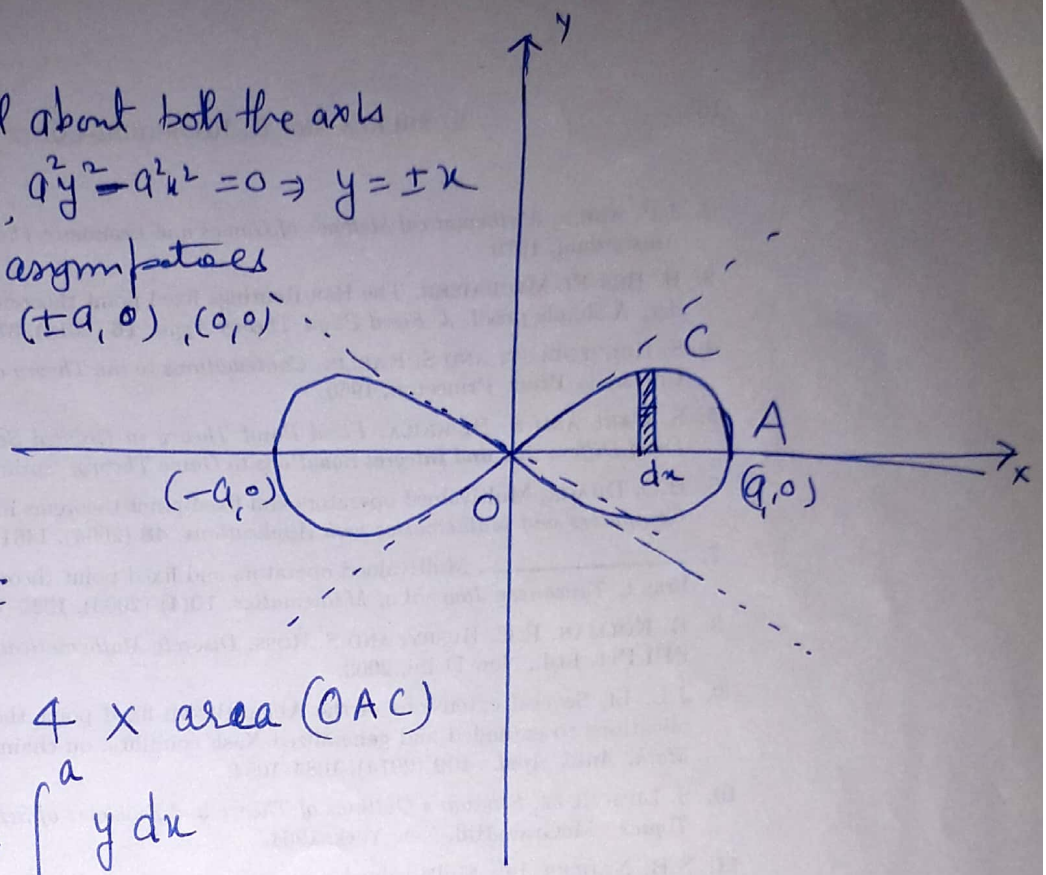
Q - Find the whole area of the curve $a^2y^2 = x^2(a^2-x^2)$ within it.

Sol:- (1) Symmetrical about both the axis

(2) Tangent at $(0,0)$, $a^2y^2 - a^2x^2 = 0 \Rightarrow y = \pm x$

(3) No parallel asymptotes

(4) Point of intersection $(\pm a, 0), (0,0)$



$$y^2 = \frac{x^2(a^2-x^2)}{a^2}$$

$$y = \frac{x}{a}\sqrt{a^2-x^2}$$

Required area = 4 × area (OAC)

$$= 4\int_0^a y\,dx$$

$$= 4\int_0^a \sqrt{\frac{a^2x^2-x^4}{a^2}}\,dx$$

$$= 4\int_0^a \frac{x}{a}\sqrt{a^2-x^2}\,dx$$

putting $x = a\sin\theta$, $dx = a\cos\theta\,d\theta$

$$= \frac{4}{a}\int_0^{\pi/2} a\sin\theta\sqrt{a^2(1-\sin^2\theta)}\,a\cos\theta\,d\theta$$

$$= \frac{4a^3}{a}\int_0^{\pi/2} \sin\theta\cdot\cos\theta\cdot\cos\theta\,d\theta$$

$$= 4a^2\int_0^{\pi/2} \sin\theta\cos^2\theta\,d\theta = 4a^2\cdot\frac{\lfloor 2\,\lfloor 3/2}{2\lfloor 5/2}$$

$$= \frac{4a^2\cdot 1\cdot\frac{1}{2}\sqrt{\pi}}{2\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}} = \frac{4a^2}{3}$$