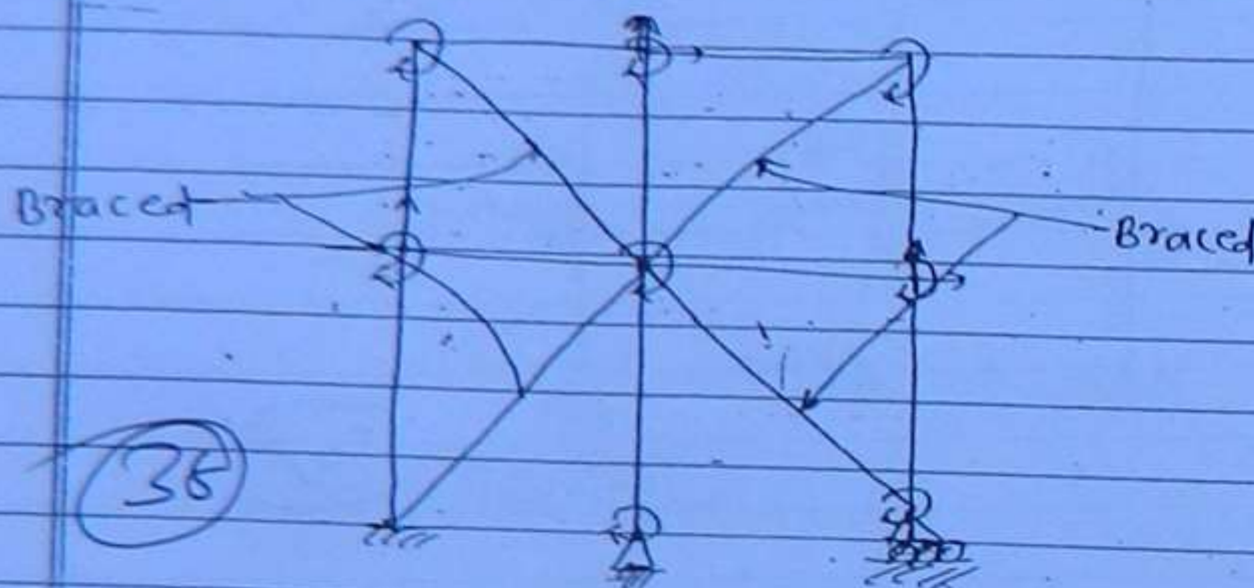
$D_K = 3J - r_e + \cancel{r_r}^{0} - m$

$\Rightarrow 3 \times 9 - 8 - 4$

$D_K \Rightarrow 27 - 12 = 15$

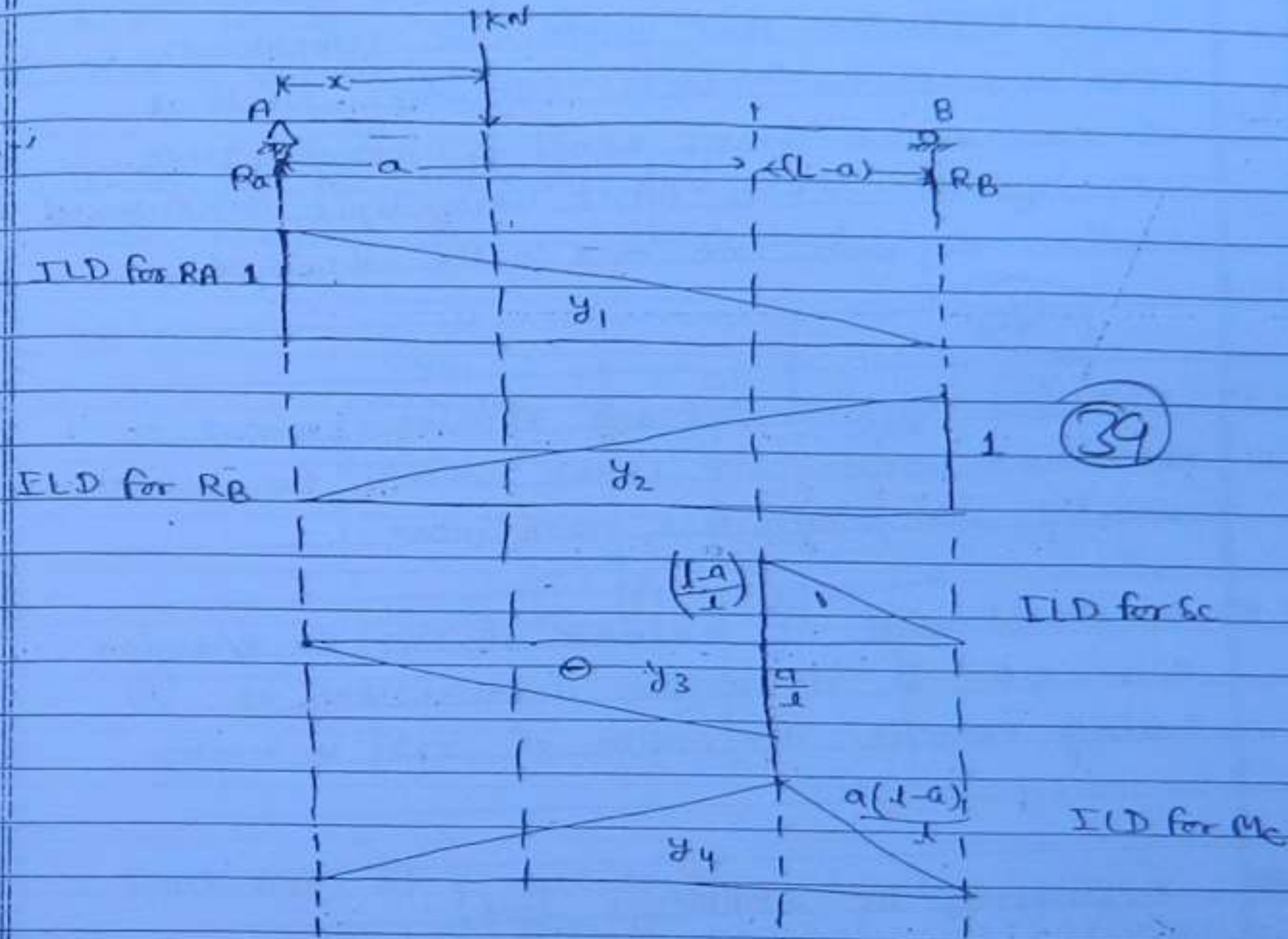
Only Beam are Axial Rigid.

Note In a elastic fram is connected by rigid Bracing than at all the braced joint linear displacement will not be permitted ($\Delta x=0$, $\Delta y=0$) member are elastic there are rigid raced.



38

INfluence Line Diagramme



Note:- ① In a simply supported Beam or girder if a concentrated load moves so than max^m. B.M. occurs always below the Position of load. And In a fixed Beam If a Concentrated load moves the max^m B.M. always occurs at the support

② The distance b/w two adjustant Point of contraflexure is called Focal length

③ The Porsion of a Beam or girder in which shear force is constant is known as shear span.

"Influence line Diagramme represent variations of the stress function such as reaction, shear force, Bending moment, slope or deflection at a point when unit concentrated load moves from one end to the other end"

40

Note ① ILD for reaction, S.F and Bending moment in determinate Beam is Linear whereas for indeterminate ~~the~~ Beam it is non linear.

② The Diamension of ordinate of ILD for Reaction and S.F. is dimension less where as Bending moment dimension of ILD is meter.

③ Application of Influence line :-

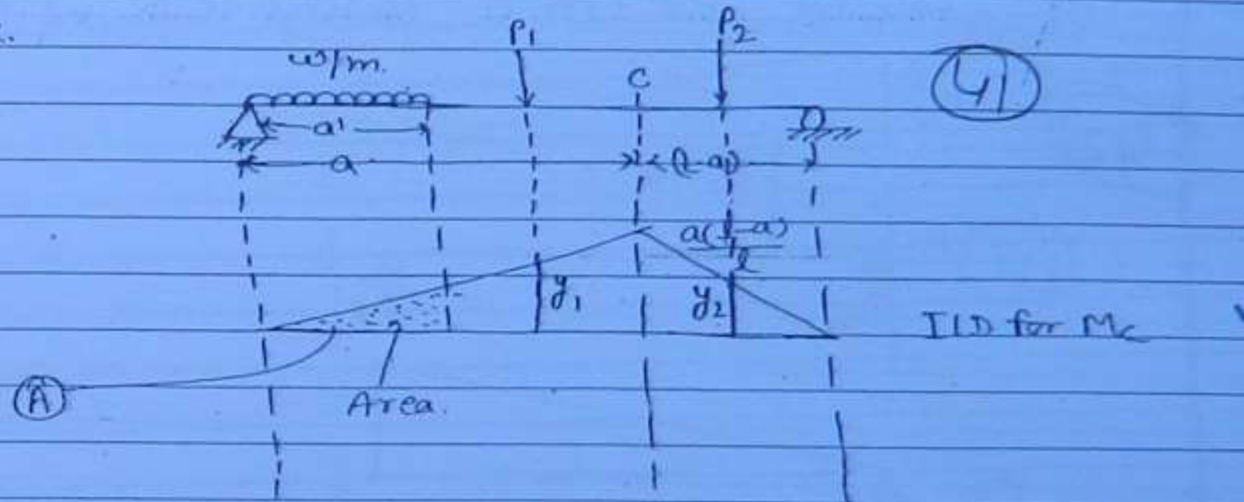
① ILD Can be use to find Position of live load which will Produced max. ~~volume~~ Value of a stress function. (load should place placed at that Point where ordinate of ILD is max)

Max. (-ve) SF at C will occur when load is placed just ~~of~~ to the left of C.

(2) ILD can be to used to study the effect of moving load on the C structure.

(3) ILD can be use to find total value of a stress function for a given load system in.

Ex.



Total B.M. at C due to given loading

$\Rightarrow$ (w × Area of ILD below udl) + $P_1 y_1 + P_2 y_2$

$$\boxed{M_C \Rightarrow w \times A + P_1 y_1 + P_2 y_2}$$

Muller Breslau Principle

(Application for determinate & Indeterminate Both)

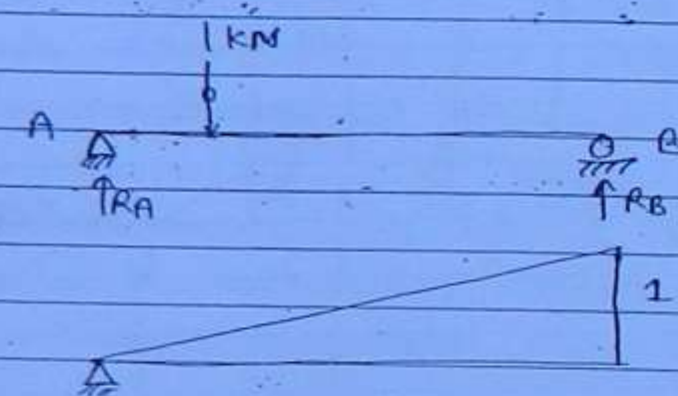
It gives qualitatives Influence line dia. for a stress function (reactions, SF & B.M.)

"The ILD for any stress function in a structure is represented by its deflected shape obtained by removeing the restrained offered by that stress function a introducing a directly related generalised displacement in the dir^n that stress function"

"Influence line dia. any stress function can be obtained by given unit displacement in dirn of that stress function and removing that stress function the resultant shape of beam ILD"

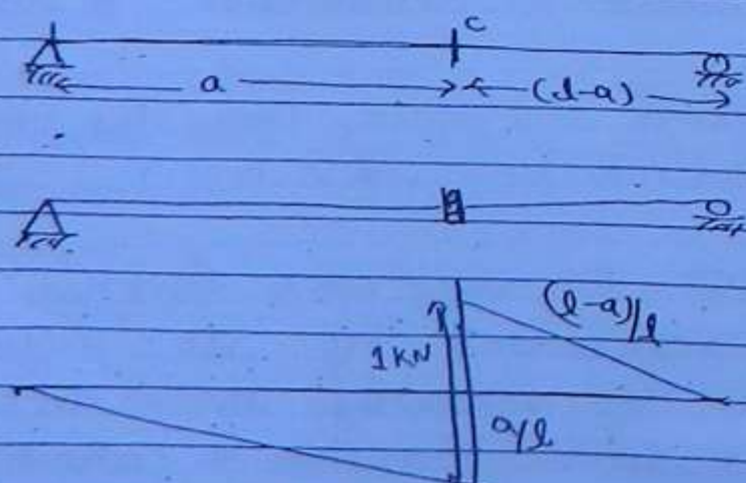
eg. ILD for R_B in a simply supported Beam.

42

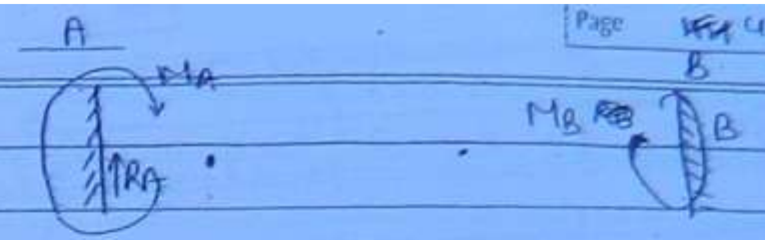


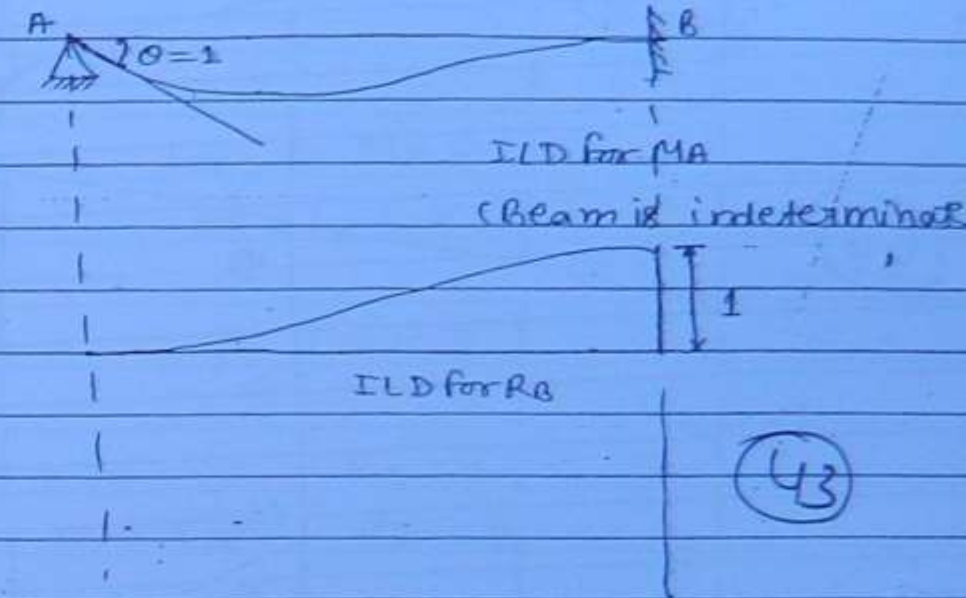
ILD for R_B

ILD for R_B can be obtained by removing R_B and giving unit displacement of B in the direction of R_B. The resultant deflection shape of beam will be ILD.



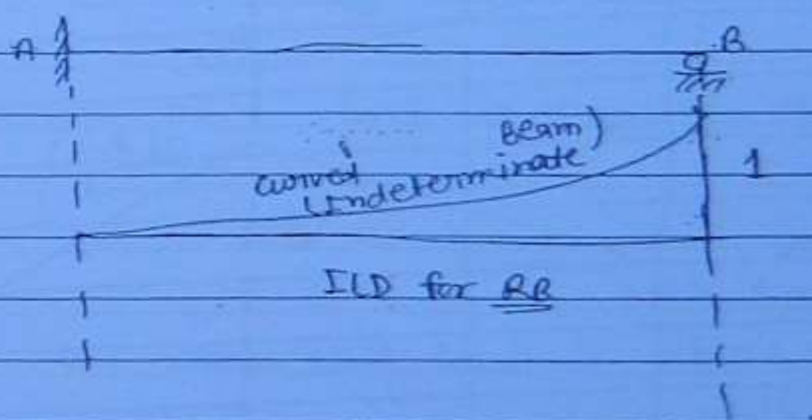
eq.1





43

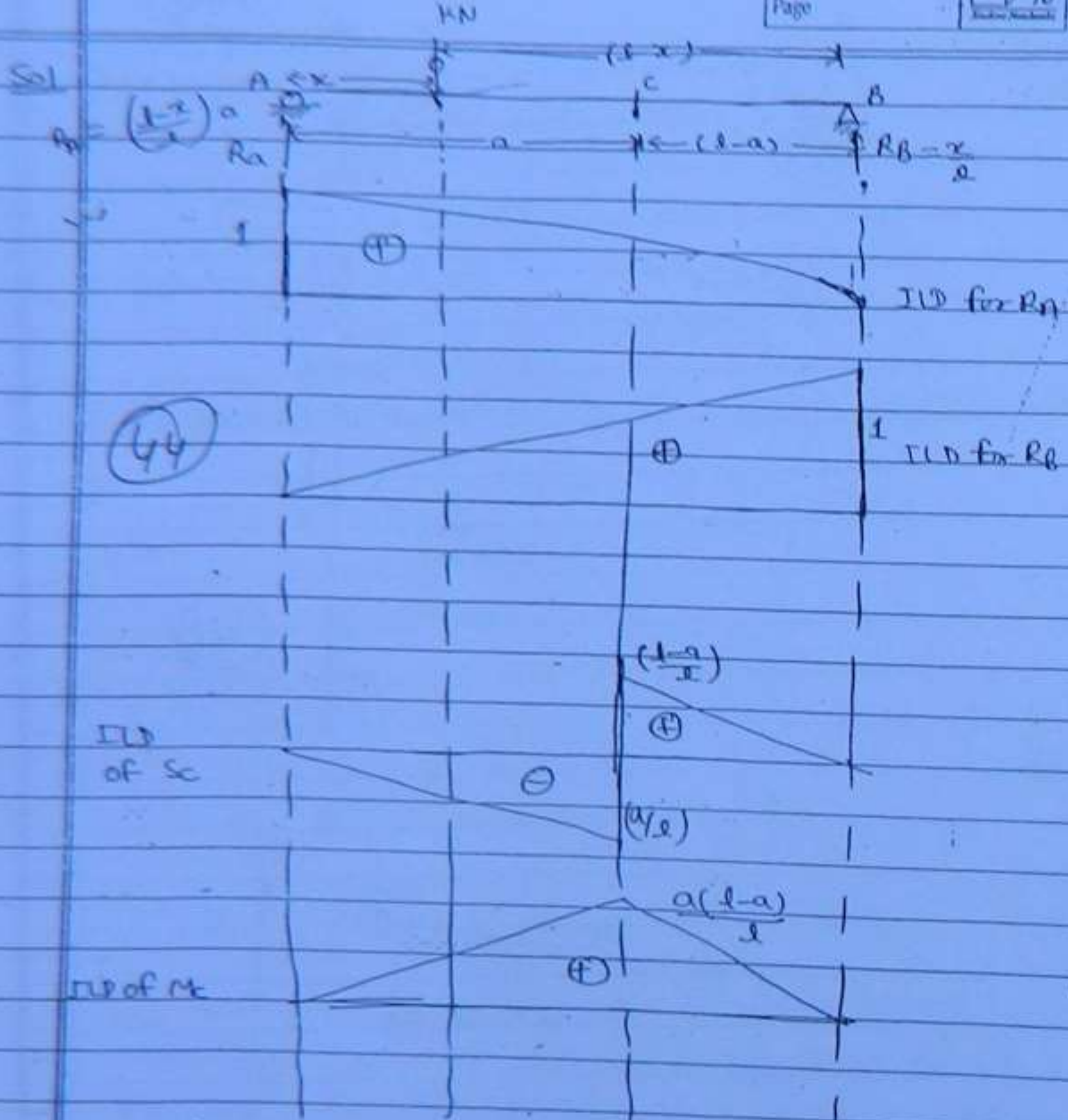
eq.2



Q. Using Ist Principal ~~Draw~~ Influence line diagram ~~for~~ for R_A, R_B, S_c & M_c for simply supported ~~Beam~~ span L is shown in figure.

Sol

$R_A = \left(\frac{l-x}{l}\right)$



ILD for RA

To find RA, Take $\Sigma M_B = 0$

$$R_A \cdot l - 1(l-x) = 0$$

$$R_A = \left(\frac{l-x}{l}\right)$$

when 1 kN load is at A $(x=0)$

$$R_A = 1$$

when 1 KN load is at RB $(x=l)$

$$R_A = 0$$

45

ILD for RB :-

Take $\Sigma M_A = 0$

$$R_B \times l - 1 \cdot x = 0$$

$$R_B = \frac{x}{l}$$

when $x = 0$, $R_B = 0$

when $x = l$; $R_B = 1$

* ILD for Sc

Case - I when unit load is the left of c

$$S_c = -R_B = -\frac{x}{l}$$

$0 \leq x \leq a$

$S_c = 0$ when 1KN load is at a

$S_c = \frac{-a}{l}$ when 1KN load is just to left of c

Case - II when unit load is ~~the~~ in CB

$S_c = +R_A \Rightarrow \frac{l-x}{l}$ $a \leq x \leq l$

at $x = a$ $S_c = \frac{l-a}{l}$

at $x = 0$ $S_c = 0$

* ILD for M_C :-

Case - I when unit load is in AC

$$M_C = R_B \times (l-a)$$

46

$$M_C = \frac{x}{l}(l-x) \qquad 0 \le x \le a$$

if $x=0$ (01 kN is at A)

$$M_C = 0$$

if $x=a$, $M_C = \frac{a(l-a)}{l}$

ILD for M_C :-

when unit load is in CB

$$M_C = R_A \cdot a$$

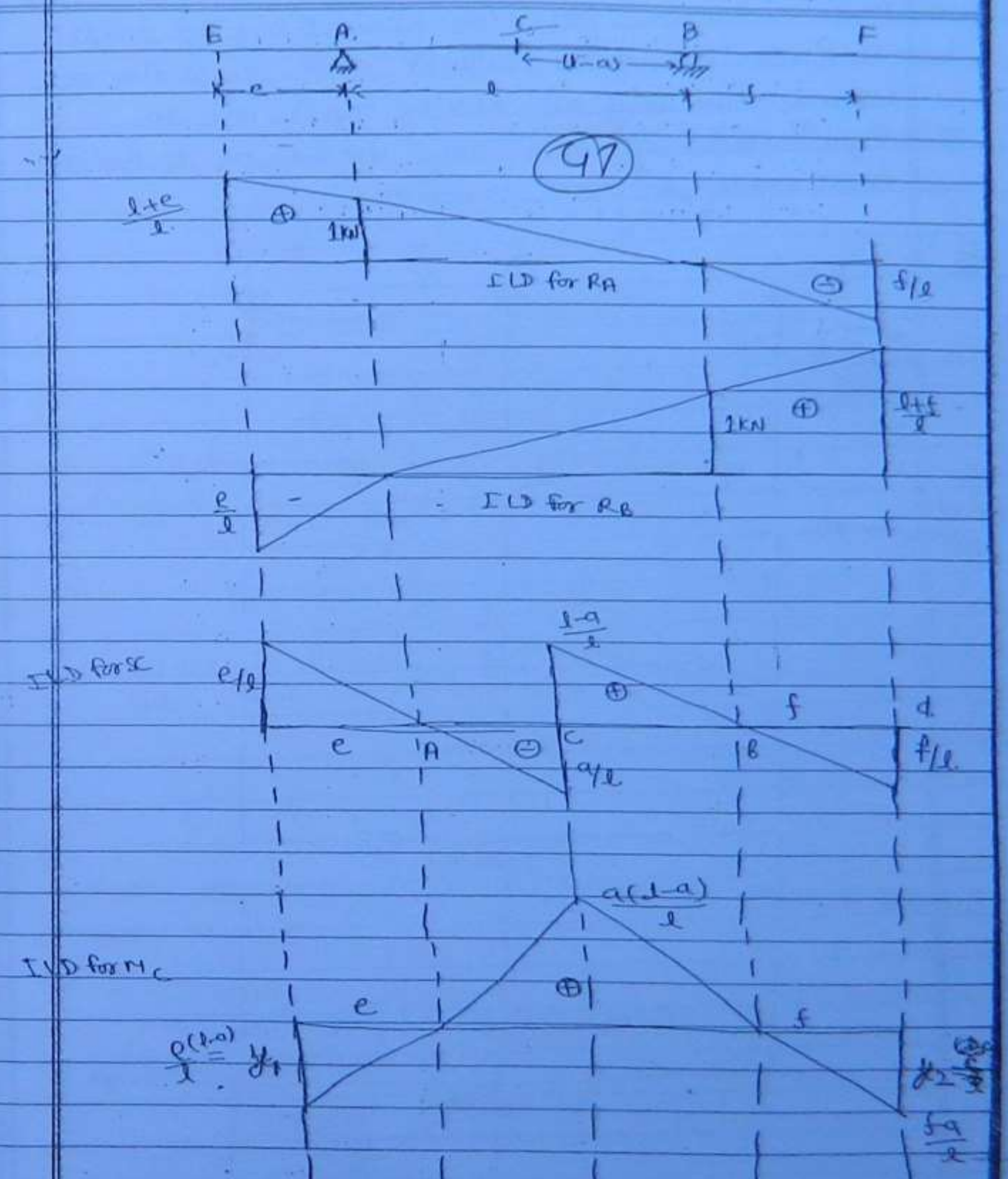
$$\Rightarrow \left(\frac{l-x}{l}\right) a \qquad a \le x \le l$$

if $x=a$

$$M_C = \frac{(l-a)}{l} a$$

if $x=l$, $M_C = 0$

Note:- ILD for a overhanging Beam can be obtained by linear extension of ILD for simply supported Beam.

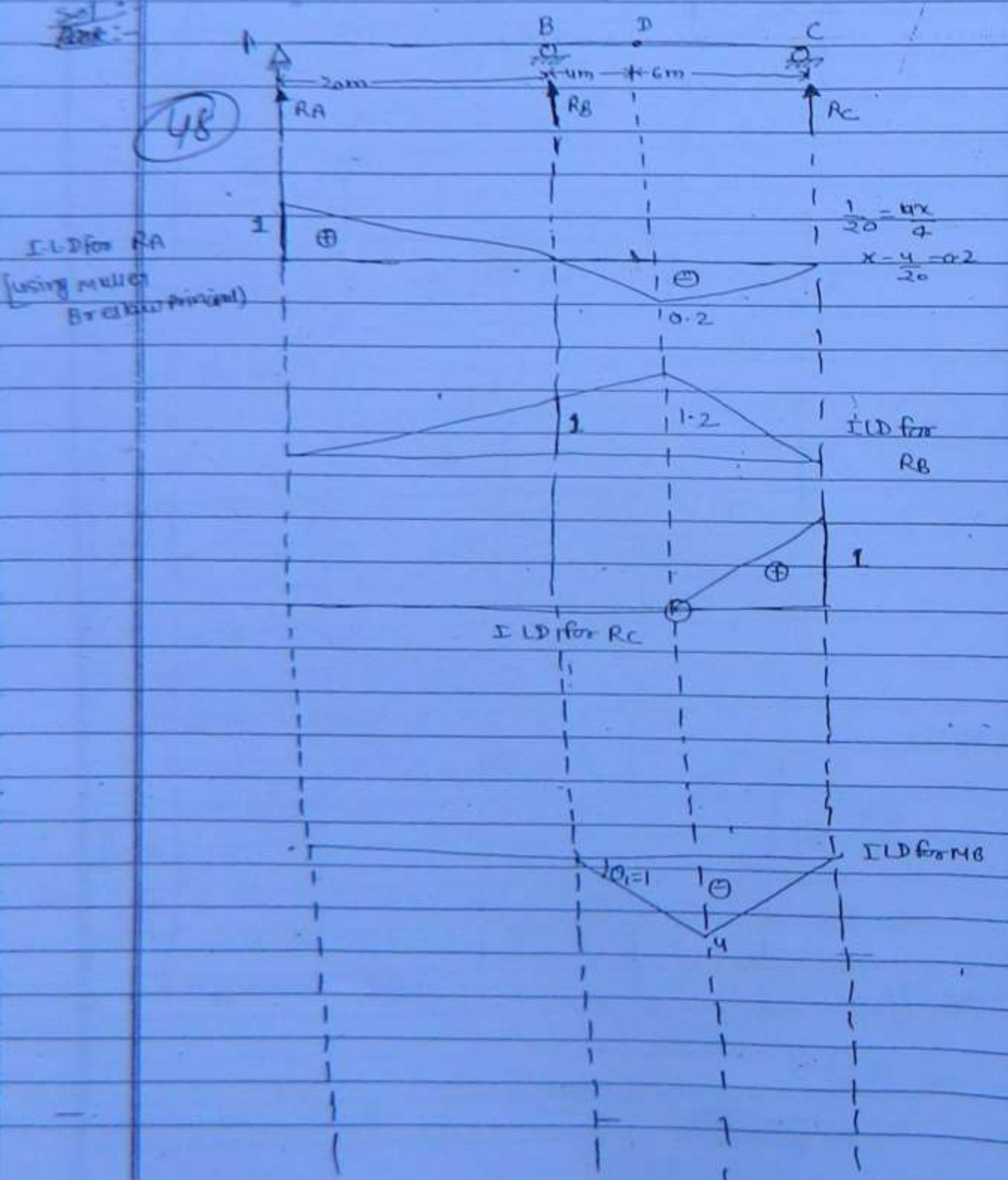


$\tan\theta = \frac{y_1}{e} \Rightarrow$

$\Rightarrow \frac{\frac{e(l-a)}{l}}{e} \Rightarrow \frac{e(l-a)}{l}$

Q. Draw Influence Dia. for R_A, R_B and R_C and M_B and also find max^m value of above stress function When a live load of 10 kN/m moves left to Right which may have any length so as to produce max^m value of stress function.

Sol:-



R_A will be maximum when only span AB is loaded

R_{Amax} = loading Rate × Area of ILD in AB

(Q9)

$$\Rightarrow 10 \times \frac{1}{2} \times \overset{10}{\cancel{20}} \times 1$$

$$\Rightarrow 100\ kN$$

loading in BC Produces -ive Reaction at A. if entire span is ABC is loaded. that net reaction of A will be :

$$R_{Anet} = 10 \times [\text{Net Area of ILD}]$$

$$\Rightarrow 10 \times \left[+\frac{1}{2} \times 20 \times 1 - \frac{1}{2} \times 10 \times 0.2\right]$$

$$\Rightarrow 10 \times [10 - 1] \Rightarrow 90\ kN$$

Maxm. Reaction at B will occur when load is present entire span ABC

$$R_{Bmax} \Rightarrow 10 \times \frac{1}{2} \times 30 \times 1.2$$

$$\Rightarrow 180\ kN.$$

R_{Cmax} will occur. when. atlest span BC is loaded.

$$R_{Cmax} \Rightarrow 10 \times \frac{1}{2} \times 1 \times 6^{5}$$

$$\Rightarrow 30\ kN$$

Max^m B.M. at B will be (-ve) i.e. hogging when atleast span BC is loaded.

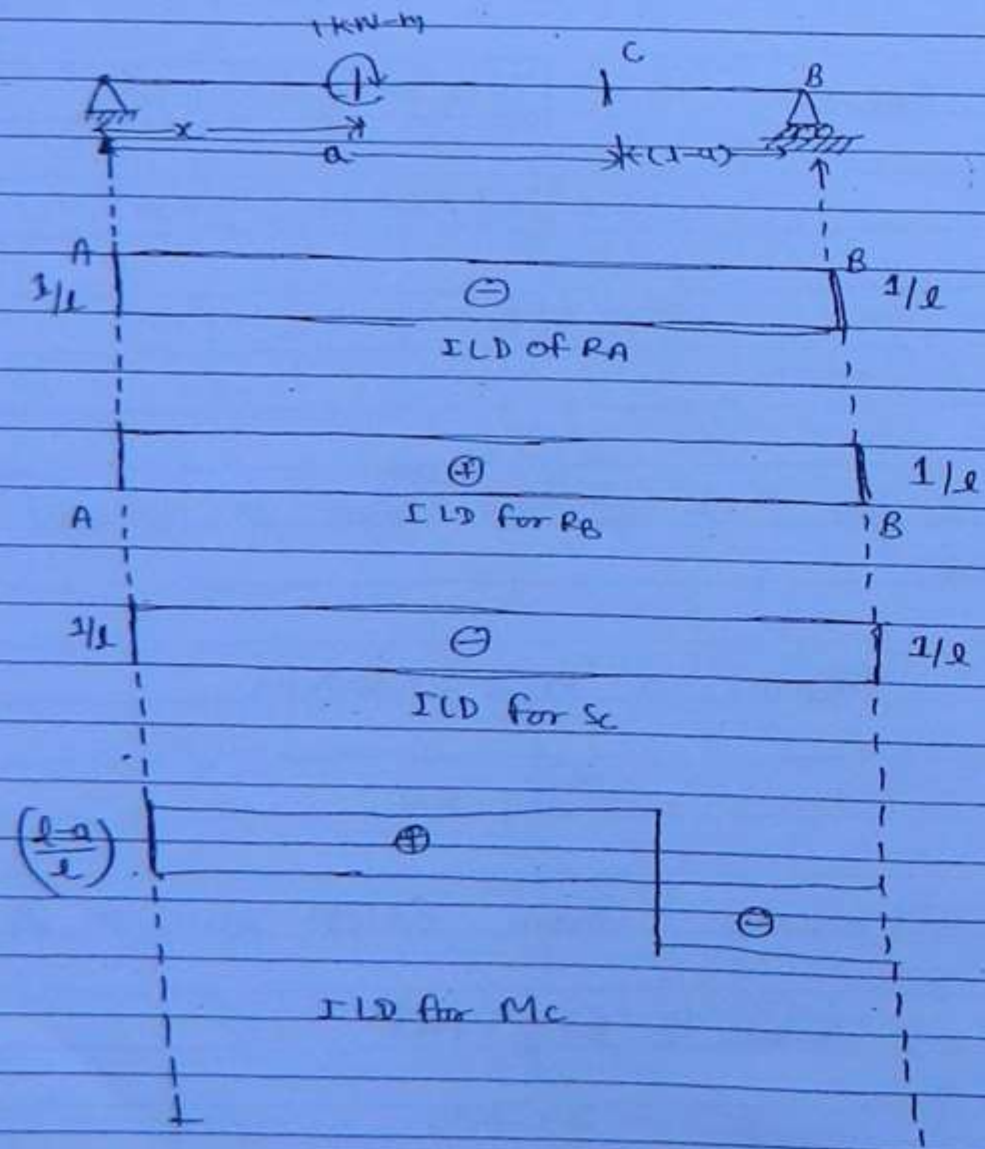
$$M_{B\,max} \Rightarrow 10 \times -\frac{1}{2} \times 4 \times 10$$

$$\Rightarrow -200 \text{ KN-m}$$

56

Q Draw ILD for simply supported girder of length l for RA, RB, SC, MC when a unit concentrated moment moves from A to B. as shown in fig.

Sol



$\Sigma M_B = 0$

$$R_A \cdot l + 1 = 0$$

$$R_A = -\frac{1}{l}$$

$\Sigma F_y = 0$ (57)

$$R_A + R_B = 0$$

$$R_B = +\frac{1}{l}$$

ILD for Sc :-

$$S_C = -\frac{1}{l}$$

ILD for Mc :-

Case - 1   When 1 kN-m to the left of C

$$M_C = R_B \times (l - a)$$

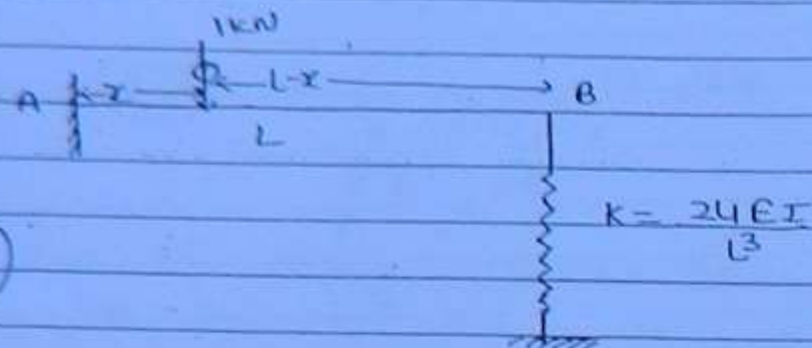
$$M_C = \frac{1}{l}(l-a) \Rightarrow \left(\frac{l-a}{l}\right)$$

Case - II   When 1 kN-m is to the right of C

$$M_C = R_A \cdot a$$

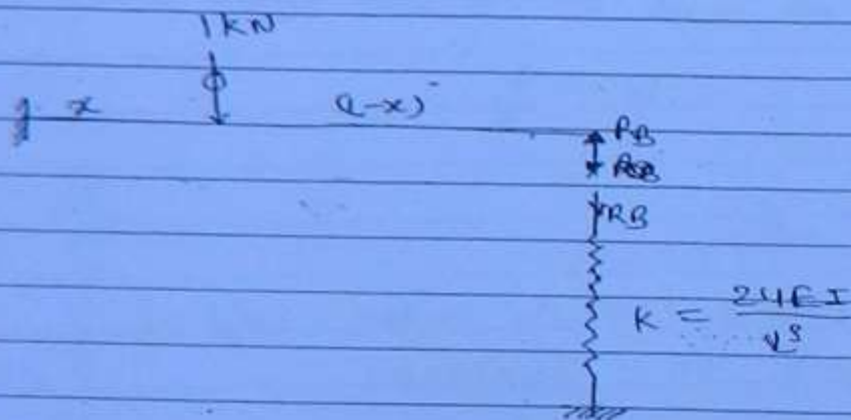
$$\Rightarrow -\frac{1}{l} a \Rightarrow \frac{-a}{l}$$

Q. A Cantilever beam AB a supported over a spring as shown in fig. Draw Influence lince Dig. for for R_B and M_A when unit concerted load moves from A to B.



52

Sol:-



let internal Reaction at B is R_B

Downward deflection at B for AB = defl^n of spring

$$\frac{1\cdot x^3}{3EI} + \frac{1\cdot x^2 (L-x)}{2EI} - \frac{R_B L^3}{3EI} \Rightarrow \frac{R_B}{K}$$

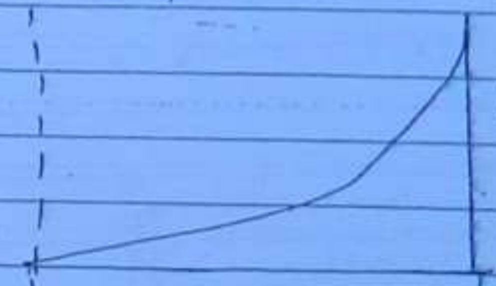
$$\frac{x^3}{3EI} + \frac{x^2 L}{2EI} - \frac{x^3}{2EI} - \frac{R_B L^3}{3EI} = \frac{R_B}{K}$$

$$\boxed{R_B = \frac{4x^2(3L-x)}{9L^3}}$$

when $x = 0$, $R_B = 0$

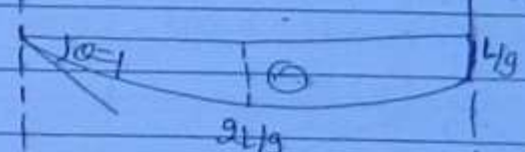
$x = L$, $R_B = \frac{8}{9}$

53



ILD for RB

ILD for MA



$M_A = R_B \cdot L - 1 \cdot x$

$\Rightarrow \frac{4x^3}{9L^3}(3L-x)L - x$

$\Rightarrow \frac{4x^3}{9L^2}(3L-x) - x$

If $x = 0$ $M_A = 0$

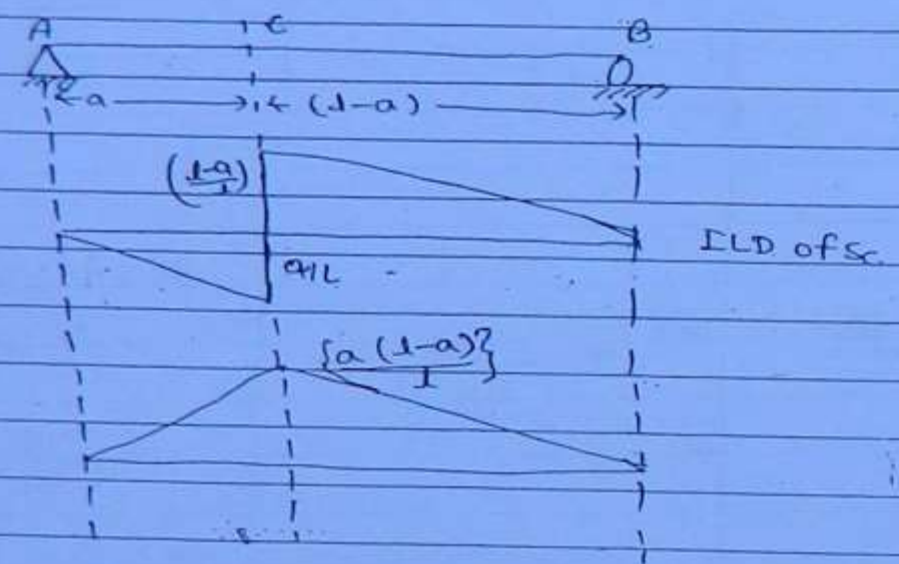
$x = L$ $M_A = -4/9$

If $x = 4/9$ $M_A = \frac{-2L}{9}$

Effect of Rolling load :-

Case-1 Q. Find Maximum shear force and B.M. at a section C. when a UDL Passes over the girder Having its length less than length of girder

(54)

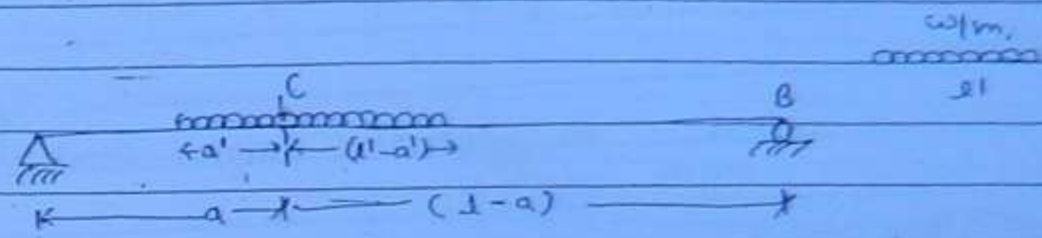


Max^m (-ive) Shear force at C will occur when head of the load is just to C load moving left to right.

Max^m (+)ive S.F. at C will occur when tail of the load is just to C. load moving Right to left.

Max^m B.M. at C will occur average loading to the left of C is equal to average load Right of C. its means section C devide the load in this same raito as it devide to the spane.

let length of load is l'



55

$$\frac{a'}{a} = \frac{l'-a'}{l-a}$$

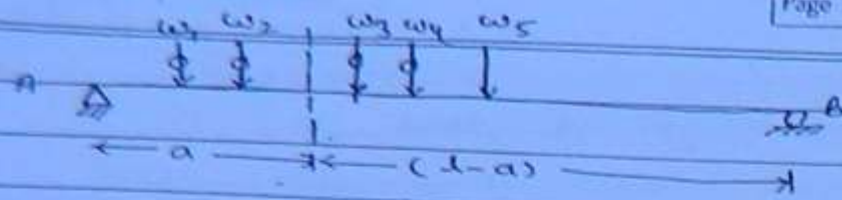
Note ① The absolute max^m -ive S.F. will occur at support B when load is just to the left of B.

Max^m (+ive) S.F. occur at A when load is just to be Right of A.

② due to moving UDL absolute max^m B.M. will occur at the centre of spane when avg. loading equal an Both side of center means Load is Symmetrically placed at but the centre

Case-2 when a series of concreted wheel load crosses a simply supported girder then ~~absolute maximum bending moment occur at the centre of span~~ max^m bending moment any section C occur when loading is placed such that average loading two the left of section. C is equal to average loading to the Right of section C.

—



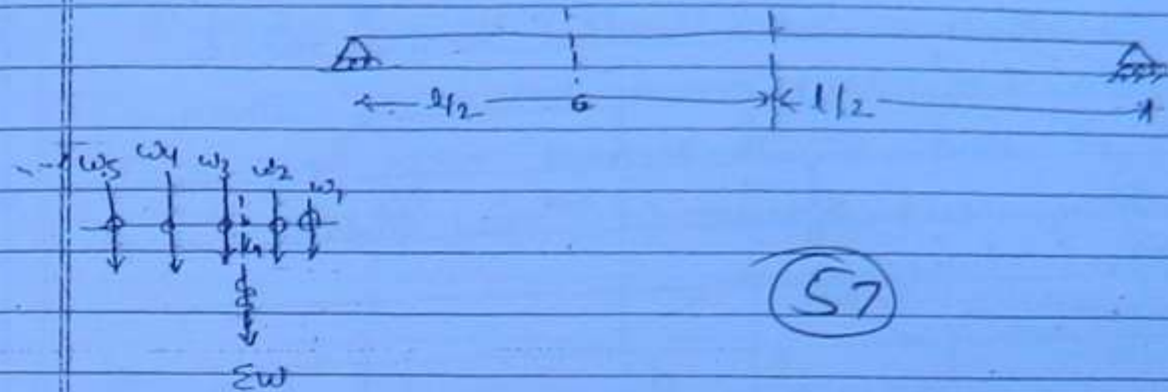
$\frac{\text{load}}{\text{length}}$ in left and Right Part is equal

(56)

due to Series of wheel load absoulte max. bending moment occurs below any one of the wheel load and never between two wheel load when the position of load such that centre of the span is located mid-way b/w C.G. of load system and load. considerstation. load under consideration is either that load which is nearest to c.g. i.e. that w_3 or that load which is next nearest to C.G. (w_2)

If $w_3 > W_2$ where w_3 is nearest load to c.g. than it is certian that maxm Bending moment will occur below w_3 but if $W_2 > W_3$ and w_2 is next nearest load to C.G. than maxm BM may occur either below w_2 or below w_3, which ever is greater.

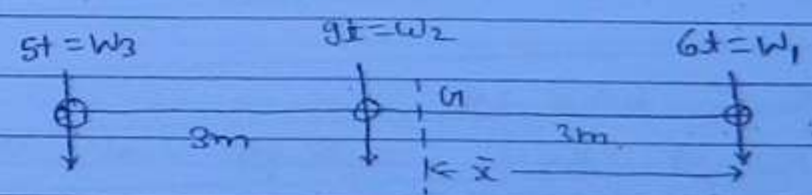
If C.G. of load system (G) Consider with any of wheel load say w_3 than absoulte maxm bending moment will occur w_3 when w_3 is located just over centre.



57

Q: A series of 3 wheel load 5 tonne, 9 and 6 tonne speared 3m center to centre cross over a simply supported girder with a span of 10 m. when load move left to right with 6 tonne load leading than find position and magnitude of max. B.M. which may occur anywher on the girder (that is means absolute maxm value.)

Sol



Distance of C.G. of loading system W.

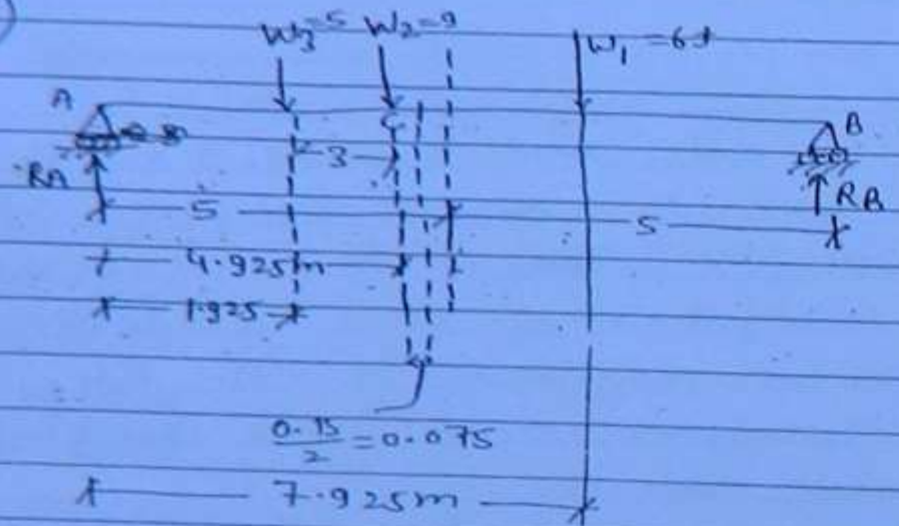
$$\bar{x} = \frac{W_1\bar{x}_1 + W_2\bar{x}_2 + W_3\bar{x}_3}{W_1 + W_2 + W_3}$$

$$\bar{x} = \frac{6\times0 + 9\times3 + 5\times6}{5+9+6}$$

$$\bar{x} = 2.85\ m$$

The load nearest to C.G. is w_2 which is havier than W_1 which is next nearest to C.G. there-for max^m B.M. will occur below w_2. When centre of span C is located mid-way b/w C.G. of load system G and load under consideration (w_2)

(58)



$R_A + R_B = 6+5+9$

$R_A + R_B \Rightarrow 20$ —— (1)

$\Sigma M_A = 0$

$R_B \times 10 - 6 \times 7.925 - 9 \times 4.925 - 5 \times 1.925 = 0$

$R_B = 10.15\,t$

$R_A = 9.85\,t$

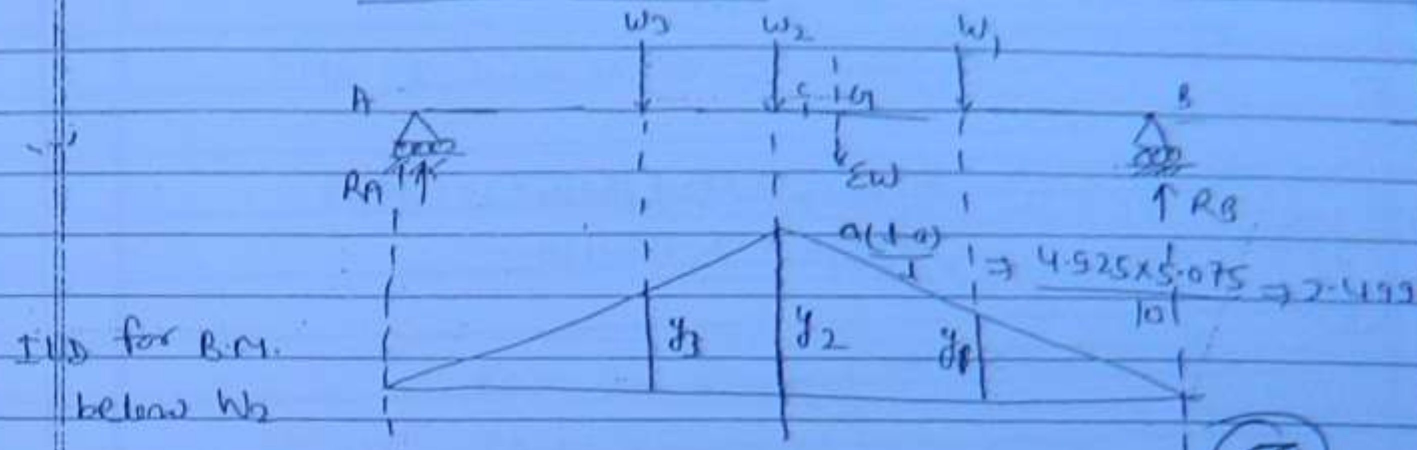
B.M. below W_2

$R_A \times 4.925 - 5 \times 3 \Rightarrow 0$

$R_A = 9.85 \times 4.925 - 15 = 0$

$\boxed{M_{max} = 33.5\,t\text{-}m}$ occur 4.925 m from A

Method 2 use of ILD



(59)

$$\frac{y_3}{y_2} = \frac{1.925}{4.925} \Rightarrow \frac{1.925}{4.925} \times 2.499$$

$$y_3 \Rightarrow 0.976$$

$$\frac{y_1}{y_2} = \frac{2.075}{5.075} \Rightarrow y_1 = 1.02$$

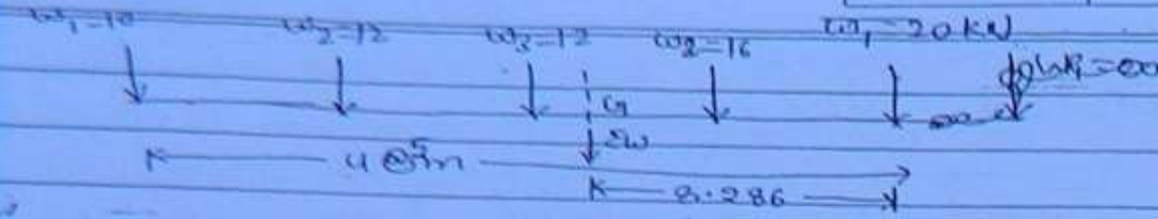
Total B.M. below W_2 ⇒

$$W_1 y_1 + W_2 y_2 + W_3 y_3$$

$$\Rightarrow 6 \times 1.02 + 9 \times 2.499 + 5 \times 0.976$$

$$\Rightarrow 33.5 \text{ t-m.}$$

Q. 5 Point loads of 10KN, 12KN, 12 KN, 16 KN and 20 KN Space at about 5m centre to centre rolle over a simply supported girder of 80 m. The load move left to right with 20 KN load leading. than Calculate position and magnitude of maxm B.M. which may occur any where on the girder.



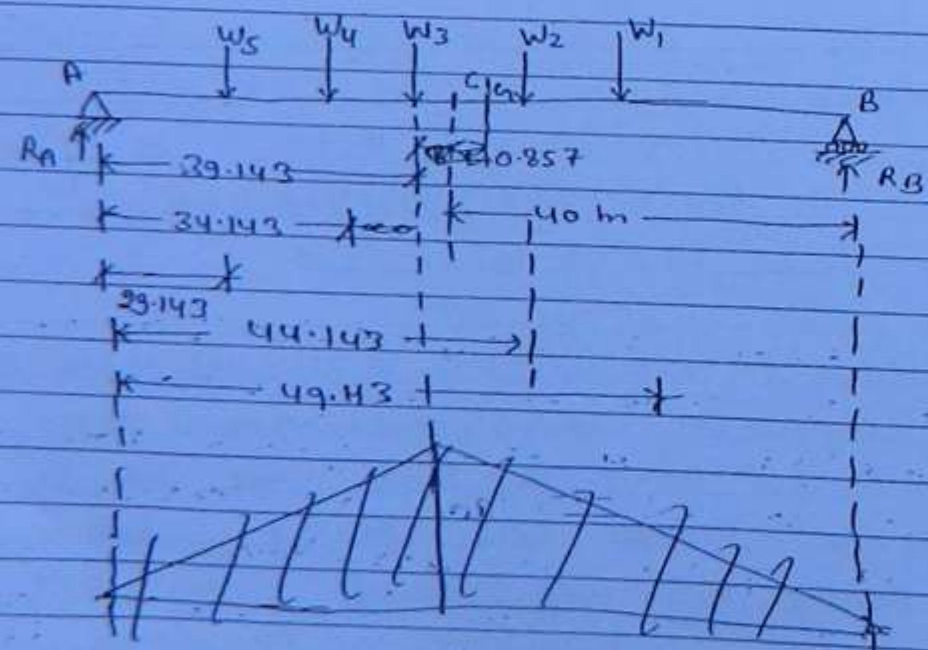
$$\bar{x} = \frac{20\times0 + 16\times5 + 12\times10 + 12\times15 + 10\times20}{20+16+12+12+10}$$

60

$\bar{x} = 8.286$ m

∵ w_3 is nearest to C.G. but w_3 is less than w_2 $(w_3 < w_2)$ but where w_2 is next nearest to C.G. hence absolute max^m B.M will occur either below w_3 or below w_2 which ever gives grater value.

Case-1 Consider max^m Bending moment occur below w_3. the centre span should lie b/w C.G. of load system load under consideration (W_3)



$$R_A + R_B = 20+16+12+12+10 = 70$$

$\Sigma M_A = 0$

61

$$R_B \times 80 - 20\times 49.143 - 16\times 44.143 - 12\times 39.143 - 12\times 34.143 - 10\times 29.143 = 0$$

$$R_B = 35.753\,kN$$

$$R_A = 70 - 35.73 = 34.25\,kN$$

B.M. below w_3

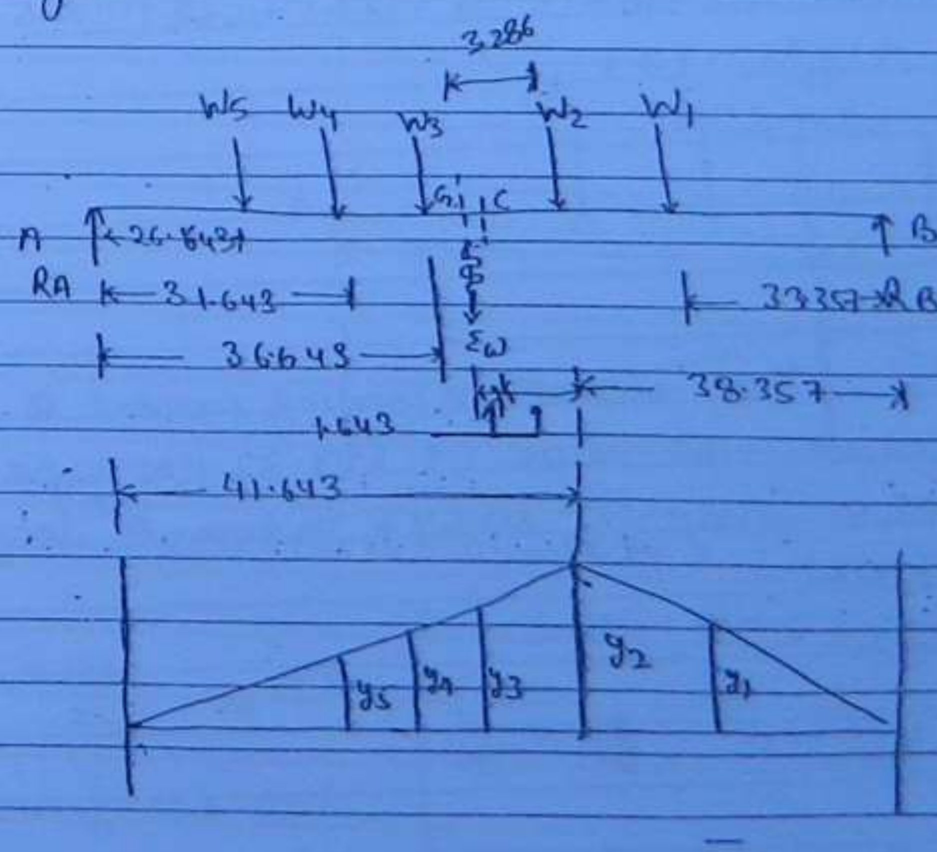
$$\Rightarrow R_A \times 39.143 - 10\times 10 - 12\times 5$$

$$\Rightarrow 34.25\times 39.143 - 100 - 60$$

B.M. below w_3 $\Rightarrow 1180.65$ kN-m.

Metho 2 Consider max^m B.M. occur below W_2

Hence, centre of span to be located between c.g. of load system and W_2.



$$y_2 = \frac{a(l-a)}{l} \Rightarrow \frac{41.643 \times (80 - 41.643)}{80}$$

$$y_2 \Rightarrow \frac{41.643 \times 38.357}{80}$$

62

$$y_2 = 19.966$$

$$\frac{y_1}{y_2} = \frac{33.357}{38.357} \Rightarrow y_1 = 17.367$$

$$\frac{y_3}{y_2} = \frac{36.643}{41.643} \Rightarrow y_3 = 17.568$$

$y_4 = 15.1715$ $\qquad$ $y_5 = 12.774$

Total BM below w_2

$\Rightarrow w_1 y_1 + w_2 y_2 + w_3 y_3 + w_4 y_4$

$\Rightarrow 20 \times 17.363 + 16 \times 19.966 + 12 \times 17.568 + 12 \times 15.1755$

$+ 10 \times 12.774$

$\Rightarrow 1187.33$ kN

The absolute bending moment is greater of the above two value i.e. 1187.33 kN-m which occur at a distance 41.643 m from A below load 16 kN (W_2)

Work book

1 b	7 a.	13 b	19 d
2 (b)	8 d.	14 c	20 only (c) correct
3 b	9 b	15 b	21 c
4 b	10 a	16 d	22 d
5 a	11 c	17 d	23 c
6 c	12 d	18 b	24 a
25 d	26 b	27 d	28 c
25 (a)			

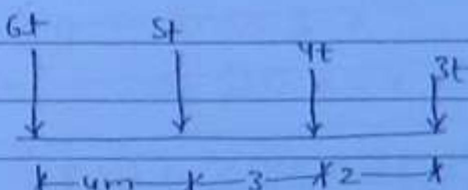
63

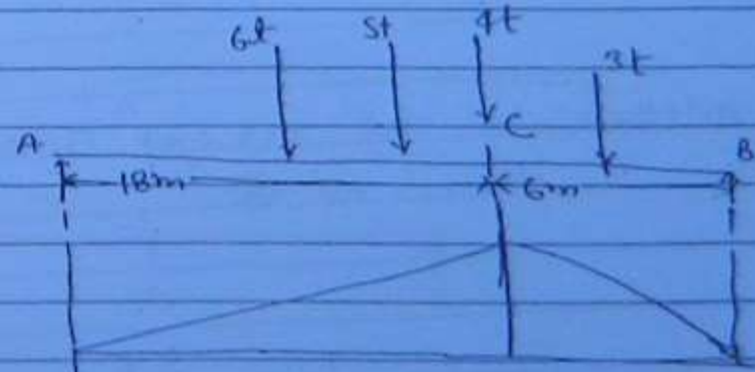
(6) load × $\frac{1}{2} \times \overset{8}{\cancel{16}} \times 3$

$\Rightarrow 2 \times 24$

$\Rightarrow 48$ m Ans

(9)





$M_{c_1} = 3 \times y_1 + 4 \times y_2 + 5 y_3 + 6 y_4$

$M_{c_1} \Rightarrow 3 \times 3 + 4 \times 4.5 + 5 \times 3.75 + 6 \times 2.75$

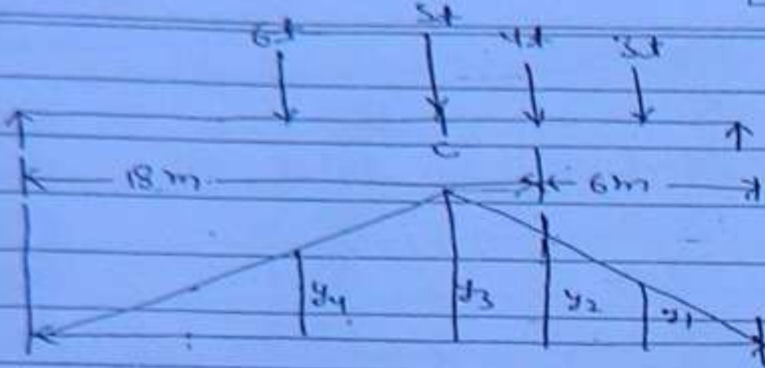
$\Rightarrow 62.25$

$y_2 = \frac{18 \times 6}{24} \Rightarrow 4.5$

$y_1 = \frac{y_1}{y_2} \Rightarrow \frac{4}{6} = 3$

$\frac{y_3}{y_2} = \frac{15}{18}$ $\quad y_3 = \frac{15}{18} \times 4.5 = 3.75$

$y_4 = \frac{11}{8} \times 3.75 \Rightarrow 2.75$



64

$y_3 = 4.5$

$\frac{y_1}{y_3} \Rightarrow \frac{1}{6}\ 4.5 \Rightarrow \frac{4.5}{6} \Rightarrow 0.75$

$\frac{y_2}{y_3} = \frac{3}{6} \quad y_2 \Rightarrow 2.25$

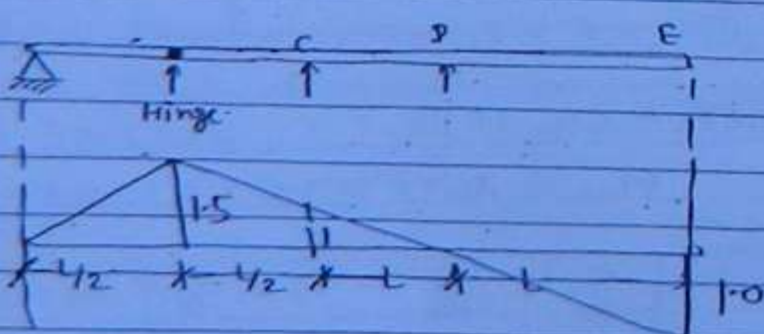
$y_4 = \frac{14}{18} \times 4.5 = 3.5$

$M = 3 \times 0.75 + 2.25 \times 4 + 5 \times 4.5 + 6 \times 3.5$

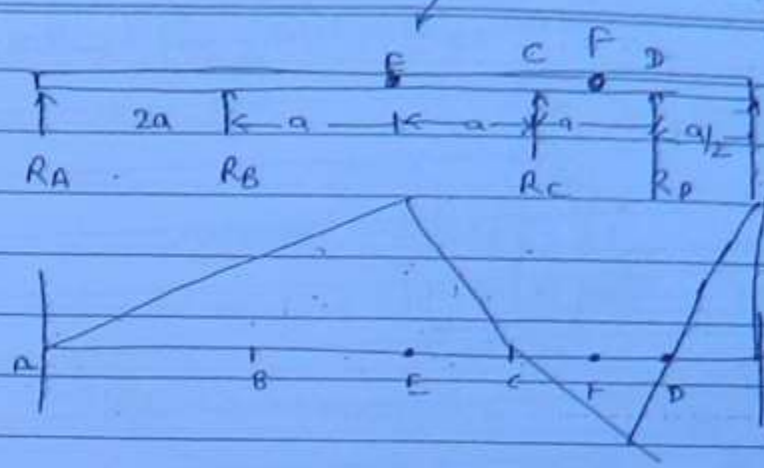
$M = 54.75$

(10) indeterminta the ILD Not linear than Ans - (A)

(15)



(17)



65

(19)

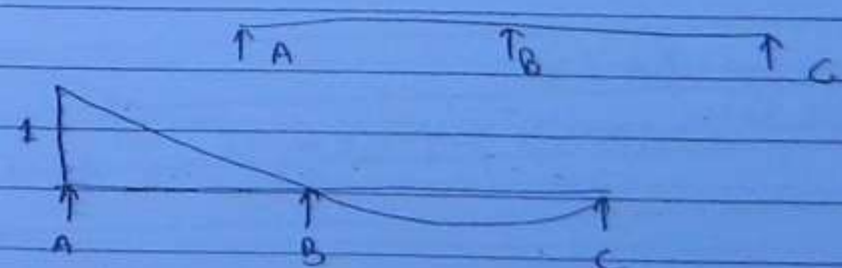
$R = f(x)$

$M_A = R \cdot l - 1 \cdot x$

$\Rightarrow f(x)\, l - x$

$\Rightarrow = [x - fn(x) l]$

(20) (a)(i)

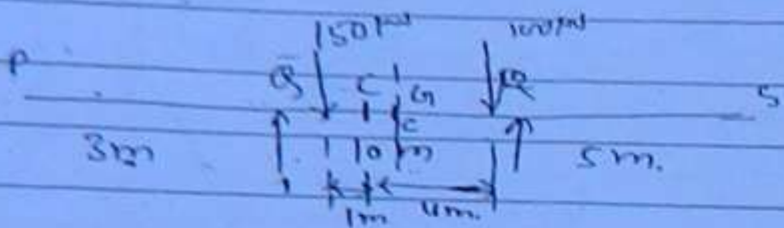


(ii)

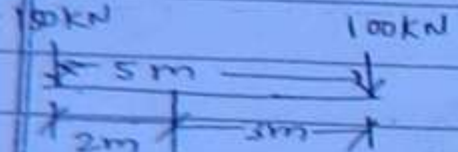
Only C statement is correct and a14 statement wrong.

(21) than determinate structure and Ans C

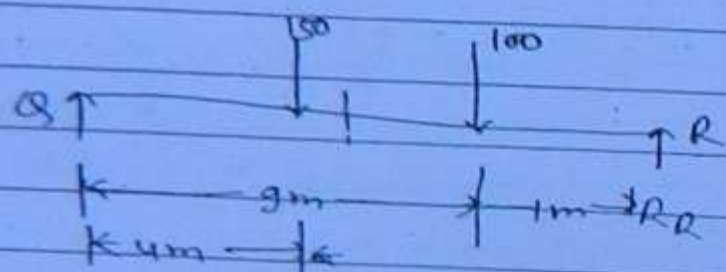
(23)



(66)



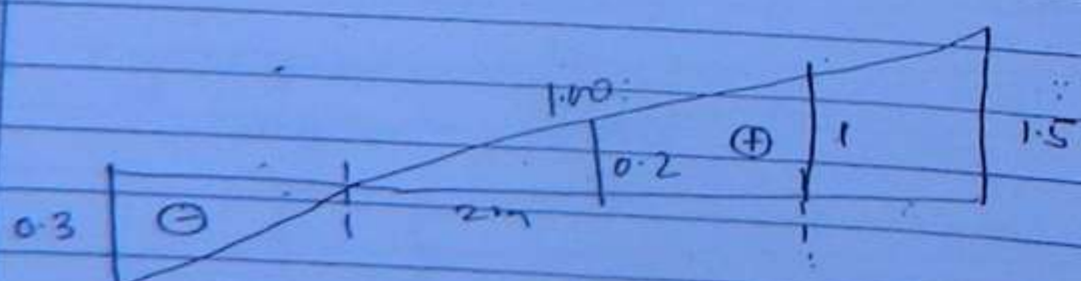
$$\bar{x} = \frac{150\times 0 + 100\times 5}{150\times 100} \Rightarrow \frac{500}{250} \Rightarrow 2$$



$$R_R \times 10 - 100\times 9 - 150\times 4 = 00$$
$$R_R = 150$$
$$R_Q = 100$$

$$M_{max} \Rightarrow R_Q \times 4$$
$$= 100\times 4 = 400 \text{ kN-m}$$

(24)



ILD for Reaction of R

Case-I $\Rightarrow 100\times1.5+150\times1$

$\Rightarrow 300$ (max) Ans

Case-II ~~150×100~~ 150×1.5

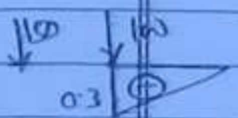
$\Rightarrow 225$

Case-III $-150\times0.3+100\times0.2$ (67)

$\Rightarrow -25$ Ans

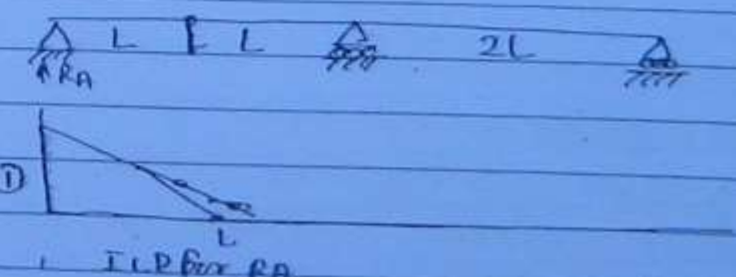
than 150 kN out of (−) B.M. than.

$-100\times0.3 \Rightarrow -30$.



(25) $150\times5 = 750$

(26)



(a) ILD for RA — not Ans

(b) ans. (a) not ans

(b) $S_B = R_A - 1$

$R_A\times L - 1(L-x) = 0$

$R_A = \frac{(L-x)}{L}$

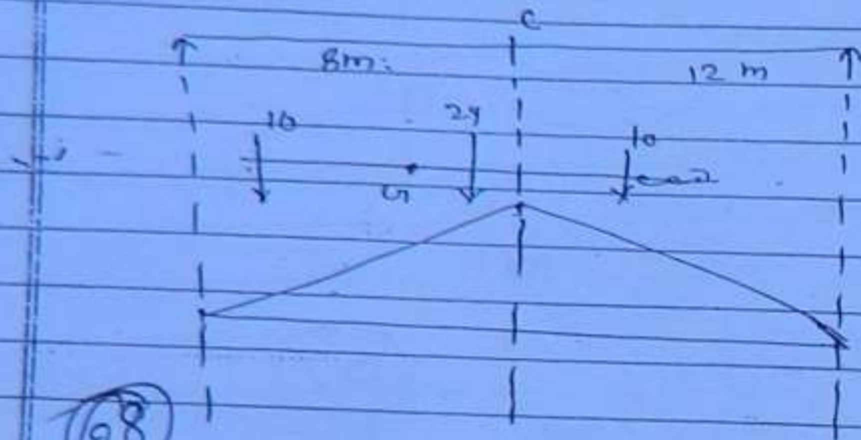
$S_B = R_A - 1$

$S_B = R_A - 1 \Rightarrow \frac{L-x}{L} - 1 \Rightarrow -\frac{x}{L}$

At $S_B = -1$

Page

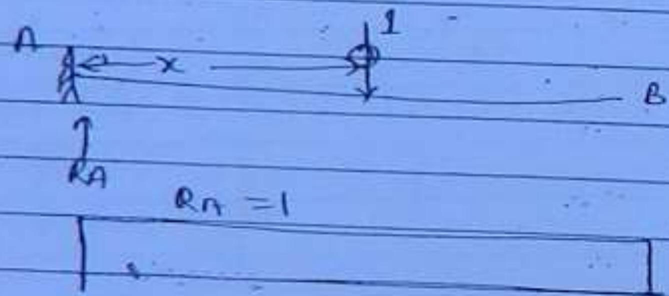
(27)



(68)

Ans - d

28 Statement (A)



ILD for SA

S.F.D.

Statment B

ILD for MA

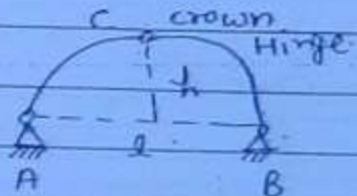
ILD for L

Ans - (C)

ARCHES

Arches (69)

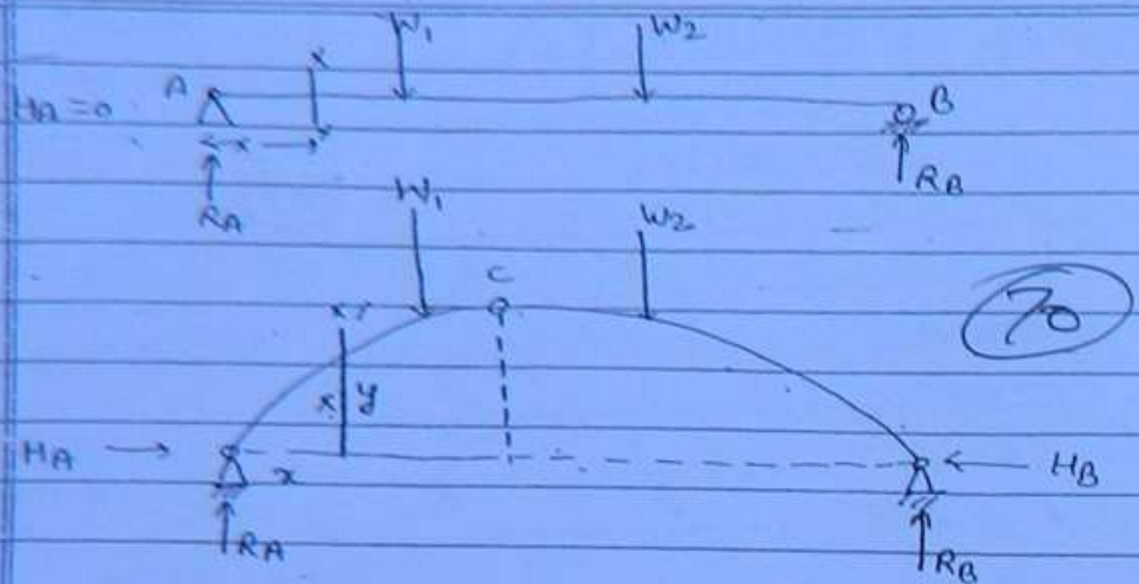
3-Hinged | 2Hinged | Fixed



→ A & B support Hinges | Indeterminate to 1st order | Indeterminate to 3rd order

→ Determinar or
Abutment Hinge
or
Springinge

→ Arches structure able to resist S.F., B.M. & Axial Thrust All. As compare to simply supported beam of equal span ~~upo~~ under equal load free Hinged arch requries smaller section because in Arch. Horizonatal reactions are developed which reduced the B.M. Because B.M. Produce by vertical reaction is sagging and B.M produse by Horizontal reaction is Hogging.



B.M. at x-x in Beam $= R_A \cdot x$

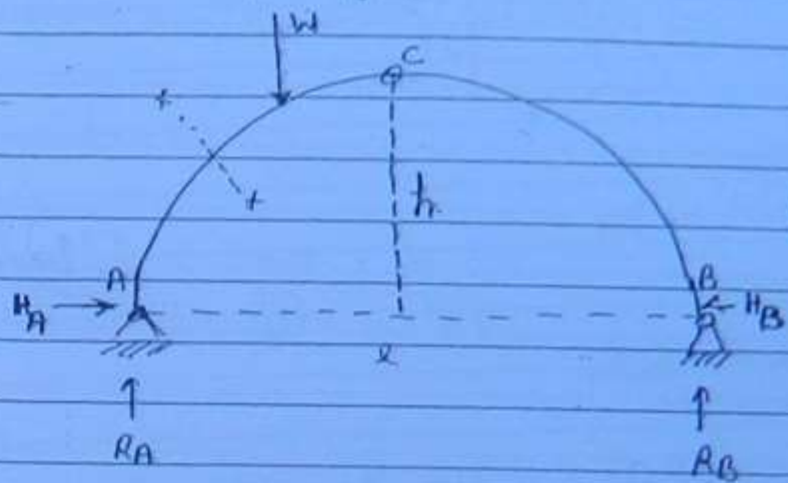
B.M. at x-x in Arch $= R_A \cdot x - H_A \cdot y$

Arch Moment ⇒ Beam Moment − H moment

Generaly for long span Arch structure are economical because Net ~~Be~~ B.M. Moment developed in Arches ~~o~~ is less than net B.M. developed in Beam hence thiner section is need however for small span Beams are prefered.

In case of multi story structure and from asthetic consideration arches are less perffect.

Analysis of three-Hinged arches :-



71

~~Total Reaction for total equation~~

Total Reation = 4

Total Equation = 3+1

$\Sigma F_x = 0$ $\quad$ $\Sigma F_y = 0$ $\quad$ $\Sigma M_z = 0$ $\quad$ $M_c = 0$

$D_s = 0$ $\quad$ Stable & Determinate.

