ELECTROMAGNETIC WAVES

Introduction.

Clerk maxwell stated that the magnetic field is not only produced by electric current but also by a time varying electric field. He concluded his idea on the basis of his experiment, when he was finding magnetic field at point outside and inside a capacitor connected to a time-varying current, using Ampere's Circuital law. He found that the results were not consistent. Maxwell pointed out that Ampere's circuital law becomes consistent only if along with conduction current, there is an additional current called by him, the displacement current.

Maxwell formulated a set of equations involving electric and magnetic fields and their sources; the charge and current. These equations are known as Maxwell's equations. These equations along with Lorentz force formula explain mathematically all the basic laws of electromagnetism.

Maxwell through his equations came out with a most important prediction, i.e., the existence of electromagnetic waves, which can propagate in space with a speed very close to the speed of light (3×10^8 m/s). He concluded light is an electromagnetic wave. In 1885 Hertz demonstrated experimentally, the existence of electromagnetic waves. Later on Marconi used these waves in establishing a communication between two locations at a distance of about 50 km, without using the wires between the two locations. This led to discovery of wireless communication and Marconi was considered as a father of wireless communication.

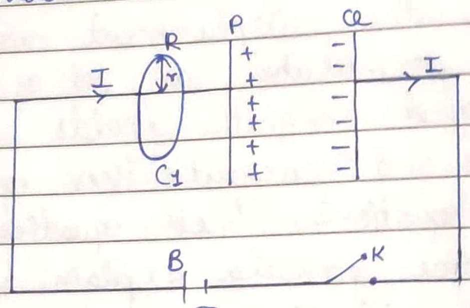
Displacement Current.

Acc. to Ampere Circuital law, the line integral of magnetic field $\vec{B}$ around any closed path is equal to μ_0 times the total current threading the closed path, i.e.,

$$\oint_C \vec{B} \cdot \vec{dl} = \mu_0 I$$

Maxwell in 1864 argued that the relation is logically inconsistent.

So he made observation:—



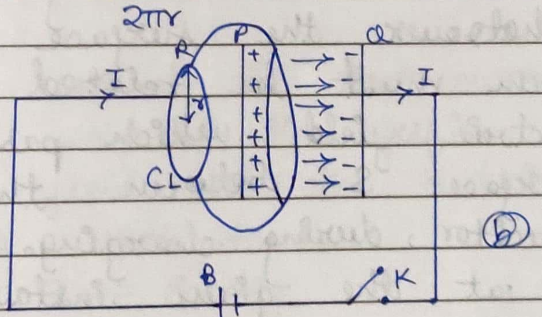
(a)

Consider, a parallel-plate capacitor having plates P and Q connected to battery. When K is pressed, the conduction current flows through connecting wires. The capacitor starts storing charge. As the charge on the capacitor grows, the conduction current in the wires decreases. When the capacitor is fully charged, the conduction current ~~starts~~ stops flowing in the wires. During charging of capacitor, there is no conduction current in the ~~wires.~~ plates of capacitor

During charging, let at an instant, I be the conduction current in the wires. This current will produce magnetic field around the wires which can be detected by using a compass needle.
let, us find magnetic field at R which is at ⊥ distance r from wire, in a region outside the parallel plate capacitor. for this we consider a plane circular loop C_1, of radius r, whose centre lies on wire and its plane is perpendicular to the direction of current carrying wire. The magnitude of the magnetic field is same at all points on the loop and is acting tangentially along the circumference of the loop. If B is the magnetic field at R, then using Ampere's circuital law, ~~pro~~ for loop C_1,

$$\oint_{C_1} \vec{B}\cdot\vec{dl} = \oint_{C_1} B\,dl\cos 0^\circ = B\cdot 2\pi r = \mu_0 I \text{ or}$$

$$B = \frac{\mu_0 I}{2\pi r}$$



Now, we consider different surface i.e., tiffin box shaped surface without lid with its circular rim, which has the same boundary as that of loop C_1. The box does not touch to the connecting wire and plate P of capacitor. The flat circular bottom S of tiffin box lies in between the capacitor plates. No conduction current is passing through the tiffin box surface S, therefore I = 0. On applying Ampere's circuital law to loop C_1 of this tiffin box surface,

$$\oint_C \vec{B} \cdot \vec{dl} = B \cdot 2\pi r = \mu_0 \times 0 = 0$$

$$B = 0$$

We note there is a magnetic field at R calculated through one way and no magnetic field at R, calculated another way. Since, this contradiction arises from the use of Ampere's circuital law, hence Ampere's circuital law is logically inconsistent. Maxwell argued that the above inconsistency must be due to something missing. The missing term must be such that one gets the same magnetic field with whatsoever the surface is used. The term must be related with a changing electric field which passes through the surface S between the plates of capacitor, during charging.
If at the given instant of time, q is the charge on the plate of capacitor and A is the plate area of capacitor, the magnitude of electric field between the plates of capacitor is,

$$E = \frac{q}{\varepsilon_0 A}$$

This field is perpendicular to surface S. It has the same magnitude over the area A of the capacitor plates and becomes zero outside the capacitor.

