Q1

Let A be an $m \times n$ matrix (where $m, n \in \mathbb{N}$) and let A' be a matrix obtained by performing a row operation on A.

Choose the true statement from the following:

1. If A has linearly independent rows then A' must have linearly independent rows.
2. If A' has linearly independent rows then A must have linearly independent rows.
3. A has linearly independent rows if and only if A' has linearly independent rows.
4. None of the above.

Correct Answers: 1, 2 and 3

Q2

Let v_1, v_2 and v_3 be three non-collinear vectors in $\mathbb{R}^3$. Let

$$S = \{c_1 v_1 + c_2 v_2 + c_3 v_3 \mid c_1, c_2, c_3 \in (0, 1), c_1 + c_2 + c_3 = 1\}$$

Choose any correct statement from the following:

1. S is a plane in $\mathbb{R}^3$ which passes through the origin
2. $S = \text{Span}\{v_1, v_2, v_3\}$
3. $\text{Span}\{v_1, v_2, v_3\} \subset S$
4. S is the interior of a triangle whose vertices are v_1, v_2 and v_3
5. S is the interior of a triangle which passes through the origin and whose vertices are v_1, v_2 and v_3
6. $S \subsetneq \text{Span}\{v_1, v_2, v_3\}$
7. $S \not\subset \text{Span}\{v_1, v_2, v_3\}$

Correct Answers: 4 and 6

Q3

Let A be an $m \times n$ matrix (where $m, n \in \mathbb{N}$). Identify the correct contrapositive and converse of the following statement (upto logical equivalence): *If* $A\mathbf{x} = 0$*, for every* $\mathbf{x} \in \mathbb{R}^n$*, then* $A = 0$*.*

1. Contrapositive: If $A \neq 0$ then there exists a vector $\mathbf{x} \in \mathbb{R}^n$ such that $A\mathbf{x} \neq 0$.
   Converse: If there exists a vector $\mathbf{x} \in \mathbb{R}^n$ such that $A\mathbf{x} \neq 0$, then $A \neq 0$.
2. Converse: If $A \neq 0$ then there exists a vector $\mathbf{x} \in \mathbb{R}^n$ such that $A\mathbf{x} \neq 0$.
   Contrapositive: If $A = 0$ then $A\mathbf{x} = 0$ for every $\mathbf{x} \in \mathbb{R}^n$.
3. Contrapositive: If $A \neq 0$ then $A\mathbf{x} \neq 0$ for every $\mathbf{x} \in \mathbb{R}^n$.
   Converse: If there exists a vector $\mathbf{x} \in \mathbb{R}^n$ such that $A\mathbf{x} \neq 0$, then $A \neq 0$.
4. Converse: If $A \neq 0$ then $A\mathbf{x} \neq 0$ for every $\mathbf{x} \in \mathbb{R}^n$.
   Contrapositive: If there exists a vector $\mathbf{x} \in \mathbb{R}^n$ such that $A\mathbf{x} \neq 0$, then $A \neq 0$.

Answer to Q3

Option 1

Q4

Let A be an $m \times n$ matrix (where $m, n \in \mathbb{N}$). Let A' be the RREF of A.

Decide whether the following statement is true or false:

If one of the entries of A' is a number which is other than 0 or 1 then the columns of A must be linearly dependent.

Answer: True