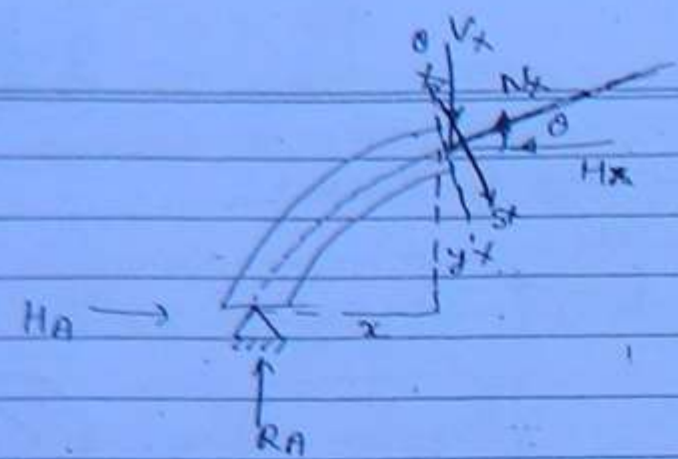
If there is no Horizontal load

than $\Sigma F_x = 0$

$H_A - H_B = 0$

$H_A = H_B = H$ (say)

following 3 types of forces are developed.



72

① B.M. at X–X

$$M_x = R_A \cdot x - H_A \cdot y$$

②. S.F. at X–X / Radial Shear

$$S_x = V_x \cdot \cos\theta - H_x \cdot \sin\theta$$

③ Normal Thrust / Axial thrust (N_x) X–X

$$N_x = V_x \sin\theta + H_x \cos\theta$$

$$\tan\theta = \frac{dy}{dx}$$

For Parabolic Arch

$$\boxed{y = \frac{4h}{l^2}(l-x)x}$$

* *** $\boxed{\frac{dy}{dx} = \tan\theta \Rightarrow \frac{4h}{l^2}(l-2x)}$

=

To compute V_x & H_x at X-X consider free body equilibrum on ~~Both~~ left of X-X

$\Sigma f_x = 0$

73

$$H_A - H_x = 0$$

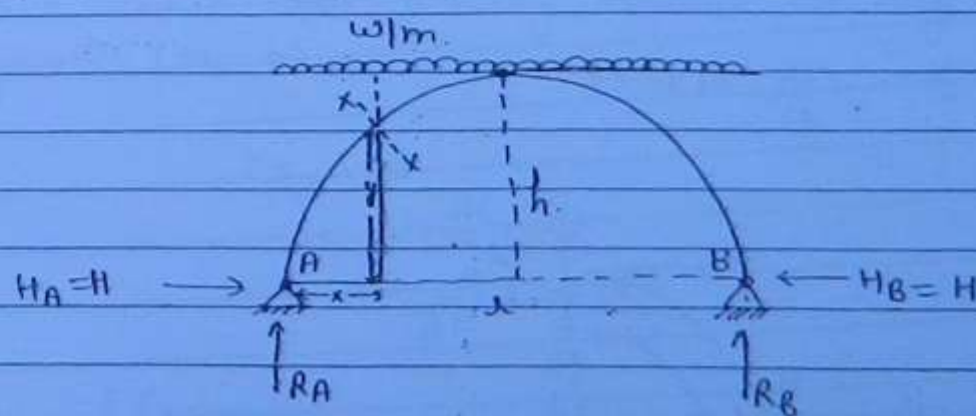
$$\boxed{H_A = H_x}$$

$\Sigma f_y = 0$

$$R_A - V_x = 0$$

$$\boxed{R_A = V_x}$$

Case-1 Three Hinge Parabolic Arch subjected to UDL over entire span.



$\Sigma F_x = 0$ — — (i)

$$H_A - H_B = 0$$

$$H_A = H_B = H$$

$\Sigma f_y = 0$

$$R_A + R_B - wl = 0 \quad \text{— (ii)}$$

$$\Sigma M_B = 0$$

$$R_A \times L - w \times L \times \frac{L}{2} = 0$$

$$R_A = \frac{wL}{2}$$

$$R_B = \frac{wL}{2}$$

74

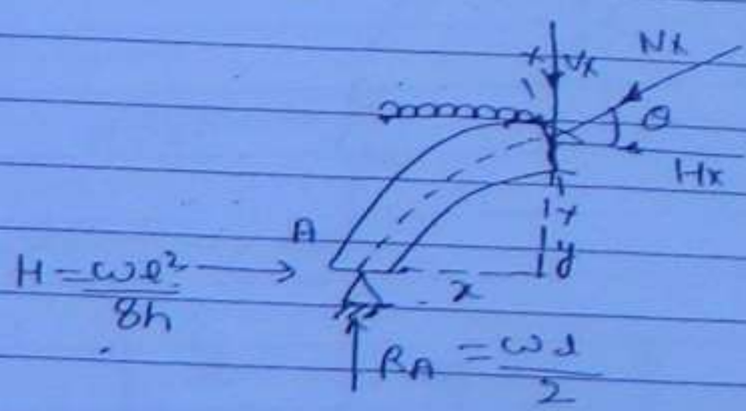
taking moment about C

$$R_A \cdot \frac{l}{2} - H \cdot h - w \cdot \frac{l}{2} \cdot \frac{l}{4} = 0$$

$$\frac{wl^2}{4} - H h - \frac{wl^2}{8} = 0$$

* Imp. $\boxed{H = \frac{wl^2}{8h}}$ Imp. *

$$y = \frac{4h}{l^2} x(l-x)$$



B.M. at X–X

$$M_x = R_A \cdot x - H \cdot y - w x \cdot \frac{x}{2}$$

put the value

$$M_x = \frac{wl}{2} \cdot x - \frac{wl^2}{8h} \times \frac{4h}{l^2} x(l-x) - \frac{wx^2}{2}$$

75

$$M_x = \frac{wl}{2} x - \frac{wl}{2} x + \frac{wx^2}{2} - \frac{wx^2}{2}$$

$$\boxed{M_x = 0}$$

Imp

> "In 3-Hinge Parabolic Arch Subjected to u.d.l over entire Span Bending moment at any section is zero." $(M_x = 0)$

$$\Sigma F_x = 0$$

$$H - H_x = 0$$

$$H = H_x = \frac{wl^2}{8h}$$

$$\Sigma F_y = 0$$

$$R_A - w \cdot x \cdot V_x = 0$$

$$V_x = R - w \cdot x = \frac{wl}{2} - wx$$

Radial Shear / SF

$$S_x = V_x \cos\theta - H_x \sin\theta$$

$$= \left(\frac{wl}{2} - w \cdot x\right)\cos\theta - \frac{wl^2}{8h} \cdot \sin\theta$$

$$S_x = \cos\theta \cdot w \left[\frac{l}{2} - x - \frac{l^2}{8h}\tan\theta\right]$$

$$y = \frac{4h}{l^2}x(l-x)$$

76

$$\frac{dy}{dx} = \tan\theta = \frac{4h}{l^2}(l-2x)$$

$$S_x = \cos\theta \cdot w \left[\frac{l}{2} - x - \frac{l^2}{8h_2}\cdot\frac{4h}{l^2}(l-2x)\right]$$

$$S_x = \cos\theta \cdot w \left[\cancel{\frac{l}{2}} - \cancel{x} - \cancel{\frac{l}{2}} + \cancel{x}\right]$$

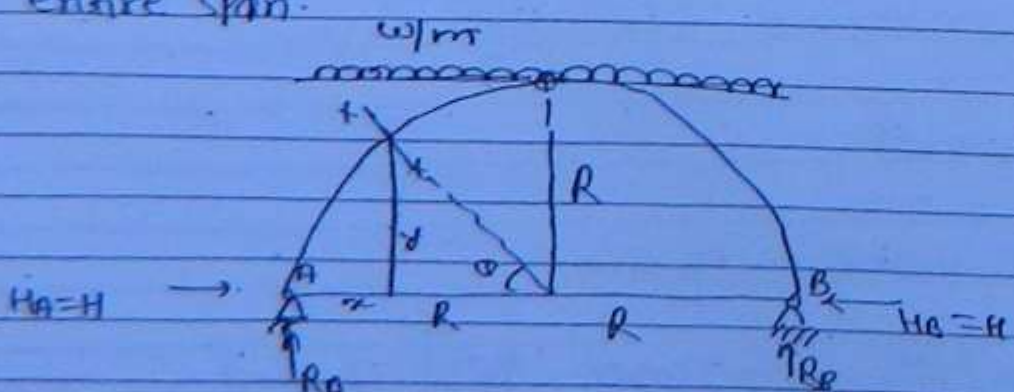
$$\boxed{S_x = 0}$$

"it means 3-Hinge Parabolick arch with UDL on with entire span is free from S.F. and B.M."

S_x and $M_x = 0$

However, Normal Thrust will exist

Case-2 3-Hings semi-circle Arch subject to udl over entire span.



ΣF_y

$$R_A + R_B - \omega \cdot 2R = 0$$

$$R_A + R_B = 2\omega R$$

77

$\Sigma M_A = 0$

$$R_B \times 2R - \omega \times \cdot 2R \cdot R = 0$$

$$R_B = \omega R$$

$$R_A = \omega R$$

Moment about A $\quad M_C = 0$

$$R_A \times R - H \cdot R - \omega R \cdot \frac{R}{2} = 0$$

$$\omega R \cdot R - HR - \frac{\omega R^2}{2} = 0$$

$$HR = \frac{\omega R^2}{2} - \omega R^2 \Rightarrow \frac{\omega R^2}{2}$$

$$\boxed{H = \frac{\omega R}{2}}$$

Horizontal Reaction is $\frac{1}{2}$ of vertical Reaction.

BM. of $x-x$.

$$M_x = R_A \cdot x - H \cdot y - \omega \cdot x \cdot \frac{x}{2}$$

$$\frac{y}{R} = \sin\theta$$

$$y = R\sin\theta$$

$$\frac{R-x}{R} = \cos\theta$$

$$R - x = R\cos\theta$$

$$x = R(1-\cos\theta)$$

(78)

$$M_x = R_A \cdot x - H_y \cdot \omega \cdot x \cdot \frac{x}{2}$$

$$\Rightarrow \omega R \cdot R(1-\cos\theta) - \frac{\omega R}{2}(R\sin\theta) - \frac{\omega}{2} R^2 (1-\cos\theta)^2$$

$$M_x = \omega R^2 \left[1-\cos\theta - \frac{\sin\theta}{2} - \frac{1}{2}(1-\cos\theta)^2 \right]$$

$$\Rightarrow \omega R^2 \left[1-\cancel{\cos\theta} - \frac{\sin\theta}{2} - \frac{1}{2} - \frac{\cos^2\theta}{2} + \cos\theta \right]$$

$$M_x = \omega R^2 \left[\frac{1}{2} - \frac{\sin\theta}{2} - \frac{\cos^2\theta}{2} \right]$$

$$M_x = \frac{\omega R^2}{2} \left[\cancel{\sin\theta}\ 1 - \sin\theta - \cos^2\theta \right].$$

$$M_x = \frac{\omega R^2}{2} \left[\sin^2\theta - \sin\theta \right]$$

$$M_x = -\frac{\omega R^2}{2} \left[\sin\theta - \sin^2\theta \right]$$

for M_{max} $\frac{dM}{d\theta} = 0$

$$\Rightarrow [\cos\theta - 2\sin\theta \cdot \cos\theta] = 0$$

$$\cos\theta(1-2\sin\theta) = 0$$

$\sin\theta = \frac{1}{2}$ $\quad$ or $\cos\theta = 0$

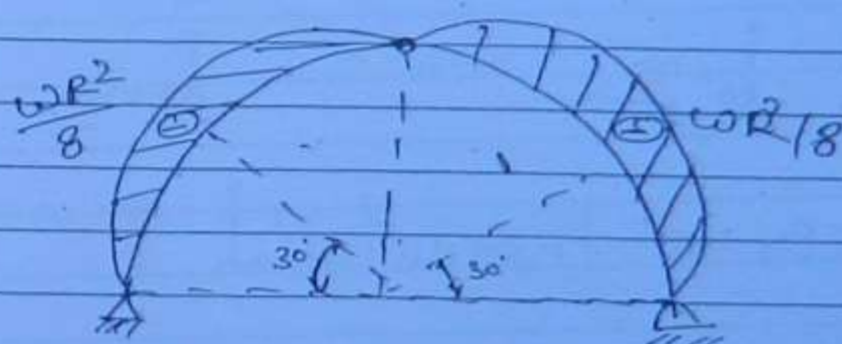
$\theta = 30$ $\quad$ or $\cancel{\cos}\ \theta = 90^\circ$

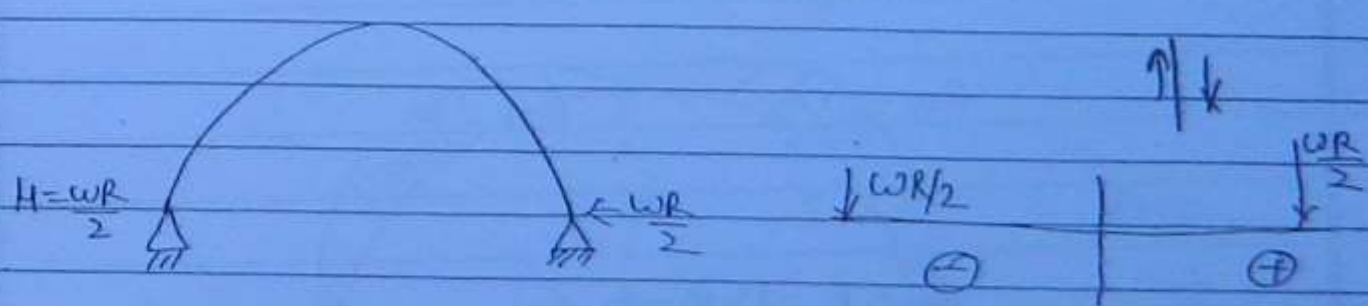
max^m Horizontal B.M. occurs at $\theta = 30°$

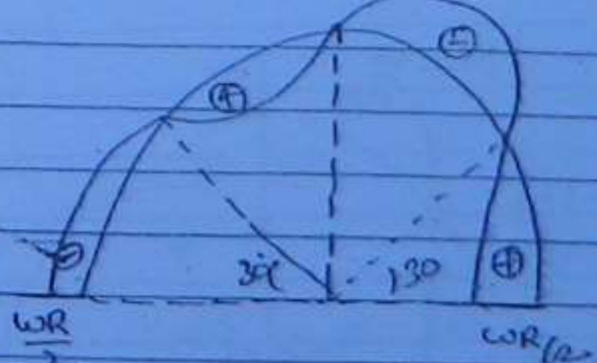
$$M_{max} = -\frac{wR^2}{2}\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$\boxed{M_{max} = -\frac{wR^2}{8}}$$

79



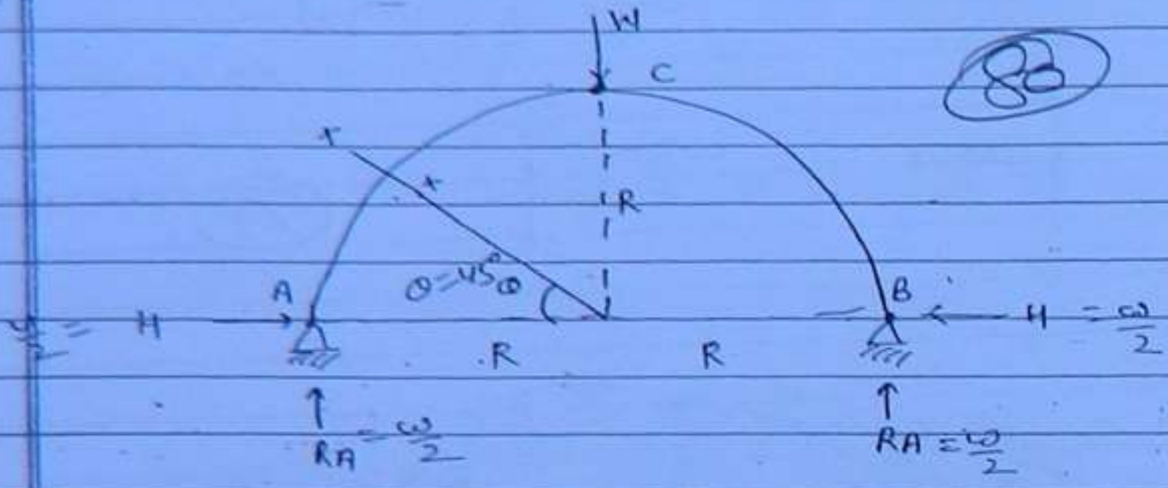




S.F.D

Q. for 3 -Hings semi-circular Arch subject to concented load w at the crown find horizontal thrust & S.F. and B.M.

Sol



(88)

$\Sigma Fy = 0 \quad RA + RB - w = 0$

$\Sigma MA = 0$

$RB \times 2R - w \times R = 0$

$RB = \frac{w}{2} \qquad RA = \frac{w}{2}$

$Mc = 0 \qquad RA \times R - H \cdot R = 0$

$\frac{w}{2} \times R - H \cdot R = 0$

$\boxed{H = \frac{w}{2}}$

$M_x = RA \cdot x - Hy$

$\Rightarrow \frac{w}{2}(x - y)$

$M_x = \frac{w}{2}\left[R(1-\cos\theta) - R\sin\theta\right]$

$$\boxed{M_x = \frac{\omega R}{2}\left[1-\cos\theta - \sin\theta\right]}$$

$\therefore$ at $\theta = 0$ $\quad M_x = 0$

at $\theta = \pi/2$ $\quad M_x = 0$ $\quad$ 81

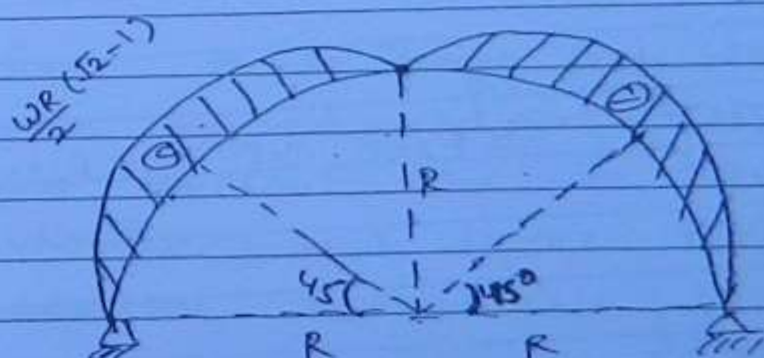
$$\frac{dM_x}{d\theta} = \frac{\omega R}{2}\left[0 + \sin\theta - \cos\theta\right] = 0$$

$$\Rightarrow \sin\theta - \cos\theta = 0$$

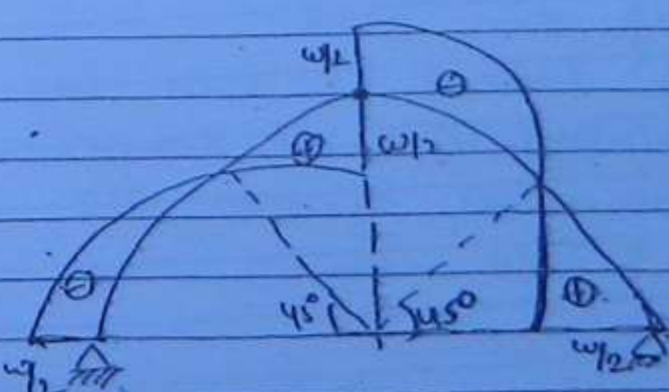
$$\theta = 45^\circ$$

$$M_{max} = \frac{\omega R}{2}\left[1 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow \frac{-\omega R}{2}\left(\sqrt{2}-1\right)$$



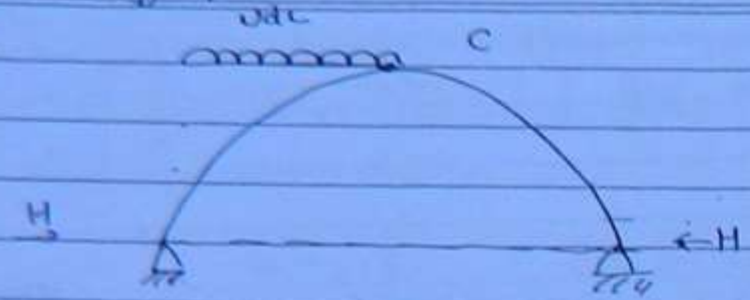
BMD

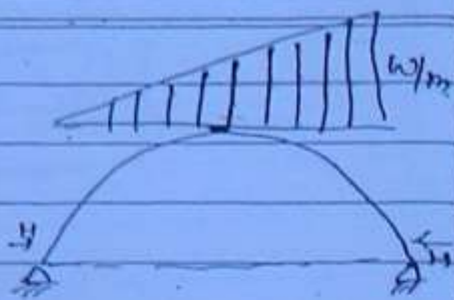


SFD

UdL for Half span.

H.W



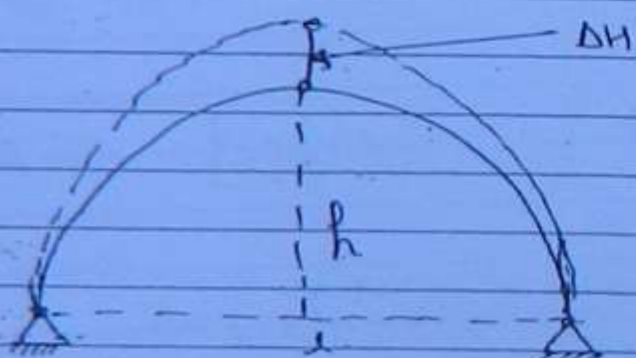


$$H = \frac{wl^2}{16h}$$

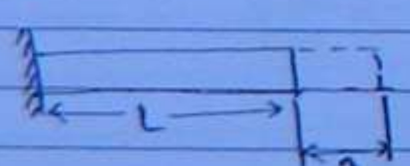
(82)

$$H = \frac{wl^2}{16h}$$

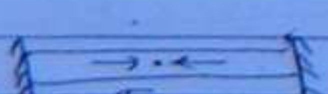
Temperature effect on 3-hinged Arches :-



$$\Delta h = \left(\frac{l^2 + 4h^2}{4h}\right)\alpha T$$ = Free $\exp^n$ in Rise of Arch

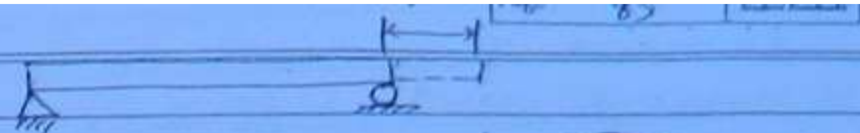


$\Delta = L\alpha T$



$\Delta = 0$

$\sigma = E\alpha T$



A 3-hinged Arch is a determinate structure and due to Rise in temp. vertical Rise (h) well increase by Δh which is called free expansion.

There will be two cases.

Case-1 If Arch is initially unloaded ~~but~~ when horizontal and vertical Reaction are not present then due to rise in temp. vertical heigh will increase by Δh but horizontal and vertical reaction at the supports will remains zero.

Case-2 If before temp. change Arch is loaded due to which ~~two~~ horizontal and vertical Reactions are ~~pres~~ present due to rise in temp. vertical height increase and Hence Horizontal thrust ~~regit~~ ~~reducted~~ redued. $\left(H \propto \frac{1}{h}\right)$

There will be no change in vertical reaction let arch is loaded with udl due to load Horizontal thrush $\left(H = \frac{wl^2}{8h}\right)$. if due to temp. rise by $T°C$ 3-Hinge Horizontal thrust is dH

$$dH = \frac{wl^2}{8}\left(-\frac{1}{h^2}dh\right)$$

$$\boxed{\frac{dH}{H} = -\frac{dh}{h}}$$

It means % decrease in Horizontal thrust is equal to % increase in vertical Rise.

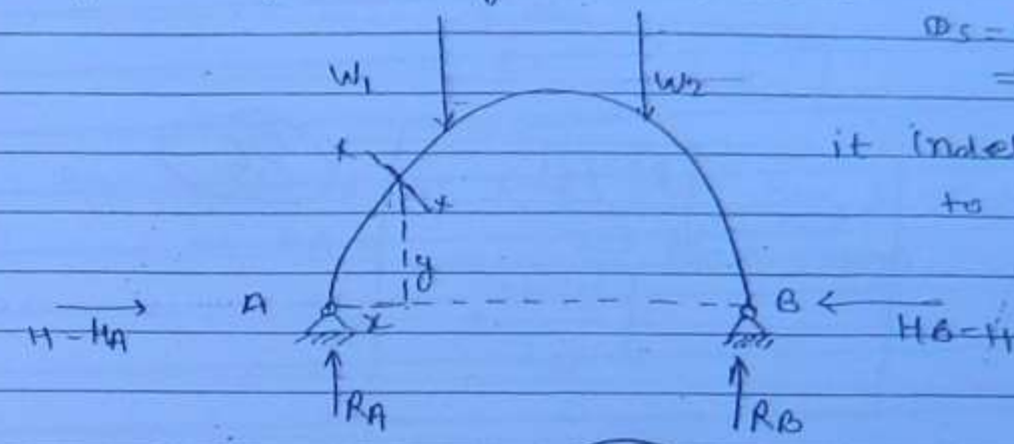
(84)

NOTE:-

Generally due to temp. effect determinate structure undergoes free expn. Hence no reactions are produced due to temp. change. provided structure is unloaded but in in-determinate structure due to temp. change reactions are produced.

As this a 2-Hinge arch is an indeterminate structure Hence due to temp. change Horizontal Thrust is induced at the supports even when arches unloaded. And Horizontal thrust will be increase if Arch was initially loaded.

(11) Analysis of 2-Hinged Arches :-



$D_s = r_e - 3$
$\Rightarrow 4 - 3 = 1$
it indeterminate to first order

(85)

2- Hinge arch indeterminate Structure Hence additional compatibility equ^n is required. Since Support A and B are not moveable. Hence $\boxed{\left(\frac{\partial U}{\partial H}\right) = 0}$ U is total strain energy. and Since effect of Bending moment is much greater, ~~th~~ greater than normal thrust and Share force then U is bending Strain Energy.

$$\therefore U = \int \frac{M_x^2\, ds}{2EI}$$

$$M_x = R_A \cdot x - H \cdot y$$
$$\Rightarrow M - H \cdot y$$

M = B.M. due to vertical forces only this is called = Beam moment

$$U = \int \frac{(M - Hy)^2\, ds}{2EI}$$

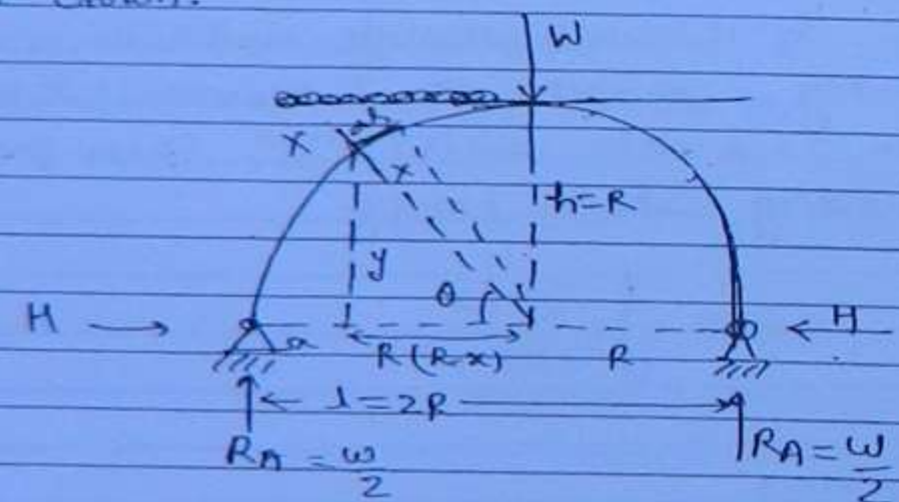
$$\frac{\partial U}{\partial H} = \int \frac{\cancel{2}(M - Hy)(-y)\, ds}{\cancel{2}EI} = 0$$

$$\frac{\partial U}{\partial H} = -\int \frac{M \cdot y\, ds}{EI} + H \int \frac{y^2\, ds}{EI} = 0$$

$$\boxed{H = \frac{\int \frac{M\, y\, ds}{EI}}{\int \frac{y^2\, ds}{EI}}}$$ (86)

Que - I

Find Horizontal thrust 2-Hinge semi-circular Arch. Subjected to a vertical load W at the crown.



$$\frac{y}{R} = \sin\theta$$

$$y = R\sin\theta$$

$$\frac{R-x}{R} = \cos\theta$$

$$R - x = R\cos\theta$$

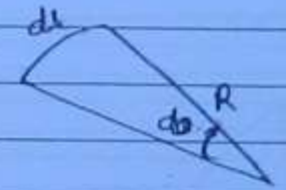
$$x = R(1-\cos\theta)$$

$$H = \frac{\int \frac{M \cdot y\, ds}{EI}}{\int \frac{y^2 ds}{EI}}$$ (87)

M = Beam Moment at x-x [BM due to vertical force only]

$\Rightarrow RA \cdot x$

$\Rightarrow \frac{\omega}{2} \cdot x$



$ds = R \cdot d\theta$

$$H = \frac{\cancel{2} \int_0^{\pi/2} \frac{\frac{\omega}{2} R(1-\cos\theta) R\sin\theta\, R \cdot d\theta}{\cancel{EI}}}{\cancel{2} \int_0^{\pi/2} \frac{(R\sin\theta)^2 R\, d\theta}{\cancel{EI}}}$$

$$H = \frac{\frac{\omega}{2} R^2 \int_0^{\pi/2} (1-\cos\theta) d\theta}{R^3 \int_0^{\pi/2} \sin^2\theta\, d\theta}$$

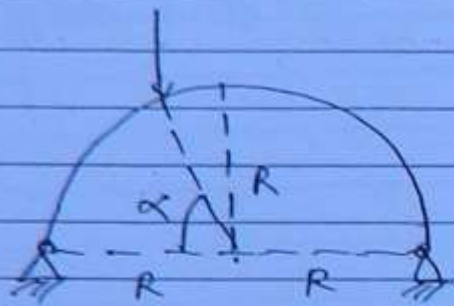
$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$H = \frac{\frac{\omega}{2} R^2 \left[\theta + \sin\theta\right]_0^{\pi/2}}{R \frac{R^3}{2} \left[1-\cos 2\theta\right]_0^{\pi/2}} \Rightarrow$$

* $\boxed{H = \frac{\omega}{\pi}}$ *

Special case-I If Concentrated load & radius vector α as shown in fig. than Horizontal thrust H is

$$\boxed{H = \frac{W}{\pi}\sin^2\alpha}$$

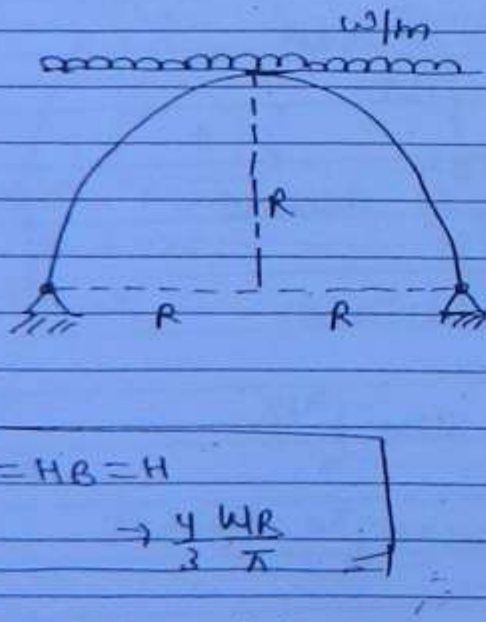


88

Special case-II

2 Hinge semi-circular Arch subjected to UDL over entire spane

EI Constant



$$\boxed{H_A = H_B = H \rightarrow \frac{4}{3}\frac{WR}{\pi}}$$

(III) If udl acts over half span, or UVL acts over entire span than Horisental thrust is

$$\boxed{H = \frac{2}{3}\frac{WR}{\pi}}$$

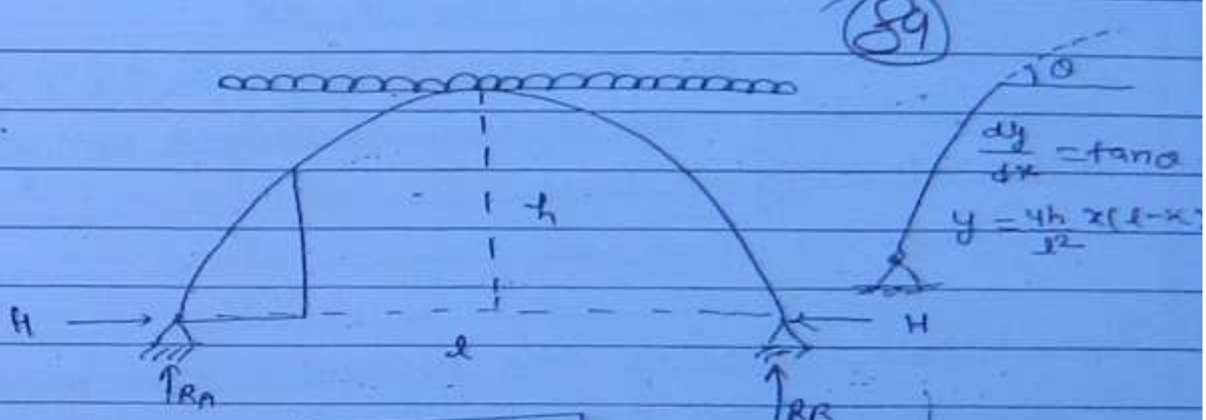
Case-2 2-Hinged Parabolic Arch Subjected to udl over entire Span.

Find Horizontal thrust at the supports if moment of inertia varies as $I = I_0 \sec\theta$

when θ is angle of tangent with the Horizontal

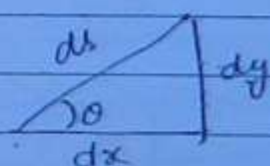
I_0 = moment of Inertia at the Crown

89



$$\boxed{I = I_0 \sec\theta}$$

$$R_A = R_B = \frac{wl}{2}$$



$$H = \frac{\int \frac{M^0 y\, ds}{EI}}{\int \frac{y^2\, ds}{EI}}$$

$$\frac{dx}{ds} = \cos\theta$$

$$ds = dx \cdot \sec\theta$$

$$H = \frac{2\int_0^{l/2} \frac{\left(\frac{wl}{2}\cdot x - \frac{wx^2}{2}\right)\cdot \frac{4h}{l^2}x(l-x)\cdot dx \sec\theta}{EI_0 \sec\theta}}{2\int_0^{l/2} \frac{\left[\frac{4h}{l^2}x(l-x)\right]^2 dx \sec\theta}{EI_0\sec\theta}}$$

$$H = \frac{w l^2}{8h} \frac{\cancel{\int_0^{l/2} x^2 (l-x)^2 dx}}{\cancel{\int_0^{l/2} x^3 (l-x)^2 dx}}$$

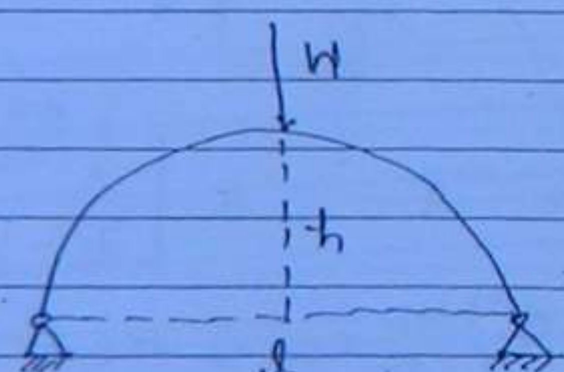
$$\boxed{H = \frac{w l^2}{8H}}$$

90

which is Same as in 3-Hinge parabolic Arch There Bending moment and shareforce will be zero at any section.

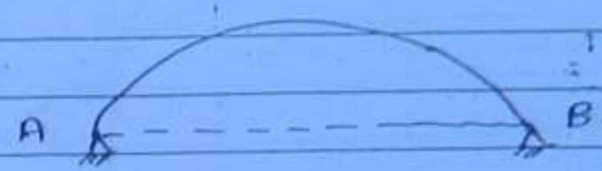
Two-Hinged parabolic Arch subjected to concentrated load w at the crown moment of Inertia $I = I_0 \sec\theta$

$$\boxed{H = \frac{25}{128} \frac{w l}{h}}$$



* Tempeture Effect in 2-Hinged Arch:- *

(9)



Due to rise in temp. the horizontal thrust induced given by

$$\boxed{H = \frac{L\alpha T}{\int \frac{y^2 \cdot ds}{EI}}}$$

Special Case-I

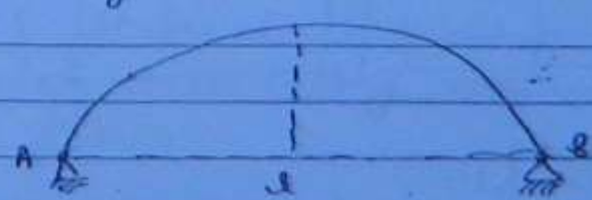
2-Hinged semi-circular Arch sub. to Rise in temp. by T°C

$$\boxed{H = \frac{4EI\alpha T}{\pi R^2}}$$

for Semi-Circular Arch EI is Constant.

Special Case-2 for 2-Hinged Parabolic Arch

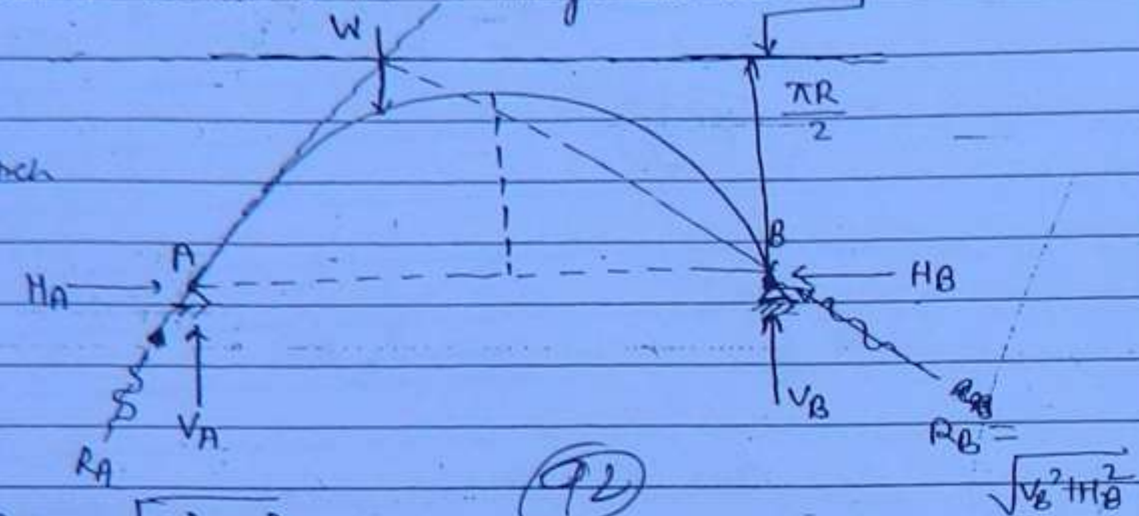
$$\boxed{H = \frac{15}{8}\frac{EI_o\alpha T}{h^2}}$$



$I_o \Rightarrow$ moment of Inertia at the crown.

* Reaction locus in 2-Hinge Arches:

2 Hinged Semi circular Arch

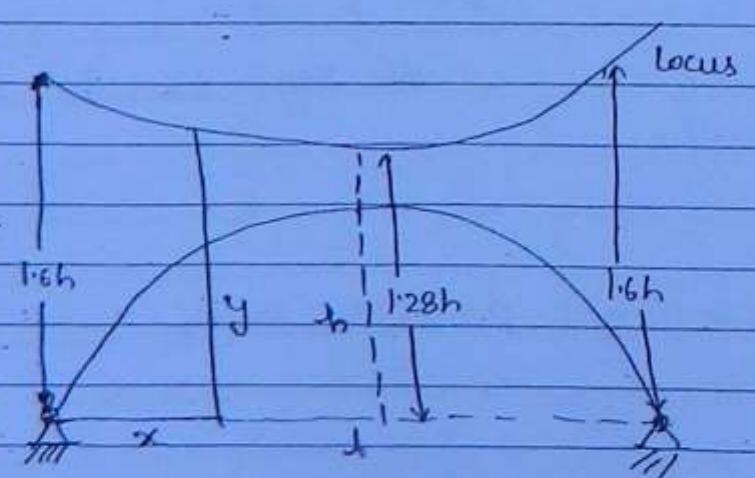


$R_A = \sqrt{V_A^2+H_A^2}$

Case-II 2 Hinge semi circular Arch.

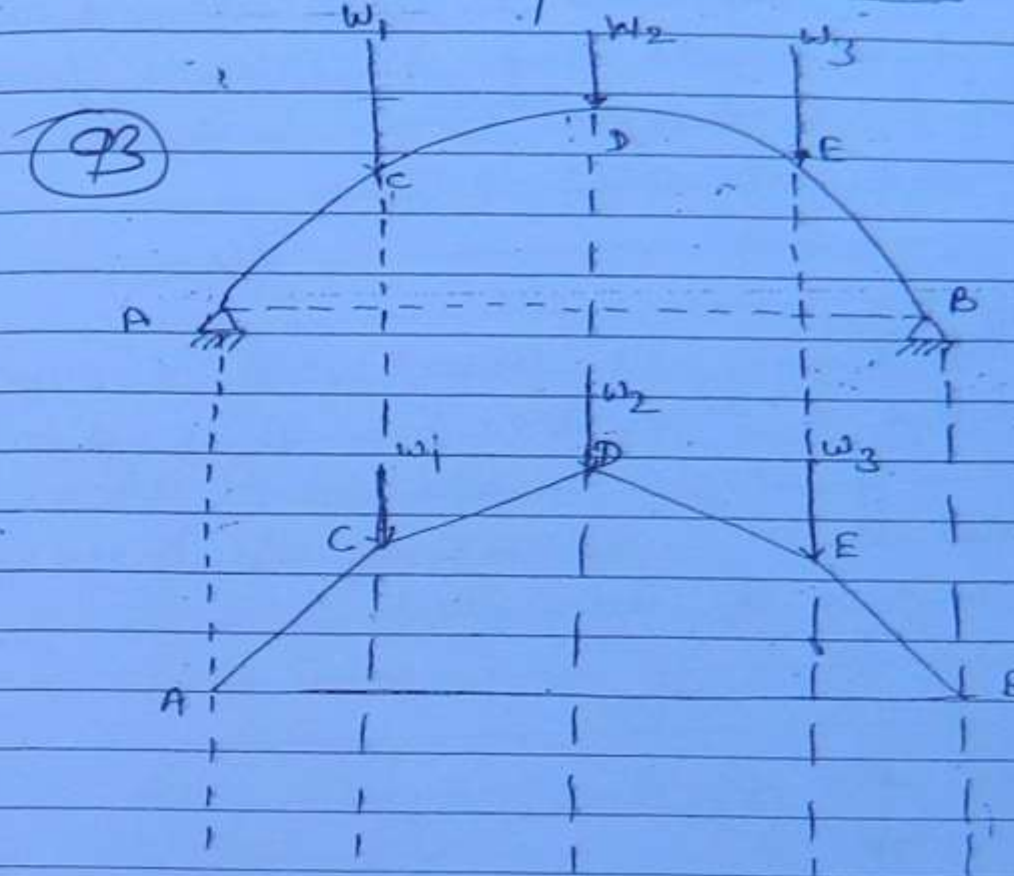
"Reaction locus is the locus of point of the intersection of 2-Resultant Reactions at the support when a point load moves from one end to the other end. for a 2-Hinged Semi-circular Arch reaction locus is a Straight line, whereas for a 2-Hinged parabolic Arch reaction locus is a curve.

Parabolic Arch



$$y = \frac{1.6hl^2}{l^2+lx-x^2}$$

Linear Arch / Theoretical Arch.



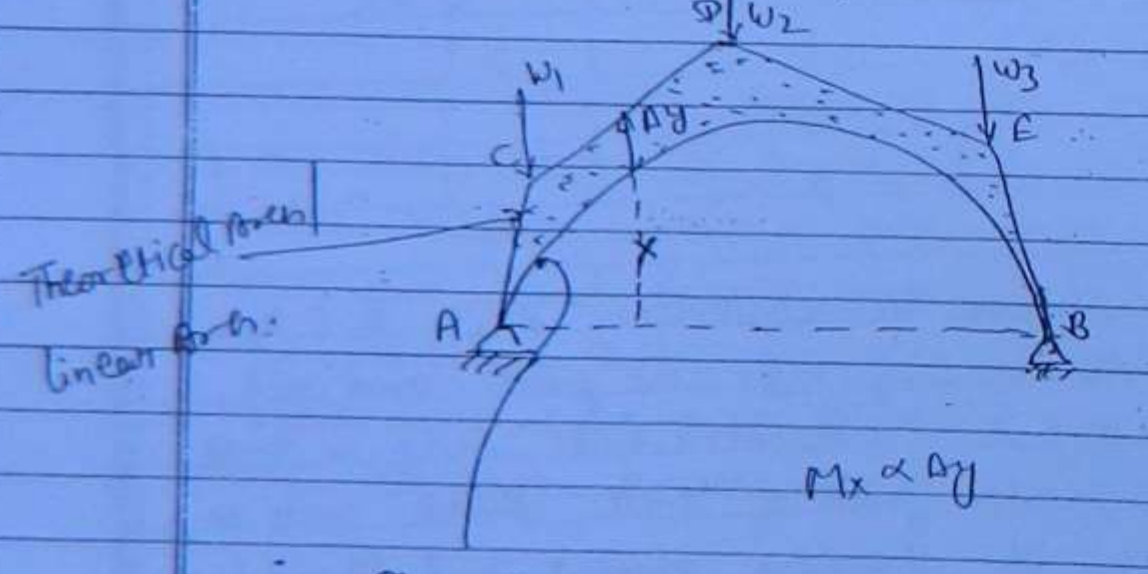
Imgiane a Structure A, C, D, E & B Subjected to loads W_1, W_2, W_3 at joints and all joints A, C, D, E & B to pin connected like a truss. if loads Act on the joints of truss there will be only Axial force No Bending moment and Shear force the shape of the above structure is similar to ~~fute~~ Funicular Polygon.

A linear Arch / theoretical Arch is a Imaginary structure which has Pin joint member having shape of funicular Polygon. the member of linear Arch carry only axial compression force & No S.F. & B.M

Since Funicular polygon depends on loading & with the shape in position of load, ∴ Shape of funicular polygon changes Hence it is difficult to constructed a funicular polygon.

94

Eddy's Theorem :- If linear Arch is superimposed over the given Arch, then B.M. at the any section on the given arch is proportional to the ordinate of the intercept b/w Given Arch & Linear Arch.



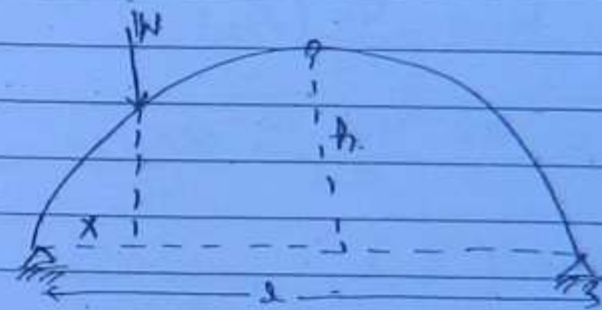
$M_x \propto \Delta y$

Work Book

(1) c	(7) a	(13) a	(19) c
(2) a	(8)	(14) c	
(3) b	(9) b	(15) b	
(4) c	(10) a	(16) a	
(5) a	(11) b	(17) E only (1) wright	
(6) c	(12) a	(18) B	

95

(2)



max^m B.M. will occur below occur load is at a distance x.

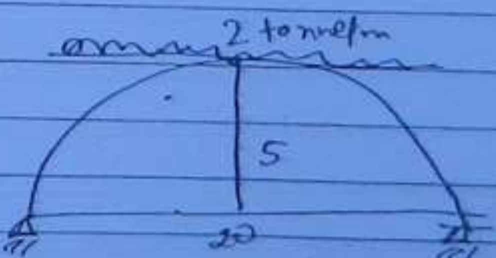
$$y = \frac{4h}{l^2} x(l-x)$$

$$M_x = R_A \cdot x - H_A \cdot y$$

for M_{max} $\frac{dM}{dx} = 0$

$$\boxed{x = 0.211L}$$

(6)

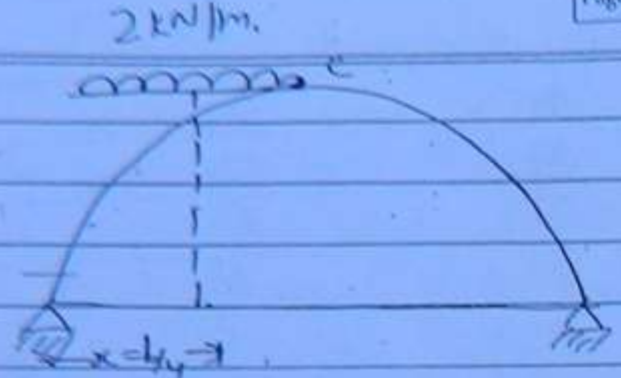


$$H = \frac{\omega l^2}{8H}$$

$$H = \frac{2 \times 20 \times 20}{8 \times 5}$$

$$H = 20$$

(17)



96

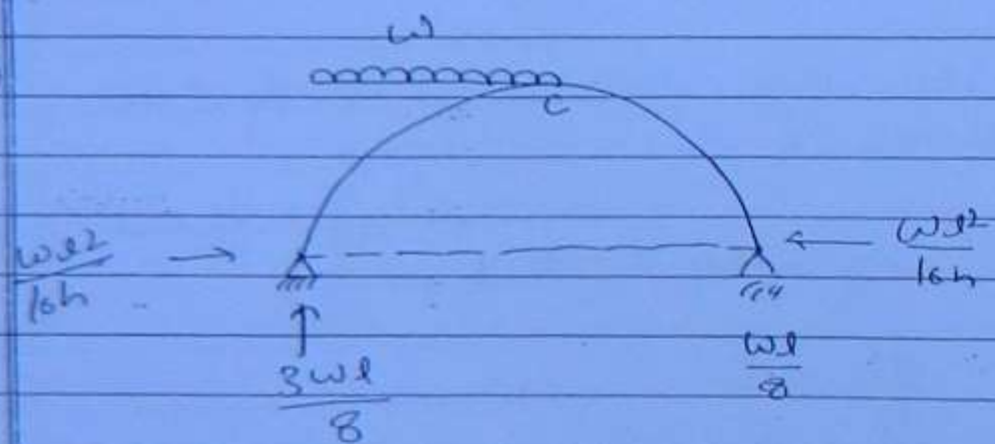
(i) $\theta = \tan^{-1}(6)$

$$\tan\theta = \frac{4h\,(\cancel{l}/2)}{l\cancel{l}}$$

$$\tan\theta = \frac{\cancel{4}^{2}h}{\cancel{2}l} = \frac{2\times 10}{40} = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\tfrac{1}{2}\right)$$

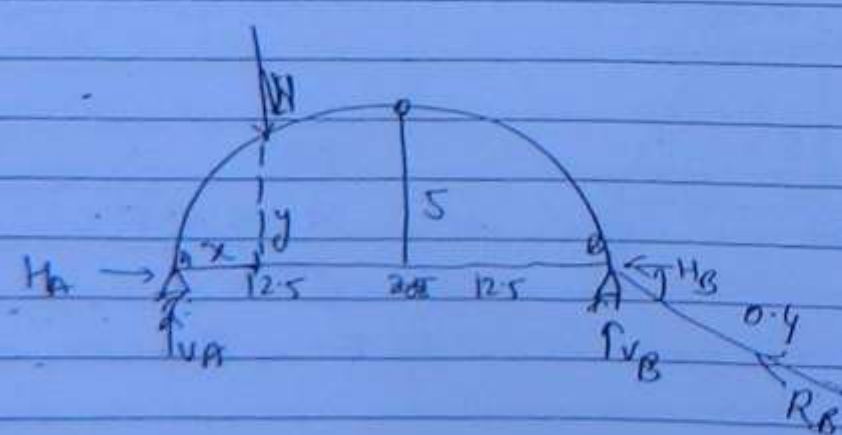
(18)



$$M_c = \frac{wl}{8}\times\frac{l}{2} - \frac{wl^2}{16h}\times h$$

$$M_c = 0$$

(19)



loe=

$$H_B = R_B \cos\theta$$
$$V_B = R_B \sin\theta$$

$$\tan\theta = \frac{V_B}{H_B}$$

97

$M_C = 0$

$V_B \times 12.5 - H_B \times 5 = 0$

$$\frac{V_v}{H_B} = \frac{5}{12.5} = 0.4$$

-: Methods of Analysis :-

Methods of Analysis

- Exact Methods
  - force method / method of consistent deformation / Flexibility mtd / Compatibility mtd
    - Ex. · Strain Energy / min^n potential Energy method / Castigliano's Theorm.
    - → unit load method
    - → Clapeyron's three moment thearm.
    - → virtual work method
    - → column analogy method
    - → Flexibility method
  - Displacement mtd / stiffness mtd / Equilibrium mtd
    - Ex.
    - → moment distribution method / Hard Cross mtd / Mtd. of successive approximation
    - → Slop defl^n method / G.A. maney mtd
    - → Kani's method / Method of Iteration
    - → stiffness matrix method.
- Approximate method
  - Ex. · Cantilever mtd
    - · Factor method
    - · portal mtd.

98

Difference b/w force method & Stiffness method.

F/F/C (force / Flexibility / Compatibility)	D/S/E (Displacement / Stiffness / Equilibrium)
→ Basic unknowns are taken redundent forces which may support reaction and member forces	→ Basic unknowns are taken Joint displacement (Δ & θ)
→ The no. of Redudent forces = No. of Degree of Static indeterminacy (D_s)	→ the no. of displacement at joint / load is equal to degree of kinematic indetermancy (D_K). In Rigid frame & Beam, effect of Axial force is negelated.
→ To determine Redudent to compatibility equ. are written no. of compactibility equ^n is equal to degree of Static intetermany (d_s)	→ To find Joint Displacement Joint eq^m conditions and shear equ^n are written. No. of equcation equal to Degree of kinematic indetermay
→ force method is suitable when $\boxed{D_S < D_K}$	Displacement method is suitable when $\boxed{D_K < D_S}$

99

Note-

① For Indeterminate Structure to be analysis by exact methods rigidity, stiffness of member should be known. if Indeterminate structure are to be analysis for which stiffness is not known than approximate method can be used

② If Ds and Dk Both are heigh such as case of multistory building & multispane than neigh ~~...~~ neither force method nor displacement method is appopriate, under such condition approximate analysis is used.

Tao

③ methods of Analysis of Indeterminate Structure may be applied for determinat structure also. But Simple Problem may given lengthy computation & solution.

Principal of superposition:-

Assumption:-

→ material is Isotropic, Homogeneous, Linear elastic in which hook's law is followed.

→ Temp. is constant

→ supports are unyielding.

Principal:- In a Beam, Truss or frame which may be determinate or indeterminate the resultent stress function due to multiple loading is equal to some of effect of indivisal loading.

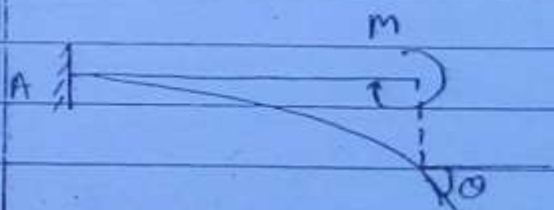
Note It is valid for those stress function which have linear relationship with loading

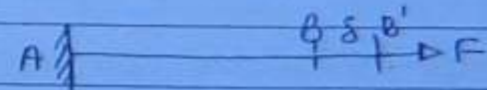
Castigliano Theorem:-

Theorem-1:-

In any structure the material of which is linear elastic, temp. is constant and supports are unyielding than first partial derivate of total strain energy with respect to any displacement component is equal to force apply in the direction of that displacement.

$$\frac{\partial U}{\partial \Delta} = P$$



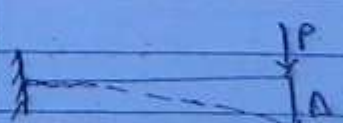


$$\frac{\partial U}{\partial \delta} = F$$

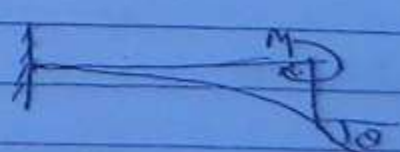
$$\frac{\partial U}{\partial \theta} = M$$

Theorem-2:-

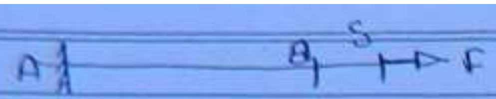
In a Beam, truss or fram (any structure) the material of which is elastic, temp. is constant, supports are unyielding the first partial derivate of total strain energy with respect to any force is equal to displacement in the dir^n of the force.



$$\frac{\partial U}{\partial P} = \Delta$$



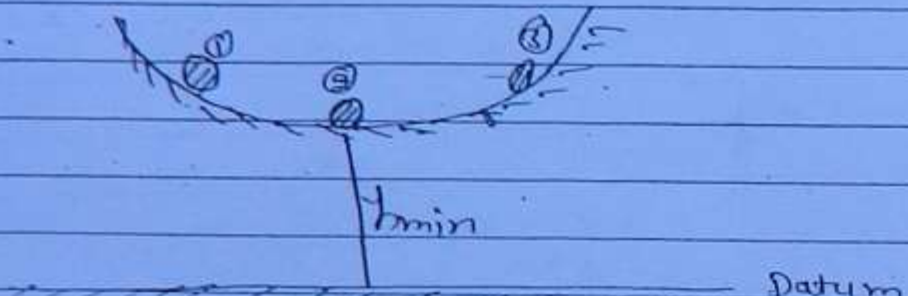
$$\frac{\partial U}{\partial M} = \theta$$



$$\frac{\partial U}{\partial P} = \delta$$

$$\frac{\partial U}{\partial P} = \delta$$ 102

Principle of minimum potential energy:-



$U_{min} = mgh_{min}$

Statement:- "Of All the geometrically Compatible State of structure which Satisfied deflection boundary Conditions force equ^n requirement will have final Stable Condition. when total potential energy in the system is min^n."

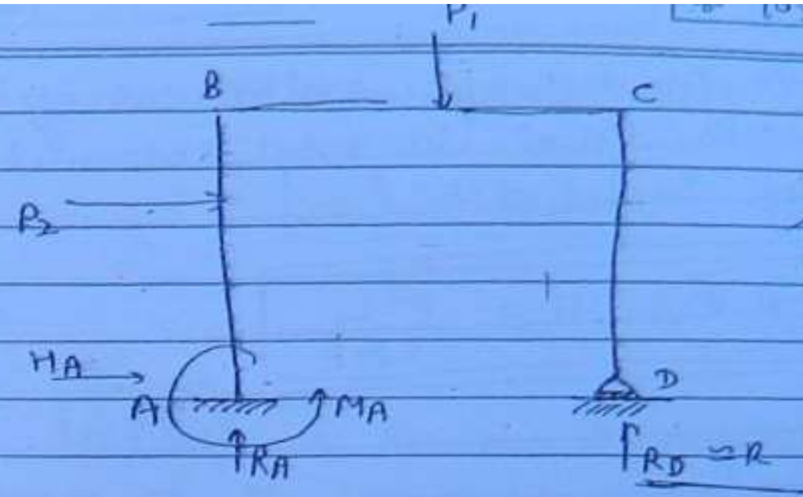
or

other words:-

If a Structure is loaded and there are redundant reaction for which infinite solution may be possible than true solution will be that for which total ~~minimum strain~~ potential energy min^m.

Let In above frame Redundant is $R_B = R$. True solution of R is such that U is min^n

$$\Rightarrow \boxed{\frac{\partial U}{\partial R} = 0}$$ → Same as Castigliano theorem



$D_s = 4 - 3 = 1$

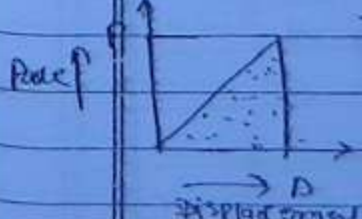
No. of Redundante $= D_s = 1$

~~Betti's Laws~~

<u>Betti's Low / Relaigh Theorem:-</u>

"In any structure the material of which is elastic and follows hook's law in which supports are unyielding, temp. is constant, external virtual work done by P system of forces during the displacement cause by Q system of forces is equal to external virtual workdone by Q system of forces when deformation is cause by P system of forces.

→ Δ is displacement given by in its direction. Then work done by P is realwork done

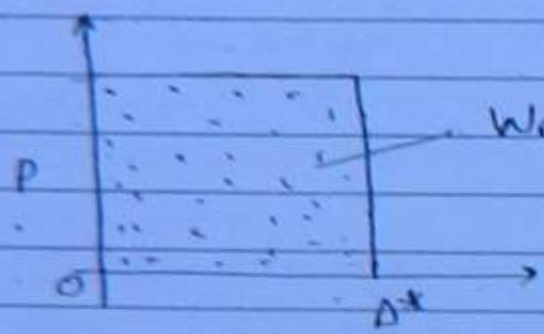


$$W_e = \frac{1}{2} \cdot P \cdot \Delta$$

If there is no loss of work in heat of or transmission it is equal to total Internal Strain energy Stored. (U_i)

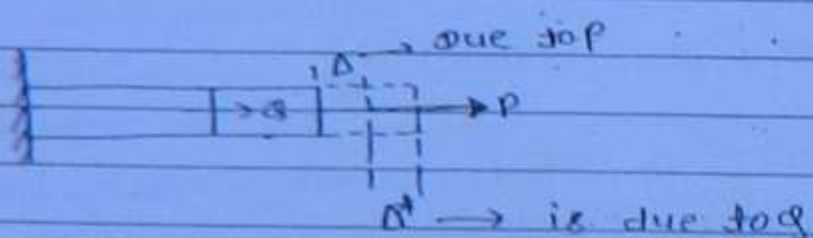
$$\boxed{W_e = \frac{1}{2} \cdot P \cdot \Delta = U_i = \frac{P^2 L}{2AE}}$$

(204)



$W_e^* = P \cdot \Delta^*$

Virtual work



Let Δ_1^* is displacement given in the direction of force P by any other force Q then external work done by P is ($W_{e_1}^*$)

$$\boxed{W_{e_1}^* = P \cdot \Delta_1^*}$$

Δ_2^* is displacement given by P in the direction force Q then external virtual work done by Q is

$$\boxed{W_{e_2}^* = Q\,\Delta_2^*}$$

According to Relaigh thearem | Be.tt's law

$$\boxed{We_1^* = We_2^*}$$

105

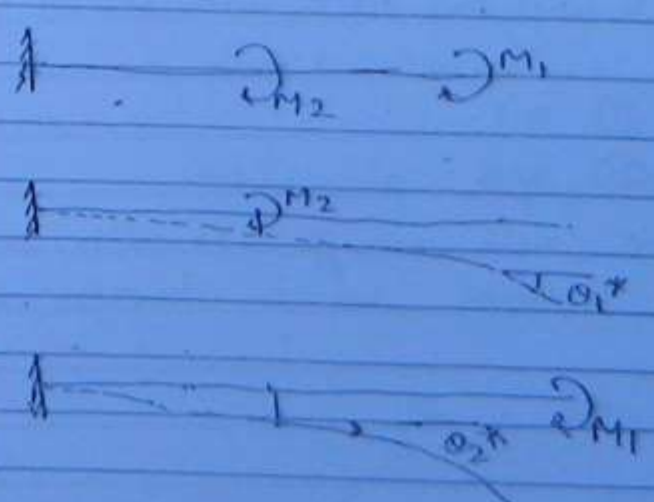
Special Case(s):-

Let θ_1^* is Rotational | Angular displacement in direction of M_1 (moment) produced by moment M_2 acting at some other point. And let θ_2^* is Rotational in direction of M_2 caused by moment M_1, then external virtual workdone by M_1 is equal to external virtual workdone by M_2.

Ex. Virtual workdone by M_1 = Ex. Virtual work done by M_2

$$\boxed{M_1 \theta_1^* = M_2 \theta_2^*}$$

if displacement is in direction of force work done will be +ive. If displacement is in opposite direction of force then work done will be (−ive)



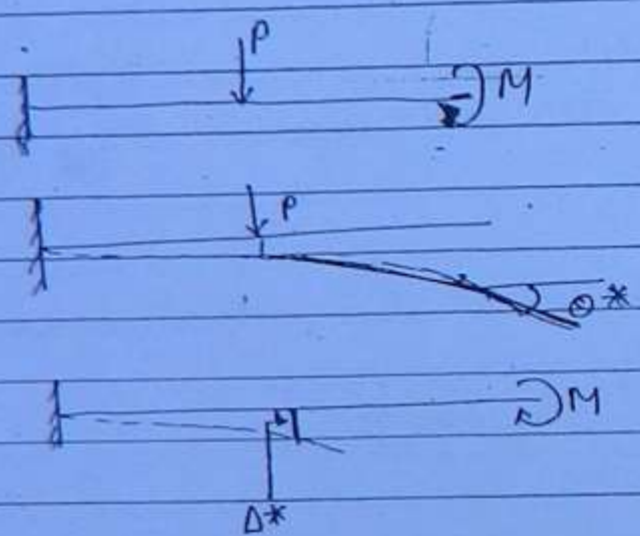
$$We_1^* = M_1 \theta_1^*$$

$$We_2^* = M_2 \theta_2^*$$

Special Case-II :- If θ^* is angular displacement ~~of~~ in the directim of moment in due to a linear force P acting at any point and let Δ^* is linear displacement in the direction of force P due to moment in. acting at Some other point. then virtual workdone by m is equal to virtual workdone by P

$$M\theta^* = P\Delta^*$$

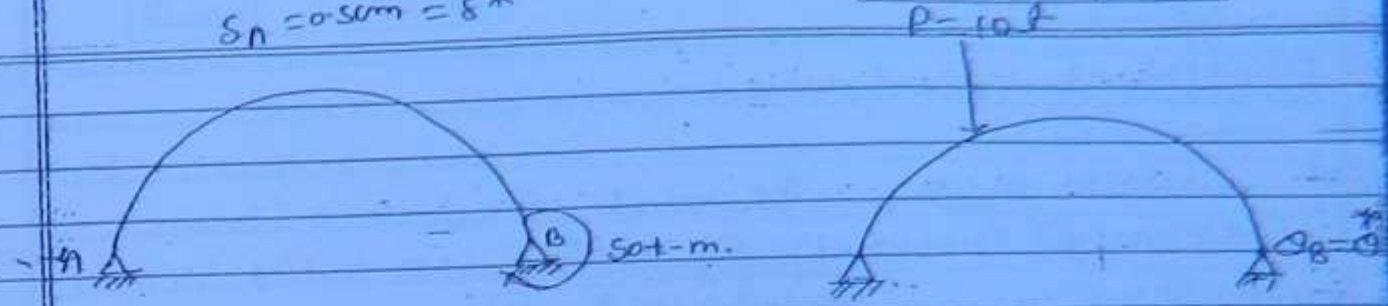
(106)



Ex. for the semi-circular two-hinge Arch as Shown in fig. a moment of 50 ~~to~~ tonne-cm applied at B which Produces a displacement of 0.5 cm at A. if a Concentrated load of 10 tonnes is applied at A then rotation at B in the arch will be.

(A) 0.01
(B) 0.1
(C) 0.05
(D) 0.5

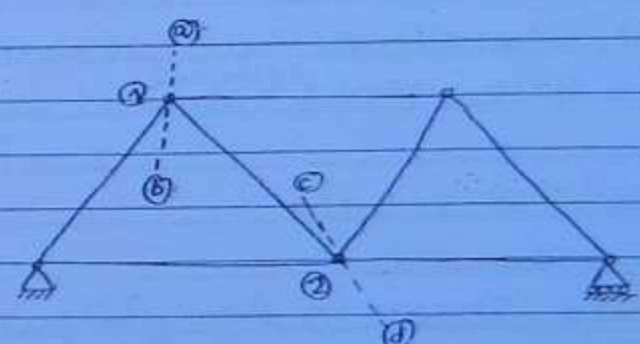
$\delta_A = 0.5cm = \delta^*$



$$M\theta^* = P \cdot \Delta^*$$
$$50 \times \theta^* = 10 \times 0.5$$
$$\boxed{\theta^* = 0.1}$$

107

Maxwell Reciprocal theorem :-



→ It is an Special Case of Bett's Law in which P=Q
let δ_{12} is displacement at Point 1 in the dir^n of ab due to point P placed at point ② in the direction of cd.

let δ_{21} is displacement at point ② in the direction of cd due to ~~Point~~ load P and in the direction of ab at point 1. then.

$$\delta_{12} = \delta_{21}$$

Point where disp. is measured

Point where force is applied.

Principal of virtual work :-

Case-I Elastic bodies :- The total virtual work done (External + Internal) for deformable bodies is equal to zero.

$$\boxed{W_e^* + W_i^* = 0}$$

Internal virtual workdone by internal forces in (generally five) because internal deformation is in opposite dirn to the internal forces

$$|W_e^*| = |W_i^*| = U_i^*$$

108

Case-2 Rigid bodies :- Rigid bodies do not have internal deformation therefore internal virtual workdone is zero, hence total external virtual work done is zero.

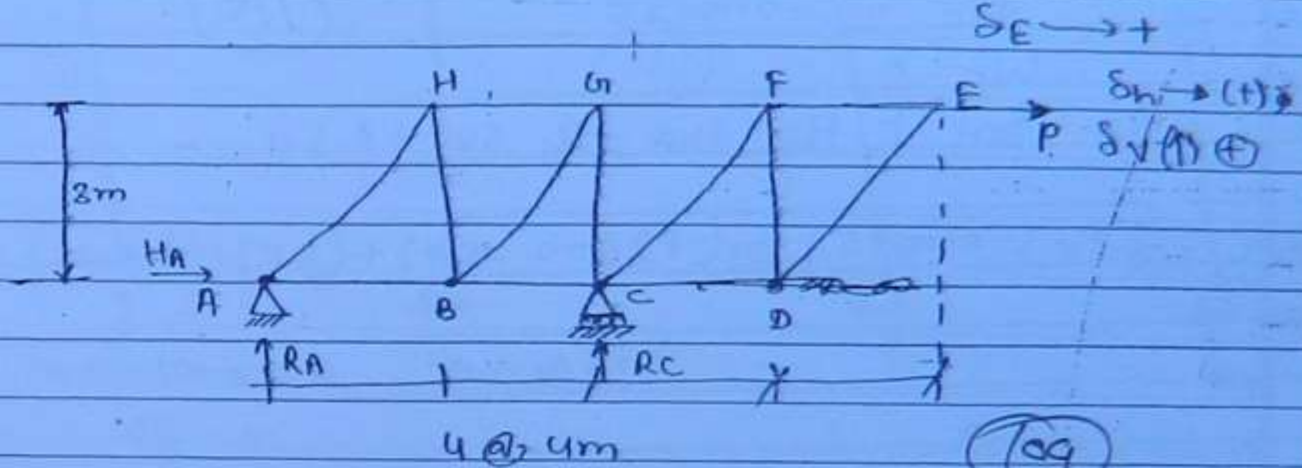
$$W_e^* = 0$$

For the truss shown in fig. compute Horizontal component of displacement at joint E when following movement of support occur is ① support A moves horizontaly by 0.0050 form Right to Left And 0.0075 vertically down

② support C move vertically down by 0.0025m.

Note that there is no change in the length of any member, because all member are considered axialy rigid.

Joint displacement ~~is~~ is due to movement of support only.



To find horizontal displacement at E, say δ_E due to support settlement. Consider ~~video~~ force P in horizontal direction at E

$\Sigma f_x = 0$

$$H_A + P = 0$$
$$H_A = P$$

$\Sigma f_y = 0$

$$R_A + R_C = 0$$

$\Sigma M_A = 0$

$$R_C \times 8 - P \times 3 = 0$$
$$R_C = \frac{3}{8} P$$

If joint displacement are given in the direction of reaction, forces than external virtual work will be done. For rigid body total external virtual work done is zero.

$\delta_{hA} = -0.0050$

$\delta_{VA} = -0.0075$

$\delta_{Vc} = -0.0025$ (1/0)

$\therefore W_e = 0$

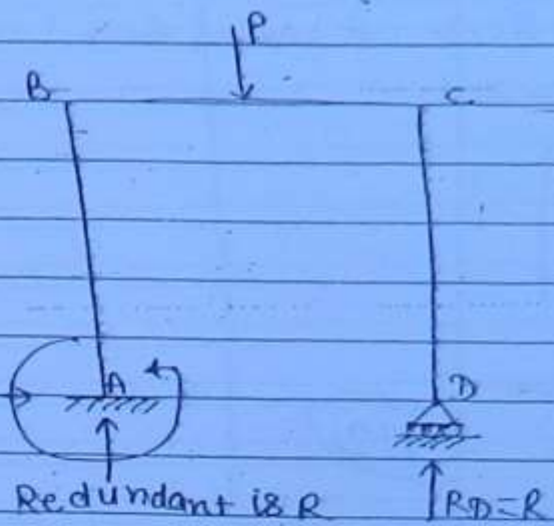
$$R_A \cdot \delta_{VA} + H_A \cdot \delta_{hA} + R_C \cdot \delta_{VC} + P \cdot \delta_E = 0$$

$$-\frac{3}{8}P \cdot (-0.0075) + (-P)(-0.0050) + \left(\frac{3}{8}P\right)(-0.0025) + P\delta_E = 0$$

$$\Rightarrow P\delta_E = -0.006785P$$

$$\Rightarrow \boxed{\delta_E = -0.006785 \text{ m.}} \ (\leftarrow)$$

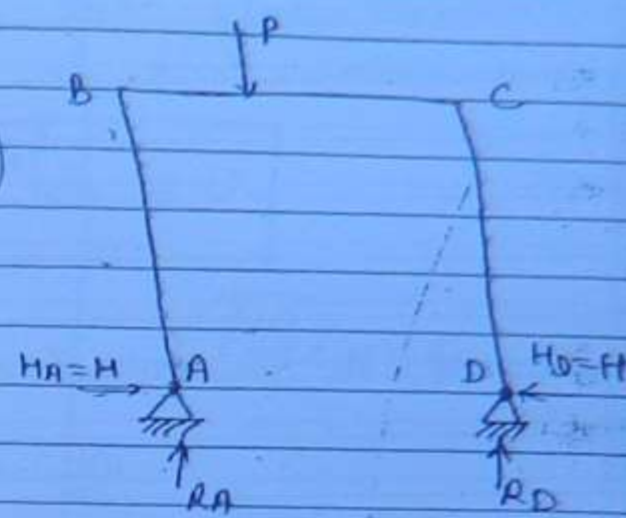
STRAIN - ENERGY METHOD



Redundant is R

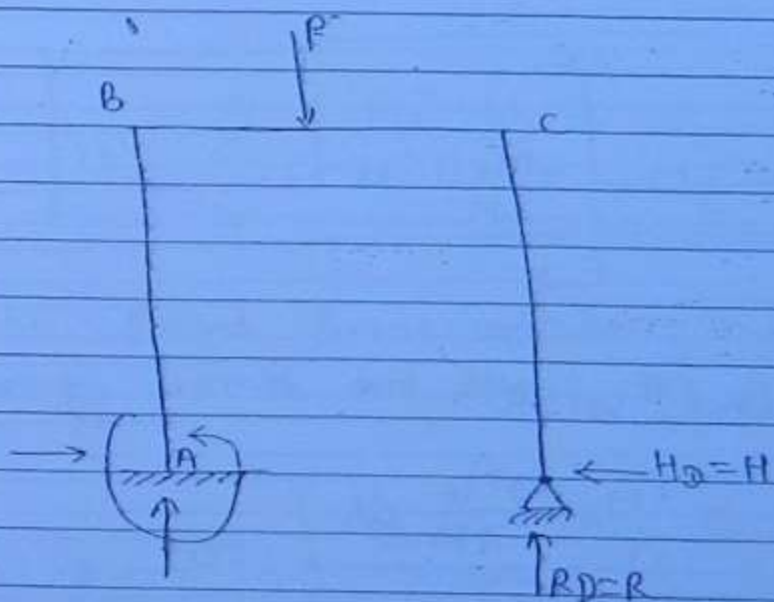
$\frac{\partial U}{\partial R} = 0$

111



Redundant is H

$\frac{\partial U}{\partial H} = 0$



Redundants are H & R

$$\frac{\partial U}{\partial H} = 0 \qquad \frac{\partial U}{\partial R} = 0$$

Note:- (1) If is Structure has Redundant reaction then for Stable equilibrium its total strain energy w.r.to redundant forces should be minimum.

$$\frac{\partial U}{\partial R} = 0 \qquad \frac{\partial U}{\partial H} = 0$$

② The strain energy considered is due to bending moment only and the effect of axial force and shear force is neglected.

$$\boxed{U = \sum \int \frac{M_x^2\, ds}{2EI}}$$

if H is redundant then.

112

$$\frac{\partial U}{\partial H} = 0$$

$$\Rightarrow \frac{\sum \cancel{2} M_x \cdot \frac{\partial M_x}{\partial H} \cdot ds}{\cancel{2} EI}$$

$$\Rightarrow \frac{\sum \int M_x \frac{\partial M_x}{\partial H} \cdot ds}{EI}$$

if Redundant is R

$$\frac{\sum \int M_x \frac{\partial M_x}{\partial R}\, ds}{EI} = 0$$

if Redundant is MA

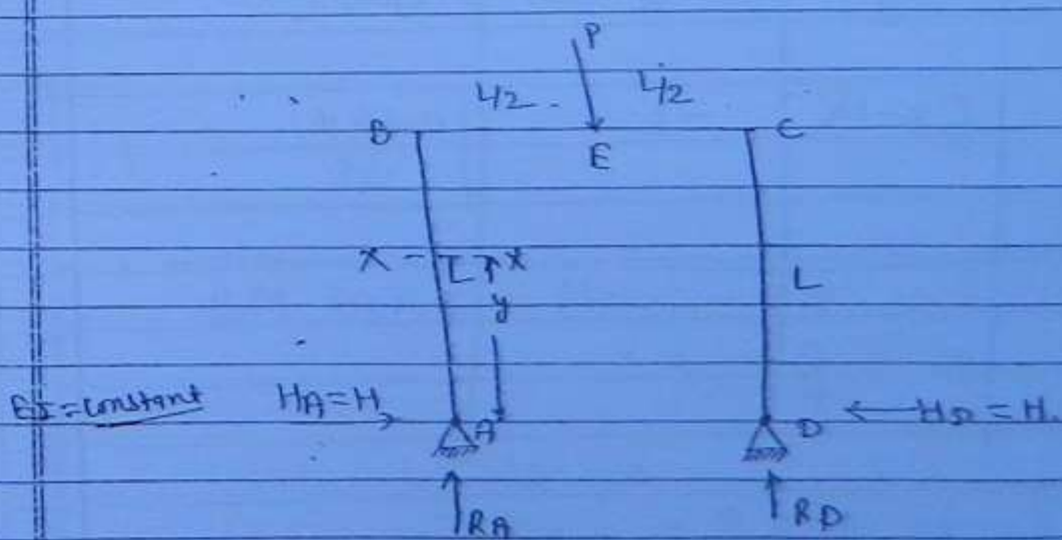
$$\frac{\sum \int M_x \frac{\partial M_x}{\partial M_A} \cdot ds}{EI} = 0$$

member	M_x	$\frac{\partial M_x}{\partial H}$	Limit of Integration.

