Submission 5

The respondent's email (**deepanshu21249@iiitd.ac.in**) was recorded on submission of this form.

Question *

Let $V = \mathcal{C}[0,1]$, the vector space of all real-valued continuous functions defined on the closed unit interval.

Let $W = \text{Span}\{\sin^2 x, \cos^2 x, 1, \cos 2x\} \subset V$.

(Please note that the symbol $\sin^2 x$ here denotes the *function* which maps $x \mapsto \sin^2 x$ for all values of x in the interval $[0,1]$. Similary $\cos^2 x, \cos 2x$.)

Which of the following is a basis of W?

$\{1 + 5\sin^2 x \cos^2 x, 5\cos^2 x\}$

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$\{1 + 5\sin^2 x, 5\cos 2x\}$

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$\{\sin^2 x, \cos^2 x, 1\}$

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$\{\sin 2x, \cos 2x\}$

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$\{\sin^2 x, \cos^2 x, \cos 2x\}$

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$\{c_1 \sin^2 x, c_2 \cos^2 x \mid c_1, c_2 \in \mathbb{R}\}$

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Question *

Let V be the vector space of all sequences $(a_n)_{n\in\mathbb{N}}$ of real numbers. Let s be the Fibonacci sequence. Let S be the set of all constant sequences, i.e.

$$S = \{(a_n) \in V \mid a_j = a_1, \forall j > 1\}$$

Let $W = \text{Span}(\{s\} \cup S)$. What is the dimension of W?

- ○ 1
- ○ 4
- ○ 0
- ○ 3
- ○ 2
- ○ -1
- ◉ infinity

Question *

Let $V = \mathbb{P}$, the vector space of all polynomials in one variable (say x), having real coefficients.

Given a non-trivial finite-dimensional subspace W of V, define

$$h(W) = \max\{\text{degree}\, p(x) \ : \ p(x) \in W\}$$

(non-trivial means $W \neq \{0\}$)

Suppose W_0 is any subspace of V such that

$$h(W_0) = \left\lceil e^{e^{e^5}} \right\rceil$$

(Here, $\lceil\rceil$ denote the greatest integer function, or ceiling function.)

Select the true statement(s) from the following:

$$\dim W_0 < \lceil e^{e^{e^5}} \rceil + 2$$

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$$\dim W_0 \leq \lfloor e^{e^{e^5}} \rfloor + 1$$

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$$\dim W_0 = \lceil e^{e^{e^5}} \rceil$$

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$$\dim W_0 < e^{e^{e^5}}$$

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$$\dim W_0 = \infty$$

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Question *

Let $V = \mathcal{C}(\mathbb{R})$, the vector space of all continuous real-valued functions defined on $\mathbb{R}$.
Let $a, b, c \in \mathbb{R} \setminus \{0\}$. Let

$$S = \{ax^2 + bx + c\} \subset V$$

(Here $ax^2 + bx + c$ denotes the function $x \mapsto ax^2 + bx + c$.)

Select the true statement(s) from the following:

- [ ] S is a linearly dependent subset of V if and only if the quadratic equation ax^2 + bx + c = 0 has exactly one real root.
- [ ] S is a linearly independent subset of V if and only if the quadratic equation ax^2 + bx + c = 0 has exactly one real root.
- [x] S is a linearly independent subset of V.
- [ ] S is a linearly independent subset of V if and only if the quadratic equation ax^2 + bx + c = 0 has no real roots.
- [ ] S is a linearly dependent subset of V if and only if the quadratic equation ax^2 + bx + c = 0 has two real roots.
- [ ] S is a linearly dependent subset of V if and only if the quadratic equation ax^2 + bx + c = 0 has no real roots.
- [ ] S is a linearly independent subset of V if and only if the quadratic equation ax^2 + bx + c = 0 has two real roots.
- [ ] S is a linearly dependent subset of V.