The electric flux through the surface S is,

$$\phi_E = \vec{E}\cdot\vec{A} = EA\cos 0^\circ$$

$$= \frac{1}{\varepsilon_0}\frac{q}{A} \times A = \frac{q}{\varepsilon_0}$$

If $\frac{dq}{dt}$ is rate of change of charge with time on the plate of the capacitor, then,

$$\frac{d\phi_E}{dt} = \frac{d}{dt}\left(\frac{q}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\frac{dq}{dt}$$

or

$$\frac{dq}{dt} = \varepsilon_0 \frac{d\phi_E}{dt}$$

Here, $\frac{dq}{dt}$ = current through surface S corresponding to changing electric field = I_D, called Maxwell's displacement current.

Displacement current is that current which comes into play in the region in which the electric field and the electric flux is changing with time.

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

This displacement current is the missing term in Ampere's circuital law. Maxwell pointed, there must be displacement current I_D, along with conduction current I in the closed loop as $(I+I_D)$ has the property of continuity, although individually they may not be continuous.

Maxwell modified Ampere's circuital law in order to make the same logically consistent. He stated Ampere's circuital law to the form,

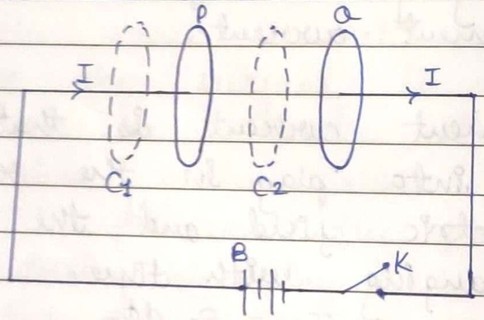
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 (I+I_D) = \mu_0\left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

This is called Ampere Maxwell's law.

Continuity of current.

$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0 (I+I_D)$$

The sum of the conduction current and displacement current $(I+I_D)$ has the important property of continuity along any closed path although individually they may not be continuous.



Consider a parallel plate capacitor having plates P and Q, being charged with battery B. During the time, charging is taking place, let at an instant, I be the conduction current flowing through the wires. let, C1 and C2 be the two loops, which have exactly the same boundary as that of the plates of capacitor C1 is little towards left and C2 is little towards right of plate P of parallel plate capacitor. Due to battery B, let the ~~induction~~ conduction current I be flowing through the lead wires at any instant; but there is no conduction current across the capacitor gap, as no charge is transported across this gap.

for loop C1, there is no electric flux,

$$\phi_E = 0 \text{ and } \frac{d\phi_E}{dt} = 0$$

$$I = I_D = I + \epsilon_0 \frac{d\phi_E}{dt} = I \quad \text{—(i)}$$

for loop C2, there is no conduction current,

$$I = 0$$

$$I + I_D = I_D$$

$$I_D = \epsilon_0 \frac{d\phi_E}{dt} \quad \text{—(ii)}$$

at a given instant if q is magnitude of charge on the plates of capacitor of area A, then $\vec{E}$ in the gap between two plates of capacitor is,

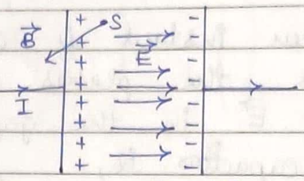
$$E = \frac{q}{\epsilon_0 A} \qquad \left(\because, E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}\right)$$

$\therefore$, Electric flux, $\phi_E = EA = \frac{q}{\epsilon_0 A} \times A = \frac{q}{\epsilon_0}$

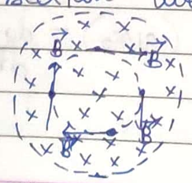
from (i), $I + I_D = \epsilon_0 \frac{d}{dt}(q/\epsilon_0) = \frac{dq}{dt} = I$

Thus, we conclude $(I+I_D)$ has the same value on the left and right side of plate P of the parallel plate capacitor. Hence, $(I+I_D)$ has the property of continuity although individually they may not.
In any general medium, both conduction current and displacement current will be present, giving rise to the total current. In a conducting medium, conduction current dominates over disp. current whereas in an insulating medium, disp. current dominates over the conduction current. Displacement current has same physical effects as the conduction current. For example, during the charging of capacitor, if we measure the $\vec{B}$ at point S between capacitor, it is found to be the same as that just outside the capacitor at point R.

The electric field $\vec{E}$ and magnetic field $\vec{B}$ between plates as,



The direction of $\vec{E}$ is from +ve to -ve plate whereas direction of $\vec{B}$ at S is perpendicular to plane of the paper. The cross-section view of $\vec{E}$ & $\vec{B}$ are



where electric field is shown by crosses and magnetic field is shown by tangents to a circle in same plane.

Consequences Of Displacement Current.
Or Prediction of Electromagnetic Waves.

Faraday's law states,
magnitude of emf induced in coil = rate of change of magnetic flux linked with it.

Therefore,
existence of emf shows the existence of electric field.

Faraday concluded that a changing magnetic field with time gives rise to an electric field.

The maxwell's concept that a changing electric field with time gives rise to displacement current which also produces a magnetic field similar to that of conduction current. It is infact, a symmetrical counterpart of the Faraday's concept, which led Maxwell to conclude that the displacement current is also a source of magnetic field. It means, the time varying electric and magnetic fields give rise to each other. From these concepts, Maxwell concluded the existence of electromagnetic wave in a region where electric and magnetic fields were changing with time.

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