113

$$\boxed{U = \sum \int \frac{M_x^2 \cdot ds}{2EI}}$$

Ex. Analysis the Portal frame, ~~as~~ is shown in fig. using strain energy method and draw the B.M.D.



$\Sigma f_y = 0$

$R_A + R_D - P = 0 \qquad R_A + R_D = P$

$\Sigma M_D = 0$

$R_A \times L - P \times \frac{L}{2} = 0$

$R_A = P/2$

$R_D = P/2$

114

$\Sigma f_x = 0$

$H_A - H_D = 0$

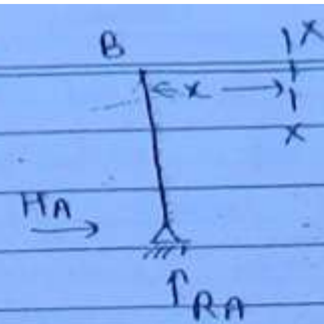
$H_A = H_D = H$ (say)

$\frac{\partial U}{\partial H} = 0$

$$\Rightarrow \frac{\Sigma \int M_x \cdot \frac{\partial M_x}{\partial H} \, ds}{EI} = 0 .$$

Member	Mx	$\frac{\partial M_x}{\partial H}$	Limit
AB	$-Hy$	$-y$	0 to L
BC	$\frac{P}{2}x - HL$	$-L$	0 to $L/2$

$M_x = -Hy$



$$M_x = R_A \times x - H \times L$$

$$\Rightarrow \cancel{R_A}\ \frac{P}{2}x - HL$$

$$\delta_{AB} = \sum \int \frac{M_x \cdot \frac{\partial M_x}{\partial H} \cdot ds}{EI} + 0$$

H/S

$$\Rightarrow 2\int_0^L \frac{(-Hy)(-y)\cdot dy}{EI} + 2\int_0^{L/2} \frac{(\frac{P}{2}x - HL)(-L)dx}{EI}$$

$$\Rightarrow \frac{2H}{EI}\left[\frac{y^3}{3}\right]_0^L + \frac{2}{EI}\left[\frac{P}{2}\frac{x^2}{2}L + HL^2x\right]_0^{L/2}$$

$$\cancel{\frac{2}{EI} H \frac{L^3}{3} + \frac{2}{EI}\left[\frac{PL^3}{4} + HL^3\right]}$$

$$\cancel{\frac{2L^3H}{EI} + \frac{2}{EI}\frac{PL^3}{4} + \frac{2HL^3}{EI}}$$

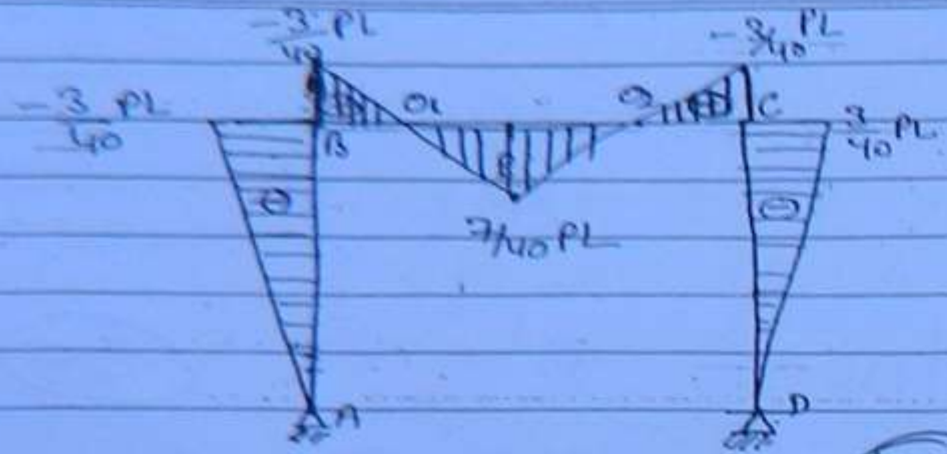
$$\boxed{H \Rightarrow \frac{3P}{40}}$$

B.M. In AB $M_x = -Hy$

$$\Rightarrow -\frac{3}{40}P\cdot y$$

✓ $M_A = 0$

✓ $M_B = -\frac{3P}{40}L \Rightarrow -\frac{3}{40}PL$



116

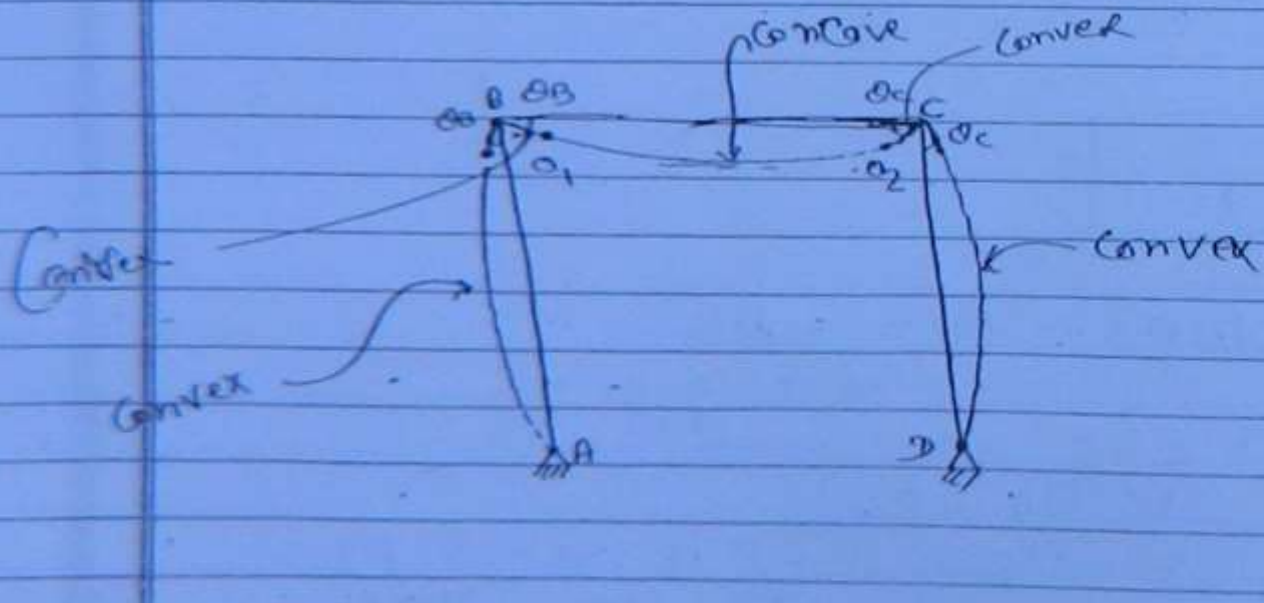
BM PE

$$M_x = \frac{P}{2}\cdot x - HL$$

$$M_B\ (x=0) := 0 - HL \Rightarrow -\frac{3}{40}PL$$

$$M_E\ (x=L/2) \Rightarrow \frac{P}{2}\times\frac{L}{2} - \frac{3}{40}PL$$

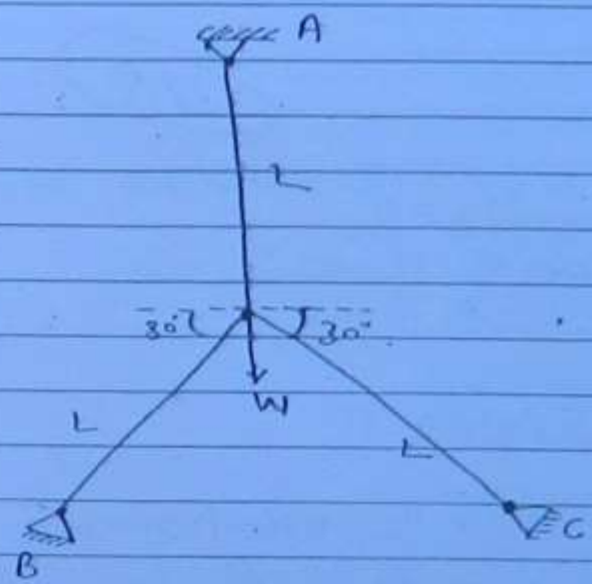
$$\Rightarrow +\frac{7}{40}PL$$



Q. 3- Identically Rods OA, OB and OC are in same vertical plane and support load W as shown in fig. forces in all members and Calculate vertical moment of joint O. Area of each member is a & all joints is pin connected.

Sol

AE = constant



117

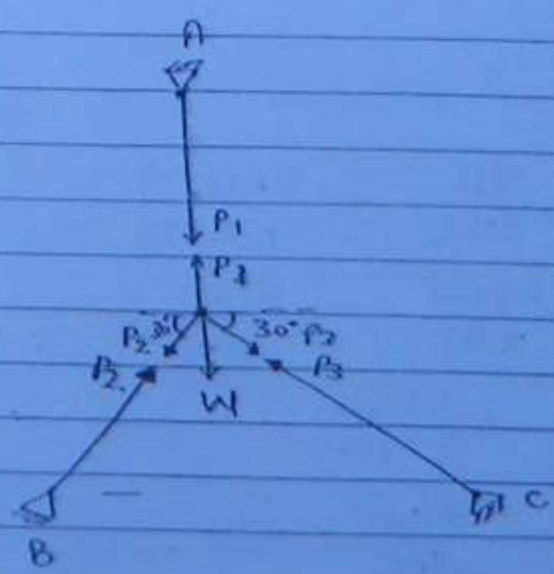
2D Truss

$D_S = m + r_e - 2j$

$\Rightarrow 3 + 6 - 2 \times 4$

$\Rightarrow 1$

let forces in members are P_1, P_2, P_3 in OA, OB, OC Respectively



Date / / Page

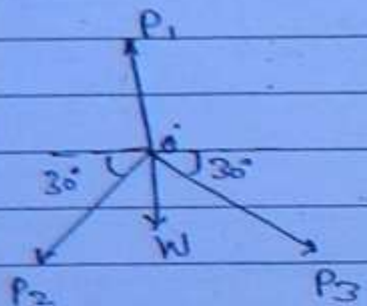
let P_1 is Redundent force:-

$$\frac{\partial U}{\partial P_1} = 0 \quad \text{---} \quad (1)$$

$$U = U_{OA} + U_{OB} + U_{OC}$$

$$U \Rightarrow \frac{P_1^2 L}{2AE} + \frac{P_2^2 L}{2AE} + \frac{P_3^2 L}{2AE}$$

118



$$\Sigma F_x = 0$$

$$P_3 \cos 30° - P_2 \cos 30° = 0$$

$$P_2 = P_3$$

$$\Sigma F_y = 0$$

$$P_1 - W - P_2 \sin 30° - P_3 \sin 30 \Rightarrow$$

$$P_1 - W - \frac{P_2}{2} - \frac{P_3}{2} = 0$$

$$\boxed{P_1 - W = P_2 = P_3}$$

$$U = \frac{P_1^2 L}{2AE} + \frac{2(P_1 - W)^2 L}{2AE}$$

$$\frac{\partial U}{\partial P_1} = 0 \Rightarrow \frac{\partial U}{\partial P_1} \Rightarrow \frac{2P_1 L}{2AE} + \frac{4(P_1 - W)L}{2AE}$$

$$\Rightarrow \frac{\cancel{2}P_1 L}{\cancel{2}AE} + \frac{\overset{2}{\cancel{4}}P_1 L}{\cancel{2}AE} - \frac{\overset{2}{\cancel{4}}\omega L}{\cancel{2}AE}$$

$$\Rightarrow \frac{3P_1 L}{AE} - \frac{2\omega L}{AE}$$

$$\frac{3P_1 \cancel{L}}{\cancel{AE}} = \frac{2\omega \cancel{L}}{\cancel{AE}}$$

$$\boxed{P_1 = \frac{2W}{3}} \quad (T) \qquad (+ive) = \text{Tension}$$

$$P_2 = P_3 = P_1 - W$$

$$\Rightarrow \frac{2W}{3} - W$$

$$\boxed{P_2 = P_3 = -\frac{W}{3}} \quad (C) \qquad \text{Compression (-ive)}$$

(119)

Method - 1

Vertical deflection of OA

$$\Rightarrow \text{Elongation of OA}$$

$$\Rightarrow \frac{P_1 L}{AE}$$

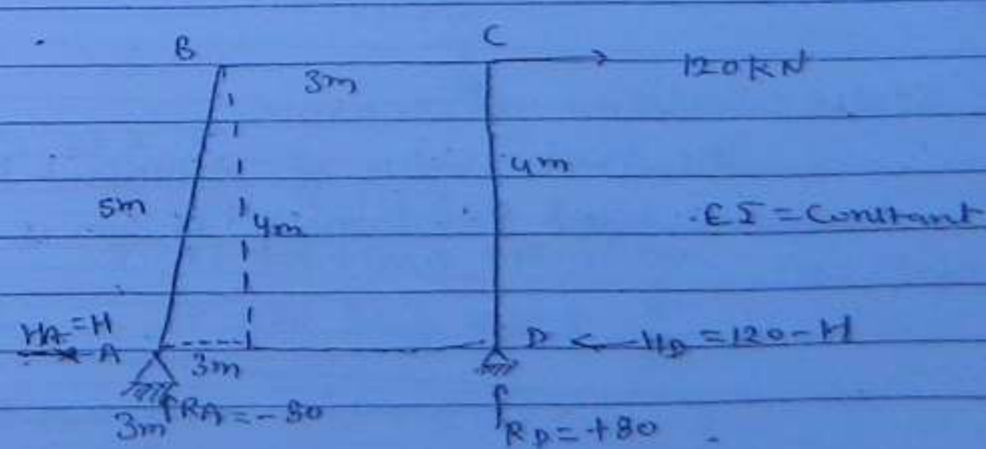
$$\Rightarrow \frac{\frac{2}{3} P \cdot L}{AE} \Rightarrow \frac{2PL}{3AE}$$

2nd method

U is Total Strain Energy Stored.

$$\frac{\partial U}{\partial W} = \Delta_0$$

Q Analaysis a portal frame shown in fig. using strain energy method. Draw Bending moment diagram.



(re) reaction = 4

$D_{se} = r_e - 3 = 4 - 3 = 1$

$D_{si} = 0$

$\Sigma F_x = 0$

$H_A + H_D = 120$ $\{H_A = H\}$

$H + H_D = 120$

$H_D = 120 - H$

~~$\Sigma F_x = 0$~~

~~$H_A + H_D = 120$~~ 120

~~$H_A = 120 - H$.~~

$\Sigma F_y = 0$

$R_A + R_D = 0$ —(ii)

$\Sigma M_A \Rightarrow$

$R_D \times 6 - 120 \times 4 = 0$

$R_D = \frac{120 \times 4}{6}$

$\boxed{R_D = 80 \text{ KN}}$

$\boxed{R_A = -80 \text{ KN}}$

Say H is REDUndant Reaction

for stable condition of structure

The True value of Redundent H is when total Pot. Energy of sym is $min^m = 0$

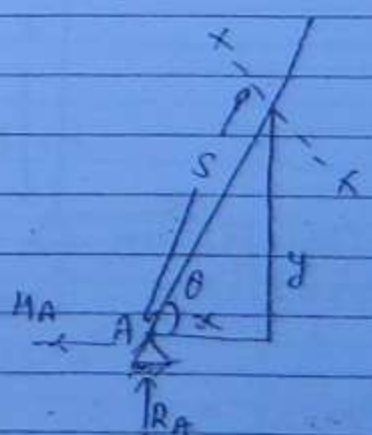
$$\frac{\partial U}{\partial H} = 0$$

$$U = \Sigma \int_0 \frac{M_x^2 \, dS}{2EI}$$

$$\frac{\partial U}{\partial H} = \sum \int \frac{M_x \frac{\partial M_x}{\partial H} \partial S}{2EI} \quad (21)$$

member	M_x	$\frac{\partial M_x}{\partial H}$	Limit of integration
AB	$-48S + 0.8HS$	$-0.8S$	0 to 5
BC	$-80(3+x)+4H$	4	0 to 3
DC	$-(120-H)y$	y	0 to 4

(1) For member AB



B.M. at x ⇒

$$M_x = R_A \cdot x + H \cdot y$$

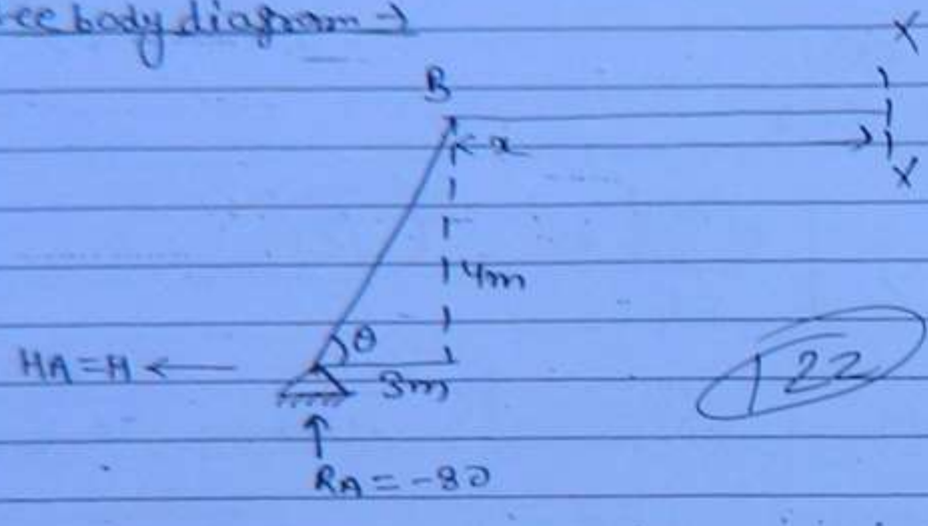
$$\Rightarrow (-80)\, S\cos\theta + H \cdot S\sin\theta$$

$$\Rightarrow -80\, S \cdot \frac{3}{5} + H \cdot S \cdot \frac{4}{5}$$

$$\boxed{B.M.\ at\ x \Rightarrow -48S + 0.8H \cdot S.}$$

(2) At member BC

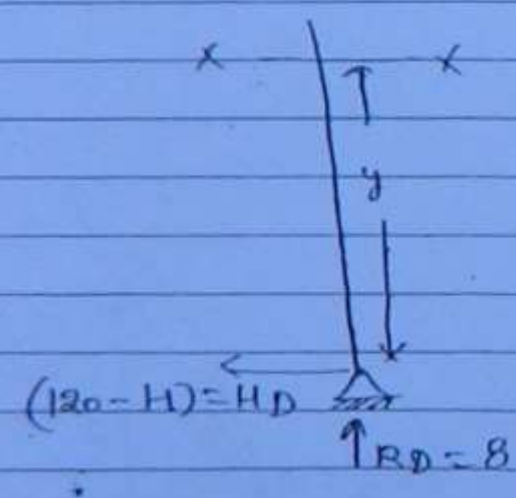
free body diagram →



$M_x = R_A(3+x) + H\,4$

$M_x \Rightarrow R\ -80\ (3+x) + 4H$

(3) At the member CD



$M_x = -(120-H)\,y$

Equation:-

$$\Sigma \frac{\int M_x \frac{\partial M_x}{\partial H}\, dl}{EI} = 0$$

Page 123

$$\Rightarrow \int_0^5 \frac{(-48s + 0.8Hs)\,0.8s\,ds}{EI} + \int_0^3 \frac{\{-80(3+x)+4H\}\,4\,dx}{EI} + \int_0^4 \frac{-(120-H)y \cdot y\,dy}{EI}$$

$$H = 88.33\ kN$$

B.M. in AB

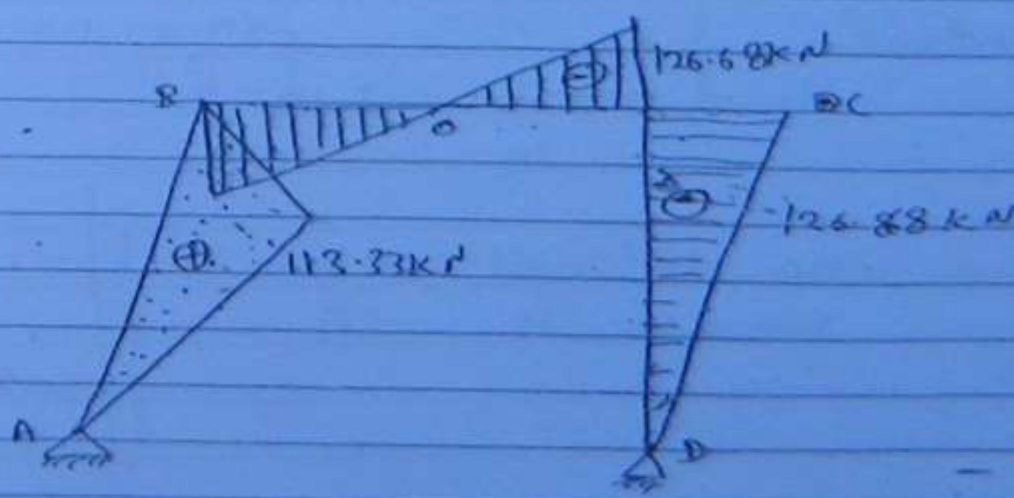
$$M_x = -48s + 0.8Hs$$
$$M_A = 0$$
$$M_B = -48 \times 5 + 0.8 \times 88.33 \times 5$$
$$\Rightarrow +113.33$$

B.M. in BC

$$M_x = -80(3+x) + 4H$$
$$M_B = -80(3+0) + 4 \times 88.33 = +113.33$$
$$M_C = -80(3+3) + 88.33 \times 4 = -126.68$$

in BM in BC

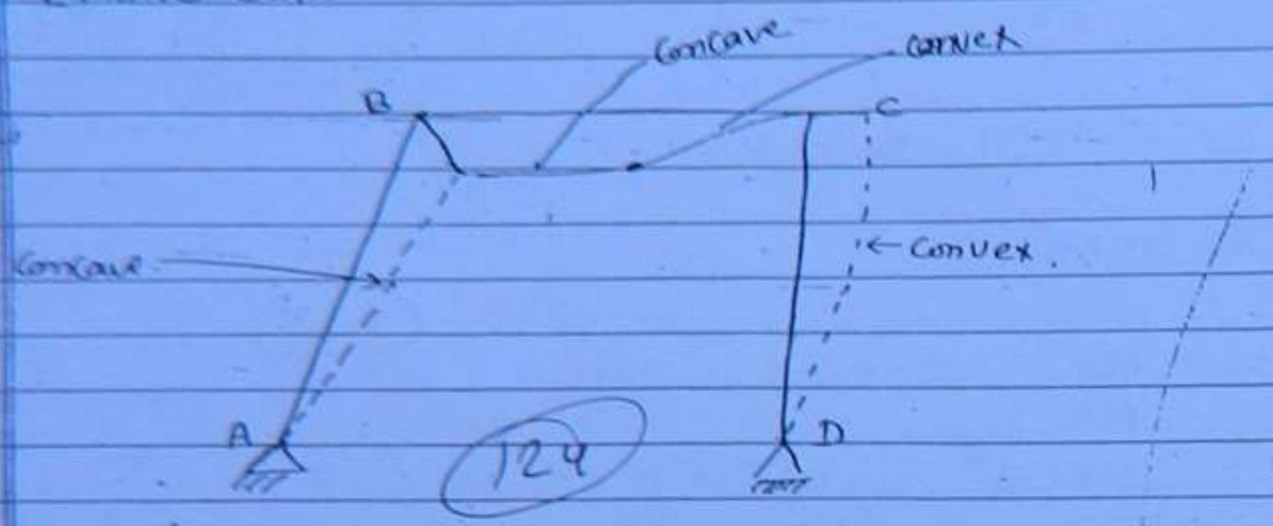
$$M_D = 0$$
$$M_C = -126.68$$



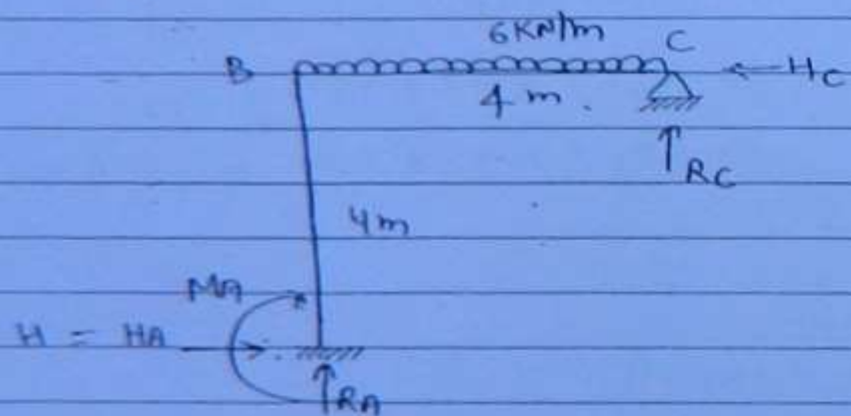
0 → Contra Flexure.

(Sagging → concave
Hogging → convex) Elastic curve

Elastic curve.



Q. Analysis a Portal frame shown in fig using strain Energy method. {EI = constant}



EI = Constant

No. of reaction $= 3+2 = 5$

$D_S = 5-3 = 2$

$\Sigma F_x = 0 \quad H_A - H_C = 0$

$H_A = H_C = H \quad —(1)$

Consider Redundants are H & MA

$R_A + R_C - 6 \times 4 = 0$

$\therefore R_A + R_C = 24$ KN ——— (11)

$\Sigma M_B = 0$

$M_A + R_A \times 4 - H_A \times 4 - 6 \times 4 \times 2 = 0$

$M_A + R_A \times 4 - H4 - 48 = 0$

$4R_A = 48 - M_A + 4H.$

$R_A = 12 - 0.25 M_A + H.$

125

from eqn (2)

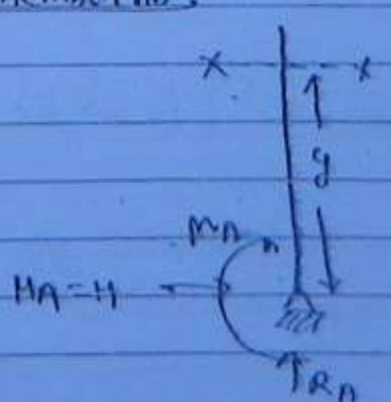
$R_C = 24 - R_A$

$R_C = 24 - 12 + 0.25 M_A - H.$

$R_C = 12 + 0.25 - H$

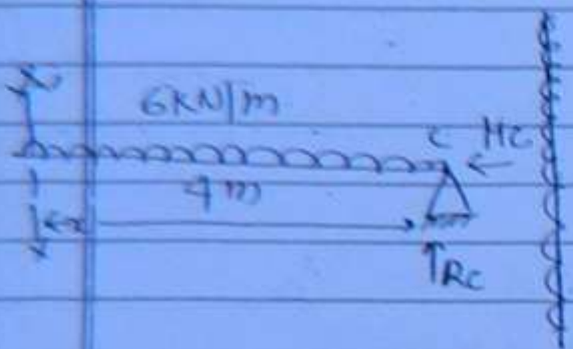
Member	Mx	$\partial Mx / \partial H$	$\partial Mx / \partial M_A$	Limit.
AB	$M_A - H \cdot y$	$-y$	1	0 to 4
BC	$(12 + 0.25 M_A - H)x - 3x^2$	$-x$	$-0.25x$	0 to 4

for member AB :-



$M_x = M_A - H \cdot y$

in foam ~~R~~ CB



$M_x = \cancel{R_c x}$

$R_c \cdot x - \omega \cdot x \cdot \frac{x}{2}$

$M_x = (12 + 0.25M_A - H)x - \frac{6x^2}{2}$

$$M_x = (12 + 0.25\,M_A - H)x - 3x^2$$

126

Equation:-

$\frac{\partial U}{\partial H} = 0$

$$\Sigma \int \frac{M_x \frac{\partial M_x}{\partial H} \cdot ds}{EI} = 0 \quad \text{--- (1)}$$

$\frac{\partial U}{\partial M_A} = 0$

$$\Sigma \int \frac{M_x \cdot \frac{\partial M_x}{\partial M_A}\, ds}{EI} = 0 \quad \text{--- (2)}$$

$$\Rightarrow \int_0^4 \frac{(M_A - Hy)(-y)\, dy}{EI} + \int_0^4 \frac{\left[(12 + 0.25M_A - H)x - 3x^2\right](-x)\, dx}{EI} = 0$$

--- (1)

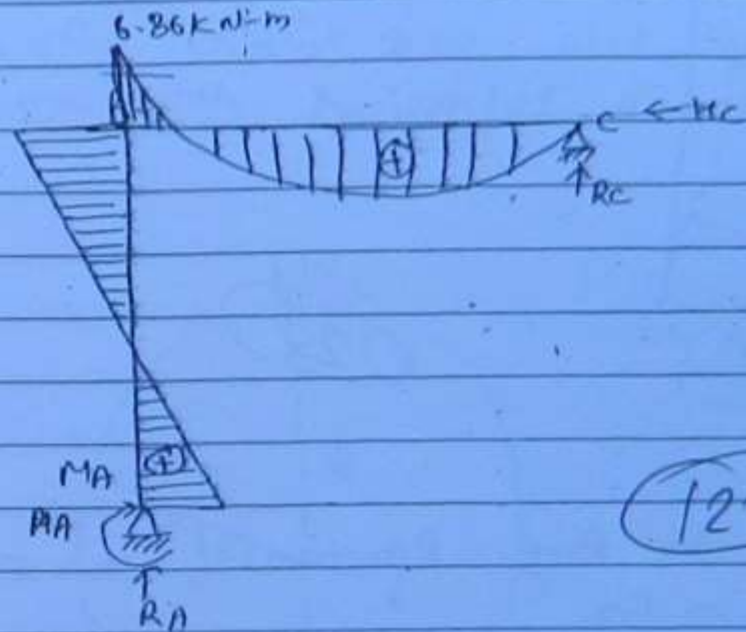
$$\Rightarrow \int_0^4 \frac{(M_A - Hy) \cdot 1 \cdot dy}{EI} + \int_0^4 \frac{\left[(12 + 0.25M_A - H)x - 3x^2\right]\left[0.25x\right] dx}{EI} = 0$$

--- (2)

Solving (1) & (2)

$M_A = +3.43$ kN-m.

$H = 2.57$ kN



(127)

At BM at AB

$M_x = M_A - Hy$

$\Rightarrow 3.43 - 2.57y$

$M_A \Rightarrow +3.43$

$M_B \Rightarrow +3.43 - 2.57 \times 4$

$\Rightarrow -6.86$

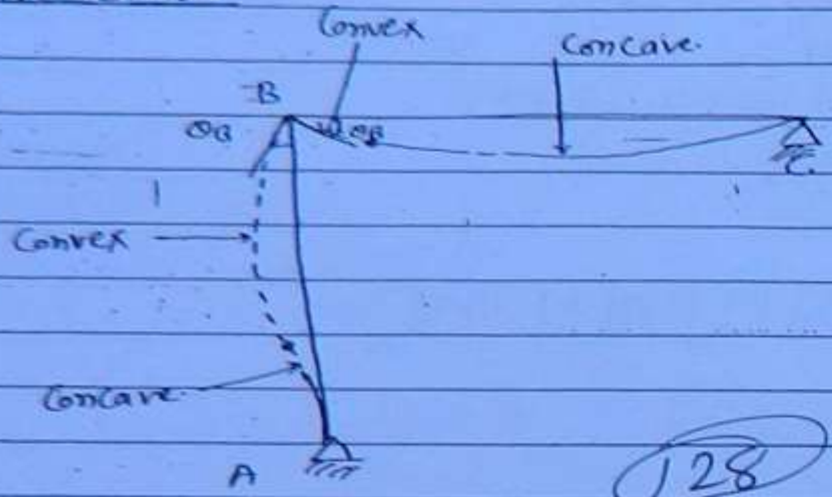
BM in CB

$M_x = (12 + 0.25M_A - H)x - 3x^2$

$M_C = 0$

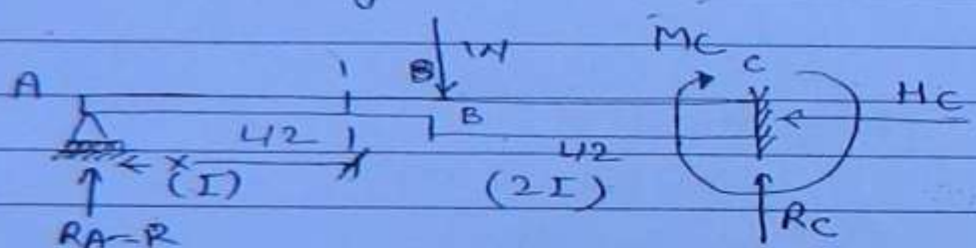
$M_B = -6.86$

Elastic curve



128

Q. Determine the Prop- Reaction of the cantilever beam shown in fig.



$$\frac{\partial U}{\partial R} = 0$$

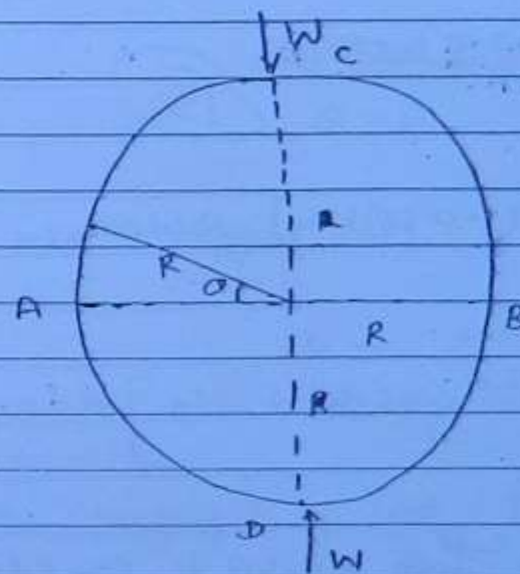
$$U_{AB} = \int_0^{L/2} \frac{(M_x)^2 \, dx}{2EI} \Rightarrow \int_0^{L/2} \frac{(R \cdot x)^2 \, dx}{2EI}$$

$$U_{BC} = \int_{L/2}^{L} \frac{\left[R_A x - W(x - L/2)\right]^2 dx}{2E(2I)}$$

$$\boxed{Ans = \frac{5}{8} W}$$

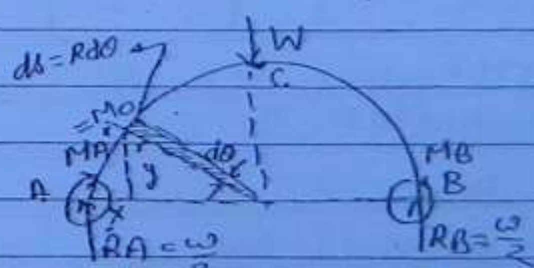
Q. A Ring of Radius R having uniform cross-section (EI = constant) is subjected to 2 equal & opposite load W as shown in fig. Determine maxm. Sagging & Hogging B.M. and also find Decrease in length of vertical Dia. and Position of contraflexture with horizontal diameter.

Sol



(129)

for the analysis let us cut a Ring in two equal part. At the level of AB.



$H_A = H_B = 0$

For True value of Redundant M_o =

$$\frac{\partial U}{\partial M_o} = 0$$

U for HAlf Ring

$$U = 2\int_0^{\pi/2} \frac{M_x^2\, ds}{2EI}$$

BM at XX

$M_x \Rightarrow R_A \cdot x + M_0$

$M_x \Rightarrow \frac{\omega}{2} R(1-\cos\theta) + M_0$

$$U = 2\int_0^{\pi/2} \frac{\left[\frac{\omega}{2} R(1-\cos\theta) + M_0\right]^2 R\,d\theta}{2EI}$$

130

$$\frac{\partial U}{\partial M} = \frac{1}{EI}\int_0^{\pi/2} 2\left[\frac{\omega}{2} R(1-\cos\theta) + M_0\right] \cdot R \cdot d\theta$$

$$\frac{\partial U}{\partial M} \rightarrow \frac{2}{EI}\int_0^{\pi/2} \left[\frac{\omega}{2} R - \frac{\omega R\cos\theta}{2} + M_0\right] R\,d\theta$$

$$\frac{\partial U}{\partial M} = \frac{2}{EI}\int_0^{\pi/2} \frac{\omega}{2} R^2 d\theta - \frac{\omega}{2} R\cos\theta\, d\theta + M_0 R\,d\theta$$

$$\Rightarrow \left[\frac{\omega}{2} R^2 \frac{\pi}{2}\right] - \left[-\frac{\omega}{2} R^2 1\right] + \left[M_0 R \frac{\pi}{2}\right]$$

$$0 \Rightarrow \frac{\omega}{2} R^2 \frac{\pi}{2} + \frac{\omega}{2} R^2 + M_0 R \frac{\pi}{2}$$

$$M_0 R \frac{\pi}{2} \Rightarrow -\frac{\omega R^2}{2} + \frac{\omega R^2}{2}\frac{\pi}{2}$$

$$M_0 R \frac{\pi}{\cancel{2}} \Rightarrow -\frac{\omega R^2}{\cancel{4}\,2}(\pi - 2)$$

$$\boxed{M_0 \rightarrow -\frac{\omega R}{2\pi}(\pi - 2)}$$

$$M_x = \frac{\omega R}{2}\left(1-\cos\theta + M_0\right] \quad -①$$

for M_x to be max | min^m.

$$\frac{\partial M_x}{\partial \theta} = 0$$

$$\frac{\omega R}{2}(0+\sin\theta)+0 = 0$$

$$\sin\theta = 0$$

131

$$\theta = 0$$

Hence, max^m Hogging B.M. is at A & B.

$$M_0 = -\frac{\omega R}{2\pi}(\pi-2)$$

$\cos\theta = 0$

from eq.(1) Max (Sagging) $\rightarrow \frac{\omega R}{2}\left[1-0\right] + \frac{-\omega R}{2\pi}(\pi-2)$

$$\Rightarrow \frac{\omega R}{2} - \frac{\omega R}{2} + \frac{\omega R}{\pi}$$

$$\Rightarrow \frac{\omega R}{\pi}$$

for Point of contraflexture

$$M_x = 0$$

$$\Rightarrow \frac{\omega R}{2}(1-\cos\theta) + M_0 = 0$$

$$\Rightarrow \frac{\omega R}{2}(1-\cos\theta) - \frac{\omega R}{2\pi}(\pi-2) = 0$$

$$\boxed{\theta = \cos^{-1}(2/\pi)}$$

Vertical Deflection of CD

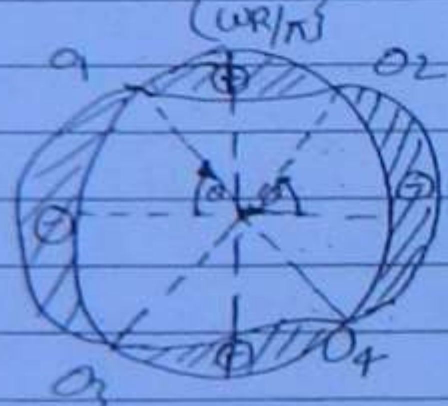
$$\Delta_{CD} = \frac{\partial U}{\partial W}$$

$$U = 4\int_0^{\pi/2} \frac{M_x^2\, ds}{2EI}$$

$$U = 4\int_0^{\pi/2} \frac{\left[\frac{WR}{2}(F - \cos\theta) + M_0\right]^2 R\cdot d\theta}{2EI}$$

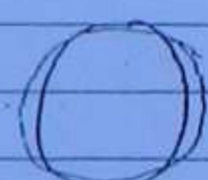
1.32

$$\Delta_{CD} = \frac{WR^3}{4\pi EI}(\pi^2 - 8)$$



$\theta = \cos^{-1}\left(\frac{2}{\pi}\right)$

$\theta = 50.45^{\circ}$



Note:- In a closed str. Resuldant ~~moment~~ member is internal and in open str. Redundant member is external.

MOMENT DISTRUBTION METHOD

is also called →

→ Hard – cross method

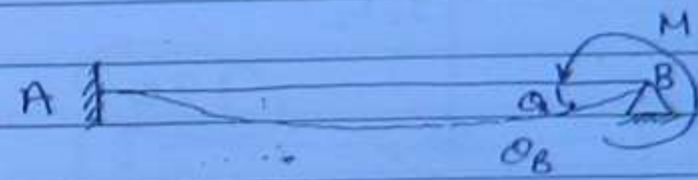
→ Mtd. of successive approximatio

→ Base of stiffness concept.

133

Stiffness:-

Case-1



$$\theta_B = \frac{ML}{4EI} \quad \text{---- when far End is fixed}$$

$$M = \frac{4EI}{L} \cdot \theta_B \quad \text{if } \theta_B = 1, \text{ than } M - K$$

(Stiffness

Stifness of $BA = \frac{4EI}{L}$, when far end A is fixed.

$$\theta_B = \frac{ML}{3EI} \qquad \theta_A = \frac{ML}{6EI}$$

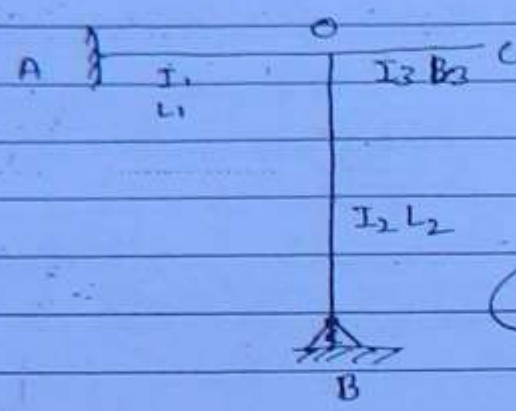
$$M = \frac{3EI}{L} \cdot \theta_B$$

If $\theta_B = 1$, $M = K$

stiffness of $BA = \frac{3EI}{L}$ ---- When far end A is Hinged.

Date / /
Page

→ If far end is free, than stiffness of the member will be zero.



(134)

k_1 stiffness of $OA = \frac{4E_1I_1}{L_1}$ - - - for End (A) is fixed.

k_2 Stiffness of $OB = \frac{3E_2I_2}{L_2}$ - - - for (B) is fined

k_3 Stiffness of CE $= 0$ - - - for C is free.

Total stiffness of all member $= \Sigma K$

$$\Sigma K = k_1 + k_2 + k_3$$

$$\Sigma k = \frac{4E_1I_1}{L_1} + \frac{3E_2I_2}{L_2} + 0$$

$$\Sigma K = K = \frac{4E_1I_1}{L_1} + \frac{3E_2I_3}{L_2}$$

Relative stiffness:-

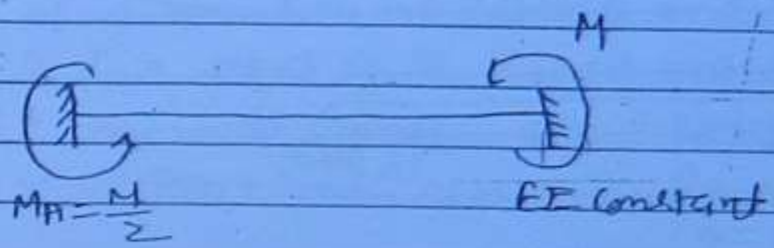
$$R.S \Rightarrow \frac{\text{stiffness}}{4E}$$

R.S. for OA $= \frac{I_1}{L_1} \;\Big|\; \left(\frac{I}{L}\right)$ - - - - - - when far end is fix

R.S. for OB $= \frac{3}{4}\frac{I_2}{L_2}$ or $\frac{3}{4}\frac{I}{L}$ — — ,, — ,, Hinge

R.S. for OC $= 0$

135



$$\frac{M_A}{M} = \frac{1}{2} = \text{Carry over factor.}$$

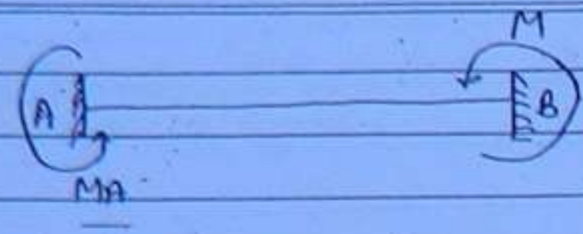
M_A = Carry over moment.

→ it is always in the direction of applied moment.

→ If a moment m is applied at B (near end) then carry over moment at far end which is fixed will be in the direction of applied moment. The Ratio of carry over moment to the applied moment is called carry over factor.

→ If far end A is Hinged or free, then carry over moment is zero, Hence carry of factor is zero.

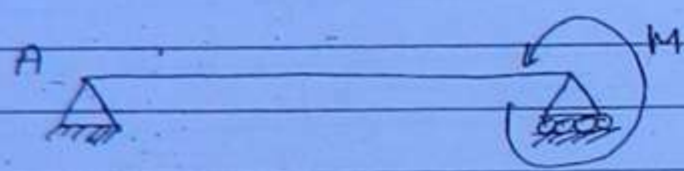
①



$\frac{MA}{M} = \frac{1}{2}$ carryover factor

136

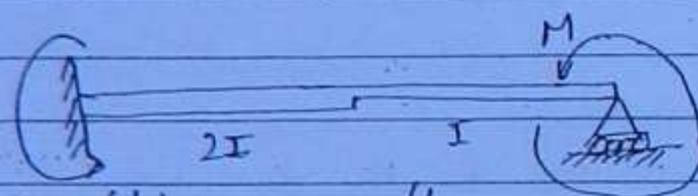
②



$MA = 0$

$C.O.F = 0$

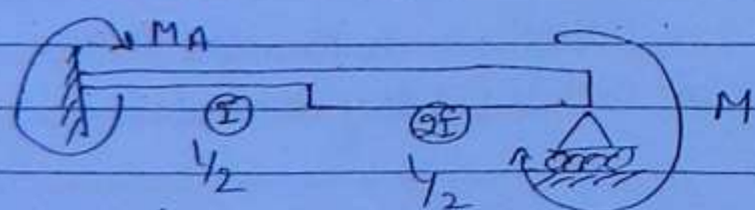
③



$MA = (\frac{L}{2})$ $(L/2)$

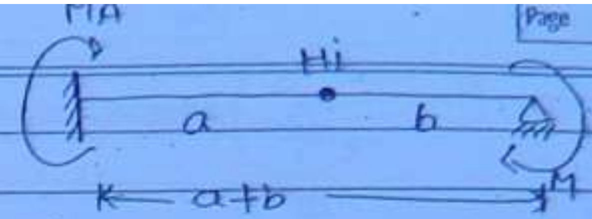
$\frac{MA}{M} = C.O.F > \frac{1}{2}$

(IV)



$\frac{M_A}{M} = C.O.F < \frac{1}{2}$

(v)



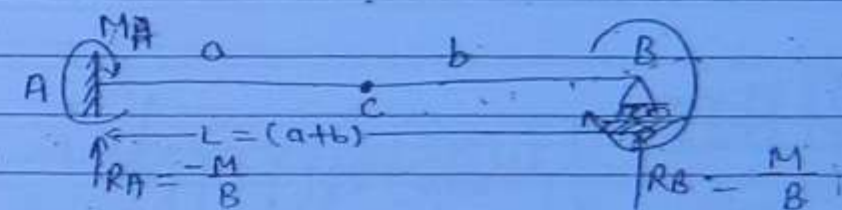
, EI = Constant

→

$$\frac{M_A}{M} = C.O.F. = \frac{a}{b}$$

137

Q Find stifness of a member at joint B and carry over factor if far end A is fixed and ~~Bottom~~ Beam contain's internal Hinge at C. Near and B is Rollar support as shown in fig.



$M_C = 0$

$$R_B \times b - M = 0$$

$$R_B = \frac{M}{B}$$

$$C.O.F = \frac{M_A}{M}$$

$\Sigma M_A = 0$

$$R_B \times (a+b) - M - M_A = 0$$

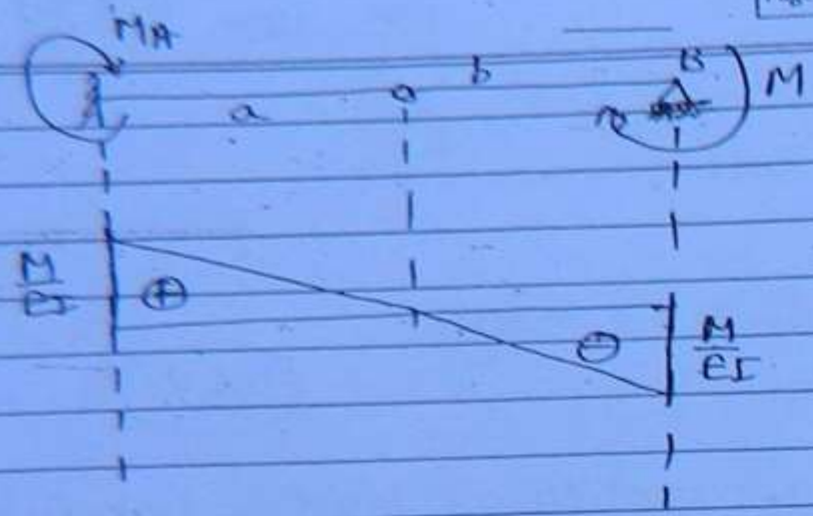
$$M_A = R_B(a+b) - M$$

$$M_A = \frac{M}{b}(a+b) - M$$

$$M_A = \frac{M}{b} \cdot a + \cancel{M} - \cancel{M}$$

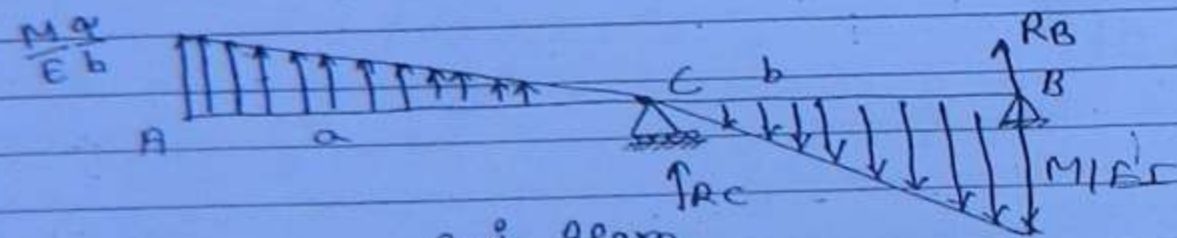
$$M_A = M\frac{a}{b}$$

$$C.O.F. = \frac{M_A}{M} = \frac{a}{b}$$



$\frac{M}{EI}$ or curvature Diagram for given beam.

138



Conj. Beam

Slope at B in real Beam = S.F. at B in conj Beam = $-R_B$

To find R_B take $\Sigma M_c = 0$

$$R_B \times b - \frac{M}{EI} \times \frac{1}{2} \times b \times \frac{2}{3} b - \frac{1}{2} \times a \times \frac{M}{EI} \cdot \frac{a}{b} \times \frac{2}{3} a = 0$$

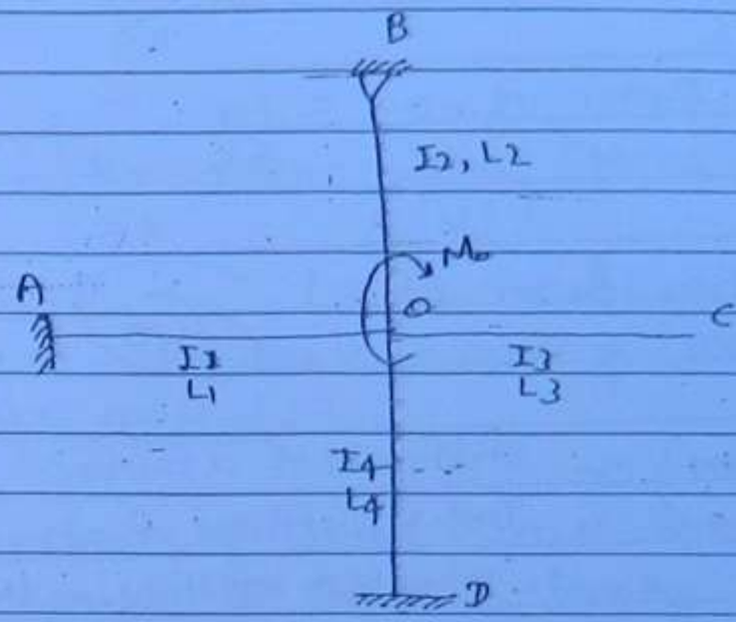
$$R_B = \frac{M b}{3EI} + \frac{M}{3EI} \frac{a^3}{b^2}$$

$$\Rightarrow \frac{M}{3EI}\left[\frac{b^3 + a^3}{b^2}\right]$$

$\theta_B = 1 \quad M = K$

$$K = \left(\frac{3EI\, b^2}{a^3 + b^3}\right) \text{ Ans}$$

Distribution theorem :-



(139)

O = Rigid joint

K_1 = Stiffness of OA $= \dfrac{4EI_1}{L_1}$

K_2 = Stiffness of OB $= \dfrac{3EI_2}{L_2}$

K_3 = Stiffness of OC = 0

K_4 = Stiffness of OD $= \dfrac{4EI_4}{L_4}$

Total stiffness of all member at O

$$\Sigma K = K_1 + K_2 + K_3 + K_4$$

At joint O

Distribution factor of OA $= \dfrac{K_1}{\Sigma K}$

Distribution factor of OB $= \dfrac{K_2}{\Sigma K}$

Distribution factor of OC $= \frac{K_3}{\Sigma K}$

Distribution factor of OD $= \frac{K_4}{\Sigma K}$

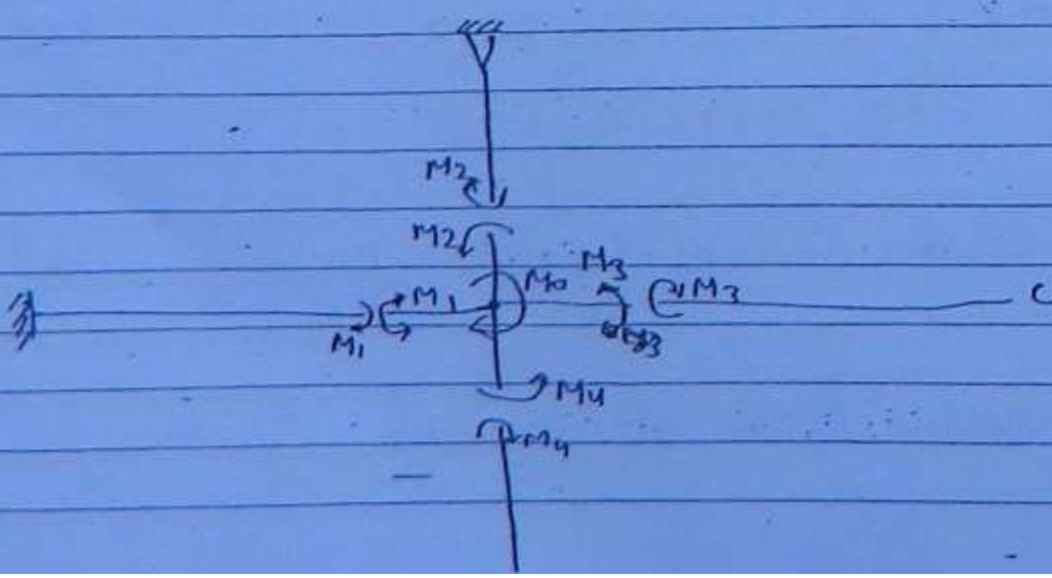
Distribution factor $\Sigma K = 1$ T40

D.F. (Distribution factor) :- $\frac{\text{Stiffness of a factor}}{\text{Total stiffness of all member metting at that joint}}$

D.F. $\Rightarrow \frac{\text{R.S. of a member}}{\text{T.R.S.}}$

moment Distribution Theorm :-
theorm →

if At A rigid ~~to~~ Joint where more thal 1 member ar metting ~~other~~ & and external moment is applied M₀, then it gets distribution in all member in the direction of applied moment in proportiona of stiffness of each members



$$M_1+M_2+M_3+M_4=0$$

$$M_1 = \frac{K_1}{\Sigma K} \times M_0 \qquad M_2 = \frac{K_2}{\Sigma K} \times M_0$$

$$M_3 = \frac{K_3}{\Sigma K} \times M_3 : \qquad M_4 = \frac{K_4}{\Sigma K} \times M_0$$

141

→ Procedure to draw Bending moment dia in Stiffness method :-

→ If the no. of Spans / stories are large then force method is Complex & Lengthy such case stiffness cases use

In A stiffness method each span is considered ~~to a~~ seperatly to draw BMD At the Both ends final moment are found each member which are called end moments. which can be found using any of the stiffness method.

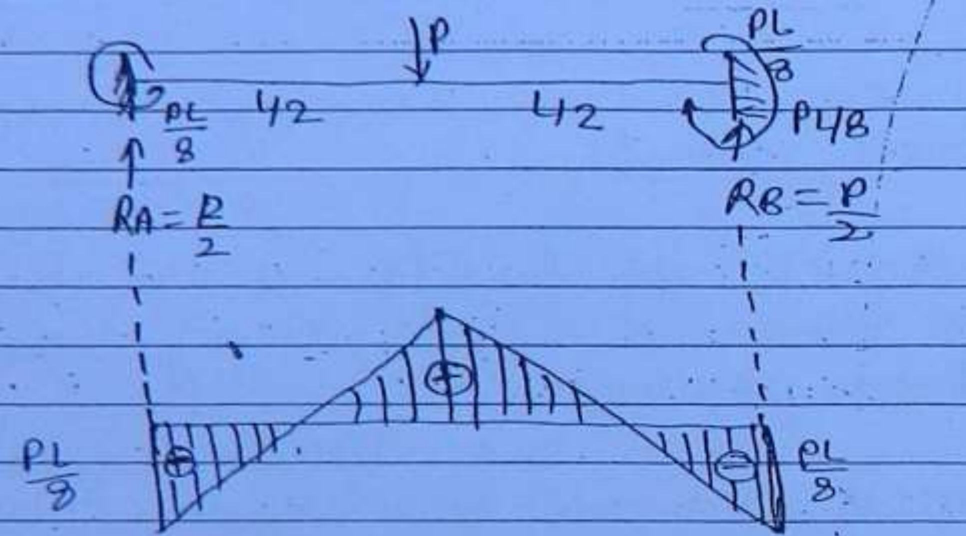
for every member end moment dia. is drawn in which Sagging moment (+ive) are Plotted Below the Beam / Inside the frame.

And Hogging moment are plotted above the Beam / outside the frame.

for Each member Considering free simply supported b with given loading free BMD / simply supported dia. is drawn saperately. In which sagging above the beam is positive (+ive) Hogging is taken below the beam (-ive) inside the frame.

To obtain final diagram. End moment dia. and free BMD are superImposed.

The Common are is Cancelled.
At net Diagram is Rig. resulted magnituc.

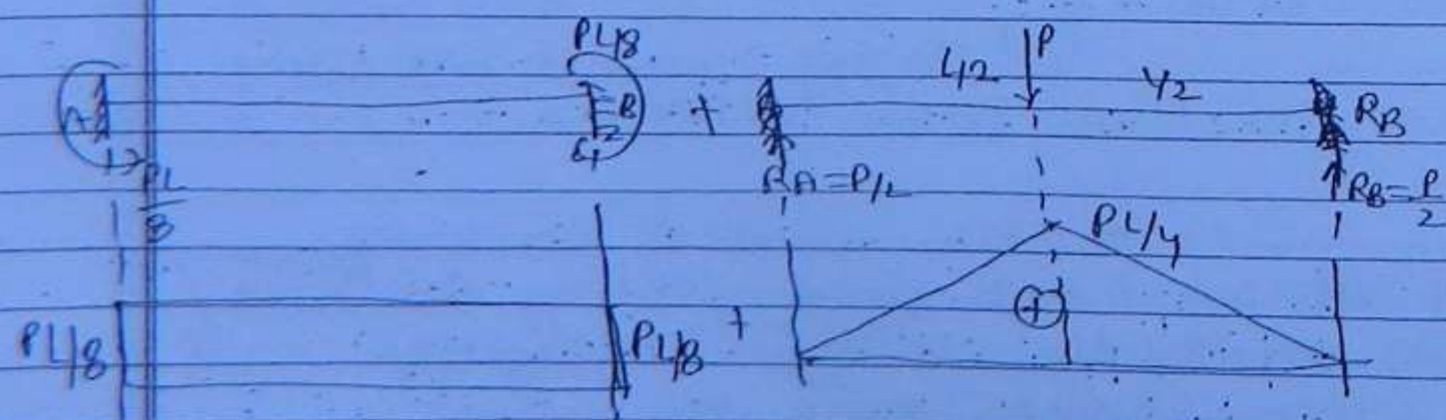


in BM. in AC

$$M_x = R_A \cdot x - \frac{PL}{8}$$

$$M_A = \frac{-PL}{8}$$

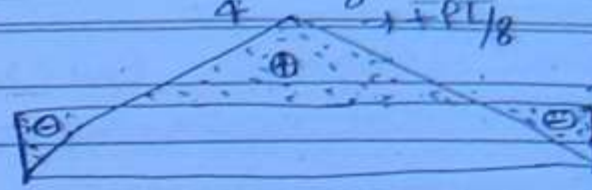
$$M_C = \frac{P}{2} \cdot \frac{L}{2} - \frac{PL}{8} \Rightarrow \frac{+PL}{8}$$



End mom. Diagram. S.S. BMD. / for BMD free.

$\frac{PL}{4} - \frac{PL}{8} \rightarrow + PL/8$

Total Dig. $\Rightarrow$



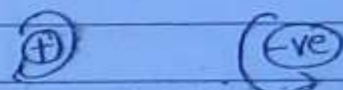
$\frac{PL}{8}$ $PL/8$

final B.M.D

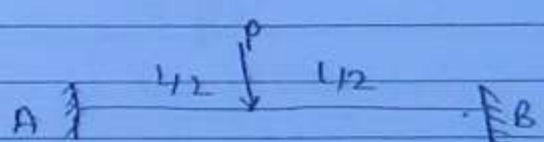
143

Sign. Convention. for Computation of end moment.

All Clock-wise moment will be taken ~~the~~ (+ive) and anti-clock wise moment will be (-ive)



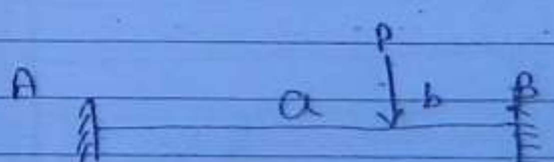
Fixed end moment for Standered loading condition:-

(1) 

$\overline{M}_{AB}$ = Fixed End Moment at A for AB.

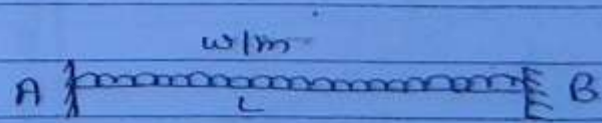
$\Rightarrow \frac{-PL}{8}$

$\overline{M}_{BA} = +\frac{PL}{8}$

(2) 

$\overline{M}_{AB} = -\frac{Pab^2}{L^2}$

$\overline{M}_{ba} = \frac{+Pa^2b}{L^2}$

(3) 

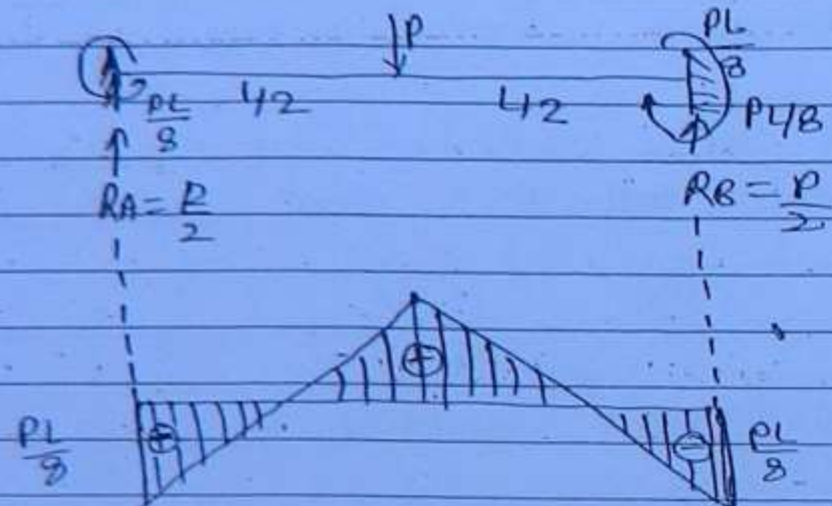
$\overline{M}_{AB} = \frac{-wl^2}{8}$

$\overline{M}_{BA} = \frac{WL^2}{8}$

To obtain final diagram. End moment dia. and free BMD are superimposed.

The common are is cancelled. At net Diagram is Rig resulted magnituc.

144

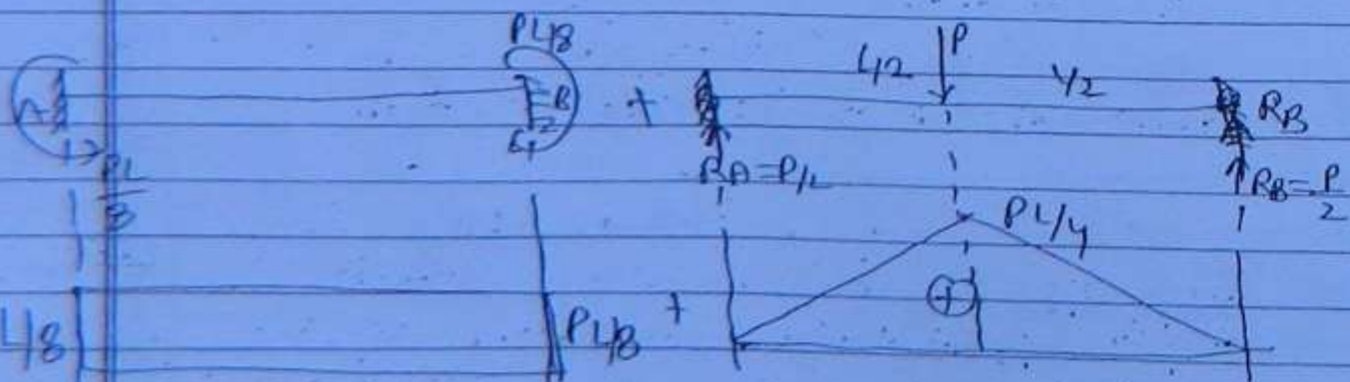


in BM. in AC

$$M_x = R_A \cdot x - \frac{PL}{8}$$

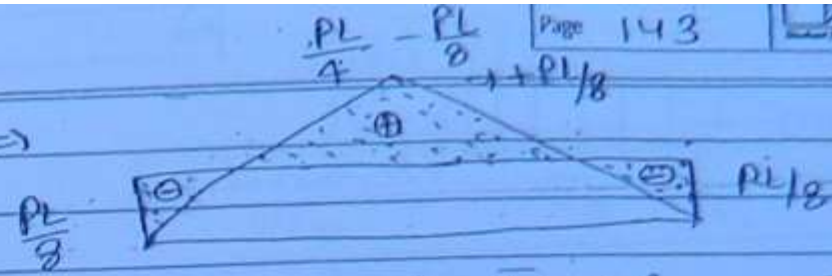
$$M_A = \frac{-PL}{8}$$

$$M_C = \frac{P}{2} \cdot \frac{L}{2} - \frac{PL}{8} \Rightarrow \frac{+PL}{8}$$



End mom. Diagram. S.S. BMD / free BMD.

Total Dia. =>

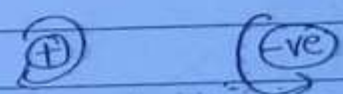


final B.M.D

145

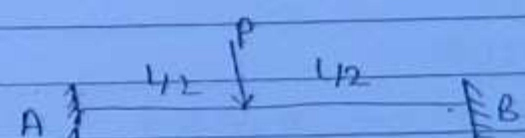
sign. Convention. for Computation of end moment.

→ All Clock-wise moment will be taken (+ive) and anti-clock wise moment will be (-ive)



Fixed end moment for standered loading condition:-

(1)

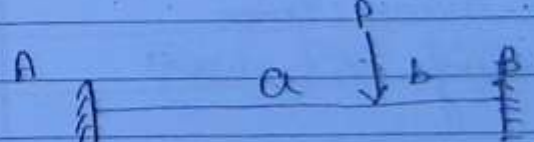


$\overline{M}_{AB}$ = Fixed End Moment at A for AB.

$\Rightarrow \frac{-PL}{8}$

$\overline{M}_{BA} = +\frac{PL}{8}$

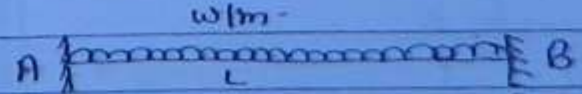
(2)



$\overline{M}_{AB} = -\frac{Pab^2}{L^2}$

$\overline{M}_{ba} = +\frac{Pa^2b}{L^2}$

(3)

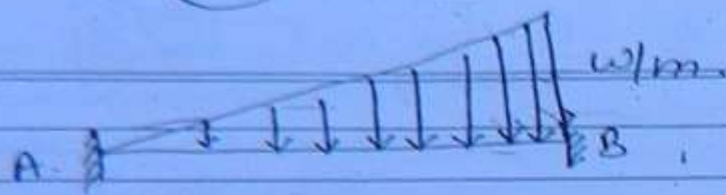


$\overline{M}_{AB} = \frac{-wL^2}{8}$

$\overline{M}_{BA} = \frac{wL^2}{8}$

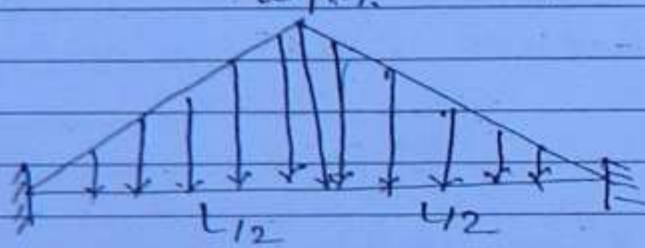
(-ive) (+ive)

(4)



$$\overline{M}_{AB} = -\frac{wL^2}{30} \qquad \overline{M}_{BA} = +\frac{wL^2}{30}$$

(5) 146

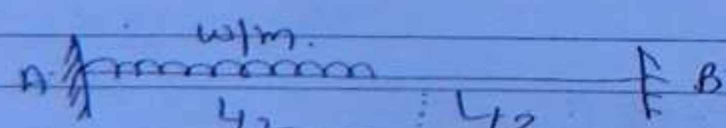


$$\overline{M}_{AB} = -\frac{5}{96} wL^2$$

$$\overline{M}_{BA} = +\frac{5}{96} wL^2$$

→ if arrow of loading points on the Reference face than loading is taken the (+ve) and if arrow of loaing Point away from Refernence face then loading is (-ive)

(6)

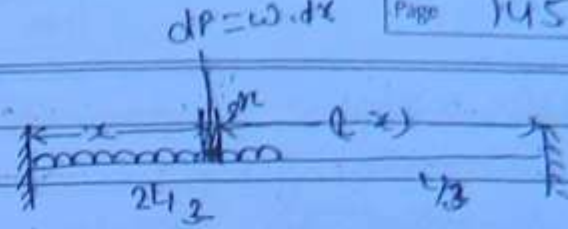


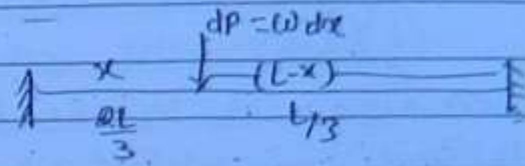
$$\overline{M}_{AB} = -\frac{11}{192} wL^2$$

$$\overline{M}_{BA} \Rightarrow +\frac{5}{192} wL^2$$

Note :-

(7)





147

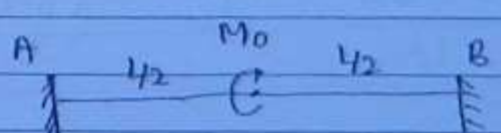
$$d\overline{M}_{AB} = \frac{-dP \cdot x \cdot (L-x)^2}{L^2} \qquad \left\{ \frac{\omega a b^2}{L^2} \right\}$$

$$\Rightarrow \frac{-(\omega \cdot dx) \cdot x (L-x)^2}{L^2}$$

$$\overline{M}_{AB} = \int d\overline{M}_{AB} = -\int_0^{2L/3} \frac{\omega \cdot x (L-x)^2}{L^2} \cdot dx$$

$$\overline{M}_{BA} = \int d\overline{M}_{BA} - \int_0^{2L/3} \frac{\omega \cdot x^2 (L-x) dx}{L^2}$$

(8)



(only direction. not sagging & hogging than tive value)

$$\overline{M}_{AB} = \frac{+M_o}{4} \qquad M_o \rightarrow \text{+ive if } \circlearrowright$$

$$M_o \rightarrow \text{-ive if } \circlearrowleft$$

$$\overline{M}_{BA} = \frac{+M_o}{4}$$

(9)

$$\overline{M}_{AB} = \frac{M_o b (3a-L)}{L^2}$$

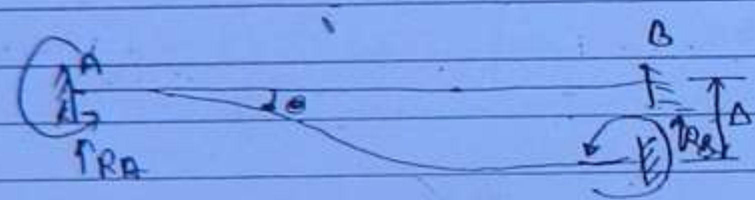
$$\overline{M}_{BA} = \frac{M_o \cdot a \cdot (3b-L)}{\cdot \cdot}$$

→ Note that if $a > L/3$, then $\overline{M_{AB}} > 0$, $\overline{M_{AB}}$ is in the direction of M_0.

148

→ if $b > L/3$, then $\overline{M_{BA}}$ is in the direction of M_0
Hence, if M_0 acts in middle third strip, then fixed end moment at A & B. Both will be in dirn of M_0.

(15) Effect of settlement of support



$$\overline{M_{AB}} = \frac{-6EI\Delta}{L^2}$$

$$\overline{M_{BA}} = -\frac{6EI\Delta}{L^2}$$

$$\Sigma F_y = 0$$

$$R_A + R_B = 0$$

$$\Sigma M_B = 0$$

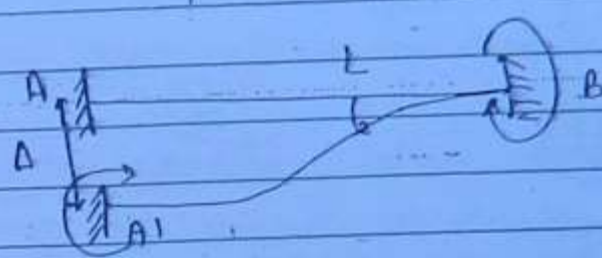
$$R_A \cdot L - \frac{6EI\Delta}{L^2} - \frac{6EI}{\Delta L^2} = 0$$

$$R_A = \frac{12EI\Delta}{L^3}$$

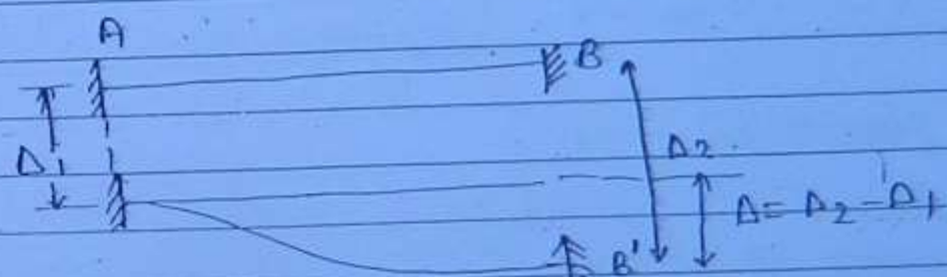
$$\boxed{\text{S.F. in AB} = +R_A = \frac{12EI\Delta}{L^3}}$$

Note :- if settlement of support cause clockwise Rotation to the member than fixing moment ~~development~~ developed at both ends will be anticlock-wise and vice-versa.

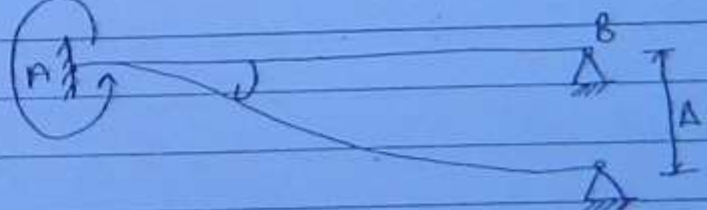
(49)



$$\overline{M_{AB}} = \overline{M_{BA}} = \frac{+6EI\Delta}{L^2}$$



(11)



$$M_{AB} = -\frac{3EI\Delta}{l^2}$$

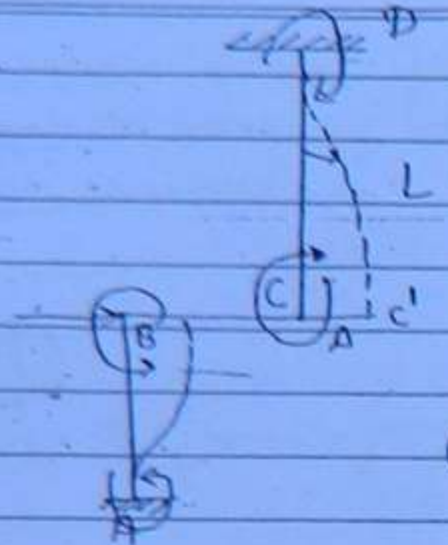
$$\overline{M_{BA}} = 0$$

(12)

$$\overline{M_{AB}} = \overline{M_{BA}} = 0$$

Page

(13)



150

$$\overline{M_{AB}} = \overline{M_{BA}} = \frac{-6EI\Delta}{L^2}$$

$$\overline{M_{CD}} = \overline{M_{DC}} = \frac{+6EI\Delta}{L^2}$$

$$\overline{M_{BC}} = \overline{M_{CB}} = 0$$

(14)

Procedure:-

(1) Find fixed end moment for each member for both ends. Considering colckwise moment tive & Anti- clockwise moment (-ive)

(2) Find distribution factor for each member at those joint where more than one member meet & Joints are rigid.

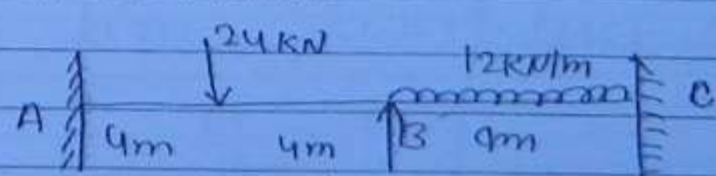
(3) Distribute the moment after balancing at all joint accorting to there Distribution factor & transfer carry over moment to the for ends if fair ends are fixed.

(4) find final end moment at the ends of tabular Calculation tive, clockwis, -ive means Anticlockwise

157

(V) Draw end moment diagram and super Imposed S. S. dia. to obtained resultant moment diagram in B.M.D tive sagging -ive - Hogging

Q For the beam shown in fig. draw B.M.D using Hardy's gross method.



EI = Constant

→ Fixed end moment

$$\bar{M}_{AB} = -\frac{PL}{8} = \frac{-24 \times 8}{8} \Rightarrow -24$$

$$\bar{M}_{BA} = +\frac{PL}{8} = +24$$

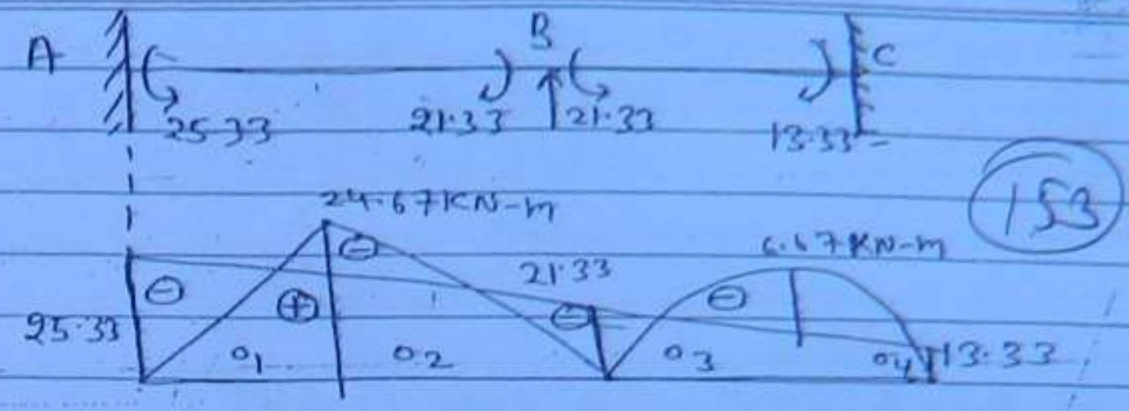
$$\bar{M}_{BC} = -\frac{wl^2}{12} \Rightarrow \frac{-12\times 4^2}{12} \to -16$$

$$\bar{M}_{CB} = \frac{+wl^2}{12} = +16$$

Joint	member	Stiffness	Total Stiffness	$D.F = \frac{K}{\Sigma K}$
B	BA	$4EI/8$	$\frac{3}{2}EI$	$1/3$
	BC	$4EI/4$		$2/3$

152

	A	B	B	C
		$1/3$	$2/3$	
	-24	$+24$	-6	$+16$
		-2.67	-5.33	
	-1.33			-2.67
final	-25.33	$+21.33$	$+21.33$	$+13.33$

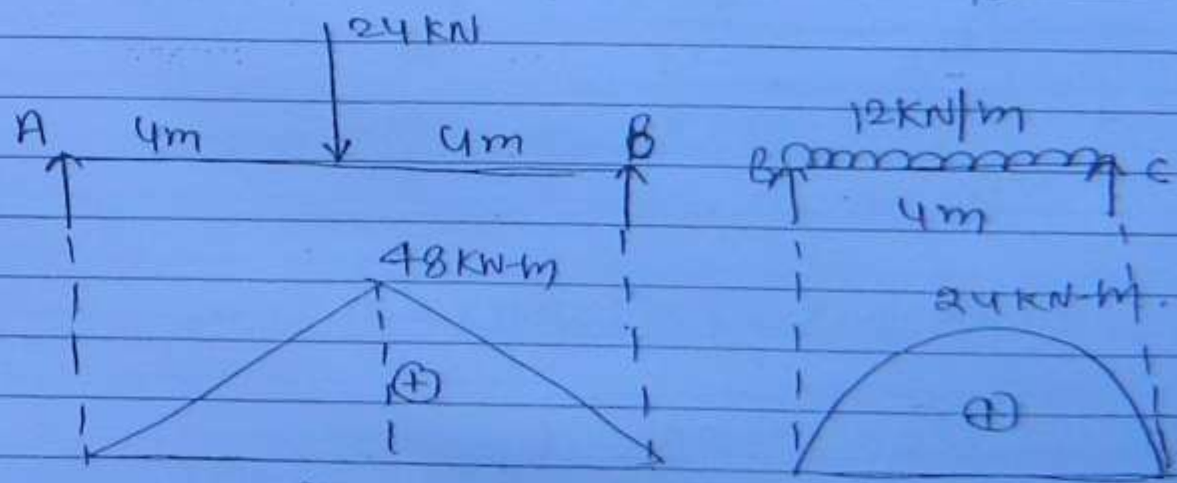


End moment dia

Resultant B.M. at centre of AB $= 48 - \left(\frac{25.33+21.33}{2}\right)$

$\Rightarrow +24.67$

of BC $= 24 - \left(\frac{21.33+13.33}{2}\right) \Rightarrow +1.67$

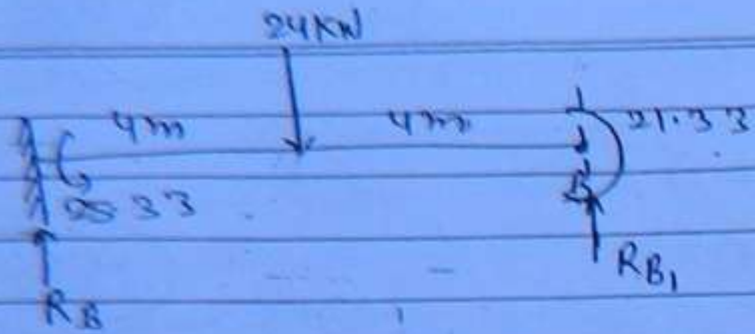


$\frac{PL}{8} = 48$

$\frac{wL^2}{8} = 24$

Computation of Reactions →

→ To find R_A consider free body equilibrium just to Left of B.



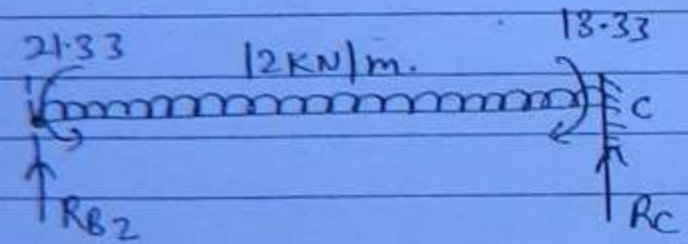
154

$$\Sigma M_B = 0$$

$$R_A \times 8 - 25.33 - 24 \times 4 + 21.33 = 0$$

$$R_A = 12.5\,KN.$$

To find R_c consider free body equlibrium of span BC



$$\Sigma M_B = 0$$

$$-R_c \times 4 + 13.33 + 12 \times 4 \times 2 - 21.33 = 0$$

$$\boxed{R_c = 22\,KN}$$

→ find $R_B = 0$

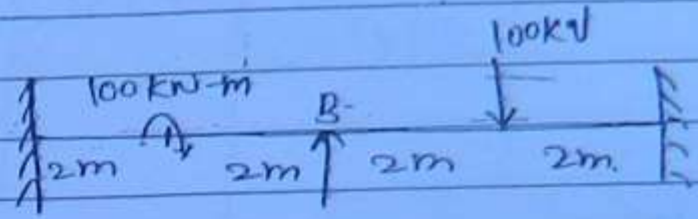
$$\Sigma f_y = 0$$

$$R_A + R_B + R_c - 24 - 12 \times 4 = 0$$

$$\boxed{R_B = 37.5\,kN}$$

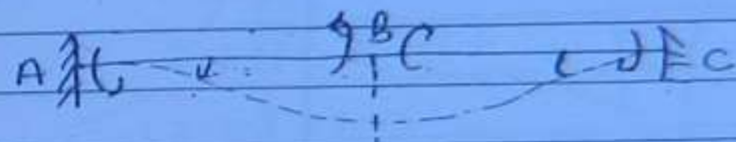
Prob: Draw B.M.D for Beam ABC shown in fig. Support B settles down by 1m EI = constant i.e. equal to 48000 kN/m²

155



$\Delta B = 1 mm \downarrow$

$EI = 48000\ kN\text{-}m^2$



$$\bar{M}_{AB} = +\frac{M_0}{4} - \frac{6EI\Delta}{L^2}$$

$$\Rightarrow \frac{+100}{4} - \frac{6 \times 48000 \times 1 \times 10^{-3}}{2}$$

$$\Rightarrow 25 - 18 \Rightarrow +7$$

$$\bar{M}_{BA} = \frac{+M_0}{4} - \frac{6EI}{L^2} \Rightarrow +7$$

$$\bar{M}_{BC} = -\frac{PL}{8} + \frac{6EI\Delta}{L^2} \Rightarrow \frac{-100 \times 4}{8} + 18$$

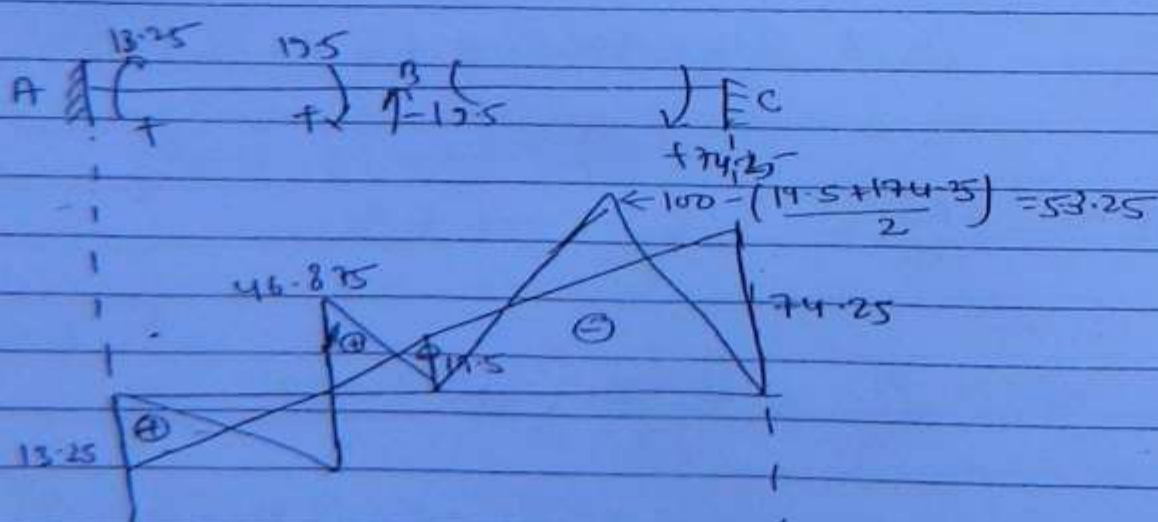
$$\Rightarrow -32$$

$$\bar{M}_{CB} = \frac{+PL}{8} + \frac{6EI\Delta}{L^2} = +50 + 18 = +68$$

Joint	member	R.stiff	T.R.S	D.F. = R.S/T.R.S
	BA	I/4		1/2
B			2I/4	
	BC	I/4		1/2

156

	A	B		C	
		1/2	1/2		
	+7	+7	−32	+68	F.F.M
		+12.5	+12.5		Balancing Moment B.M.
	+6.25			+6.25	
Total	+13.25	+19.5	−19.5	+74.25	

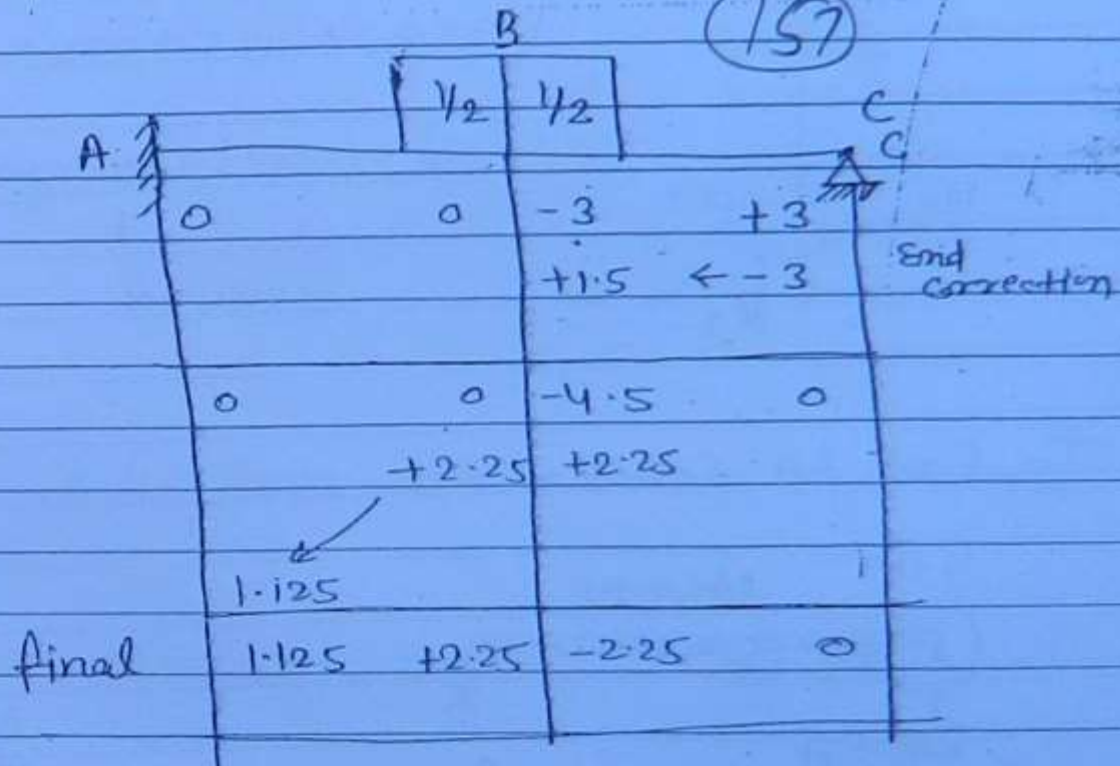


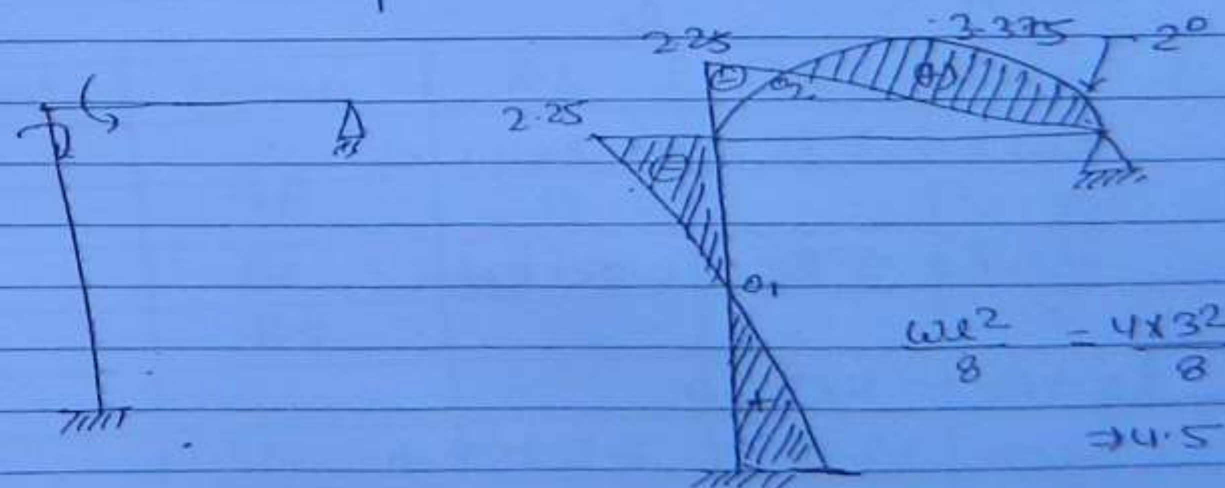
Net BM at Centre of AB

$= 7 - 50 - 3.125 = 53.125$

Joint	member	R.S.	T.R.S.	D.F.
B	BA	$I/4$	$\frac{2I}{4}$	$1/2$
	BC	$3/4 \times I/3$		$1/2$

157





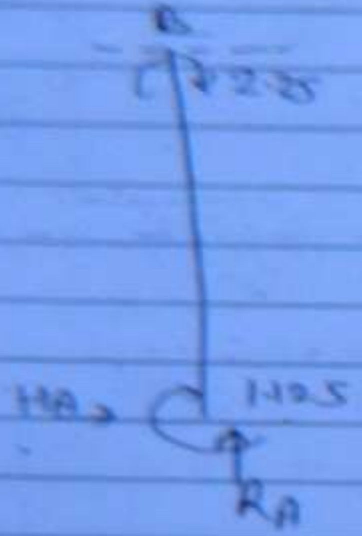
$\frac{wl^2}{8} = \frac{4 \times 3^2}{8}$

$\Rightarrow 4.5$

1.125 Res. BM at centre of BC

$\Rightarrow 4.5 - \frac{2.25}{2} = 3.375$

To find the consider force body equilibrium of AB



158

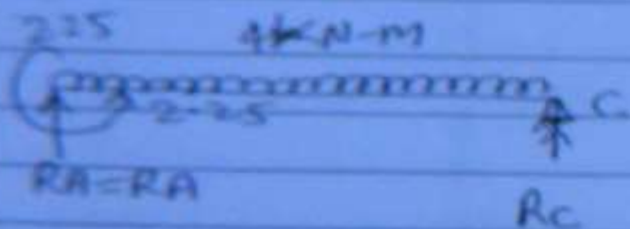
$\Sigma M_B = 0$

$-HA \times 4 + 1.125 + 2.25 = 0$

$HA = \frac{1.125 + 2.25}{4}$

$= 0.84\,kN$

To find RA consider F.B equli of BC



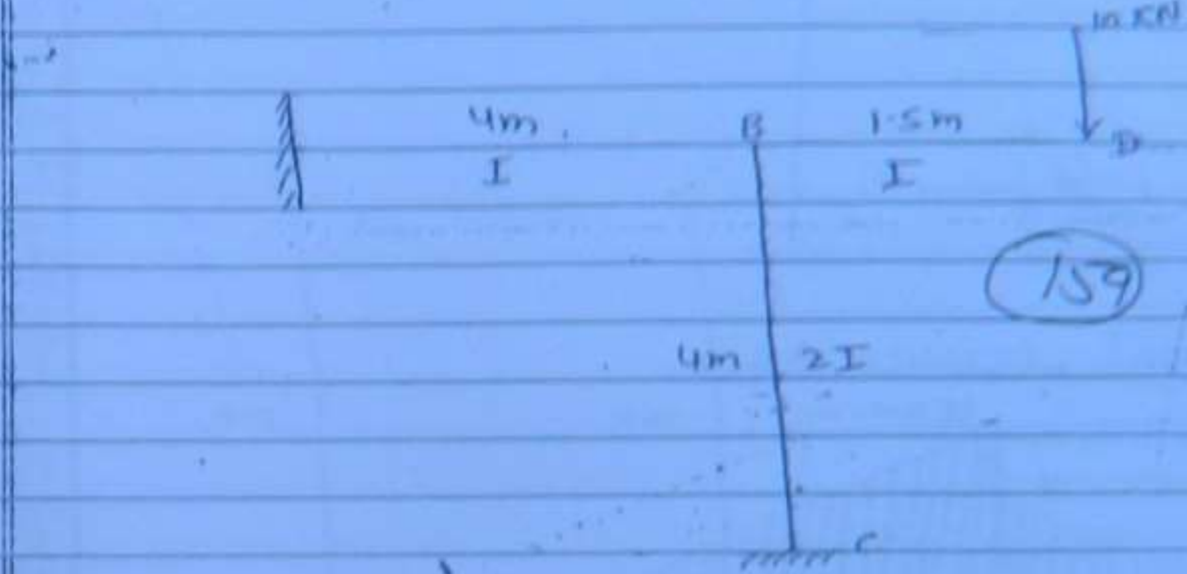
$R_B \times 3 - 2.25 - 4 \times 3 \times 1.5 = 0$

$R_B = R_A = 6.75\,kN$

$R_A + R_C - 4 \times 3 = 0$

$R_C = 12 - 6.75 = 5.25\,kN$

Q. Analysis frame shown in fig. using M.D.M.
Draw B.M.D. and elastic curve.



159

$\bar{M}_{AB} = 0 = \bar{M}_{BA}$

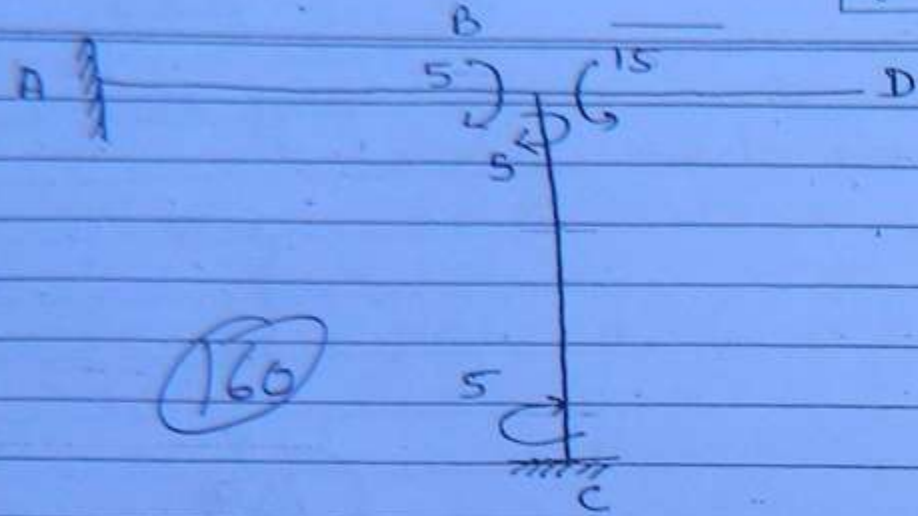
$\bar{M}_{BC} = 0 = \bar{M}_{CB}$

$\bar{M}_{BD} = -15$

$\bar{M}_{DB} = 0$

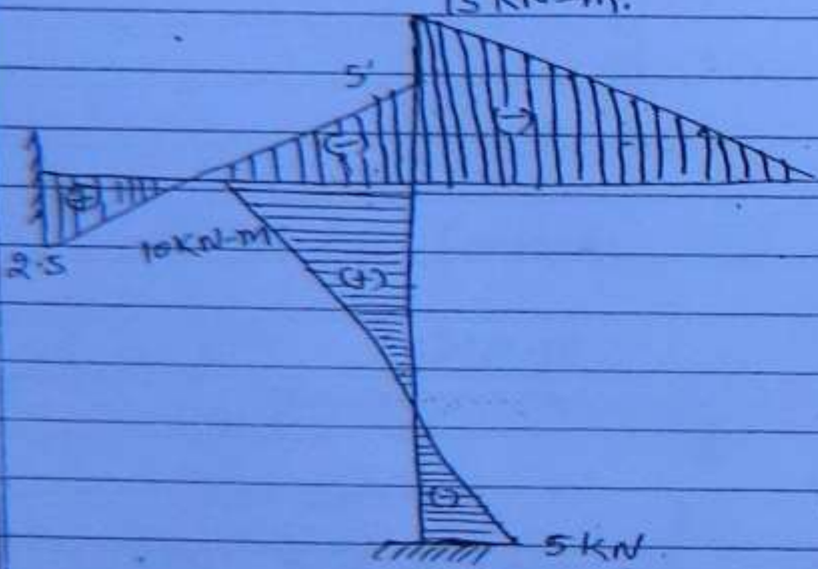
Joint	Member	R.S.	T.R.S	D.F = R.S/T.R.S
B	BA	I/4	3I/4	1/2
	BC	2I/4		2/5
	BD	0		0

	1/3	2/3			
0	0	0	0	-15	0
	+5	+10			
2.25			+5		
Final +2.5	+5	+10	+5	-15	0

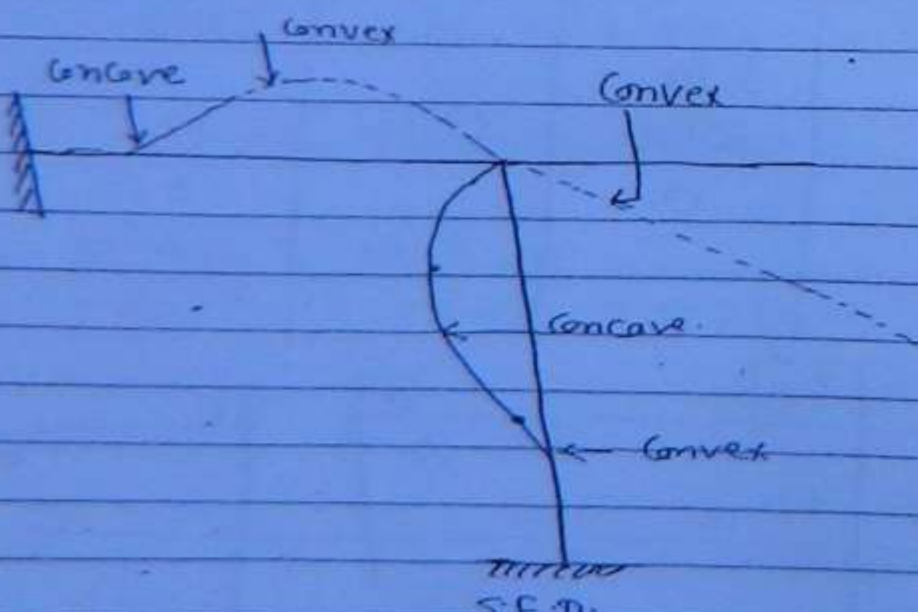


160

→ Horizontal member AB →



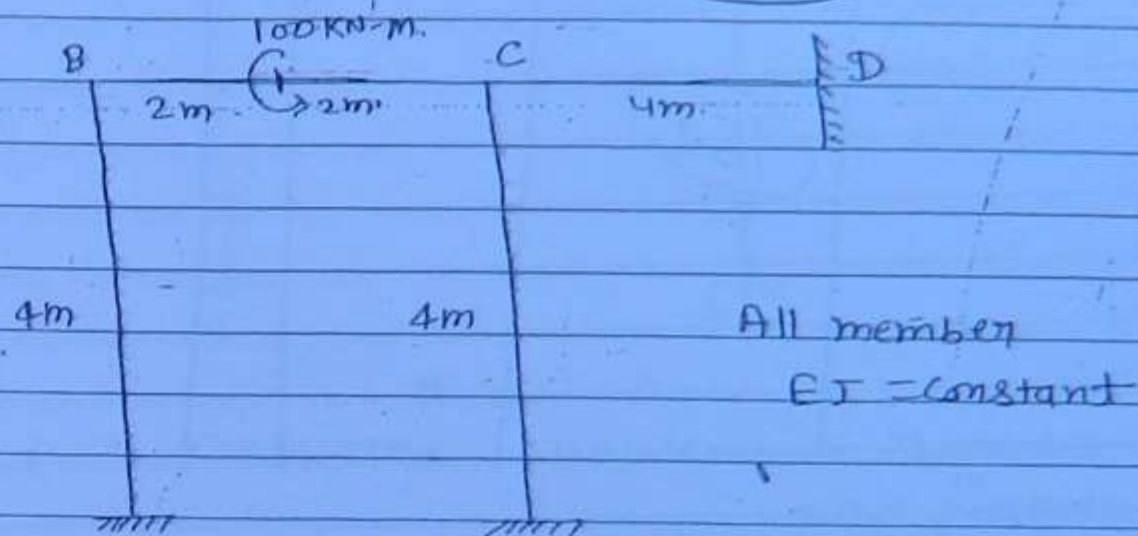
B.M.D.



S.F.D.

Q.5 Analysis given Frame shown in Fig. using moment Distribution Method and Draw Bending moment Diagram.

(161)



$$\bar{M}_{BC} = \frac{M_0}{4} = \left(-\frac{100}{4}\right) = -25$$

$$\bar{M}_{CB} = -25$$

other, member are zero

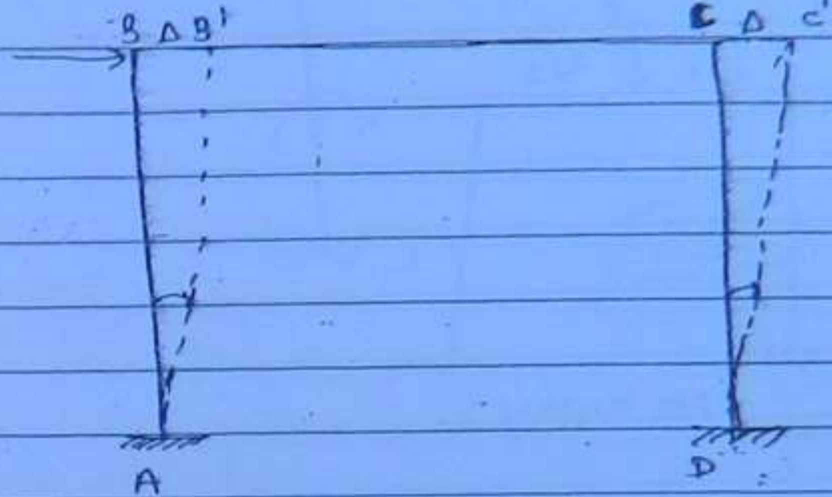
Joint	Member	R.S.	T.R.S	D.F.
B	BA	$I/4$		$1/2$
	BC	$I/4$		$1/2$
C	CB			$1/3$
	CD			$1/3$
	CE			$1/3$

Date / /
Page

					E		C
	1/2	1/2	1/3	1/3		1/3	
	0	-2.5	2.5	0		0	0
	+12.5	+12.5	+8.33	+8.33		+8.33	
+6.25		+4.17	+6.25		+4.17		+4.17
	-2.08	-					

162

SWAY Analaysis (Joint sway)



$$BB' = CC' = \Delta$$

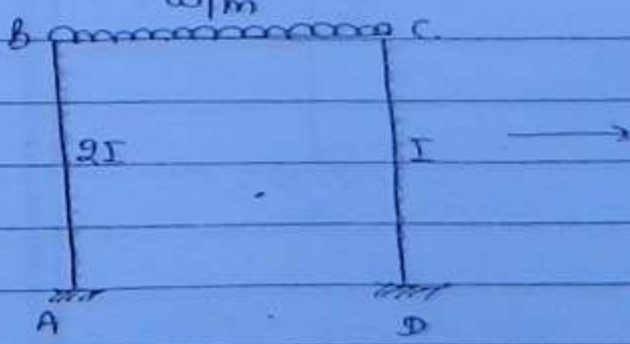
$\Delta \rightarrow$ Transverse Displacement of member or shear Displacement

$$\bar{M}_{AB} = \bar{M}_{BA} = -\frac{6EI\Delta}{L^2}$$

$$\bar{M}_{CD} = \bar{M}_{DC} = -\frac{6EI\Delta}{L^2}$$

$$\bar{M}_{BC} = \bar{M}_{CB} = 0$$

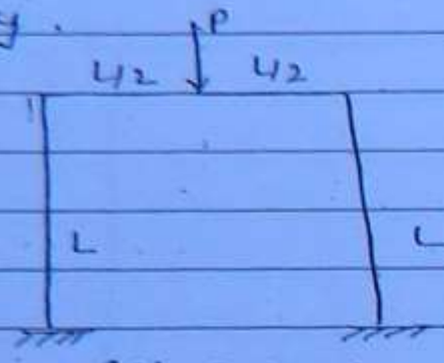
Thumb rule to find direction of sway.



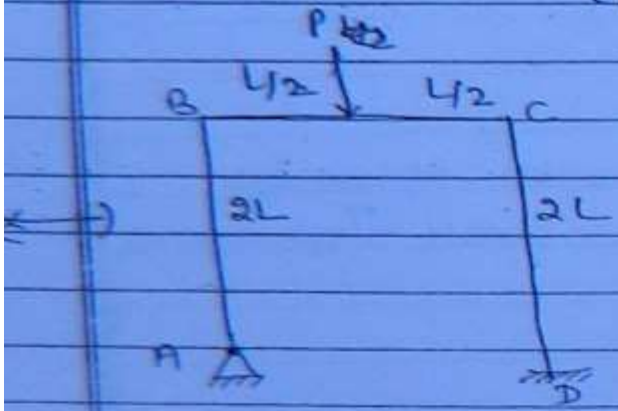
Sway Towards right Because stiffness of CD is less than stiffness of AB.

Loading is symm. but stiffness is not Symm.

If loading, stiffness and support condition are symm. about the centre of frame, there will be no sway.



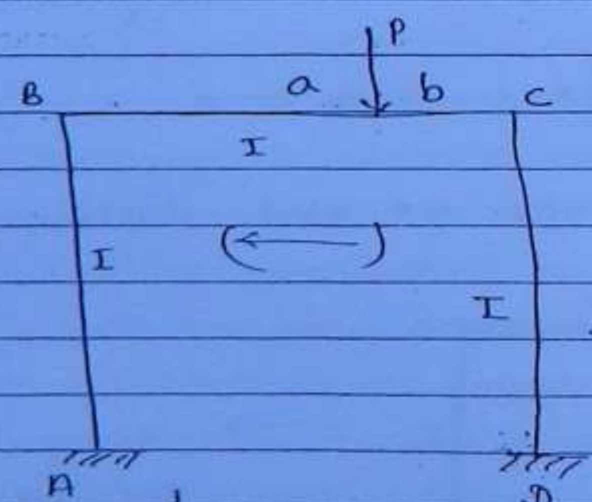
(No sway)

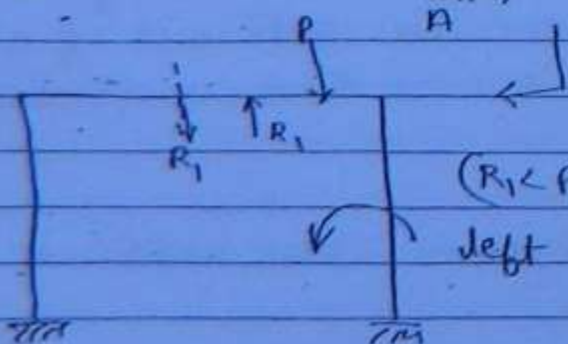


Sway towards left because stiffness of AB is less than stiffness of CD.

→ Sway towards Left if $\boxed{a>b}$

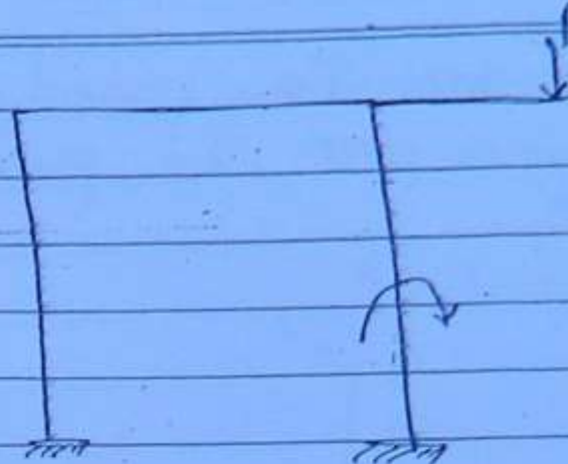
→ Sway toward Right if $\boxed{a<b}$





$(R_1 < P)$

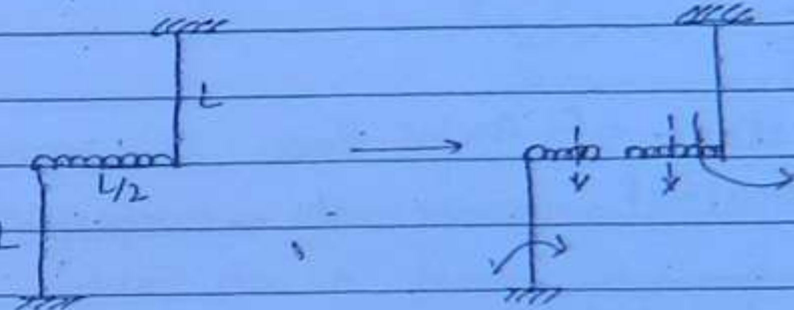
left side sway



Toward Right.

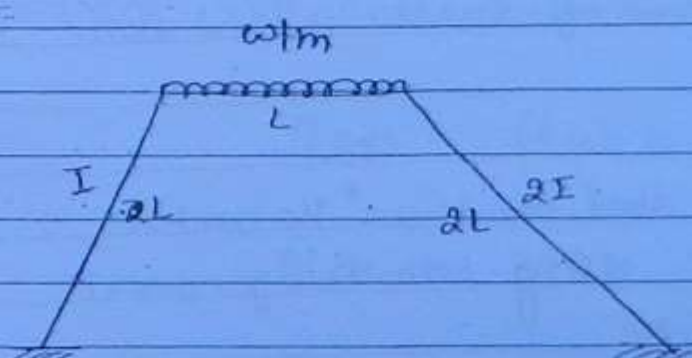
165

(⟶)
Sway
direction



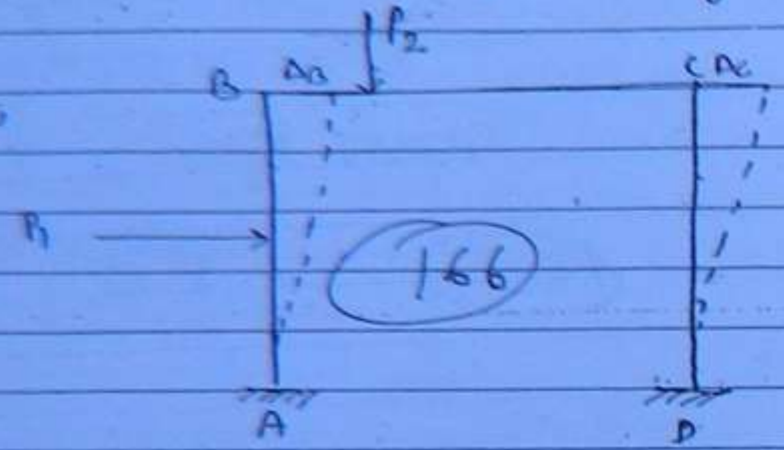
(EI Constant)

Resultant sway Towards Right: (→)



Sway may occur, which can be found by Computation.

Procedure to Analysis sway cases when Δ is not known



$\Delta_B = \Delta_C = \Delta$

that is effect of Axial force are neglected
i.e. length of member do not change.

① initially analysis of frame is done neglected sway. It's called non sway analysis. In non sway analysis find fixed end moment due to loading, Distribution factor at joints and compute the final ends moment at the end of table. called non. sway moments

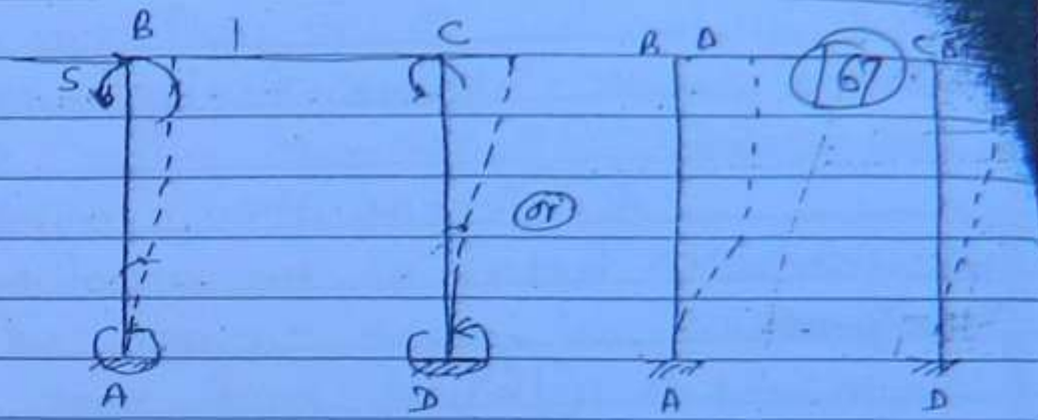
② if frame is vertical ~~that~~ ~~then~~ then find Horizontal Reaction at the support using Non-sway moment & given loading and check

$$\Sigma f_x = H_A + H_D + P_1$$

if $\Sigma F_x = 0$ there will be no sway but $\Sigma f_x \neq 0$ than Sway force will be Present and sway force (S) is equal to ΣF_x.

$$\boxed{S = \Sigma f_x}$$

(3) To find the effect of sway force remove the loading and apply sway force at the Panal



Under sway force Str. will go sway Δ as shown in fig. due to Δ fixing moment will be produce in those member for which transveres displacment is produced.

If Δ Cause Clockwise Rotational to the member than fixing moments produced will Anti-clockwise and vice-versa.

$$\bar{M}_{AB} = \bar{M}_{BA} = -\frac{6EI\Delta^2}{l^2}$$

$$\bar{M}_{BC} = \bar{M}_{CB} = 0$$

$$\bar{M}_{CD} = \bar{M}_{DC} = -\frac{6EI\Delta^2}{l^2}$$

Since Δ is not known, Hence following Procedure is addopted. find Ratio of fixing moments Produce it's ratio.

$$\bar{M}_{AB} : \bar{M}_{BA} : \bar{M}_{BC} : \bar{M}_{CB} : \bar{M}_{CD} : \bar{M}_{CD}$$

$$\Rightarrow -\frac{6EI\Delta}{l^2} : -\frac{6EI\Delta}{l^2} : 0 : 0 : -\frac{6EI\Delta}{l^2} : -\frac{6EI\Delta}{l^2}$$

$$\Rightarrow -1 : 0 : 0 : 0 : -1 : -1$$

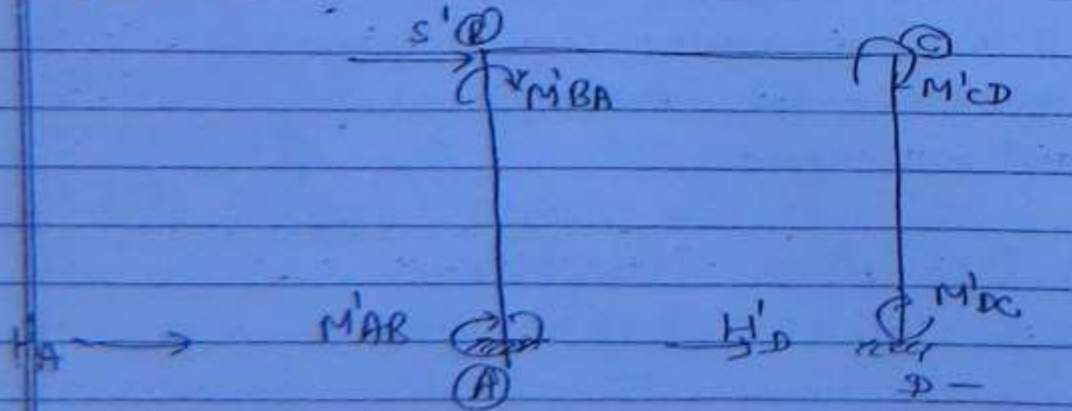
Adoppted any fixing moments in above ratio
say −10 : −10 : 0 : 0 : −10 : −10

(4) Entre above assume fixing moment in this table.
& distrube them accourding to the distribution factor, at the end of table.
There ~~are~~

168

A		B		C		D
	−10	−10	0	0	−10	−10
Column (a)	M'_{AB}	M'_{BA}	M'_{CD}	M'_{CB}	M'_{CD}	M'_{DC}

→ The moment given in column (a) are not actual sway moment, which are due to same sway force S'.



$$\Sigma F_x = 0$$

$$\boxed{S' + H_A + H_D = 0} \quad \text{—— (1)}$$

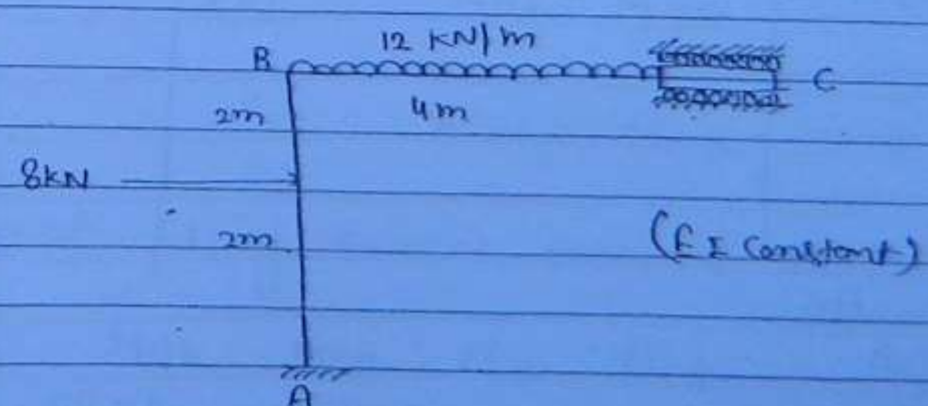
S' is known from equ (1) H_A' and H_D' can be computed by considering free body equilibrium of AB & CD with moments given in column A.

(5) The moment obtain in column (a), are due to same sway faces (S') as found above ~~there~~ Hence actual sway moment actual sway force (S) will be

$$\boxed{\frac{\text{Column (a)} \times S}{S'}}$$

(6) Final end moment will be algebraic sum of Non sway moment (Step-1) & actual sway moment obtain frame previous (Step 5)

Q. Analysis the frame shown in fig. the use Hardy cross method of moment distribution.



Since support C can carry moment Hence stiffness of BC is equal to $\frac{4EI}{L}$

Joint	member	Relative Stiffness	total Stiffness	Distribution factor
B	BA	I/4	$\frac{2I}{4}$	1/2
	BC	I/4		1/2

170

$$\bar{M}_{AB} = \frac{-PL}{8} \Rightarrow -\frac{8 \times 4}{8} \Rightarrow -4$$

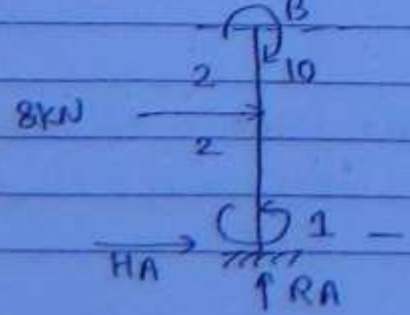
$$\bar{M}_{BA} = +4 \qquad \bar{M}_{BC} = \frac{-\omega L^2}{12}$$

$$\bar{M}_{BC} = \frac{-12 \times 4 \times 4}{12}$$

$$\bar{M}_{BC} = -16 \qquad \bar{M}_{CB} = +16$$

	A	B (1/2)	B (1/2)	C
	-4	+4	-16	+16
		+6	+6	
	+3			+3
no sway moment	-1	+10	-10	+19

free body equilibrium AB



$\Sigma M_B = 0$

$-H_A \times 4 - 1 - 8 \times 2 + 10 = 0$

$H_A = -1.75$

$H_C = 0$

171

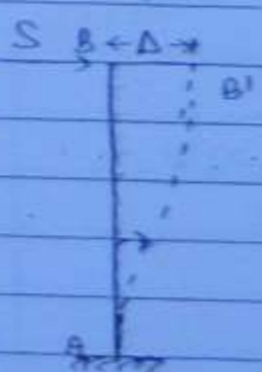
$\Sigma F_x = 0$

$8 + H_A = 8 - 1.75 \Rightarrow 6.25 \ (\rightarrow)$

→ since ΣF_x is not equal to zero Hence sway will occur sway force

$S = \Sigma F_x = 6.25 \text{ kN} \ (\rightarrow)$

Part of question:- SWAY ANALYSIS



$\bar{M}_{AB} = \bar{M}_{BA} = \frac{-6EI\Delta}{L^2}$

$\bar{M}_{BC} = \bar{M}_{CB} = 0$

Ratio of Fixing moment produced due to Δ.

$\bar{M}_{AB} : \bar{M}_{BA} : \bar{M}_{BC} : \bar{M}_{CB}$

$\frac{-6EI\Delta}{L^2} : \frac{-6EI\Delta}{L^2} : 0 : 0$

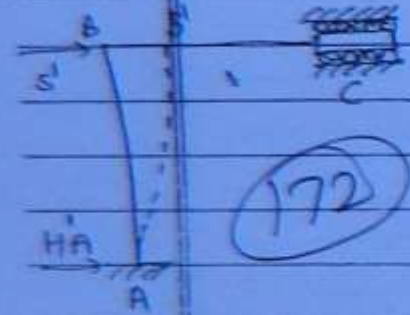
$-1 : -1 : 0 : 0$

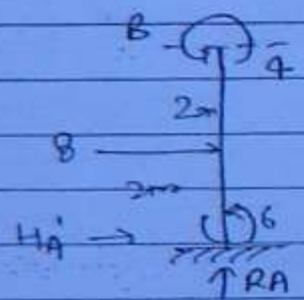
$-8 : -8 : 0 : 0$

		$\frac{1}{2}$	$\frac{1}{2}$	
	-8	-8	0	0
		+4	+4	
	+2			+2
Column (a)	-6	-4	+4	+2
ASM	-15	-10	+10	+5
table 1 USM	-1	+10	-10	+19
total	16	0	0	+24

the moment in given colum (A) are due to some sway force S' such that

$$S' - H_A' = 0$$





172

$M_B = 0$

$-H_A \times 4 - 6 - 4 = 0$

$H_A' = 2.5$

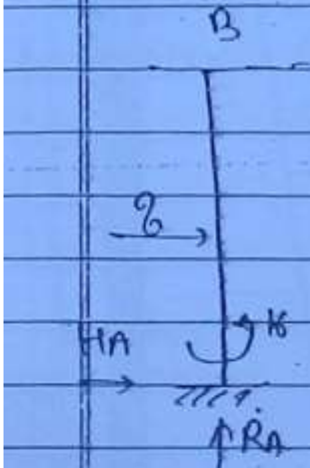
from (1)

$S' - 2.5 = 0 \quad \boxed{S' = 2.5}$

the moments given in column A are due to some sway force S' Hence actual sway moment → $\frac{\text{Column (a)} \times S}{S'}$

			table-1 USM	Net sway mom
M_{AB}	$\Rightarrow \frac{-6 \times 6.25}{2.5} \Rightarrow$	-15	-1	-16
M_{BA}	$\Rightarrow \frac{-4 \times 6.25}{2.5} =$	-10	+10	0

$M_{CD} = \frac{+4 \times 6.25}{2.5} = +10$	-10	0
$M_{DC} = \frac{+2 \times 6.25}{2.5} \Rightarrow +5$	$+19$	$+24$



173

$\Sigma M_B = 0$

$-H_A \times 4 - 16 - 8 \times 2 = 0$

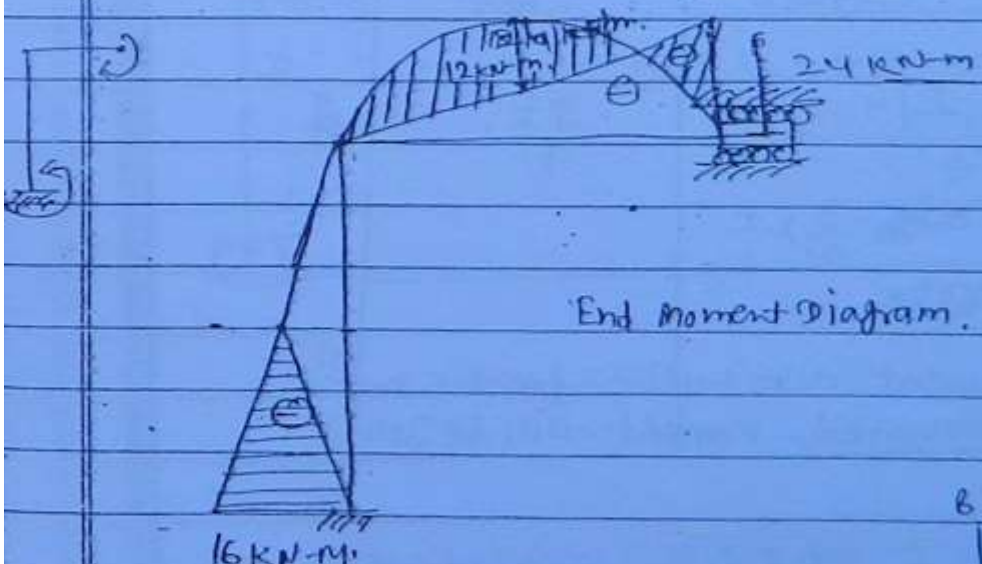
$H_A = 8$

$H_C = 0$

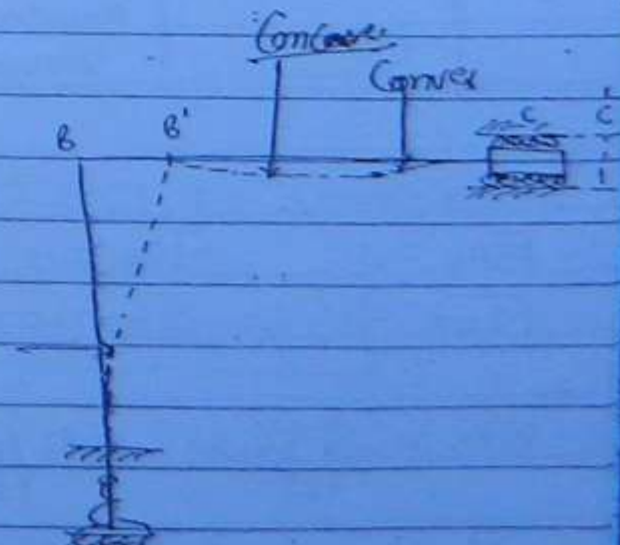
$\Sigma f_x = 0$

$8 + H_A + H_C = 0$

$8 - 8 = 0$ (OK)



End moment Diagram.

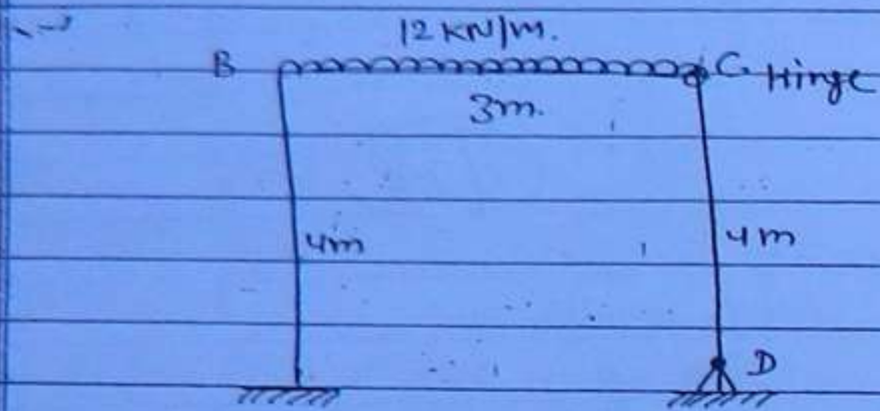


$BB' = CC'$

$\frac{M}{I} = \frac{E}{R}$

Elastic curve.

Q. Analysis the frame shown in fig. and draw the BMD use moment distribution method.



174

Joint	Member	Relative Stiffness	total stiffness	D.F.
B	BA	$I/4$	$\frac{2I}{4}$	$\frac{1}{2}$
	BC	$\frac{3}{4} \times \frac{I}{3}$		$\frac{1}{2}$
C	CB			
	CD	moment distribution factor not required. moment will be zero.		

$\bar{M}_{AB} = 0 \qquad \bar{M}_{BA} = 0$

$\bar{M}_{CD} = 0 \qquad \bar{M}_{DC} = 0$

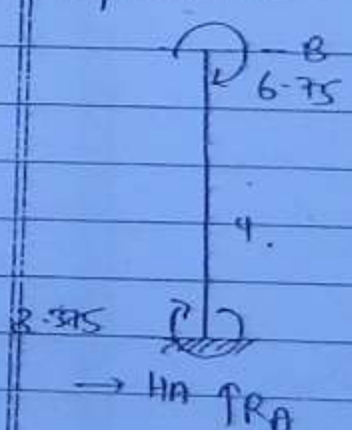
$$\bar{M}_{BC} = -\frac{wl^2}{12} \Rightarrow -\frac{12 \times 3 \times 3}{12} \Rightarrow -9 \text{ KN/m}$$

$$\bar{M}_{CB} = \frac{wl^2}{12} = \frac{+12 \times 3 \times 3}{12} \Rightarrow 9 \text{ KN/m.}$$

	A		B (1/2)	B (1/2)	C		D
	0	0	-9	+9	0		0
End Correctn			-4.5 ←	-9			
Corrected val	0	0	-13.5	0	0		0
	0	6.75	6.75	0	0		0
	+3.375						
Non sway moment :	+3.375	+6.75	-6.75	0	0		0

175

free body dia AB



$$-HA \times 4 + 6.75 + 3.375 = 0$$

$$HA = \frac{6.75}{4} = 2.53$$

$$\Sigma M_C = 0$$

$$H_D = 0$$

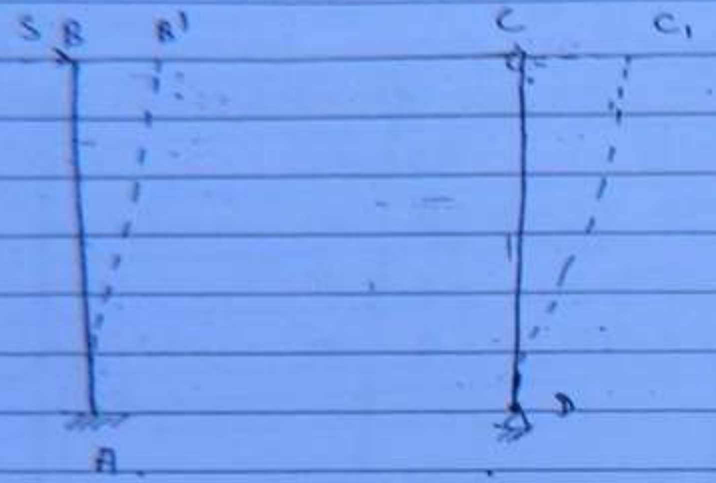
$$\Sigma f_x = 0$$

$$HA + H_D = 2.53 + 0 = 2.53 \neq 0 \text{ (sway)}$$

$$\boxed{S = \Sigma f_x = 2.53 \text{ kN } (\rightarrow)}$$

Remove the loading and applied sway force.
Right = 2.25 kW

Sway



(176)

Fixing moment due to sway

$$\bar{M}_{AB} = \bar{M}_{BA} = \frac{-6EI\delta}{l^2}$$

$\bar{M}_{BC} = \bar{M}_{CB} = 0$

$\bar{M}_{CD} = \bar{M}_{DC} = 0$ {Both ~~are~~ ends are Hinged.}

Fixing moment are

$\bar{M}_{AB} : \bar{M}_{BA} : \bar{M}_{BC} : \bar{M}_{CB} : \bar{M}_{CD} : \bar{M}_{DC}$

$\Rightarrow$ ~~6EIδ~~

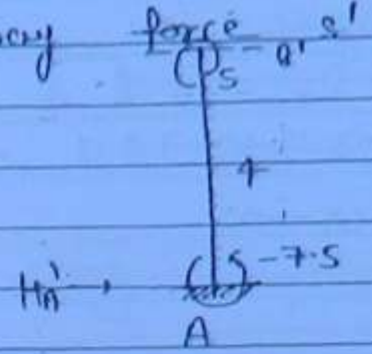
$\Rightarrow -1 : -1 : 0 : 0 : 0 : 0$

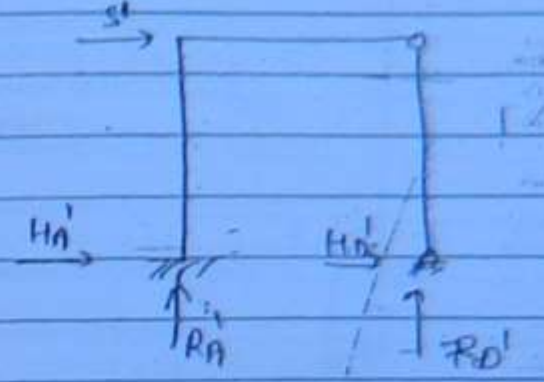
$\Rightarrow -10 : -10 : 0 : 0 : 0 : 0$

A	B ($\frac{1}{2}$)	B ($\frac{1}{2}$)	C	C	D
−10	−10	0	0	0	0
	+5	+5			
+2.5					
−7.5	−5	+5	0	0	0 0

Given A

Moment obtain in column A are due to some sway force s'





for AB

$$H_A' + H_D' + S' = 0 \quad - - - - - (1)$$

Taking moment About B (177)

$$\Sigma M_B = 0$$

$$-H_A' \times 4 - 7.5 - 5 = 0$$

$$H_A' = -3.125 \qquad H_D' = 0$$

from equ (1) $\quad H_A' + H_D' + s' = 0$

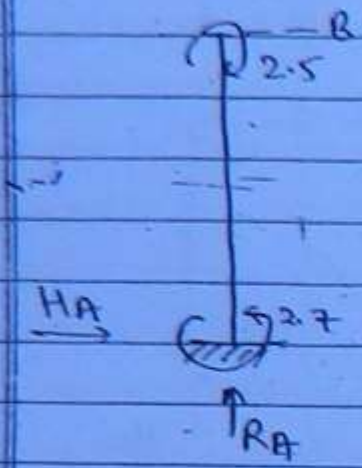
$$-3.125 + 0 + s' = 0$$

$$\boxed{S' = 3.125 \text{ kN}}$$

actual sway moment

Column(a) $\times S$ / s'

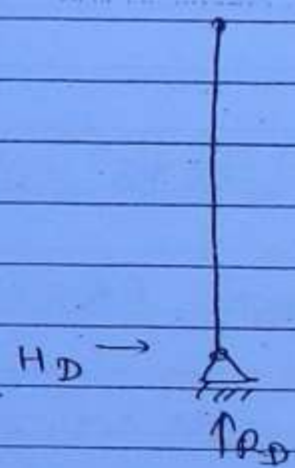
		NSM table 1	Final moment ~~Net~~ sway
$M_{AB} = \frac{-7.5 \times 2.53}{3.125}$	$\Rightarrow -6.07$	$+3.375$	-2.75
$M_{BA} \Rightarrow \frac{-5 \times 2.53}{3.125}$	$= -4.05$	$+6.75$	$+2.75$
$M_{BC} = \frac{5 \times 2.53}{3.125}$	$= +4.05$	-6.75	-2.75
$M_{CB} = 0$	$= 0$	0	0
	$= 0$	0	0
$M_{CD} = 0$	$= 0$	0	0
$M_{DC} = 0$			



$\Sigma M_B = 0$

$-H_A \times 4 - 2.7 + 2.7 = 0$

$H_A = 0$



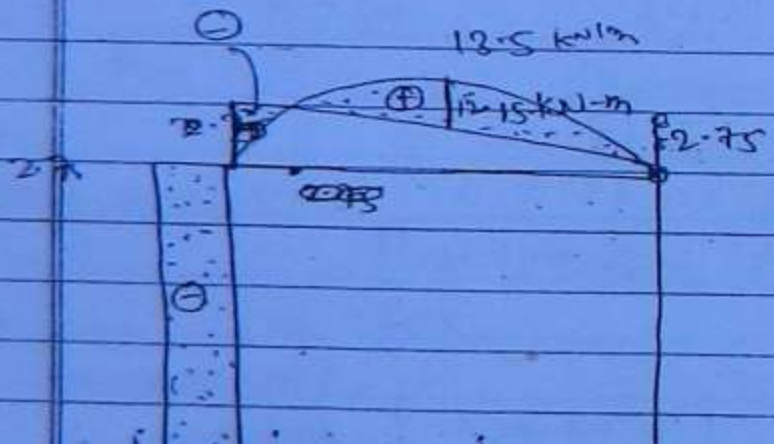
$\Sigma M_C = 0$

$H_D = 0$

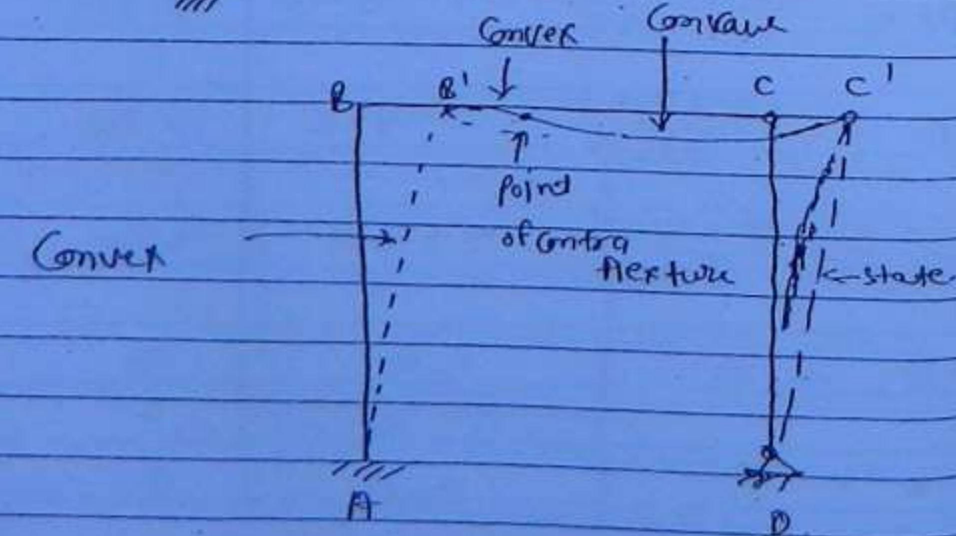
178

$\Sigma F_x = H_A + H_D$

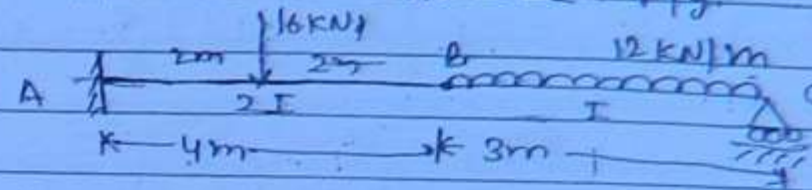
$= 0$



$13.7 - \frac{2.75}{2} = 12.15$



Q. Analysis the Beam Shown in fig.



$$\cancel{\bar{M}_{AB} = \frac{-Wab}{L} \rightarrow \frac{-16\times2\times2}{4} \rightarrow -16\,kN/m}$$

$$\cancel{\bar{M}_{BA} = \frac{Wab}{L} \rightarrow \frac{16\times2\times2}{4} = 16\,kN\,m}$$

$$\bar{M}_{AB} = \frac{-PL}{8} \rightarrow -\frac{16\times4}{8} \rightarrow -8$$

$$\bar{M}_{BA} = +8$$

179

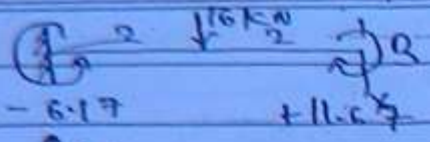
$$\bar{M}_{BC} = -\frac{WL^2}{12} \rightarrow -\frac{12\times3\times3}{12} \rightarrow -9$$

$$\bar{M}_{CB} = +9$$

memeber	Join	R.S	T.R.S	D.F.
B	BA	$2I/4$	$\frac{3I}{4}$	$2/3$
	BC	$\frac{3}{4}\times\frac{I}{3}$		$\frac{1}{3}$

	A	B (2/3)	B (1/3)	C	
	-8	+8	-9	+9	
			-4.5 ←	-9	End Correction
	-8	+8	-13.5	0	Corrected value
		+3.66	+1.83		
	+1.83				
Non sway moment	-6.17	-11.67	11.67	0	

Consider free body equm AB

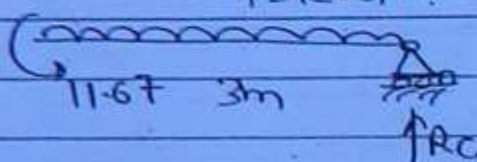


$$\Sigma M_B = 0$$

$$R_A \times 4 - 6.17 - 16 \times 2 + 11.67 = 0$$

180

$$R_A = 6.625 \ (\uparrow)$$



$\Sigma M_B = 0$

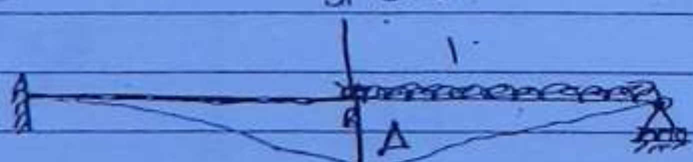
$$R_C \times \cancel{8}^{3} - 11.67 + \underline{12 \times 3} \times 1.5 = 0$$

$$R_C = 14.11 \ (\uparrow)$$

$$\Sigma F_y = 16 + 12 \times 3 - 6.625 - 14.11$$

$$S = \Sigma F_y \Rightarrow 31.6265 \ (\downarrow)$$

Sway Analysis :-



Due to Δ fixing moment

$$\bar{M}_{AB} = \frac{-6E(2I)\Delta}{4^2} \Rightarrow \bar{M}_{BA} = \frac{-\cancel{6}^{3} E.\cancel{2}I\Delta}{\cancel{4}_{2} \times 4}$$

$$\bar{M}_{BC} = +6\frac{3EI\Delta}{3^2} \rightarrow$$

$$\bar{M}_{CB} = 0$$

$\bar{M}_{AB} : \bar{M}_{BA} : \bar{M}_{BC} : \bar{M}_{CB}$

$-\frac{3}{4}EI\Delta : \frac{3}{4}EI\Delta : \frac{8}{3}EI\Delta : 0$

~~$\frac{3}{4} : \frac{3}{4} : 1 : 0$~~

~~$-4 : -4$~~

(181)

Ratio of fixing moment

$\bar{M}_{AB} : \bar{M}_{BA} : \bar{M}_{BC} : \bar{M}_{CB}$

$\Rightarrow -\frac{12EI\Delta}{16} : -\frac{12EI\Delta}{16} : +\frac{3EI\Delta}{9} : 0$

$\Rightarrow -\frac{3}{4} : -\frac{3}{4} : \frac{1}{3} : 0$

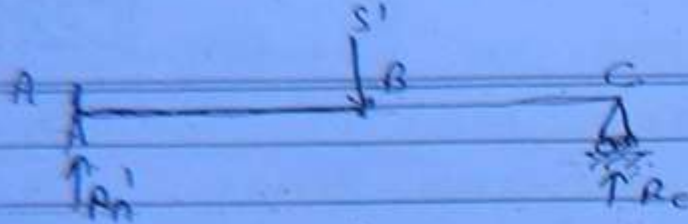
$\Rightarrow -9 : -9 : +4 : 0$

$\Rightarrow -18 : -18 : +8 : 0$

	A		B		C
		$\boxed{2/3}$	$\boxed{1/3}$		
	-18	-18	+8		0
		+6.67	+3.33		
	+3.33				
Column (a)	-14.67	-11.33	+11.33		0

moments obtain a column a after due to some sway force S'

Page 10

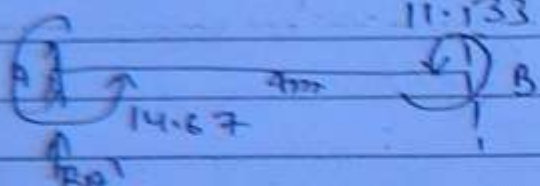


$\Sigma f_y = 0$

$S' - R_A' - R_C' = 0$

182

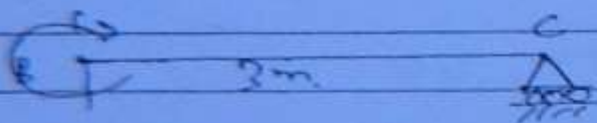
$$\boxed{S' = R_A' + R_C'}$$



$\Sigma M_B = 0$

$R_A' \times 4 - 14.67 - 11.33 = 0$

$R_A' = +6.5\ Km$



$-R_C' + 3 + 11.33 = 0$

$R_C' = 3.77$

$S' = 6.5 + 3.77$

$\Rightarrow 10.27\ KN(\downarrow)$

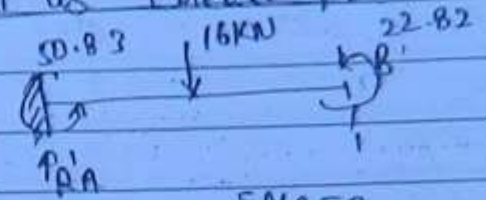
$D = 11.67$

actual sway moment

		NSM	Final
$M_{AB} = \dfrac{-14.67 \times 31.625}{10.27}$	$\Rightarrow -44.65$	-6.17	-50.82
$M_{BA} = \dfrac{-11.33 \times 31.625}{10.27}$	$\Rightarrow -34.45$	$+11.67$	22.82

$M_{BC} = \frac{11.33 \times 31.625}{10.27} \rightarrow 34.45 \quad | -11.67 \quad | +22.82$

∴ let us check for

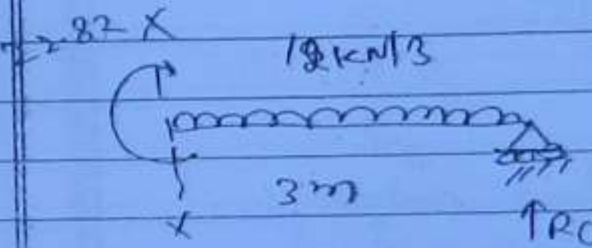


(183)

$\Sigma M_B = 0$

$R_A \times 4 - 16 \times 2 - 22.82 - 50.83 = 0$

$R'_A = 26.41 \text{ kN}$



$\Sigma M_B = 0$

$- R'_C \times 3 + \cancel{22.82}\ 12 \times 3 \times 1.5 + 22.82 = 0$

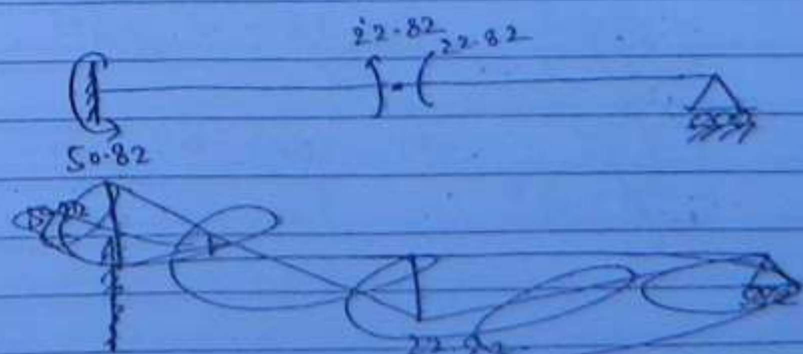
$R_C = + 25.61$

$\Sigma F_y \Rightarrow R_A + R_C = 16 + 12 \times 3$

$\Rightarrow 16 + 39$

$26.4 + 25.6 \Rightarrow 52$

$\cancel{52} \rightarrow \cancel{52} = 0$ Check save.



Centre ordinate and moment = $-\frac{50.84 + 22.82}{2}$

$\Rightarrow -25.41$

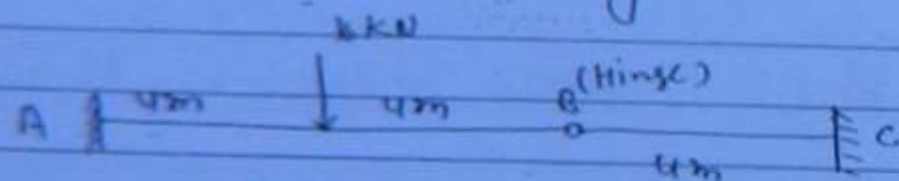
184

$M_{AB} = \frac{PL}{8} \Rightarrow \frac{16 \times 4}{8} = 8$

for UDL $M_{BC} \Rightarrow \frac{wl^2}{8} \Rightarrow \frac{12 \times 3^2}{8} = 13.5$

ESE :- Using moment distribution method analysis the Beam and Draw the Bending moment diagram.

EI = Constant



$\overline{M_{AB}} = \frac{-PL}{8} \Rightarrow \frac{16 \times 8}{8} \Rightarrow -16\ 8\ kN$

$\overline{M_{BA}} \Rightarrow -16\ kN$

$M_{BC} = M_{CB} = 0$

Distribution factor

~~Member~~ ~~Joint~~ ~~R.S~~ ~~TRS~~ ~~D.F.~~

~~B~~ ~~BA~~ ~~$\frac{3}{4} \times \frac{I}{8} \Rightarrow \frac{3}{32} I$~~

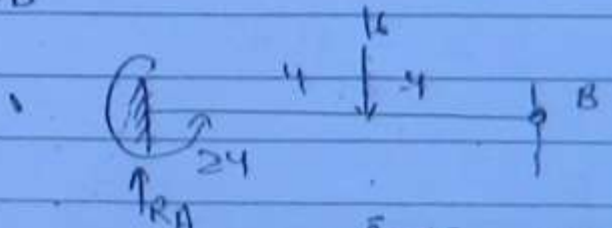
~~BC~~ ~~$\frac{3}{4} \times \frac{I}{8} = \frac{3}{8} I$~~

Distribution factor (0) than B is Hinge

-16	+16	0	0
-8 ←	-16		
NM -24	0	0	0

185

Consider AB



$\Sigma M_B = 0$

$R_A \times 8 - 24 + 16 \times 4 = 0$

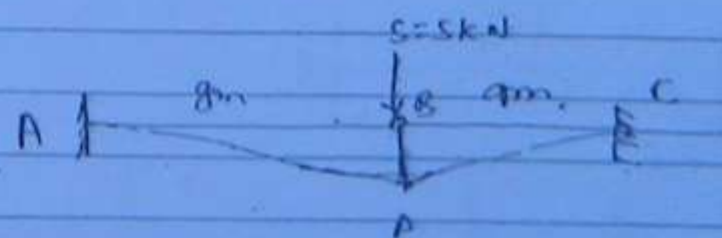
$R_A = \frac{64+24}{8} = 11\,kN$

Consider BC ⇒

$R_C = 0$

$\Sigma f_y = 16 - 11 - \cancel{8} = 0$

$-5\,(\downarrow)$



$\bar{M}_{AB} = \frac{-3EI\Delta}{L^2} = \bar{M}_{BA}$

$\bar{M}_{BC} = 0 = \bar{M}_{BD} \quad \bar{M}_{CB} = \frac{3EI\Delta}{L^2}$

Fixed moment

$$\overline{M_{AB}} : \overline{M_{BA}} : \overline{M_{BC}} : \overline{M_{CB}}$$

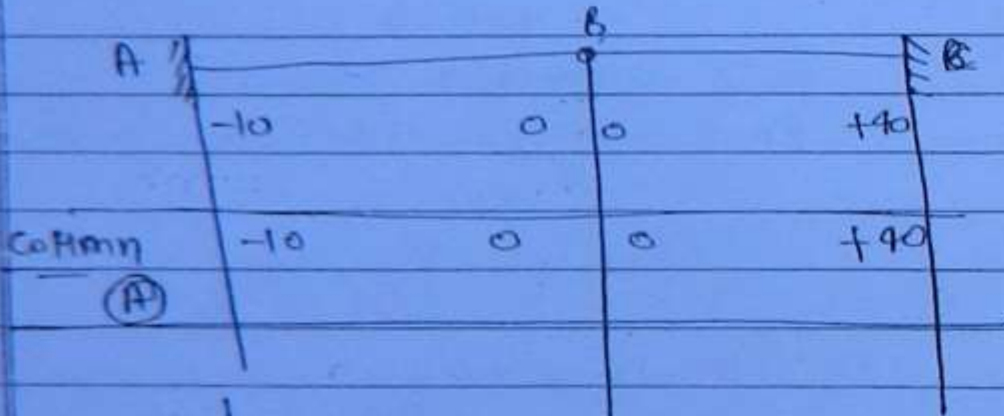
$$\Rightarrow -\frac{3EI\Delta}{L^2_{8^2}} : 0 : 0 : +\frac{3EI\Delta}{4^2}$$

$$\Rightarrow -\frac{1}{64} : 0 : 0 : \frac{1}{16}$$

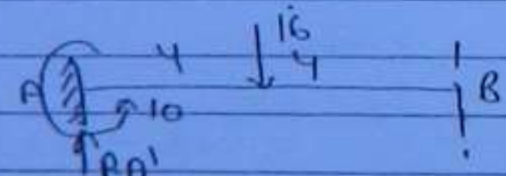
(186)

$$\Rightarrow -1 : 0 : 0 : +4$$

Say $\Rightarrow -10 : 0 : 0 : +40$



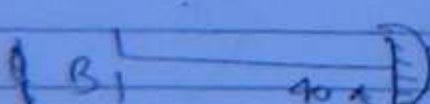
Consider AB



$\Sigma M_B = 0$

$$R_A' \times 8 - 10 = 0$$

$$R_A' = +1.125$$



$\Sigma M_B = 0$

$$-R_C' \times 4 + 40 = 0$$

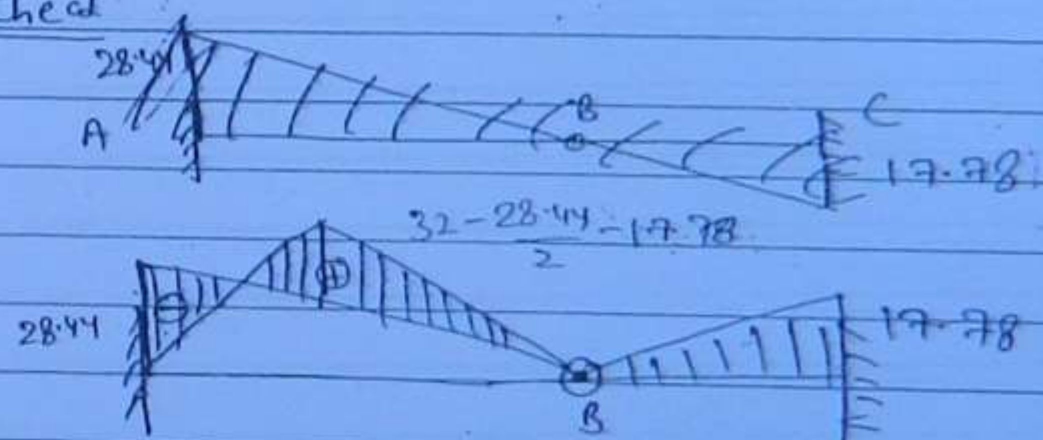
$$R_C' = 10$$

$S' = R_A' + R_C'$
$= 1.25 + 10$
$S' \Rightarrow 11.25$

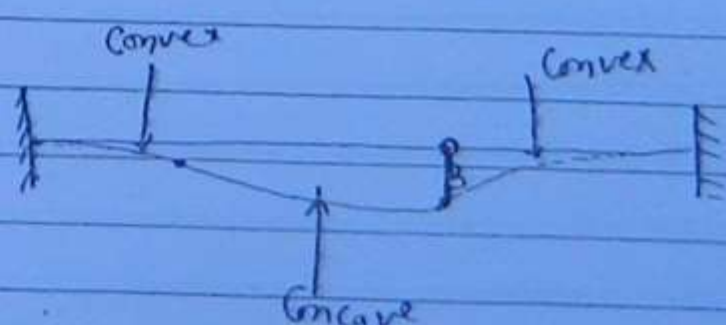
187

actual sway moments		NSM	Net moments
$M_{AB} = \frac{-10 \times 5}{11.25}$	$= -4.44$	-24	28.44
$M_{BA} =$	$= 0$	0	0
$M_{BC} =$	$= 0$	0	0
$M_{CB} = \frac{40 \times 5}{11.25}$	$\Rightarrow 17.78$	0	17.78

Let us check

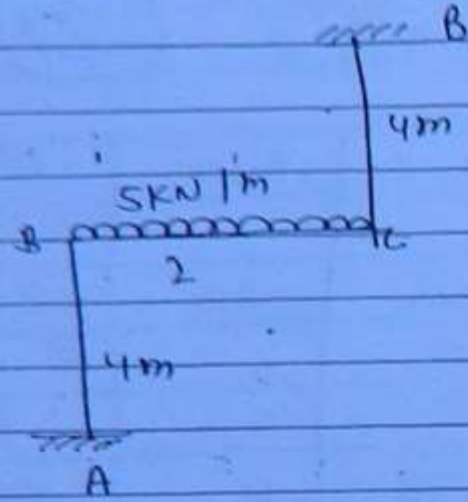


M_{AB2} $\frac{PL}{4} \Rightarrow \frac{16 \times 8}{4} \Rightarrow 32$



Q Analyse the frame shown in fig. using moment Distribution method.

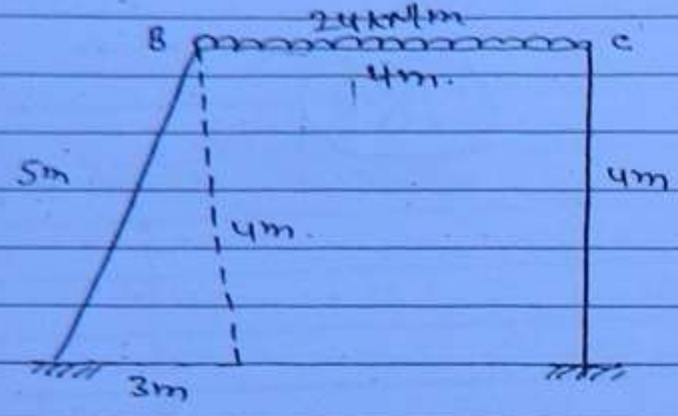
(188)



EI = constant.

189

8. for a the frame shown in fig draw the BM dia using Hard-cross method. EI dia.



190

$\bar{M}_{AB} = \bar{M}_{BA} = 0$

$$\bar{M}_{BC} = -\frac{wl^2}{12} \Rightarrow \frac{-24\times4^2}{12} \Rightarrow -32$$

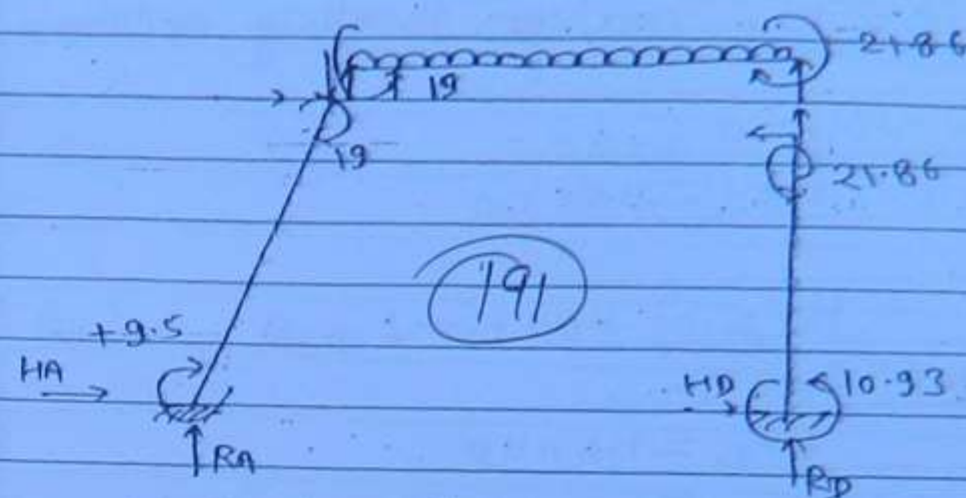
$$\bar{M}_{CB} = +32$$

$$\bar{M}_{CD} = \bar{M}_{DC} = 0$$

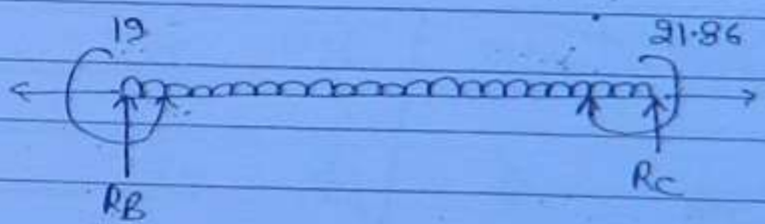
Distribution factor.

Joint	member	Relative stiffness	Total Relative stiffness	DF
B	BA	I/5	$\frac{9I}{20}$	4/9
	BC	I/4		5/9
C	CB	I/4	$\frac{1I}{2}$	1/2
	CD	I/4		1/2

		4/9	5/9	1/2	1/2	
	0	0	-32	+32	0	0
		14.22	17.77	-16	-16	
	7.11		-8	8.89		-8
NSM	+7.5	+12	-19	+21.86	-21.86	-10.93



→ To find HA in inclined member AB RA is requried since, their is no vertical loading in AB Hence. RA = RB , Hence RB can be completed by taking free body di'equilibrium of BC.



$\Sigma M_C = 0$

$$R_B \times 4 - 19 - 24 \times 4 \times 2 + 21.86 = 0$$

$$R_A = R_B = 47.285 \text{ kN}$$

To find HA, Consider free body equilibrium of AB

$\Sigma M_B = 0$

$$-H_A \times 4 + R_A \times 3 + 9.5 + 19 = 0$$

$$H_A = \frac{3 \times 47.289 + 9.5 + 19}{4}$$

$$H_A = 42.58 \; (\rightarrow)$$

To find H_D, consider freebody equilibrium of DC

$$\Sigma M_C = 0$$

$$-H_D \times 4 - 10.93 - 21.86 = 0$$

$$H_D = -81.97 (\rightarrow)$$

Net Horizontal force :-

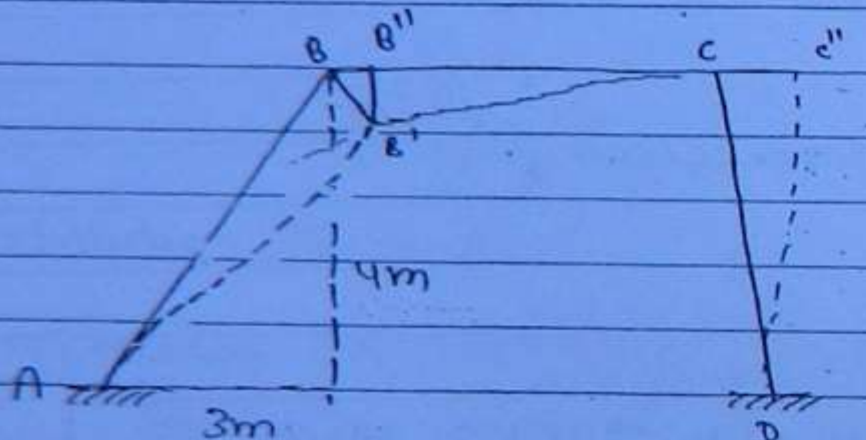
$$\Sigma F_x = H_A + H_D$$

$$\Rightarrow 42.598 - 8.197$$

192

$$\Rightarrow 34.398 \text{ KN} (\rightarrow)$$

Sway analysis :-



let $BB' = \Delta$

$$BB'' = \Delta \cos\theta = \frac{4}{5}\Delta$$

$$B'B'' = \Delta \sin\theta = \frac{3}{5}\Delta$$

Horizontal comp. of sway at B & C are equal

$$BB' = CC'$$

$$CC' = \frac{4}{5}\Delta$$

193

→ Due to sway fixing moment are developed.

$$\bar{M}_{AB} = \bar{M}_{BA} = \frac{-6EI\Delta}{5^2} \Rightarrow \frac{-6EI\Delta}{25}$$

$$\bar{M}_{BC} = \bar{M}_{CB} = \frac{6EI(3/5\Delta)}{4^2} \rightarrow \frac{18}{80}EI\Delta$$

$$\bar{M}_{CD} = \bar{M}_{DC} = -\frac{6EI(4/5\Delta)}{4^2} \Rightarrow \frac{-24}{80}EI\Delta$$

Ratio of fixing moment due to sway

$$\bar{M}_{AB} : \bar{M}_{BA} : \bar{M}_{BC} : \bar{M}_{CB} : \bar{M}_{CD} : \bar{M}_{DC}$$

$$\Rightarrow -\frac{6}{25} : -6/25 : 18/80 : 18/80 : -24/80 : -\frac{24}{80}$$

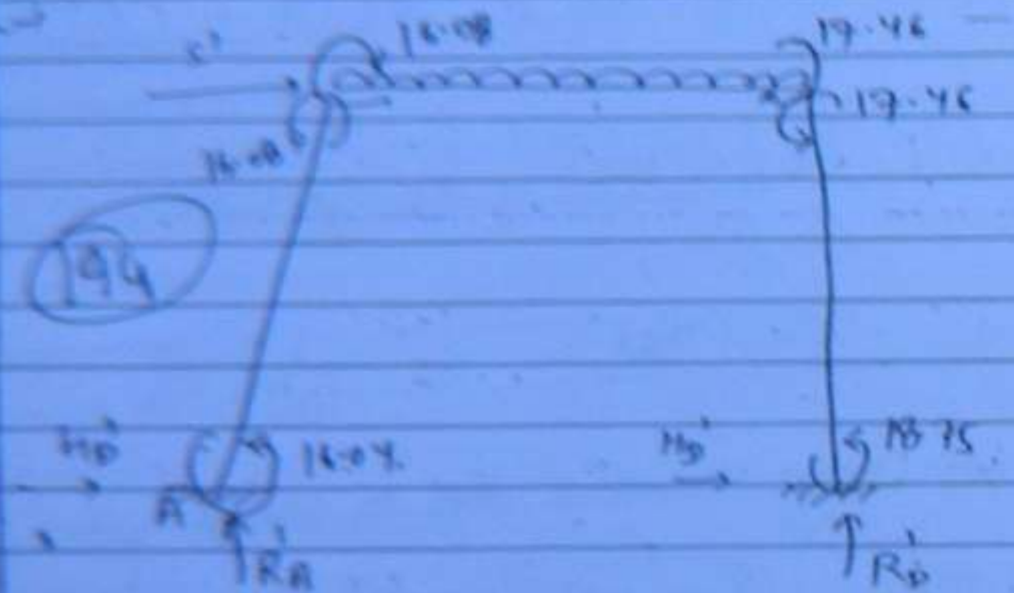
$$\Rightarrow -\frac{1}{25} : -\frac{1}{25} : \frac{3}{80} : \frac{3}{80} : -\frac{1}{20} : -\frac{1}{20}$$

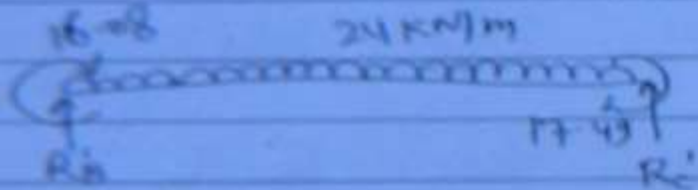
$$\Rightarrow -16 : -16 : +15 : +15 : -20 : -20$$

letus distribute the above moment.

	4/9	5/9	1/2	1/2	
-16	-16	15	15	-20	-20
	.44	.56	2.5	2.5	
.22		1.25	0.28		1.25
-16.04	-16.08	+16.88	17.46	-17.46	-18.7[illegible]

Let us mom. given in case same sway force Both.





$R_A' = R_B'$

To find R_B' consider span BC'

$R_B' \times 4 + 16.08 + 17.49 = 0$

$R_B' = R_A' = -8.385$ kN

To find H_A' consider span AB → $\Sigma M_B = 0$

$R_A' \times 3 - H_A' \times 4 - 16.04 - 16.08 = 0$

$H_A' = -14.32$

195

To find H_b' consider span CD

$\Sigma M_c = 0$

$-H_b' \times 4 - 18.75 - 19.46 = 0$

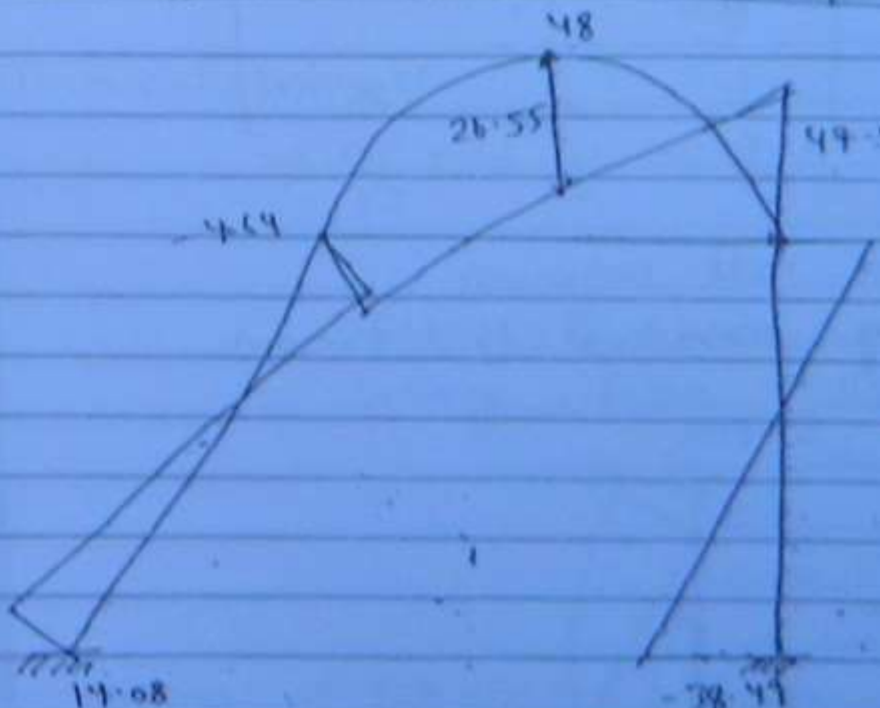
$S' + H_A' + H_b' = 0$

$S' - 14.32 - 9.65 = 0$

$\boxed{S' = 23.97\,kN}$

moment given in col ② even due to S'. Hence actual sway moment

Actual sway moment	-23.58	-23.64	+23.64	+25.67	-25.67	-27.9
Column ② x S' | S'						
NSM	+9.5	+19	+19	+21.86	-21.86	-10.93
final moment	+14.08	-4.64	+4.67	+47.53	-47.53	-38.49



$M_{BC} = \frac{wl^2}{8}$

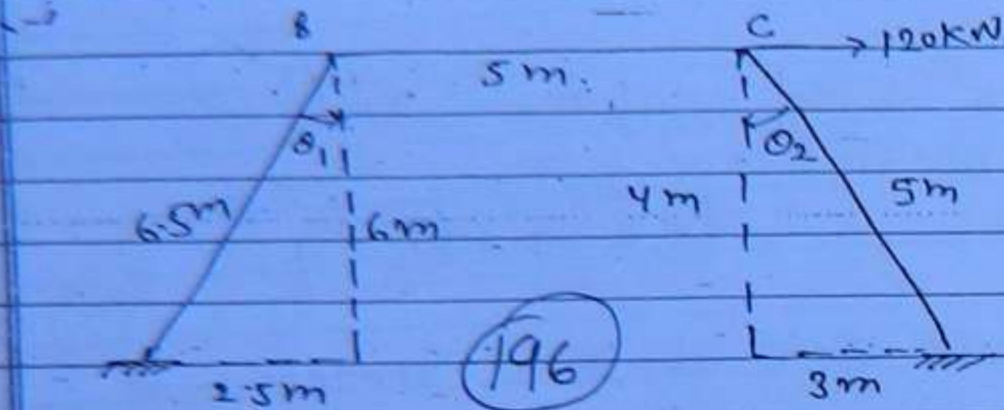
$\Rightarrow \frac{24 \times 4^2 \times 4}{8}$

$\Rightarrow 48$

$M\ centre = 48 - \left(\frac{-4.64 + 47.53}{2}\right) = 26.55$

Q.

Analysis the frame shown in fig. using MDM.



196

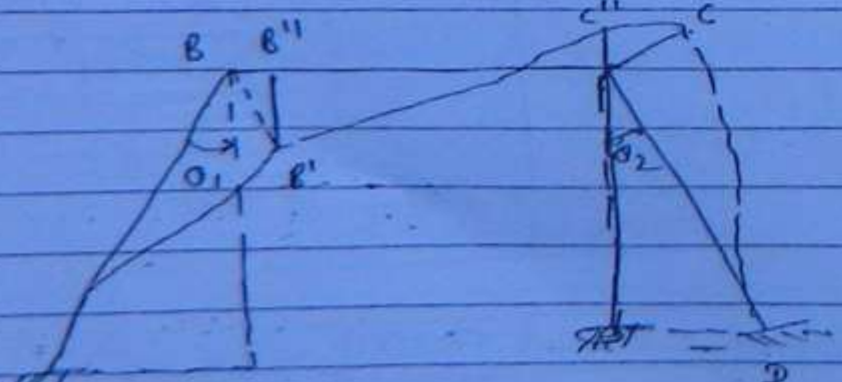
Distribution factor.

Joint	Member	R.S	T.R.S.	D.f.
B	BA	I/6.5	$\frac{23}{6.5}$ I	10/23
	BC	I/5		13/23
C	CB	I/5	$\frac{2}{5}$ I	1/2
	CD	I/5		1/2

fix. End moment all member.

Hence Non sway moment not Present

S = 120



let $BB' = \Delta$

$$BB' = \Delta \cos\theta_1 = \frac{6\Delta}{6.5} \rightarrow \frac{12}{13}\cos\theta$$

197

$$B'B'' = \Delta \sin\theta_1 \Rightarrow \frac{2.5}{6.5}\Delta = \frac{5}{13}\Delta$$

Horizontal displacement B = Horizontal displacement C

$$BB'' = c''c'$$

$$c''c' = \frac{12}{13}\Delta$$

$$\frac{c'c''}{cc'} - \cos\theta_2$$

$$cc' = \frac{c'c''}{\cos\theta_2} = \frac{12/13\,\Delta}{4/5} = \frac{15}{13}\Delta$$

$$cc'' = cc'\sin\theta_2$$

$$\Rightarrow \frac{15}{13}\Delta \times \frac{3}{5} = \frac{9}{13}\Delta$$

Sway of AB $= BB' = \Delta$

Sway of BC $= BB'' + cc''$

$$\Rightarrow \frac{5}{13}\Delta + \frac{9}{13}\Delta$$

$$\Rightarrow \frac{14}{13}\Delta$$

Sway of CD $= cc' = \frac{15}{13}\Delta$

Page 10

fixing moment due to sway:-

$$\bar{M}_{AB} = \bar{M}_{BA} = \frac{-6EI\Delta}{6.5^2}$$

198

$$\bar{M}_{BC} = \bar{M}_{CB} = \frac{6EI\Delta}{5^2} = \frac{6EI(14/13)\Delta}{5^2} \rightarrow \frac{84EI\Delta}{325}$$

$$\bar{M}_{CD} = \bar{M}_{DC} = \frac{-6EI(15/13\Delta)}{5^2} \Rightarrow \frac{90}{325}\Delta$$

Ratio of fixing moment:-

$$\Rightarrow \frac{-1}{6.5^2} : \frac{-1}{6.5^2} : \frac{14}{13\times5^2} : \frac{14}{13\times5^2} : \frac{-15}{13\times5^2} : \frac{15}{13\times5^2}$$

$$\Rightarrow \frac{3}{169} : \frac{-3}{169} : \frac{14}{325} : \frac{14}{325} : \frac{15}{325} : \frac{15}{325}$$

$$\Rightarrow \frac{-100}{13} : \frac{-100}{13} : +14 : +14 : -15 : -15$$

$$\Rightarrow -7.69 : -7.68 : 14 : 14 : -15 : -15$$

$$\Rightarrow -15.38 : -15.38 : 28 : 28 : -30 : 30$$

Distribute above movement according to D.F. and obtain Col (a) letus movement given in Col (a) are due to some sway force s', such that

$$s' + H_D + H_A = 0$$

199

find H_A' And H_D', R_A' and R_D' Are requrid R_A', R_D' Can be found By using free body equ^m at span BC with the given in Column (a) hence s' as known.

$$\text{final sway moment} = \frac{\text{coal (a)} \times s}{s'}$$

Moment sway moment are not given then final sway moment are final moment.

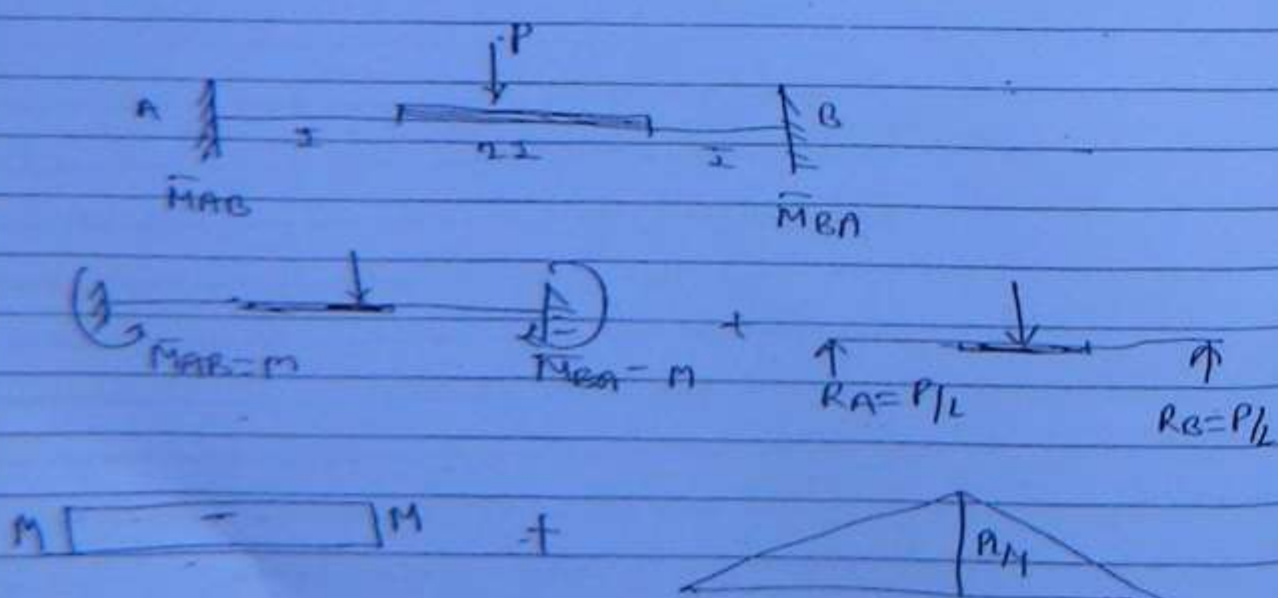
And moment dia will be final B.M.D. be there is no simply support dia.

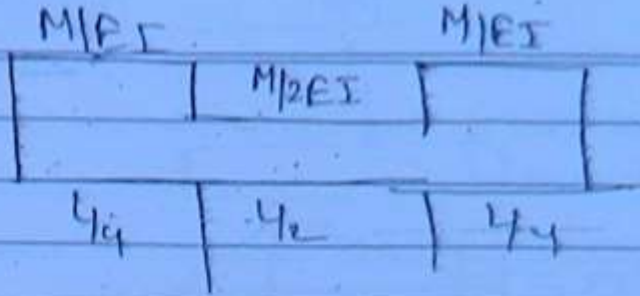
Chapter – 4

1	c	14	B	27	c	40	
2	c	15		28	b	41	
3	c	16	B	29	a	42	
4	a	17	C	30	c	43	
5	a	18	b	31		44	
6	d	19	a	32		45	
7	d	20	b	33		46	
8	a	21	c	34		47	
9		22	b	35		48	
10		23	d	36		49	
11		24	b	37		50	
12		25	b	38		51	
13	c	26	a	39		52	
53	B	58		62		65	
54		59		63		66	
55		60		64		67	d
56		61		68	d	69	b
57		70	b	71	c	72	a
73	b	74	d				

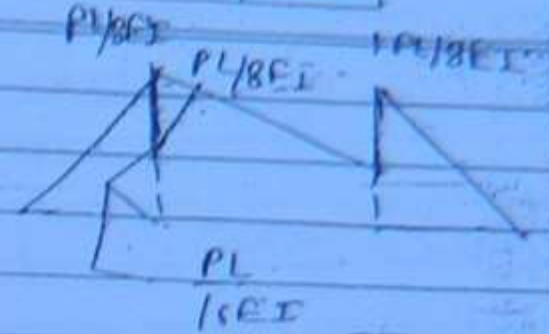
200

80





(f)



201

Area moment method.

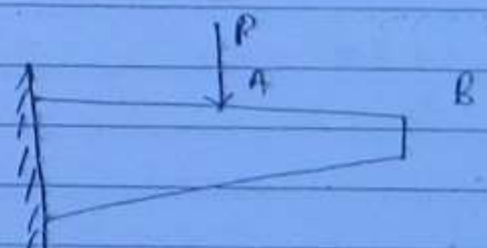
Change in slope from A to B = 0

$\theta_{AB} = 0$

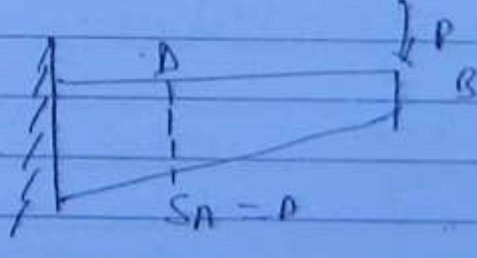
$$\Rightarrow -\left[\frac{M}{EI}\times\frac{L}{4} + \frac{M}{2EI}\times\frac{L}{2} + \frac{M}{EI}\times\frac{L}{4}\right] +$$

$$\left[\left(\frac{1}{2}\times\frac{L}{4}\times\frac{PL}{8EI}\right)\times 2 + \frac{1}{2}\times\frac{PL}{16EI}\times\frac{L}{2}\times\frac{PL}{16EI}\times\frac{L}{2}\right]$$

(27)

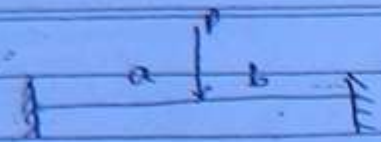


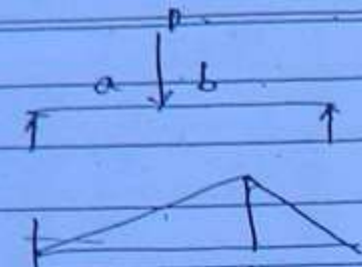
$\delta_B = 0$



Ans ⇒ A

(31) (a)





$M_{AB} = \frac{-Pab^2}{L^2}$

$M_{BA} = \frac{Pa^2b}{L^2}$

$\frac{Pab^2}{L^2} + \frac{Pa^2b}{L^2} \Rightarrow \frac{Pab(a+b)}{L^2} \Rightarrow \frac{Pa}{b} = M$

$\frac{Pab}{L} = M$

(32) ~~4F~~

$\theta A + \theta B + \theta C + \theta D$

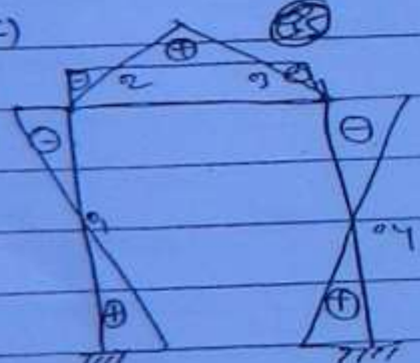
$\frac{4EI}{L} + \frac{4EI}{L} + \frac{3EI}{L} + 0$

$\Rightarrow \frac{11EI}{L}$

(33) (a)

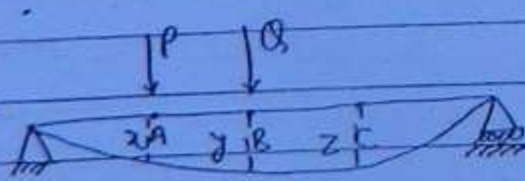
(34) (c)

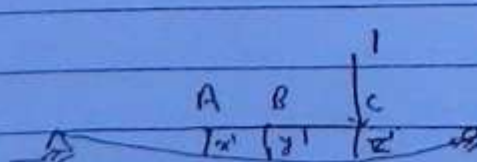
(35)



(d)

(36)



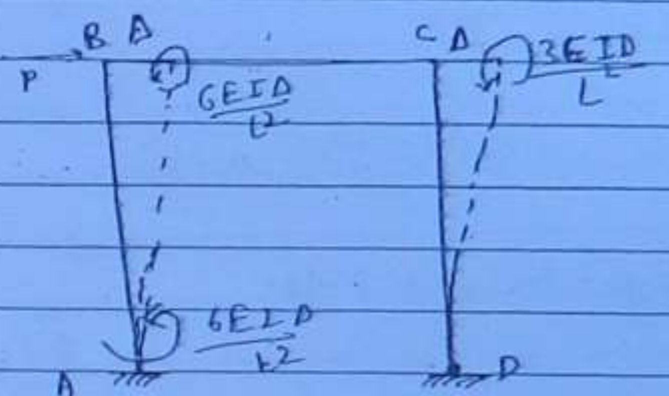


so virtual work done by unit load = virtual work done by P, Q

$$1 \cdot z = Px' + Qy'$$

$$\boxed{z = Px' + Qy'} \quad (b)$$

(d)



$\Sigma M_B = 0$ in FBD of AB

$$L \times -H_A + \frac{6EI\Delta}{L^2} + \frac{6EI\Delta}{L^2} =$$

$$H_A = \frac{-12EIL}{L^3}$$

$\Sigma M_C = 0$ in CD

$$-H_D \times L - \frac{3EI\Delta}{L^2} = 0$$

$$H_D = -\frac{3EI\Delta}{L^3}$$

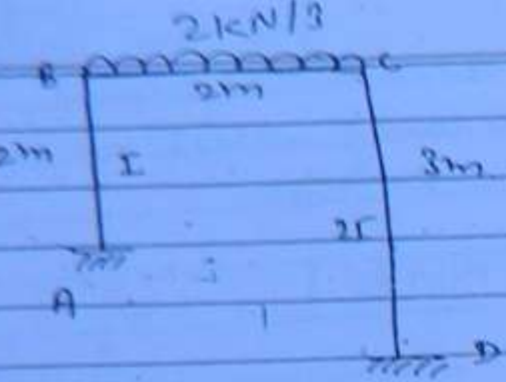
$$P + H_A + H_D = 0$$

$$P - \frac{12EI\Delta}{L^3} - \frac{3EI\Delta}{L^3} = 0$$

$$\boxed{P = \frac{15E\Delta I}{L^3}}$$

Page 10

(40) (c)

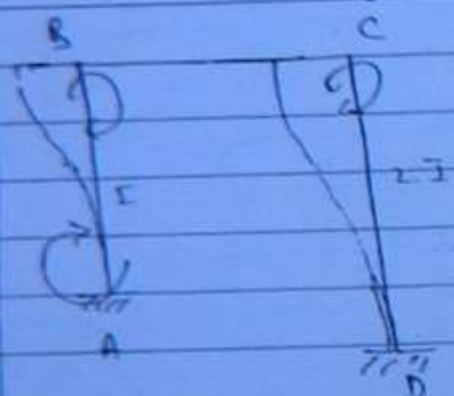


(202)

stiffness of BA $\Rightarrow \frac{4EI}{2}$

Stiffness of CD $= \frac{4E(2I)}{3} \Rightarrow \frac{8EI}{3}$

fram will sway toward left.



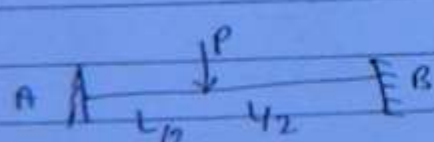
$\bar{M}_{AB} = \bar{M}_{BA} = \frac{+6EI\Delta}{L^2(2^2)}$

$\bar{M}_{CD} = \bar{M}_{DC} = \frac{+6E(2I)\Delta}{3^2}$

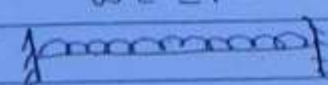
$\frac{\bar{M}_{AB}}{\bar{M}_{CD}} = \frac{\frac{1}{2}}{2/9} \Rightarrow \frac{9}{8}$

(41) (b)

(42) (c)



$w \cdot L = P = w = P/L$



$\frac{PL}{8} = 860$

$PL = 480$

$\bar{M} = \frac{wL^2}{12}$

$\Rightarrow \frac{P}{L} \times \frac{L^2}{12} \Rightarrow \frac{PL}{12}$

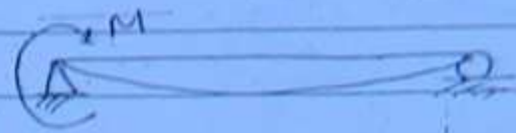
$\Rightarrow \frac{480}{12}$

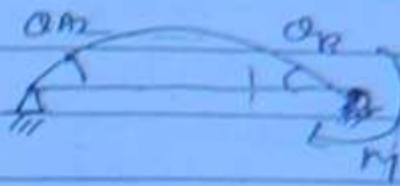
$\Rightarrow 40 \text{ kNm.}$

(43) (d)

(44) (c) (45) (c) (40)

(46)





$\theta_{A_1} = \frac{ML}{3EI}$ $\theta_{A_2} = \frac{ML}{6EI}$

$$\text{Net } \theta_A = \theta_{A_1} - \theta_{A_2} \Rightarrow \frac{ML}{3EI} - \frac{ML}{6EI} = \frac{ML}{6EI}$$

(d)

(47) (a) (48) (c) (49) (c)

(50) (c) (51) (b) (53) (c) (54) d

(55) c

(56) (d)

stiffness of BC = $4\,(EI/L)$

BA = $3\,(EI/L)$

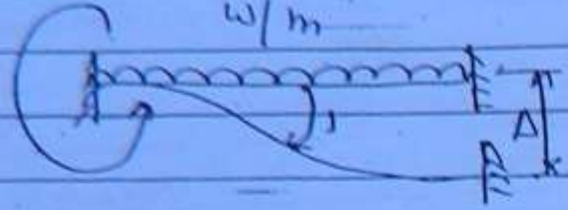
total 7

D.F. $\Rightarrow$ 4/7, 3/7

(57) (A)

Page

(58) (a) (59)



204

$$\bar{M}_{AB} = +\frac{\omega L^2}{12EI} - \frac{6EI\delta}{l^2}$$

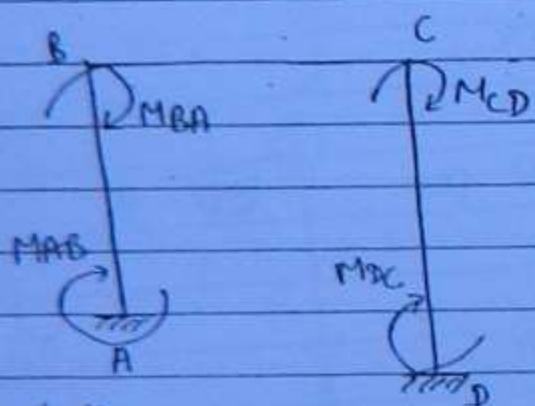
$$\Rightarrow \frac{\omega L^2}{12EI} - \frac{6EI}{l^2}\frac{\omega L^4}{72EI}$$

$$\Rightarrow 0$$

(d.)

(60)

(a)



Shear Eq^n.

$\Sigma F_x = 0$ $\quad H_A + H_D + P = 0$

$\Sigma M_B = 0$ for AB

$$-H_A \times L + M_{AB} + M_{BA} = 0$$

$$H_A = \frac{M_{AB} + M_{BA}}{L_1}$$

for DC

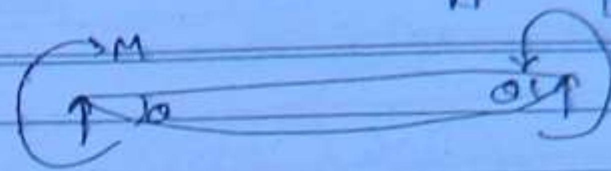
$\Sigma M_c = 0$

$$H_D = \frac{M_{CD} + M_{DC}}{L_2}$$

$$\frac{M_{AB} + M_{BA}}{L_1} + \frac{M_{CD} + M_{DC}}{L_2} + P = 0 \quad (a)$$

205

(61)

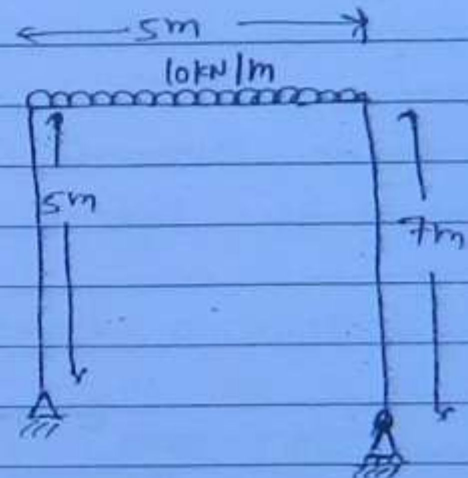


$\theta = \frac{M}{2EI}$ $\qquad M = \frac{2EI\theta}{L}$

if $\theta = 1$

$M = k$ $\qquad \boxed{k = \frac{2EI}{L}}$ $\qquad$ (b)

(62) $EI = 81380\ kN\text{-}m^2$

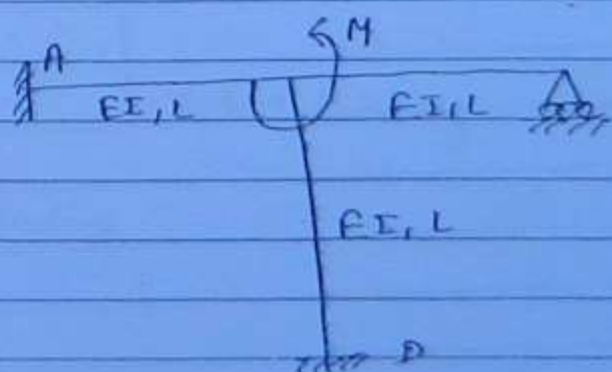


(a)

(63) d

(64)

(b)



Stiffness of All member at B $= \frac{4EI}{L} + \frac{4EI}{L} + \frac{3EI}{L}$

$k = \frac{11EI}{L}$

$M = K\cdot\theta$ $\qquad \boxed{\theta = \frac{ML}{11EI}}$

$M = \frac{11EI}{\ }\cdot\theta$

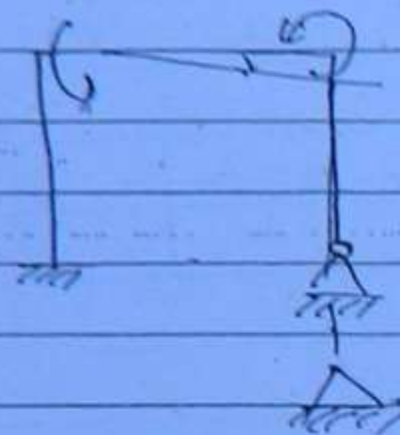
Page

(65) (a)

206

(66) (b)

(67) d



Slope Deflection method

→ Based on stiffness concept | Equilibrium Concept | Displacement Concept

207

→ Given By G.A. Maney

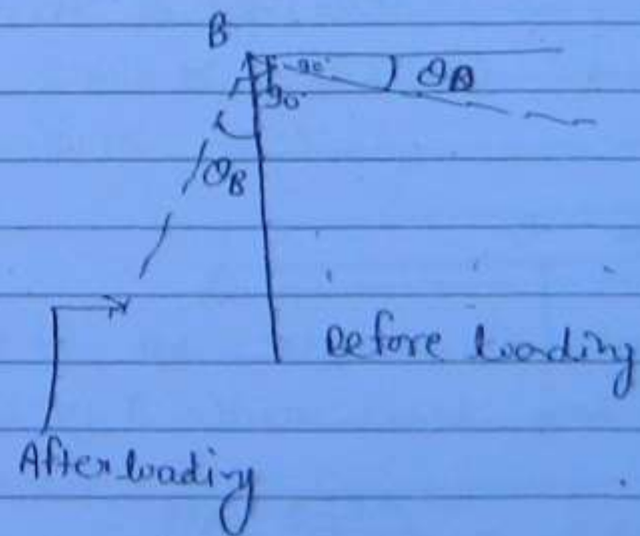
→ Basic ~~unit~~ unknown are joint Displacement (θ & Δ)

→ Joint Displacement are found using join equilibrium condition & shear equcation

→ Joint Displacement are relate to the member forse i.e. moments, such equcation are called slop-Deflection equcation.

→ Joint are considered rigid i.e. angle b/w member metting at rigid joint do not change after loading.

→ Joint rotates a whole



Derivation of Slope Deflection Eqn:-

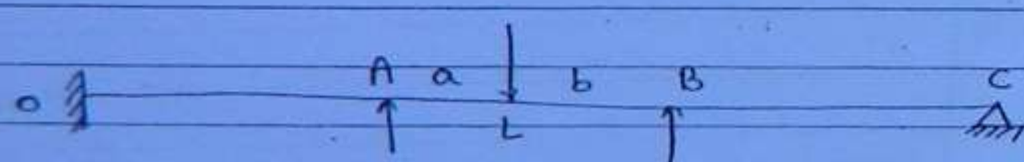
① Sign convention:- 208

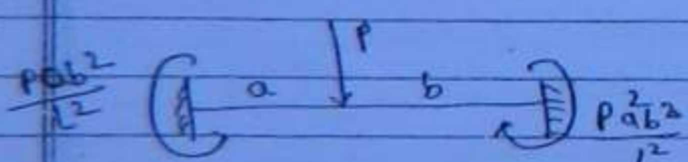
(i) End moments:- (+) clockwise
(−) Anticlockwise

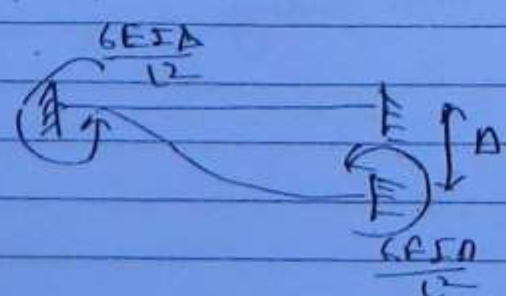
(ii) Slope (Rotation of joints)
(+)θ Clockwise
(−)θ Anticlockwise

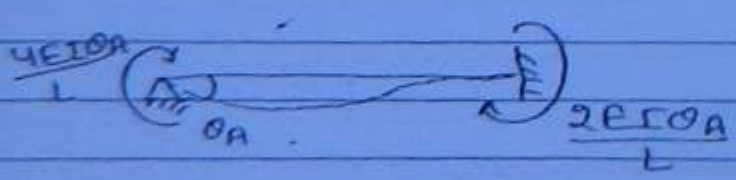
(iii) Linear Displacement:-
Those Δ, δ which will produce clockwise rotation to the member $\xrightarrow[\text{treated}]{\text{will be}}$ (+)ive

Those δ which produce anticlockwise rotation to the member $\xrightarrow[\text{be treated}]{\text{will}}$ (−) ive.



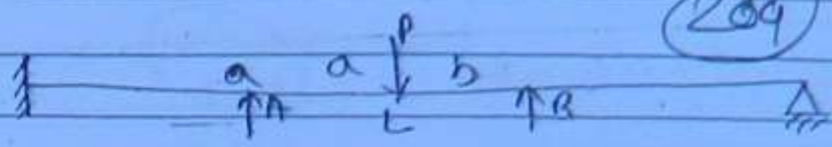






Derivation of slope Deflection eqn.

204



Step-I Consider A & B to be fixed support then due to given loading fixed end moment will be $\overline{M}_{AB}$ and $\overline{M}_{BA}$

$$\overline{M}_{AB} = -\frac{Pab^2}{L^2}$$

Step-II If joint A ~~Rooted~~ Rotates by θ_A then moment Produced at A will be $= \frac{4EI}{L} \times \theta_A$

Carry over moment of $\theta_B = \frac{2EI}{L} \times \theta_B$

3) If supports B by Rotates by θ_B than moment Produce B will be $\frac{4EI}{L}\theta_B$ than Carry over moment of A will be $\frac{2EI}{L} \times \theta_B$

(4) if one of the support settles (say) support B settels by Δ then fixed and moment Produced to settlement

$$\overline{M}_{AB} = \frac{6EI\Delta}{L^2} \qquad \overline{M}_{BA} = \frac{-6EI\Delta}{L^2}$$

where Δ is treated (+ive) if it Produces Clockwise rotation to the member.

* Final Effect of all the above 4 Cases.

→ Final moment in AB at A   210

$$M_{AB} = \bar{M}_{AB} + \frac{4EI}{L}\cdot\theta_A + \frac{2EI\theta_B}{L} - \frac{6EI\Delta}{L^2}$$

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\Delta}{L}\right]$$ --- force-Disp relation

Final moment at B in AB

$$M_{BA} = \bar{M}_{BA} + \frac{2EI}{L}\left[2\theta_B + \theta_A - \frac{3\Delta}{L}\right]$$

$M \Rightarrow$ +tive ↻

$\theta \Rightarrow$ +tive ↻

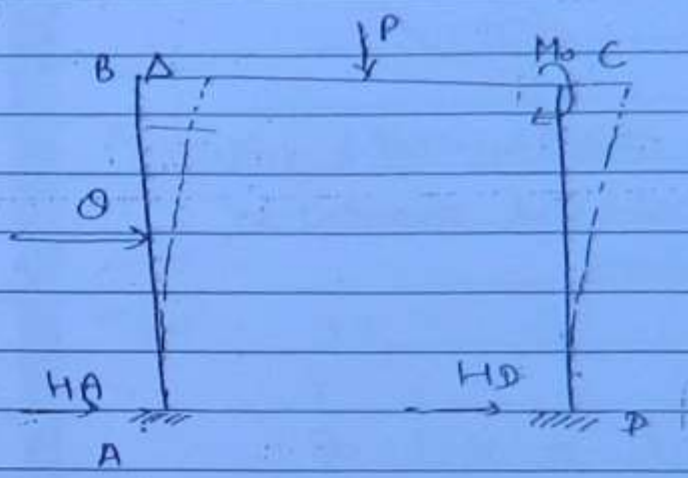
$\Delta \Rightarrow$ +tive if Produce ↻ rot^n to the member

The Joints Displace-ments (θ, Δ) are basic unknown which are to be determine using joint equilibrium conditions. and shear equ^n. the no. of joint equilibrium condition will be equal to no. of rotational displacement(θ)

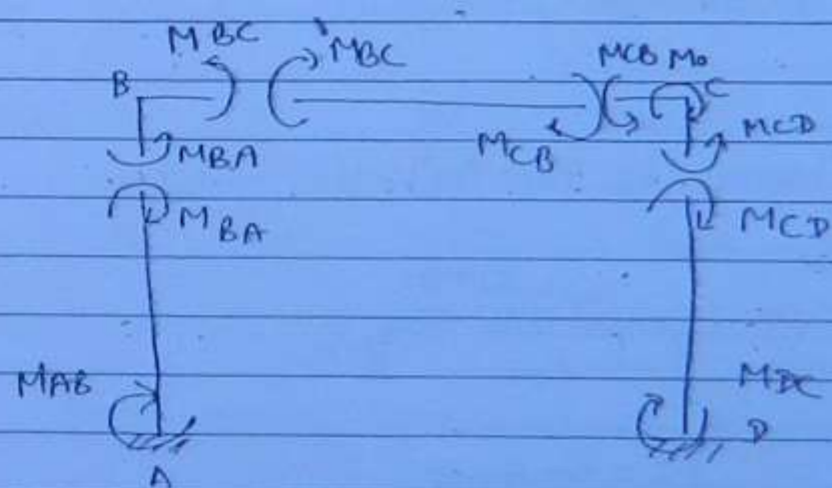
no. of Shear equation will be equal to No. of shear displacement (Δ).

→ In this analysis members are considered inextensible (axialy rigid)

(211)



$\theta_B, \theta_C, \Delta \quad (\Delta_B = \Delta_C = \Delta)$



Eq^n of joint B (joint equi^m condition)

$$\Sigma M_B = 0$$

$$-M_{BA} - M_{BC} = 0$$

$$\boxed{M_{BA} + M_{BC} = 0} \quad \text{------ (I)}$$

Joint eqn at C, $\Sigma M_C = 0$

$$-M_{CB} - M_{CD} + M_0 = 0$$

$$\boxed{M_{CB} + M_{CD} = M_0} \quad \text{------ (II)}$$

Shear Equation $\Sigma f_x = 0$

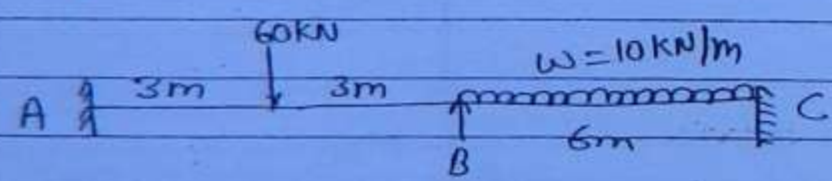
$$\boxed{H_A + H_D + Q = 0} \quad ----(iii)$$

212

- Solving eqn (I) (II) & (III) θ_C, θ_B, and Δ will be known. and Hence final end moment in each member will can be found.

- Bending moment dia is similar to that of moment distribution method.

Q. for analysis the beam by slope Deflection method



EI = constant

Fixed End moment

$$\bar{M}_{AB} = \frac{-PL}{8} \Rightarrow \frac{60 \times 6}{8} \Rightarrow -45\text{-m}$$

$$\bar{M}_{BA} = +\frac{PL}{8} = +45\text{-m}$$

$$\bar{M}_{BC} = \frac{-wL^2}{12} \rightarrow \frac{10 \times 6^2}{12} \Rightarrow -30 \text{ KN-M}$$

$$\bar{M}_{CB} = +30 \text{ KN-m}$$

Slop Deflection equcation.

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\Delta}{L}\right]$$

$$M_{AB} = -45 + \frac{2EI}{6}(\theta_B) \quad \text{—— (i)}$$

$$M_{BA} = 45 + \frac{2EI}{6}(2\theta_B) \quad \text{—— (ii)}$$

$$\bar{M}_{BC} = -30 + \frac{2EI}{6}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$$

$$\bar{M}_{BC} = -30 + \frac{2EI}{6}(2\theta_B) \quad \text{—— (iii)}$$

$$\bar{M}_{CB} = 30 + \frac{2EI}{6}(\theta_B) \quad \text{—— (iv)}$$

Joint Equm equcation at B

$$M_{BA} + M_{BC} = 0$$

$$45 + \frac{2EI}{3}(2\theta_B) + (-30) + \frac{2EI}{3}(2\theta_B) = 0$$

$$\frac{4}{3}EI\theta_B = 30 - 45$$

$$\therefore \quad EI\theta_B = -15$$

$$\theta_B = -15 \times \frac{3}{4} \rightarrow \frac{-45}{4EI}$$

$$\boxed{\theta_B = \frac{-45}{4EI}}$$

substituting θ_B in eqn (i)(ii) (iii) (iv)

$M_{AB} = -45 + \frac{2EI}{6} \times -\frac{45}{\frac{4}{2}EI}$ (214)

~~MAB~~ $\Rightarrow$ ~~$-45+0.33$~~ $\Rightarrow$ ~~-48.75~~

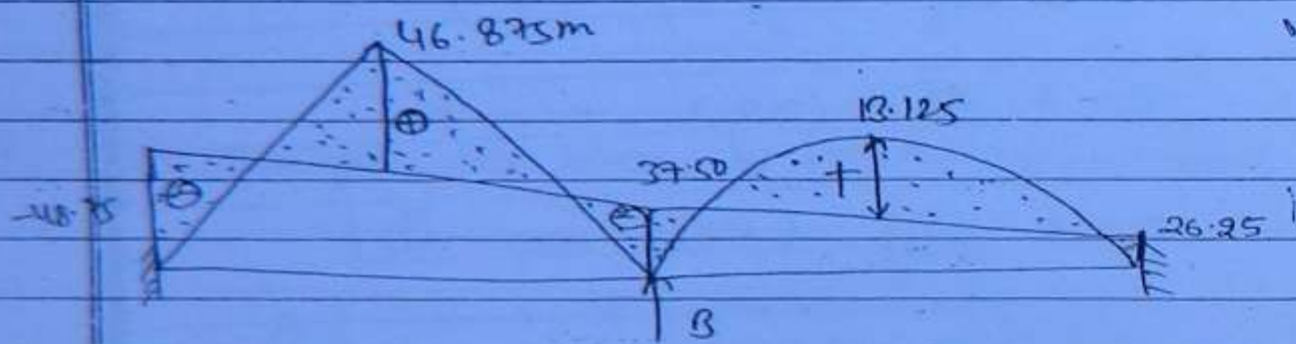
~~MBA~~ $\Rightarrow$

$M_{AB} = -48.75$

$M_{BA} = +37.50$

$M_{BC} = -37.50$

$M_{CB} = +26.25$



Simply support at Centre of AB = $\frac{PL}{4} = \frac{60 \times 6}{4} = 90$

Net B.M. at centre AB $\Rightarrow$

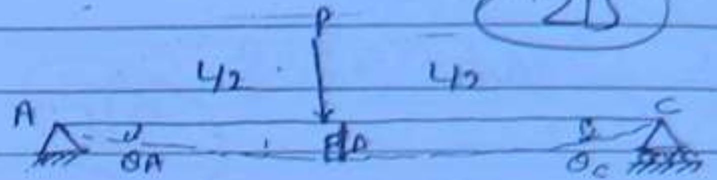
$$90 - \frac{(48.75 + 37.5)}{2} = 46.875$$

Simply supported in centre BC $= \frac{wL^2}{8} = \frac{10 \times 6^2}{8} = 45$ KN-M

Net BM at Centre BC $\Rightarrow 45 - \frac{(37.5 + 26.25)}{2} = +13.125$

Q. Analysis the beam shown in fig. using slop deflection method.

215



EI = Constant

Note :- Above simply supported beam is determinate which can be analysis using equilibrium condition alone, However if method of indeterminate analysis are requried to use than any of the stiffness method may be applied but computations will be lengthi.

Those slope deflection method consider above structure is made of two member AB and BC hence B is a assumed rigid ~~point~~ joint which is unsupproted.

there will be 4 unknown

$\theta_A, \theta_B, \theta_C, \Delta_B$

Sol Fixed End moment

$$\bar{M}_{AB} = \bar{M}_{BA} = \bar{M}_{BC} = \bar{M}_{CB} = 0$$

Slop deflection equation.

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$\Rightarrow 0 + \frac{2EI}{L/2}\left(2\theta_A + \theta_B - \frac{3\Delta}{L/2}\right) - (1)$$

$M_{AB} = $ ~~$\frac{4EI\,\theta_A}{L}$~~

$$\text{or } \frac{4EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right) - (1)$$

$$M_{BA} = 0 + \frac{\cancel{4EI}\ 2EI}{L/2}\left(2\theta_B + \theta_A \cancel{\pm} \frac{3\Delta}{L/2}\right) \quad —(2)$$

$$\bar{M}_{BA} = \frac{4EI}{L}\left(2\theta_B + \theta_A \pm \frac{6\Delta}{L}\right.$$

216

$$M_{BA} = 0 + \frac{4EI}{L}\left(2\theta_B + \theta_A - \frac{6\Delta}{L}\right) \quad —(ii)$$

$$M_{BC} = 0 + \frac{4EI}{L}\left(2\theta_B + \theta_C + \frac{6\Delta}{L}\right) \quad —(iii)$$

$$M_{CB} = 0 + \frac{4EI}{L}\left(2\theta_C + \theta_B + \frac{6\Delta}{L}\right) \quad —(iv)$$

Joint Eqm equcation.

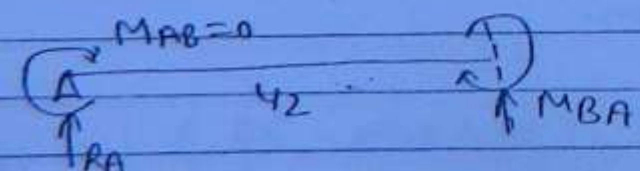
$M_{AB} = 0$ — (A)

$M_{BA} + M_{BC} = 0$ — (B)

$M_C = 0$ — (C)

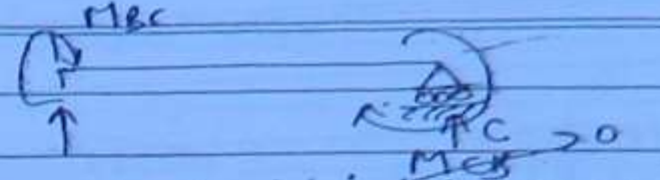
Shear eqy. $R_A + R_C - P = 0$ — (D)

for R_A & R_B



$$R_A \times L/2 + \cancel{M_{AB}}^{\,0} + M_{BA} = 0$$

$$\boxed{R_A = \frac{M_{BA}}{2}}$$



217

$$-R_C \times \frac{L}{2} + M_{BC} + M_{CB} = 0$$

$$R_C = \frac{M_{BC}}{L/2}$$

put in D

$$-\frac{M_{BA}}{L/2} + \frac{M_{BC}}{L/2} - P = 0$$

$$-M_{BA} + M_{BC} = \frac{PL}{2} \quad \text{—— } (D')$$

Sloving equ^n (A) (B) (C) & (D')

$$\theta_A = \frac{PLa^2}{16EI}$$

$$\theta_B = 0 \qquad \Delta = \frac{PL^3}{48EI}$$

$$\theta_C = -\frac{PL^2}{16EI}$$

on subsuiting value In equ (i) (ii) (iii) (iv)

$$M_{AB} = 0$$

$$M_{BA} = \frac{-PL}{4}$$

$$M_{BC} = \frac{+PL}{4}$$

$$M_{CB} = 0$$

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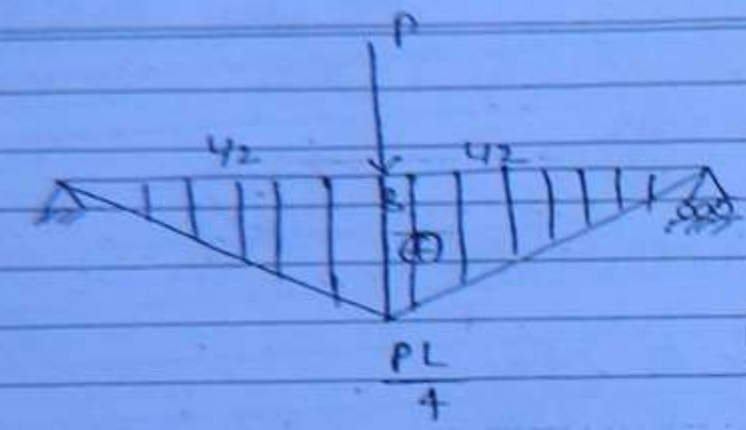
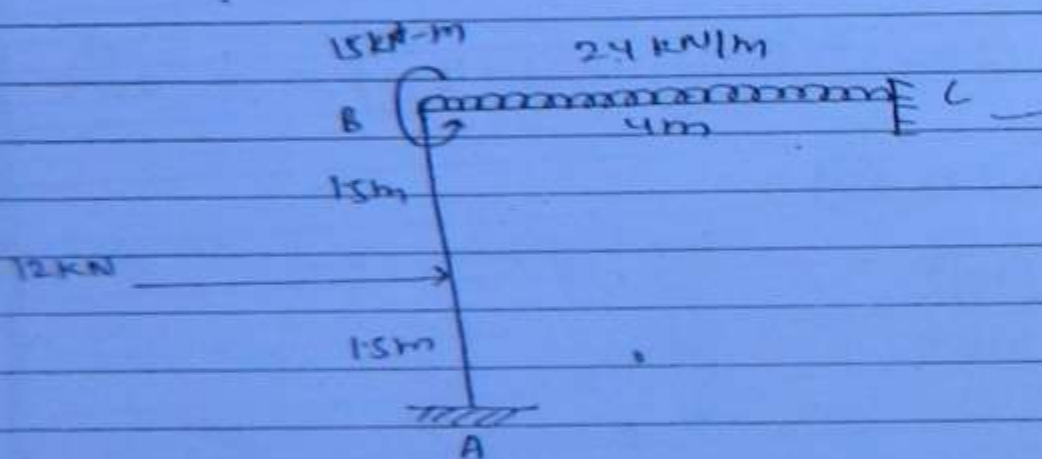


218

Q. Analysis the frame shown in fig. using Slope Deflection method



Sol Fixed End moment

$$M_{AB} \Rightarrow \frac{-ML}{8} \Rightarrow \frac{-12 \times 3}{8} \Rightarrow -4.5 \text{ kN-m}$$

$$M_{BA} = +4.5 \text{ kN-m}$$

$$M_{BC} = \frac{-wl^2}{12} = -\frac{24 \times 4 \times 4}{12} \Rightarrow -32 \text{ kN-m}$$

$$M_{CB} = +32 \text{ kN-m}$$

Sor deflection equ^{m}.

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L}\left(2\cancel{\theta_A}^{0} + \theta_B - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$$M_{AB} \Rightarrow -4.5 + \frac{2EI}{3}(\theta_B) \quad \text{— ①}$$

$$M_{BA} = 4.5 + \frac{2EI}{3}(2\theta_B) \quad \text{— ②}$$

$$M_{BC} = \bar{M}_{BC} + \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_C}^{0} - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$$\Rightarrow -32 + \frac{\cancel{2}EI}{\cancel{4}\,2}(2\theta_B)$$

$$M_{BC} \Rightarrow -32 + EI(\theta_B) \quad \text{— ③}$$

$$M_{CB} = +32 + \frac{\cancel{2}EI}{\cancel{4}\,2}(\theta_B) \quad \text{— ④}$$

Joint Equ^{m} equation →

$$M_{BA} + M_{BC} = \cancel{0}\ -15$$

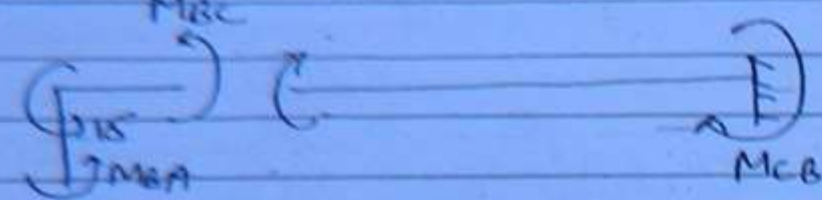
$$4.5 + \frac{2EI}{3}(2\theta_B) - 32 + EI\theta_B = -15$$

$$4.5 + 1.33EI\theta_B - 32 + EI\theta_B = -15$$

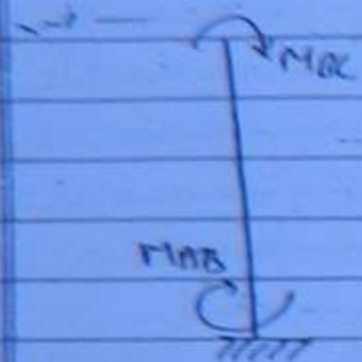
$$2.33EI\theta_B = -15 + 32 - 4.5$$

$$EI\theta_B \Rightarrow \cancel{12.5}\ 12.5$$

$$\boxed{\theta_B = \frac{\cancel{18.24}\ 5.36}{EI}}$$



220



$\Sigma M_B = 0$

$-M_{BA} - M_{BC} - 15 = 0$

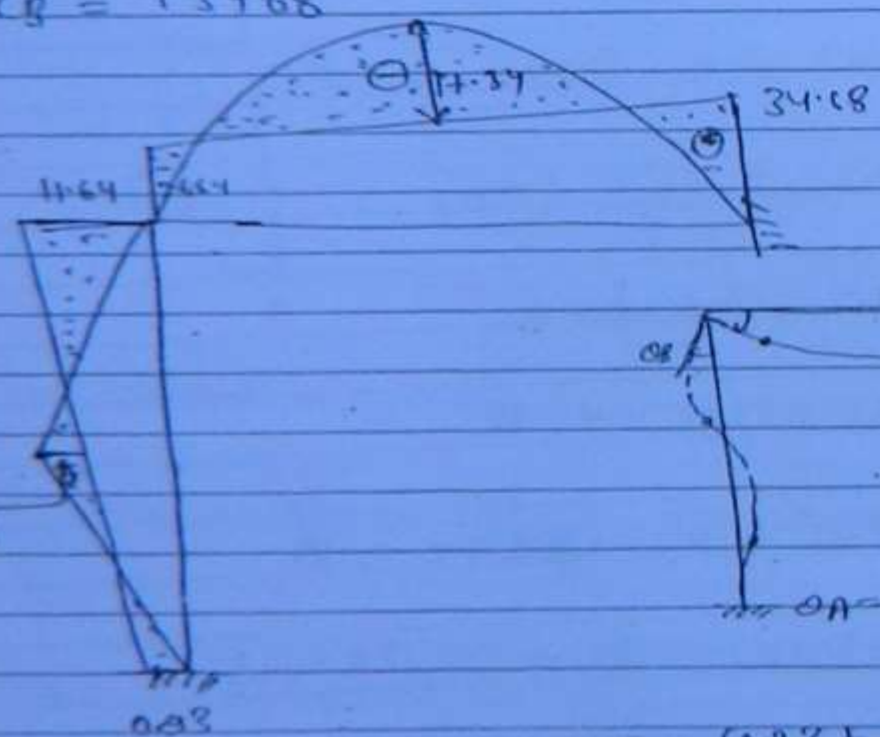
$\boxed{M_{BA} + M_{BC} = 15}$

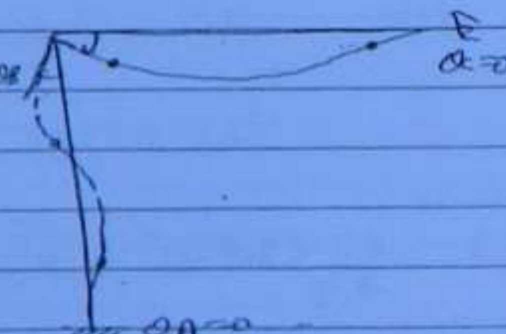
$M_{AB} = -0.93$

$M_{BA} = +11.64$

$M_{BC} = -26.64$

$M_{CB} = +34.68$





$9 - \left(\frac{0.93 + 11.64}{2}\right) = 2.71$

$AB = \frac{PL}{4} \Rightarrow \frac{12 \times 3}{4} \Rightarrow 9$

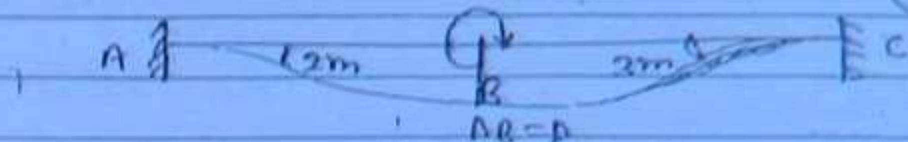
$BC = \frac{wl^2}{8} = \frac{24 \times 4^2}{8} = 48 \rightarrow 48 - \left(\frac{26.64 + 34.68}{2}\right)$

$\Rightarrow 17.34$

Q Analysis the Beam ABC using Slop. Deflection method.

221

24 KN-m



EI Constant.

Consider B is a unsupported joint. unknown.

θ_B, Δ_B

θ_B & $(\Delta_B = \Delta)$ are unknown

Fixed End moments:-

$$\bar{M}_{AB} = \bar{M}_{BA} = \bar{M}_{BC} = \bar{M}_{CD} = 0$$

~~moment~~ Slope deflection equ^n

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L}\left(2\cancel{\theta_A}^{0} + \theta_B - \frac{3\Delta}{L}\right) = 0$$

$$\Rightarrow 0 + \frac{2EI}{2}\left(\theta_B - \frac{3\Delta}{2}\right)$$

$$M_{AB} \Rightarrow EI\left(\theta_B - \frac{3\Delta}{2}\right) \quad \text{—— (1)}$$

$$M_{BA} = 0 + \frac{2EI}{2}\left(2\theta_B - \frac{3\Delta}{2}\right) \quad \text{—— } \left(\frac{2\Delta}{L}\right)$$

$$M_{BA} = EI\left(2\theta_B - \frac{3\Delta}{2}\right) \quad \text{—— (2)}$$

$$M_{BC} = \bar{M}_{BC} + \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_C}^{0} + \frac{3\Delta}{L}\right)$$

$$\bar{M}_{BC} = 0 + \frac{2}{3}EI\left(2\theta_B + \Delta\right) \quad \text{—— (3)}$$

$$M_{CB} = 0 + \frac{2EI}{3}\left(\theta_B + \Delta\right) \quad \text{—— (4)}$$

Joint equ.

$$M_{BA} + M_{BC} = 24$$

$$EI(2\theta_B - 1.5\Delta) +$$

$$M_{BA} + M_{BC} = 24$$

$$EI(2\theta_B - 1.5\Delta) + \frac{2}{3}EI\left(2\theta_B + \frac{3\Delta}{3}\right) = 24$$

$$2\theta_B EI - 1.5\Delta EI + 1.32 EI\theta_B + 0.66\Delta EI = 24$$

$$3.32\,\theta_B EI - 0.84\Delta = 24$$

$$3.32\theta_B - 0.84\Delta = \frac{24}{EI} \quad \text{—— (A)}$$

M_{AB}

R_A

$$R_A \times 2 + M_{AB} + M_{BA} = 0$$

$$R_A = \frac{(M_{AB} + M_{BA})}{2}$$

222

Shear equation $\Sigma F_y = 0$

$$R_C = \frac{M_{BC} + M_{CB}}{3}$$

$$\Rightarrow \frac{0.66EI(2\theta_B + \Delta) + 0.66EI(\theta_B + \Delta)}{3}$$

$$\Rightarrow \frac{1.32\theta_B EI + 0.66EI\Delta + 0.66EI\theta_B + 0.66EI\Delta}{3}$$

$$\Rightarrow 0.44\theta_B EI + 0.22EI\Delta + 0.22EI\theta_B + 0.22EI\Delta$$

$$\Rightarrow 0.66EI\theta_B + 0.44EI\Delta$$

$$R_B = -\left[\frac{M_{AB} + M_{BC}}{2}\right]$$

$$\Rightarrow -\frac{1}{2}\left[EI(\theta_B - 1.5\Delta) + EI(2\theta_B - 1.5\Delta)\right]$$

$$\Rightarrow -\frac{1}{2}\left[EI\theta_B - 1.5\Delta EI + 2EI\theta_B - 1.5EI\Delta\right]$$

$$\Rightarrow -1.5EI\theta_B + 1.5EI\Delta = 0$$

223

$$0.66EI\theta_B + 0.44EI\Delta = -1.5EI\theta_B + 1.5EI\Delta$$

$$2.16EI\theta_B = 1.06EI\Delta$$

$$\Delta = 0.350B$$

Put (2)

$$3.32\theta_B - 0.35\theta_B = \frac{24}{EI}$$

$$2.97\theta_B = \frac{24}{EI}$$

$$\theta_B = \frac{8.06}{EI}$$

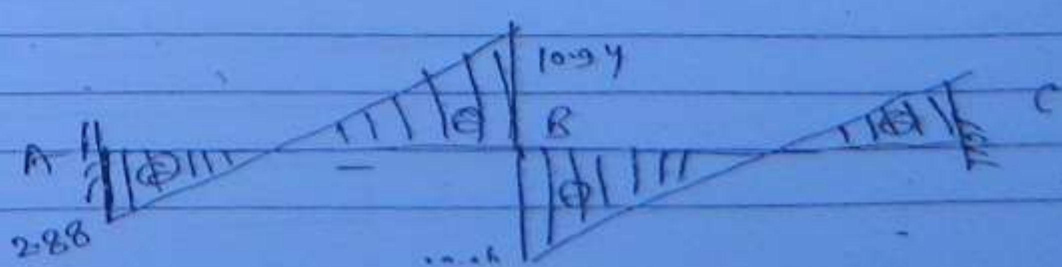
$$\Delta = \frac{3.456}{EI}$$

$$M_{AB} = \frac{M_0\, b(3a-L)}{L^2}$$

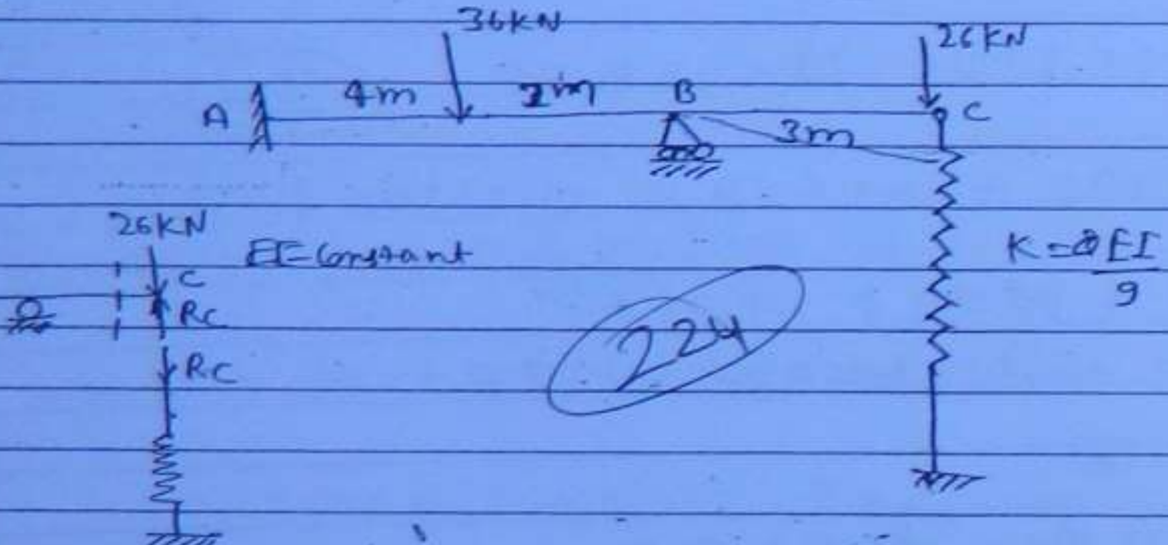
$$\Rightarrow \frac{24\times 3\,(3\times 2-5)}{5^2}$$

$$\Rightarrow \frac{24\times 3\times 1}{25} \rightarrow +2.88$$

$M_{AB} = 2.88$, $M_{BA} = 10.94$, $M_{BC} = 13.06$, $M_{CB} = 7.2$



Q. Analysis the Beam shown in fig. using slop. Deflection method. Joint C is supported over a spring with stiffnes $K = \frac{EI}{9}$



Sol

~~Fixed end moment~~

$\overline{M_{AB}} = \frac{wab}{L}$ Unknown are $\theta_B, \theta_C, \Delta_C$

$R_C = K \cdot \Delta_{spring}$

$\Delta_{spring} = \Delta_C$ $R_C = K \cdot \Delta_C \Rightarrow \frac{EI}{9}\Delta_C$

Fixed end moment →

$$M_{AB} = \frac{-wab^2}{L^2} \Rightarrow -\frac{36 \times 4 \times 2 \times 2}{6 \times 6} \Rightarrow -16 \text{ KN-m}$$

$$M_{BA} = \frac{36 \times 4 \times 4 \times 2}{36 \times 6} \Rightarrow +32 \text{ KN-m}$$

$\overline{M}_{BC} = 0$

$\overline{M}_{CB} = 0$

Slop Deflection eqy

225

$$M_{AB} = -16 + \frac{2EI}{6}\left(2\cancel{\theta_A}^{0} + \theta_B - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$$\Rightarrow -16 + \frac{EI}{3}\left(\theta_B - \cancel{\frac{3\Delta}{L}}^{0}\right]$$

$$M_{AB} = -16 + \frac{EI\theta_B}{3} \quad \text{— (i)}$$

$$M_{BA} = +32 + EI\,\frac{2\theta_B}{3} \quad \text{— (ii)}$$

$$M_{BC} = 0 + \frac{2EI}{12}\left(2\theta_B + \theta_C - \frac{3\Delta_C}{L}\right]$$

$$\Rightarrow 0 + \frac{2EI}{3}\left(2\theta_B + \theta_C - \frac{3\Delta_C}{3}\right)$$

$$M_{BC} = \frac{2EI}{3}\left(2\theta_B + \theta_C - \Delta_C\right) \quad \text{— (iii)}$$

$$M_{CB} = \frac{2EI}{3}\left(2\theta_C + \theta_B - \Delta_C\right) \quad \text{— (iv)}$$

Joint equcation at B

$$M_{BA} + M_{BC} = 0 \quad \text{— (A)}$$

$$32 + EI(0.66)\theta_B + 0.66EI\left[2\theta_B + \theta_C - \Delta_C\right) = 0$$

$$32 + 0.66EI\theta_B + 1.32EI\theta_B + 0.66EI\theta_C - 0.66\Delta_C = 0$$

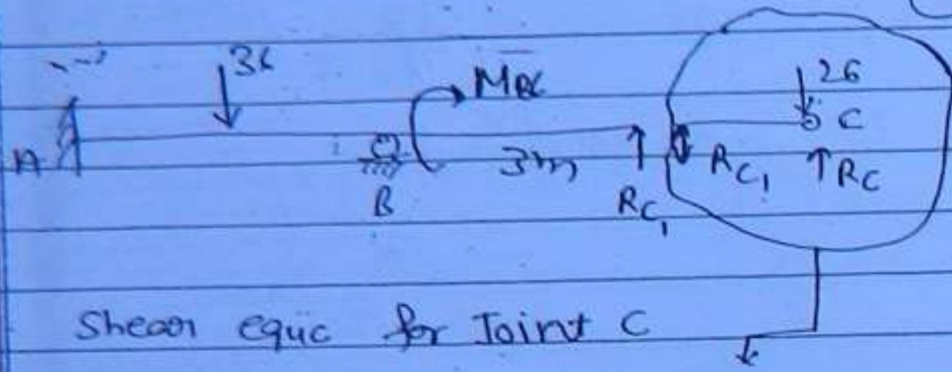
$$32 + 1.98EI\theta_B + 0.66EI\theta_C - 0.66\Delta_C = 0$$

$$1.98EI\theta_B + 0.66EI\theta_C - 0.66\Delta_C = -32 \quad \text{— (A)}$$

Joint eq^n A + C

$M_{CB} = 0$ — (B)

226



Shear equ for Joint C

$$R_C - 26 - R_{C_1} = 0$$

$$\frac{EI}{9}\Delta_C - 26 - R_{C_1} = 0$$

$\Sigma M_B = 0$

$$-R_{C_1} \times 3 + M_{BC} = 0$$

$$R_{C_1} = \frac{M_{BC}}{3}$$

Shear eq^n will be

$$\frac{EI}{9}\Delta_C - 26 - \frac{M_{BC}}{3} = 0 \quad \text{— (C)}$$

$M_{CB} = 0$

$$0.66EI(2\theta_C + \theta_B - \Delta_C) = 0 \quad 0.66EI$$

$$1.32EI\theta_C + 0.66EI\theta_B - \Delta_C = 0$$

Sloving eq^n (A) (B) & (C)

$$\theta_B = \frac{6}{EI} \qquad \theta_C = \frac{60}{EI}$$

$$\Delta_C = \frac{126}{EI}$$

$M_{AB} = -16 + \frac{EI}{\cancel{3}} \times \frac{\cancel{6}\,2}{EI} \Rightarrow -14$ kN-m

$M_{BA} = 32 + \frac{\cancel{EI}\,2}{\cancel{3}} \times \frac{\cancel{6}\,2}{EI}$ 227

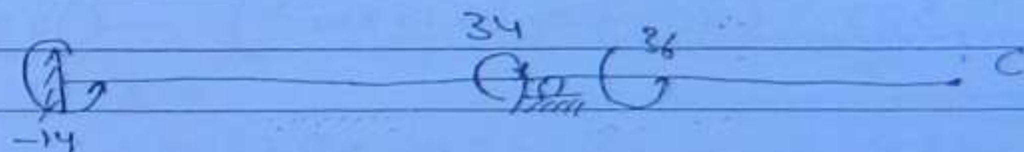
$\Rightarrow 34$ kN-m

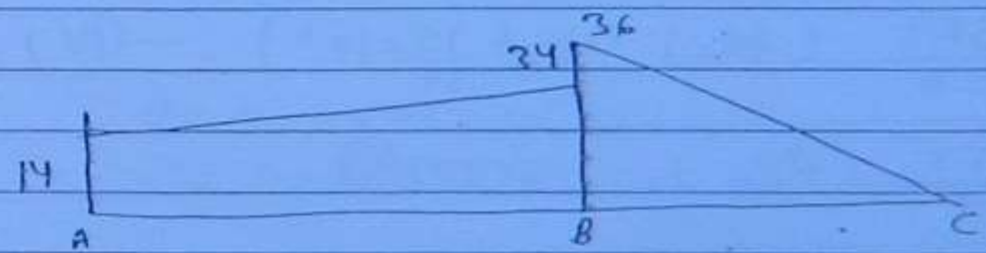
$M_{BC} \Rightarrow \frac{2}{3} EI \left(2 \times \frac{6}{EI} + \frac{60}{EI} - \frac{126}{EI} \right]$

$\Rightarrow \frac{2}{\cancel{3}} \cancel{EI} \times \frac{-\cancel{54}\,18}{\cancel{EI}}$

$M_{BC} \Rightarrow -36$

$M_{CB} = 0$

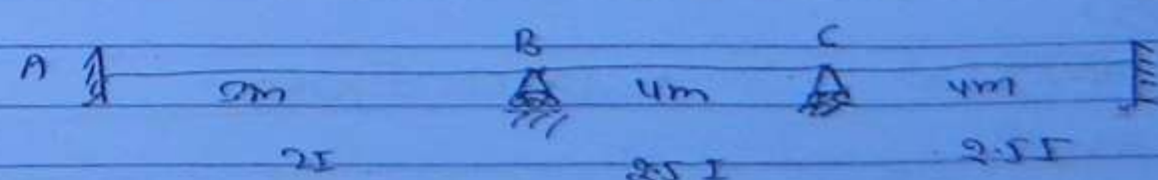




2021

Q. Analysis the Continous beam shown in fig using Slope deflection method. support B settels down 5mm.

$E = 2 \times 10^5$ N/mm2 $\quad I = 36 \times 10^6$ mm



$I = \ldots \times 10^{\ldots}\ mm^2$

$I = 36 \times 10^6\ mm^4$

$\Delta_B = 5mm.$

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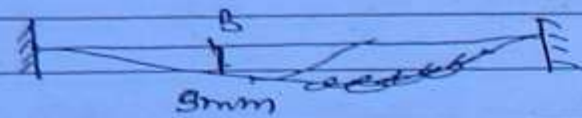
Page	

228



$$M_{AB} = \bar{M}_{AB} + \frac{2E(2I)}{3}\left(2\cancel{\theta_A}^{0} + \theta_B - \frac{3 \times 5 \times 10^{-3}}{3}\right)$$

$$M_{AB} = \frac{4EI}{3}\left(\theta_B - 5 \times 10^{-3}\right) \quad \text{— (i)}$$

$$M_{BA} = \frac{4EI}{3}\left(2\theta_B - 5 \times 10^{-3}\right) \quad \text{— (ii)}$$

$$\left\{\frac{3\Delta}{L}\right\}$$

$$M_{BC} = 0 + \frac{2E(2.5I)}{4}\left(2\theta_B + \theta_C + \frac{3 \times 5 \times 10^{-3}}{4}\right)$$

$$M_{BC} = \frac{5EI}{4}\left(2\theta_B + \theta_C + \frac{15}{4} \times 10^{-3}\right) \quad \text{— (iii)}$$

$$M_{CB} = \frac{5EI}{4}\left(2\theta_C + \theta_B + \frac{15}{4} \times 10^{-3}\right) \quad \text{— (iv)}$$

$$M_{CD} = \frac{5EI}{4}\left(2\theta_C\right) \quad \text{— (v)}$$

$$M_{DC} = \frac{5EI}{4}\left(\theta_C\right) \quad \text{— (vi)}$$

Joint equ At B

$$M_{BA} + M_{BC} = 0$$

$$1.33EI\left(2\theta_B - 5 \times 10^{-3}\right) + 1.25EI\left(2\theta_B + \theta_C + 3.75 \times 10^{-3}\right)$$

$\Rightarrow$ 1.33

$\Rightarrow$ ~~$1.33 \times 2 \times 10^5 \times$~~ 36×10^6 $\therefore$ or ~~$1.25 \times 2 \times 10^5 \times 36$~~ $\times 10^6$

$\Rightarrow$ ~~9576×10^{11}~~

$\Rightarrow$ ~~19152×10^{11}~~

$$2.66EI\theta_B - 6.66\times10^{-3}EI + 2.5EI\theta_B + 1.25EI\theta_C + 4.69\times10^{-3}EI = 0$$

$$5.16EI\theta_B + 1.25EI\theta_C - 1.97\times10^{-3}EI = 0$$

$$5.16\theta_B EI + 1.25EI\theta_C = 1.97\times10^{-3}EI \quad -(A)$$

229

At joint C

$$M_{CB} + M_{CD} = 0$$

$$1.25EI(2\theta_C + \theta_B + 3.75\times10^{-3}) + 1.25EI(\theta_C 2) = 0$$

$$2.50EI\theta_C + 1.25EI\theta_B + 4.68\times10^{-3}EI + 2.5EI\theta_C = 0$$

$$5EI\theta_C + 1.25EI\theta_B + 4.68\times10^{-3}EI = 0$$

$$1.25EI\theta_B + 5EI\theta_C = -4.68\times10^{-3}EI$$

$$\theta_B = +6.49\times10^{-4}$$
$$\theta_C = -1.099\times10^{-3}$$

$M_{AB} = -41.77$ $\quad$ $M_{CB} = +19.78$

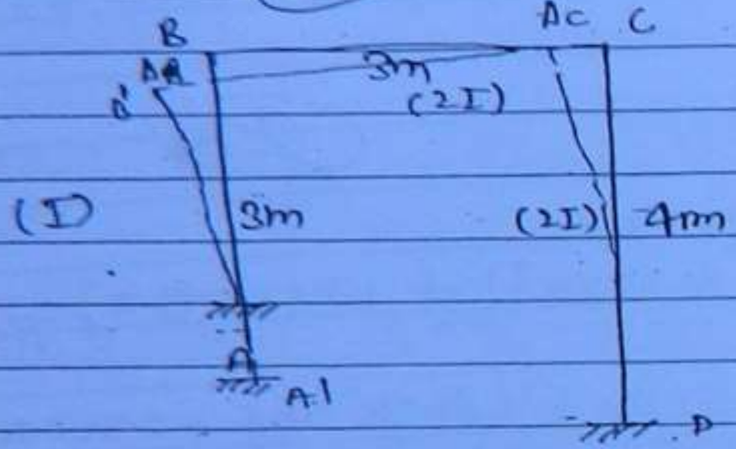
$M_{BA} = -35.53$ $\quad$ $M_{CD} = -19.78$

$M_{BC} = +35.33$ $\quad$ $M_{DC} = -9.89$

To

Q. For the frame shown in fig. support A settles by 15 mm vertically down. Analyse the frame by slope-deflection method. And draw the BMD dia. Given that $EI = 48000$ kN-m.

230



vertical Δ of B = 15mm

Horizontal Δ of B = Δ (unknown)

Δ of C = Δ (11—)

FEM =

Δ MAB

$$M_{AB} = 0 + \frac{2EI}{L}\left(2\theta_A + \theta_B + \frac{3\Delta}{L}\right) \quad (-3\Delta/L)$$

$$M_{AB} = \frac{2EI}{3}(\theta_B + \Delta) \quad \text{—— (i)}$$

$$M_{BA} = \frac{2EI}{3}(2\theta_B + \Delta) \quad \text{—— (ii)}$$

$$M_{BC} = 0 + \frac{2E(2I)}{3}\left(2\theta_B + \theta_C + \frac{3 \times 15 \times 10^{-3}}{3}\right)$$

$$M_{BC} \Rightarrow \frac{4EI}{3}(2\theta_B + \theta_C + 15 \times 10^{-3}) \quad \text{(iii)}$$

$$M_{CB} = \frac{4EI}{3}(2\theta_C + \theta_B + 15 \times 10^{-3}) \quad \text{—— (IV)}$$

$$M_{CD} = 0 + \frac{2E(2I)}{4}\left[2\theta_C + \overset{0}{\theta_D} + \frac{3\Delta}{4}\right]$$

$$M_{CD} \Rightarrow EI\left[2\theta_C + \frac{3\Delta}{4}\right] \quad \text{— (V)}$$

$$M_{DC} = EI\left[\theta_C + \frac{3\Delta}{4}\right] \text{ or } [0.75\Delta] \quad \text{— (VI)}$$

231

Joint equ^n At B

$$M_{BA} + M_{BC} = 0$$

$$0.67EI(2\theta_B + \Delta) + 1.33EI(2\theta_B + \theta_C + 15\times10^{-3}) = 0$$

$$\Rightarrow 1.34EI\theta_B + 0.67EI\Delta + 2.66EI\theta_B + 1.33EI\theta_C + 19.95\times10^{-3} = 0$$

$$\Rightarrow 4EI\theta_B + 2EI\Delta + 19.95\times10^{-3} = 0 \quad \text{— (P)}$$

At joint C

$$M_{CB} + M_{CD} = 0$$

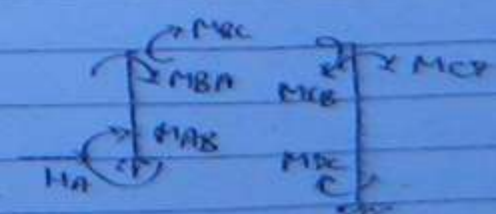
$$1.33EI(2\theta_C + \theta_B + 15\times10^{-3}) + EI(2\theta_C + 0.75\Delta) = 0$$

$$2.66EI\theta_C + 1.33\theta_B + 19.95\times10^{-3} + 2EI\theta_C + 0.75\Delta EI = 0$$

$$4.66EI\theta_C + 1.33\theta_B + 0.75\Delta \pm 19.95\times10^{-3} = 0 \quad \text{— (Q)}$$

Shear equ^n $H_A + H_D = 0$

To find H_A, consider FBD of AB



$$\Sigma M_B = 0$$

$$-H_A\times3 + M_{AB} + M_{BA} = 0$$

$$H_A = \frac{M_{AB} + M_{BC}}{3}$$

To find H_D, consider FBD of CD

$\Sigma M_C = 0$

$-H_D \times 4 + M_{CD} + M_{DC} = 0$

232

$$H_D = \frac{M_{CD} + M_{DC}}{4}$$

$$H_A = \frac{M_{AB} + M_{BA}}{3}$$

$$\Rightarrow \frac{1}{3}\{0.67EI\theta_B + 0.67\Delta + 1.34EI\theta_B + 0.67EI\Delta\} = 0$$

$$\Rightarrow \frac{1}{3}[2EI\theta_B + 1.34\Delta] = 0$$

$$\Rightarrow 0.67\,EI\theta_B + 0.45\Delta = 0 \quad \text{— (x)}$$

$$H_D = \frac{M_{CD} + M_{DC}}{4}$$

$$\Rightarrow \frac{1}{4}[2EI\theta_C + 0.75\Delta EI + EI\theta_C + 0.75\Delta]$$

$$\Rightarrow 0.75\,EI\theta_C + 1.5\Delta = 0 \quad \text{— (y)}$$

~~eq~~ x

$$4EI\theta_B + 2EI\Delta + 19.95\times10^{-3} = 0$$

~~θ_D~~ =

M_{AB} = +91.63 kN-m

M_{BA} = −65.82 "

M_{BC} = +65.82

M_{CB} = +116.35

M_{CD} = −116.35

M_{DC} = +81.93

Prosn (Q, 7, 9, 16 . Page-167)

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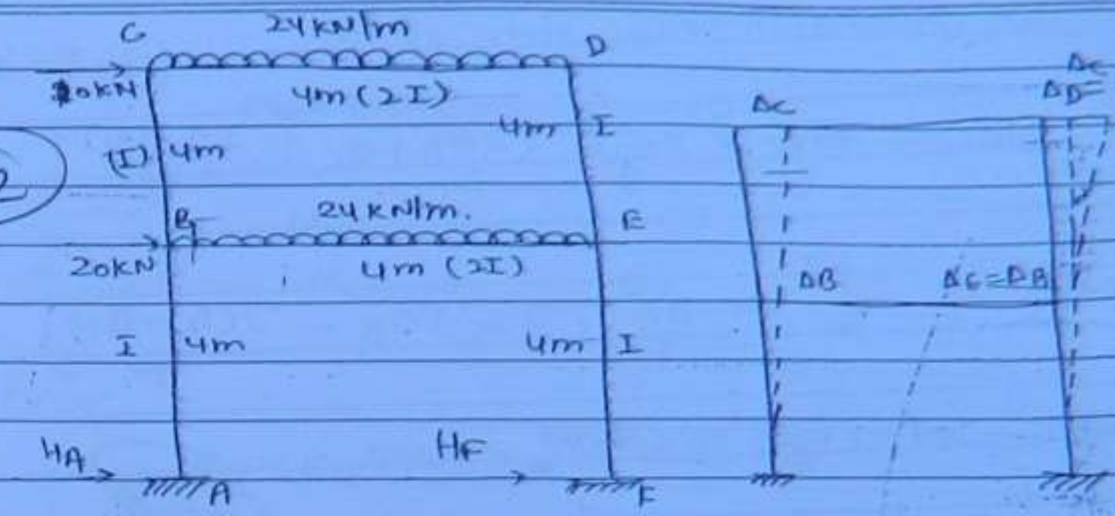
Page 237

q-

232



unknown $\theta_B, \theta_C, \theta_C\ \theta_D, \Delta_B, \Delta_C$

Joint equation. -

At joint B :-

$M_{BA} + M_{BC} + M_{BE} = 0$ — (i)

At joint C

$M_{CB} + M_{CD} = 0$ — (ii)

At joint D

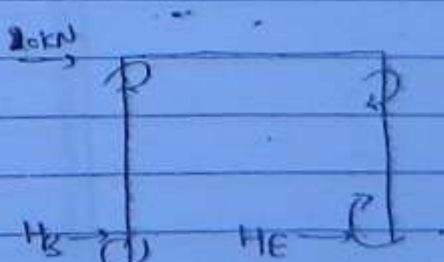
$M_{DE} + M_{DC} = 0$ — (iii)

At joint E

$M_{ED} + M_{EB} + M_{EF} = 0$ — (iv)

Shear equation :- $\Sigma F_x = 0$

$H_A + H_F + 10 + 20 = 0$ — (5)



$\Sigma F_x = 0$

$H_B + H_E + 10 = 0$ — (6)

→ Slop Deflection equation of BC.

$$M_{BC} = \overline{M}_{BC}^{\,0} + \frac{2EI}{4}\left(2\theta_B + \theta_C + \frac{3(\Delta_2 - \Delta_1)}{4}\right)$$

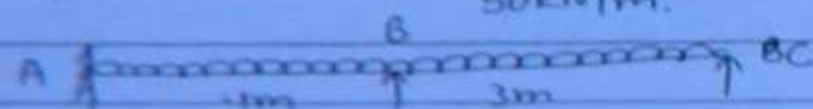
$$M_{CB} = \overline{M}_{CB}^{\,0} + \frac{2E}{4}\left(2\theta_C + \theta_B - \frac{3(\Delta_2 - \Delta_1)}{4}\right)$$

Slope Deflection equation for AB

$$M_{AB} = 0 + \frac{2EI}{4}\left(2\theta_A^{\,0} + \theta_B - \frac{3\Delta}{4}\right)$$

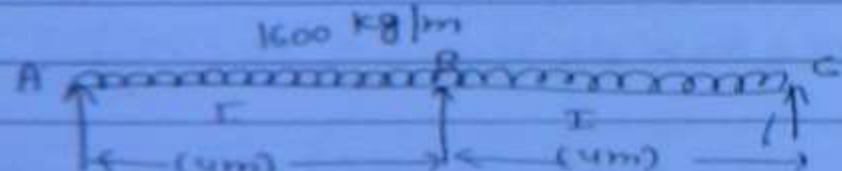
235

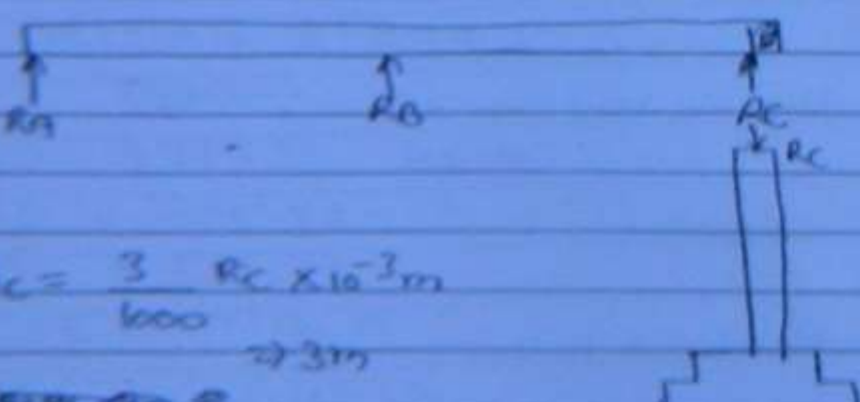
Q.16



At B sinks 5mm

Q.14(b)





$$\Delta_C = \frac{3}{1000} R_C \times 10^{-3} m$$

⇒ 3m

$\theta_B, \theta_C, \theta_A, \theta_C$ unknown

$\Sigma M_B = 0$

$$M_{BA} + M_{BC} = 0 \quad \text{——} \quad (1)$$

235

$$M_{AB} = 0 \quad \text{——} \quad (2)$$

$$M_{CB} = 0 \quad \text{——} \quad (3)$$

$\Sigma F_y = 0$

$$R_A + R_B + R_C = 1600 \times 8 \quad \text{——} \quad (4)$$

-: TRUSSES :-

236

Assumption

- Pin joint frames
- All joints are connected by frictionless Hinges
- All member are straight.
- loading is applied only at joint.
- self weight of member is neglected
- Member of truss carry only Axial forces (Compⁿ / Tension) No S.F. & B.M.
- Hook's law is valid, The material is isotropic, Homogeneous and Linear Elastic.

TRUSS

- 2D Trusses (Plane truss)
  - determinate ($D_S = 0$)
  - Indet (Redundant truss) ($D_S > 0$)
    1. Force method / unit load method / Strain Energy method / Compatibility method
    2. Displacement method / Equilibrium method / stiffness method
    3. Graphical methods.
- 3D Trusses (Space Truss)
  - determinate
  - indeter.

Method
- Method of Joint
- Method of Section
- Graphical method

237

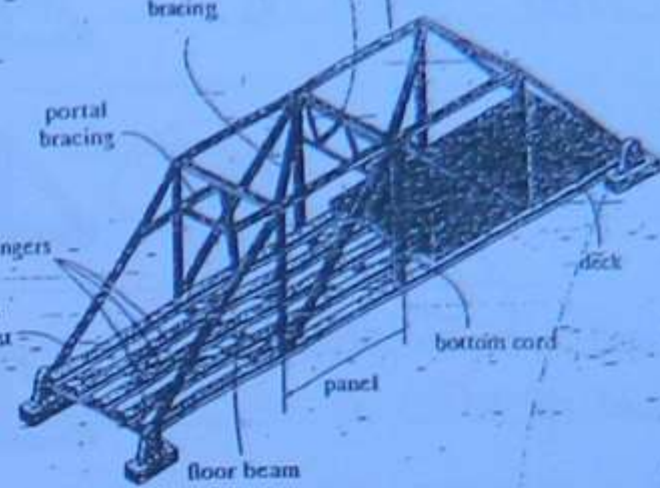
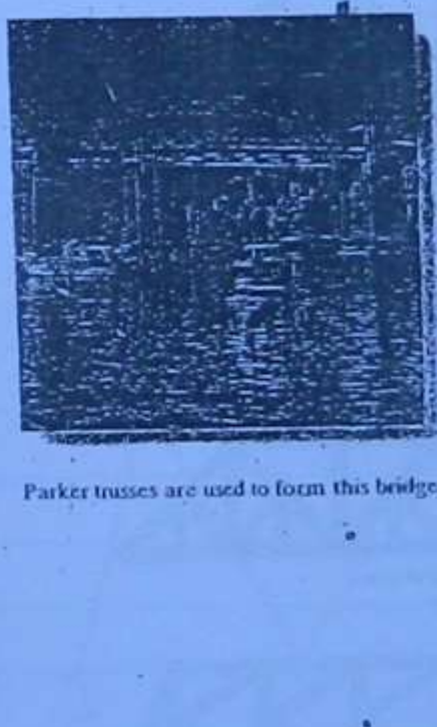


Fig. 3-4



Parker trusses are used to form this bridge.

Bridge Trusses. The main structural elements of a typical bridge truss are shown in Fig. 3–4. Here it is seen that a load on the *deck* is first transmitted to *stringers*, then to *floor beams*, and finally to the joints of the two supporting side trusses. The top and bottom cords of these side trusses are connected by top and bottom *lateral bracing*, which serves to resist the lateral forces caused by wind and the sidesway caused by moving vehicles on the bridge. Additional stability is provided by the *portal* and *sway bracing*. As in the case of many long-span trusses, a roller is provided at one end of a bridge truss to allow for thermal expansion.

A few of the typical forms of bridge trusses currently used for single spans are shown in Fig. 3–5. In particular, the Pratt, Howe, and Warren trusses are normally used for spans up to 200 ft (61 m) in length. The most common form is the Warren truss with verticals, Fig. 3–5c. For larger spans, a truss with a polygonal upper cord, such as the Parker truss, Fig. 3–5*d*, is used for some savings in material. The Warren truss with verticals can also be fabricated in this manner for spans up to 300 ft (91 m). The greatest economy of material is obtained if the diagonals have a slope between 45° and 60° with the horizontal. If this rule is maintained, then for spans greater than 300 ft (91 m), the depth of the truss must increase and consequently the panels will get longer. This results in a heavy deck system and, to keep the weight of the deck within tolerable limits, *subdivided* trusses have been developed. Typical examples include the Baltimore and subdivided Warren trusses, Figs. 3–5e and 3–5*f*. Finally, the K-truss shown in Fig. 3–5g can also be used in place of a subdivided truss, since it accomplishes the same purpose.

BRIDGE TRUSSES!

238

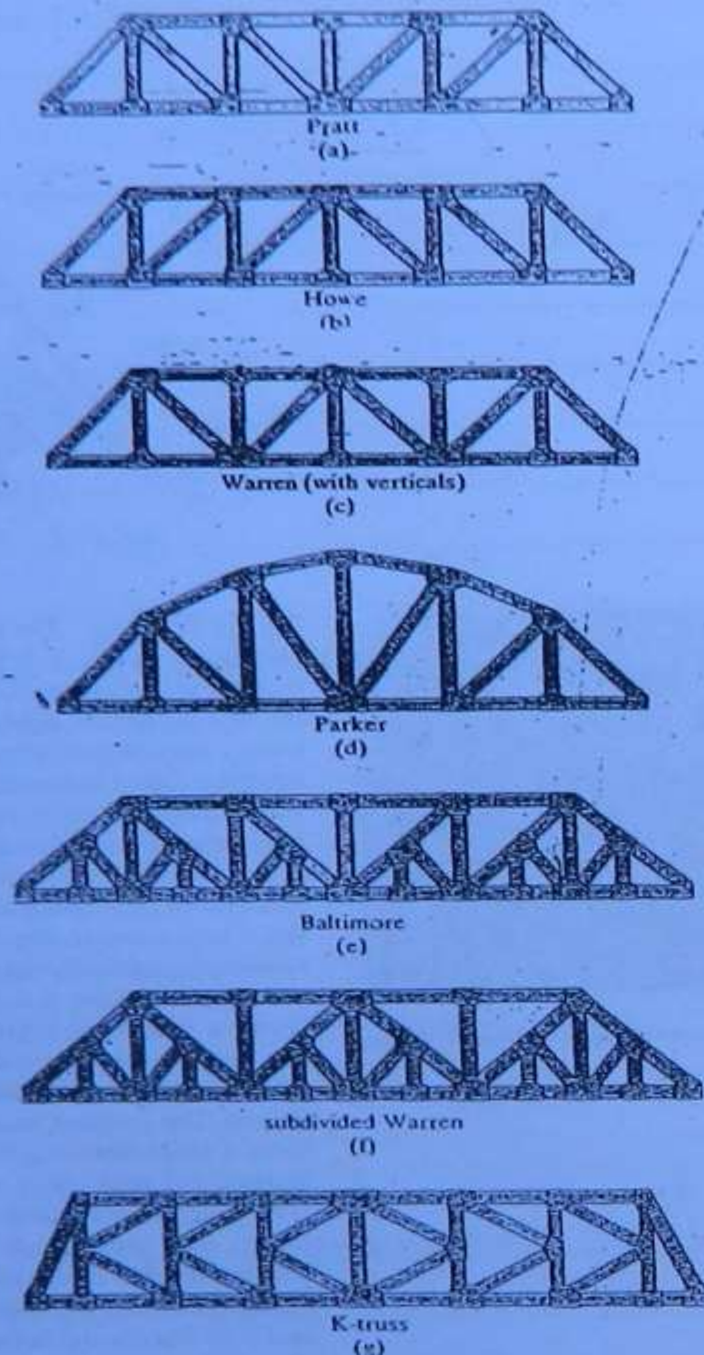


Fig. 3-5

Equilibrium method | stiffness method

(3) Graphical method:

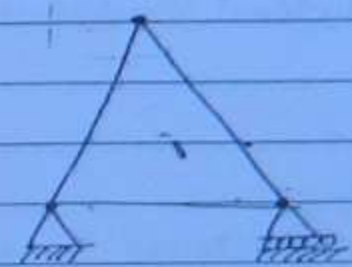
Uses of Trusses →

(1) Roof Truss (2) Bridge Truss

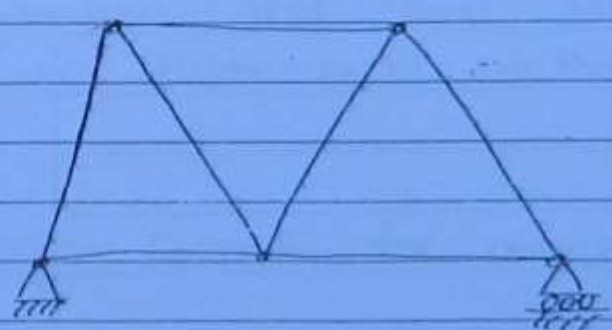
Type of Truss →

(1) Simple Truss :-

The simple Truss Consists of Triangular Block. The simplest frame work is a Triangular with three member.

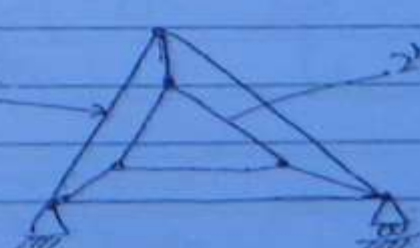


Simple Truss

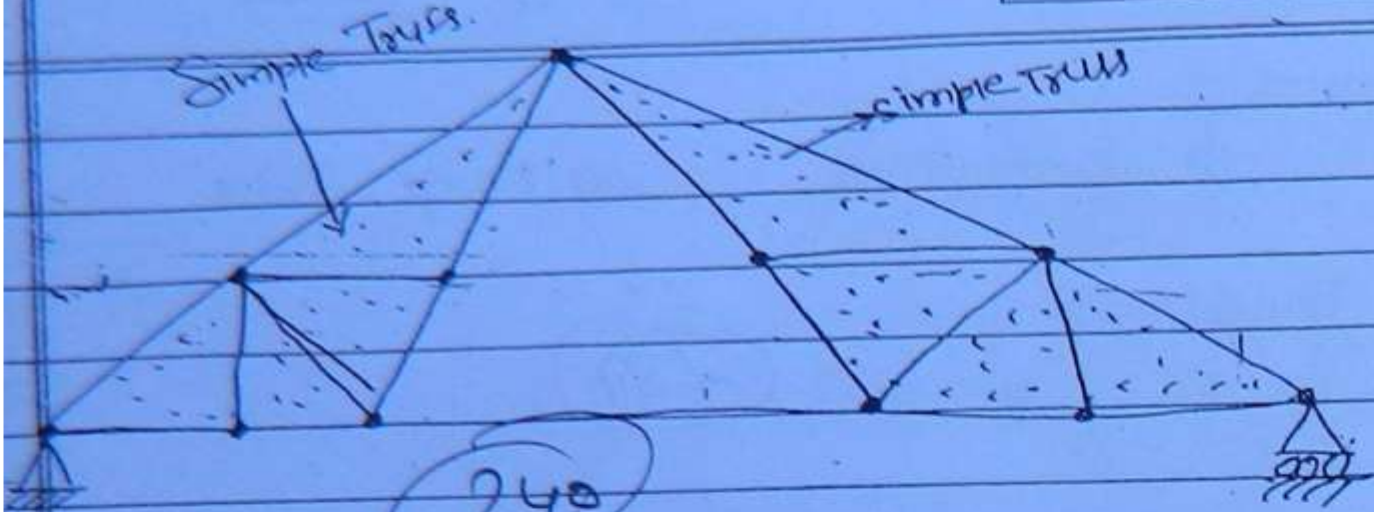


Compound Truss :- It is formed by 2 or more simple Truss. It is often used for longer spans and is proved cheaper than a simple single truss for the same span.

Principal Simple Truss

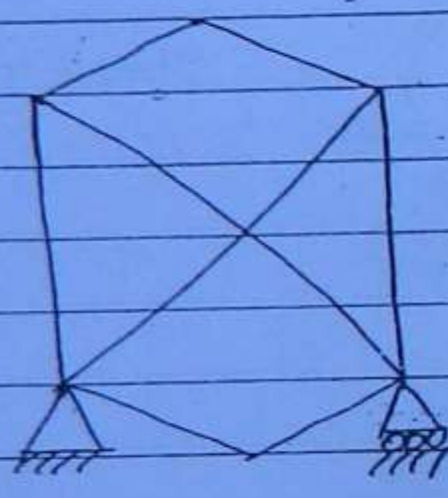


2nd secondary Simple Truss



240

<u>Complex Truss</u> :- If Truss is neither simple nor compound than it will be complex truss. Generally a complex Truss has polygenol structure.



complex truss

Analysis of Trusses :-

241

Step-I check the degree of Static indeterminacy

$$\boxed{D_S = (m + r_e) - 2J}$$

→ if $D_S = 0$ than Truss is called determinate perfect frame.

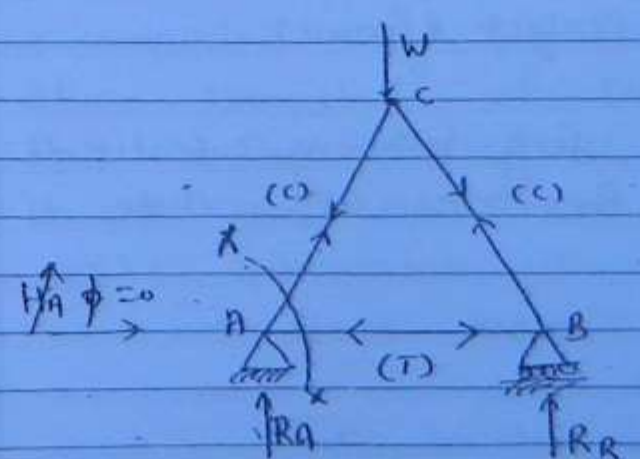
for such Trusses equilibrium equation are use for analysis. And method of joint and method of Section can be use.

Step-II :-

if $d_s \neq 0$ than Truss is called indeterminate Redudants

+ Redudanted may be internal and external. To analysis such Truss ~~method~~ of force method or displace-ment method can be use.

Sign Convention for member forces:-



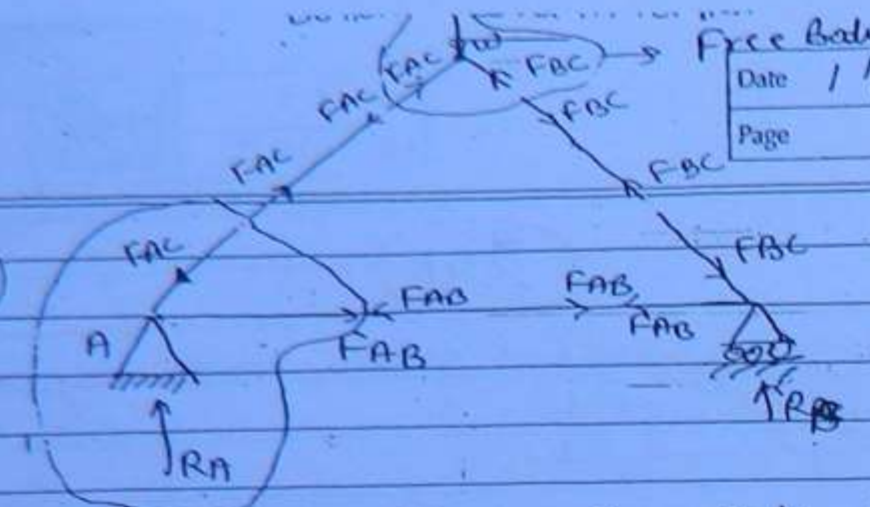
+ → Tension
− → Compression.

AC ⇒ →← Compression

AB ⟹ ←→ Tension

242

Free Body dia of C.



Member Free Body

Joint free Body Diagram.

→ At joints Tension force is represented by Arrow moveing away from joint.

→ Compression force Points towards the joint.

→ In 2 Dimension Case (Plane Truss) for equlibrium of each joint followint to Condition should be Satisfied

① $\Sigma F_x = 0$

② $\Sigma F_y = 0$

Analysis of determinate ~~frame~~ plane Trusses :- ($D_S = 0$)
(Perfect frame)

Step-1 > find support reactions using following 3 conditions of static equilibrium for truss as a whole.

(i) $\Sigma F_x = 0$

(ii) $\Sigma F_y = 0$

(iii) $\Sigma M_z = 0$

Step-7 > To determine member forces use either method of joint or method of section.

(243)

(i) method of joint →

This method is suitable when force in all member of trusses are required. the free body diagram of each joint is considered and following 2 condition of static equilibrium are use to find member forces

① $\Sigma F_x = 0$ ② $\Sigma F_y = 0$

• This method is applicable only when no. of unknown force at a joint is ≤ than or equal to two. In this method the judicious selection of joint is done in such a way no. of unknown at joint are ≤ 2

(ii) method of section →

It is sutiable when forces are required only in two member i.e. 1,2,3 After computation of support reaction two find member forces an imgainary section is cut such that not more than 3 member will be cut accross the section. i.e. unknown are not more than 3.

The section assume must cut those member in which forces are required. the section can be vertical, inclined, horizontal of zig-zag. free body equilibrium either from left Portion or for right Portion to the Section is considered. And only following 3 condition are applied.

244

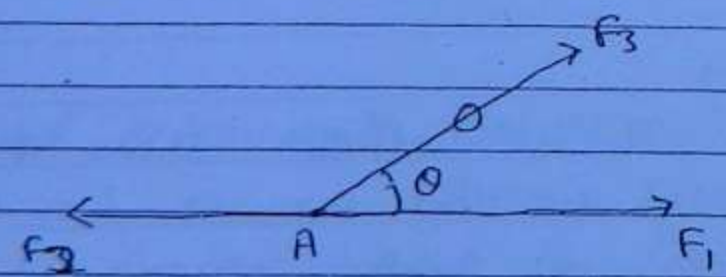
① $\Sigma F_x = 0$ ② $\Sigma F_y = 0$ ③ $\Sigma M_z = 0$

It means at a time more than 3 unknown force can be found.

for obj / Imp

Thumb rule to find members which carry Zero forces

1) If at a joint only three member meet Two of than are co-linear there is no external force and Reaction at that joint than 3rd member (no-Colinear member) will carry zero force.



$\phi \neq 0$
$\phi \neq 180$

$\boxed{F_3 = 0}$ ∵ $\Sigma F_y = 0$

$F_3 \sin\theta = 0$

$\boxed{F_3 = 0}$

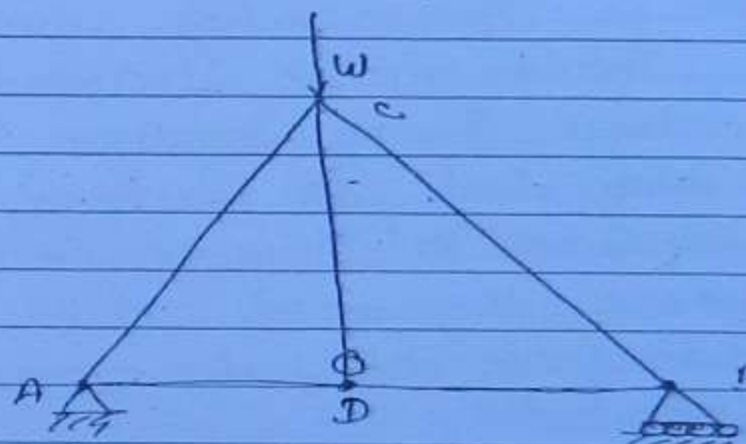
furthere notic that under such case both Colinear member will have equal and alike forces.

$$\boxed{F_1 = F_2}$$

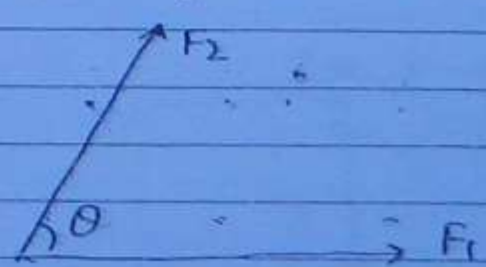
245

(either both will be tensile or Compression or Both will be zero)

eg.



(2) If At Joint only two member are present which are not Co-linear and their is no external load or reaction at the Joint then both member will carry zero forces.



$\phi \neq 0$

$\neq 180$

$\Sigma F_y = 0$

$+ F_2 \sin\theta = 0$

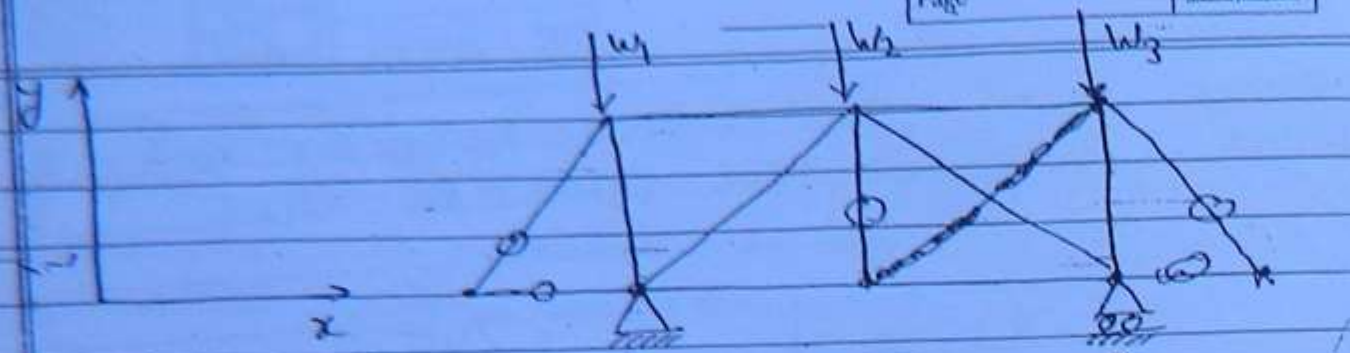
$\boxed{F_2 = 0}$

$\Sigma F_x = 0$

$F_1 + F_2 \cos\theta = 0$

$F_1 + 0 = 0$

$\boxed{F_1 = 0}$

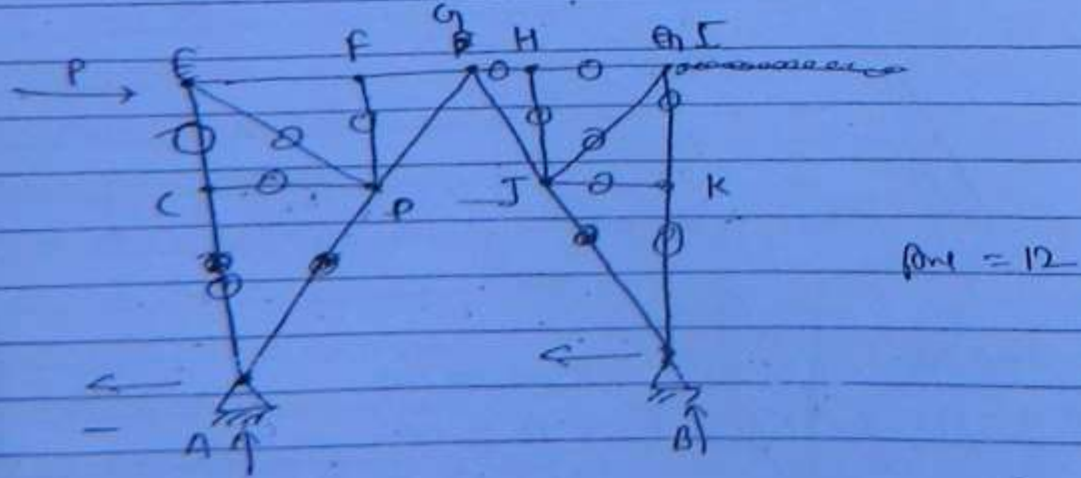


246

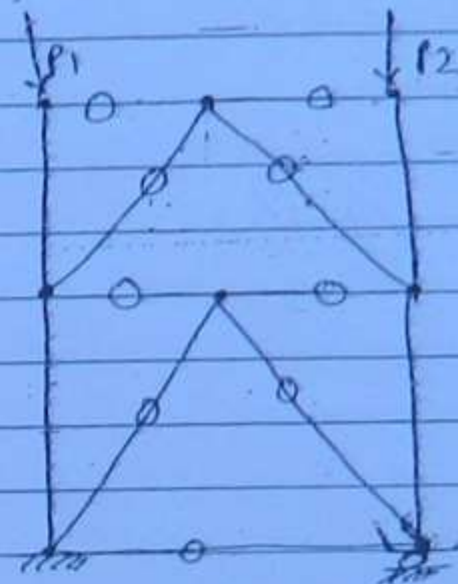
Note:- If sum of the members in a Truss Carry zero force at a given loading than such member should not be removed Because under change loading condition such member may carry load. moreover a members are remove DSI may become (<0) less than 0. Hence gematery of the truss will not be preserved, under general loading condition.

Q. In this truss shown in fig. Ideenty the no. of member which carry zero forces.

(a) 8 (b) 10 (c) 12 (d) 14



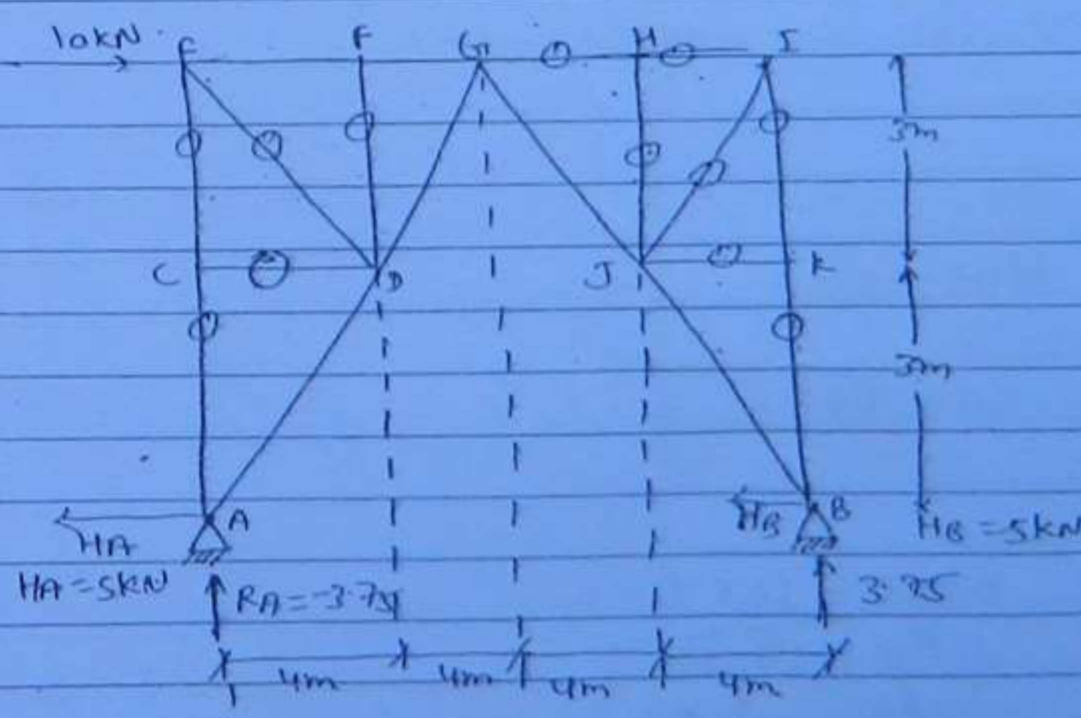
Q. Identify the zero force member in the frame. Shown in fig.



247

(A) 12   (B) 8   (C) 9   (d) None of these

Q. Analysis the truss given below using method of joint and find forces in all member.



$r_e = 4$

$D_{se} = r_e - 3 \Rightarrow 4 - 3 = 1$

$D_{si} = m - (2J - 3)$

$\Rightarrow 18 - (2 \times 11 - 3)$

$\Rightarrow 18 - 19$

$\Rightarrow -1$

248

$D_s = D_{se} + D_{si}$

$\Rightarrow 1 - 1 \Rightarrow 0$

$\Sigma F_x = 0$

$\Sigma F_y = 0 \quad , \Sigma M_A = 0$

$+$

$\boxed{\Sigma M_{B(G)} = 0}$

→ in this case all member concerment at G
Hence truss can be cut in 2 part and
each part can be rotated about G.

$\Sigma F_x = 0$

$H_A + H_B = 0$

$\Sigma F_y = 0$

$R_A + R_B = 0$

$\Sigma M_A = 0$

$R_B \times 16 - 10 \times 8 = 0$

$$\boxed{\begin{aligned} R_B &= +3.75 \\ R_A &= -3.75 \end{aligned}}$$

$\sum M_G = 0$

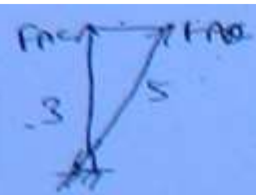
$$R_B \times 8 - H_B \times 6 = 0$$

$$H_B = \frac{R_B \times 8}{6} \Rightarrow \frac{3.75 \times 8}{6} = 5\,kN$$

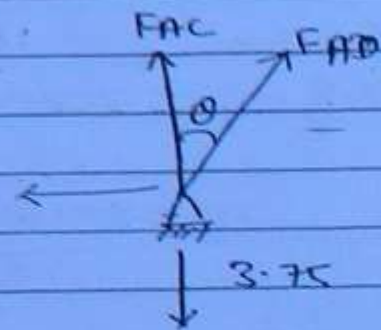
$$\boxed{H_A = 5\,kN, \quad H_B = 5\,kN}$$

249

member	Force +(T) −(C)	
AC	0	
AD	6.25	
CD	0	
CE	0	
DE	0	
DF	0	
DG	6.25	
EF	−10	
FG	−10	
GH	0	
HI	0	
GJ	−6.25	
HJ	0	
IJ	0	
JK	0	
IK	0	
JB	−6.25	
KB	0	



Consider joint A



250

$\sin\theta = \frac{L}{K} = \frac{4}{5}$

$\Sigma fx = 0$

$FAD \sin\theta - 5 = 0$

$FAD = \frac{5}{\sin\theta} = \frac{5}{4/5} \rightarrow \frac{25}{4}$

$FAD = 6.25 \; (+)$

$\Sigma fy = 0$

$FAC + FAD \cos\theta - 3.75 = 0$

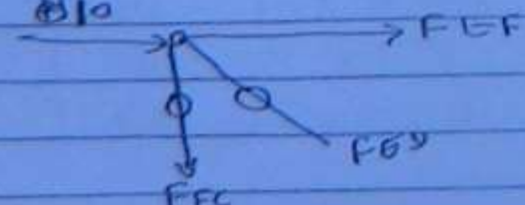
$FAC = 3.75 - FAD\cos\theta$

$\Rightarrow 3.75 - 6.25 \cdot \frac{3}{5}$

$FAC \Rightarrow 3.75 - 3.75 = 0$

Consider joint D At D member AD & DG are co-linear and other member share zero forces. Hence both co-liner force. AD & DG will have equal forces

Joint E



$\Sigma f_x = 0$

$$F_{EF} + 10 + F_{CD}\cos\theta = 0$$

$$F_{EF} + 10 + 0 = 0$$

$$F_{EF} = -10$$

251

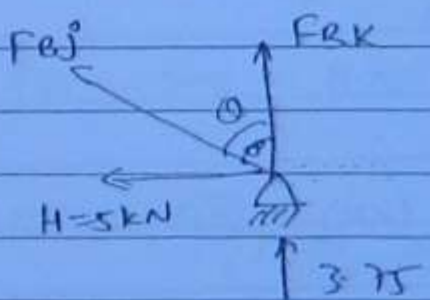
$\Sigma F_y = 0$

F_{BC}

At joint F

$$F_{EF} = F_{FG}$$

<u>Consider joint B</u>



$\Sigma f_x = 0$

$$\cancel{F_{BR} + F_{Bj}\cos\theta + 3.75 = 0}$$

$$-5 - F_{Bj}\sin\theta = 0$$

$$F_{Bj} = \frac{-5}{\sin\theta} = -6.25$$

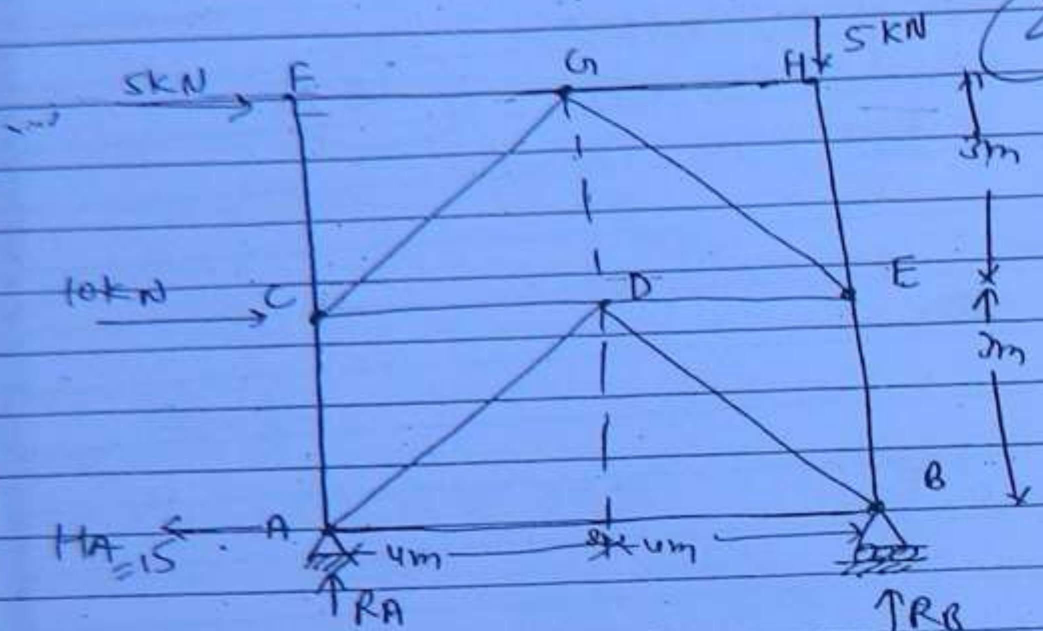
$\Sigma f_y = 0$ $\quad 3.75 + F_{BK} + F_{Bj}\cos\theta = 0$

$$3.75 + F_{BK} + \cancel{-3.7} -6.25 \times \frac{1}{2} = 0$$

$$F_{BK} = 0$$

Q → Find forces in all member of truss

252



$m = 13 \quad re = 3$

$DS = m + re - 2j$

$\Rightarrow 13 + 3 - 2 \times 8$

$\Rightarrow 0$

Serial No.	Member	Force (kN) (+) T (−) C
1	AB	7.5
2	AC	+1.875
3	AD	+9.375
4	CD	−12.5
5	CG	+3.125
6	CF	0
7	DE	2.5
8	BD	−9.375
9	BE	−6.875
10	FG	−5
11	GH	0
12	EG	−3.125
13	HE	−5

$\Sigma F_x = 0$

$5 + 10 - HA = 0$

$HA = 15$

253

$\Sigma F_y = 0$

$RA + RB = 5$

~~RA~~

$\Sigma M_A = 0$

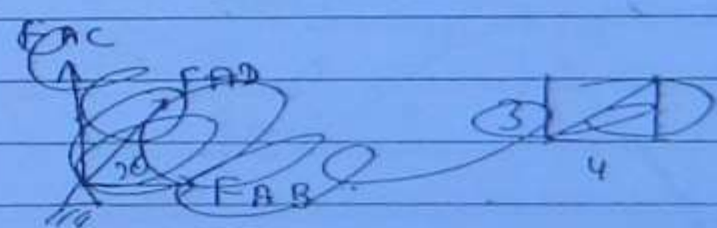
$- RB \times 8 + 5 \times 8 + 10 \times 3 + 5 \times 6 = 0$

$RB = \dfrac{-40 + 30 + 30}{8}$

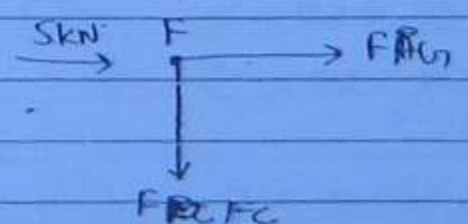
$RB \Rightarrow \dfrac{+90}{8} \Rightarrow 12.25$

$RA = -7.5$

At ~~ya~~ ~~Consider~~ ~~member~~ (A)



(1) Consider member F



$\Sigma F_x = 0$

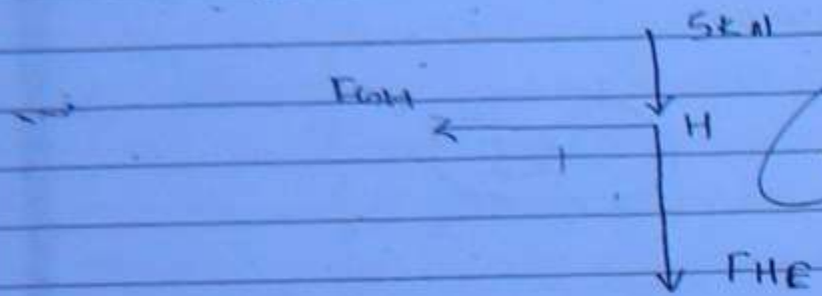
$F_{AG} + 5 = 0$

$F_{AG} = -5$

ΣF_y

$F_{AC} = 0$

(3) Consider Joint H



254

$\Sigma F_x = 0$

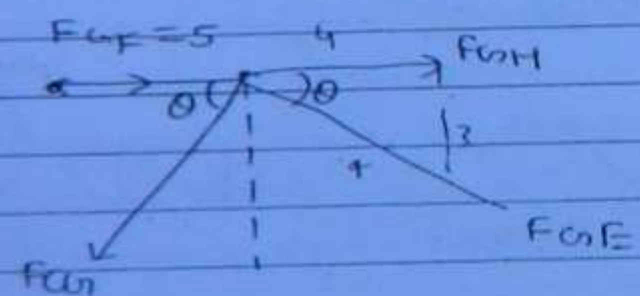
$F_{GH} = 0$

$\Sigma F_y = 0$

$-F_{HG} - 5 = 0$

$F_{HG} = -5$

Consider Joint G



$\Sigma F_x = 0$

$F_{EG}\cos\theta + 5 - F_{CG}\cos\theta = 0 \quad -(i)$

$F_{EG}\cos\theta - F_{CG}\cos\theta = -5$

$\Sigma F_y = 0$

$-F_{CG}\sin\theta - F_{EG}\sin\theta = 0 \quad -(ii)$

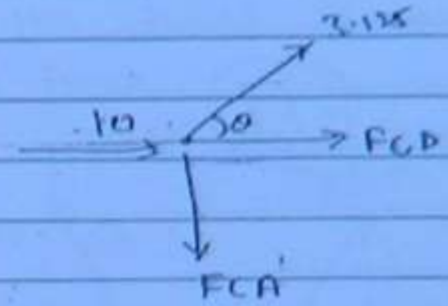
$F_{CG} = -F_{EG}$

$-F_{CG}\cos\theta - F_{CG}\cos\theta = -5$

$-2F_{CG}\cos\theta = -5$

$+2F_{CG} \times 4/5 = +5 \qquad F_{CG} = \frac{-5\times5}{8} = \frac{-25}{8} = -3.125$

Joint C



253

$\Sigma F_y = 0$

$3.125 \sin\theta - F_{AC} = 0$

$F_{AC} = 3.125 \times \frac{3}{5}$

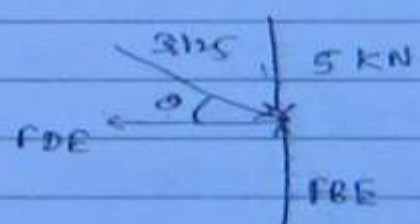
$F_{AC} = +1.875$

$\Sigma F_x = 0$

$F_{CD} + 3.125 \cos\theta + 10 = 0$

$F_{CD} = -12.5$

Consider joint E



$\Sigma F_y = 0 \quad F_{DE} - 3.125 \cos\theta = 0$

$F_{DE} = 3.125 \times \frac{4}{5}$

$F_{DE} = 2.5$

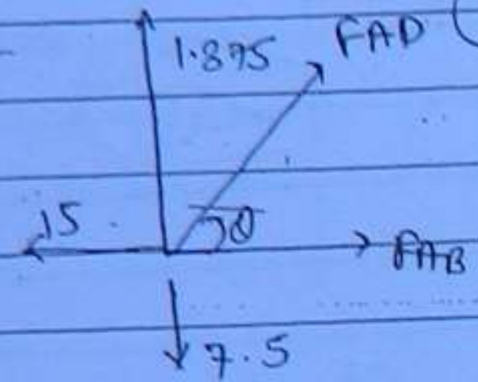
$\Sigma F_y = 0 \quad -F_{BE} - 3.125 \sin\theta = 0$

$F_{BE} = -6.875$

Page

(c) Consider joint A

256



$\Sigma F_y = 0$

$+ \cancel{1.875 + FAD}$

$1.875 + FAD \sin\theta - 7.5 = 0$

$1.875 + FAD \times \frac{3}{5} - 7.5 = 0$

$FAD \Rightarrow +9.375$

$\Sigma F_x = 0$

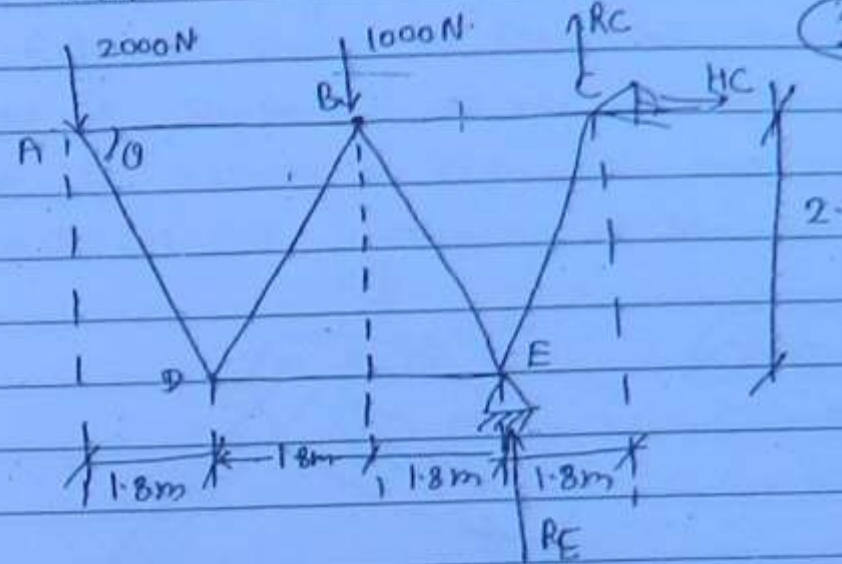
$-15 + FAB + FAD \cos\theta = 0$

$FAB \Rightarrow 15 - 9.375 \times \frac{4}{5}$

$FAB \Rightarrow +7.5$

Q Analysis the truss shown in fig. & list forces in the table.

(257)



$\Sigma F_x = 0$

$H_C = 0$ $\quad$ $DS = 0$ (perfect frame)

$\Sigma F_y = 0$

$R_E + R_C = 3000\,N$

$\Sigma M_C = 0$

$R_E \times 7.8 - 2000 \times 7.2 - 1000 \times 3.6 = 0$

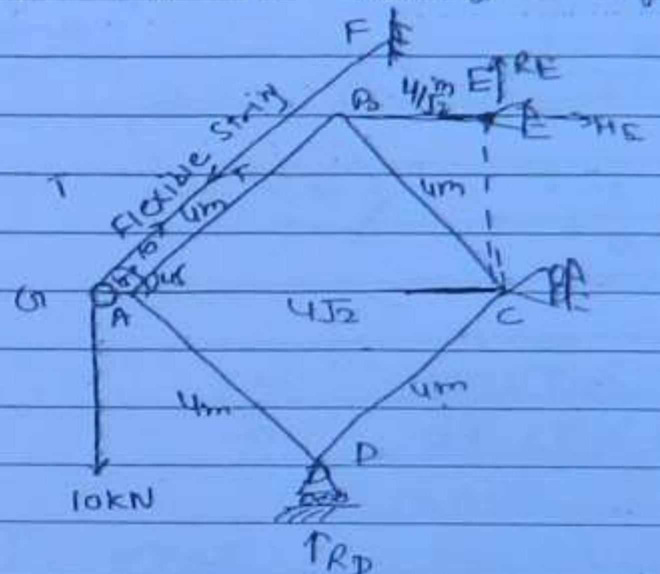
$$R_E = \frac{14400 + 3600}{4}$$

$R_E = 10000\,N$

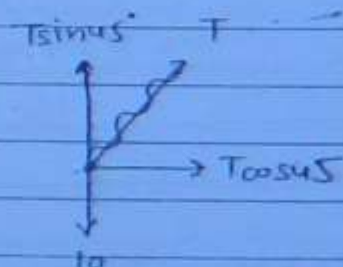
$R_C = -7000\,N$

258

Q- A Spire Truss ABCD Shown in fig. Carry load of 10 KN Apply through a Spring GF which is a passing over a frictionless pully as shown in fig. GF & AB are parallel. The truss is supported on roller at C & D. A link BE is shown in fig- is part of truss. find forces in all member of the truss



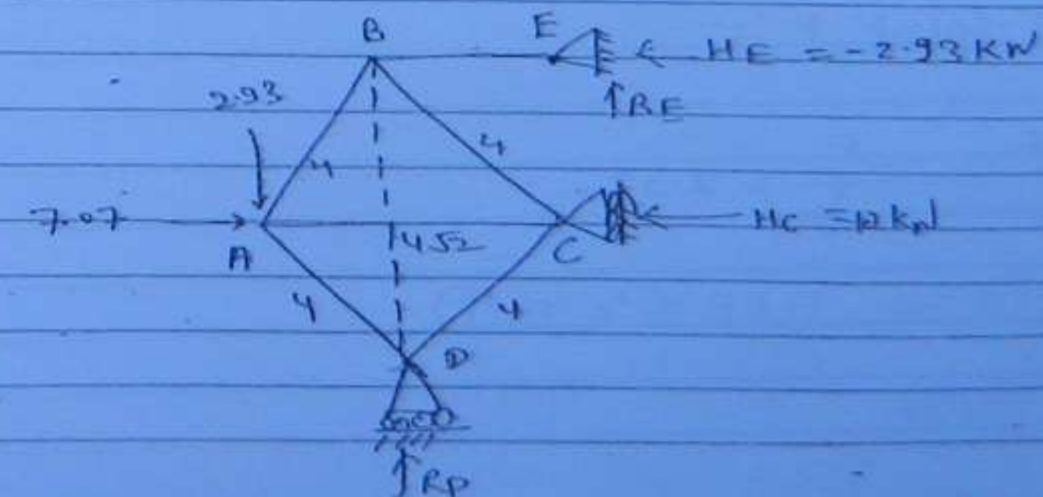
259



Net vertical force at G $\Rightarrow 10 - \frac{10}{\sqrt{2}}$

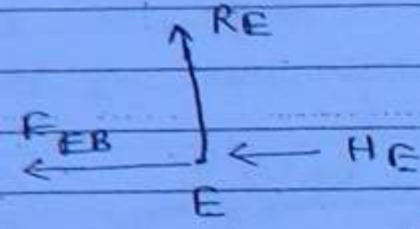
$\Rightarrow 2.93$ kN

Net Horizontal at G $\Rightarrow \frac{10}{\sqrt{2}} = 7.07$



$$D_s = m + r_e - 2J$$
$$\Rightarrow 6\times 4 - 2\times 5$$
$$\Rightarrow 0$$

Joint At E



260

$\Sigma F_y = 0$

$$R_E = 0$$

$\Sigma F_D = 0$

$$R_D + R_E - 2.93 = 0$$
$$R_D = 2.93 \text{ kN}$$

$\Sigma F_x = 0$

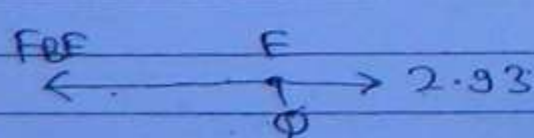
$$7.07 - H_C - H_E = 0$$
$$H_C + H_E = 7.07 \quad -(1)$$

$\Sigma M_C = 0$

$$R_D \times \frac{4}{\sqrt{2}} - 2.93 \times 4\sqrt{2} - H_E \times \frac{4}{\sqrt{2}} = 0$$

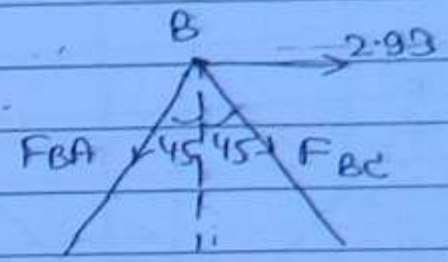
$$\boxed{H_E = -293 \text{ kN}}$$

Consider joint F :-



$\Sigma F_x = 0 \quad F_{BE} = 2.93$ (T)

Joint B



261

$\Sigma f_x = 0$

$$2.93 + F_{BC}\sin\theta - F_{BA}\sin\theta = 0$$

$$2.93 + \frac{F_{BC}}{\sqrt{2}} - \frac{F_{AB}}{\sqrt{2}} = 0 \quad \text{—— (i)}$$

$\Sigma f_y = 0$

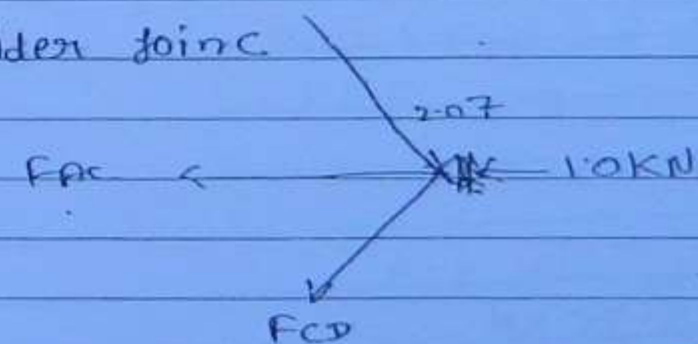
$$-\frac{F_{AB}}{\sqrt{2}} - \frac{F_{BC}}{\sqrt{2}} = 0 \quad \text{—— (ii)}$$

from (i) & (ii) $\qquad F_{AB} = -F_{BC}$

$\cancel{+\frac{F_{AB}}{\sqrt{2}}} -$ $\qquad F_{AB} = +2.07$

$F_{BC} = -2.07$

(3) Consider joint C

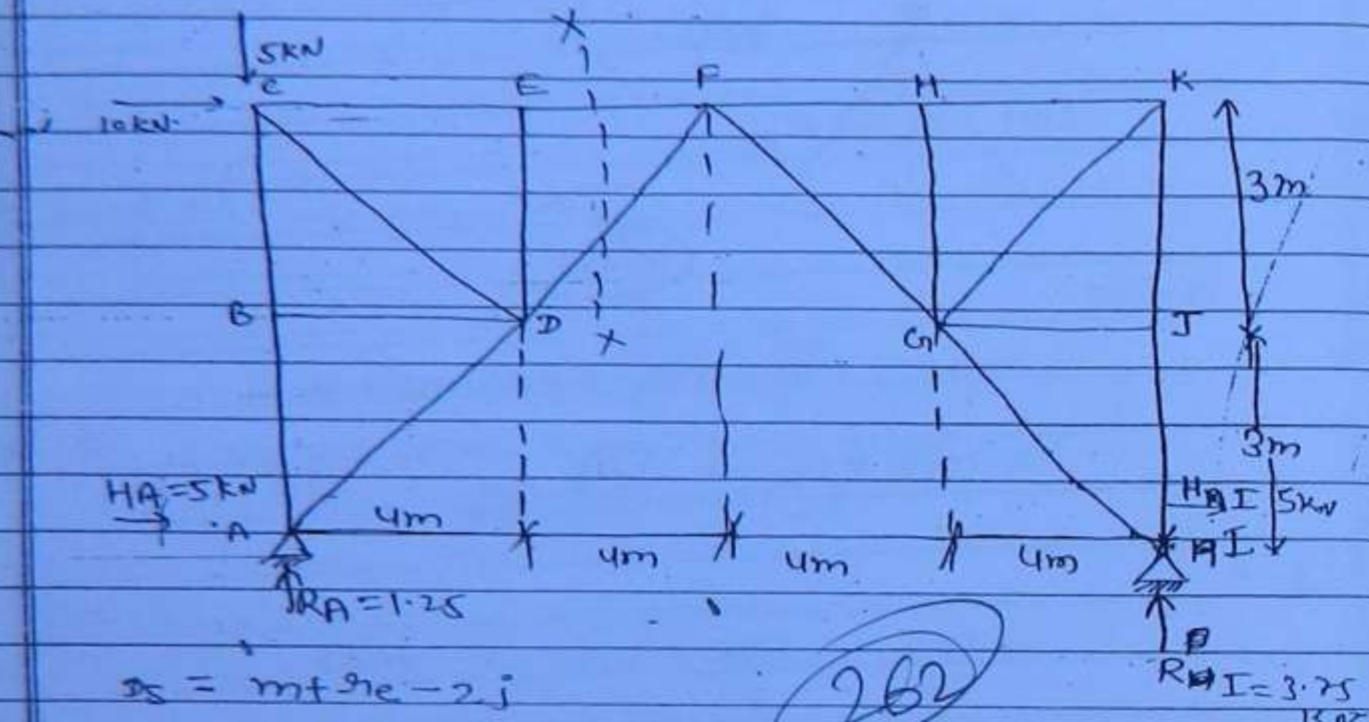


$F_{CD} = -2.07$

$F_{AC} = -7.07$

(method of section)

Q Calculate forces in the members EF & DF.



262

$$D_s = m + r_e - 2j$$

$$\Rightarrow 18 + 4 - 2 \times 11 = 0$$

$\Sigma F_x = 0 \quad H_A + H_I = 10$

$\Sigma F_y = 0 \quad R_A + R_I = 5$

$\Sigma M_A = 0$

$$-R_I \times 16 + 10 \times 6 = 0$$

$$R_I = \frac{60}{16} = 3.75 \text{ KN}$$

$$R_A = 1.25 \text{ KN}$$

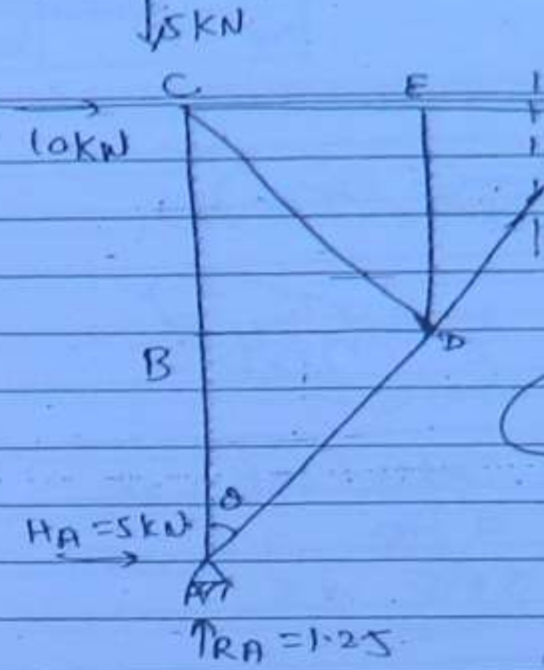
$$\boxed{R_I = 3.75 \quad R_A = 1.25}$$

$\Sigma M_F = 0$

$$R_I \times 8 - H_I \times 6 = 0$$

$$H_I = \frac{3.75 \times 8}{6}$$

$$\boxed{H_I = 5 \quad H_A = 5}$$



263

$\Sigma F_y = 0$ $\quad R_A - 5 + F_{DF}\cos\theta = 0$

$$1.25 - 5 + F_{DF} \times \frac{3}{5} = 0$$

$$F_{DF} = +6.25 \text{ kN}$$

$\Sigma F_x = 0$

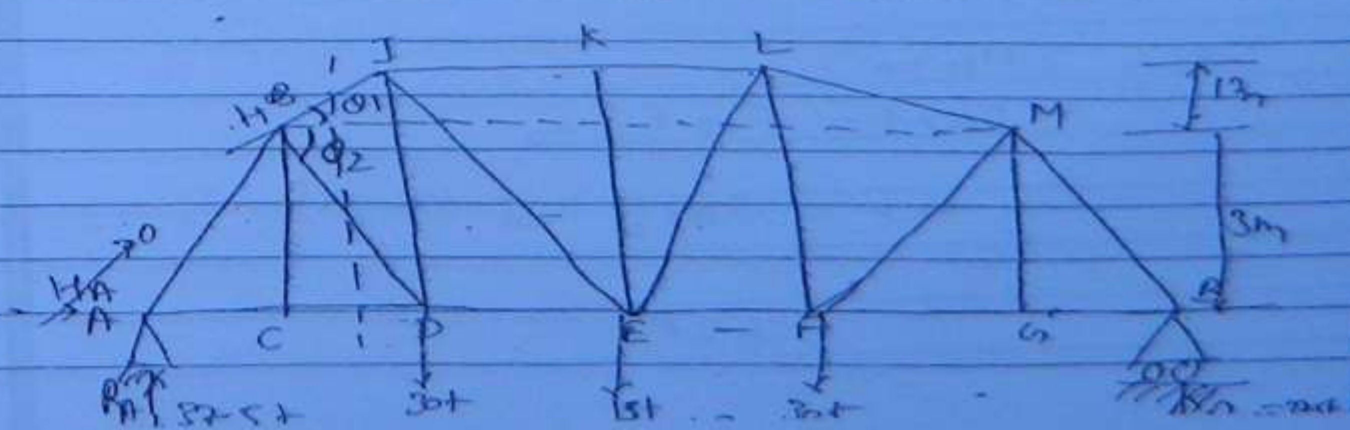
$$F_{EF} + F_{DF}\sin\theta + 10 - 5 = 0$$

$$F_{EF} = 5 - 10 - F_{DF}\cdot\sin\theta$$

$$\Rightarrow -5 - 6.25 \times \frac{4}{5}$$

$$F_{EF} \Rightarrow -10$$

Q Find forces in member HD and HJ.



$Ds = m + re - 2J$

→ (All Are triangular Block)

$Ds = 0$

284

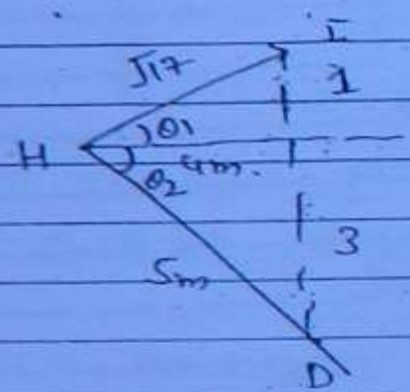
$\Sigma Fx = 0 \quad HA = 0$

$\Sigma Fy = 0$

$RA + RB = 30 + 10 + 30$

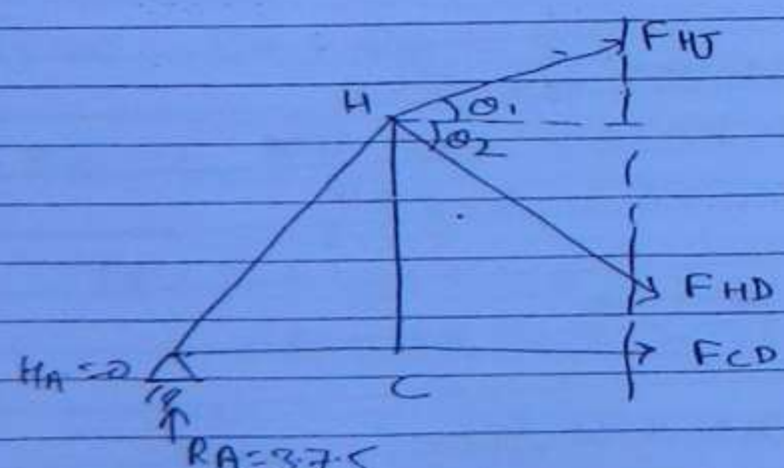
$RA = RB \quad 2RA = 70$

$RA = 37.5 = RB$



$\sin\theta_1 = \frac{1}{\sqrt{17}} \qquad \cos\theta_1 = \frac{4}{\sqrt{17}}$

$\sin\theta_2 = \frac{3}{5} \qquad \cos\theta_2 = \frac{4}{5}$



Taking $\Sigma M_H = 0$

$RA \times 4 - FCD \times 3 = 0$

$FCD = RA \times \frac{4}{3} \Rightarrow 37.5 \times \frac{4}{3}$

$FCD = 150$

$\Sigma F_x = 0$

$F_{CD} + F_{HD} \cos\theta_2 + F_{HJ} \cos\theta_1 = 0$

$50 + F_{HD} \frac{4}{5} + F_{HJ} \frac{4}{\sqrt{17}} = 0$ — ①

(0.8) (0.9701)

$\Sigma F_y = 0$

$R_A + F_{HJ} \sin\theta_1 - F_{HD} \sin\theta_2 = 0$

$37.5 + F_{HJ} \times \frac{1}{\sqrt{17}} - F_{HD} \times \frac{3}{5} = 0$ — ②

(0.2427) 0.6

265

from equ ① & ②

$F_{HD} = 31.25\ t$

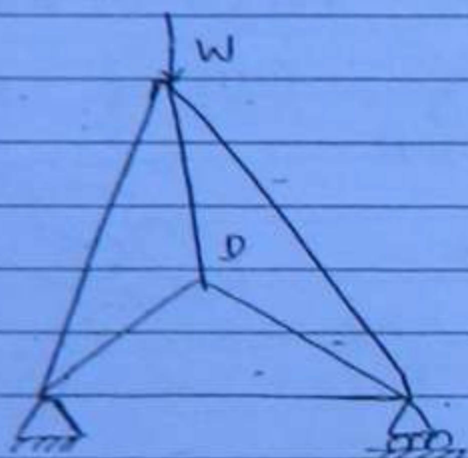
$F_{HJ} = -77.30\ t$

Analaysis of indeterminate truss

$D_s > 0$

$= 1$ or higher degree.

→ Fore method | unit load method | strain energy method | use of castigliano's thearums.



266

$D_s = m + r_e - 2j$

$\Rightarrow 6 + 3 - 2 \times 4 \Rightarrow 9 - 8 = 1$

$D_{se} = 0$

$D_{si} = m - (2J - 3)$

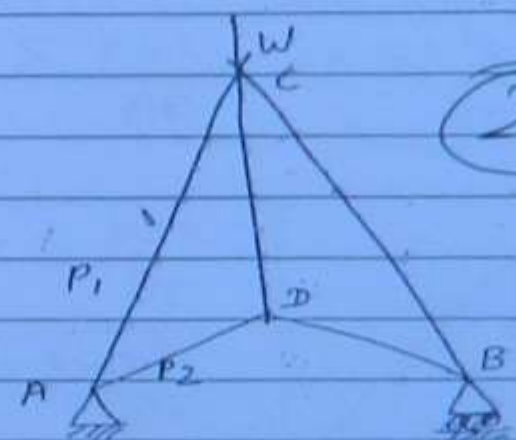
$\Rightarrow 6 - (2 \times 4 - 3)$

$\Rightarrow 1$

P-1 Find degree of static indeterminacy and check D_{se} & D_{si}, if degree of indet. is '1' it means there is one redudent reaction / force. identify the redundent in above case. redundent is internal force / force member / member force.

let Redudentent force is force in member AB.

Step-2 due to given load forces in truss member are S_1, S_2 ------ S_n which are requried to compute. If redudent member is remove the rest of the truss or truss will be determinate & stable. due to given loading in such determinate truss after removable of redundent, let force in member are P_1, P_2 ----- P_n. which canbe found by method of joint, method of section.

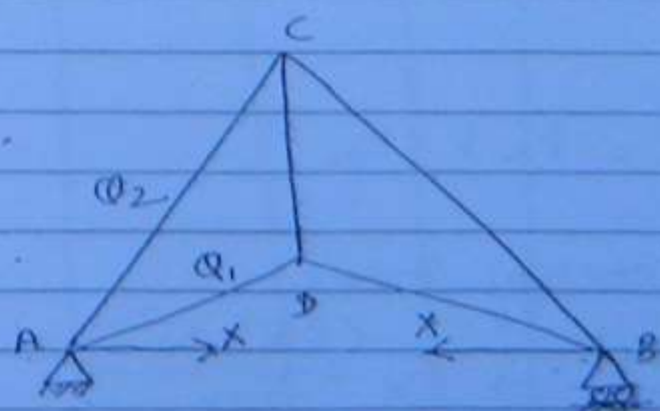


267

$\therefore$ force in AB = 0

Step-3

Remove loading and assume the value of redudent force is 'X' since force in AB is 'X' hance apply 'X' force in A & B in the direction of AB

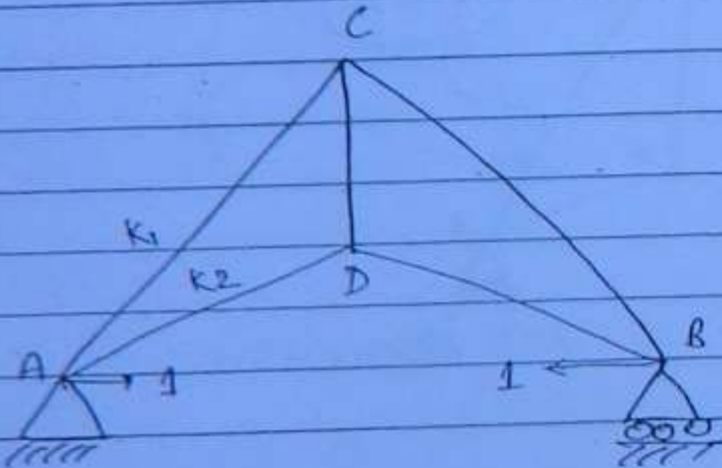


let due to x force in AB force developed in other member are $Q_1, Q_2 ---- Q_n$

$$\boxed{Q_{AB} = X}$$

268

Step-4 if 'x' is taking common and unit load is applied, at A & B in the direction of AB, then forces in member are $K_1, K_2, K_3 ----$ K_n than $Q = KX$



$$\boxed{Q = KX}$$

Step-5 The final force in truss member $S_1 = P_1 + Q_1$

$$S_1 = P_1 + X \cdot K_1$$
$$S_2 = P_2 + X \cdot K_2$$
$$\vdots$$
$$S_n = P_n + X K_n$$

the total strain energy stored

$$U = \sum \frac{S^2 L}{2AE} \Rightarrow \sum \frac{(P + K \cdot X)^2 \cdot L}{2AE}$$

The true of of redudent x will be that for that the total strain energy stored in the system in min

$$\frac{\partial U}{\partial x} = 0 \qquad 269$$

$$\frac{\Sigma 2(P + KX) \cdot K \cdot L}{2AE} = 0$$

$$\boxed{\Sigma \frac{PKL}{AE} + X \Sigma \frac{K^2 L}{AE} = 0}$$

$$\boxed{X = \frac{-\Sigma \frac{PKL}{AE}}{\Sigma \frac{K^2 L}{AE}}}$$

Summary:-

member	P	K	L	AE	$\frac{PKL}{AE}$	$\frac{K^2L}{AE}$	$S = R + KX$
					$\Sigma \frac{PKL}{AE}$	$\Sigma \frac{K^2L}{AE}$	

(1) Check of D_s and identify Redudent
(2) Remove the Redudent find P force system due to given loading
(3) Remove the loading and applied unit load in the direction of redudent And find K force system In all the member.
(4) Find value of Redudent force X

$$\boxed{X = \frac{-\sum \frac{PKL}{AE}}{\sum \frac{K^2L}{AE}}}$$

270

(5) Find Final force in the member
$$\boxed{S = P + K \cdot X}$$

Special case →
→ if $D_s = 2$ means there are two redudent say Redudent force are (X_1 & X_2) Remove both the Redundants and find P force system in the determinate truss

→ Apply unit load in the direction of first Redudent and find K force system in all member.

→ Now Apply unit load in the direction of Second Redundent and find K' system of force in All members.

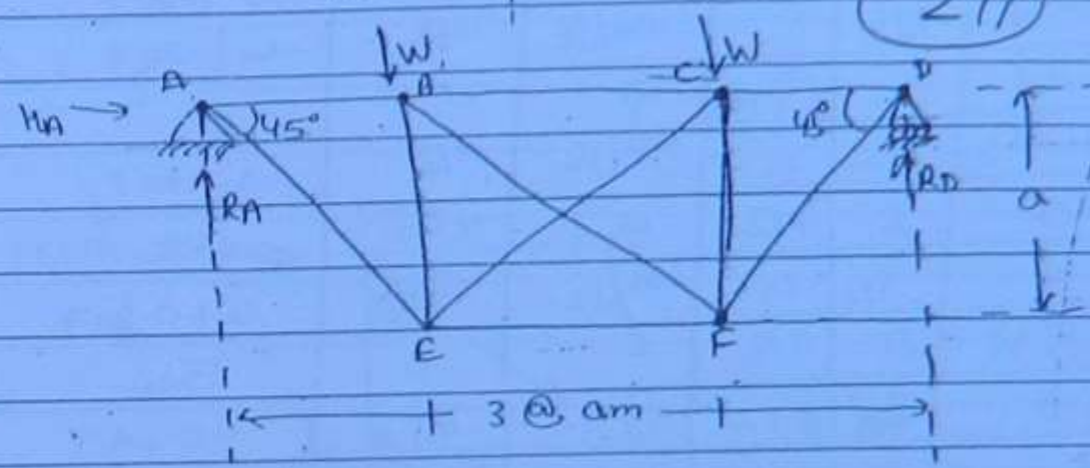
$$\frac{\partial U}{\partial X_1} = 0 \qquad X_1 = \frac{-\sum \frac{PKL}{AE}}{\sum \frac{K^2L}{AE}} \qquad\qquad X_2 = \frac{-\sum \frac{PK'L}{AE}}{\sum \frac{K'^2L}{AE}} \qquad \frac{\partial U}{\partial X_2} = 0$$

Q20 V.Imp Q

~~Analyse~~ Analysis the truss shown in below and find force in all member if AE is axial.

Rigidity → $\frac{AE}{L}$ = Axial stiffness.

271

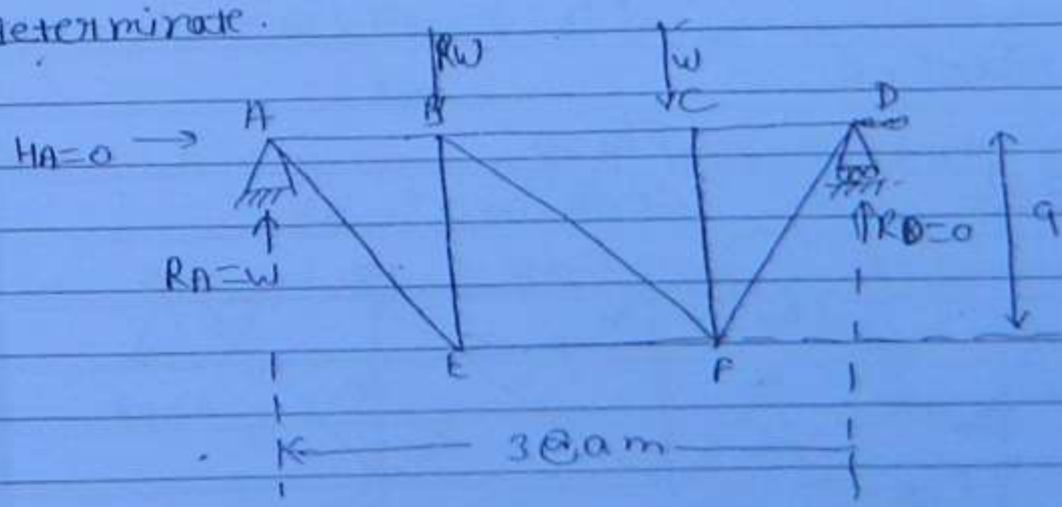


$Ds = m + re - 2j$ $\quad$ $Dse = 0$

$= 10 + 3 - 2 \times 6$ $\quad$ $Dsi = 1$

$Ds \Rightarrow 13 - 12 = 1$

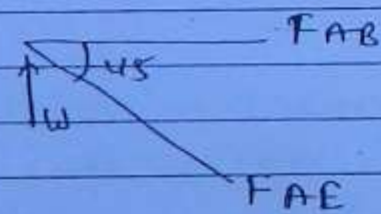
Let the Redundent member is ~~EC~~ EC. if Redundent is ~~Boom~~ Remove, than rest of the truss will be determinate.



	member	P	K	L	PKL	K^2L	$S = P + K \cdot X$
1	AB	$-w$	0	a	0	0	$-w$
2	AE	$\sqrt{2}W$	0	$a\sqrt{2}$	0	0	$2\sqrt{w}$
3	BC	$-W$	$-1/\sqrt{2}$	a	$wa/\sqrt{2}$	$a/\sqrt{2}$	$-w+0.207$
4	BE	$-W$	$-1/\sqrt{2}$	a	$wa/\sqrt{2}$	$a/\sqrt{2}$	$-w+.207$
5	CD	$-W$	0	a	0	0	$-w$
6	CF	$-W$	$-1/\sqrt{2}$	a	$wa/\sqrt{2}$	$a/\sqrt{2}$	$-w+.207$
7	EC	0	1	$a\sqrt{2}$	0	$a/\sqrt{2}$	-0.293
8	EF	W	$-1/\sqrt{2}$	a	$wa/\sqrt{2}$	$a/\sqrt{2}$	$w+0.207$
9	FD	$\sqrt{2}W$	0	$a\sqrt{2}$	0	0	$\sqrt{2}w$
10	BF	0	1	$a\sqrt{2}$	0	$a\sqrt{2}$	-0.293
				ΣPKL	$\sqrt{2}wa$	$2a(\sqrt{2}+1)$	
						ΣK^2L	

272

(1) Consider joint A



$\Sigma F_x = 0$

$F_{AB} + F_{AE}\cos 45^\circ = 0$

$F_{AB} = -\sqrt{2}w \dfrac{1}{\sqrt{2}}$

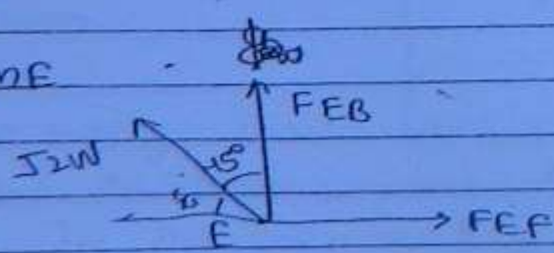
$F_{AB} = -w$

$\Sigma F_y = 0$

$F_{AE}\sin 45^\circ = w$

$F_{AE} = \sqrt{2}w$

(2) Consider joint E



$\Sigma F_y = 0$

$F_{EB} + \sqrt{2}w\cos 45^\circ = 0$

$F_{EB} = -\sqrt{2}w \times \dfrac{1}{\sqrt{2}} \Rightarrow -w$

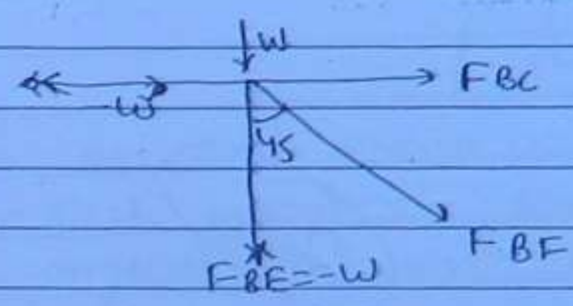
~~ΣFx=0~~ $\Sigma F_x = 0$

$$F_{EF} + \sqrt{2}\,w \sin 45^\circ = 0$$

$$F_{EF} = W$$

(3) Joint B

223



$\Sigma F_y = 0$

$$F_{BE} + \cancel{F_{BF} \cos 45}$$

$$-w + w + P_{BF} \cos 45 = 0$$

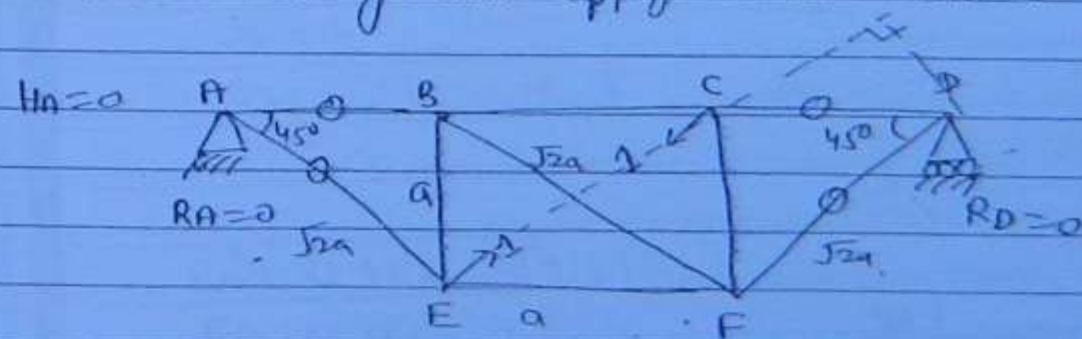
$$P_{BF} = 0$$

$\Sigma F_x = 0$

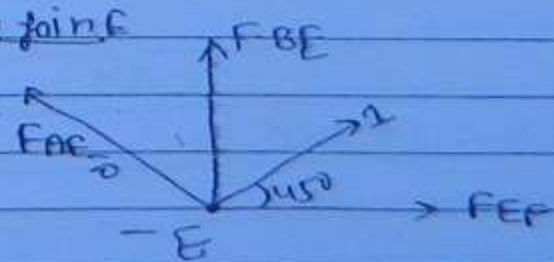
$$-w + F_{BC} + F_{BF} \cos 45 = 0$$

$$F_{BC} = -w$$

Step 3 Remove loading and apply unit load at E & C



Consider joint E



$\Sigma F_y = 0$

$$F_{BE} + 1 \sin 45 = 0$$

$$\boxed{F_{BE} = \frac{-1}{\sqrt{2}}}$$

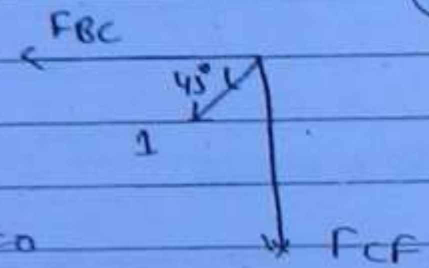
$\Sigma F_x = 0$

$$F_{EF} + 1 \cdot \sin 45 = 0$$

$$\boxed{F_{EF} = -\frac{1}{\sqrt{2}}}$$

Consider point C

2574



$\Sigma F_y = 0$

$$+F_{CF} - 1 \sin 45 = 0$$

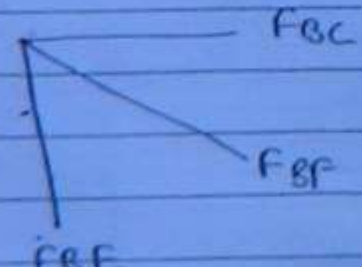
$$\boxed{F_{CF} = +1/\sqrt{2}}$$

$\Sigma F_x = 0$

$$F_{BC} - 1 \cos 45 = 0$$

$$\boxed{F_{BC} = 1/\sqrt{2}}$$

Consider poin B



$\Sigma F_x = 0$

$$F_{BF} = 1$$

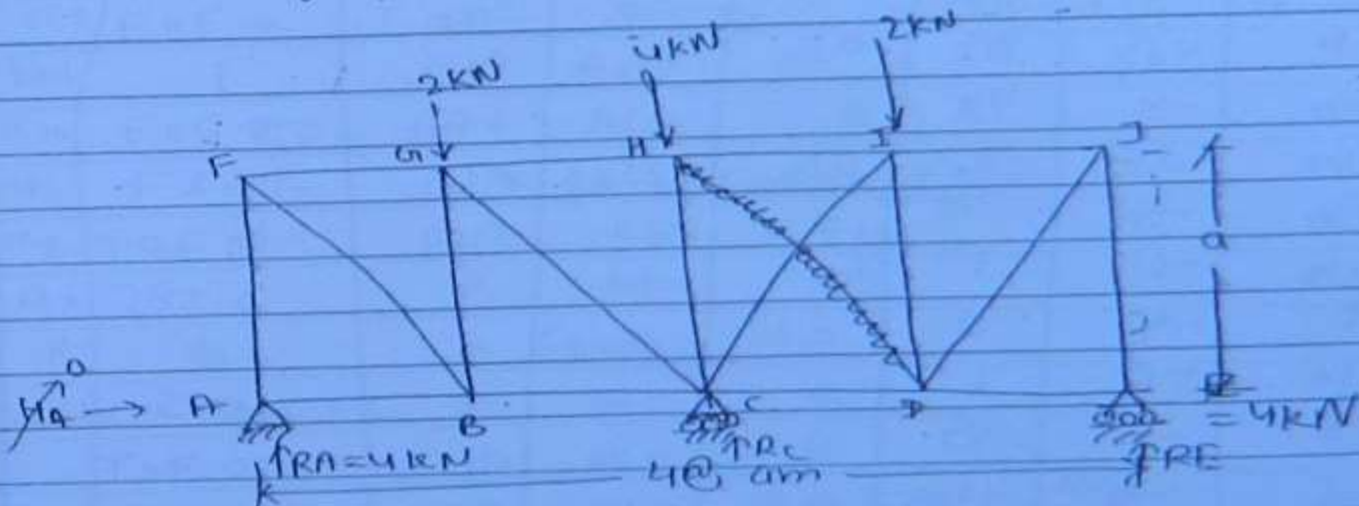
$$x = \frac{-\sum \frac{PKL}{AE}}{\sum \frac{K^2 L}{AE}}$$

275

$$X = \frac{-\sqrt{2}\,\omega a/AE}{\sqrt{2}\,2a(\sqrt{2}+1)/AE} \Rightarrow \frac{-\omega\sqrt{2}}{2\sqrt{2}(\sqrt{2}+1)}$$

$$X \Rightarrow 0.293\,W$$

Q. Analysis the truss shown in Fig and find the force in each member, the diagonal member makes angle of 45° with AE constant.



$r_e = 4$

$D_{se} = r_e - 3 \Rightarrow 1$ $\quad D_{si} = m - (2J - 3)$

$\Rightarrow 17 - (2\times10 - 3)$

$= 0$

$R_A + R_C + R_E = 2+4+2$

$R_A + R_C + R_E \Rightarrow 8$

$\sum M_A = 0$

$R_E \times 4a - 2\cdot3a - 4\cdot2a - 2\cdot a = 0$

since Dse is External, Hence External Reaction is indeterminant

(Rc is indeterminate)

276

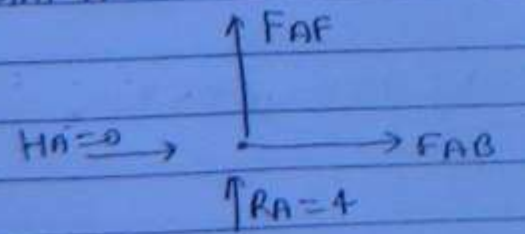
→ find a force system, remove Redundent (Say Rc)

$R_A = R_E = 4$

S.No	member	P	K	L	PKL	K^2L	S=P+KX	
1	AB	0	0	a	0	0	0	a
2	AF	−4	1/2	a	−2a	1/4 a	−0.707	0.72
3	BC	4	−1/2	a	−2a	a/4	0.707	0.72
4	BF	$4\sqrt{2}$	$1/\sqrt{2}$	$a\sqrt{2}$	$-4\sqrt{2}a$	$a/\sqrt{2}$	1	1.018
5	BG	−4	1/2	a	−2a	+a/4	−0.707	−0.72
6	CG	$2\sqrt{2}$	$-1/\sqrt{2}$	$a\sqrt{2}$	$-2\sqrt{2}a$	$a/\sqrt{2}$	−1.827	−1.827
7	FG	−4	1/2	a	−2a	a/4	−0.707	−0.72
8	GH	−6	1	a	−6a	a	0.585	0.51
9	HC	−4	0	a	0	0	−4	−4
10	IH	−6	1	a	−6a	a	0.585	
11	JI	−4	1/2	a	−2a	a/J	−0.707	
12	CI	$2\sqrt{2}$	$-1/\sqrt{2}$	$a\sqrt{2}$	$-2\sqrt{2}a$	$a/\sqrt{2}$	−1.827	
13	DI	−4	1/2	a	−2a	+a/4	−0.707	
14	DJ	$-4\sqrt{2}$	$1/\sqrt{2}$	$a\sqrt{2}$	$-4\sqrt{2}a$	$a/\sqrt{2}$	1	
15	DC	4	−1/2	a	−2a	a/4	0.707	
16	EJ	−4	1/2	a	−2a	1/4 a	−0.707	
17	ED	0	0	a	0	0	0	
					$\Sigma PKL = -44.97a$	$6.828a$		

$\Sigma K^2L =$

Consider joint A



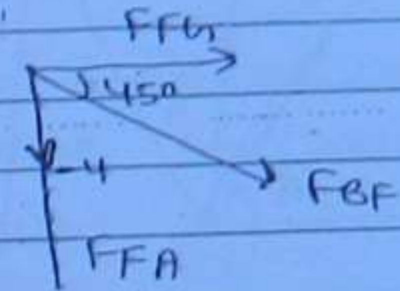
$\Sigma F_x = 0$ $\qquad$ $\Sigma F_y = 0$

$F_{AB} = 0$ $\qquad$ $F_{AF} + 4 = 0$

$F_{AF} = -4$

277

Consider Point F



$\Sigma F_y = 0$

$F_{BF} \sin 45 - 4 = 0$

$F_{BF} = 4 \times \sqrt{2}$ $\qquad$ $F_{BF} = 4\sqrt{2}$

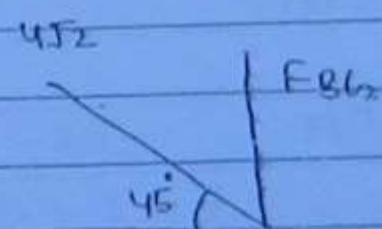
$\Sigma F_x = 0$

$F_{GF} + F_{BF} \cos 45 = 0$

$F_{GF} + 4\sqrt{2} \times \frac{1}{\sqrt{2}} = 0$

$F_{GF} = -4$

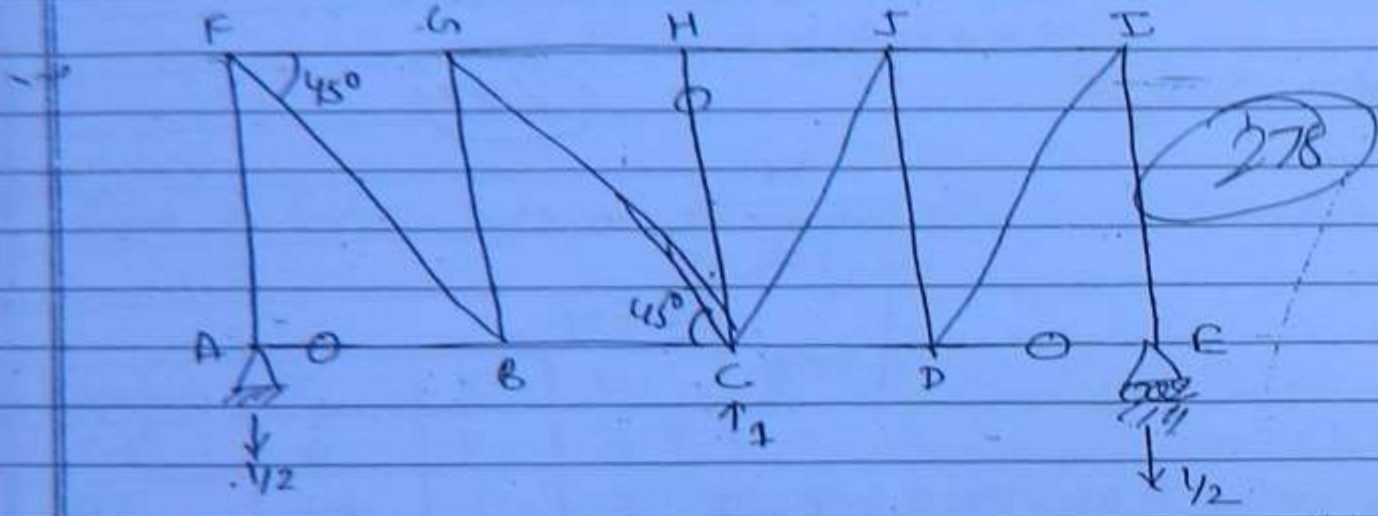
Consider point B



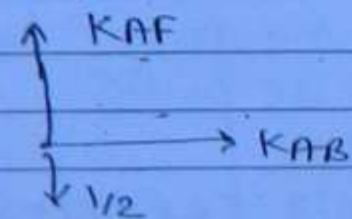
$\Sigma F_y = 0$ $\quad$ $F_{BG} + 4\sqrt{2} \sin 45 = 0$

$F_{BG} \Rightarrow -4$

→ For K System of force apply will load at C a unit load



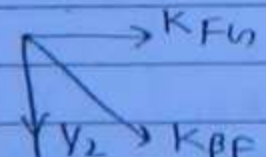
Consider joint A



$K_{AB} = 0$

$KAF = 1/2$

Consider joint F



$\Sigma F_y = 0$

$$-K_{BF}\frac{1}{\sqrt{2}} - \frac{1}{2} = 0$$

$$K_{BF} = -\frac{1}{\sqrt{2}}$$

$\Sigma F_x = 0$

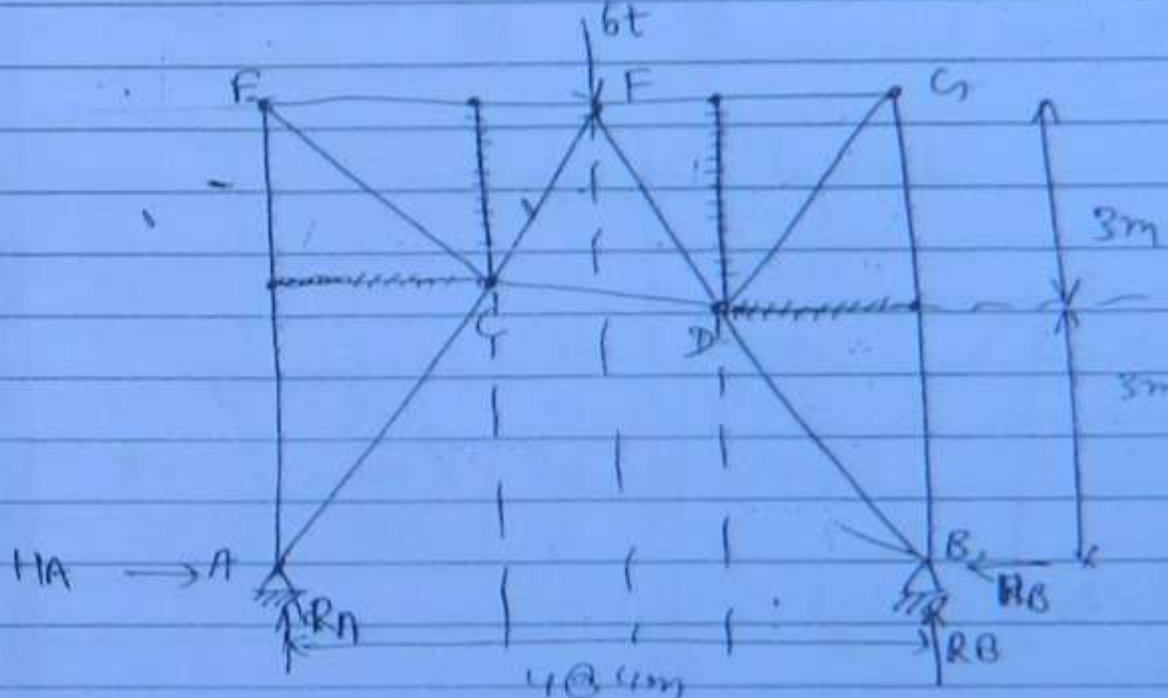
$K_{FG} = 1/2$

$$X = \frac{-\Sigma PKL/AE}{\Sigma K^2L/AE} \Rightarrow -\left(\frac{-44.978}{6.8289}\right)$$

279

$$\Rightarrow +6.5852$$

Q. for the truss shown in fig all member have same axial Rigidity Calculate Horizontal and vertical Reaction at supports.



$D_S = m + re - 2J$ $\quad\quad$ $D_{se} = re - 3$

$\Rightarrow 11 + 4 - 2 \times 7$ $\quad\quad$ $= 4 - 3$

$D_S = 1$ $\quad\quad$ $= 1$

$\Sigma F_x = 0$

$HA = HB = H$

$\Sigma F_y = 0$

$RA + RB = 6$

$\Sigma M_A = 0$

$$R_B \times 16 - 6 \times 8 = 0$$

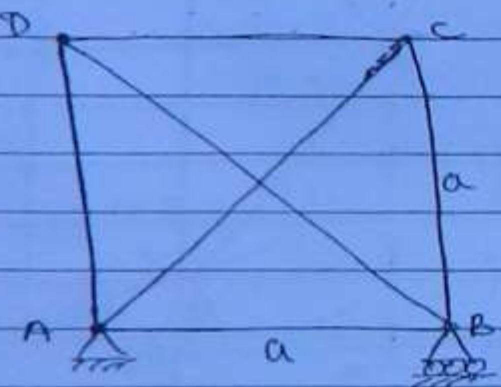
$$R_B = \frac{6 \times 8}{16} = 3\,kN$$

$R_A = R_B = \cancel{3\,kN}\ 3t$

→ Let the Redundent is H ($H_A = H_B = H$)

$H = 3.27\,t$

Lack of Fit Problems



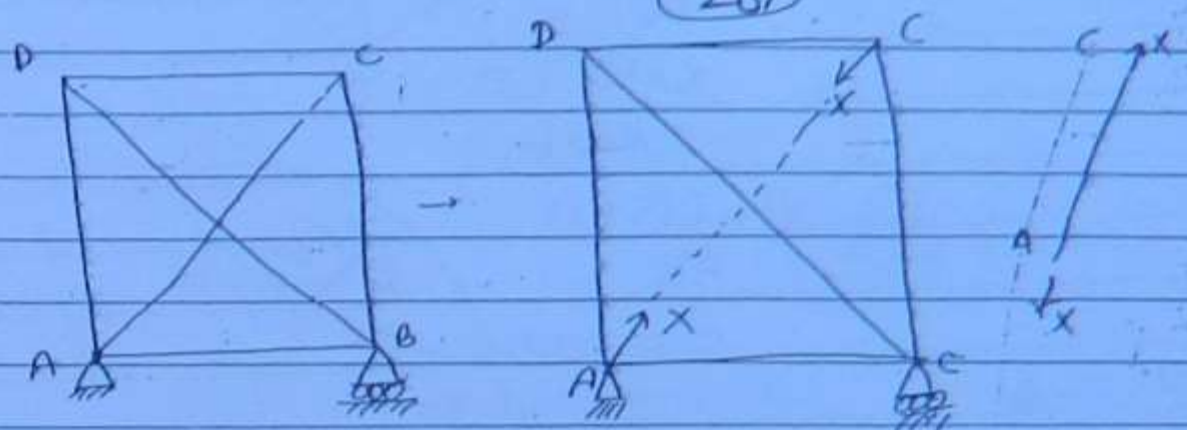
AE = constant

→ The Truss members are febreacted in the factory and assembeled in the field. If any one member is febricacted either Δ too short or Δ too long. If defective member is force fully fitted. then all members may be sub. to forces.

→ Let a square truss ABCD is to be assembled but member AC is feberacted Δ too short.

for force full filling of AC let us Apply x force in AC.

Hence joints A & C will be also under x force in direction AC. find forces in all members due to x force in AC

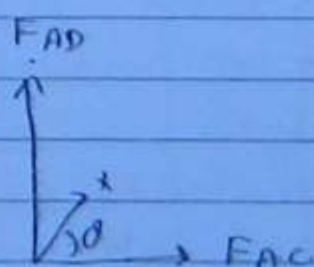


Member	force (Q)	L	Q^2L
AB	$-x/\sqrt{2}$	a	$x^2a/2$
AD	$-x/\sqrt{2}$	a	$x^2a/2$
AC	x	$a\sqrt{2}$	$x^2a\sqrt{2}$
BC	$-x/\sqrt{2}$	a	$x^2a/2$
BD	x	$a\sqrt{2}$	$x^2a\sqrt{2}$
CD	$-x/\sqrt{2}$	a	$x^2a/2$
		ΣQ^2L	$2x^2a + 2\sqrt{2}x^2a \Rightarrow 2x^2a(\sqrt{2}+1)$

Consider joint A

$R_A \pm R_B = 0$

$H_A = 0$



$\Sigma F_x = 0$

$F_{AB} + x\cos 45^\circ = 0$

$F_{AB} = -\frac{x}{\sqrt{2}}$

$\Sigma F_y = 0$

$F_{AD} + x\sin 45 = 0$

$F_{AD} = -\frac{x}{\sqrt{2}}$

Total Strain Energy in All member of Truss :-

$$U = \Sigma \frac{Q^2 L}{2AE}$$

$$\Rightarrow \frac{\cancel{2} x^2 a (\sqrt{2}+1)}{\cancel{2} AE}$$

$$U = \frac{x^2 a (\sqrt{2}+1)}{AE}$$

282

$$\frac{\partial U}{\partial x} = \Delta$$

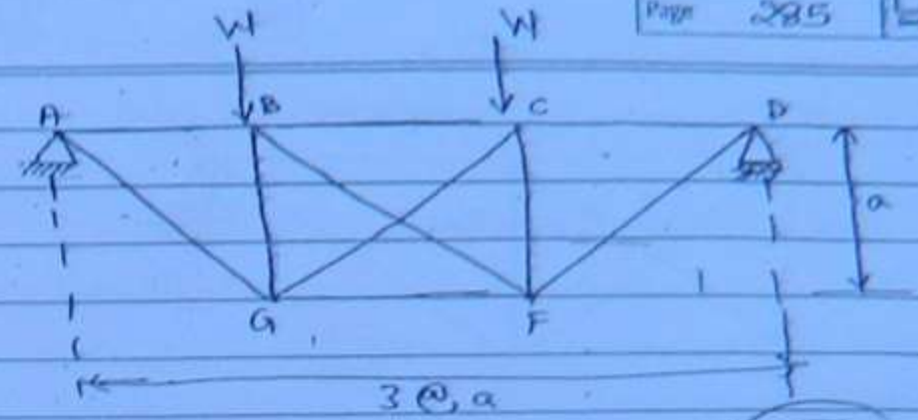
$$\frac{2x a (\sqrt{2}+1)}{AE} = \Delta$$

$$\boxed{x = \frac{\Delta \cdot AE}{2a(\sqrt{2}+1)}}$$

Note :- The Above forces Shown in the table Are due to lakeof fit effect when their is no loading in the truss. if loading Effect is also to be Considered, when then seperate analysis as per pervious discusion will be requried and Combined effect algebraic sumof lake of fit effect and loading Effect.

Q:- (i) fig. Shows a Pin joint a truss A Having Constant axial stiffness for each member. Calculate the axial force in member gf.

(ii) Calculate the force in same member gf if it is febreacted 0.1% too short when structure Carrying no loading.



283

So. loading effect

$$D_S = m + r_e - 2J$$
$$\Rightarrow 10 + 5 - 2\times 6$$
$$\Rightarrow 13 - 12 = 1$$

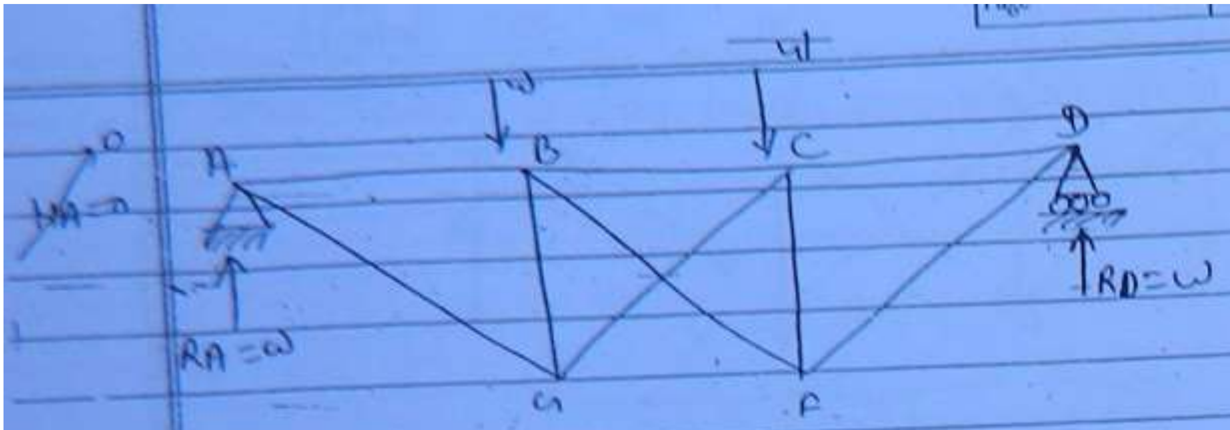
$D_{Se} = 0$

$D_S = 1$

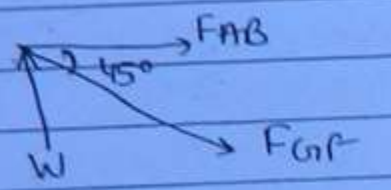
Hence redundent is internal force let member GF is redundent.

To find P ~~force~~ force system remove the member and find ~~forces~~ forces in all member by method of joint.

S.No	P Member	member	K	PK	K^2	$Q = KX'$	Q^2
1	$-w$	AB	0	0	0	0	0
2	$w\sqrt{2}$	AG	0	0	0	0	0
3	$-2w$	BC	1	$-2w$	1	X'^2	X'^2
4	$-2w$	BG	1	$-2w$	1	X'^2	X'^2
5	$w\sqrt{2}$	BF	$-\sqrt{2}$	$2w$	2	$-\sqrt{2}X'$	$2X'$
6	$-w$	CD	0	0	0	0	0
7	$-2w$	CF	1	$-2w$	1	X'	X'^2
8	$w\sqrt{2}$	CG	$-\sqrt{2}$	$-2w$	2	$-\sqrt{2}X'$	$2X'^2$
9	0	GF	1	0	1	X'	X'^2
10	$w\sqrt{2}$	DF	0	0	0	0	0



① consider joint a



284

~~$\Sigma f_x = 0$~~ $\Sigma f_y = 0$

$$\omega - F_{GF} \sin 45^\circ = 0$$

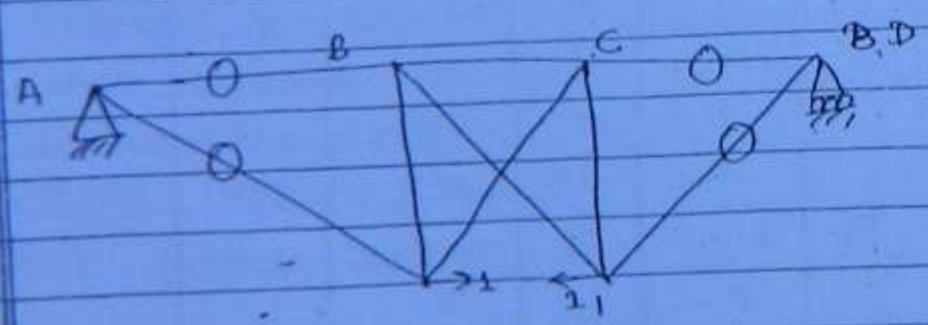
$$F_{GF} = \omega\sqrt{2}$$

$\Sigma f_x = 0$

$$F_{AB} + \omega\sqrt{2} \cos 45^\circ \Rightarrow$$

$$F_{AB} = -\omega\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$F_{AB} = -\omega$$



No reaction, because there are internal reaction

To find k force system remove given loading and apply unit load at G&F. At find k force method of joint.

Stiffness $\frac{AE}{L}$ = constant.

X = Redudent force in GF will be.

$$X \Rightarrow \frac{-\sum \frac{PKL}{AE}}{\sum \frac{K^2L}{AE}} \Rightarrow \frac{-\sum PK}{\sum K^2}$$

285

$$\Rightarrow \frac{-\cancel{10}^{5} w}{\cancel{8}_{4}} \rightarrow 1.25w$$

Step 2

$$\Delta = \frac{0.1}{100} \times a$$

$$\Delta = \frac{a}{1000}$$

Let due to lack of fit force in member GF is X'

$$\text{Hence } \frac{\partial U}{\partial X'} = \Delta = \frac{a}{1000}$$

$\frac{AE}{L}$ = const for all member

$$U = \sum \frac{Q^2L}{2AE} = \frac{L}{2AE} \sum Q^2$$

Q_1, Q_2 --- Q_n are forces in Truss member due to X' force in GF

$$U = \frac{L}{2AE} \sum Q^2 \Rightarrow$$

due to unit force in ~~dir~~ ~~of~~ K_1, K_2, K_3 --- K_n are the force in truss member, Hence due to x' forces in ~~of~~ forces in truss member will be

$$Q_1 = K_1 x'$$
$$Q_2 = K_2 x'$$
$$\vdots$$
$$Q_n = K_n x'$$

286

(a)

$$U = \frac{L}{2AE} \Sigma Q^2 \qquad \text{let } \frac{AE}{L} = C$$

$$U \Rightarrow \frac{1}{2C} \times 8x'^2$$

$$\frac{\partial U}{\partial x'} \Rightarrow \frac{2}{2C} \times 8x' \Rightarrow \frac{8x'}{C} = \frac{a}{1000}$$

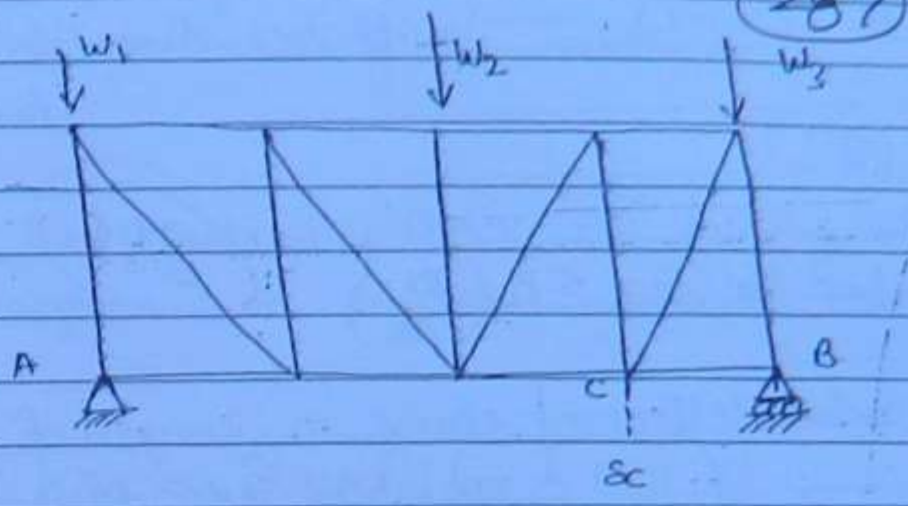
$$x' = \frac{aC}{8000} \Rightarrow \boxed{\frac{a\,AE}{L\,8000} = x'}$$

Deflection of Truss joints Methods

1. force method | unit load method | maxwell's m | Strain Energy method
2. Stiffness method | Equilibrium method
3. Graphical method.
   (a) Williot Mohr Diagram } Applicable
   (b) Bar Chain method } for Truss

unit load method | Maxwell's method :-

287



→ Let vertical Deflection of joint C is requried.

Procedure

Step 1 due to given load system find P-force system in all members unit method of joint say member forces are $P_1, P_2 ---- P_n$.

Step 2 Remove given loading and Apply unit load at C in the direction of desired defllicection (vertical) and find K force system say due to unit load at C member forces are $K_1, K_2 ---- K_n$.

Step-3

the vertical deflection at C is given as

$$\delta c = \frac{\Sigma PKL}{AE}$$

if δ is (+ive) $\longrightarrow$ it is in the direction of unit load.

if δ is (−ive). $\longrightarrow$ opposite direction of unit load.

Special case →

288

$$\delta_c = \Sigma P\left(\frac{PL}{AE}\right)$$

$\frac{PL}{AE}$ = Axial deflection | change in length of any member due to force P in that member

$$P \longrightarrow (+)\ T$$
$$P \longrightarrow (-)\ C$$

$$\boxed{\delta_c = \sum_{i=1}^{n} k_i \Delta_i}$$

if tempr of some of the member is change Δ_i will be due to temp effect

$$\Delta_i = (L\alpha T)_i$$

T = Temp. Change $^\circ C$
(+) if T is ↑
(−) if T is ↓

α = coeff. of Termal change

→ Δ_i in any member i may be due to defective manufacturing also if member is manufacture.

A to long $\xrightarrow{\Delta_i}$ +ive

$\xrightarrow{\Delta_i}$ −ive

→ ~~Moment~~ movement of truss joint is also occur due to settlement of support, for such conditions virtual work method may be applied.

289

The combind effect of loading and temp. change will be algebraic sum of individual effect

$$\boxed{\delta_C = \sum_{i=1}^{n} K_i\left(\frac{PL}{AE} + L\alpha T\right)}$$

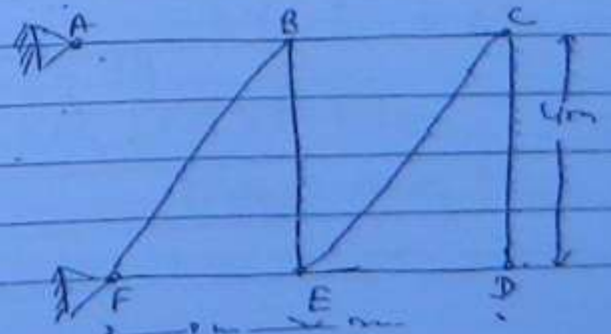
The computation may be ~~perf~~ performed in tabular form.

member	P	K	L	AE	PKL/AE
					$\sum \frac{PKL}{AE}$

IES 2011

Q

3 member of the Truss shown in the fig. below have been cut either too long or too short. Calculate the vertical displacement of joint D due to discripancies length of these member by unit load method. All member have same cross-section.



AE constant

$\Delta_{AB} = -3\,mm$

$\Delta_{BC} = +2\,mm$

$\Delta_{EF} = +1\,mm$

vertical deflection of joint Δ_D

$$\Delta_D = \sum_{i=1}^{n} K_i \Delta_i$$

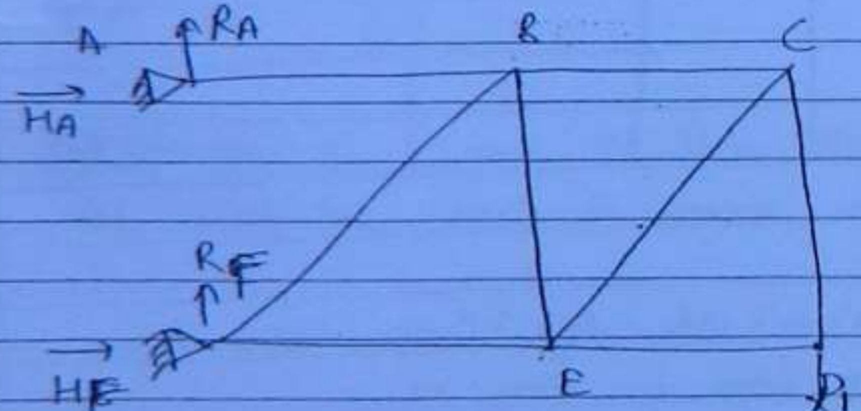
290

$$\Delta_D = K_{AB} \cdot \Delta_{AB} + K_{BC} \Delta_{BC} + K_{EF} \Delta_{EF} + \overset{0}{\cancel{K \Delta_{\text{other member}}}}$$

$$\boxed{\Delta_D = K_{AB} \Delta_{AB} + K_{BC} \Delta_{BC} + K_{EF} \Delta_{EF}}$$

$$D_S = m + r_e - 2j$$
$$\Rightarrow 8 + 4 - 2 \times 6$$
$$\Rightarrow 0$$

→ find K force is due to unit load at D in vertical direction.



$M_B = 0$
$R_A \times 3 = 0$
$R_A = 0$

or

At joint A
$\Sigma F_y = 0$
$R_A = 0$

$\Sigma F_y = 0$
$\overset{0}{\cancel{R_A}} + R_F - 1 = 0$

$R_F = 1$

$\Sigma F_x = 0$

$H_A + H_F = 0$

291

$\Sigma M_F = 0$

$H_A \times 4 + 1 \times 6 = 0$

$H_A = -1.5$

$H_F = +1.5$

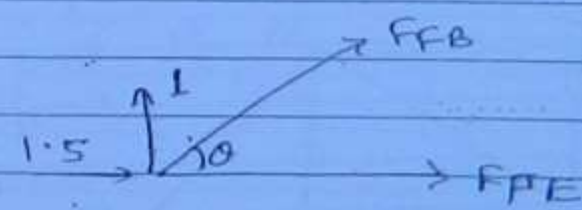
Consider joint A

$\Sigma F_x = 0$

$F_{AB} - 1.5 = 0$

$F_{AB} = 1.5$

Consider Joint F



$\Sigma F_y = 0$

$K_{FB} \cdot \sin\theta + 1 = 0$

$K_{FB} = -\frac{1}{4} \times 5 = -1.25$

$\Sigma F_x = 0 \qquad 1.5 + K_{FE} + K_{FB} \cdot \cos\theta = 0$

$K_{FE} = -1.5 + 1.25 \times \frac{3}{5}$

$= 0.75$

Consider joint B

$\Sigma F_x = 0$

$K_{BC} + 1.25 \cos\theta - 1.5 = 0$

$\boxed{K_{BC} = +0.75}$

$\Delta_D = K_{AB} \cdot \Delta_{AB} + K_{AC} \cdot \Delta_{BC} + K_{EF} \cdot \Delta_{EF}$

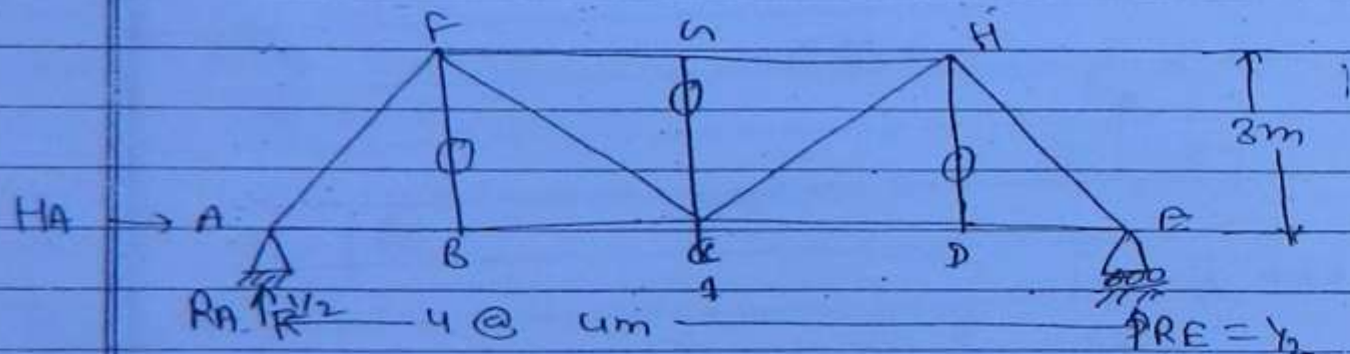
$\Rightarrow (+1.5) \times (-3) + (0.75)(2) + (0.75)(4)$

$\boxed{\Delta_D = -6\ mm}$ ↑

292

∵ Δ_D is (−) it means Δ_D is in opposit Direction of unit load.

Q. Compute vertical ~~di~~ compenet of the displacement of joint C due to decrease in temp. by 50°C in bottom chord member only given that $\alpha = \frac{1}{150000}/°C \Rightarrow 6.67 \times 10^{-5}$



$D_S = m + r_e - 2j$

$= 13 + 3 - 2 \times 8 = 0$

vertical deflection of joint C

$$\Delta_C = \sum_{i=1}^{n} K_i \Delta_i$$

Δ_i = change in length of member due to temp change.

~~Since~~ Since Temp. change is only Bottom chord member Hence for other member $\Delta_i = 0$

293

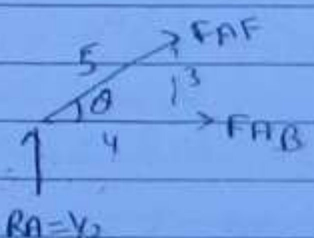
$$\delta_C = K_{AB}\Delta_{AB} + K_{BC}\Delta_{BC} + K_{CD}\Delta_{CD} + K_{DE}\Delta_{DE}$$

$$\Delta_{AB} = \Delta_{BC} = \Delta_{CD} = \Delta_{DE} = l\alpha T$$

$$\Rightarrow 4 \times 6.67\times10^{-6} \times (-50) \times 10^{3}$$

$$\Rightarrow -1.33\,mm$$

Consider Joint A & force system.



~~$\Sigma f_x = 0$~~ ~~$\Sigma f_y = 0$~~

~~$F_{AF}\sin\theta + 1/2 = 0$~~

~~$F_{AB}\cos\theta =$~~ ~~$F_{AF} \times \frac{3}{5} = -\frac{1}{2} \Rightarrow F_{AF} =$~~

$\Sigma f_y = 0$

$$F_{AF}\sin\theta + \frac{1}{2} = 0$$

$$F_{AF} = -\frac{1}{2} \times \frac{5}{3} \Rightarrow -\frac{5}{6} = 0.833$$

$\Sigma f_x = 0$

$$F_{AB} + F_{AF}\cos\theta$$

$$F_{AB} = 0.833 \times \frac{4}{5} \Rightarrow \frac{5}{6} \times \frac{4}{5} \Rightarrow \frac{2}{3}$$

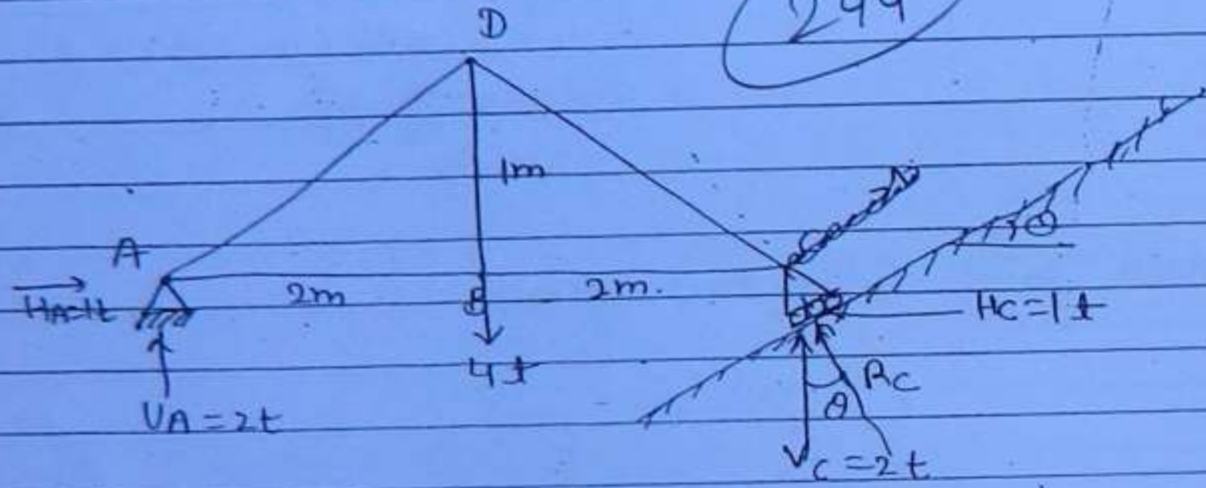
$$F_{AB} = F_{CD} = F_{DE}$$

$$\delta_C = \Sigma k_i \Delta_i$$

$$\Rightarrow 4 \times \frac{2}{3} \times -1.33 \Rightarrow -3.55\,mm\ (\uparrow)$$

Q A small truss shown in fig. has all pin jointed member having Area of cross section $20 cm^2$ each. The plane of Rollar is inclined and is parallel to AD. determine movement of joint C when vertical load of 4 t is applied at B given that $E = 2 \times 10^6\ Kg/cm^2$.

294



$V_C = R_C \cos\theta$

$H_C = R_C \sin\theta$ $\qquad \frac{H_C}{V_C} = \tan\theta = \frac{1}{2}$

$$\boxed{H_C = \frac{V_C}{2}}$$

$\Sigma M_A = 0$

$V_C \times 4 - 4 \times 2 = 0$

$V_C = 2t$

$H_C = 1t$

$\Sigma f_x = 0$

$H_A - H_C = 0$

$H_A = H_C = 1t$

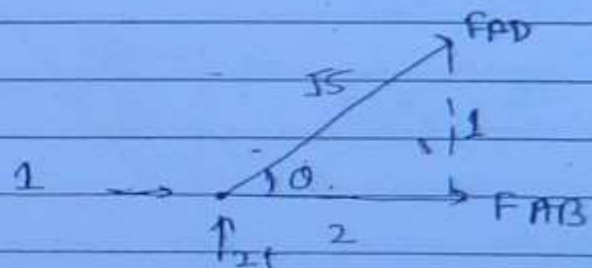
$\Sigma f_y = 0 \quad V_A + V_C - 4 = 0$

$V_A = 2t$

member	P t	K	L (m)	PKL
AB	+3	$\sqrt{5}/2$	2	$6.708 \times 1000 \Rightarrow 6708$
AD	−4.47	0	$\sqrt{5}$	~~[illegible]~~
BC	+3	$\sqrt{5}/2$	2	$6.708 \Rightarrow 6708$
BD	+4	0	1	$0 \Rightarrow 0$
CD	−4.47	0	$\sqrt{5}$	$0 \Rightarrow 0$
				$\Sigma PKL = 13146$ kg-m

① Joint A

2995



$\Sigma Fx = 0$

$$FAB + FAD\cos\theta + 1 = 0$$

$$FAB + FAD \frac{2}{\sqrt{5}} + 1 = 0$$

$$FAB = -1 + 4.472 \times \frac{2}{\sqrt{5}} \Rightarrow +3\,kN$$

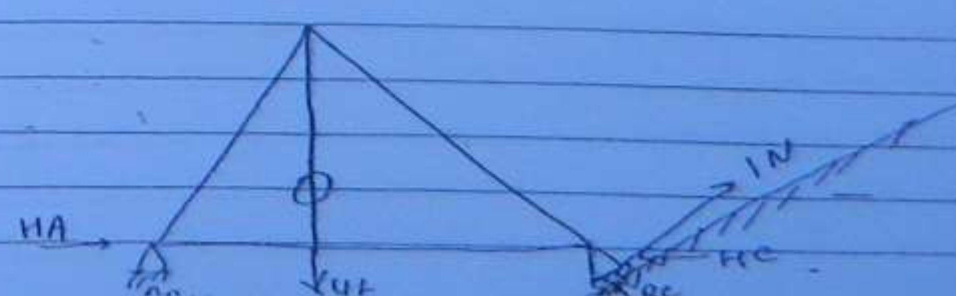
$\Sigma Fy = 0$

$$FAD \sin\theta + 2t = 0$$

$$FAD = -2 \times \sqrt{5}/1$$

$$\boxed{FAD \Rightarrow -4.472}$$

To find Deflection in the direction of plane Apply unit load ∥ to the plane. Unit load when θ with the horizontal.



Page
296

$\Sigma M_C = 0$

$V_A \times 4 = 0$

$\underline{V_A = 0}$

$\Sigma F_y = 0$

$V_A + V_C + 1 \sin\theta = 0$

$V_C = -1 \sin\theta$

$\boxed{V_C \Rightarrow -1 \times \frac{1}{\sqrt{5}} = -\frac{1}{\sqrt{5}}}$

$\boxed{H_C = \frac{V_C}{2} \Rightarrow -\frac{1}{2\sqrt{5}}}$

$\Sigma F_x = 0$

$H_A + 1 \cdot \cos\theta + H_C = 0$

$H_A = -\frac{2}{\sqrt{5}} - \frac{1}{2\sqrt{5}} \Rightarrow -\frac{\sqrt{5}}{2}$

$\Delta_C = \frac{\Sigma PKL}{AE}$

$P \rightarrow Kg$

$A = cm^2$

$E = Kg/cm^2$

$L = m.$

Δ_C will be in

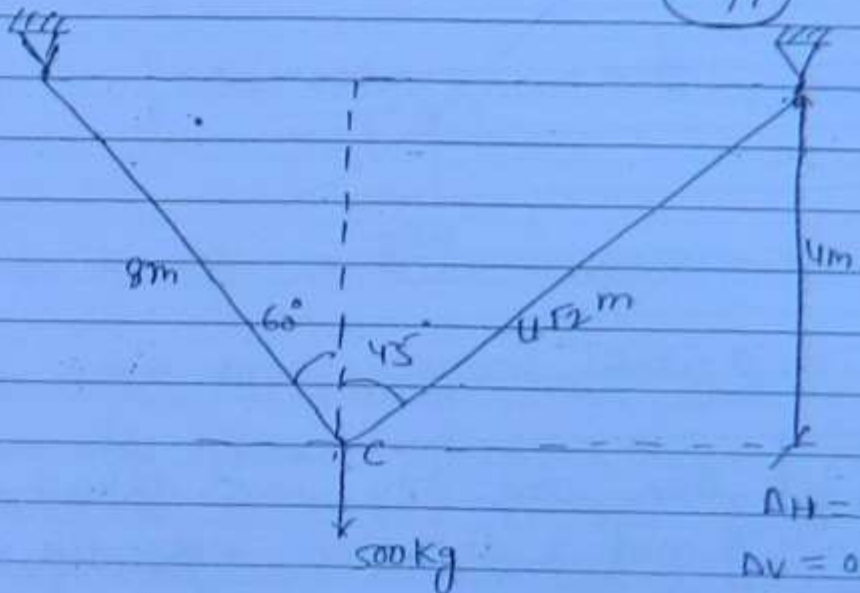
$\Delta_C = \frac{13416}{20 \times 2 \times 10^6}$ m.

$\Rightarrow \cancel{3.354 \times 10^{-3}}$

$\Delta C \rightarrow 0.32865$

Q. 2 Straight bars AC & BC 8mm in dia meat at joint C. Calculate Horizontal and vertical component of deflection if all Joints are Hinge an vertical load of 500 kg is apply at joint C. young modulus 2×10^6 kg/cm².

(297)



$\Delta H = 0.828$ mm (→)

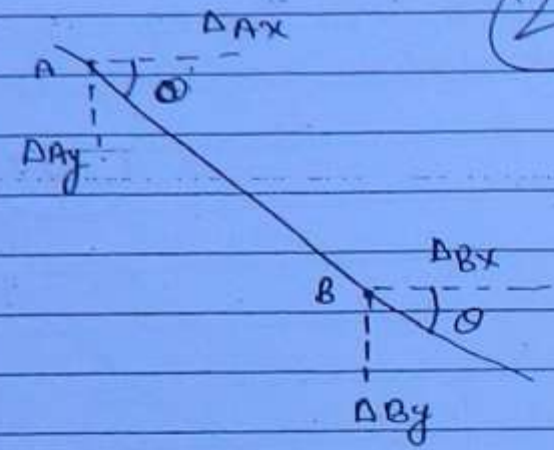
$\Delta V = 0.4329$ mm (↓)

Ans:-

Stiffness Method to solve Truss

Force - Displacement Relation →

298



Consider AB is a truss member and A & B are truss joint.

let Δ_{Ax} and Δ_{Ay} are displacement of joint A in x and y direction Respectly. similarly Δ_{Bx} and Δ_{By} are displacment of joint B.

The displacement of joint A in the direction of member AB :-

$$\Delta_A = \Delta_{Ax}\cos\theta + \Delta_{Ay}\sin\theta$$

Displacement of joint B in the direction of member AB →

$$\Delta_B = \Delta_{Bx}\cos\theta + \Delta_{By}\sin\theta$$

θ is angle of member AB with Horizontal

Axial displacement of member AB ⇒

$$\Delta_A - \Delta_B$$

$$\Rightarrow \Delta_{Ax}\cos\theta + \Delta_{Ay}\sin\theta - \Delta_{Bx}\cos\theta - \Delta_{By}\sin\theta$$

$$\Delta_{AB} \Rightarrow (\Delta_{Ax} - \Delta_{Bx})\cos\theta + (\Delta_{Ay} - \Delta_{By})\sin\theta$$

$$\Delta_{AB} = \Delta_B - \Delta_A$$

$$\Delta_{AB} = (\Delta_{Bx} - \Delta_{Ax})\cos\theta + (\Delta_{By} - \Delta_{Ay})\sin\theta$$

$$\Rightarrow \frac{P_{AB} \cdot L}{AE} = (\Delta_{Bx} - \Delta_{Ax})\cos\theta + (\Delta_{By} - \Delta_{Ay})\sin\theta$$

$$\Rightarrow \boxed{P_{AB} = \frac{AE}{L}\left[(\Delta_{Bx} - \Delta_{Ax})\cos\theta + (\Delta_{By} - \Delta_{Ay})\sin\theta\right]}$$

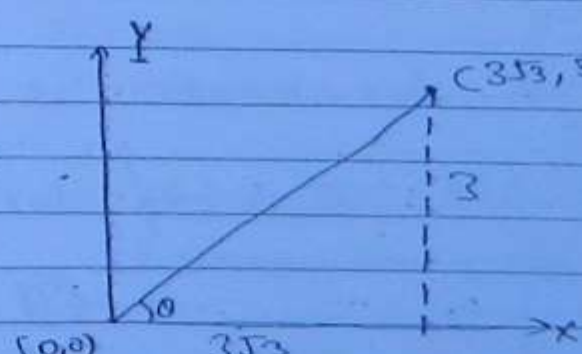
$$\Rightarrow K\left[(\Delta_{Bx} - \Delta_{Ax})\cos\theta + (\Delta_{By} - \Delta_{Ay})\sin\theta\right]$$

$$K = \frac{AE}{L} \Rightarrow \text{Axial displacement}$$

Q- A BAR A, B of a pin jointed Truss line in the x, y plane has the ~~given~~ cordinates of A (0,0) and cordinates of B $(3\sqrt{3}, 3)$ in m. joint A & B are displace and the displacement in x-y direction are $\Delta_{Ax} = 1$ $\Delta_{Ay} = \sqrt{3}$ $\Delta_{Bx} = 2$ $\Delta_{By} = 2\sqrt{3}$ mm in A Axial stiffness of BAR is 100 kN/m. ~~the~~ force induced in the bare above displacement is ~~100~~

(a) 100 KN (B) 200 KN
(c) 173 KN (d) 273 MN

Sol



$$\tan\theta = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

$$\cos\theta = \sin 30 = \sqrt{3}/2$$

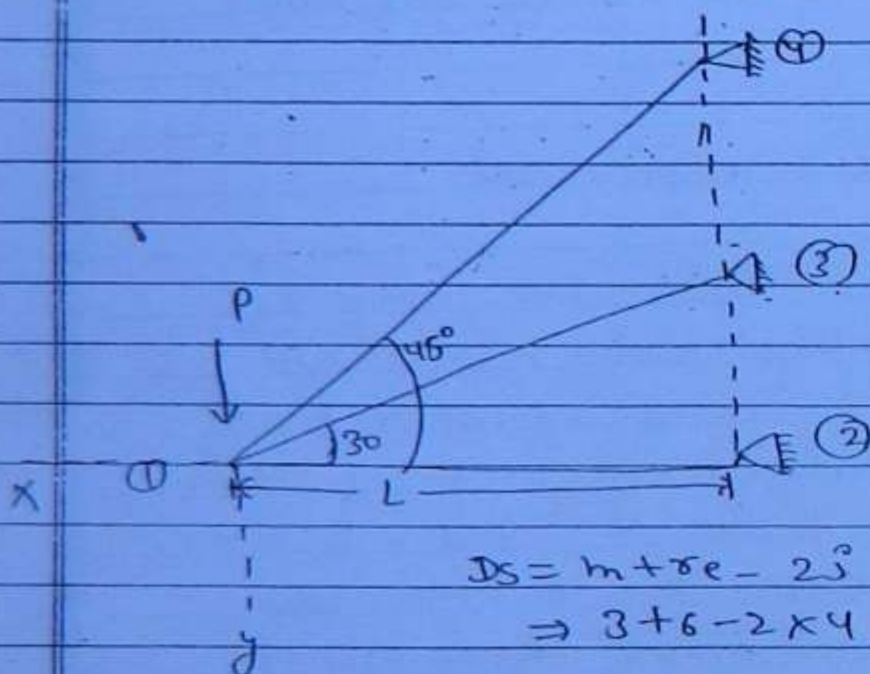
$$\sin\theta = \sin 30 = \frac{1}{2}$$

$$P_{AB} = 100\left[(2-1)\times\frac{\sqrt{3}}{2} + (2\sqrt{3} - \sqrt{3})\times\frac{1}{2}\right]$$

$$\Rightarrow 100\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] = 100\sqrt{3} \Rightarrow 173\,kN$$

300

Q. for three bar truss shown in fig. compute vertical displacement of node 1. using stiffness method.



$AE = \text{const.}$

$L_{12} = L$

$L_{13} = 2L/\sqrt{3}$

$L_{14} = \sqrt{2}L$

$$D_S = m + r_e - 2j$$

$$\Rightarrow 3+6-2\times4 \Rightarrow 9-8 = 1$$

$$D_K = 2j - r_e$$

$$\Rightarrow 2\times4 - 6 \Rightarrow 2 \begin{cases} \Delta_{1x} \\ \Delta_{1y} \end{cases}$$

for Displacement Relations for member 1,2

$$P_{12} = \frac{AE}{L}\left[(\cancel{\Delta_{2x}} - \Delta_{1x})\cos 0^\circ + (\cancel{\Delta_{2y}} - \Delta_{1y})\sin 0^\circ\right]$$

$$P_{12} = \frac{AE}{L}\left[(-\Delta_{1x})\right] \Rightarrow \text{---} \quad (1)$$

$$P_{13} \Rightarrow \frac{AE}{2L/\sqrt{3}} \left[(\cancel{\Delta_3 x}^{\,0} - \Delta_1 x)\cos 30^\circ + (\cancel{\Delta_3 y}^{\,0} - \Delta_1 y)\sin 30^\circ \right]$$

361

$$\Rightarrow \frac{\sqrt{3}}{2}\,\frac{AE}{L}\left[-\Delta_1 x \cdot \frac{\sqrt{3}}{2} - \frac{\Delta_1 y}{2} \right]$$

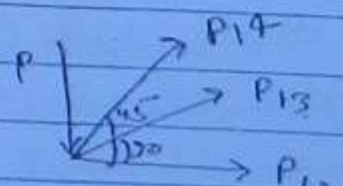
$$\Rightarrow -\frac{\sqrt{3}}{4}\,\frac{AE}{L}\left[\sqrt{3}\,\Delta_1 x + \Delta_1 y \right] \quad \text{— (2)}$$

for member P_{14}

$$P_{14} = \frac{AE}{\sqrt{2}L}\left[(\cancel{\Delta_4 x}^{\,0} - \Delta_1 x)\cos 45^\circ + (\cancel{\Delta_4 y}^{\,0} - \Delta_1 y)\sin 45^\circ \right]$$

$$P_{14} = -\frac{AE}{2L}\left[\Delta_1 x + \Delta_1 y \right] \quad \text{- - - (3)}$$

To find joint displacement node 1 consider free body equilibrium of node 1.



$\Sigma F_x = 0$

$$P_{12} + P_{13}\cos 30^\circ + P_{14}\cos 45^\circ = 0 \quad \text{- - (1)}$$

$\Sigma F_y = 0$

~~$P_{13}\sin 45$~~

$$P_{12}\sin 30^\circ + P_{14}\sin 45^\circ - P = 0 \quad \text{— (2)}$$

Solving equ (1) & (2)

$$\boxed{\Delta_1 y = -\frac{3.28PL}{AE}}$$

MATRIX METHOD (302)

Flexibility matrix method (force mtd)

Stiffness method (Displacement m

Properties of matrix If a matrix is [A]mxn. then it means are m rows and n columns. Flexibility matrix & Stiffness matrix will be alway a square matrix

(2) A unit matrix is that which has unit at element alonge the Diagonal.

(3) A Diagonal matrix is that which has all member zero other than digonal member.

(4) the magnitude of a matrix represented by its determinate.

(5) A determinate will have zero value. any two rows and any two column idential.

(6) the ~~sing~~ sign of determinate will charg if element of any two rows or any two column are interchange.

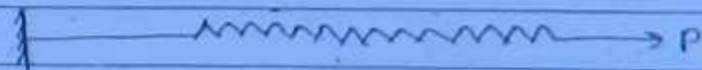
(7) The inverse of a matrix A is

$$[A]^{-1} = \frac{\text{Adj}\cdot[A]}{|A|}$$

303

Adj [A] matrix obtain by transpose of cofactor of matrix A.

Flexibility and Stiffness :-



$$\Delta = \frac{P}{K} \qquad \text{or} \qquad f = \frac{\Delta}{P}$$

$$K = \frac{P}{\Delta}$$

K → stiffness
f – flexibility.

Stiffness is equal to force required to produce unit displacement ~~is~~ flexibility is displacement produce by unit force.

Properties of flexibility and Stiffness matrix :-

→ f & K matrix always square matrix [m×m]

$$[A]_{n\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

→ The diagonal element will be always non-zero and non-negative i.e. always Positive.

→ The matric will be always symmetrical about the diagonal. it means the transpose of matrix will be unchange it is because of applicability of maxwell's Reciprocal Theorm.

304

$f_{12} = f_{21}$ or $K_{12} = K_{21}$

$f_{xy} = f_{yx}$ $\quad$ $K_{xy} = K_{yx}$

$a_{xy} = a_{yx}$

→ Procedure to developed flexibility matrix.

$$[f_n]_{n \times n} = \begin{bmatrix} f_{11} & f_{12} \text{ --- } f_{1n} \\ f_{21} & f_{22} \text{ --- } f_{2n} \\ \vdots & \\ f_{n1} & f_{n2} \text{ --- } f_{nn} \end{bmatrix}$$

f_{11} = Displacement in the direction ① due to unit force in the direction of ①

f_{21} ⇒ Displace. in the dir^n ② due to unit force in the direc ①

f_{xy} ⇒ " ——— " ——— ⓧ ——— " ——— " ——— ⓨ

f_{xy}

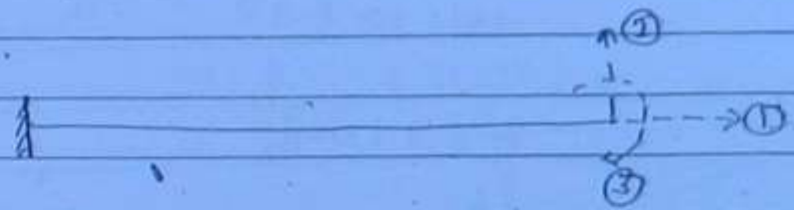
Direction of Disp. Measured $\quad$ Direction of unit force applied.

Step 1 To generenate first column element of flexibility matrix give / apply unit force in dir^n ① only and measure the ~~direction in~~ displacement in direction ①, ② - - - ⓝ

305

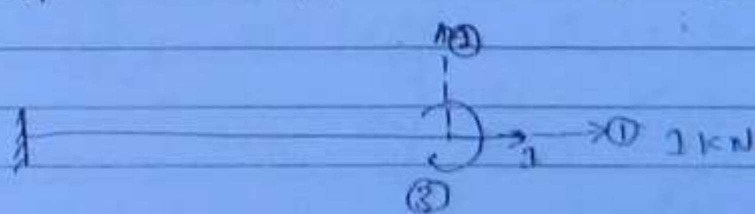
Step 2 To obtain second column apply unit force in dir^n ② only & repeat the procedure.

Q.



L, A, E, I constant

→ To develop first column of flexibility matrix apply unit force in direction ① only and measure the displacement in all the cordinate direction.



f_{11} = Displacement in dir ① due to unit force in the dir^n ①

→ $\frac{L}{AE}$

f_{21} = Disp. in dir ② due to unit force in dir ①

= 0

$f_{31} = 0$

→ To generate second colume of flexibility matrix apply unit force in dir^n of ② only and measured the displacement in all the cordinate direction.

$f_{12} = 0$ $\qquad f_{22} = \dfrac{L^3}{3EI}$ $\qquad f_{32} = -\dfrac{L^2}{2EI}$

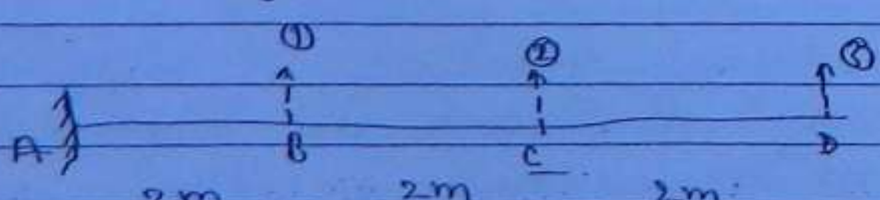
→ To generate 3 column of flexibility aply unit force in the dirⁿ ③

$f_{13} = 0$ $\qquad f_{23} = \dfrac{-L^3}{2EI}$ $\qquad f_{33} = \dfrac{L}{EI}$

$$[f] = \begin{bmatrix} L/AE & 0 & 0 \\ 0 & L^3/3EI & -\dfrac{L^2}{2EI} \\ 0 & -\dfrac{L^3}{2EI} & \dfrac{L}{EI} \end{bmatrix}$$

→ diagonal member are Non zero and non-negative.

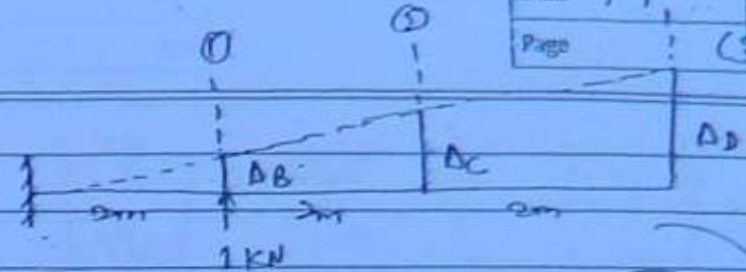
Q. Develop flexibility matrix for Cordinate shown in fig.



EI = Constant

Date / / Page 10/10

① first colum.



$f_{11} = \frac{1\cdot(2)^3}{3EI} \Rightarrow \frac{8}{3EI}$

307

$f_{21} = \cancel{\Delta_B + \theta_B \cdot L_{BC}}$

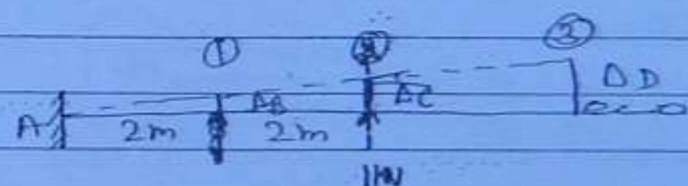
$\Rightarrow \frac{8}{3EI} + \frac{1\cdot(2)^2}{2EI} \times 2 \Rightarrow \frac{20}{3EI}$

$f_{31} = \Delta_D = \Delta_B + \theta_B \times L_{BD}$

$\Rightarrow \frac{8}{3EI} + \frac{1(2)^2}{2EI} \times 4 \Rightarrow \frac{32}{3EI}$

second column

②



$\cancel{f_{21} = \frac{1(4)^3}{3EI}}$

$f_{12} = \frac{20}{3EI} = f_{21}$

$f_{22} = \frac{PL^3}{3EI} \rightarrow \frac{1\ (4)^3}{3EI} \Rightarrow \frac{64}{3EI}$

$f_{32} \Rightarrow \frac{64}{3EI} + \frac{48}{3EI} \rightarrow \frac{112}{3EI}$

③ Third Collumn

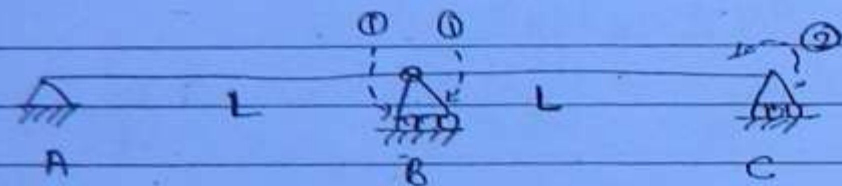
$f_{13} = \frac{32}{3EI}$ $f_{23} = \frac{112}{3EI}$

$f_{33} = \frac{1\ 6)^3}{\ } = \frac{216}{3EI}$

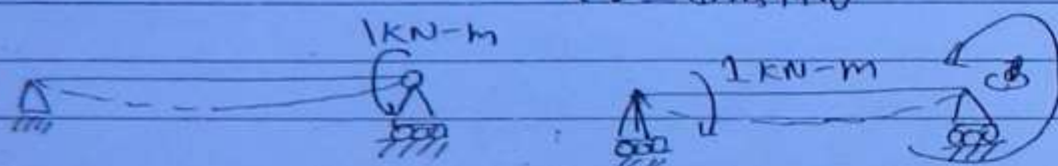
$$[f] = \frac{1}{3EI}\begin{bmatrix} 8 & 20 & 32 \\ 20 & 64 & 112 \\ 32 & 112 & 216 \end{bmatrix}$$

308

Q Developed flexibility matrix for the beam cordinate shown in fig.



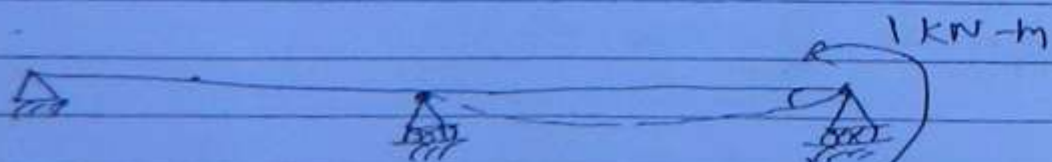
EI = constant



$\theta_{B_1} = \frac{L}{3EI}$ $\theta_{B_2} = \frac{L}{3EI}$ $\theta_C = \frac{L}{6EI}$

$f_{11} = \frac{L}{3EI} + \frac{L}{3EI} = \frac{2L}{3EI}$

$f_{21} = \frac{6L}{6EI}$



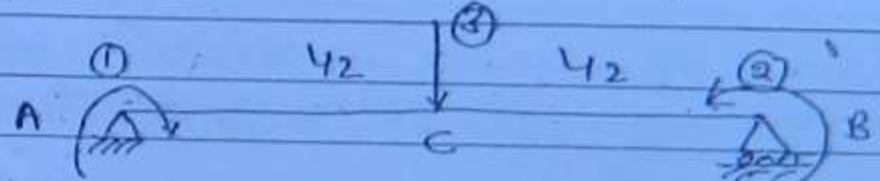
$\theta = \frac{L}{3EI}$ $f_{12} = \frac{L}{6EI}$ $f_{22} = \frac{L}{3EI}$

$$[f] = \begin{bmatrix} \frac{2L}{3EI} & \frac{L}{6EI} \\ L/6EI & L/3EI \end{bmatrix}$$

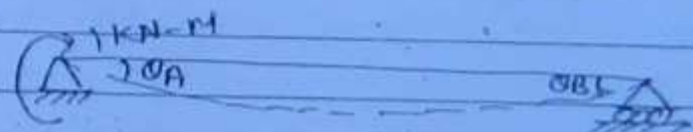
309

$$[f] = \frac{L}{6EI}\begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$$

Q Developed the flexibility matrix for the beam with cordinate shown in fig.



EI = Constant

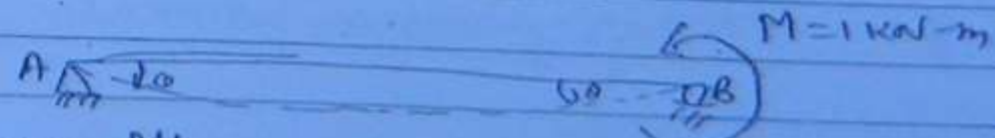


$A = \frac{ML}{3EI}$ $\theta_B = \frac{ML}{3EI}$

$f_{11} = \frac{L}{3EI}$

$f_{31} = \frac{L^2}{16EI}$

$f_{21} = L/6EI$



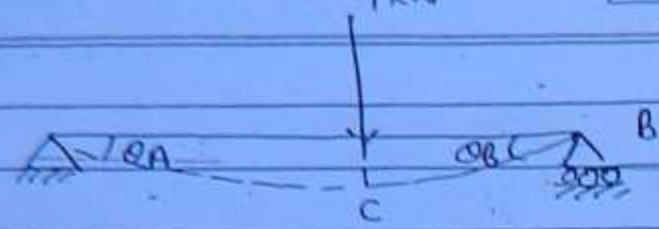
$\theta_A = \frac{ML}{6EI}$ $\theta_B = \frac{ML}{6EI}$

$f_{12} = \frac{L}{6EI}$

$f_{32} = \frac{L^2}{16EI}$

$f_{22} = \frac{L}{3EI}$

Page



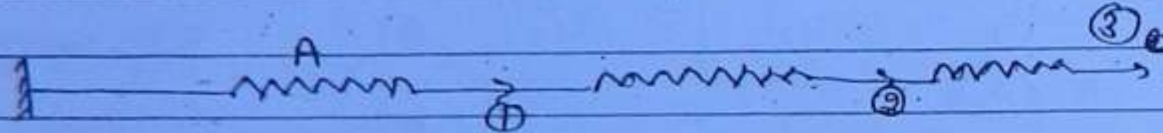
$\theta_A = \frac{L^3}{16EI}$ $\qquad \theta_B = \frac{L^3}{16EI}$ $\qquad \delta_C = \frac{L^3}{48EI}$

$f_{13} = \frac{L^2}{16EI}$ $\qquad f_{23} = \frac{L^2}{16EI}$

310

$f_3 = \frac{L^3}{48EI}$

Q. ~~Develop~~ flexibility matrix method for the spring system shown in fig:



$f_A = 0.05$ cm/kN

$f_B = 0.10$ "

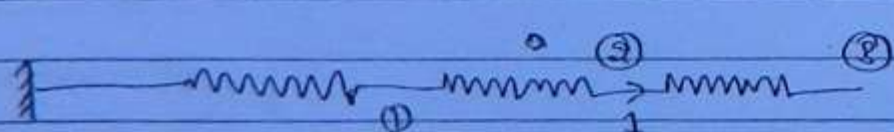
$f_C = 0.20$ "

$f = \frac{\Delta}{P} \Rightarrow \Delta = f \times P$

$f_{11} = f_A \times 1 \Rightarrow 0.05$ cm.

$f_{21} = \delta_2 = \delta_1 = 0.05$ cm

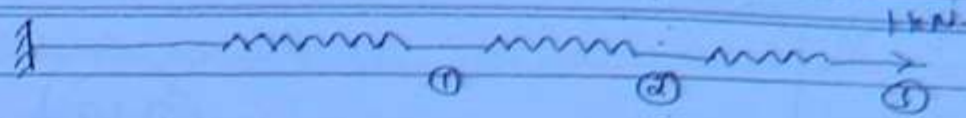
$f_{31} = \delta_3 = \delta_1 = \delta_2 \Rightarrow 0.05$ cms.



$f_{12} = 0.05$ cm

$f_{22} = 0.05 + 0.10 \Rightarrow 0.15$ cm

$f_{32} = 0.15$ cms



$f_{13} = 0.05$ cm/kN

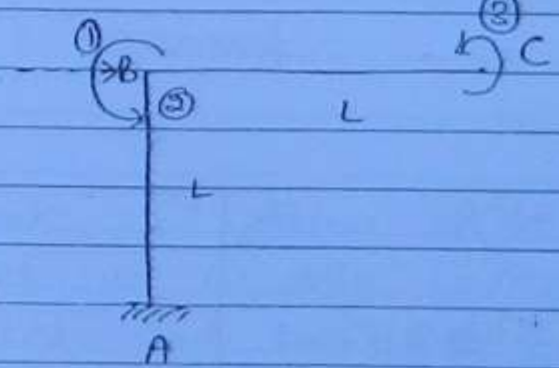
$f_{23} = 0.05 + 0.10 = 0.15$

$f_{33} = 0.05 + 0.10 + 0.20 = 0.35$

③,1

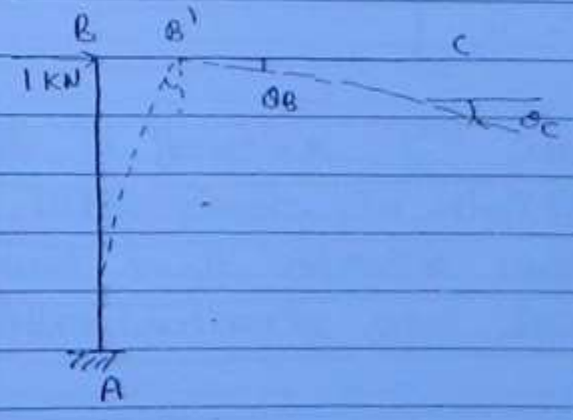
$$[f] = \begin{bmatrix} 0.05 & 0.05 & 0.05 \\ 0.05 & 0.15 & 0.15 \\ 0.05 & 0.15 & 0.35 \end{bmatrix}$$

Q. Develop flexibility matrix for cantilever beam with cordinate marked in fig.



EI = Constant

Sol:-

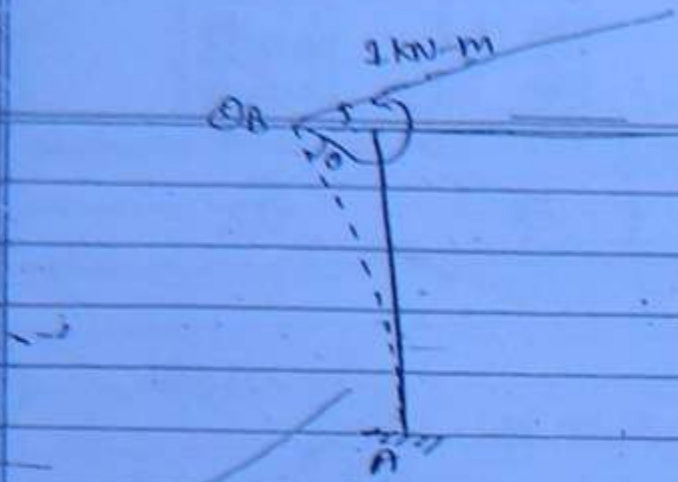


$f_{11} = BB' = \frac{L^3}{3EI}$

$f_{21} = \frac{-L^2}{2EI}$

$f_{31} = \frac{-L^2}{2EI}$

Date / /
Page



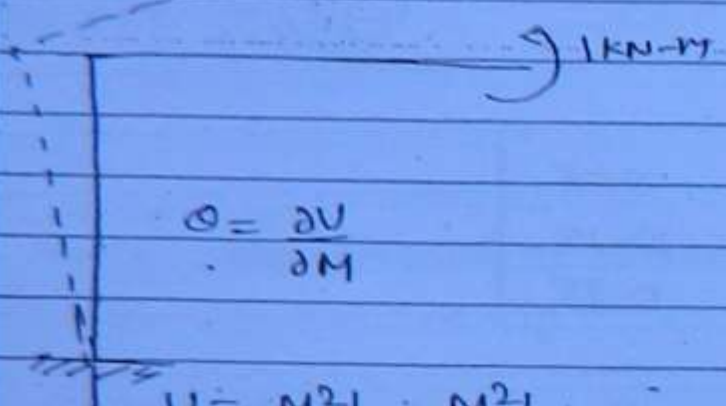
C $\theta_B = \frac{L}{EI}$

3/2

$f_{12} = \frac{-L^2}{2EI}$

$f_{22} = L/EI$

$f_{32} = \frac{L}{EI}$



$F_{13} = \frac{-L^2}{2EI}$

$\theta = \frac{\partial U}{\partial M}$

$F_{23} = \frac{L}{EI}$

$U = \frac{M^2 L}{2EI} + \frac{M^2 L}{2EI}$

$F_{33} = \frac{2L}{EI}$

$\Rightarrow \frac{2M^2 L}{2EI}$

$\frac{\partial U}{\partial M} = \frac{2ML}{EI} \Rightarrow \frac{2L}{EI}$

$$[f] = \frac{L}{EI}\begin{bmatrix} L^3/3 & -L^2/2 & -L^2/2 \\ -\frac{L^2}{2} & 1 & 1 \\ -L^2/2 & 1 & 2 \end{bmatrix}$$

Stiffness matrix

Procedure to develop Stiffness matrix →

$$K = \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & & & \vdots \\ K_{n1} & & & K_{nn} \end{bmatrix}$$

K_{11} = Force dev / req. at (1) due to unit disp. at (1) only
K_{21} = " " (2) " " (1)

K_{xy} = Force dev. / req at (x) due to unit disp. at (y) only

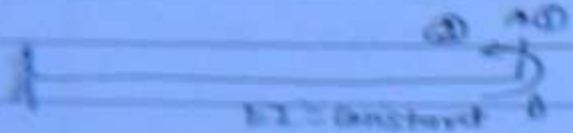
K_{xy}: x → Direction of force measured; y → Direction of unit disp.

To generate 1st column of stiffness matrix give unit displacement at (1) only it means all other cordinate are locked. and measure force devlop / requrid in other cooridinate direction.

To generate 2nd column of stiffness give unit displacement in dir^n (2) only and measure the forces in all other cordinate direction. when all other cordinate are locked.

Q. develop stiffness matrix for beam AB with coordinate shown in fig.

sol→



(3/4)

$$K_{11} = R_B = \frac{12EI}{L^3}$$

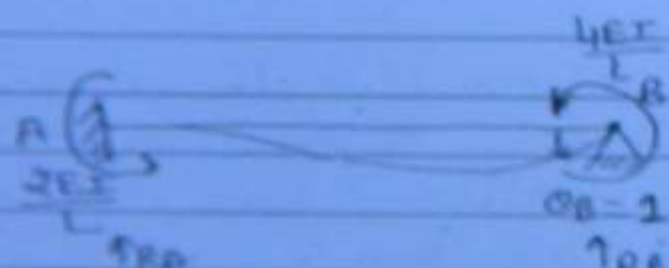
$$K_{21} = \frac{-6EI}{L^2}$$

$\Sigma M_A = 0$

$$-R_B \times L + \frac{6EI}{L^2} + \frac{6EI}{L^2} = 0$$

$$R_B = \frac{12EI}{L^3}$$

2nd



$\Sigma M_A = 0$

$$-R_B \times L - \frac{4EI}{L} - \frac{2EI}{L}$$

$$R_B = \frac{-6EI}{L^2}$$

$$K_{12} = R_B = \frac{-6EI}{L^2}$$

$$K_{22} = \frac{+4EI}{L}$$

$$[K] = \frac{EI}{L}\begin{bmatrix} 12/L^2 & -6/L \\ -6/L & 4 \end{bmatrix}$$

Ans

Q: Generate stiffness matrix for the beam with coordinate shown in fig.

(315)

Sol:

$K_{11} = \frac{4EI}{L}$

$K_{21} = \frac{-2EI}{L}$

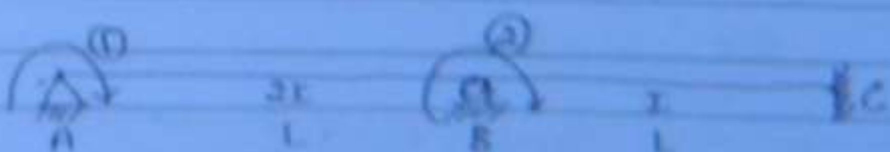
$K_{12} = \frac{-2EI}{L}$

$K_{22} = \frac{4EI}{L}$

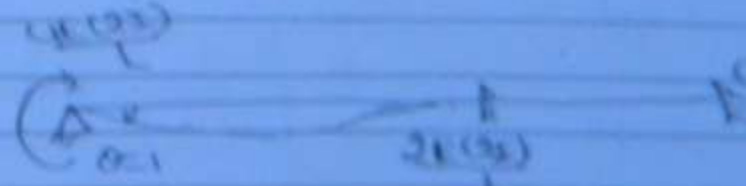
$$[k] = \frac{EI}{L}\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$$ Ans.

$$k = \frac{2EI}{L}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Q: develop stiffness matrix for a beam shown in fig.

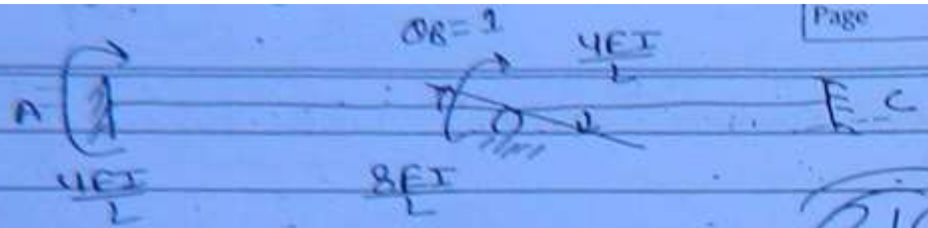


Sol:



$K_{11} = 8EI/L$

$K_{21} = 4EI/L$



316

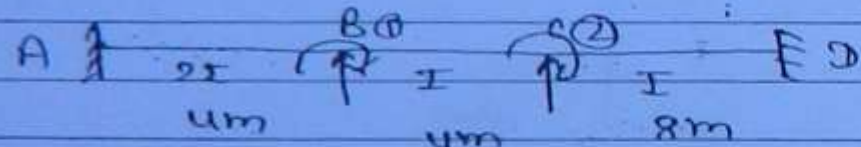
$K_{12} = 4EI/L$

$$K_{22} = \frac{8EI}{L} + \frac{4EI}{L} = \frac{12EI}{L}$$

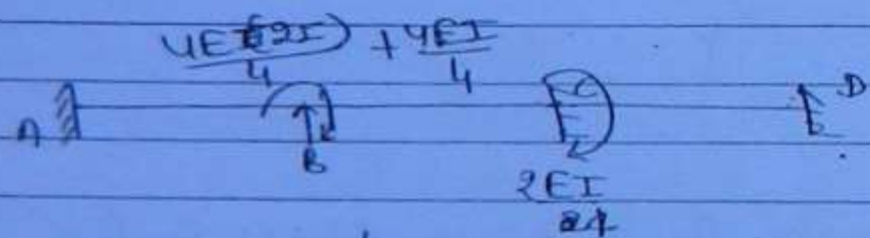
$$K = \begin{bmatrix} 8EI/L & 4EI/L \\ 4EI/L & 12EI/L \end{bmatrix}$$

$$K = \frac{4EI}{L}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

Q Develop stiffness matrix for beam shown in fig.

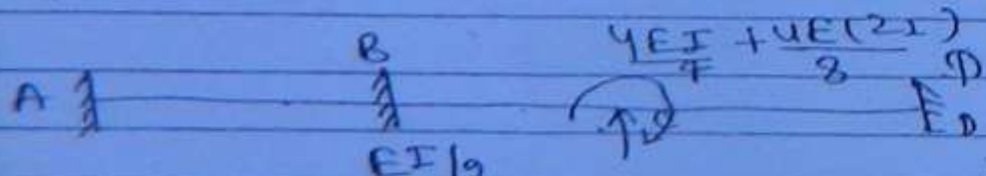


Sol



$K_{11} = 2EI + EI = 3EI$

$K_{12} = \frac{EI}{2} = 0.5EI$



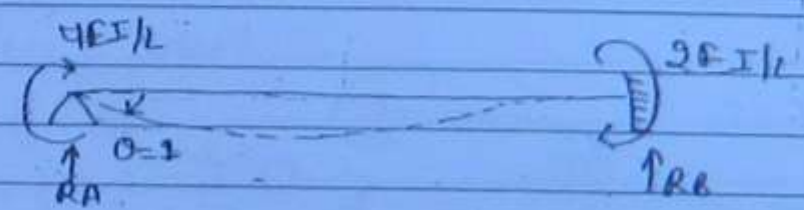
$K_{12} = \frac{EI}{2} = 0.5EI$

$K_{22} = 2EI$

$$K = EI\begin{bmatrix} 3 & 0.5 \\ 0.5 & 2 \end{bmatrix}$$ Ans

Q. Devlop Stiffness metrix for the Beam with cordinate shown in fig.

Sol)

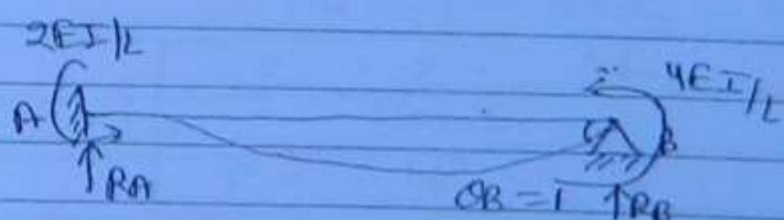


$\Sigma M_B = 0$

$$R_A \times L + \frac{4EI}{L} + \frac{2EI}{L} = 0$$

$$R_A = \frac{6EI}{L^2} \qquad R_A + R_B = 0 \qquad R_B = +\frac{6EI}{L^2}$$

$$K_{11} = \frac{4EI}{L} \qquad K_{21} = \frac{-2EI}{L} \qquad K_{31} = \frac{-6EI}{L^2} \quad K_{41} = +\frac{6EI}{4}$$



$$K_{12} = \frac{-2EI}{L} \qquad K_{22} = \frac{4EI}{L} \qquad K_{32} = \frac{6EI}{L^2} \quad K_{42} = \frac{-6EI}{L^2}$$

$$[K] = \frac{EI}{L}\begin{bmatrix} 4 & -2 & -6 & 6 \\ -2 & 4 & 6 & -6 \end{bmatrix}$$

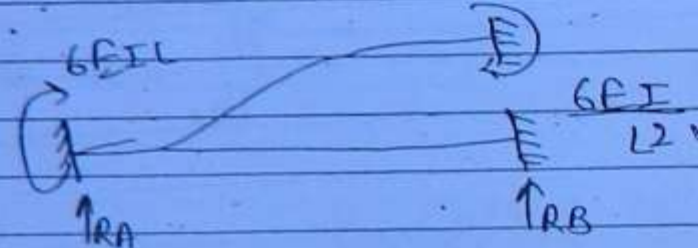
$$[K] = \frac{2EI}{L}\begin{bmatrix} 2 & -1 & -3 & 3 \\ -1 & 2 & 3 & -3 \end{bmatrix}$$

(318)

$\Sigma M_B = 0$ $\quad R_A = \frac{12EI}{L^3}$ $\quad R_A = \frac{-12EI}{L^3}$

$K_{13} = -\frac{6EI}{L^2}$ $\quad K_{23} = +\frac{6EI}{L^2}$ $\quad K_{33} = \frac{12EI}{L^3}$

$K_{43} = \frac{-12EI}{L^3}$



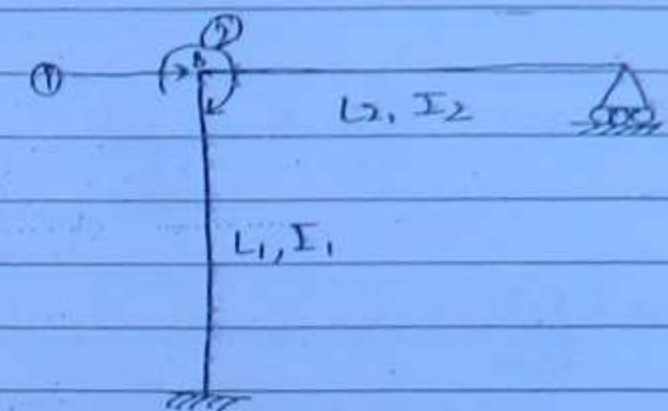
$R_A = -12EI/L^3$

$R_B = \frac{12EI}{L^3}$

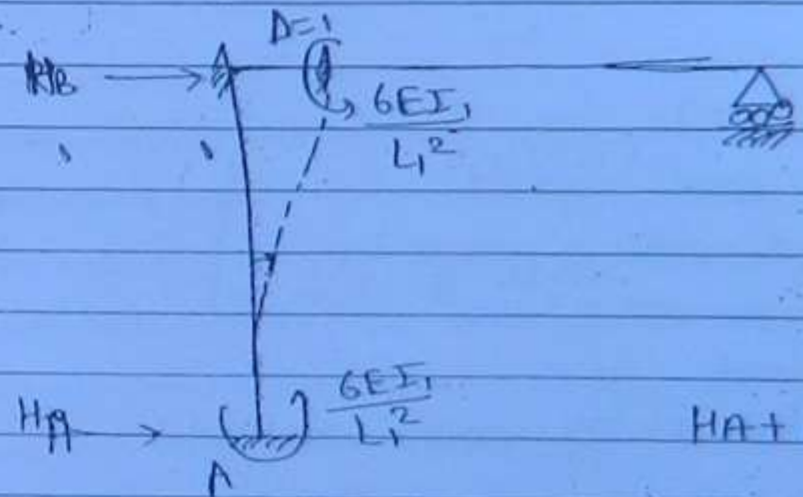
$K_{14} = \frac{+6EI}{L^2}$ $\quad K_{24} = \frac{-6EI}{L^2}$ $\quad K_{34} = \frac{-12EI}{L^3}$ $\quad K_{44} = \frac{+12EI}{L^3}$

$$K = \frac{2EI}{L}\begin{bmatrix} 2 & -1 & -3 & 3 \\ -1 & 2 & 3 & -3 \\ \frac{-3}{L} & \frac{3}{L} & \frac{6}{L^2} & \frac{-6}{L^2} \\ \frac{3}{L} & \frac{-3}{L} & \frac{-6}{L^2} & \frac{6}{L^2} \end{bmatrix}$$

Q. Devlop stiffness Matrix for the frame Shown in fig. with Cordinate indicated.



319



$$H_A + H_B = 0$$

$$\Sigma M_B = 0$$

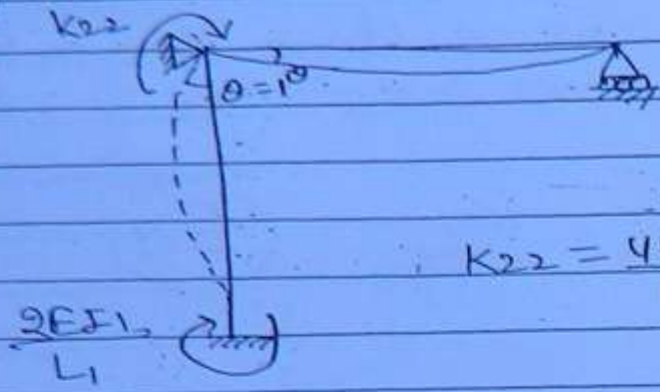
$$-H_A \times L_1 - \frac{6EI_1}{L_1^2} - \frac{6EI_1}{L_1^2}$$

$$H_A = \frac{-12EI_1}{L_1^3}$$

$$H_B = \frac{+12EI_1}{L_1^3}$$

$$K_{11} = \frac{12EI_1}{L_1^3} \qquad K_{22} = \frac{-6EI_1}{L_1^2}$$

to generate 2^{nd} column:-



320

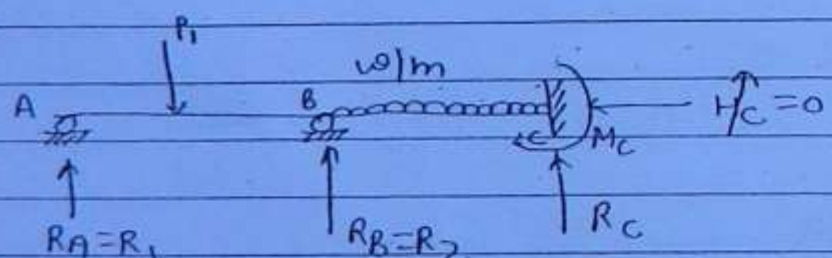
$$K_{22} = \frac{4EI_1}{L_1} + \frac{3EI_2}{L_2}$$

$$K_{21} = \frac{-6EI_1}{L_1^2}$$

$$K_{22} = \frac{4EI_1}{L_1} + \frac{3EI_2}{L_2}$$

$$K = \begin{bmatrix} \frac{12EI_1}{L_1^3} & -6EI_1/L_1^2 \\ -6EI_1/L_1^2 & \frac{4EI_1}{L_1} + \frac{3EI_2}{L_2} \end{bmatrix}$$

Flexibility matrix method.



Step-1 determine degree of static indetermincy in beam neglected axial forces.

$$D_s = D_e + D_{si}^{\,0}$$
$$= r_e - 3$$
$$\Rightarrow 5 - 3 = 2$$

in above structure $D_s = 2$ Hence identify Redudent Say Redundant are R_A & R_B (R_e).
(R_1)

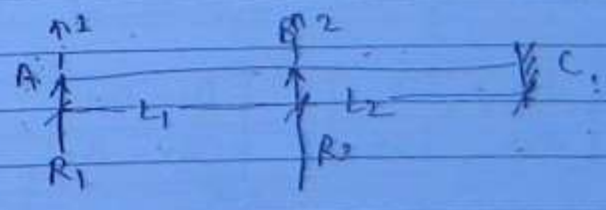
Step 2 Remove the redundent and assign one Coordinate in the direction of each Redundent the No. of Co-orident will be equal to r_s that is no. of Redundent.

327

develop flexibility matrix for the above assign Co-ordinates.

Step 3 Remove the Redundent and find the deflections in the direction of assign ~~Cordi~~ Coordinates due to given load system. when the Redundent are release, the str. are determinate which is called release structure basic determinate Structure. let due to given loading the deflections in the direction of Co-ordinate are Δ_{1L}, Δ_{2L} - - - -

Step-4 Remove the given loading and apply the Redudent Reaction in the direction of assigned Co-ordinate.



And find displacement in the direction of co-ordinate due to Redundent let Δ_{1R}, Δ_{2R} --- --- Δ_{nR}. R the displacements cause by Redundent in the direction of co-ordinate.

$$\Delta_{1R} = f_{11}R_1 + f_{12}R_2$$
$$\Delta_{2R} = f_{21}R_1 + f_{22}R_2$$

322

$$\begin{bmatrix} \Delta_{1R} \\ \Delta_{2R} \end{bmatrix} = \begin{bmatrix} f \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

$$\begin{bmatrix} \Delta_{1R} \\ \Delta_{2R} \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

ep-5 the final displacement in the direction of co-ordinate will be $\Delta_1 = \Delta_{1L} + \Delta_{1R}$

$$\Delta_2 = \Delta_{2L} + \Delta_{2R}$$

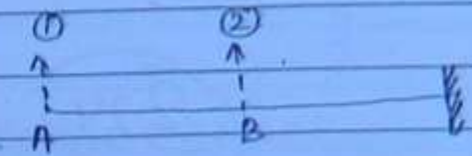
but final displacement are

$$\Delta_{1R} = \cancel{\Delta_1}^{0} - \Delta_{1L}$$
$$\Delta_{2R} = \cancel{\Delta_2}^{0} - \Delta_{2L}$$

$$\begin{bmatrix} \Delta_{1R} \\ \Delta_{2R} \end{bmatrix} = \begin{bmatrix} \Delta_1 - \Delta_{1L} \\ \Delta_2 - \Delta_{2L} \end{bmatrix} \Rightarrow \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} \Delta_1 - \Delta_{1L} \\ \Delta_2 - \Delta_{2L} \end{bmatrix}$$

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} \Delta_1 - \Delta_{1L} \\ \Delta_2 - \Delta_{2L} \end{bmatrix}$$

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = [f]^{-1} \left[\overset{0}{\Delta} - \Delta_L \right]$$

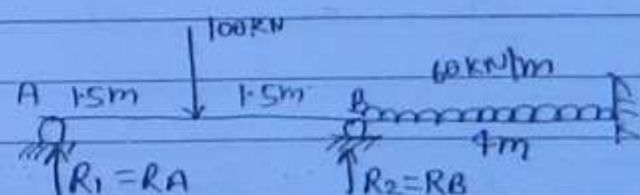


323

f_{11} = displacement at ① due to unit force at ①
f_{12} = displacement at ① due to unit force at ②

$$\Delta_{1R} = f_{11} \cdot R_1 + f_{12} \cdot R_2$$
$$\Delta_{2R} = f_{21} \cdot R_1 + f_{22} \cdot R_2$$

Ex. Analysis the Beam shown in fig. find the Reaction using flexibility matrix method.

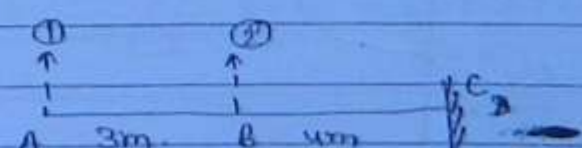


EI = Constant

$D_S = r_e - 3 \Rightarrow 5 - 3 = 2$

Let Redundent are R_A & R_B.

Assign one coordinate in dirn of each Redundent.

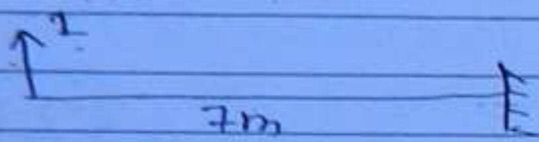


1st Column of flexibility matrix

$$f_{11} = \frac{l^3}{3EI} \Rightarrow \frac{7^3}{3EI} = \frac{343}{3EI}$$

$$f_{21} = \frac{136}{3EI}$$

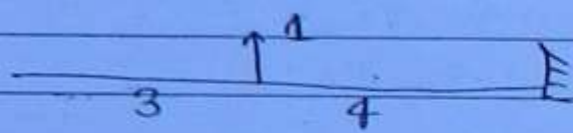
1st column of flexibility matrix



324

$$f_{11} = \frac{1 \cdot 7^3}{3EI} \rightarrow \frac{343}{3EI}$$

2nd column of flexibility matrix



$$f_{12} = \frac{1 \cdot 4^3}{3EI} + \frac{1 \cdot 4^2}{2EI} \times 3$$

$$\Rightarrow \frac{64}{3EI} + \frac{24}{EI} \rightarrow$$

$$f_{22} = \frac{1 \cdot 4^3}{3EI} = \frac{64}{3EI}$$

$$[f] = \begin{bmatrix} \frac{343}{3EI} & \frac{136}{3EI} \\ \frac{136}{3EI} & \frac{64}{3EI} \end{bmatrix}$$

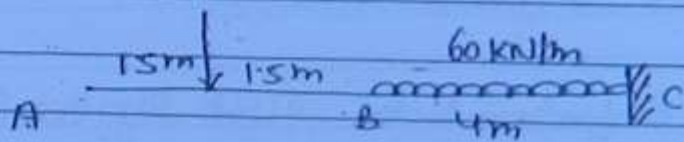
$$[f]^{-1} = \begin{bmatrix} \frac{64}{3EI} & -\frac{136}{3EI} \\ -\frac{136}{3EI} & \frac{343}{3EI} \end{bmatrix}$$

$$\rightarrow \frac{343}{3EI} \times \frac{64}{3EI} - \left(\frac{136}{3EI}\right)^2$$

$$[f]^{-1} = EI \begin{bmatrix} 0.055 & -0.118 \\ -0.118 & 0.297 \end{bmatrix}$$

325

in Basic Relase structure find Δ_{1L} and Δ_{2L} due to given loading in the direction of assign Co-ordinate.



$$\Delta_{1L} = -\left[\frac{100(5.5)^2}{3EI} + \frac{100(5.5)^2}{2EI} \times 1.5 + \frac{60 \times (4)^3}{8EI} + \frac{60 \times 4^3}{6EI} \times 3 \right]$$

$$\Delta_{1L} = \frac{-11654.58}{EI}$$

$$\Delta_{2L} = -\left[\frac{100 \times (4)^3}{3EI} + \frac{100(4)^2}{2EI} \times 1.5 + \frac{60 \times 4^4}{8EI} \right]$$

$$\Delta_{2L} = -\frac{5253.33}{EI}$$

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = [f]^{-1} \begin{bmatrix} 0 - \Delta_{1L} \\ 0 - \Delta_{2L} \end{bmatrix}$$

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} 0.055EI & -0.118EI \\ -0.118EI & 0.297EI \end{bmatrix} \begin{bmatrix} 0 + 11654.58/EI \\ 0 + \frac{5253.33}{EI} \end{bmatrix}$$

$$R_1 = 0.055\,EI \times \frac{11654.58}{EI} + (0.118\,EI) \times \frac{5253.33}{EI}$$

$R_1 = 21.11$ kN

326

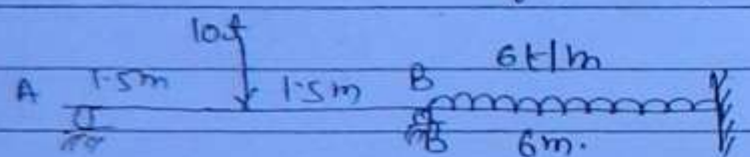
$R_2 = 185$ kN

Stiffness matrix method→

<u>Step-1</u>:- Neglecting Axial deformation find degree of Kinematic Indetermancy

$$D_K = 3J - r_e - m \Rightarrow 3\times 3 - 5 - 2 = 9 - 7 = 2$$

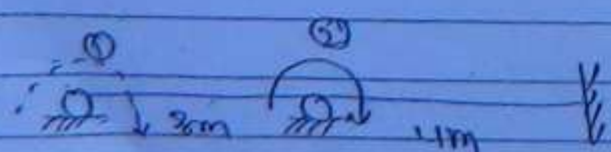
(Axial rigid member)



$D_K = 2$ there are two independent displacement i.e. degree of freedom.

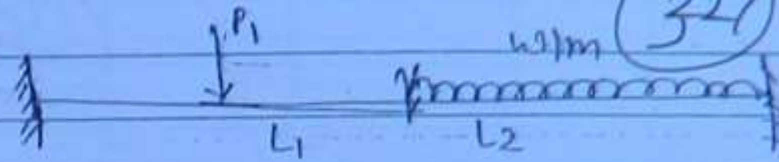
assign one Co-ordinate in the direction of each displacement hence No. of Co-ordinate will be equal to D_K.

<u>Step-2</u> Find stiffness matrix for the assign Co-ordinates and determine $[K]^{-1}$



Step 3 Let P_1' and P_2' are forces of the moment required / develop at cordinate ① & ② to lock the structure due to given loading

327



$$\bar{M}_{AB} = P_1' = -\frac{PL_1}{8}$$

$$\bar{M}_{BA} + \bar{M}_{BC} = P_2' = +\frac{PL_1}{8} - \frac{wl_2^2}{12}$$

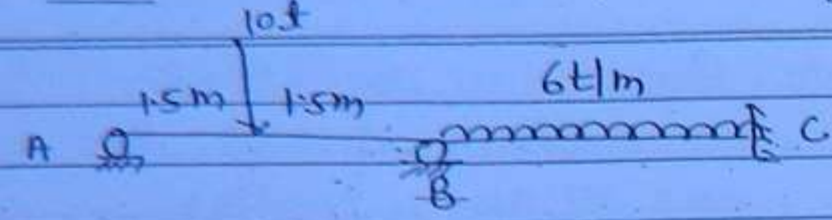
Let P_1, P_2 ------ are final forces / moment available in the direction of assign co-ordinate. if No external load or moment acts in direction of origin co-ordinate, then final value P_1 and $P_2 = 0$

$$\boxed{\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} - \begin{bmatrix} K_1^{\circ} \end{bmatrix}^{-1} \begin{bmatrix} \cancel{P_1}^{0} - P_1' \\ \cancel{P_2}^{0} - P_2' \end{bmatrix}}$$

P_1' and P_2' Nothing that fixed end moment at co-ordinate ① and ② when structure is locked due to given loading.

If co-ordinates are linear then P' and P_2' will be Reactions due to given loading in the direction of co-ordinate.

Q.

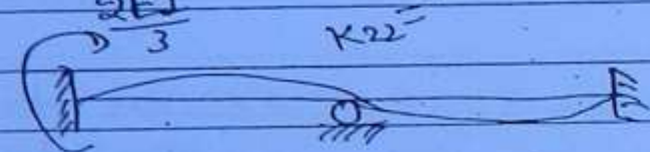


328

$K_{11} = \frac{4EI}{3}$

$K_{21} = \frac{2EI}{3}$

$K_{22} = \frac{4EI}{3} + \frac{4EI}{4} = \frac{7EI}{3}$



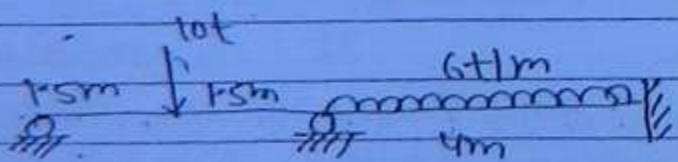
$K_{21} = 2EI/3$

$K_{22} = \frac{7EI}{3}$

$$[K] = \begin{bmatrix} \frac{4EI}{3} & \frac{2EI}{3} \\ 2EI/3 & \frac{7EI}{3} \end{bmatrix}$$

multiply

$$K^{-1} = \begin{bmatrix} 7/8EI & -1/4EI \\ -1/4EI & 1/2EI \end{bmatrix}$$

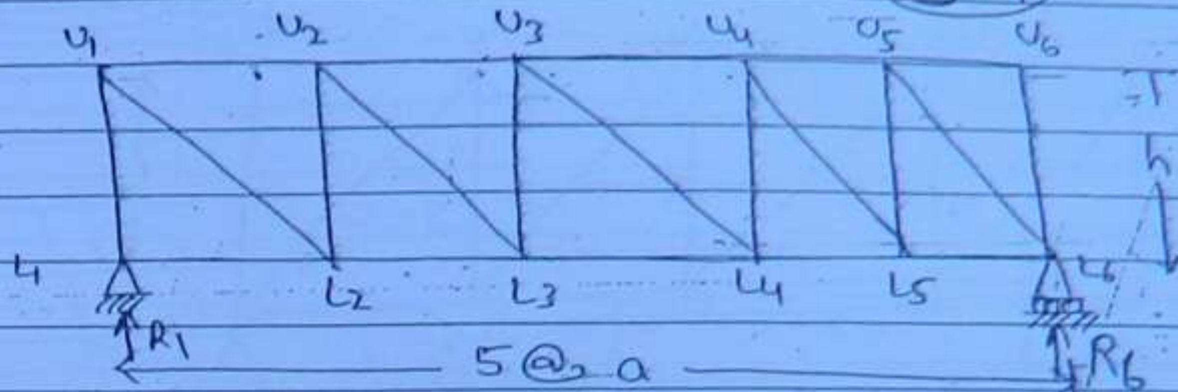


$P_1' = \bar{M}_{AB} = -\frac{10 \times 3}{8} \Rightarrow -3.75$

$P_2' = \bar{M}_{BA} + \bar{M}_{BC} = +3.75 - \frac{6 \times 4^2}{12}$

$\Rightarrow 3.75 - 8 \Rightarrow -4.25$

ILD for Truss



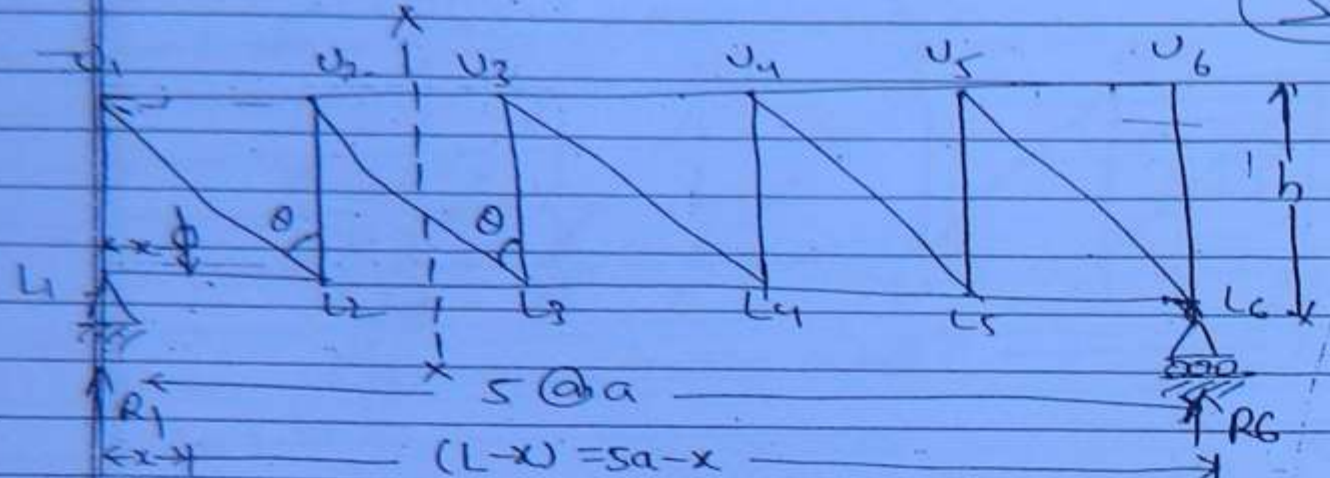
→ In simply supported trusses when unit load moves from ① end to the other end then top chord member are in comp. Bottom chord are in tension And in inclined member force change ~~in~~ ~~the~~ for comp to tension.

When unit load is left of member than force is inclined member is compression and when unit load is right of the member than force is tension. in vertical members force changes tension to comp.

if load moves over top chord then truss is Called Deck type truss and If load moves over bottom Chord then truss is Called through type

Page 330

Q. Draw ILD for U_2U_3, L_2L_3 and L_2 and L_3



consider unit load place at a distance x

$L = 5a$

$$\Sigma M_6 = 0$$

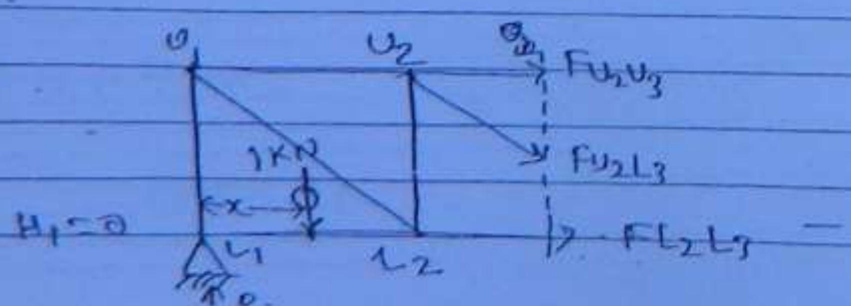
$$R_1 \times L - 1(L-x) = 0$$

$$R_1 = \frac{L-x}{L} \rightarrow \frac{5a-x}{5a}$$

$$R_6 = \frac{x}{L} = \frac{x}{5a}$$

ILD for L_2L_3

Case-1 When unit load is to the left of ~~L_2~~ U_2 U_2L_2. Consider free body dia. to the left of x-x



To find F in L_2L_3 take moment about U_2

$$\Sigma M_{U_2} = 0$$

$$R_1 \times a - 1 \times (a-x) - F_{L_2L_3} \cdot h = 0$$

$$F_{L_2L_3} = \frac{R_1 \cdot a - (a-x)}{h}$$

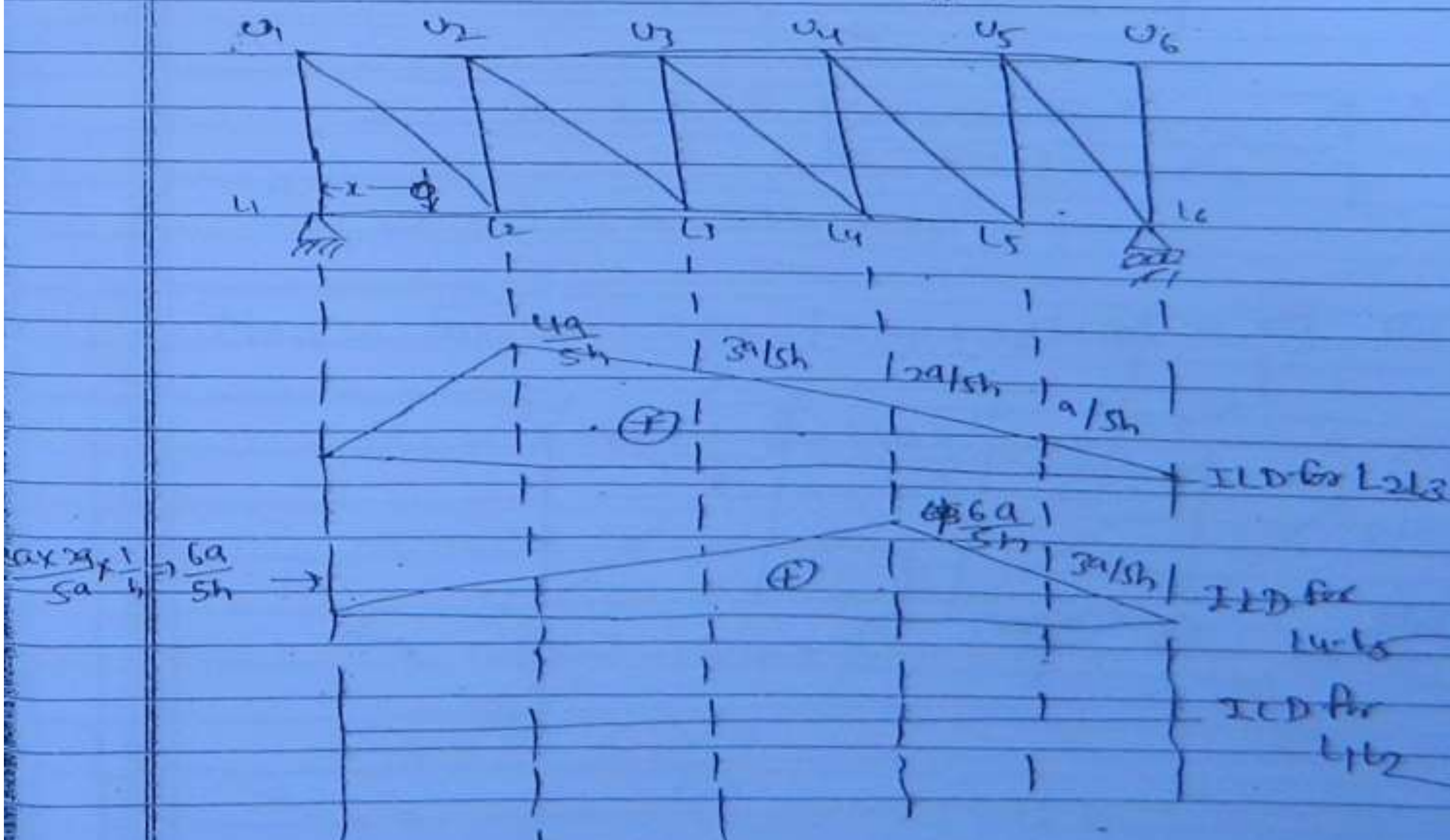
$$F_{L_2L_3} = \frac{\left[\frac{5a-x}{5\cancel{a}} \cdot \cancel{a}\right] - (a-x)}{h}$$

$$F_{L_2L_3} = \frac{5a - x - 5a + 5x}{5h} \Rightarrow$$

$$\boxed{F_{L_2L_3} = \frac{4x}{5h}}$$

when $x = 0$, $F_{L_2L_3} = 0$

when $x = a$ $F_{L_2L_3} = \frac{4a}{5h}$ (+) Tension.



Page 10

Case-II when unit load is in $L_2 L_6$

332

$$\Sigma M_{U_2} = 0$$

$$R_1 \times a - F_{L_2L_3} \times h = 0$$

$$F_{L_2L_3} = \frac{R_1 \cdot a}{h}$$

$$F_{L_2L_3} = \left[\left(\frac{5a-x}{5a}\right)a\right]\frac{1}{h}$$

$$F_{L_2L_3} \Rightarrow \frac{(5a-x)}{5h}$$

$\left(\frac{a_1 \times a_2}{a_1 + a_2}\right) \times \frac{1}{h}$

Revise formula

$\frac{a \times 4a}{a + 4a} \times h$

$\Rightarrow \frac{4a^2}{5ah} \Rightarrow \frac{4a}{5h}$

if $x = a$

$$F_{L_2L_3} = \frac{4a}{5h}$$

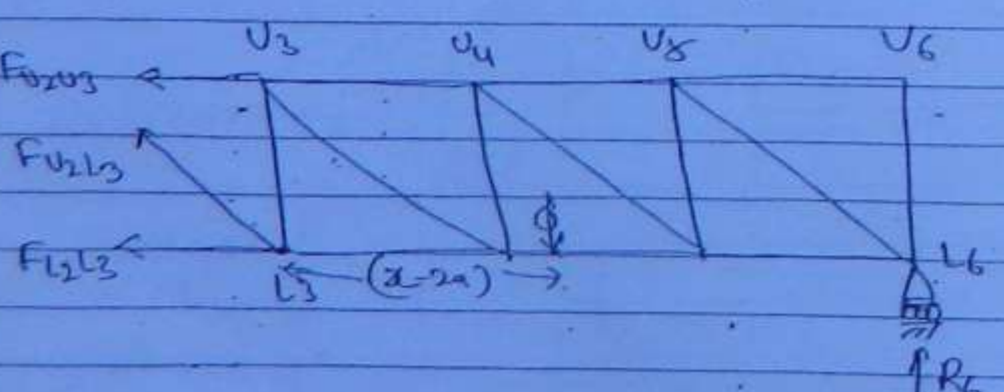
if $x = 2a$

$$F_{L_2L_3} = \frac{3a}{5h}$$

if $x = 0$ $\quad x = 5a$

$$F_{L_2L_3} = 0$$

ILD for $U_2 U_3$



333

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} K \end{bmatrix} \begin{bmatrix} \cancel{P_1}^{=0} - P_1' \\ \cancel{P_2}^{=0} - P_2' \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} 7/8EI & -1/4EI \\ -1/4EI & 1/2EI \end{bmatrix} \begin{bmatrix} 0 + 3.75 \\ 0 + 4.25 \end{bmatrix}$$

$$\Delta_1 = \theta_A = +\frac{2.22}{EI}$$

$$\Delta_2 = \theta_B = +\frac{1.187}{EI}$$

To find final end moment write slope deflection equations.

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{3}\left[2\theta_A + \theta_B - \frac{3\cancel{\Delta}^{=0}}{L}\right] \rightarrow ①$$

$$\Rightarrow -3.75 + \frac{2EI}{3}\left[\frac{2\times 2.22}{EI} + \frac{1.187}{EI}\right]$$

$$M_{AB} \Rightarrow -3.75 + 3.75 = 0$$

$$M_{BA} = \bar{M}_{BA} + \frac{2EI}{L}\left[2\theta_B + \theta_A - \frac{3\cancel{\Delta}^{=0}}{L}\right]$$

$$\rightarrow +3.75 + \frac{2EI}{3}\left[\frac{2\times 1.187}{EI} + \frac{2.22}{EI}\right]$$

$$\rightarrow +6.81 \text{ t-m}$$

$$M_{BC} = \bar{M}_{BC} + \frac{2EI}{4}\left[2\theta_B + \cancel{\theta_C}^{0} - \frac{3\cancel{\Delta}^{=0}}{L}\right]$$

$$\rightarrow -8 + EI \times \frac{1.187}{EI}$$

$$\rightarrow -6.81 \text{ tm}$$

$$M_{CB} = +8.55$$

Page

Chepter= 6

1. c
2. d
3. a
4. d
5. c
6. c
7. b
8. A
9. b
10. a
11. ~~d~~a
12. c
13. ~~a~~ d.c

334

335

Case 1 when unit load is in L_1, L_3

$$\Sigma M_{L_3} = 0$$

$$F_{U_2U_3} \times h + R_6 \times 3a = 0$$

$$F_{U_2U_3} = -R_6 \times \frac{3a}{h} \Rightarrow -\frac{x}{5a} \times \frac{3a}{h}$$

$$\Rightarrow -\frac{3}{5}\frac{x}{h}$$

case-2 when load is in $L_3 L_6$

$$\Sigma M_3 = 0$$

$$R_6 \times 3a + F_{U_2U_3} \times h - 1(x - 2a) = 0$$

$$F_{U_2U_3} = \frac{-R_6 \times 3a + (x - 2a)}{h}$$

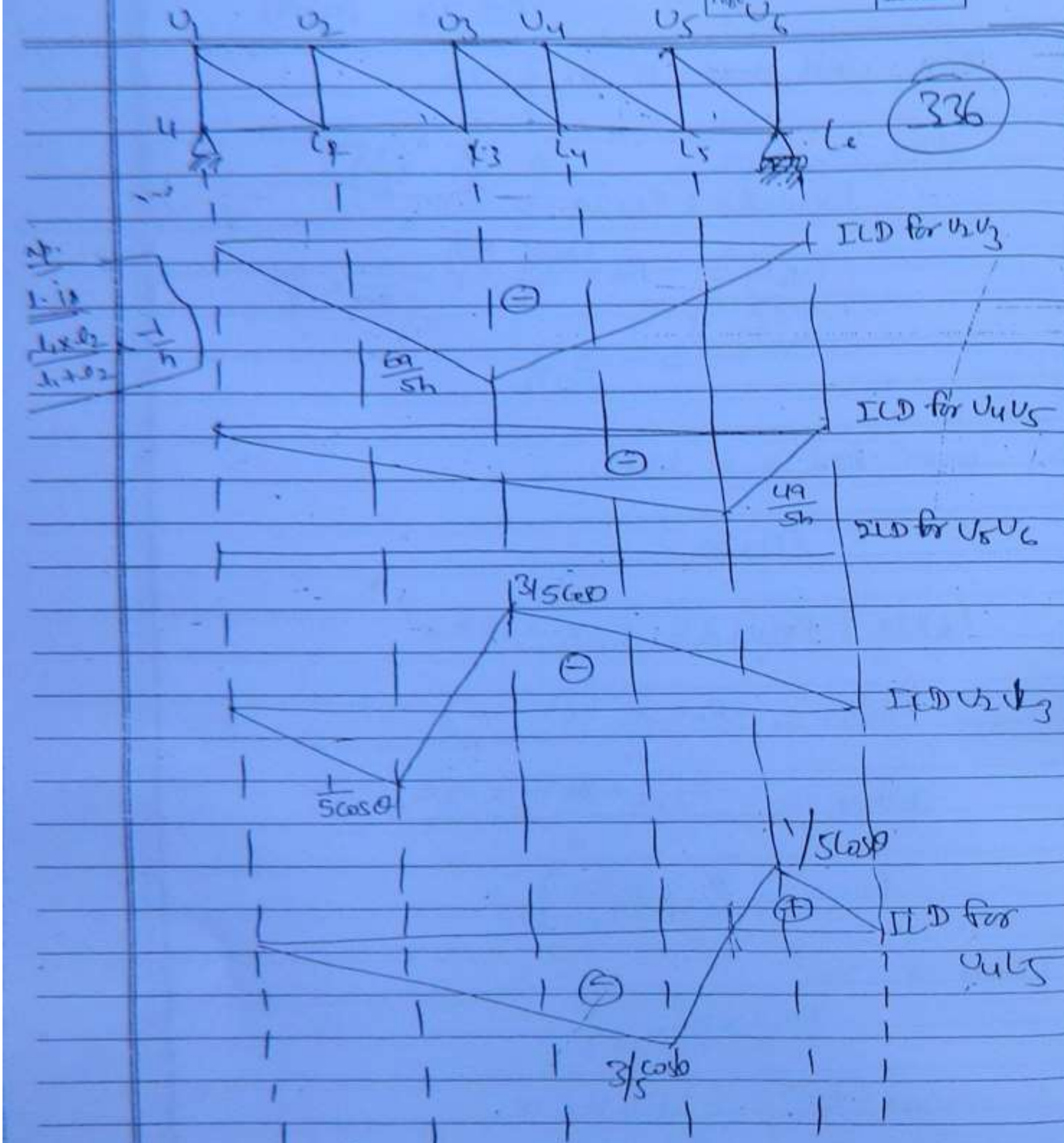
$$F_{U_2U_3} = \frac{-x/5a \times 3a + x - 2a}{h}$$

$$F_{U_2U_3} \Rightarrow \frac{2}{5}\left(\frac{x - 5a}{h}\right)$$

When $x = 2a$

$$F_{U_2U_3} = -\frac{6}{5}\frac{a}{h}$$

when $x = 5a$ $\quad F_{U_2U_3} = 0$

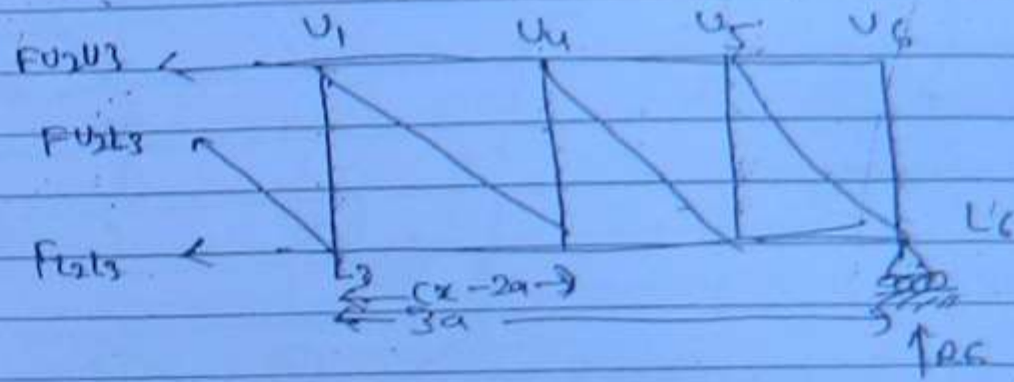

U1
U2
U3
U4
U5
U6
L1
L2
L3
L4
L5
L6
ILD for U2U3
69/5h
ILD for U4U5
49/5h
ILD for U5U6
3/5cosθ
ILD U2U3
1/5cosθ
1/5cosθ
ILD for U4L5
3/5cosθ

in
ILD↑ U_2L_3 :-

Inclined member makes angle θ with the vertical

(337)

Case-1 when unit load is in L_1L_2



$\Sigma F_y = 0$

$$F_{U_2U_3}\cos\theta + R_6 = 0$$

$$F_{U_2U_3} = \frac{-R_6}{\cos\theta} = \frac{-x}{5a} \times \frac{1}{\cos\theta}$$

Case-2 When 1KN is in L_3L_6

$$F_{U_2L_3}\cos\theta + R_6 - 1 = 0$$

$$F_{U_2U_3} = \frac{1 - x/5a}{\cos\theta}$$

If $x = 2a$

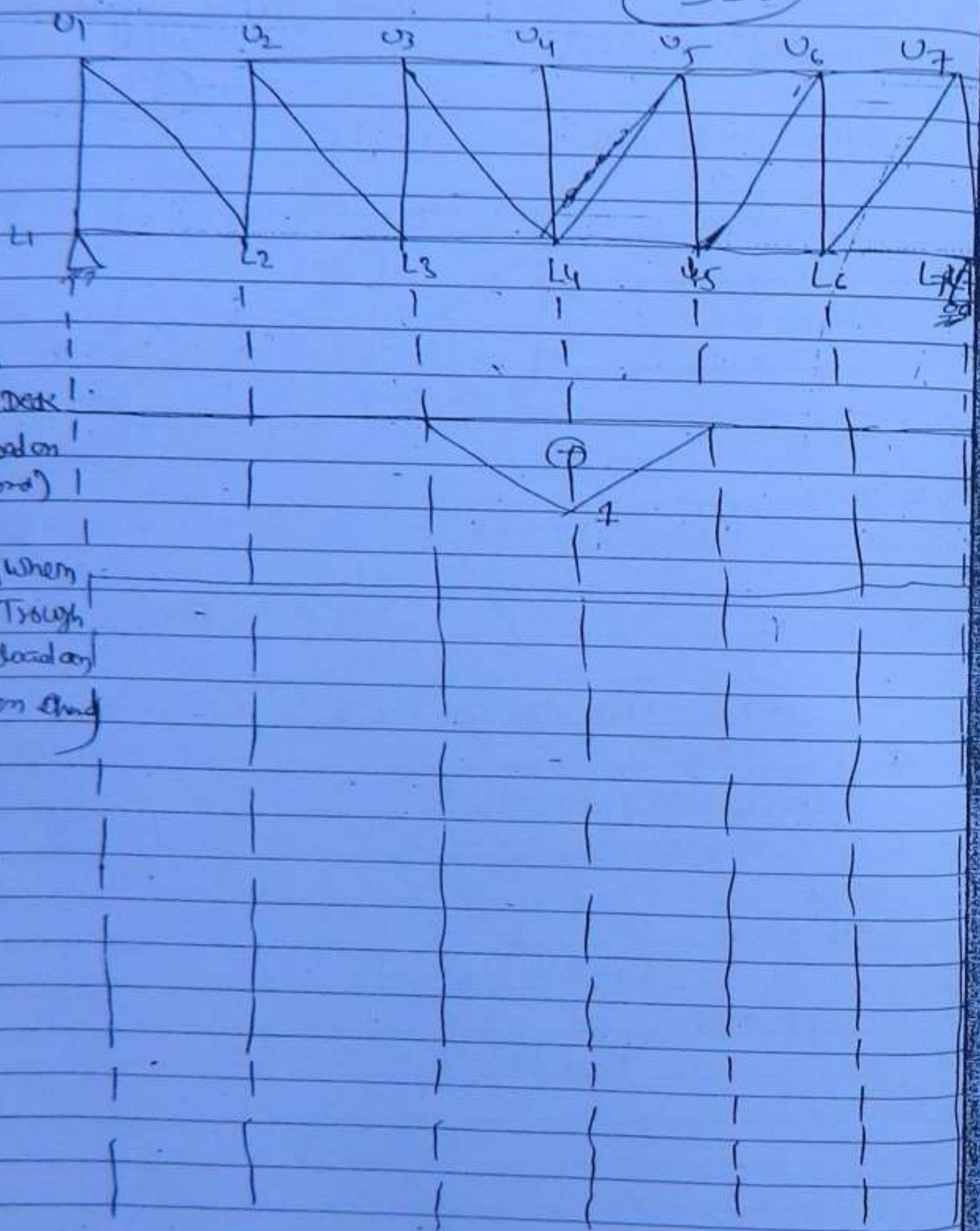
$$F_{U_2L_3} = \frac{3}{5\cos\theta}$$

If $x = 5a$

$$F_{U_2L_3} = 0$$

Q

338



ILD for U3L4 when truss is Deck type [Unit Load on Top chord]

ILD for U3L4 when truss is Trough type [Unit load on Bottom chord]

Date / /
Page 341

Mobiler 5

339

(1) b
(2) d
(3) d
(4) b (by method of section)
(5) d
(6) d
(7) c
(22) 92

(8) b
(9) a
(10) b
(11) None (e)
(12) b
(13) c
(14) d
(23) b
(25) b

(15) c
(16) a b
(17) b
(18) b
(19) c
(20) a
(21) a
(24)

(22) for max tension live load should be only in tension ~~zon~~ zone.

d $\Rightarrow$ D.L (Net Area) + L.L × (Area)

$\Rightarrow 20 \times [(-10 + 20] + [10 \times 20]$

$\Rightarrow 400$ kN

To

340